DEVELOPMENT AND APPLICATION OF DIRECT TRAFFIC ESTIMATION METHOD

Yehuda Gur, Department of Systems Engineering, University of Illinois at Chicago Circle, and Chicago Area Transportation Study

The paper describes the following major problems encountered in the process of programming and testing the direct traffic estimation method: how to estimate contributions to volumes of individual zones, how to deal with assymetric networks, how domain overlap occurs and can be identified, and how to improve results by smoothing the boundaries between main and prime domains. The major conclusions of the first-stage testing of the model are as follows: (a) The model produces reliable expressway volume estimates that may be used for planning and analysis; (b) currently, it is not recommended that the model be used for estimating arterial volumes, and, in case such a use is made, the range of possible errors should be kept in mind; and (c) application of the model gives an excellent method for describing the probable sources of traffic on a given link and the function of the link in serving the travel demand as a part of the network. Visual representation of the results using the available auxiliary programs is of special help.

WITHIN the framework of urban transportation planning, estimates of traffic volumes on major arteries have special importance. They are essential inputs for both planning and design and for evaluation of alternative plans and programs. Traditionally, link volumes on the highway network are estimated through the chain of land use-trip generation-conversion-distribution-assignment (4). This method is aimed at and is relatively efficient for estimating link volumes for the whole region or large parts of it. However, where volumes estimates are required for a small number of links, running the full assignment is quite inefficient.

In cases where information on sources of traffic on particular links is required, the same problem occurs. The assignment package can supply this information but only through a tedious and expensive process. The direct traffic estimation method (DTEM) is efficient in the analysis of separate links, and can be used where such an analysis is needed.

This method was developed by Schneider in 1965 (1). It was computer programmed and applied by the Tri-State Transportation Commission (TSTC) in 1967 (2). In this method, link volumes are estimated by examination of the link itself, not the interchange it serves. The link volume is estimated as a function of the potential of the region to generate traffic on the link and the availability of parallel links to serve this traffic. The method also gives a measure of the relative importance of different zones in the region as sources for traffic on the link. Analysis of each link is done completely independently of analysis of other links.

After careful examination of the program being used by TSTC, it was found that the use of this program by the Chicago Area Transportation Study (CATS) would require major changes either in the CATS data system or in the program. This was due to differences in the geographical identification system and network coding rules used in the 2 studies. Since DTEM was intended to be used by CATS, together with the existing assignment program, it was decided to reprogram DTEM so that it would accept the existing CATS assignment input tapes.

A program was written in SIMSCRIPT for the CDC 6000 series computers. (Currently, a FORTRAN version of the program is being written.) Current size limitations are 19,999 nodes maximum, 9,999 zones maximum, and

$$CM = 12{,}000 + N^{*} \{[2 + (C/2)]\} + 2Z$$

where

CM = central memory size (words) (50,000 for the CDC 6400 and 130,000 for the CDC 6600),
N = number of nodes,
Z = number of zones, and
C = maximum number of links connected to 1 node.

Together with the main program, a system of auxiliary programs, mainly in FORTRAN, was prepared. These programs enable efficient calibration, preparation of input, printing of summaries, and plotting of output. A users' and reference manual was also prepared for these programs.

THE THEORY

The theory of the direct traffic estimation method was developed in another paper (1). (In the Appendix some theoretical difficulties are discussed, and modifications to the original theory are suggested.) Here it is intended to give only the logic of the method, definition of terms, basic assumptions, and major results of the theory. This section also describes a number of problems in application of the theory and the way these problems are dealt with.

Terminology

Figures 1 and 2 show graphical description of some of the following terms:

1. Point of interest (POI). A point on the road network (not an intersection) whose volume is being estimated (Fig. 1).
2. Link of interest (LOI). The link on which the POI is located. For convenience in reference, the direction of the link is designated as north-south.
3. Main nodes. The nodes at the ends of the link of interest.
4. Turning links. The links connected to the main nodes, excluding the main link. The turning links end with the turning nodes.
5. Path value (F_{ij}). The generalized cost of travel between 2 points, i and j. It is defined as the value of the minimum path between the points on the coded network.
6. Main boundary. A line through the POI, approximately perpendicular to the link of interest, that divides the region into north and south domains. The path value from the POI to any point on the main boundary is the same going north or south.
7. Domain (d). A definite area within the region (Fig. 2).
8. Main path. The minimum path from the POI (to a point).
9. Prime path. The minimum path from the main boundary (to a point).
10. Main domains (n, s). The north domain (n) is part of the region that is most easily reached from the POI going north. (That is, the best path from the POI to any point in the north domain passes through the main node.) Similarly, the south main domain is defined. Hence, the main boundary may be defined as the indifference line between the northbound and the southbound main paths (Fig. 2).
11. Prime domains (n′, s′). The prime domains are parts of the main domains (n, s) for which the main path is also the prime path.
12. Decay function G(·). A decreasing function of the friction between 2 points.
13. Domain integral (Id).

$$Id = \sum_{j \in d} G(F_{oj}) \times V_j \qquad (1)$$

where

F_{oj} = path value between zone j and POI, and
V_j = number of trip ends at zone j.

The summation is done over the finite area d (a zone, subregion, or region).

Assumptions

1. Complete symmetry exists in volumes and path values throughout the systems; i.e., $Q_{ij} = Q_{ji}$, where Q_{ij} is 1-way volume between i and j, and $F_{oj} = F_{jo}$, where F_{oj} is directional path value between POI and zone i.
2. Each trip end sends and receives exactly 1 trip daily.
3. A northbound vehicle at the POI has its origin in the south domain and its destination in the north domain.

(In all statements following, northbound can be replaced by southbound by replacing n by s, s by n, n′ by s′, and so on.)

4. A northbound trip originating in s′ is defined as free. Its destination may be anywhere in the north.
5. A northbound trip originating out of s′ has by definition a prime path better than the main path. If it uses the link of interest, it implies that its destination is in n′. Such trips are called fixed.
6. The relative possibilities of trips through the POI terminating in different domains are proportional to the domain integrals. That is,

$$P\{D = j | D \in d,\ j \in d\} = I_j/I_d \tag{2}$$

where D is the destination of the trip.

Calculation of Volumes

Based on the previous assumptions, the daily 2-way volume through the POI has been found to be

$$(I'_n \cdot I_s + I'_s \cdot I_n)/(I_n + I_s) \tag{3}$$

Partial Volumes

Equation—Let d be a subdomain of the north domain, so that $d \in n$, $d' \in n'$, $d' \in d$, and $d' \geq \phi$. The total 2-way volume from this domain through the POI is

$$Q = (I_d \times I'_s + I'_d \times I_s)/(I_n + I_s) \tag{4}$$

If we sum up the partial volumes to all the subdomains in the north domain, we get

$$Q = [1/(I_n + I_s)] * [I'_s \cdot I_n + I_s \cdot I'_n] \tag{5}$$

This equation is used to calculate turning volumes and the contribution of each zone in the link volume.

Turning Movements—Clearly, the path from any point in the region to the POI passes through exactly 1 turning link (as defined earlier). Hence, the subdomains (parts of the prime and the main domains) that belong to each turning link may be found, and the turning movement calcuated by using Eq. 5.

Volumes for Individual Zones—Equation 4 may be used to find 2-way volumes from individual zones through the POI. As usual, a zone is represented on the network as a point (load node) with a given number of trip ends on it. In this case, the complete zone belongs or does not belong to a prime domain. This might cause some inaccuracies in the estimation of the distribution of individual zones to the links volume. In order to partly overcome this problem, the program enables a breakdown of zones between the prime and nonprime domains. In this case, a zone may belong partly to the prime domain. Equation 4 covers the 2 cases.

Asymmetric Networks

Clearly, the assumption of symmetry of volumes and path values should be approximately true in order to allow successful application of this method. This is especially important near the POI. Hence, estimation of directional peak volumes or of volumes on 1-way links in the network is impossible. On the other hand, inclusion of 1-way links in the network is allowed as long as they do not cause substantially different path values to and from the POI.

Direction of Paths

Theoretically, the path values between any 2 points are the same in both directions. Unfortunately, 1-way links do exist and cause a number of small problems that are described below.

One-Way Turning Links—The main tree is built from the POI outward (Fig. 3). Therefore, if a turning link is 1 way inward (case 1), the calculated turning volume on this link will always be 0. If a turning link is 1 way outward (case 2), then its calculated volume may be positive.

One-Way Boundary Links—The main boundary is defined by a building a tree from the POI. The main domain of each node is determined according to the main node it is connected to. For the 2-way boundary link A-B, the following relation always holds:

$$| F_{oA} - F_{oB} | \leq T_{AB} \tag{6}$$

where F_{oI} is the path value of I from POI, and T_{AB} is the link friction. If the boundary link is 1 way, this is not always true. In this case, it may happen that the boundary passes around a 1-way boundary link.

Sensitivity of Volume to Path Values

Errors in path values from the POI to different zones appear due to errors in coding the network or in link friction values. These errors may be classified as either systematic or random errors. They may affect the estimated volume by changing the domain integral values of zones or by transferring zones between domains.

Random Errors—In the case of random errors in link friction values, the errors due to changing zone values are generally very small. The errors due to transferring zones may be quite large. These errors usually consist of a transfer of a zone in or out of the prime domains. This happens quite often, because generally the path value from the POI to a zone is never much smaller than the best alternative path from the main boundary. A small increase in the "main path" value may easily cause a zone to leave the prime domain. Errors due to transfers between domains are especially serious when only a few zones are included in the prime domain (in this case, the inclusion of any zone makes a large percentage difference in the calculated volume) and when one of the main domains is much larger than the other (a common case when the link is located near region boundaries). This problem is overcome by breaking zones between main and nonmain domains as follows:

$$Z' = \begin{cases} 0 & \text{if } Z'' \leq 0 \\ Z'' & \text{if } 0 < Z'' \leq 1 \\ 1 & \text{if } Z'' > 1 \end{cases} \tag{7}$$

where

$Z'' = 0.5 + k\ [(F'_A - F_{oA})/F_{oA}]$,

Z' = percentage of the zone (i.e., the trip ends in the zone) that belongs to the prime domain,

F'_A = path value on the prime tree,

F_{oA} = path value on the main tree (from the POI), and

k = specified parameter (currently used k = 0.1/0.2).

By this method, errors in volume due to misspecification of domains still cause considerable error, but much smaller than before. In the computer program, this formula may be applied through a special optional routine. Experimenting with this method shows substantial improvements in estimation of volumes on minor links with almost no effect on high-volume links. The problem of 0-volume links (due to nonexistence of prime domain) vanished.

Systematic Errors—Systematic errors in the path values cause changes of domain integral values. The danger of such errors exists especially when the network used for calibration is substantially different from the network used for estimation. This problem is discussed later.

Domain Overlap

Sometimes, due to peculiarities in networks, a certain area belongs by definition to the prime domains of 2 parallel adjacent links. This is called domain overlap. An example of such a condition is shown in Figure 4. There, area A belongs to the prime domains of the 2 points of interest I and II. Clearly, the existence of domain overlap is in contradiction with the logic of the theory. It causes overestimation of volumes on the links involved.

The existence and the extent of domain overlap in the analysis of any specific link are practically impossible to predict. They may be checked by plotting the domain boundaries of adjacent links. Generally speaking, areas with irregular networks are more prone to have difficult overlap problems. (Thus, in the Chicago area, which has relatively uniform network configuration, the problem is not especially serious.)

The domain overlap problem may be solved by merging 2 links into 1, estimating the joint volume, and then externally or otherwise redistributing the volume. A detailed study and implementation of correction procedures for this problem are yet to take place.

Concluding Remarks

The direct traffic estimation method uses practically the same inputs as the traditional method: trip ends, decay function, and coded network. Yet, the 2 methods are significantly different in the ways used to analyze the data. The DTEM is advantageous in solving individual links. But, it has a number of disadvantages. The more important are the unavailability of capacity constraints and the inability to apply the method for assymetric loadings. Only careful and continual use of the method will enable its practicality as a planning tool to be evaluated and its most effective place within the set of available travel demand prediction models to be defined.

CALIBRATION AND TESTING

DTEM was calibrated and tested with CATS 1965 data. The major findings are as follows:

1. The negative exponential function (Eq. 8) may be used as the decay function.

$$G(F_{oJ}) = \exp(-\beta * F_{oJ}) \qquad (8)$$

where

F_{oj} = travel friction between the point of interest and zone j,
β = coefficient, to be determined by the calibration; and
$G(\cdot)$ = decay function.

2. The optimal β for estimation are $\beta(1) = 0.06\ f^{-1}$ for expressways, and $\beta(2) = 0.08\ f^{-1}$ for arterials, where f is the CATS friction measure. (A unit friction represents approximately 24 sec on a free network.)
3. Expressway estimated volumes are close to counted volumes with a standard error of less than 12 percent of the mean.
4. Estimated arterial volumes have a standard error of 47 percent of the mean. The error is correlated negatively with the counted volume and number of lanes.

Method and Analysis

Ideally, the model may be calibrated by analyzing the distribution origins of vehicles traveling on the link of interest. Comparing the actual distribution with the distribution implied by the model (as calculated by Eq. 4) allows both the form of decay function and its parameters to be found and checked. Such an analysis may increase substantially our understanding of urban travel and the performance of the model. This analysis requires data on origins of trips using the link. These data are available only through a roadside interview. Unfortunately, such data were not readily available for the present analysis (and are practically unavailable for expressways because of technical difficulties in collection.)

In the calibration procedure used here, the decay function was chosen externally. Volumes on sample links were calculated by using different coefficients in the function. The optimal coefficients were chosen so as to minimize the sum of deviations between the estimated and the counted volumes. Later, the volumes estimated by using the chosen coefficients were analyzed to find the level of accuracy and the error properties of the estimates.

The Data

The model was calibrated and tested with CATS 1965 data. These data include an updated coded highway network, estimated number of zonal trip ends, and counted volumes on major roads. The volume counts on this set of data were carefully prepared and intensively checked. A sample of 11 expressway links and 22 arterial links was chosen for analysis. The sample covers a wide range of locations, geometry, and link volumes.

The Decay Function

Two decay functions were tested for use: the A function, developed by Schneider (3), and the negative exponential function. The two were shown earlier to fit DTEM (1, 3). In preliminary runs, no advantage of either of the 2 functions was apparent. Hence, it was decided to use the simpler one, i.e., the negative exponential function (Eq. 8).

It should be remembered that it is possible that other functions may prove to be better. Until more analysis is done (preferably by using extensive data from roadside interviews), no final conclusion can be made. However, it should be noted that the negative exponential is the only decay function that is completely consistent with the theory (see Appendix).

Analysis of Results

The basic theory presumes that 1 decay function may be used for all route types. However, it was found both by CATS (Figs. 5 and 6) and by TSTC (2) that the use of 1 decay function causes underestimation of expressway volumes or overestimation of arterial volumes or both. Use of 1 decay function for the 2 cases would require manual adjustment of results. Such adjustments may be avoided, or at least decreased, by a separate analysis of the 2 link types. Such a separation was done in this analysis.

Figure 1. Part of region around point of interest.

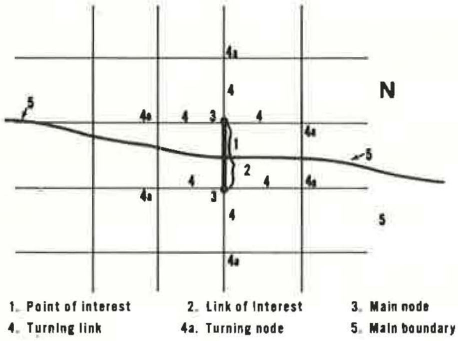

Figure 2. Definition of domain.

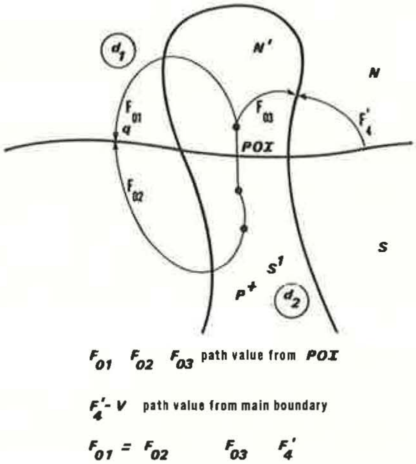

Figure 3. One-way turning links.

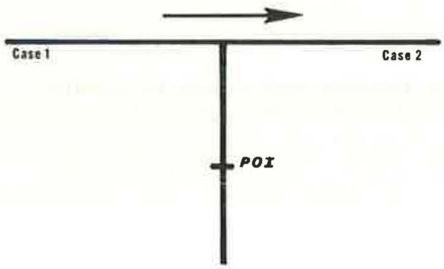

Figure 4. Domain overlap.

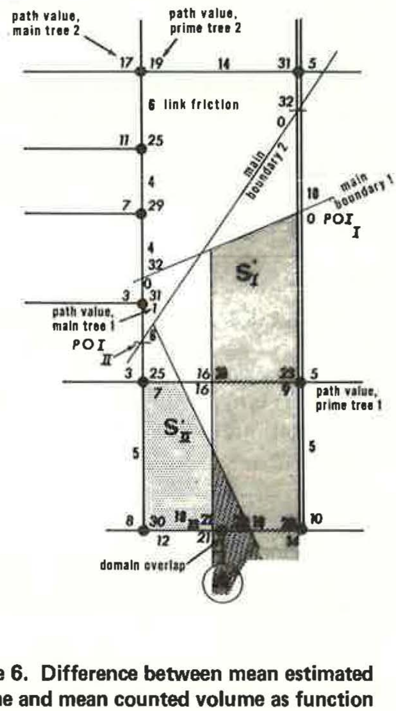

Figure 5. Difference between mean estimated volume and mean counted volume as function of β for expressways.

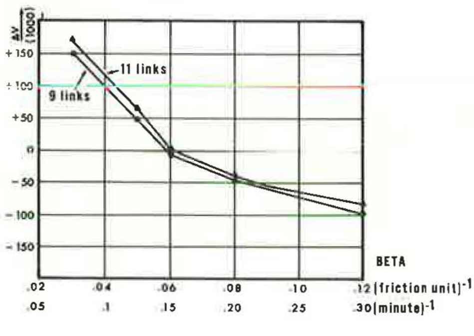

Figure 6. Difference between mean estimated volume and mean counted volume as function of β for arterials.

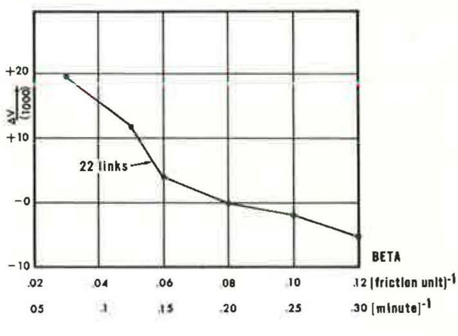

Table 1. Test results.

Item	Expressways 11-Link Sample	Expressways 9-Link Sample[a]	Arterials, 22-Link Sample
Mean volume, vehicles per day, 2-way			
Counted	120,850	123,630	16,240
Estimated	124,200	117,880	16,090
Mean error	3,350	-5,750	-150
Standard deviation of volumes, vehicles per day, 2-way			
Counted	59,200	63,800	9,340
Estimated	52,100	51,800	9,300
Standard error of estimate, vehicles per day, 2-way	23,900	14,600	7,810
Standard error, percent	20.0	11.8	47
Range of volumes, vehicles per day, 2-way	8,600 to 216,600	8,600 to 216,600	200 to 38,300
Range of errors, vehicles per day, 2-way	-17,300 to 59,400	-17,300 to 17,300	-14,000 to 13,100
Correlation of error with counted volumes			-0.42

[a]The 9-link sample is identical to the 11-link sample after 2 links are dropped that show large domain overlap.

Choice of Optimal β—Figures 5 and 6 show that the mean error is quite sensitive to the value of β. This sensitivity is greater in expressways than in arterials. Because of this sensitivity, the choice of β is dictated by the results and is straightforward. Directly from the graphs we get $\beta(1) = 0.06$ (f^{-1}) for expressways, and $\beta(2) = 0.08$ (f^{-1}) for arterials.

Domain Overlap—The original sample includes 11 expressway links. The errors in estimates of these links are given in Table 1. Two of these links were respectively the express and the local lanes of the same section of an expressway (Dan Ryan Expressway on the south side of Chicago). Analysis of the domain boundaries of the links revealed a strong domain overlap (as described earlier). This caused overestimation of the volumes on these 2 links.

Because this situation (of parallel local and express lanes) is unique and identifiable, it was decided to drop the 2 links from further analysis. It is expected that using the means suggested earlier, i.e., estimation of the joint volume on the 2 links, will enable solution of this problem.

Error of Estimate, Expressways—A summary of the error properties is given in Table 1. The following points are of interest:

1. A relatively wide range of volumes was checked.
2. The standard error of estimate is 14,600 vehicles/day, 2-way or 11.8 percent of the mean volume. The maximum error in the sample is 17,300 vehicles/day.
3. The variance of the estimated volumes is lower than the variance of the counted volumes. This may be caused by a tendency of the model to underestimate high-volume links and overestimate low-volume links.

The error of 17,300 vehicles/day (the maximum observed) is less than the capacity of 1 expressway lane. Such a level of accuracy is well within the range acceptable for planning. Although the accuracy of estimates depends also on the errors in other stages of the modeling (network speeds and zonal trip end estimates), it seems that this total approach is relatively reliable for estimating expressway volumes.

Error of Estimate, Arterials—A summary of errors in estimates of arterial volumes is given in Table 1. The following observations are of interest:

1. The range of volume tested is quite high—200 through 38,000 vehicles/day. The results show that further breaking of this group may be helpful.
2. The standard error of estimate is 47 percent of the mean volume or 7,810 vehicles/day.

3. The range of errors is from -14,000 to 13,100 vehicles/day.
4. A significant negative correlation (-0.42) was found between the error and the counted volumes. This shows, as expected before, that the model underestimates high-volume links and vice versa.

Although this level of accuracy may be sufficient for certain purposes in transportation analysis, it is by no means satisfactory. More work on the model is required in order to improve these estimates.

Network Speeds—Multiplication of link friction values by a constant is equivalent to multiplying by the same constant. Seemingly, a systematic increase or decrease in link friction values is equivalent to these multiplications. Such systematic changes in friction occur while a transfer is made among theoretical, free, loaded, and congested networks. (These are not accurate terms but refer to the travel friction under different link loadings.) Because of the high sensitivity of the volumes to β (as shown in Fig 5), it is important that, in both the network used in the calibration stage and the network used for prediction, the friction corresponds to the same level of load on the network. The lack of capacity constraints in DTEM will always cause certain inaccuracies in volume estimates. However, systematic bias, which may be caused by using 2 differently loaded networks, is at least as erroneous.

Correlation Analysis of Errors—The errors in the arterial volume estimates were analyzed in order to find the reasons for the high errors and, possibly, to find ways of correcting them. The following results are of interest:

1. Negative correlations exist between the error and the counted volumes, capacity class, or number of lanes. This fact may be explained, at least partly, by lack of capacity constraints. It is reasonable to assume that, given the increase in link friction due to increase in volume, the prime domains of the small links will decrease and cause the total volume on these links to go down faster than on other links.
2. The error is negatively correlated with distance to the nearest parallel expressway.

Correction for these 2 variables through the regression equation causes a decrease in the standard error of estimates of approximately one-half. Nevertheless, it is not recommended that these corrections be used without further study of the causes of errors in the model.

Conclusions

1. DTEM produces reliable estimates of expressway volumes. These estimates may be used for planning and analysis.
2. Currently, it is not recommended that DTEM be used for estimating arterial volumes. In case such a use is made, the range of possible errors should be kept in mind.
3. Application of the model gives an excellent method for describing the probable sources of traffic on a given link and the function of the link in serving the travel demand as a part of the network. Visual representation of the results using the available auxiliary programs is of special help.

Recommended Further Research

Some theoretical problems and directions for research are suggested elsewhere (3 and the Appendix). In this section, concentration is on operational research in the framework of the existing theory.

1. It is suggested that the decay function be carefully examined by using data from roadside interviews when and where such data are available.
2. An intensive examination of error properties of arterial estimates should be made. Large reductions in the existing error levels are very likely.
3. A procedure that enables merging of links with overlapping prime domains into 1 equivalent link should be developed and tested. Such a procedure may be included within the program by a separate auxiliary subroutine.

4. This model is the first one used by CATS in which absolute path values and not relative values only are used for predicting link volumes. A comparative study of scaling of the 3 main networks (1965, current, and 1985) should be carried out in order to ensure compatibility among the networks.
5. Use of the available graphic output for production of predicted flow diagrams on major links should be studied.

It is expected that a large part of the existing problems in the model may be solved by conducting the research recommended here. Besides the solution of the specific problems, this research may serve as an excellent tool for improving understanding of urban travel characteristics.

REFERENCES

1. Schneider, M. Direct Estimation of Traffic Volume at a Point. Highway Research Record 165, 1967, pp. 108-116.
2. Brown, G. J. H., and Woehrle, A. H., Jr. Program, Inputs and Calibration of the Direct Traffic Estimation Method. Highway Research Record 250, 1968, pp. 18-24.
3. Schneider, M. Access and Land Development. HRB Spec. Rept. 97, 1968, pp. 164-177.
4. Final Report. Chicago Area Transportation Study, Vols. 1, 2, and 3, 1962.

APPENDIX

MODIFICATION OF THE VOLUME EQUATION

Schneider developed the volume formula as follows (1):

1. A hypothetical trip end is put near the point of interest. It is shown that

$$Q = 1/(P_n + P_s) \tag{9}$$

where P_n and P_s are respectively the probabilities that any northbound or southbound vehicle at the POI will choose the hypothetical trip end as its destination. Q is the daily 1-way volume through the POI.

2. "Free" vehicles are defined as vehicles that begin their trips inside the prime domain. The remaining vehicles are called "fixed." The proportions of the free vehicles (called A_s and A_n) in the southbound and northbound traffic streams are found. This is done by equating the number of trips originating in the south prime domain with the number terminating there (1, Eqs. 12, 13, and 14).
3. The probabilities P_n and P_s are later calculated as the weighted average of the probabilities that either a fixed or a free vehicle will occupy the hypothetical destination (1, Eqs. 9, 10, and 11).
4. P_n and P_s are substituted in Eq. 1 to get the solution (1, Eq. 16).

In the original representation of the direct traffic estimation method, Schneider (1) found the 1-way daily volume on a link to be

$$Q_1 = [(I_n - I_s)/(I_n + I_s)]\,[1 - (1 - r_n)\cdot(1 - r_s)] \tag{10}$$

where

I_d = the domain integral of d,
n, n', s, s' = 4 domains,
$r_n = I_{n'}/I_n$, and
$r_s = I_{s'}/I_s$.

Or, substituting the r's gives

$$Q_1 = [1/(I_n + I_s)]\cdot[I_n'I_s + I_s'I_n - I_s']\tag{11}$$

Brown and Woehrle (2), in applying the method, give the formula for calculation of 2-way link volume as

$$Q_2 = [1/(I_n + I_s)]\cdot[I_n'I_s + I_s'\cdot I_n]\tag{12}$$

They refer to Schneider (1) as the source of their formula but give no explanation for the differences. Schneider himself suggested (1, p. 115) that modifications to his Eq. 16 were required and were being implemented but did not specify why and how they were to be done.

The analysis here results in a third volume formula as shown below.

Choice of Destination and Route by Northbound Free Trip

A typical situation is presented and is shown in Figure 2.

A domain d_2 is located in the south prime domain. A point p is found so that every trip from d_2 to the north has to pass through it. Here, the choice of the exact destination and the route of such a trip are discussed. Because all the vehicles going to the north from d_2 pass through point p, it will be used as a point in which the trips are observed, ensuring that no trips are eliminated from consideration. Equation 13 gives

$$P\{D\in d\} = I_d^*/I_n^*\tag{13}$$

where D is the destination of the trip, d is some domain in the north, and I* is the domain integral with respect to p.

Clearly, at p, the trip is completely free to choose a route. If d is in the north prime domain, it is very likely that the trip will use the path through the POI. When d is not in the north prime (for example, $d = d_1$ in Fig. 1), other paths may be considered. In particular, the possibility of crossing the main boundary at q (Fig. 2) cannot be ignored. q is the point on the main boundary nearest to d_1. This possibility should be accounted for because it is considered a valid choice while the symmetric case is analyzed (q is the POI, trip originates at d_1). Without unnecessary complications, it may be assumed that the probabilities of crossing at the POI and at q are each equal to ½.

Putting this in probability notation gives the following:

1. For any trip originating in s′ and going to n,

$$P\{X = POI \mid D\notin n'\} = 1/2\tag{14}$$

$$P\{X = POI \mid D\in n'\} = 1\tag{15}$$

$$P\{D\in d\} = I_d^*/I_n^*\tag{16}$$

where X is the crossing point of the main boundary. Taking d = n′, gives

$$P\{D\in n'\} = I_{n'}^*/I_n^*\tag{17}$$

$$\begin{aligned}P\{X = P\phi I\} &= 1/2\cdot(1 - I_{n'}^*/I_n^*) + I_{n'}^*/I_n^*\\ &= (I_n^* + I_{n'}^*)/(2\cdot I_n^*)\end{aligned}\tag{18}$$

2. For those trips that also pass through the point of interest,

$$P\{D \in n'\} = (I^*_{n'}/I^*_n) \cdot P\{X = POI\}$$
$$= (2 \cdot I^*_{n'})/(I^*_{n'} + I^*_n) \qquad (19)$$

In case the decay function being used is the negative exponential, it always gives

$$I^*_{n'}/I^*_n = I'_n/I_n$$

and Eq. 19 can be modified into

$$P\{D \in n\} = (2 \cdot I'_n)/(I_n + I'_n) \qquad (20)$$

without any loss of generality. For convenience of notation, define

$$r_n = (2 \cdot I_n')/(I_n + I_n')$$
$$r_s = (2I_s')/(I_s + I_s') \qquad (21)$$

Volume Formula

Using the steps described in the preceding section gives

$$Q_nA_n = Q_s(1 - A_s) + Q_sr_sA_s$$
$$Q_sA_s = Q_n(1 - A_n) + Q_nr_nA_n \qquad (22)$$

(parallel to 1, Eqs. 12 and 13).

The solution of this system (clearly, $Q_n = Q_s$) gives

$$A_n = r_s/[1 - (1 - r_s) \cdot (1 - r_n)]$$
$$A_s = r_n/[1 - (1 - r_s) \cdot (1 - r_n)] \qquad (23)$$

(parallel to 1, Eq. 14.)

$$P_n = I_o\{A_n[2/(I_n + I_n')] + (1 - A_n) \cdot (1/I_n')\}$$
$$P_s = I_o\{A_s \cdot [2/(I_s + I_s')] + (1 - A_s) \cdot (1/I_s')\} \qquad (24)$$

(parallel to 1, Eqs. 10 and 11).

Putting Eq. 23 into Eq. 24 gives

$$P_n = [2I_o/(I_n + I_n')]/[r_n + r_s - r_n \cdot r_s]$$
$$P_s = [2I_o/(I_s + I_s')]/[r_n + r_s - r_n \cdot r_s] \qquad (25)$$

To conclude, take $I_o = 1$ and put Eq. 25 into Eq. 12, which gives

$$Q_3 = [I_n \cdot I_s' + I_s \cdot I_n']/[I_n + I_s + I_n' + I_s'] \tag{26}$$

where Q_3 is the 1-way volume through the POI.

Some Observations

A major shortcoming of this approach is the necessity of analyzing the problem at the origin instead of at the POI. But this procedure seems necessary to make the theory of DTEM completely valid. Because of this procedure, it is necessary to use the negative exponential function as the only possible decay function (for the crucial passage from Eq. 19 to Eq. 20). Only the negative exponential function has a "complete lack of memory" required to assume that the residual trip length distribution is always equal to the total trip length distribution.