TRAFFIC ASSIGNMENT WITHOUT BIAS

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Traffic assignment procedures based on shortest time path tend to overassign traffic to high-speed facilities. Because of this, 2 assignment formulas were developed that, taken as parameters in the assignment process, eliminate the overall bias. One formula, known as the time-distance factor, was described in an earlier publication. The formula introduced in this paper, called the exponential factor, is based on a combination of link distance and speed raised to an exponential power. Both were tested by using data from 9 different studies in 6 states. When assignments were based on time, the mean overassignment to the high-speed facility was 45 percent. At the optimum adjustment in the formulas this difference was eliminated, and the percent rms error was less than a third of the error in assignments by the shortest time path. A count of assignment errors in individual zone-to-zone movements revealed that in assignments by the shortest time path there were 18 times as many overassignment errors as underassignment errors. At the optimum assignments by the 2 formulas, the number of overassignment and underassignment errors was about in balance, and the total number of assignment errors was only about 60 percent of that based on assignments by shortest time. Also the traffic volumes assigned closely matched those actually counted.

•BASIC to any procedure for assigning traffic to a network by the shortest path is the development of a measure or parameter that will in general route a trip to the correct highway. If the measure used gives close to an optimum assignment, then various calibration and restraint procedures to improve the assignment further will involve only very minor changes in speed. On the other hand, if the measure of the spatial relation in itself has a strong bias, then the procedures will involve much larger adjustments to overcome the bias. The result will be a coarser assignment with regard to actual traffic and speeds.

In a previous paper (1) it was shown that the use of travel time for this measure results in too many trips being assigned to the high-speed facility. Conversely, the use of shortest distance gives too few trips to the high-speed facility.

One of the purposes of this paper is to document more thoroughly those statements by using a greatly expanded amount of data and new research techniques and then, by applying these statistical techniques, to ascertain the proportion of time and distance in a time-distance (TD) factor previously described (1) that will give the best assignment.

The principal purpose is the introduction of a new parameter, the exponential factor, for traffic assignment. It gives results quite similar to those obtained by use of the TD factor, but, because the formula involving the use of speed to an exponential power is completely different, it should prove of interest and eventually of considerable value to the transportation planning profession.

THE TD FACTOR

The formula for the link length for assignment purposes is based on a combination of time and distance, as follows:

$$\text{TD factor} = TP + D\,(1.00 - P)$$

where

T = time, min;
D = distance, miles; and
P = decimal fraction of weight assigned to time.

When $P = 1.0$, the assignment is based on travel time; when $P = 0$, the assignment is based on distance. Between these 2 extremes, any value of P can be tried until one is found that will give the best results as determined by the root mean square error for the entire network. Once a value of P is decided on, the same constant value of it is used for the entire network assignment.

THE EXPONENTIAL FACTOR

The formula for the link length for traffic assignment purposes is based on a combination of distance and speed raised to an exponential power, as follows:

$$\text{Exponential factor} = D(K/V)^N$$

where

D = link distance,
V = speed, and
K = arbitrarily selected constant.

It is believed that the value of the exponent N will never exceed the range of $N = 0$ to $N = 1$.

Suppose that V is expressed in miles per hour, D in miles, and $K = 60$. When $N = 1$, the expression becomes $60D/V$, which is the formula for travel time in minutes. When $N = 0$ the fraction $(K/V)^0$ becomes 1 and the formula ends with only the link distance.

For a sample computation of an exponential factor using an exponent of 0.5, assume that the times, distances, and resulting speeds between a pair of zones are as follows (1):

Facility	Time (min)	Distance (miles)	Speed (mph)
Freeway	12.5	8.3	39.8
Alternate	13.2	6.4	29.1

$$\text{Freeway exponential factor } (N = 0.5) = 8.3\ (60/39.8)^{0.5} = 10.20$$
$$\text{Alternate exponential factor } (N = 0.5) = 6.4\ (60/29.1)^{0.5} = 9.19$$

Because the alternate route has the smallest assignment factor, it will be assigned the trips.

The exponential factors for various values of N are shown in Figure 1. The data are plotted on full logarithmic scale so that the curves will be straight lines. Figure 2 shows a comparison of the exponential factor and the TD factor. The exponential factor with $N = \frac{1}{2}$ agrees quite closely with the TD factor using $\frac{1}{3}$ T and $\frac{2}{3}$ D. In the exponential factor curves, the constant K had been set at 60 in order to give an assignment factor of 1 at 60 mph. (With $N = 1$, this also gives the curve for travel time.) As assignments are based on the relative link lengths, K could just as well be 1. If we make $K = 1$, the exponential factor becomes simplified to the following:

$$\text{Simplified exponential factor} = D/V_N$$

SOURCES OF DATA

To test the formulas required the following for each zone-to-zone movement, which represents a group of trips, all originating in one zone and terminating in another: total number of trips, trips using freeway, travel time and distance via freeway, and travel time and distance via alternate route. Published information was available from

Figure 1. Comparison of exponential factors for values of N.

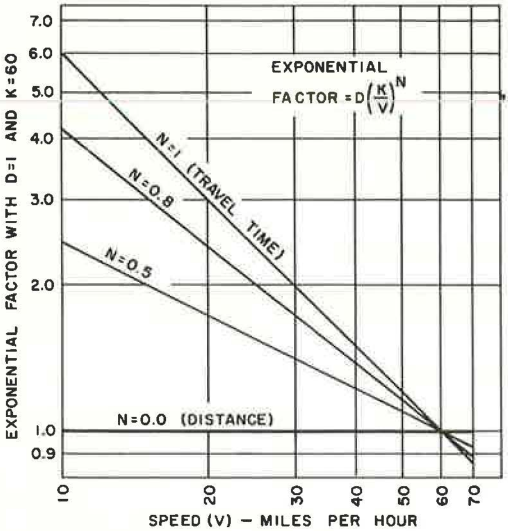
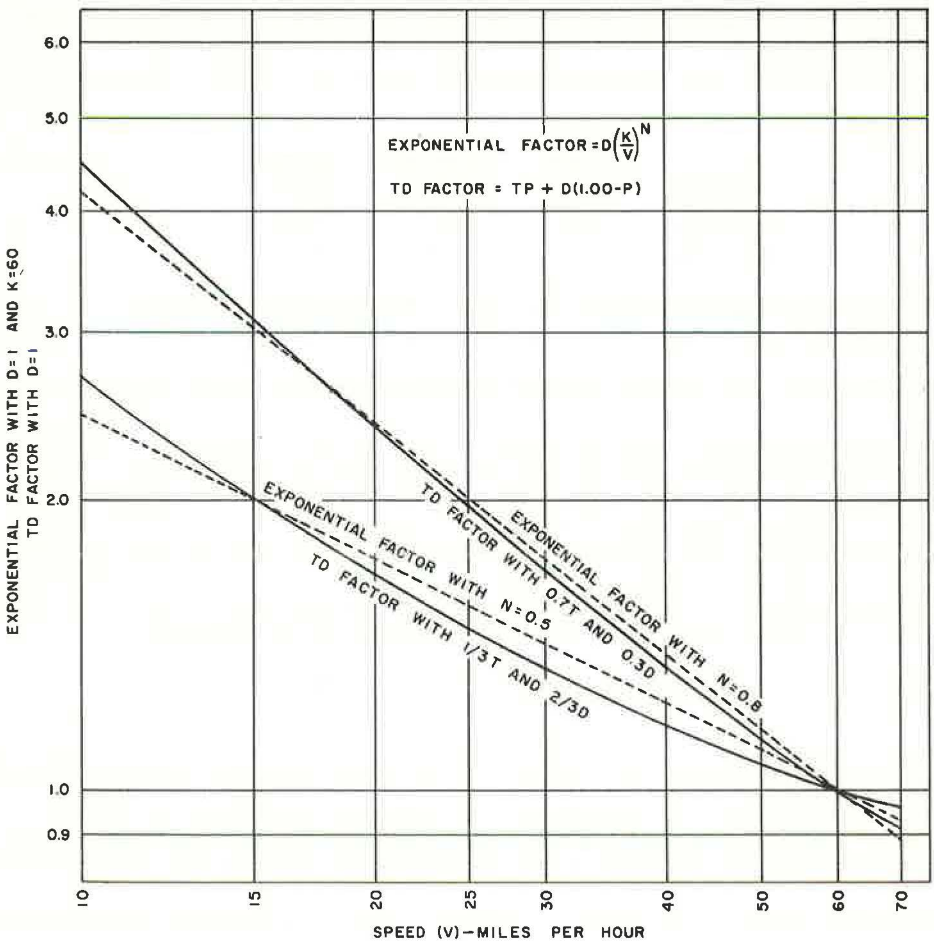


Table 1. Trips between zones and on facilities.

Facility	Trips Between Zones	Trips on Facility	Zone-to-Zone Movements
Urban freeway			
Gulf	22,556	8,413	19
Alvarado	92,278	23,856	154
Cabrillo	61,140	27,060	101
Oceanside	6,904	2,823	53
Shirley	19,756	8,152	88
Small city bypass			
Kokomo	5,526	2,053	69
Lebanon	6,110	4,464	50
Rural Freeway			
Kansas	6,054	3,967	46
South Dakota	7,388	4,388	88

Figure 2. Comparison of exponential and TD factors.



8 separate studies. In addition, an unpublished report on the Gulf Freeway in Houston was obtained from the Texas Highway Department. This made a total of 9 sources of data from 6 states, as follows:

Urban Freeways
- Gulf Freeway, Houston (2)
- Alvarado Expressway, San Diego (3)
- Cabrillo Freeway, San Diego (3)
- Oceanside-Carlsbad Freeway, California (3)
- Shirley Highway, Arlington and Fairfax Counties, Virginia (4)

Small City Bypasses
- Kokomo, Indiana (5)
- Lebanon, Indiana (5)

Rural Freeways
- Interstate 70, Kansas (6)
- Interstate 29, South Dakota (7)

METHOD OF ANALYSIS

Data from these studies were used to write a computer program that provided a tabulation of the following data for each study: Number of zone-to-zone movements, number of trips actually using freeway, number of trips assigned to freeway, percentage of zone-to-zone trips assigned to freeway, number of trips assigned to freeway as a percentage of actual trips using freeway, number of zone-to-zone movements erroneously overassigned to freeway, number of zone-to-zone movements erroneously underassigned to freeway, and total number of zone-to-zone movements erroneously assigned. The actual and assigned trips are self-explanatory. The erroneously assigned trips can be described as follows.

In an assignment by shortest path method, there are only 2 assignments possible. Either all zone-to-zone trips are assigned to the freeway, or no trips are assigned to it. For example, suppose that 49 percent of the trips represented in a zone-to-zone movement actually use the freeway. Then the best possible assignment to the freeway by this procedure would be 0 trips, and such an assignment would be considered correct. However, if all of the trips were assigned to the freeway, the assignment would be incorrect and the computer would classify the assignment as a plus error. Examples of the assignment classifications that are possible for the percentages of zone-to-zone trips actually using the freeway and assigned to the freeway are as follows:

Actual	Assigned	Classification
49 and under	0	Correct
49 and under	100	Plus error
51 and over	0	Minus error
51 and over	100	Correct

The assignment data were computed and tabulated in increments of 0.1 for the value of P in the TD factor and of N in the exponential factor. This gave 11 assignments for each factor for each study. In addition, in the range from 0.46 to 0.24, assignments were made in increments of 0.02, which improved accuracy in the optimum range.

ANALYSIS RESULTS

Total Traffic Assigned

As indicated in the earlier report on the TD factor, all assignments to the higher type of facility were too high by the shortest time path and too low by the shortest distance. The percentages for each of the 9 highways are given in Tables 2 and 3. The percentages for each highway at the optimum exponential factor and TD factor for the group are also given. Additional detail on number of trips assigned is given in Table 1. The tables give assignments at intervals of 0.2 in values of N and P, plus the

Figure 3. Percentage of actual trips assigned by exponential factor for values of N in intervals of 0.1.

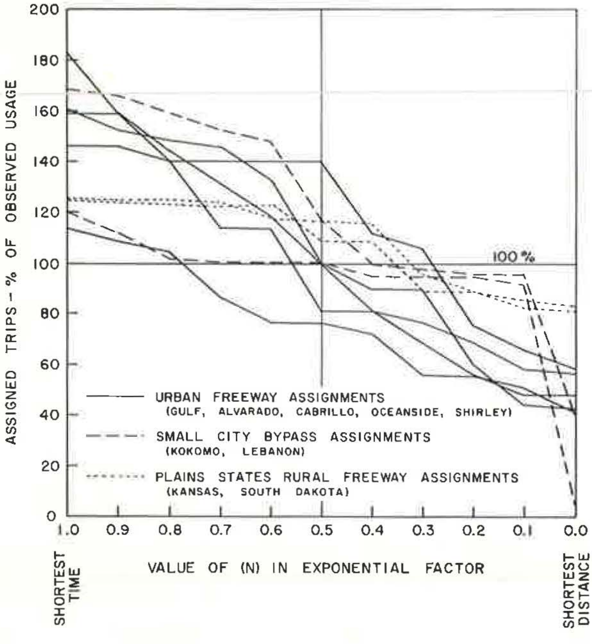


Table 2. Percentage of actual trips assigned by exponential factor for values of N in intervals of 0.2.

Facility	1.0[a]	0.8	0.6	0.46[b]	0.4	0.2	0.0[c]
Urban freeway							
Gulf	159	141	114	81	81	56	41
Alvarado	183	144	119	101	90	60	42
Cabrillo	161	149	133	94	80	69	54
Oceanside	146	140	140	115	113	75	58
Shirley	114	105	75	72	71	55	48
Small city bypass							
Kokomo	169	159	148	104	100	96	39
Lebanon	120	102	101	96	95	95	4
Rural freeway							
Kansas	125	124	123	108	108	89	82
South Dakota	126	125	118	117	116	90	81
Mean	145	132	119	99	95	76	50

[a]Shortest time. [b]Optimum. [c]Shortest distance.

Figure 4. Percentage of actual trips assigned by TD factor for values of P in intervals of 0.1.

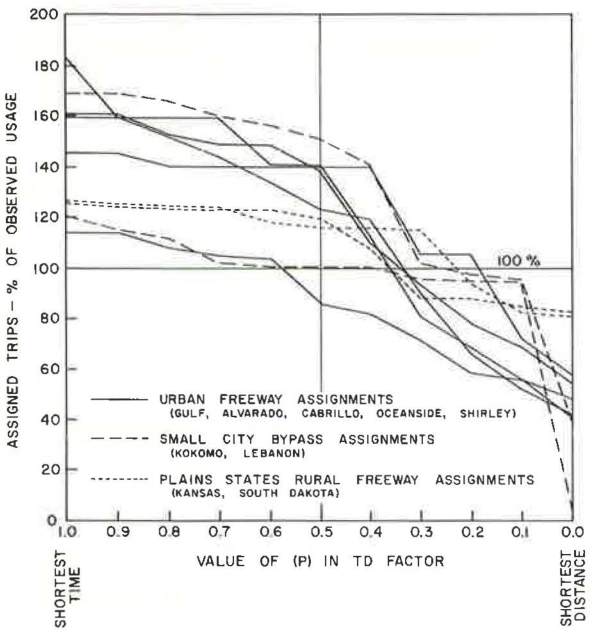


Table 3. Percentage of actual trips assigned by T factor for values of P in intervals of 0.2.

Facility	1.0[a]	0.8	0.6	0.4	0.32	0.2	0.0[b]
Urban freeway							
Gulf	159	159	141	114	81	69	41
Alvarado	183	152	134	119	90	66	42
Cabrillo	161	153	148	111	97	77	54
Oceanside	146	140	140	140	113	106	58
Shirley	114	109	104	82	72	58	48
Small city bypass							
Kokomo	169	166	156	141	115	98	39
Lebanon	120	112	101	101	101	95	4
Rural freeway							
Kansas	125	124	123	108	107	89	82
South Dakota	126	125	118	117	115	94	81
Mean	145	138	129	115	95	84	50

[a]Shortest time. [b]Shortest distance.

assignments at the optimum values of N and P. Computations, however, were made at intervals of 0.1, and plots of the assignment data for the exponential factor and the TD factor on this basis are shown in Figures 3 and 4. The number of lines, one for each study, tends to slightly obscure the basic trend; therefore, Figures 5 and 6 show the mean values with boundary lines at 1 standard deviation from the mean. Assignments of 99 percent of the observed number of trips are obtained at a value of $N = 0.46$ in the exponential factor and $P = 0.32$ in the TD factor. As N and P were computed at intervals of 0.02 in the optimum range, the 99 percent figure is the closest one based on an actual assignment. At N and P values that give an average assignment close to the actual volumes, the dispersion about the mean, as measured by the standard deviation, is about a third less than the dispersion when assignments are by the shortest time path.

Root Mean Square Error

The formula for the computation of the rms error is as follows:

$$\text{rms} = \sqrt{[\Sigma(X_{GC} - X_A)^2]/(N - 1)}$$

where

X_{GC} = total vehicles counted on a given route,
X_A = assigned volume to the route, and
N = number of routes.

It is possible to interpret the results of the computation given above better in this case if the reader can review a sample computation. The following one is for shortest time path.

Facility	Facility Type	Actual Trips (percent)	Assigned Trips (percent)	Difference	Difference^2
Gulf	Freeway	37	59	-22	484
	Alternate	63	41	+22	484
Alvarado	Freeway	26	47	-21	441
	Alternate	74	53	+21	441
Cabrillo	Freeway	44	71	-27	729
	Alternate	56	29	+27	729
Oceanside	Freeway	41	60	-19	361
	Alternate	59	40	+19	361
Shirley	Freeway	41	47	-6	36
	Alternate	59	53	+6	36
Kokomo	Bypass	37	63	-26	676
	Alternate	63	37	+26	676
Lebanon	Bypass	73	87	-14	196
	Alternate	27	13	+14	196
Kansas	Freeway	66	82	-16	256
	Alternate	34	18	+16	256
South Dakota	Freeway	59	75	-16	256
	Alternate	41	25	+16	256
	Total	900	900	0	6,870

Total alternates and freeways = 18
Average trips on each, percent = 900/18 = 50
rms error $= \sqrt{6,870/17} = 20.1$
Percent rms error = $(20.1 \times 100)/50 = 40.2$

Figure 5. Mean percentage of actual trips assigned by exponential factor for values of N in intervals of 0.1.

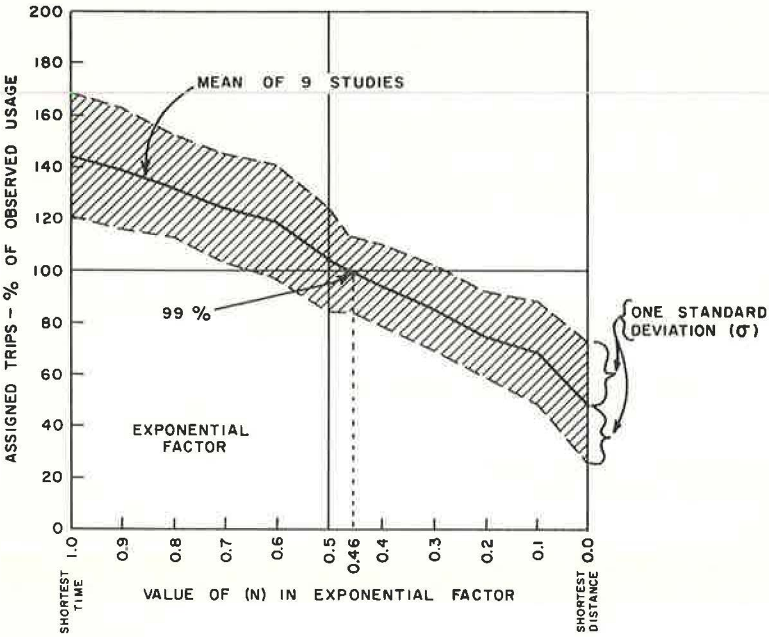


Figure 6. Mean percentage of actual trips assigned by TD factor for values of P in intervals of 0.1.

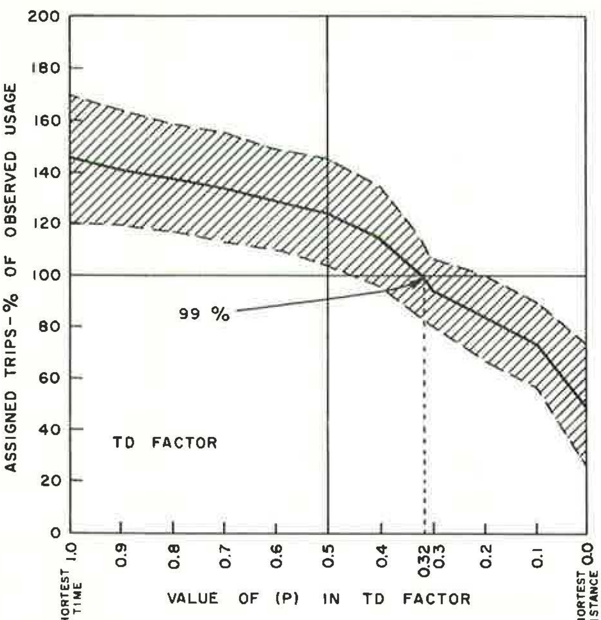


Figure 7 shows rms errors for the exponential factor and TD factor. In the optimum range for each method, the percent rms error is less than a third of the error in assignments by the shortest time path.

Movements Incorrectly Assigned

This analysis is probably the most significant part of the report. Results based on route totals alone do not take into account the variation in accuracy that is possible from one end of the route to the other. A count of the number of zone-to-zone movements incorrectly assigned, classified according to the number of plus errors and minus errors, gives a better insight into the workings of an assignment procedure. Of course, in an all-or-nothing assignment there are only 2 possibilities: either 0 percent or 100 percent of the trips represented in a zone-to-zone movement are assigned to the facility. In this analysis, therefore, the assignment that comes closest to the actual is considered the correct one, even though numerically it may be considerably different from the actual. Thus, if 37 percent of trips represented in a zone-to-zone movement actually uses a freeway, an assignment of 0 percent would be correct, and an assignment of 100 percent would represent a plus error. If 80 percent of the trips in a zone-to-zone movement actually uses a freeway, an assignment of 0 percent would represent a minus error, and an assignment of 100 percent would be considered correct.

Tables 4 and 5 give the percentage of plus and minus errors for different values of N and P for each of the 9 facilities in this study. In assignments by shortest time nearly all of the errors are plus errors, and in assignments by shortest distance most of the errors are minus errors except on 1 facility. Stated in another way, in assignments by shortest time there are 18 times as many plus errors as minus errors, and in assignments by shortest distance there are 8 times as many minus errors as plus errors. These facts are considered by the author to be of great statistical significance. At some point between the 2 extremes, where the number of plus and minus errors is about in balance, the total number of errors averages only about 60 percent of that based on assignment by shortest time and the traffic volumes assigned closely match the actual traffic counts. The trend in percentage of assignment errors for various values of N and P is shown in Figures 9 and 10. Figure 11 shows a comparison of assignments by shortest time and shortest distance and the optimum exponential and TD factors. For the latter two, the volumes assigned were each 99 percent of the actual number of trips.

APPLICATION TO NETWORK ASSIGNMENT

Most computer programs are written for assignments by shortest time path. All that would be necessary for these procedures to be applied would be to substitute the formula for exponential factor or TD factor in the program for the computation for travel time in determining the link spacing for traffic assignment. The program should be written so that the value of N or P could be easily changed between assignment runs to get the best assignment.

There is in existence at the present time a program for the TD factor. It was prepared in 1970 and is now a part of the standard FHWA urban transportation program system battery for the IBM 360 computer. The TD factor, in principle, will be found in the instructions for the weight card (WEIGHT) in the program deck BUILDVN, although the TD factor as such is not mentioned. It can be developed by assigning a weight to time and to distance in accordance with the instructions for using the card.

The computation of network root mean square error is now available in assignment programs. This measure would appear to be a logical one in determining the optimum value of N or P for the best assignment. After the optimum assignment by this procedure is reached, some further improvement in the calibration is possible by an iterative process. However, because assigned traffic volumes will probably already be close to the actual, it would appear that the size of subsequent speed adjustments per iteration should be set somewhat lower than usual when assignments are made on the basis of travel time.

Figure 7. Root mean square error for exponential and TD factors for values of N and P.

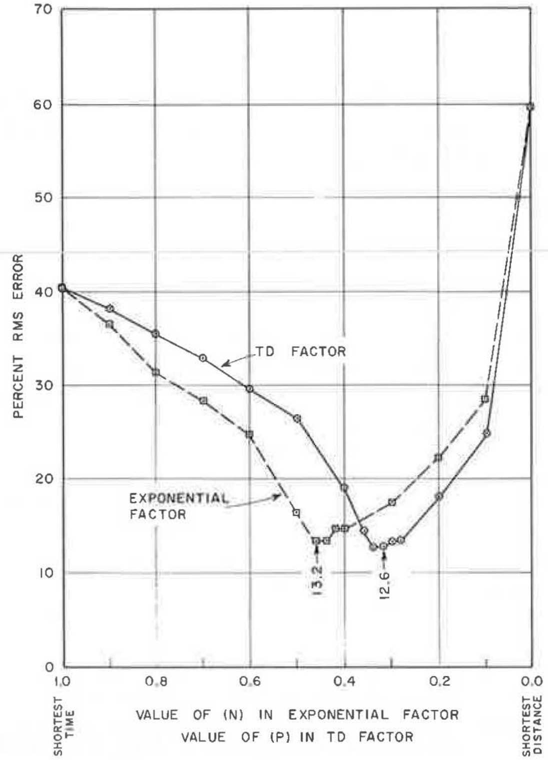


Figure 8. Percentage of zone-to-zone movements incorrectly assigned by exponential factor for values of N.

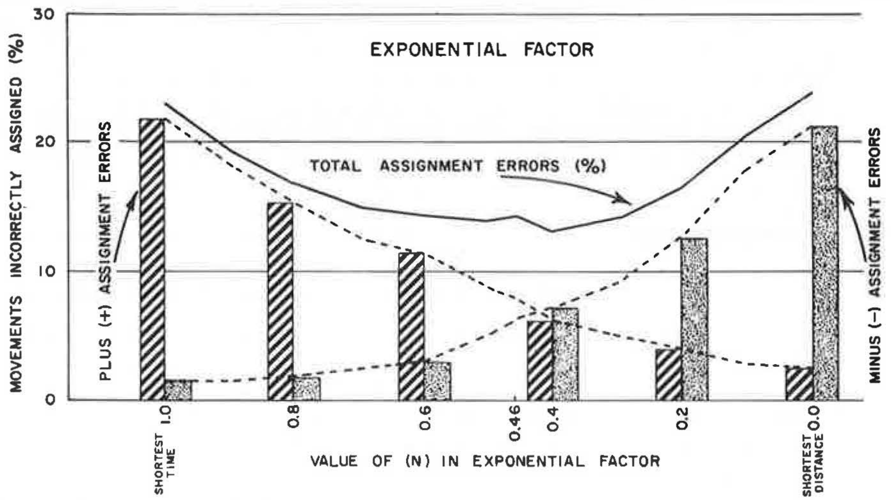


Table 4. Percentage of zone-to-zone movements incorrectly assigned by exponential factor for values of N.

	1.0[a]		0.8		0.6		0.46[b]		0.4		0.2		0.0[c]	
Facility	+	-	+	-	+	-	+	-	+	-	+	-	+	-
Urban freeway														
Gulf	16	0	11	0	5	0	0	5	0	5	0	21	0	37
Alvarado	28	1	20	2	13	2	12	4	7	5	3	10	1	17
Cabrillo	24	1	20	1	18	4	13	14	7	14	7	20	5	29
Oceanside	15	2	11	2	11	2	5	4	6	5	0	13	0	28
Shirley	9	1	7	2	3	9	3	11	3	12	2	21	2	25
Small city bypass														
Kokomo	36	0	29	0	22	0	13	1	11	1	9	3	10	9
Lebanon	38	4	12	6	8	6	8	10	4	10	4	10	0	22
Rural freeway														
Kansas	9	0	7	0	7	2	4	7	4	7	2	9	0	15
South Dakota	21	2	20	2	15	2	14	2	13	5	9	7	7	11
Mean	21.8	1.2	15.2	1.7	11.3	3.0	8.0	6.4	6.1	7.1	4.0	12.7	2.8	21.4
Total	23.0		16.9		14.3		14.4		13.2		16.7		24.2	

[a]Shortest time. [b]Optimum. [c]Shortest distance.

Figure 9. Percentage of zone-to-zone movements incorrectly assigned by TD factor for values of P.

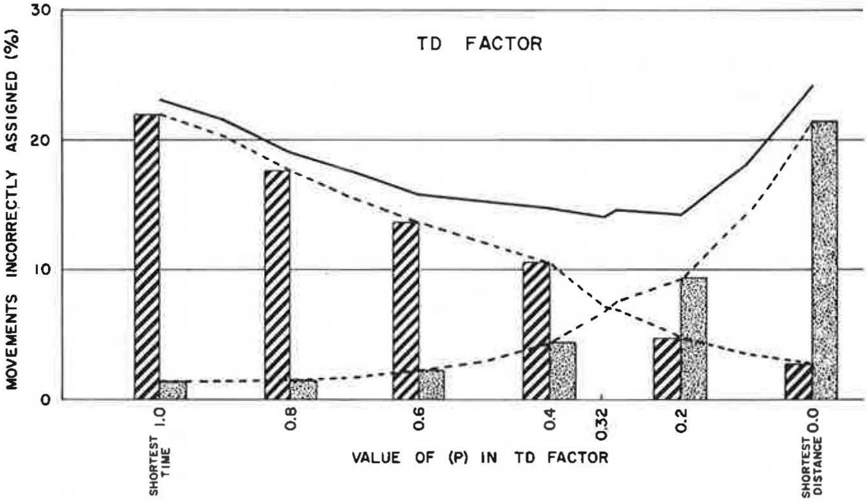
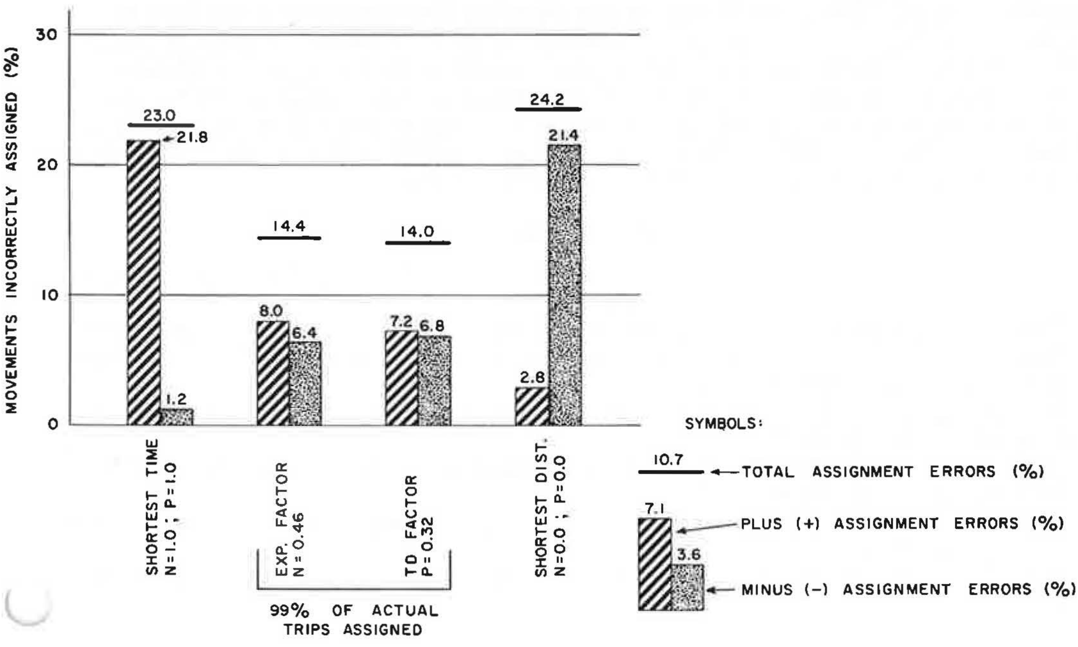


Table 5. Percentage of zone-to-zone movements incorrectly assigned by TD factor for values of P.

Facility	1.0[a] +	1.0[a] -	0.8 +	0.8 -	0.6 +	0.6 -	0.4 +	0.4 -	0.32[b] +	0.32[b] -	0.2 +	0.2 -	0.0[c] +	0.0[c] -
Urban freeway														
Gulf	16	0	16	0	11	0	5	0	0	5	0	11	0	37
Alvarado	28	0	23	2	13	2	12	4	7	5	3	10	1	17
Cabrillo	24	1	22	1	19	1	17	10	13	13	7	16	5	29
Oceanside	15	2	11	2	11	2	11	2	6	6	6	7	0	28
Shirley	9	1	7	1	7	4	3	8	3	11	2	16	2	25
Small city bypass														
Kokomo	36	0	35	0	26	0	21	1	13	1	9	1	10	9
Lebanon	38	4	18	4	8	6	8	6	8	6	4	10	0	22
Rural freeway														
Kansas	9	0	7	0	7	2	4	7	4	9	2	9	0	15
South Dakota	21	2	20	2	15	2	14	3	12	5	9	6	7	11
Mean	21.8	1.2	17.7	1.3	13.7	2.1	10.6	4.3	7.2	6.8	4.9	9.3	2.8	21.4
Total	23.0		19.0		15.8		14.9		14.0		14.2		24.2	

[a]Shortest time. [b]Optimum. [c]Shortest distance.

Figure 10. Percentage of zone-to-zone movements incorrectly assigned for selected values of N and P.



Before adopting these procedures, the planner may wish to run some test assignments using the exponential factor or TD factor. In doing so, he will probably select for the tests some readily available input data from a network previously assigned. In this event, he is cautioned to be sure that the speeds used are objectively measured speeds that have not been revised or adjusted prior to or as a part of the previous assignment runs. A number of planners realize that when assignments are by shortest time, the measured speeds give too high an assignment to high-speed facilities. As a result some of them may have either subconsciously or systematically reduced input speeds of the high-speed facilities relative to those of low speed. This may even have been done by changing the definition of the speed measurement for each type of facility. If such an adjustment in input has been made, the exponential factor or TD factor will not give satisfactory results. Some typical changes that may have been made that the planner should look for are

1. Selective manual speed reductions on some high-speed facilities, usually freeways;
2. Use of peak-hour speeds on freeways and off-peak speeds on other facilities;
3. Manual upward adjustment of arterial speeds in the CBD; and
4. Use of average speeds on rural freeways and 85 percentile speeds on other rural arterials.

The discussion given above has related strictly to assignment of trips, without regard to how the trips are distributed. The distribution could be the origin-destination data from the original survey, or it could be based on a gravity or other model. If the distribution is by a gravity model, some error may be introduced by using travel time as a basis. Logically, better results would be possible by using the optimum TD or exponential factor in the model instead of travel time.

When the exponential factor is compared with the TD factor, it is apparent that although both give strikingly better assignments than that given by the shortest time path the difference between the optimum assignment by each is not great enough to be conclusive. Either can be used with confidence, and perhaps after several full-scale assignments on complete networks a preference will gradually develop for one or the other.

ACKNOWLEDGMENT

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