

FREEZING AND HEAVING OF SATURATED AND UNSATURATED SOILS

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Analysis of the behavior of air-water, ice-water, and air-ice interfaces shows that the apparent contact angle, γ , between an air-ice interface and the wall of a soil pore ought to vary rapidly with changes in pore water pressure, P_w , and ice pressure, P_i . This paper gives the expected relationship and shows that cooling of fairly dry soil, with freezing nucleated at one spot, should cause certain pores to fill abruptly with ice, thus depleting the water content of the surrounding soil. This conclusion agrees with available data. It is suggested that the ice pressure is slightly greater than atmospheric pressure when pores fill. Limited data available in the literature indicate that a given moist soil may or may not heave; when it does, the ice lenses apparently form some distance behind the freezing front. To explain this, the concepts of primary and secondary heaving are proposed for saturated soil. Primary heaving occurs when the base of the growing ice lens coincides with the limit of freezing, and the rate of heaving is limited by the rate of heat extraction. Secondary heaving is believed to occur when the freezing extends below the nominal base of the (visible) ice lens. Ice in the frozen pores can move, relative to the particles, as an integral part of the growing lens. A solution model of secondary heaving is used to illustrate an increase of ice pressure from the freezing front to the base of the ice lens, which will occur at a point where P_i is equal to the overburden pressure. It is contended that secondary heaving produces larger heaving pressures than primary heaving and explains why previous theories underestimated the maximum heaving pressure of saturated soil. It is inferred that whenever heaving occurs as the freezing front is descending through the soil, the process must be secondary heaving. It is suggested that the development of significant heaving pressures by unsaturated soil will ordinarily involve secondary heaving and that the pressure developed is less than that developed by the same soil when it is saturated.

•RELATIVELY little attention has been given to freezing and heaving of moist soils. An earlier paper (3) reported the failure of a hypothesis to explain heaving in unsaturated soils, which leads to a new effort to understand the results along with similar results subsequently obtained by Hoekstra (8). Pursuit of this question has led to a conclusion, not as yet found elsewhere in the literature, that the apparent angle of contact between an air-ice interface and a soil particle is a variable that depends on temperature and pore water pressure. From this, it can be understood that freezing of unsaturated soils involves a process that causes some pores to abruptly fill with ice.

In relating these conclusions to limited experimental data, a concept was encountered that suggests that the current model of frost heaving in saturated soils is too simple as a general model. It is suggested that the simple model should be called primary heaving and that this is superseded by secondary heaving as the maximum heaving pressure is developed, or when the frost line is descending through the soil. It is suggested that the development of significant heaving pressures in unsaturated soils probably involves secondary heaving.

DEFINITIONS AND ASSUMPTIONS

The following are assumed to be sufficiently accurate for the purposes of this paper.

1. The freezing point of water, in the absence of free solutes, obeys a general form of the Clapeyron equation. When the deviation of the freezing point, ΔT , from 0 C is not too large,

$$\Delta T = [(P_w/\rho_w) - (P_i/\rho_i)]/(\Delta H_f/T) \quad (1)$$

where P_w is the (gauge) pore water pressure (negative in unsaturated soils), P_i is the (gauge) ice pressure, ρ is the density of the indicated phase, ΔH_f is the heat of phase transition, and T is the absolute temperature. The pore water pressure is as it would be as measured by a Tensiometer and does not represent the actual fluid pressure within an adsorbed water film.

2. The Kelvin equation for the pressure discontinuity at a curved phase boundary may be written for an air-water interface as

$$P_w - P_a = 2\sigma_{aw}/\bar{r}_{aw} \quad (2a)$$

where P_a is the pressure in the soil air, hereafter taken to be zero (gauge), σ_{aw} is the surface tension of the air-water interface, and \bar{r}_{aw} is the mean curvature of the interface, taken to be positive when the curvature is centered on the water side of the interface.

For an ice-water interface,

$$P_i - P_w = 2\sigma_{iw}/\bar{r}_{iw} \quad (2b)$$

where the mean radius of curvature is taken to be positive when centered on the ice side of the interface.

For an air-ice interface,

$$P_i - P_a = 2\sigma_{ai}/\bar{r}_{ai} \quad (2c)$$

where the mean radius of curvature is taken to be positive when centered on the ice side of the interface. It will be assumed that the curvature of the air-ice interface in soil pores changes rapidly through sublimation or condensation when there are exposed areas of adsorbed film water very close by.

3. When a soil particle has an adsorbed film of liquid water, it will be assumed that the air-water interface will always meet the film at an angle of 0 deg. It will be assumed that an air-water interface will always meet an air-ice interface at an angle of 0 deg. Finally, it will be assumed that an ice-water interface will always meet an adsorbed film at an angle of 180 deg. The first assumption is commonplace. The second seems plausible, if not inevitable. The third is consistent with experimental results if the others are true (9).

4. It will be assumed that $\sigma_{aw}:\sigma_{iw}:\sigma_{ai} = 2.20:1:3.20$. The relationship between σ_{aw} and σ_{iw} is based on the experimental results of Koopmans and Miller (9). The relationship with σ_{ai} follows from the assumed contact angle between the air-water and the ice-water interfaces. If the air-water interface had had its handbook value for 0 C in the experiments cited, the respective values would be related as 72.3:33.1:105.4. If the real value of σ_{aw} was some 5 or 10 percent lower, as it probably was in the absence of extraordinary precautions in the soil system involved, then the other values should be reduced by the same factor, thus keeping the ratios the same.

APPARENT CONTACT ANGLE OF THE AIR-ICE INTERFACES WITH A SOIL PARTICLE

Given the assumptions in the preceding section, the expected configuration of the junction of the surface of mineral particle (having an adsorbed film) with an air-ice interface can be constructed as a function of P_i and P_w or of P_w and T as they are related to P_i in Eq. 1. Three examples are shown in Figure 1. The first is constructed

to give an apparent angle of contact, γ , of 90 deg, and it is seen that the ice pressure must be intermediate between the air pressure and the pore water pressure, i.e., less than atmospheric pressure. The second is constructed for equality of air and ice pressures, for which $\gamma = 68$ deg. The third is constructed for radii of curvature for air-water and air-ice interfaces that are of the same relative magnitudes as the first example, but where the sign of the air-ice curvature is reversed; i.e., the ice pressure is greater than atmospheric pressure, and the angle γ is 57 deg.

These constructions are for two-dimensional surfaces, for which the mean curvature is twice the radius of a circular cross section shown in the figure. From inspection of the figures it can be shown that

$$\gamma = \cos^{-1} \left[\frac{|\bar{r}_{aw}| - |\bar{r}_{iw}|}{|\bar{r}_{aw}| + |\bar{r}_{iw}|} \right] \quad (3a)$$

or, with substitutions from Eqs. 2a, 2b, and 2c,

$$\gamma = \cos^{-1} \left[\frac{(P_i/P_w) - (1 - \sigma_{iw}/\sigma_{aw})}{(P_i/P_w) - (1 + \sigma_{iw}/\sigma_{aw})} \right] \quad (3b)$$

In Eq. 3a, absolute values of \bar{r} are used to avoid confusion arising from sign conventions. Equation 3b is shown graphically in Figure 2.

AIR-WATER-ICE EQUILIBRIUMS IN PORES

Haines (7) described the accepted model of drying and wetting of soils in which the relationship between water content and P_w is controlled by pore geometry and surface tension of the air-water interface and by the history of wetting and drying. In principle, a similar exercise could be performed with the air-water-ice system for freezing and thawing of pore ice in unsaturated soil. This is a much more difficult task because of the additional phase and the variable contact angle for the air-ice interface and soil particles as described in the preceding section. At this time, only some general conclusions and relationships will be mentioned.

Inspection of Figure 1 shows that air and ice can coexist in a plane-parallel or cylindrical pore only if $\gamma = 90$ deg, $\bar{r}_{iw} = -\bar{r}_{aw} = d/2$ where d is the pore diameter, and the air-ice interface is reduced to a single point that the other two interfaces also have in common. [The interfaces (Fig. 1) in symmetric 2-dimensional pores can be visualized by laying the edge of a vertical mirror across the figure and rotating until the air-ice interface and its image make a smooth curve.] With the exception of this unique combination, the pore must be filled with ice, water, or air, depending on P_w and T .

Consider next a V-shaped pore (two dimensional) with an apex angle, θ , of $2 \times (90 \text{ deg} - 68 \text{ deg}) = 44$ deg (Fig. 3). For convenience, pores with apex angles larger than 44 deg will be referred to as wide pores, whereas those with apex angles smaller than 44 deg will be referred to as narrow pores.

The 44-deg pore is unique because, whenever P_w and T are such that $P_i = 0$ in Eq. 1, the air-ice interface will be flat and will intersect the respective pore walls at 68 deg without regard to how full the pore is. Thus, the width of the interface does not enter into the determination of permissible combinations of P_w and T at which air, ice, and water can coexist in the 44-deg pore.

If θ of a V-shaped pore is other than 68 deg or 0 deg, there will be a unique position of the interface in the pore determined by \bar{r}_{ai} and γ . In a narrow pore ($\theta < 44$ deg), the ice pressure can never equal or exceed atmospheric pressure when air is present in the pore. Conversely, in a wide pore, the ice pressure can never equal or be less than atmospheric pressure if air is present. Thus, in unsaturated soil, ice cannot form in a wide pore until all narrow pores have filled with ice, if freezing is nucleated in each.

Figure 1. Junction of (2-dimensional) air-ice, ice-water, and air-water interfaces at a surface having an adsorbed film of water.

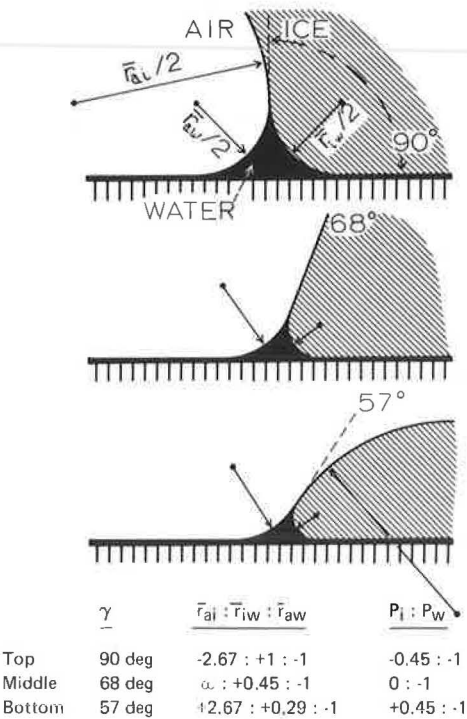


Figure 2. The apparent angle of contact, γ , of an air-ice interface with a film of water adsorbed on a particle as function of ice pressure and pore water pressure.

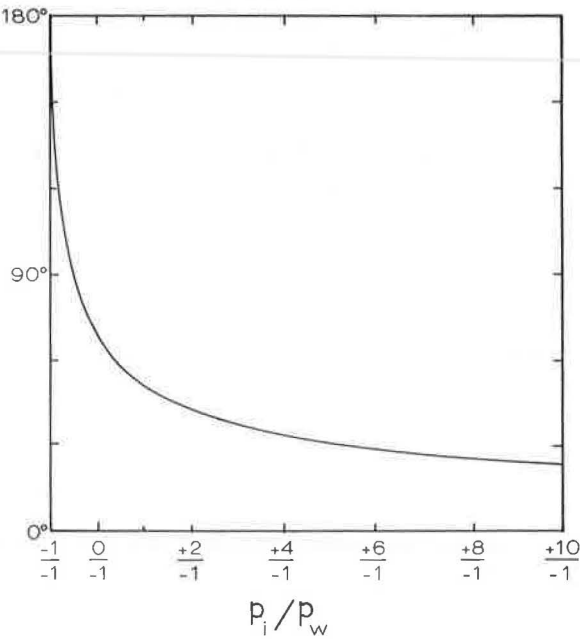
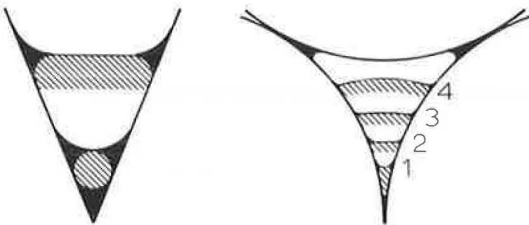


Figure 3. Air, ice, and water in pores. Left: Two of an infinite number of positions possible in a 44-deg pore when $P_i = 0$. Right: Stages of growth of an ice wedge as T decreases; P_w held constant.



Inspection of Figure 1 (with the aid of a mirror) shows that a contact angle of $\gamma = 57$ deg cannot be achieved in a V-shaped pore if $\theta < 2 \times (90 \text{ deg} - 57 \text{ deg}) = 66 \text{ deg}$, which is to say that the ice pressure in a 66-deg pore cannot exceed that which satisfies the ratio $P_i: P_w = +0.45: -1$. If we put this in more general terms, $\gamma \geq (90 \text{ deg} - \theta/2)$, and the limit of the ratio $P_i: P_w$ is as

$$\left[1 + \left(\frac{\sigma_{iw}}{\sigma_{aw}} \right) \frac{(\sin \theta/2) + 1}{(\sin \theta/2) - 1} \right] : [-1] \quad (4)$$

If the equilibrium conditions in the critical pore ($\theta = 44 \text{ deg}$) are perturbed, the results can be anticipated. If T is lowered (and the water content is held constant), P_i and γ must remain constant, and P_w must decrease. If T is lowered slightly (and P_w is held constant), the ice phase will diminish and disappear, leaving only water appropriate to the imposed P_w .

In a narrow V-shaped pore ($\theta < 44 \text{ deg}$), we can flatten the concave air-ice interface and decrease γ by lowering the temperature while holding P_w constant. Both effects require the pore to fill with ice until the width of the interface is sufficient to satisfy the new curvature and contact angle, at which point the interface will come to rest.

In a wide V-shaped pore, we can increase the convexity of the air-ice interface and decrease γ by lowering the temperature while holding P_w constant. Again, both effects will be satisfied by increasing the volume of the ice phase to some new rest position of the interface. In both, a small change in temperature may have a large effect on the volume of ice formed.

The generalized shape of a soil pore may begin with a very narrow angle at the point of contact between two particles, with the θ increasing rapidly away from the point of contact; however, the angle of the walls converges again on entering the open space between several particles. In a pore that diverges with distance away from a particle contact point, there is a transition from a narrow pore to the critical 44-deg angle to a wide pore.

Consider a pore with low water content, in which the air-water interfaces (before cooling) lie deep in the crevices (in the "narrow" region). Assume that the pore water is in equilibrium with water in a much larger body of soil such that the changes in P_w in one pore are buffered by the surrounding soil. Let the soil be cooled until stable ice can be formed in one water wedge, which is isolated from its neighbors and interfaces with them only via adsorbed films and the vapor phase, such that the nucleation of ice at the desired spot does not nucleate freezing elsewhere. As the temperature is gradually lowered, the ice wedge will grow (Fig. 3), and the ice-water interface rises in the crevice. The ice pressure will rise (assuming P_w stays constant) toward atmospheric pressure and beyond. However, at some point, any of three events may occur that will cause the entire pore to rapidly and irreversibly fill with ice: The periphery of the growing ice wedge may reach a neighboring water wedge, nucleating freezing there; the air-ice interface may encounter the surface of a particle on the far side of the pore; or the residual air space may become so narrow that it fills by a Haines jump. Both of the first two events are equivalent to "narrowing" the pore in terms of the divergence of the particle surfaces intersected by the air-ice interface, and, if the temperature is held constant, the pore must fill with ice. In the third, the water imbibed will promptly freeze. Because ice will reach atmospheric pressure in a two-dimensional wedge where the divergence of the walls is only 44 deg (and still deeper in a wedge between two spherical particles—where another component of curvature exists), it seems almost certain that the ice pressure will exceed atmospheric pressure before most pores can fill with ice. It appears, however, that most pores will fill at an ice pressure only slightly greater than atmospheric pressure.

The abrupt filling of a pore will cause an abrupt drawdown of P_w in the surrounding pores and a local rise of temperature. Both have the effect of reducing P_i until P_w and T return to ambient values, which delays the filling of neighboring pores with ice that is nucleated by the freezing of the first pore. As conditions return to normal, the

surrounding pores will follow the same sequence and will fill. The net result, then, is a growing body of ice-filled pores surrounded by ice-free pores that are being depleted of water. This expectation agrees with experimental observations (3, 8, 9).

If the soil is fairly wet at the outset such that water wedges are already in contact, it would seem that ice could propagate ahead of the ice-filled pores; however, as pores to the rear of the freezing front fill with ice, the drawdown will shift conditions toward those described for the freezing of a drier soil and the freezing front will be overtaken by the zone of filling. The freezing of fairly wet soils has not been visualized in detail.

It is to be expected that some pores will be bypassed without filling. The bypassed pores remain as sinks of water, buffering restoration of P_w . When a column is frozen from one end, however, the pores should eventually fill except that isolated air bubbles may be cut off.

A separate exercise, which must focus on the pore necks, is required to trace the thawing of an ice-filled pore in unsaturated soil. This will not be discussed here except to remark that again "jumps" involving abrupt melting and emptying are to be expected and that they occur at higher T (or lower P_w) than the filling jump so that there will be pronounced hysteresis effects.

The above analysis apparently differs from one offered by Globus (5). He evidently saw the phenomenon as one in which the appearance of ice grains in the voids shifted the pore size distribution downward (while lowering liquid water content), thereby greatly reducing the mean curvature of air-water menisci, which lowers P_w drastically.

HEAVING OF UNSATURATED SOILS

When an ice wedge exceeds atmospheric pressure, it will tend to force the adjoining particles apart. Beskow (1) probably observed this effect in his account of the expansion of (unloaded) moist sand on freezing. He attributed expansion to the formation of columnar ice at particle contacts. In his experiments, pore filling evidently did not occur, but it might not be expected in sand unless cooling is slow enough to permit translocation of water.

The qualitative account suggests that pore filling probably occurs when the ice pressure exceeds atmospheric pressure. If all pores are filled at the freezing front, heaving should ensue if the overburden pressure does not exceed the pressure for pore filling (primary heaving, to be described later). If very many of the pores are bypassed, their tendency to contact will be in conflict with the tendency of ice-filled pores to expand. Therefore, the net effect may not produce sufficient thrust to overcome the overburden pressure, even if it is only the weight of the frozen soil itself, until the fraction filled reaches a critical value.

Dirksen and Miller (3) found that, when columns of moist New Hampshire silt were frozen from one end with a relatively small overburden pressure (about 0.04 kg/cm^2), heaving began whenever conditions were such that the degree of ice saturation of the pores reached about 0.9 in the zone of active accumulation behind the freezing front.

In somewhat similar experiments, which unfortunately precluded heaving, Hoekstra (8) apparently observed ice lens formation at about the same degree of pore saturation. For lenses to form in his system, it was necessary for the soil to consolidate; therefore, the pressures that developed may have been fairly high.

Neither experimental technique could reveal precise water and ice content profiles close to the freezing front. Dirksen (13) and Hoekstra (8) both inferred, however, that identifiable ice lenses formed slightly behind the freezing front. A photograph contained in Dirksen's paper shows many ice lenses in the frozen soil but none in a zone about 3-mm thick located behind the freezing front as the column approaches a steady state. This evidence is inconclusive, however, because the sample was removed and photographed some hours after the rate of heaving had dwindled below the threshold of measurement.

PRIMARY AND SECONDARY HEAVING

The current model of frost heaving in saturated soils is shown in schematic form in Figure 4, which includes pressure profiles sketched on the right (neglecting hydro-

static components) as if heaving were in progress and as if the unfrozen soil offered very small impedance to water flow. Heaving in accord with this model will be called primary heaving. The identifying feature is that the base of the growing ice lens coincides with the limit of freezing. It is inferred (4) that lens growth ceases when the conditions diminish the mean radius of curvature to some critical value, $(\bar{r}_{lw})_{cr}$ at which the interface penetrates the pore neck, so that the pore water freezes in situ. Thus, the maximum heaving pressure expected (from primary heaving) is

$$(P_t)_{max} = P_w + 2\sigma_{lw}/(\bar{r}_{lw})_{cr} \quad (5)$$

Although tests of this equation have been described as "encouraging" in samples where the range of neck sizes is small (4), it is not clear what happens when there is a large range of neck sizes. Penner (12), using samples with fairly narrow particle size distributions, concluded that the critical pore neck (as computed from Eq. 5 for his data) was smaller than the size of most of the soil pores when "pore sizes" were computed from desorption curves. One suspects from his data that all pore necks were larger than the computed critical radii and that the residual water involved was "wedge water." Penner did not consider this possibility but instead visualized an ice lens with an irregular front that therefore existed at irregular temperatures (owing to the temperature gradient). Because of this, the ice lens always terminated at pores small enough to block further ice penetration at the local temperature. It will now be suggested that a different situation was more probable.

Suppose that the ice does penetrate the pore system for some distance beyond the base of the nominal ice lens as shown in Figure 5. It is suggested that in such circumstances the nominal ice lens may continue to grow and that the ice in the frozen fringe will move with it, while the particles remain stationary. This phenomenon will be called secondary heaving.

The author has shown (10) that ice that fills a large pore between two filter papers moves readily despite appearing to be stationary. When supercooled water is on the opposite sides of the respective filters, an increase of the water pressure on one side will cause supercooled water to emerge from the opposite side. Concurrent melting and freezing on opposite sides of the pore causes the ice to be in motion between its stationary boundaries, which are the unfrozen films adsorbed on the filter paper and the ice-water menisci in the filter pores. Thus, in principle, the ice in the frozen fringe could also move as an integral part of the ice body that includes the nominal ice lens. Water traverses the ice-filled pores in the ice phase and goes around the obstacles in the film phase.

The next troublesome question is the alleged failure of the particles to move with the moving ice. It is suggested that there is a pressure gradient in the ice whenever it is in contact with unfrozen films and whenever a temperature gradient exists (i. e., in the frozen fringe below the nominal ice lens) and the ice pressure increases as the temperature decreases. When the ice pressure reaches the overburden pressure, the intergranular stress reaches zero and particles will move with the ice; the base of a nominal ice lens will appear at this level. When the ice pressure is less than the overburden pressure, part of the load must be borne by the particle framework, and the particles will remain stationary. Before this contention can be accepted, it is necessary to explain how there can be a pressure gradient in the ice in the frozen fringe.

SOLUTION ANALOGS OF FROST HEAVING

Because the dynamics of adsorbed films are not easily understood or agreed on, the author has found it useful, on occasion, to think in terms of an equivalent system that involves familiar and well-understood concepts (2, 11). Heaving can be simulated by using a mental analog that employs an osmometer. The simple analog for primary heaving is shown in solid lines in Figure 6. An ice lens, in the form of a weighted piston, rests on a thin layer of aqueous solution. The ice pressure is fixed by its own weight and the load; the solution must be at the same pressure. The solution rests on a semipermeable membrane in contact with pore water at a lower pressure, P_w . The pressure difference (at equilibrium) will equal the osmotic pressure, π , of the solution

$(P_i - P_w = \pi)$. At equilibrium, the temperature of the ice-water interface, the ice pressure, and the pore water pressure below the membrane must satisfy Eq. 1. The vertical tube, S, simulates the unfrozen soil, its impedance to water flow, and the distance to the water table. If the temperature is lowered, water in the solution will be converted to ice and added to the base of the ice piston, the concentration of the residual solution will rise (unbalancing the osmometer), and pore water will be absorbed by the solution (osmosis), lifting the piston. If the temperature is held at the new level, ice growth will continue until the added weight of ice increases P_i to the point where Eq. 1 is again satisfied. Note that the pore water pressure, as measured by a Tensiometer, is not the solution pressure but is the water pressure beneath the membrane. The impedance to water movement represented by the membrane and the solution simulates the impedance offered by the unfrozen films shown in Figure 4.

To simulate secondary heaving requires that the analog be extended by adding a second stage (broken lines, Fig. 6). The lower stage now simulates conditions at the lower limit of freezing, and the upper stage simulates conditions at the upper (colder) extremity of continuous unfrozen films in the frozen fringe (simulated by the vertical tube, F), which terminate at the base of the nominal (visible) ice lens. At equilibrium, Eq. 1 must be satisfied at both levels, but the upper level is colder. At equilibrium, the maximum ice pressure at each level will have been reached, and this will be greater in the upper stage. At equilibrium (a steady state of heat flow upwards), there is no water movement; therefore, P_w should reach its hydrostatic value based on only the elevation above the water table (if second order effects such as thermo-osmosis are neglected). Consequently, we conclude that the ice pressure increases as the temperature decreases so long as the ice is in local equilibrium with the continuous film phase extending downward to the unfrozen soil. If the system is not in a steady state and heaving is in progress, there must be a hydraulic gradient across the frozen fringe, and P_w will decline more rapidly with elevation so that ice pressure will not rise as rapidly with decreasing temperature as in the static case.

The significant result is the conclusion that the maximum heaving pressure that can be developed during secondary heaving depends only on the temperature ΔT_o at the base of the nominal ice lens (the upper limit of the continuous films) and on the pore water pressure, $(P_w)_o$ (obtained from the hydrostatic equation for the elevation of the base of the nominal lens).

$$(P_i)_{\max} = \rho_i [(P_w)_o / \rho_w + \Delta H_f \Delta T_o / T] \quad (6)$$

Confirmation of this conclusion may not be easy because it may be difficult to identify the position (and temperature) of the base of the lowest nominal ice lens, and attainment of equilibrium may be extremely slow and may require sensitive instruments and great patience.

"Tertiary" heaving could occur, in which the lowest lens gradually disappears, accompanied by the growth of an overlying lens at still lower temperature. Development of tertiary heaving pressure should not begin until the lower lens has disappeared. The ultimate maximum heaving pressure could turn out to be a constant that corresponds to the temperature at which unfrozen films disappear (or cease to conduct water) or the surface temperature if it is above this limit. It seems unlikely that the ultimate heaving pressure will be established experimentally in a reasonable length of time unless the rates of equilibration are faster than expected.

Because impedance to flow of water through the frozen fringe limits the rate of secondary heaving, increasing the temperature gradient will reduce the width of the fringe and its impedance. This suggests that the rate of secondary heaving would turn out to be inversely proportional to the thickness of the fringe, i.e., directly proportional to the temperature gradient in the fringe, an effect not to be confused with thermo-osmosis. It may be, however, that the heat conductivity of the frozen fringe may be the limiting factor in water transport in the frozen fringe (10), leading to the same result.

Figure 4. Primary heaving in saturated soil.

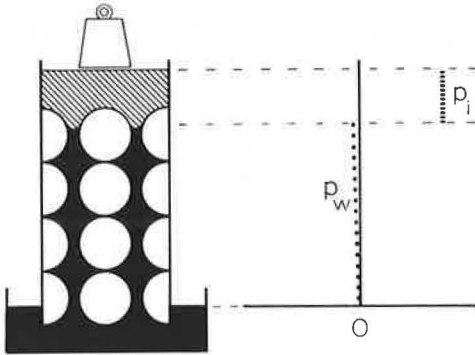


Figure 5. Secondary heaving in saturated soil.

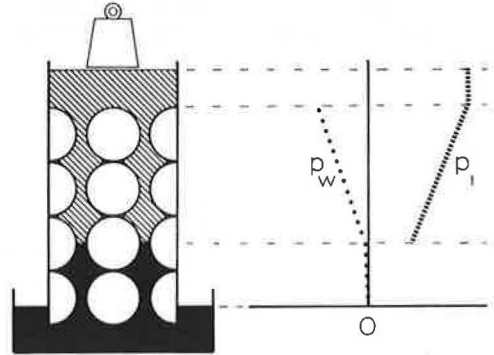


Figure 6. The solution (osmometer) analog of primary frost heaving (solid lines) with an added stage (dashed lines) to simulate conditions of secondary heaving.

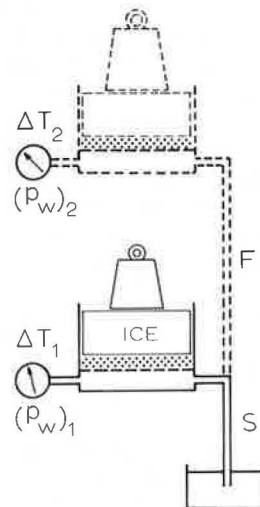
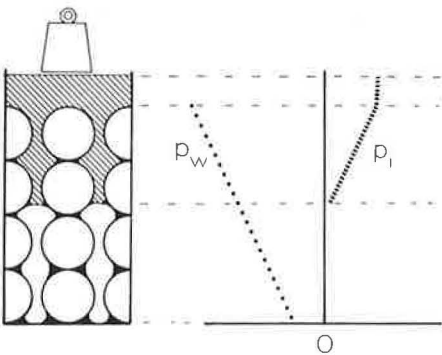


Figure 7. Secondary heaving in unsaturated soil.



HEAT OR WATER CONDUCTION LIMITATIONS ON RATE OF HEAVE

If, as proposed, primary heaving is identified with the absence of pore ice below a growing ice lens, the frost line will remain stationary because it is highly improbable that it will ever be cold enough below the lens base for spontaneous nucleation of a new ice lens at a lower depth. In other words, the rate of primary heaving should be directly proportional to the net rate of heat extraction at the frost line; i.e., heat extraction is the limiting process in determining the rate of primary heaving.

When transmission of water to the lens is the limiting process and the frost line descends with formation of successive ice lenses, the appearance of new lenses must signify at least some degree of ice penetration of the pores beneath the lowest visible lens. Hence we would infer that, whenever rate of heave is limited by water conduction and the frost line is progressing downward, secondary heaving is taking place. If this is true, secondary heaving is neither rare nor necessarily slow.

SECONDARY HEAVING IN UNSATURATED SOIL

Secondary heaving in unsaturated soil is shown in Figure 7 as if it were in progress. It differs from secondary heaving in saturated soil in that the frozen fringe terminates at an air-ice interface instead of an ice-water interface. During active freezing, the lowest water content will occur in the ice-free soil next to the freezing front. If a steady state is approached, with the temperature gradient becoming stationary, the water content gradient will ultimately reverse because of the effect of the temperature gradient in the unfrozen soil once active adsorption of water by the frozen soil has ceased. Distillation of water from the warm to the cold end of the unfrozen soil sets up a circulatory motion of the water, which returned toward the warm end in the liquid phase (6). Thus, P_w at the base of the frozen fringe depends on a more complex phenomenon than the position of the water table in the saturated soil. The maximum heaving pressure may be transient if P_w at the freezing front continues to decline as a steady state is approached. The ultimate steady-state heaving pressure developed by secondary heaving in unsaturated soil depends on the temperature at the base of the lowest nominal ice lens and on the P_w that can be sustained at the contact between the frozen fringe and the unfrozen soil in a steady state. It is anticipated that increasing the temperature gradient will increase this P_w , as will increasing the length of the unfrozen soil.

The maximum heaving pressure that can be developed by unsaturated soil will be less than that which can be developed in saturated soil but may, nevertheless, be sufficient to have important engineering and agricultural consequences. In fact, it is probably a far more extensive source of problems in agriculture than is heaving of saturated soils.

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