APPROACH TO PROBABILITY DISTRIBUTION OF VALUE OF WALKING TIME AND PEDESTRIAN CIRCULATION MODELS

Jean Francois Allouche, Bureau Central d'Etudes pour les Equipments d'Outre-Mer, Paris

•TIME is money. So goes the dictum, but not until engineers began to apply economic analysis to transportation plans was there any real concern about how much money it was worth. The quantitative concept of a value of time has been used for some time but mostly by engineers trying to improve and rationalize manufacturing processes. For those analyses, the value of time can be easily taken into account through salaries, prices, and interest rates; time saved is equivalent to labor saved or additional production or shorter hours and faster capital turnover.

But the transportation system moves people—people as consumers much more than people as agents of production—and only their behavior can tell what subjective value they assign to their own time. [In most U.S. urban areas, for instance, truck trips represent about 5 percent of all vehicle trips, and business trips amount to approximately 10 percent of person movements. Work trips, preempting leisure time and not production time, do not, at least under our present social organization, fall in the "production" category (12, p. 81).] This is more than an academic problem; indeed, one of the most important quantifiable benefits in highway programs, a part of the transportation system, consists of savings in travel time.

Attempts have been made to measure the value of time, or rather the values of time. Factual research as well as psychological inference indicates that our valuation of time is influenced by a variety of factors, ranging from the types of activities we pursue to our levels of income and the amounts of time being saved. If we restrict our discussion to time spent in transportation, the perceived value of time depends on the purposes and the conditions of travel as well as on personal factors.

Most investigations performed in the past (1, 2, 3, 4, 5, 6, 7) considered the value of time as a constant. Two papers only treated the value of time as a stochastic variable. Pratt (8), using the central limit theorem approach in statistics, hypothesized that the interplay of numerous subjective factors, including value of time and inconvenience cost, would result in a normal distribution of the "catch-all index" by which individuals compare 2 travel modes (the index being 0 when a person finds both modes equally attractive). St. Clair and Lieder (9), studying a toll versus free highway situation, assumed a normal distribution for both the value of time and the inconvenience cost (as measured by the number of speed changes on each alternative route). By successive trials, they determined, for the mean and standard deviation of both distributions, values that would yield the closest approximation to observed route choices. Their approach, however, does not allow for a statistical test of confidence of their assumption. A second round of analysis performed by Thomas on the data used in his original study (7) of commuter's values aims at defining the value of time as a variable function of income and of the amount of time saved, but it is not a probability distribution. Apparently, there has been no attempt to determine directly an empirical probability distribution for the value of time.

Also, from a different standpoint, it is remarkable that most researchers have confined their investigations to the value of driving or riding time. With the recent upturn of interest toward public transit in urban areas and the growing attention given to people-mover systems, it becomes equally important to estimate the value of walking time. (People-mover systems are very short-haul facilities and can best be described as distributor systems within major activity centers such as the CBD, shopping centers, campuses, and airports.)

The present paper proposes a method that can be used to determine the actual probability distribution of the value of walking time to motorists and has no assumption as to its mathematical form. The resulting curve then either can be used directly in modeled-choice procedures or can be submitted to statistical tests in order to determine the best mathematical approximation for incorporation in a predictive model of pedestrian behavior.

THE MODEL

The model is based on the behavior of car drivers selecting a parking location before they walk to their destinations. In a first-stage approximation, we will assume that only parking fee and walking time influence the decision-maker. This eliminates factors such as weather, environmental quality, or street gradient. It is legitimate to disregard grade if the study area is reasonably level, and, except with shoppers, it is unlikely that parking decisions take much account of the environmental quality along the path followed to on-foot destination.

Formulation

We hypothesize that a driver tends to minimize his total generalized cost (money and time).

$$C = c + xd \tag{1}$$

where

C = total cost,
c = parking fee for desired parking duration,
d = distance from garage to on-foot destination, and
x = disutility cost of walking 1 unit of distance.

If a driver selects parking facility 1, parking fee c_1, and distance d_1 from his destination, rather than facilities 2, 3, . . . , N, we assume that

$$c_1 + xd_1 \leq c_j + xd_j \qquad \text{for all } j = 2, 3, \ldots, N \tag{2}$$

Three cases are possible:

1. If $d_1 - d_j > 0$, then Eq. 2 implies that

$$x \leq (c_j - c_1)/(d_1 - d_j) \tag{3}$$

2. If $d_1 - d_j = 0$, then Eq. 2 implies that

$$c_1 \leq c_j$$

i.e., if our original assumption of rational trade-offs between time and walking is correct, the driver should select the cheaper facility. But this tells nothing about x.

3. If $d_1 - d_j < 0$, then Eq. 2 implies that

$$x \geq (c_1 - c_j)/(d_j - d_1) \tag{4}$$

Each parking decision then results in a list of inequalities. The whole set of inequalities imposes on x a lower bound or an upper bound or both. An additional constraint on x is that, inasmuch as the model is valid, no negative value should be accepted for the distance disutility cost. This assumption is validated by most people's behavior: They like to park close to their destinations. Therefore, in all cases we have

$$L \leq x \leq M \tag{5}$$

or

$$L \leq x \tag{6}$$

Graphically, the argument goes as follows (Fig. 1): Total cost for a driver parking in facility i and walking to building j is

$$g_{ij} = c_i + xd_{ij} \tag{7}$$

For a given parker, j is fixed and Eq. 7 can be represented by a straight line in the coordinate system x, g. All parking facilities can be so represented. Whatever x, the disutility cost to him, the driver will consider only the minimum cost curve [PRST (F), Fig. 1].

If the driver selects facility C [line C, Fig. 1], then x, his valuation of distance disutility, must belong in the range r to s, for this is where C is the minimum cost curve. Selection of F is the case exemplified by inequality (Eq. 6). It will deserve special treatment. Selection of A would mean that the driver has a negative distance disutility cost. Selection of E is merely unaccountable by the model in its present form. E is never the minimum-cost solution and is, therefore, termed a noncompetitive facility under the circumstances. Both of these aberrant cases will be briefly discussed hereafter.

As already mentioned, a negative disutility cost of distance is at odds with the empirical observed behavior of an overwhelming majority of drivers. Negative disutility cost values or selection of a noncompetitive facility can be interpreted without throwing the model away, however. They can arise from a nonuniform distortion in distance perception on the part of the driver. If there are only a few occurrences of drivers with negative costs, we will just dismiss the datum and tally only the percentage of people falling in that category for later use in an assignment model.

The 2 aberrant cases can also arise from what we might term blurred rather than biased perception. People are probably little sensitive to small differences in distances and may, therefore, make decisions apparently at odds with the "numbers." If negative disutility cost values are observed in significant number, we propose to investigate whether it can be traced to some slackness in sensitivity to distance by defining a sensitivity threshold, for instance.

Histogram

A histogram is a statistical representation describing the number of observations falling within a certain range (a to b). These observations are represented by a rectangle, the area of which is proportional to the number of observations and one side of which is the interval (a to b) on the horizontal axis of the graph.

Figure 2 shows a distribution with 3 groups of observations: In group 1, 10 observations are between 0 and 5; in group 2, 20 observations are between 5 and 10; and in group 3, 20 observations are between 10 and 20. Group 3, being spread over an interval twice as large as that of group 2, is assigned an ordinate twice as low.

The limit of a distribution's histogram is a probability density function when intervals multiply ad infinitum and their width tends toward 0. In other words, the histogram constitutes an approximation to the probability density function after the area under its perimeter has been normalized to 1 (by rescaling ordinates).

Such a histogram could be constructed step by step from the inequalities (Eq. 5) attached to each surveyed parker. Each parker is considered as being one observation, and this procedure does not affect the final configuration of the histogram. Parkers with the same parking location and the same on-foot destination could be considered as a group at this stage, but later considerations will call for individual processing. The range for the group is assumed to be the interval defined by the set of inequalities. If, as probable, the data come from a sample survey, each observed parker has to be weighted with the appropriate factor to expand sampled data to the whole population of parkers.

The problem of parkers with an upper unbounded disutility cost of walking (inequality, Eq. 6) can be dealt with in 2 ways.

1. By estimating an absolute highest disutility cost based on the highest upper bounds observed among other parkers and based also on common sense rationales,

Figure 1.

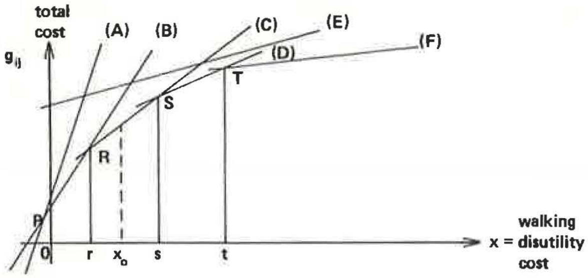


Figure 2.

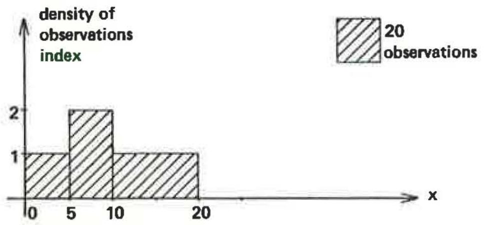


Figure 3.

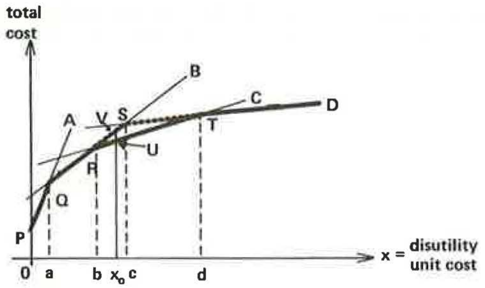
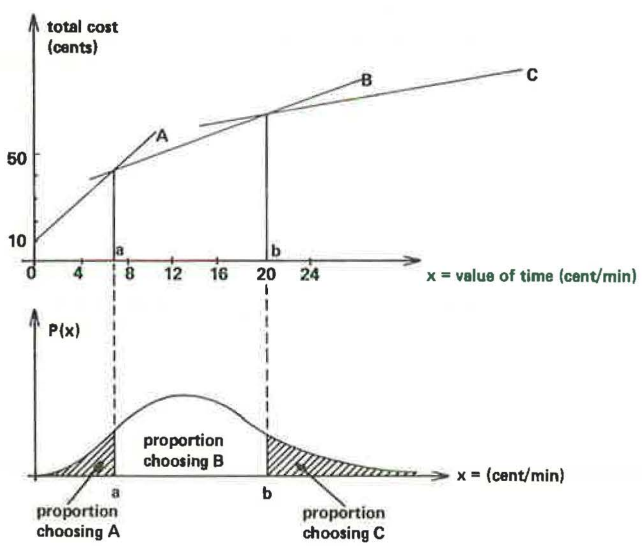


Figure 4.



e.g., a value that would make a taxi ride a preferable alternative; and

2. By calibrating a value of the upper bound that, in turn, would produce a best fit when the histogram is used for predictive purposes (this will be developed later in this paper).

Distribution of the Value of Time

In the preceding section we have proposed a method to build the distribution of the disutility cost of walking based on distance. An assessment of walking speed is necessary to derive the distribution for the value of time.

Existing studies (11) show that pedestrian travel speeds (averaged over a complete portal-to-portal walk because speeds can vary significantly from block to block during the same walk) can be considered normally distributed, with significantly different means for men (4.93 ft/sec) and women (4.53 ft/sec). Standard deviations are similar (approximately 45 percent of the value of the mean). Here again, the problem can be approached in several ways:

1. The speed is assumed to be constant and equal to the mean of the observed distributions for men and women separately, if this information is reported in the survey;
2. Walking speed is considered to be a normally distributed random variable with known mean and standard deviation and is assumed to be statistically independent of walking disutility cost; and
3. Walking speed is considered to be linearly correlated in a positive way with distance disutility cost (the rationale on which this assumption is based is developed in the brief discussion hereafter).

Regarding the first approach (constant average speed), once a unique speed has been set (possible one for each sex and purpose), multiplying the horizontal scale by that speed factor will transform dollars/ft into dollars/min, thereby yielding the distribution of the value of time. Some studies have indicated a variation of walking speed with time of day, but this seems to be related primarily to trip purpose. If warranted, the model can be applied separately to each trip purpose.

Regarding the second approach, we briefly discuss the assumption of independence between walking speed and distance disutility cost, on which it is predicated. A rapid and lively pace is often correlated with a certain liking for walking. It seems logical to assume that a great liking for walking is linked to a lower distance disutility cost. Conversely, a slow walking speed would be associated more often with a dislike for walking and a high distance disutility cost. The assumption of independence appears at best as a convenient simplification, pending a careful and much-needed test of its validity. If the kind of statistical correlation depicted above does exist but has to be overlooked for convenience reasons, the distribution of time value derived under the assumption of statistical independence will be flatter and more widespread than the "true" distribution. The computations required to develop the distribution under the assumption of statistical independence is described in the Appendix.

The rationale in favor of a correlation (i.e., the third approach) is developed in the preceding paragraph. If the first and second approaches yield unsatisfactory results, it is possible to single out some of the observed parkers and track them later to measure their average walking speed. (In a later section, tests will be suggested to evaluate the reliability of the distribution produced.) Conditional distributions of distance disutility cost could be constructed for people with given walking speeds, and their correlation with speed analyzed. This is a long and costly operation, but there is good reason to think that, if such a correlation does exist, it depends primarily on the culture or the distribution of temperamental features among the population and therefore is rather constant from place to place, at least within a culturally homogeneous domain such as the whole of North American cities. Several definitions for stability are suggested in the Appendix. At any rate, once this tedious investigation is performed, the student of different cities could dispense with this special kind of tracking survey and use only standard parking surveys for data gathering.

PRELIMINARY EVALUATION OF THE MODEL

Possibly the utmost advantage of the approach outlined in this paper is the very low cost of data collection. Practically all of the information required is available from standard parking surveys conducted at intervals in major American city center areas. Minor changes such as additional questions dealing with sex, age, or income could be accommodated at practically no cost if one wanted a stratified application of the model. Data processing would be slightly more sophisticated than for a standard parking study, but the basic program for histogram building is neither sophisticated nor very long and can be used again in different studies.

Many situations in which walking can be traded off for money in a downtown environment are of a multiple-choice nature. More than 2 modes are often available. More than 2 routes can easily be envisioned. The parking decision on which this paper is predicated also involves multiple competition as soon as there are more than 2 parking facilities open to the public. This multiple-choice character inherent to the problem thus precludes the use of discriminant analysis. On the other hand, competition among many garages is beneficial to our approach; the more facilities there are, the better our chance is to define a narrow interval containing the driver's (unknown) value of time.

The model proposed here has conceptually more explanatory power than models based on discriminant analysis, although the latter may produce relevant predictions at the aggregate level. A discriminant analysis type of model "does not specifically estimate the route choice for the individual motorist; rather, it predicts the action expected of an 'average' motorist when faced with the given route choice situation" (7 p. 59, Thomas comments about his own model only, but the statement can be applied to any model of the discriminant analysis type). On the other hand, if supplied with a decision-maker's actual value of time, our model is capable of predicting that person's decision. In our model, probabilities do not reflect people's indeterminacy or "unpredictability" (i.e., people completely and identically defined in terms of Thomas' mod formulation have to be arbitrarily assigned opposite decisions in order for his model to work satisfactorily) but rather uncertainty on our part as to their (well-defined) time valuation or other similar parameters.

Using a parking fee as the dollar element of our "total cost" function eliminates the trap of "perceived costs" in which so many previous studies have fallen when, for instance, car-operating costs are used.

We now turn to the shortcomings or difficulties we see involved in the model.

The first problem involves time perceived versus time actually spent (or, for that matter, distance perceived versus actual distance). All models share this problem. However, if the decision-maker systematically overestimates or underestimates all actual time durations by the same factor, this merely amounts to scaling down or up by that factor the value of time computed from actual duration. The revised value is the person's value for each actual unit of time and can be used sensibly for evaluation purposes. In that case, adding a question about perceived walking time in the parking survey should take care of the problem inasmuch as people are sensitive to short-time durations and capable of estimating them within a reasonably narrow range. (We refer here not to people's biases but to their own indeterminacy. It is a case of "blurred" perception.) This procedure would eliminate the need for walking speed analysis, as developed above. Both could be later synthesized in an improved version of the model, as will be discussed later. Perception biases that depend on circumstances cannot be integrated systematically in the model and will be assumed away, together with the multitude of other random influences.

The influence of the weather on the disutility cost of walking (which often takes place outdoors) does not need demonstration. Using data gathered on a rainy day would lead to the value of time spent carrying an umbrella or dodging raindrops. Although this is not uninteresting, it is suggested that the first analysis be conducted on data pertaining to moderate weather conditions. Results will be usable for a larger set of circumstances, including semicovered malls and indoor facilities.

It has been argued that short-term and long-term parkers do not have the same value of time. Although it is obvious that they do not attach the same value to parking time, is not evident that their valuations of walking time are different. A discussion in the Appendix demonstrates that, as parking duration increases, cheaper and more distant facilities become preferable. This establishes the necessity of treating each interviewed parker separately to assess his exact parking fee, either by direct questioning or by using rates and recorded parking duration. Short-term and long-term parkers could, of course, be processed separately, if so desired, and separate histograms prepared.

Drivers occupy parking facilities on a first-come, first-served basis. At certain times during the day, some facilities are saturated and the driver is faced with only a restricted supply. Figure 3 shows how this may bias the estimated range for the driver's distance disutility cost to the point where the actual value lies outside the estimated range. If all facilities were available, the minimum-cost domain is the line PQRTD, and a driver with a disutility cost x_o selects facility C. As in the earlier section, we reason that his disutility cost range is b to d. When C is removed from the supply, the minimum-cost curve becomes PQRSTD, and the driver selects facility B. If we do not know that C is out of the supply, we will interpret his decision as evidence that his disutility cost range is a to b, which is erroneous. One could keep track of facility saturation by hours of the day in the survey, but it does not seem practical at this point. Moreover, saturation is not a stable condition: C may be saturated a moment and then become open again as B becomes saturated, and so on. The bias tends to smooth out the histogram rather than change its balance. Figure 3 shows that range a to b was mistaken for the "true" range b to d. But it is so only because x_o is inferior to c. Had x_o been superior to c, the driver would have selected facility D, thereby leading us to mistakenly assume range d to $+\alpha$ instead of the true range b to d. The net result is that people who should have been distributed in the b to d interval will now be represented on either side of the histogram.

MODEL APPLICATION

The probability distribution of the value of time (or walking distance disutility cost) can be used in many applications, which can be roughly categorized in 2 groups.

1. Economic analyses estimating value of time savings accruing to pedestrians due to an improvement in pedestrian circulation, such as an overpass, a trail system, or a people mover, or evaluating alternate plans for a primarily pedestrian-oriented facility, such as a hospital, university campus, or civic center. (In case of a people mover, our curve would give a lower limit because the action of walking is considered a hardship by many, regardless of the time involved; a people mover mitigates this hardship.)
2. Pedestrian assignment models based on a total-cost-function assignment of drivers with known on-foot destinations to parking facilities (enabling a comprehensive treatment of parking schemes and pedestrian systems) and an estimation of pedestrian traffic diverted to a new facility, such as an overpass, or a people mover.

We use drivers' assignment to parking facilities as the example because it will help explain calibration procedures. Assume an average walking speed determined by the driver's physical characteristics (sex and age). Assume also 3 parking facilities A, B, and C and a group of drivers who have identical characteristics, have destinations in the same building, and are willing to purchase 1 hour of parking. Parking rates, walking distance, and time are as follows:

Facility	Parking Rate ($/hour)	Walking Distance (ft)	Walking Time (min)	Total Cost Function
A	0.10	1,250	5	$0.10 + 5x$
B	0.30	500	2	$0.30 + 2x$
C	0.50	250	1	$0.50 + x$

Figure 4 shows the minimum cost curve and the probability density function of x, the value of time, as functions of x. People with a value of time lower than a will select facility A. People with a value of time greater than b will select C. People with a value of time between a and b will select B. Their proportions are represented by the areas delimited by the probability density curve P (x), the x axis, and abcissas a and b. Knowing P (x), we can compute these proportions and can accordingly assign the group of drivers under study to the various facilities.

In summary, the inputs required for this application are the pattern of final trip destinations and parking durations; the locations of parking facilities; parking rates for each facility; and the probability density function of the disutility cost of walking, or the probability density function of the value of time, plus some information about walking speeds (average or probability distribution). All inputs but the last are currently available. The output is an assignment to parking facilities of drivers with destinations in given buildings.

CALIBRATION

So far we have mentioned only 1 degree of liberty for the model: the absolute maximum M, the value of time to be used in cases where inequalities (Eqs. 5 and 6) do not define an upper bound for x. (This section deals with value of time, but is readily applicable to the calibration of a model based on walking disutility cost.)

It is possible to define an index of performance for the model with respect to which parameter M can be adjusted. To that end, we apply the model to a parker-assignment problem in a situation where a parking survey with origin-destination data is available.

Let Y_{ij} be the number of people observed in the survey walking from parking facility i to building j (the pair i-j is called an interchange). Let X_{ij} be the number of people assigned by the model to interchange i-j. Ideally $X_{ij} = Y_{ij}$. If $X_{ij} > Y_{ij}$, then $X_{ij} - Y_{ij}$ people have been erroneously assigned, thus creating an imbalance $X_{i'j'} < Y_{i'j'}$ in another pair i′-j′. Thus,

$$I = \left(\sum_{ij} |X_{ij} - Y_{ij}|\right) \Big/ 2\sum_{ij} Y_{ij}$$

is the percentage of people erroneously assigned. (The number of pairs i-j may be very large. In some cases, it may be more practical to group interchanges by volume categories or into screen lines and apply correlation analysis to the aggregate.)

Each value of M will yield a certain value I (M) for the performance index. Trying several values of M over a reasonable range will give evidence of a trend for I (M). It can be shown that I (M) has an asymptote when M tends to infinity. This in turn will enable us to determine a value of M and to maximize I. If the fit between observed and synthesized interchange patterns is not satisfactory, another degree of liberty can be provided.

So far we have used 0 as the lower bound for ranges that lack one. But there is probably a nonzero minimum value, L, to people's value of time.

To calibrate the model with respect to M and L at the same time requires that some maximizing techniques be borrowed from nonlinear programming (such as steepest ascent or reduced gradient) so that the I (M, L) surface can be "climbed" on. They generally entail much computer time. Besides, in our problem the computation of 1 point on the surface already involves a complete assignment run plus a performance analysis (the latter is relatively inexpensive). This makes the feasibility of a double calibration dependent on the cost of running the basic assignment program.

POSSIBLE IMPROVEMENTS

Stratification of the data by trip purpose, sex, age, and income may uncover significant variations in the probability distribution of the value of time with respect to these factors. Logical considerations suggest the direction, if not the extent, of the influence of these factors on the value of time.

A joint study of walking speed v, time perception bias b, and distance disutility cost z would give information about the mutual correlations of these 3 characteristics in a given individual. Correlations between v and z and between b and v could be considered as stable among cities (within a given culture, e.g., Northern Europe or South America). They depend mainly on cultural, noneconomic features. The mathematics of a model based on such premises is developed in the Appendix.

Street grade plays a double part in decisions entailing walking. First, it increases the actual walking time, and, second, it makes walking more exhausting and thereby increases its disutility cost. In a first approximation, we may assume that this increase Δz in disutility cost is directly related to the street gradient g.

$$\begin{aligned} \Delta z &= ag && \text{if } g \geq 0 \\ \Delta z &= 0 && \text{if } g < 0 \quad \text{(walking down a street)} \end{aligned} \tag{8}$$

and total cost for a parker would now be

$$C = k\theta + tz + t(g)\,\Delta z \tag{9}$$

where t(g) is the time effectively spent walking on a street with gradient g, t is total walking time, and θ is parking duration. The ideal situation to test this model and calibrate the parameter a would be a city with several dense nuclei, one in flat terrain and another in a hilly area. A first study conducted in the flat area would provide the distribution of z. A second study in the hilly core would determine the probability distribution of a, assuming the previously derived z distribution. We are working at this time on a model that could determine directly both the z-distribution and the a-distribution.

It is interesting to note that the latter approach, if successfully developed, can be applied to any variable teamed with value of time (or distance disutility cost), provided that a scale is available for that variable. "Street attractiveness" or "environmental quality" in a flat area can be treated within this framework. The quality scale required could be based on the variations in walking speed. Hoel (11) has observed that the same pedestrian walks at varying speeds in the course of a trip, depending on the type of block he is walking along (shop windows, bank, factory, or parking lot).

CONCLUSIONS

This paper proposes a model of pedestrian behavior that can be tested in a real situation at a minimal cost, if coupled with a standard parking survey for data collection.

Most of the paper deals with how to determine the probability density distribution of the value of walking time, which is believed to be the central element for a pedestrian behavioral model applicable to a variety of situations (parking location selection, utilization of short distance people movers, or evaluation of pedestrian circulation improvements).

Valuable improvements to the model can be introduced, if necessary, by adequately designing the parking survey questionnaire.

The approach proposed allows for an incremental study design, concerned first with the value of time and then with other elements of the choice procedure such as street gradients or environmental quality.

Advantages of the model in its basic form are the low cost of data collection; its multiple-choice nature, covering a wider range of situations than binary-choice models; its "explanatory" orientation in that it proposes a rationale for pedestrian behavior instead of a more numerical correlation and remains meaningful at the level of an individual decision-maker; and its avoidance of the use of dollar costs that are ill-defined in the decision-maker's mind (such as car operating costs).

Among the model's shortcomings are the problem of time perceived versus actual time; the variability of walking speeds; the influence of the weather, which is unaccounted for; and the first-come, first-served rule of operation in the situation selected to calibrate the model that tends to distort the distribution of the value of time. Some of these drawbacks can be mitigated through investigation of particular interrelations such as that between walking speed and time perception bias, considered as permanent personal characteristics. Others are still beyond the range of analysis.

Evaluation criteria are proposed to test the model's reliability. This paper suggests that a first test be made in a medium-sized city that has a flat and rather uniform CBD. Subsequent tests can then be designed, if warranted, depending on the insufficiencies evidenced by the first one. This paper has tried to anticipate some of the problems and to suggest solutions.

REFERENCES

1. LeFevre, P. The Economic Justification of Road Construction. Road International, Summer 1956 and Autumn 1956.
2. Hummel, C. M. A Criterion Designed to Aid Expenditure Programming. HRB Bull. 249, 1960, pp. 52-60.
3. Haikalis, G., and Joseph, H. Economic Evaluation of Traffic Networks. HRB Bull. 306, 1961, pp. 39-63.
4. Warner, S. Stochastic Choice of Mode in Urban Travel: A Study in Binary Choice. University Microfilm, Inc., Ann Arbor, Mich., 1962.
5. Lisco, T. E. The Value of Commuters' Travel Time: A Study in Urban Transportation. Univ. of Chicago, Dept. of Economics, PhD dissertation, June 1967.
6. Quarmby, D. A. Choice of Travel Mode for the Journey to Work: Some Findings. Jour. Transport Economics and Policy, Vol. 1, No. 3, Sept. 1967.
7. Thomas, T. C. The Value of Time for Passenger Cars: An Experimental Study of Commuters' Values. Stanford Research Institute, Palo Alto, 1967.
8. Pratt, R. H. A Utilitarian Theory of Travel Mode Choice. Highway Research Record 322, 1970, pp. 40-53.
9. St. Clair, G. P., and Lieder, N. Evaluation of Unit Cost of Time and Strain-and-Discomfort Cost of Non-Uniform Driving. HRB Spec. Rept. 56, 1959, pp. 116-129.
10. Anderson and Bancroft. Statistical Theory in Research. McGraw-Hill, 1952.
11. Hoel, L. A. Pedestrian Travel Rates in Central Business Districts. Traffic Engineering, Jan. 1968.
12. Future Highways and Urban Growth. Wilbur Smith and Associates, 1961.
13. Oort, D. J. The Evaluation of Traveling Time. Jour. Transport Economics and Policy, Vol. 3, No. 3, Sept. 1969.

APPENDIX

DERIVATION OF DISTRIBUTION OF VALUE OF TIME ASSUMED STATISTICALLY INDEPENDENT OF WALKING SPEED

Let

v = walking speed,
t = walking time,
δ = walking distance,
x = value of time,
z = distance disutility cost,
$p(z)$ = probability density function of z among the population,
$q(v)$ = probability density function of v among the population, and
$r(x)$ = probability density function of x among the population.

$p(z)$ is assumed known through the histogram method, $q(v)$ is known from prior studies, and $r(x)$ is to be determined.

We have, by definition,

$$\delta = vt \tag{10}$$

Also, the overall cost of walking must be the same, whether computed on the basis of time or on the basis of distance.

$$xt = z\delta \tag{11}$$

Equations 10 and 11 imply that

$$x = zv \text{ for } x, z, v \geq 0 \tag{12}$$

x is the product of 2 independent stochastic variables, and its distribution is

$$r(x) = \int_{z_1}^{z_2} p(z) \cdot q(x/z) \cdot dz \tag{13}$$

z_1 and z_2, the limits of integration, are functions of the boundaries of the ranges permitted for z and v, which in turn define the range permitted for x.

If p(z) and q(v) are step functions, Eq. 13 is changed into Eq. 14.

$$r(x) = \sum_{i=1}^{N} p[(z_i + z_i+1)/2] \cdot q[2x/(z_i + z_i+1)] \quad (z_{i+1}-z_i) \tag{14}$$

where $z_1, z_2, \ldots, z_{N+1}$ are the abcissas at which p(z) jumps from one step to the next. Equation 14 is particularly relevant to the determination of r(x) by approximation, when p and q are only empirically defined, and therefore not amenable to theoretical calculus.

DEFINITION OF STABILITY OF THE RELATION BETWEEN WALKING SPEED AND DISTRIBUTION OF DISTANCE DISUTILITY COST

As mentioned earlier, there is the possibility that fast walkers are more walk-loving than average and thereby have a low distance disutility cost (or, better said, show a distance disutility cost distribution shifted toward the low values). Conversely, slow walkers would have a distance disutility cost distribution shifted toward the high values. This correlation can be formulated as follows:

$$\bar{v} = f(z) \tag{15}$$
$$\sigma_v = g(z) \tag{16}$$

where

$\bar{v}$ = mean value of the average walking speed v, and
σ_v = standard deviation of v

both for a subpopulation of given distance disutility cost z.

Equations 15 and 16 are (at least theoretically) sufficient to define the distribution of v, shown by Hoel (11) to be normal.

It is hypothesized that the correlation between v and z is stable. One definition of stability is to assume that the functions f and g are identical from city to city. But this would mean in turn that people with a given walking speed have the same average disutility cost in city A and city B. This is not obvious.

Although, it seems reasonable that people's physical characteristics (exemplified here by "walking speed") are distributed in approximately the same way in various cities, differences in social and economic conditions may influence distance or time valuations by those physically similar people located in different cities.

For instance, if wages, prices, or dividends were doubled overnight while people's preferences stayed unchanged, their distance disutility cost would also have doubled, although their walking characteristics would still be the same as before. Equations 15 and 16 would then read

$$\bar{v} = f(z') \tag{17}$$
$$\sigma_v = g(z') \tag{18}$$

where

z' = new distance disutility cost = twice old cost = 2z.

However, for everyone, disutility cost relative to the population's mean would be undisturbed by the overnight change.

$$z'/\text{mean } z' = 2z/\text{mean } (2z) = 2z/2 \text{ mean } z = z/\text{mean } z \tag{19}$$

Consequently, one way of expressing the stability of the correlation between v and z, regardless of (intercity) differences in socioeconomic conditions, is to replace Eqs. 15 and 16 with Eqs. 20 and 21, valid in both the before and after situations of the example given above.

$$\bar{v} = F(Y) \tag{20}$$
$$\sigma_v = G(Y) \tag{21}$$

where

$Y = z/\bar{z}$, and
$\bar{z}$ = mean value of z among the city population.

Taking the functions F and G to be the same in different cities is now a relatively safe assumption.

A different but somewhat similar rationale could lead to $\bar{v}$ and σ_v being functions of $Y = (z - \bar{z})/\sigma_z$; these functions, like P and G, would then be considered valid for all cities.

IMPACT OF PARKING DURATION ON TIME AND COST TRADE-OFFS

The total cost C_i to a driver selecting parking location i is

$$C_i = \theta k_i + xt_i \tag{22}$$

where

k_i = hourly rate of facility i,
t_i = walking time from facility i to on-foot destination,
θ = parking duration, and
x = value of time.

The cost differential between 2 facilities for the given driver is

$$\Delta C = C_1 - C_2 = \theta \cdot \Delta k + x \cdot \Delta t \tag{23}$$

where

$\Delta k = k_1 - k_2$, and
$\Delta t = t_1 - t_2$.

Assume that facility 1, which is optimal for duration θ_o, is competing with facility 2, which is less expensive ($\Delta k > 0$) but more distant ($\Delta x < 0$). For θ_o, ΔC is negative; i.e.,

$$\theta_o \cdot \Delta k + x \cdot \Delta t < 0 \tag{24}$$

But ΔC increases when θ increases, and when

$$\theta > -(x\ \Delta t/\Delta k) \tag{25}$$

ΔC becomes positive and facility 2 is the preferred one.

This shows that cheaper, more distant facilities become preferable when parking duration increases, if a constant value of walking time x is used. Of course, the marginal value of parking time k_i tends to decrease. This is well in accordance with observations.

DERIVATION OF DISTRIBUTION OF VALUE OF TIME, ASSUMING THAT THERE IS A CORRELATION BETWEEN WALKING SPEED AND DISTANCE DISUTILITY COST AND A PERSONALIZED SYSTEMATIC PERCEPTION BIAS

Let

v = walking speed,
T = actual walking time,
t = perceived walking time,
D = actual walking distance,
δ = perceived walking distance,
x = perceived value of time,
z = perceived distance disutility cost,
Z = actual distance disutility cost,
b = perception bias factor,
$p(Z)$ = probability density function of Z among the total population,
$r(x)$ = probability density function of x among the total population,
$q(v/Z)$ = probability density function of v among the subpopulation with distance disutility cost Z, and
$s(b)$ = probability density function of b among the total population.

The perception bias b is assumed to be statistically independent of all other personal characteristics z, v, x. The correlation between z and v is expressed through q(v/Z), as developed earlier. The distribution p(Z) is known from model application (histogram). The distribution s(b) is known from answers to a special question in the parking survey or prior studies on perceived time. Basic definitional relationships are

$$t = b \cdot T \tag{26}$$
$$\delta = b \cdot D \tag{27}$$
$$D = V \cdot T \tag{28}$$
$$xt = zd = ZD \tag{29}$$

The model described in this paper enables us to construct the histogram of Z. We now want to relate x (value of perceived time) with Z (actual distance disutility cost). Equations 26 through 29 lead to

$$x = (Z \cdot v)/b \tag{30}$$

Therefore, the probability density distributions are related as follows:

$$r(x) = \int_{Z_1}^{Z_2} \int_{V_1}^{V_2} p(Z) \cdot q(v/Z) \cdot s(Zv/x) \cdot dZ \cdot dv \tag{31}$$

As before, bounds can be imposed on v, Z, and b that will restrict the range of x. For each (permitted) value of x, the limits of integration are functions of the bounds imposed on v, Z, and b.

As shown earlier Eq. 31 can be transformed to fit the case of step functions, particularly relevant to the empirical determination of r(x).