COMPARISON OF PROBABILISTIC MODAL-CHOICE MODELS: ESTIMATION METHODS AND SYSTEM INPUTS

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Twelve models were formulated by segmenting the total travel time and total travel cost by rapid transit and by automobile in different ways or by leaving them out completely and including only socioeconomic variables in the model. These models were then estimated by using logit, probit, and discriminant analyses. The results were evaluated in 2 respects: Are there differences in performance among the methods of estimation? and Are there differences in performance among the 12 model specifications? The results indicate that there are no statistically significant differences either among the methods of estimation or among the model specifications themselves. A model that uses only 2 user characteristics, income and the number of working household members, and 1 system-related variable, a dummy variable for walk access to the transit station, performs no worse than a model that uses a whole set of system characteristics in addition to those 3 variables. Values of time significantly lower than those previously reported were found; the "best" estimate in this study is only 12 percent of the wage rate.

THERE ARE 3 objectives in this research: (a) to investigate the relative merits of the 3 methods most widely used in probabilistic modal-choice modeling or in value of time studies, logit, probit, and discriminant analyses (1, 2, 3, 4); (b) to investigate the need for trip segmentation, that is, Is there a need to differentiate among access, egress, and line-haul times and costs? and if so, How should this trip segmentation be done?; and (c) to obtain further evidence on the value of time.

Answers to the first objective are provided by estimating several different models by all 3 methods and then by assessing and comparing the forecasting accuracy of each method. For the second objective, these different models were designed by segmenting the travel costs and travel times in different ways. The forecasting accuracy of each model specification was then assessed and compared with the forecasting accuracy of the other model specifications. The third objective was accomplished as a by-product from several different models estimated by objectives 1 and 2.

BASIC MODEL AND METHODS OF ESTIMATION

A general probabilistic travel demand model can be expressed as

$$Pr\,(ij, M) = Pr\,(ij) \cdot Pr\,(M|ij) \tag{1}$$

where

$Pr\,(ij, M)$ = probability that the event (ij, M) occurs, that is, an individual makes a trip between points i and j by using mode M;

$Pr\,(ij)$ = probability that an individual makes a trip between points i and j; and

$Pr\,(M|ij)$ = probability that an individual uses mode M, given that he makes a trip between points i and j.

Clearly, in modal choice, models are estimated for $Pr(M|ij)$. In this research only binary choice, automobile versus rapid transit, is considered.

It is assumed that there is an index y that determines to which group (automobile or transit) an individual is likely to belong. This index y is constructed as follows:

$$y = a + b\,(L_{ij}^{M1} - L_{ij}^{M2}) + c\,(SE)$$

where

- a = constant term;
- b = $(1 \times k)$ vector, where k is the number of system characteristics;
- c = $(1 \times \ell)$ vector, where ℓ is the number of user characteristics;
- L_{ij}^{M} = $(k \times 1)$ vector of system characteristics describing the level of service between points i and j by mode M;
- SE = $(\ell \times 1)$ vector of user characteristics; and
- a, b, c = coefficients to be estimated.

Verbally expressed, it is hypothesized that modal differences in the level of service (i.e., differences in travel times) combined with the user characteristics are the determinants of choice of mode. [Also the ratios of system characteristics can be used (1, 5, for example).]

In the logit model, the probability that choice of mode M, the dependent variable, will equal 1 (automobile choice is denoted by 1, and transit choice is denoted by 0) is

$$\Pr(M = 1|ij) = e^{y}/(1 + e^{y}) \tag{4}$$

and similarly

$$\Pr(M = 0|ij) = 1 - \Pr(M = 1|ij) = 1/(1 + e^{y})$$

No assumptions are needed about the distributions of the variables or of y.

The probit model uses the same linear function y. If, for any given individual, $y \geq y_{crit}$, then M = 1; and if $y < y_{crit}$, then M = 0. The assumption is made that y_{crit} is normally distributed over the population. The probability that M will equal 1 is

$$\Pr(M = 1|ij) = \Pr(y_{crit} \geq y|ij) = 1/\sqrt{2\pi} \int_{-\infty}^{y} e^{-(t^2)/2}\,dt \tag{5}$$

and similarly

$$\Pr(M = 0|ij) = 1 - \Pr(M = 1|ij) = 1/\sqrt{2\pi} \int_{y}^{\infty} e^{-(t^2)/2}\,dt$$

The method of maximum likelihood is used to obtain estimates for a, b, and c in both logit and probit analysis. (A computer program originally written by John Gragg, Department of Economics, University of British Columbia, and modified by Peter Stopher for CDC 6400 was used in estimating the models.)

In discriminant analysis, no dependent variable exists. The objective is to find such linear combination of the explanatory variables that their joint distribution, the distribution of y, for the 2 groups would possess very little overlap. This discrimination rule classifies an observation at y as coming from population 1 if $f_1(y)/f_2(y) > k$, and otherwise from population 0. If we assume that y is normally distributed, as is conventionally done, then, after some serious manipulation,

$$\Pr(M = 1|ij) = e^{y+\ln(p/q)}/[1 + e^{y+\ln(p/q)}] \tag{6}$$

and

$$\Pr(M = 0|ij) = 1/[1 + e^{y+\ln(p/q)}]$$

where p and q are the a priori probabilities of group membership 1 and 0 respectively.

DATA

Data Source

The data used were originally prepared by Lisco (2) and are well documented in his thesis; only a couple of comments are in order here.

The data consist of 159 work trips made during the morning rush hour from Skokie to the Chicago Loop. Only a binary choice (automobile or rapid transit) was available to the trip-makers. The corresponding travel times and costs were compiled for each segment of an individual trip; these disaggregate figures formed the basis for the trip segmentation.

Trip Segmentation

The system shown in Figure 1 will help explain the trip segmentation procedures. (The actual system was more complicated because of train transfers needed by some of the travelers. This does not change the principles, however.) Links 1 through 7 describe the transit network, and links 8 through 10 describe the automobile network. Times and costs associated with each link were obtained for each element in the sample. These transportation system attributes are given in Table 1.

Three user attributes—income, number of workers in household, and automobile ownership—and 1 indirect system attribute—walk access to rapid transit station—were also included in the data. These socioeconomic attributes are given in Table 2.

Twelve models were estimated by combining the travel time and travel cost differences in different ways. Of the socioeconomic attributes, dummy 1 and dummy 2 are included in every model; but if income was used, then automobile ownership was not, and vice versa.

A description of the models is given in Table 3. Next to the description column, a relationship is given to indicate how the time and cost differences were combined. Access refers to the trip from home to station, egress refers to the trip from station to work, and total access means access and egress taken together. Excess refers to the time spent outside the vehicle, for either walking, waiting, or transferring. Travel time differences are transit minus automobile, and travel cost differences are automobile minus transit. Except for the socioeconomic variables, given in Table 2, model 7 is the same as the one used by Quarmby (3), model 8 is the one used by Lisco (2), and model 9 was used by TRC (6, the TRC model used ratios instead of difference of respective travel times and costs). The system variables in these models were excess time (not in Lisco's model), total time, and total cost or out-of-pocket cost (TRC model).

RESULTS

Two kinds of evaluations were made on the basis of the results: (a) What is the best method? and (b) What is the best model? Ideally, the evaluation of the methods and the models should be done with a set of data other than that used for the model calibration. However, in the present study the original body of data was already quite small (159 individuals), and splitting that would only have left too few either for the model estimation or for the control group. An alternative procedure was adopted. It involved taking random samples from the data, computing the corresponding probabilities for each individual, summing them up to the expected value, and comparing the actual and expected values. By taking enough samples and getting the expected values and their standard deviations for each method and model, one could perform statistical tests (t-tests) to see whether there are any differences among the 3 methods or among the 12 models.

Twenty samples of 20 individuals were drawn, and the t-tests were undertaken. Unfortunately, no differences either among the methods or among the models were detected this way. The hypothesis that the results are statistically equal could not thus be rejected.

It was, therefore, decided to engineer the answer to these 2 questions. First, the methods were checked for dominance. No model could be excluded because of dominance (based on all 3 methods). The decision was then made to rank the methods and the models. This ranking was based on multiple criteria. The ranking criteria were di-

Figure 1. Transportation system between Skokie and Chicago Loop.

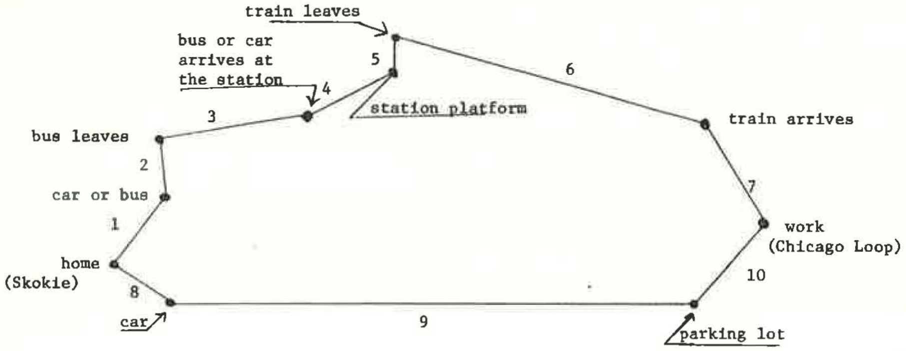

Table 1. Time and cost variables of transportation system.

Link	Description	Notation Time	Notation Cost	Note
1	Walk to car or bus	T1	—	
2	Wait for bus	T2	—	If car access to station, T2 = 0
3	Take car or bus to station	T3	C3	
4	Walk to station platform	T4	—	If walk access to station, T1 = T2 = T3 = C3 = 0
5	Wait for train	T5		
6	Line haul in train	T6	C6	C6 = train fare
7	Egress to work	T7	—	Walk was only egress mode
8	Walk to car	T8	—	
9	Drive to parking lot in Loop	T9	C9	T9 includes parking time; C9 excludes parking cost
10	Egress to work	T10	C10	Walk was only egress mode; C10 = parking cost

Table 2. Socioeconomic variables.

Description	Notation	Models Used in
Income (household or unrelated individual)	I	1 through 9
Workers per household (dummy 1, 0 if 1 and 1 if otherwise)	D1	All
Walk access to station (dummy 2, 0 if distance is 5 miles and 1 if otherwise)	D2	All[a]
Automobile ownership	A	10 through 12

[a]One-half mile was data supported walk access distance to station.

Table 3. Description of models.

Model	Time Description	Time Relationship[a]	Cost Description	Cost Relationship[a]
1	Access	T1 + T2 + T3 + T4 + T5 - T8	Access	-C3
	Line-haul	T6 - T9	Line-haul	C9 - C6
	Egress	T7 - T10	Egress	C10
2	Access	Same as model 1	Total access	C10 - C3
	Line-haul	Same as model 1	Line-haul	Same as model 1
	Egress	Same as model 1		
3	Total access	T1 + T2+T4 + T5 + T7 - T8 - T10	Total access	Same as model 2
	Line-haul	Same as model 1	Line-haul	Same as model 1
4	Excess	T1 + T2 + T4 + T5 - T8 - T10	Total access	Same as model 2
	In-vehicle access	T3	Line-haul	Same as model 1
	Line-haul	Same as model 1		
5	Excess	Same as model 4	Total access	Same as model 2
	In-vehicle	T3 + T6 - T9	Line-haul	Same as model 1
6	Excess	Same as model 4	Total access	Same as model 2
	Total	$\sum_{i=1}^{7} Ti - \sum_{i=8}^{10} Ti$	Line-haul	Same as model 1
7	Excess	Same as model 4	Total	C9 + C10 - C3 - C6
	Total	Same as model 6		
8	Total	Same as model 6	Total	Same as model 7
9	Excess	Same as model 4	Out-of-pocket	C10 - C6 - $\widehat{C3}$[b]
	Total	Same as model 6		
10	None		None	
11	None		Parking	C10
12	Excess	Same as model 4	Parking	C10

[a]See Table 1.　[b]$\widehat{C3}$ = Bus fare to station or parking cost at the station, no driving costs.

vided into 2 categories: classification criteria and criteria based on expected values. The classification criteria included 3 items:

1. Misclassified automobile users,
2. Misclassified transit users, and
3. Proportion classified correctly.

A group membership probability of ≥0.5 was used as a rule for correct group classification, and the correctly classified proportion was obtained based on these classification results. The classification criteria are given in Tables 4, 5, and 6 for the logit, probit, and discriminant methods respectively.

The expected-value criteria were computed based on the results of the 20 random samples and included the following:

1. Average absolute error, which = [(actual number of automobile users - expected| value of automobile users)/20];
2. Percent error, which = (1/20) [(actual number of automobile users - expected value of automobile users)/actual number of automobile users];
3. Standard deviation of error, and
4. Rank sum of the 20 predictions. [This item was obtained by ranking the results of 20 sample predictions (in case of a tie, the rank average was assigned for each tied value) and summing the ranks for each model. The lower the rank sum is, the better the model is.]

These results are given separately for each method in Tables 4, 5, and 6.

In the discriminant analysis method, 2 values have been given for classificatory measures. The upper ones were derived by using sample proportions as the a priori probabilities and the lower ones by using 0.5 as the a priori probability (Eq. 6). The latter a priori probability produces better classification results. The same does not hold true for the expected-value statistics (Tables 4, 5, and 6), however; but the sample proportions as a priori probabilities now give better results. In the ranking analyses reported below, the classification rankings correspond to those values obtained with 0.5 a priori probability.

Ranking Analysis of Estimation Methods

The methods were ranked separately by using the classification criteria and the expected-value criteria. Each column for each model was assigned a rank of 1 to 3. The best method was assigned the rank of 1. In case of a tie, the average of a rank sum was assigned for each tie. The ranks were then summed by column and row to yield a rank sum for each method. The inspection of the statistical measures given in Tables 4, 5, and 6 indicates that discriminant function always produces the lowest standard error. An experiment with another data set proved this is not always true. Hence, this criterion, expected-value criterion 3, was dropped from the ranking of the methods. The priority ranking of the 3 estimation methods is given in Table 7. The results of the ranking analysis indicate that the logit analysis is the best method. Examination of data given in Tables 4, 5, and 6 reveals, however, that the differences between logit and probit analyses are very small. In any case, logit analysis seems to slightly edge both probit and discriminant analyses.

Ranking Analysis of Model Specifications

A similar ranking as undertaken for the methods of estimation was performed for the 12 model specifications. In addition to the classification criteria and criteria based on expected values, the number of variables in the model was used in ranking the models to input (approximately, of course) the data collection and model estimation costs. Ranks run from 1 to 12; no averaging was done for ties, but the lowest tied rank was assigned for all ties. The results of this ranking analysis are given in Table 8.

Three comments may be made on the basis of these ranking results. First, models without any explicit system variables appear to be best accorcing to the classification criteria. Model 10 and model 11 rank as first and second. According to expected-value

Table 4. Ranking criteria for logit estimation method.

Model	Classification Criteria 1	2	3	Expected-Value Criteria 1	2	3	4
1	20	16	0.774	1.04	0.3	0.88	142.5
2	20	17	0.767	1.02	0.5	0.90	130
3	22	18	0.748	1.08	0.7	0.94	143
4	21	14	0.779	0.98	1.1	0.93	118
5	21	14	0.779	0.97	0.7	0.92	117.5
6	21	14	0.779	0.98	0.8	0.93	112
7	20	14	0.786	0.98	0.7	0.91	116.5
8	23	14	0.767	1.10	0.8	0.98	140.5
9	22	13	0.779	0.96	1.3	0.97	118.5
10	21	16	0.767	1.07	0.8	1.16	115.5
11	18	17	0.779	1.15	0	1.15	144
12	21	16	0.767	1.12	0.9	1.24	151

Table 5. Ranking criteria for probit estimation method.

Model	Classification Criteria 1	2	3	Expected-Value Criteria 1	2	3	4
1	22	16	0.762	1.05	0.5	0.87	138.5
2	21	16	0.767	1.02	0.5	0.89	129
3	22	18	0.748	1.08	0.7	0.93	141.5
4	22	14	0.773	0.99	1.1	0.92	118
5	22	15	0.767	0.99	0.9	0.91	121
6	22	14	0.773	1.00	0.9	0.91	122.5
7	20	13	0.792	0.99	1.1	0.89	119
8	23	14	0.767	1.10	0.9	0.97	144
9	22	13	0.779	0.98	1.6	0.96	119
10	21	16	0.767	1.06	0.4	1.11	110.5
11	23	15	0.762	1.15	1.4	1.10	149
12	21	16	0.767	1.11	0.1	1.20	148

Table 6. Ranking criteria for discriminant estimation method.

Model	Criteria 1	2	3	Expected-Value Criteria 1	2	3	4
1	38	8	0.710	1.09	0.8	0.48	134.5
	12	30	0.736				
2	37	7	0.729	1.08	0.7	0.50	128
	12	29	0.743				
3	36	7	0.729	1.08	0.5	0.55	119
	13	29	0.736				
4	44	5	0.691	1.05	0.7	0.44	122.5
	13	31	0.724				
5	44	3	0.704	1.06	0.8	0.42	131
	15	29	0.724				
6	44	4	0.698	1.06	0.8	0.43	137
	14	31	0.717				
7	40	6	0.710	1.07	0.5	0.44	132
	15	29	0.724				
8	40	11	0.680	1.11	0.3	0.59	123.5
	13	34	0.704				
9	37	10	0.704	1.06	0.3	0.52	130.5
	14	28	0.736				
10	58	1	0.630	1.08	0.7	0.53	124
	21	16	0.767				
11	44	5	0.691	1.10	0.9	0.50	135
	17	18	0.779				
12	47	2	0.691	1.09	0.9	0.49	146
	14	23	0.767				

Table 7. Ranking of estimation methods.

Method	Rank	Rank Sum by Criteria: Classification	Expected Value	Total
Logit	1	63.5	64.5	128
Probit	2	72	75.5	147.5
Discriminant	3	80.5	76	156.5

Table 8. Ranking of model specifications.

Rank	Model by Criteria: Classification	Expected Value	Total
1	10	7	7
2	11	5, 10	10
3	7	4	4, 5, 10
4	9	9	6
5	5	6	2

Table 9. Coefficients and standard deviations of model 5.

Variable	Logit Coef.	Logit S. D.	Probit Coef.	Probit S. D.	Discriminant Coef.	Discriminant S. D.
Constant term	-0.384	1.722	-0.337	1.040	-0.358	-
Time						
Out-of-vehicle	-0.316	0.114[a]	-0.173	0.0639[a]	-0.114	0.147
In-vehicle[b]	-0.0206	0.0559	-0.0165	0.0321	-0.0085	0.0037[a]
Total						
Cost						
Access[c]	0.0158	0.0088	0.0087	0.0052	0.006	0.0048
Line-haul	0.0414	0.0248	0.0249	0.0148	0.0170	0.0129
Total						
Income[d]	-0.0136	0.0056[a]	-0.0074	0.0033[a]	-0.0056	0.0018[a]
Automobile ownership						
2 workers per household	1.010	0.507[a]	1.087	0.284[a]	0.602	0.216[a]
Walk access	2.176	0.780[a]	1.277	0.447[a]	0.711	0.226[a]

[a]Significant at 0.05 level.
[b]Time differences are transit time minus automobile time in hundreds of seconds.
[c]Cost differences are automobile cost minus transit cost in cents.
[d]In thousands of dollars.

Table 10. Coefficients and standard deviations of model 9.

	Logit		Probit		Discriminant	
Variable	Coef.	S. D.	Coef.	S. D.	Coef.	S. D.
Constant term	1.075	1.143	0.584	0.694	0.288	—
Time						
Out-of-vehicle	-0.271	0.141	-0.145	0.0806	-0.103	1.34
In-vehicle[b]						
Total	-0.0327	0.0552	-0.0236	0.0319	-0.0157	0.0062[a]
Cost						
Access[c]						
Line-haul						
Total	0.0165	0.0088	0.0092	0.0051	-0.0068	0.0034
Income[d]	-0.0157	0.0054[a]	-0.0087	0.0031[a]	-0.0070	0.0023[a]
Automobile ownership						
2 workers per household	1.837	0.501[a]	1.032	0.278[a]	0.722	0.226
Walk access	1.996	0.769[a]	1.174	0.443[a]	0.684	0.217[a]

[a]Significant at -0.05 level.
[b]Time differences are transit time minus automobile time in hundreds of seconds.
[c]Cost differences are automobile cost minus transit cost in cents.
[d]In thousands of dollars.

Table 11. Coefficients and standard deviations of model 10.

	Logit		Probit		Discriminant	
Variable	Coef.	S. D.	Coef.	S. D.	Coef.	S. D.
Constant term	2.218	0.573	1.220	0.319	0.725	—
Time						
Out-of-vehicle						
In-vehicle[b]						
Total						
Cost						
Access[c]						
L[illegible]-haul						
[illegible]						
In[illegible]						
Automobile ownership	-1.460	0.343[a]	-0.797	0.184[a]	-0.592	0.126[a]
2 workers per household	1.660	0.476[a]	0.940	0.268[a]	0.623	0.194[a]
Walk access	1.624	0.663[a]	0.963	0.376	0.511	0.162[a]

[a]Significant at -0.05 level.
[b]Time differences are transit time minus automobile time in hundreds of seconds.
[c]Cost differences are automobile cost minus transit cost in cents.
[d]In thousands of dollars.

Table 12. Means of selected transit system variables.

Variable	Value[a]	Variable	Value[a]
Total travel time	3,624	Total travel cost	58.2
Out-of-vehicle time	749	Line-haul cost	38.2
In-vehicle time	2,875	Total access cost	20.0

[a]Time is in seconds, and cost is in cents.

Table 13. Effect of changes in travel time and cost on transit proportion.

Model	Transit Time Up 30 Percent	Automobile Time Down 23 Percent	Transit Cost Up 30 Percent	Automobile Cost Down 23 Percent	Remarks
Quarmby	- 9.0			-8.0	Reported by McGillivray
Wohl-Kraft	- 7.3	-6.2	-1.6	-2.0	Reported by McGillivray
McGillivray	-7.5		-5.6		Reported by McGillivray
Warner	-6.4		-6.4		Reported by McGillivray
Lisco	-34.6		-8.1		Change in total travel time and total travel cost
Model 7	- 8.8		-7.0		Change in total travel time and total travel cost
Model 7	-15.0				Change in excess time

Table 14. Value of time.

Model	Method	Value of Time ($/hour)	Standard Error of Value of Time	Percent of Wage Rate
7	Logit	0.71	1.17	12.5
	Probit	0.22	1.23	16.0
	Discriminant	0.83	—	14.0
8	Logit	3.49	3.02	62
	Probit	3.44	2.90	61
	Discriminant	3.60	—	64
9	Logit	0.62	0.82	11
	Probit	0.71	0.86	12.5
	Discriminant	0.65		11.5

criteria, however, models that have explicit system performance variables rank best. Model 7 and model 5 rank as the first and second.

Second, it is interesting that model 10, which has only 1 system variable (a dummy for walk access), and user attributes (car ownership, which probably is not system independent, and number of workers in the household), seems to be as good as models 5 and 7. Thus, the decision to buy a second car often signifies car choice, and the decision to reside near the transit station signifies transit choice. These choices are, of course, conditioned by socioeconomic factors such as the number of family members in the labor force and income. The effect of the extent of the public transportation system on modal choice may be negligible at present. The good performance of model 10 is not of spurious nature, as the actual survey proved (7, p. iii):

> This study indicated that diversion of Loop-bound trips ... is made mainly from other rapid transit modes, along with suburban railroads and buses, with only a small number being diverted from automobile trips.

Third, the results of the ranking analysis indicate that model 12, which included only those variables that were statistically significantly (at the 0.05 level) different from 0, consistently showed poor performance. The customary null hypothesis, b = 0, may not be a good one. There seems to be a reason to remember that 0 is a very particular coefficient; it implies no relation between the dependent and the explanatory variables. Why should a variable be excluded solely because its coefficient, which is the maximum likelihood estimate, has a wide standard error? In this study, all the variables were originally included because they should have an effect on modal choice and not because they were available in the data set. Coefficients and standard deviations are given in Tables 9, 10, and 11 for models 5, 7, and 9.

COMPARISONS WITH OTHER SIMILAR MODELS

McGillivray (5, p. 40, Table 13) computed changes in modal split for the policies of increasing relative travel time by 30 percent and increasing relative travel cost by 30 percent for 4 models. These results are reproduced here along with 3 new results for the same policies. The new results are those for Lisco's original model, for model 7 when the change is in total travel time and total travel cost, and for model 7 when the change is in excess time (Tables 12 and 13).

With regard to a 30 percent increase in transit time, it appears that the 4 results reported by McGillivray are largely identical; models developed by Quarmby, Wohl-Kraft, McGillivray, and Warner estimate the change in transit proportion to be between 6½ and 9 percent. A much different result is obtained by using Lisco's model; there the same increase in transit time would decrease the transit proportion by 34½ percent. Model 7 estimates the decrease in transit proportion to be nearly 9 percent in response to the 30 percent increase in total transit time; a result similar to those reported by McGillivray. However, if the change is in excess time, then model 7 indicates a 15 percent reduction in transit proportion.

With respect to travel costs, the results reported by McGillivray indicate that a 30 percent increase in transit cost (or 23 percent decrease in automobile cost) would decrease the transit proportion by 6 to 8 percent. There is an exception: The Wohl-Kraft model estimates this change to be about 2 percent. The results obtained by Lisco's model and model 7 give support to the 6 to 8 percent figure; the percentages for Lisco's model and model 7 are 8 and 7 percent respectively.

Three comments are in order on the basis of these results. First, model 7, where the travel time was broken down into excess and total travel time components, suggests that travelers value excess time differently from in-vehicle time. This result was obtained also by Quarmby and Kraft and Wohl. These models, Quarmby, Wohl-Kraft, and model 7, also estimate an identical magnitude for the change in transit proportion in response to a change in total travel time. Second, it is somewhat surprising that models by McGillivray and Warner on the one hand and by Lisco on the other estimate such different responses, even though the models are largely similar and do not include the excess-time term. McGillivray and Warner models give results similar to the other models mentioned above, but Lisco's model estimates modal choice to be much more sensitive to changes in travel time. Third, changes in modal split in response to changes in travel cost appear to be equal by all the modal-choice models; however, Wohl and Kraft, who use a different type of model, estimate less sensitivity with respect to travel cost.

VALUE OF TIME SAVED

Values of time saved, or more commonly values of time, were computed from model 7 (excess time, total time, and total cost), from model 8 (total time and total cost), and from model 9 (excess time, total time, and out-of-pocket cost). The results, including the standard error of the value of time and value of time as a percentage of the wage rate, are given in Table 14.

Value of time, in dollars/hour, was computed as $(b_1/b_2) \times 0.36$, where b_1 and b_2 are the estimated coefficients of total travel time difference and total cost difference respectively. Variance (b_1/b_2) is computed from the following formula:

$$\mathrm{Var}\,(b_1/b_2) = 1/b_2^4\,[b_2^2\,\mathrm{var}\,(b_1) + b_1^2\,\mathrm{var}\,(b_2) - 2b_1b_2\,\mathrm{cov}\,(b_1b_2)] + 1/b_2^3\,[b_1\,\mathrm{var}\,(b_2) + b_2\,\mathrm{cov}\,(b_1b_2)]$$

It appears from the results that all 3 methods of estimation produce approximately the same values of time. Models 7 and 9, which both have excess and total travel time variables, with the cost term being total cost difference in the former and out-of-pocket cost difference in the latter, obtain largely equal values of time. The average value of time for model 7 is 82 cents/hour and for model 9 is 66 cents/hour. These ratios are about 14 and 12 percent of the wage rate respectively.

Model 8, which has total travel time and total travel cost variables but no excess time variable, obtains a much higher value of time: approximately \$3.50/hour or 62 percent of the wage rate. (In Lisco's study the time value was 40 percent of the wage rate, and in Quarmby's 20 to 35 percent.)

The standard errors of all value of time estimates are quite large. Data given in Table 11 show that the standard errors are about equal to the values of time themselves. Two comments may be made on the basis of the results. First, the out-of-vehicle and in-vehicle times must have extremely different values because of the large difference in value of time depending on whether it was computed from a model where excess time is explicitly accounted. This result was also indicated by the results obtained in the previous section, where choice of mode was much more sensitive to out-of-vehicle than in-vehicle times. From the coefficients of models 4, 5, 6, 7, and 9, it may be inferred that out-of-vehicle time is 6 to 15 times the in-vehicle time, most of the values being around 7. [Quarmby found the out-of-vehicle time to be 2.5 to 3.0 times the in-vehicle time. In Ergün's recent study the value of walking time was estimated to be between \$4.50 and \$11.50 (8). This result is in general agreement with the findings of this paper.] Second, in spite of the large standard errors estimated for the value of time in this study, it appears that the value of time savings may not be so large as previously believed. This concerns especially the in-vehicle time. Therefore, to be realistic any economic study of a transportation improvement must consider the out-of-vehicle and in-vehicle times separately.

CONCLUSIONS

Three major conclusions of this paper are as follows:

1. The methods of estimation, commonly used in probabilistic modal-choice models, probit, logit, and discriminant analyses, all yielded comparable results. Any of them can be used with equal success.
2. At present, the modal-choice behavior of travelers appears to be only marginally influenced by the travel times and travel costs. This, in turn, implies that travel service by automobile and transit are not perfectly substitutable services and should, therefore, be modeled separately.

3. The values of travel times obtained are substantially lower than those previously reported. In particular, the value of out-of-vehicle time is much different from the value of in-vehicle time. This fact should be recognized in any economic study of a transportation improvement if travel time is given a monetary value.

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