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DEVELOPMENT AND IMPLEMENTATION OF A PARKING ALLOCATION MODEL

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This paper develops the underlying assumptions of the parking allocation model and describes the results of its calibration and application in a case study. This version of the model incorporates three of the basic variables influencing parking choice: cost, walking distance, and capacity constraint. The model is embedded in a linear programming context that uses a disutility concept to combine the effects of the trade-offs among cost, distance, and other variables. Parkers arriving during a given time period are allocated such that their joint disutility is minimized, subject to capacity and demand constraints. In general, the performance of the parking allocation model in its first operational application is encouraging. It does replicate the distribution pattern of parkers among facilities and the facility totals. Initial testing suggests that the model captures the dynamics of the parking project. Based on these results, further careful pilot applications are warranted.

•A SYSTEMS ANALYSIS of parking was structured by the authors in a previous paper (1). This parking analysis framework hinges on a parking allocation model (PAM) that simulates the choice of a parking facility by a trip-maker. The objective of this paper is to report on the experience gained in applying PAM in an initial case study.

For purposes of this discussion, it is necessary to situate this model into the broader context of the urban transportation planning process. When we recognize the simultaneous nature of the urban travel process, it is nonetheless necessary, at the present time at least, to assume a sequential process to simulate the urban travel phenomenon. In this context, the parking analysis described in this paper follows modal split and, ideally, precedes assignment. Hence, the parking analysis process assumes a fixed stock of automobile trips; the interaction between the cost and inconvenience of parking on the one hand and the demand for various modes of transportation on the other should be taken into account as part of the modal-split analysis. In other words, aggregate parking demand at a given final destination is explicitly assumed to be an exogenous input to PAM.

The purpose of PAM is to accept a stock of automobile trips to a final destination and to allocate these trips to a set of parking facilities. Given this fixed-demand context, each parker would ideally want to park at his final destination and do so at no cost. As a matter of fact, this is what happens in low-density residential areas or even in the CBD of a small community. However, in higher density centers, it is obviously not possible for each parker to achieve these ideal conditions. Thus, the concept of competition for the available parking spaces is introduced in the analysis. When the competition reaches a threshold level, parking spaces are no longer "free," and either a time limit or a price is imposed.

In this competitive environment, parkers must make choices among alternative facilities characterized by attributes such as (a) the total out-of-pocket cost of parking; (b) the spatial separation between the parking facility and the final destination; (c) the service provided by the facility in terms of waiting time, safety of the user, and protection of his vehicle; (d) the location of the parking facility with respect to the travel routes; and (e) the likelihood of finding a space. It can be hypothesized that, when selecting a facility, a user implicitly or explicitly trades off among these, and perhaps other, attributes and, hence, assigns some disutility to each of the facilities that he considers.

Each parker will attempt to minimize his own disutility. To simulate such a process in which the available choices change as each parker is allocated would theoretically require that a sequential order be assigned to each parker and that the process be repeated until all parkers are allocated to a facility. Changes in the available choices occur when the capacity of a facility is reached. To make the allocation algorithm computationally tractable (in terms of running time) requires some approximation of the process. It can be assumed that a joint disutility minimization performed over a relatively short period of time sufficiently approximates the individual disutility minimization that would occur over the same time span. In other words, parkers are grouped within a given arrival period and assigned simultaneously to parking facilities in a way that minimizes their joint disutilities. For a given time period of arrival a, this can be stated mathematically as follows:

$$\text{Minimize} \sum_j \sum_k \sum_q \sum_d Z(j,k,q,d) \times X(j,k,q,d)$$

subject to

$$\sum_j \sum_q \sum_d X(j,k,q,d) \leq s(a,k) \qquad \text{for each } k$$

$$\sum_k X(j,k,q,d) = T(j,q,d) \qquad \text{for each } (j,q,d)$$

where

- j = index identifying a zone of final destination;
- k = index identifying a parking facility;
- q = index identifying a group of parkers (by purpose or income or both);
- a = index identifying a time period of arrival;
- d = index identifying a time period of departure ($d \geq a$);
- X(j,k,q,d) = number of parkers arriving at time period a, departing at time period d, belonging to group q, destined to zone j, and allocated to facility k;
- Z(j,k,q,d) = disutility of each of the parkers;
- s(a,k) = supply (number of spaces) available at time period a in facility k; and
- T(j,q,d) = number of parkers belonging to group q, destined to zone j, arriving in period a, and departing in period d.

(This formulation assumes that parking duration is not subject to any restriction. Otherwise, the supply constraints must be slightly modified.)

A CASE STUDY

Pittsburgh, Pennsylvania, was chosen as the case study for pilot implementation of PAM. This city was chosen because (a) it is a medium-sized city; (b) it has a well-defined CBD, a feature that facilitated this initial analysis; and (c) an acceptable data base for a parking system analysis was available. A home-interview study was performed in 1967, and a parking study, consisting of an inventory, occupancy counts, and curb-side interviews, was performed in 1969 (2).

This paper will focus on the allocation of long-duration work trips for which it is assumed that workers arrive during a single period and that their durations are stratified in three categories: between 7 and 8 hours, 8 to 9 hours, and longer than 9 hours. This initial focus on the long-duration work trip is logical inasmuch as parkers for this purpose constitute from 40 to 50 percent of the total parkers in cities of greater than 500,000 population, and they consume over 70 percent of the total space-hours. Further, the dynamics of the early-arriving work-trip parkers has significant impacts on the personal business and shopping parkers who generally arrive at later time periods.

The study area (Fig. 1) was divided into 116 zones in which 74 parking facilities are open to the public. Curb parking was not considered in the analysis inasmuch as it represents a very small fraction of available space and is subject to time restrictions.

The data base required to calibrate and validate PAM involved demand data, supply data, and CBD network data. Demand data were obtained from the curb-side interview of parkers; information was obtained on the number of parkers in each facility by final CBD destination, trip purpose, and parking duration. Information on the socioeconomic characteristics of the drivers, the number of people sharing the parking cost, and the arrival time at the parking facility was not available in the survey. Supply information, including the parking rate structure and facility capacities, was available from the inventory work sheets. A detailed CBD walking network was coded in which each zone was represented by a centroid located in the middle of the zone and connected to the street network by four "dummy" walking links. Interzonal walking distances were estimated by skimming this network, whereas intrazonal walking distances were manually estimated. Information on waiting times at the parking facilities was not available; however, the three major factors influencing parking choice, i.e., parking cost, walking distance, and facility capacity, were available in the calibration data set.

DEVELOPMENT OF THE CALIBRATION DATA SETS

The joint distribution for parking cost and walking distance for work trips with a duration of 7 hours or longer is given in Table 1. Although, as might be expected, there is considerable dispersion in the data, a definite trend of decreasing parking cost with increasing walking distance is evident. This dispersion can be attributed to the following factors:

1. The data displayed cover a relatively large geographic area.
2. Low-cost parking facilities are located closer to the final destination in the periphery of the CBD than they are in the central portion.
3. Although not explicitly indicated in the survey, this joint distribution is directly influenced by the effects of available supply, i.e., capacity constraints. For example, in the core of the CBD, there are cases when the closest available facility is about 1,000 ft or more from the final destination, and the cost for this facility is still quite high.
4. Other factors such as the approach route may influence the choice of a parking facility.
5. Consumers generally lack full information concerning the available choices.

The trade-off between walking distance and parking cost is most acutely faced by those destined for the core area of the CBD, as shown in Figure 1 by Liberty Avenue, Grant Street, and Boulevard of the Allies. Plotting the joint distribution of cost and distance only for those parkers whose final destinations are within this triangular area (Table 2) reduces the dispersion and accentuates the relation between cost and distance.

To further reduce the scattering, we stratified the data by intervals of 200 ft for distances up to 3,000 ft and intervals of 500 ft for distances greater than 3,000 ft. The average parking cost for each of the distance intervals is shown in Figures 2 and 3 for the entire study area and the core area respectively. Thus, four calibration data sets, disaggregated and grouped data sets for the entire study area and the core area, were developed.

FORMULATION OF THE DISUTILITY FUNCTIONS

The PAM proposed in this paper does not lend itself to a calibration procedure as generally understood. Because of the structure of the model, the output of PAM is defined implicitly rather than explicitly. This is in contrast to, for example, a modal-split model in which the output variable, modal split, is an explicit function of the input variables. Hence, to exercise PAM requires that an initial estimate of the disutility function be obtained. However, validation must be based on the ability of the model to replicate the observed interchanges between final destinations and parking facilities together with consideration of the quality of the disutility functions.

Figure 1. Study site.

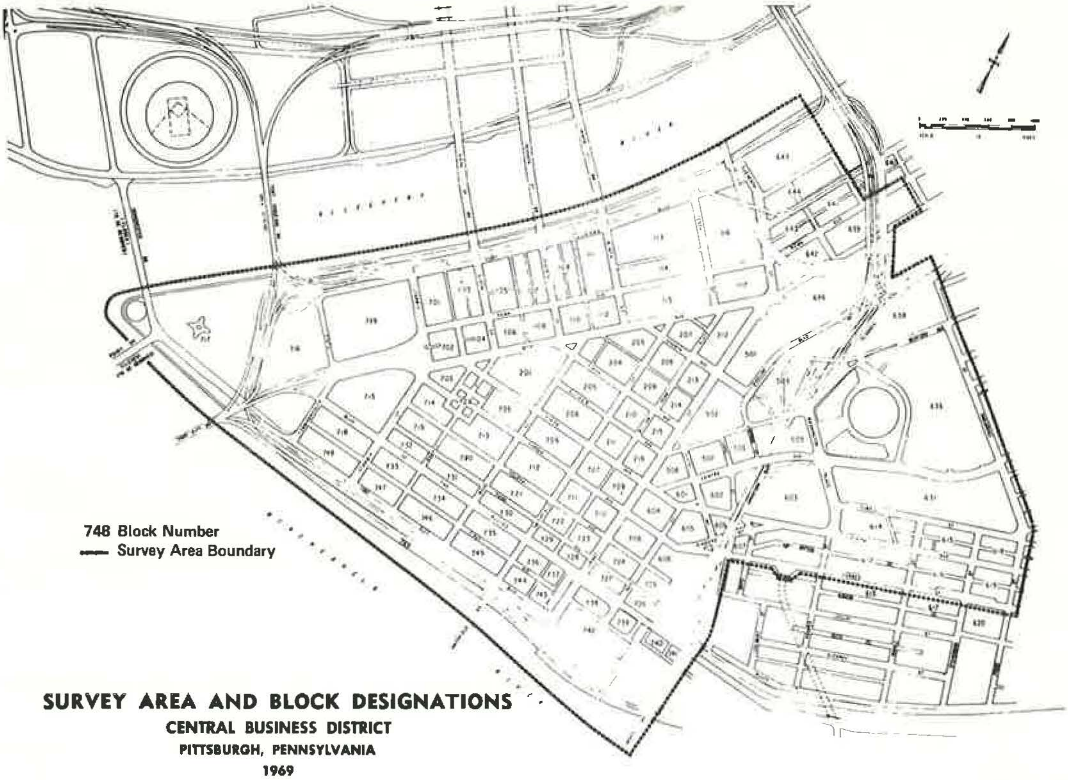


Table 1. Percentage of parkers in entire study area who will walk a given distance from and pay a given price for a parking space.

	Distance (ft)											
Cost ($)	>500	500-1,000	1,000-1,500	1,500-2,000	2,000-2,500	2,500-3,000	3,000-3,500	3,500-4,000	4,000-4,500	4,500-5,000	5,000-5,500	5,500-6,000
>1.00			1.1	1.1	1.0	4.9	7.9	2.4	1.5	0.4	0.3	0.4
1.00 to 1.50	0.2	1.5	1.5	2.5	1.1	0.5				0.2		
1.50 to 2.00	8.9	9.2	12.9	10.9	6.7	2.4		0.1				
2.00 to 2.50	6.3	2.9	6.0	1.8	1.1	0.3		0.1				
2.50 to 3.00	1.2	0.6	2.1	0.6								
3.00 to 3.50	0.6											

Table 2. Percentage of parkers in core area who will walk a given distance from and pay a given price for a parking space.

	Distance (ft)											
Cost ($)	>500	500-1,000	1,000-1,500	1,500-2,000	2,000-2,500	2,500-3,000	3,000-3,500	3,500-4,000	4,000-4,500	4,500-5,000	5,000-5,500	5,500-6,000
>1.00	0.3	2.9	5.5	1.6	2.4	5.3	4.5	1.3	0.8	0.2	0.1	0.4
1.00 to 1.50	1.9	2.9	4.7	2.4	0.7	0.3	0.3		0.1	0.2		
1.50 to 2.00	8.5	9.3	10.2	9.2	3.7	1.4	0.2	0.1	0.2			
2.00 to 2.50	5.2	3.1	4.5	1.2	0.8	0.3	0.1	0.1				
2.50 to 3.00	0.9	0.5	0.9									
3.00 to 3.50	0.3		0.2									

Figure 2. Relationship between cost and distance (entire study area).

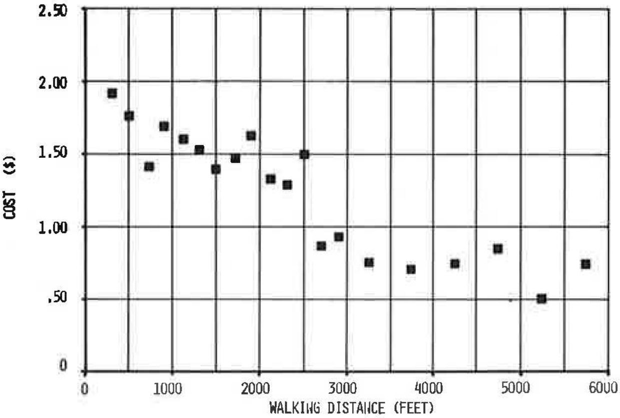


Figure 3. Relationship between cost and distance (core area only).

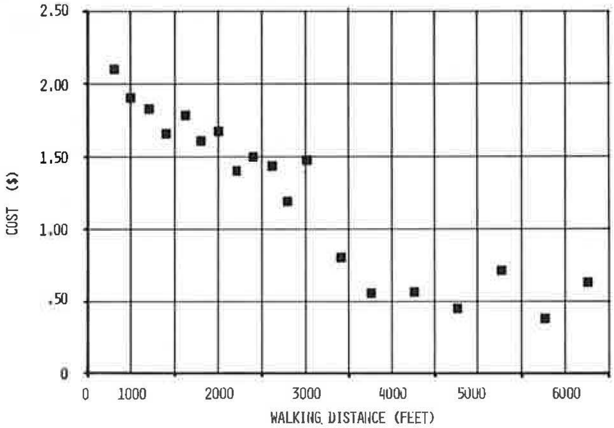


Table 3. Calibrated cost-distance relation and corresponding disutility functions.

Model Number	Cost-Distance Relationship[a]	Standard Error of Distance Coefficient	Correlation Coefficient: Log-Transformed Equation	Correlation Coefficient: Actual Equation	Disutility Function
1[b]	C = 218 - 4.5*D	0.001	0.94	0.94	Z = C + 4.5*D
2[b]	C = 45 + exp(5.587 - 0.067*D)	0.012	0.80	0.87	Z = C + 268*[1 - exp(-0.067*D)]
3[c]	C = 45 + exp(5.157 - 0.041*D)	0.002	0.88	0.88	Z = C + 174*[1 - exp(-0.041*D)]

[a]Cost is expressed in cents and distance in 100 ft. [b]Eqs. 1 and 2 pertain to the core area. [c]Eq. 3 pertains to the entire study area.

Although several variables could be considered, it has been assumed that cost and distance are the two variables that characterize a facility and determine parking choice. It has also been hypothesized that these two variables can be combined into a single measure of disutility. What can be observed by examining a plot of parking cost versus walking distance (Figs. 2 and 3) is that, despite a certain scattering, cost decreases as distance increases. Let $f(C, D) = 0$ be the relationship between cost and distance. This relationship can be readily calibrated by least squares once a functional form has been selected for $f(C, D)$.

The next step of the analysis is to determine how C and D can be combined into a single measure of disutility. When D is equal to zero or very small, disutility is equal to parking cost. If $f(C, D)$ is viewed as a trade-off between cost and distance, then the derivative of C with respect to D can be viewed as a marginal rate of substitution between cost and distance. Hence, the contribution of distance to disutility can be defined as the sum of the "substitutions" made between a given distance D* and the "ideal" distance, namely 0.

To illustrate this definition, let us assume that $f(C, D)$ is such that

$$C = -\alpha D + \beta$$

where α and β are two positive, calibrated constants. (In other words, a straight line has been fitted to the plot of cost versus distance.) In such a linear formulation, the rate of substitution of distance into cost is equal to α and, therefore, constant. Hence, the contribution to the disutility measure by the distance characteristic D* of a given facility is αD^*. If C* is the cost associated with D*, the disutility of this facility becomes

$$Z = C^* + \alpha D^*$$

A linear model is attractive because of its simplicity. However, it is unlikely that the rate of substitution should remain constant over the range of possible distances. Furthermore, the difference in the disutilities of two facilities located at distances of, say, 500 and 1,000 ft should be greater than the one corresponding to two facilities located at, say, 2,000 and 2,500 ft. To this end, two functional forms are available for $f(C, D)$, namely an exponential function where

$$C = \alpha \exp(-\beta D) \qquad (\alpha, \beta > 0)$$

and a power function where

$$C = \alpha D^{-\beta} \qquad (\alpha, \beta > 0)$$

In both cases, the marginal rate of substitution is a decreasing function of distance as can be seen from the derivatives of these functions:

$$\left|\frac{dC}{dD}\right| = \alpha\beta \exp(-\beta D)$$

and

$$\left|\frac{dC}{dD}\right| = \alpha\beta D^{-\beta-1}$$

It can be seen that the rate of substitution reaches (asymptotically) zero when distance becomes large. According to the definition given earlier, the contribution of a walking distance D* to the disutility measure becomes in the case of the exponential formulation

$$\int_{D^*}^{0} \left|\frac{dC}{dD}\right| = \alpha\left[\exp(-\beta D)\right]_{D^*}^{0} = \alpha\left[1-\exp(-\beta D^*)\right]$$

This element of the disutility function can be interpreted as follows. At zero distance, the disutility due to distance is zero. At a large or "infinite" distance, this disutility is represented by α. For intermediate distances, disutility is a fraction of α, the fraction being an exponentially decreasing function of distance. Hence, the disutility of a facility characterized by C^* and D^* is expressed as

$$Z = C^* + \alpha[1-\exp(-\beta D^*)]$$

In the case of the power function, the same steps could be followed to derive the expression of the disutility, provided that the "ideal" distance is not zero but is small, say 100 ft (otherwise, the function is not defined). If D_o is such a distance, then the disutility becomes

$$Z = C^* + \alpha\left[(D_o)^{-\beta} - (D^*)^{-\beta}\right] \qquad (D^* \geq D_o)$$

At this point, an observation should be made concerning parking cost as such and the fraction of that cost that can be substituted for in terms of distance. Examination of the data reveals that, for the long-term parkers considered, the minimum cost of parking C_o is 50 cents. Thus, given that a trip-maker has decided to park in the study area, the "substitutable" amount of his parking cost C is $C - C_o$. Alternatively, one could assume that, inasmuch as there is no parking space under C_o available in the study area, the function f(C, D) should yield no value of C under C_o. This condition implies that the asymptotic value of the exponential function should be C_o instead of 0. To this end, the dependent variable in the least-squares estimation of the parameters α and β becomes $C - C_o$. Inasmuch as the logarithmic function is not defined when its argument tends to 0, the value of C_o was set at 45 cents so that the log-linearized relationship could be calibrated by least squares.

CALIBRATION OF THE DISUTILITY FUNCTIONS

Numerical results presented in this paper should be viewed as preliminary inasmuch as application and testing of the model are under way at the present time. Several calibration runs of the cost-distance trade-off function f(C, D) have been performed. However, not all of the corresponding disutility functions have been used as input to the model. As noted earlier, the "quality" of an estimated disutility function depends not only on statistical measures (such as standard errors or correlation coefficients) but also on the extent to which the model using this function reproduces the observed allocations of parkers among parking facilities.

The present discussion focuses on the calibration of the linear and exponential disutility functions, which have actually been used as input to the parking allocation model. The results of the calibration of the cost-distance relationships and the corresponding disutility functions are given in Table 3. The first two disutility functions pertain to the triangular core of the study area, whereas the third one is representative of the entire study area. All three functions have been calibrated on cost data grouped by distance intervals as described earlier. The relatively low standard errors of the coefficients and high correlation coefficient are due, in part, to the grouping of data.

APPLICATION OF THE PARKING ALLOCATION MODEL

The three calibrated disutility functions given in Table 3 were used as input to the parking allocation model. One arrival period and three departure periods, i.e., between 7 and 8 hours, 8 to 9 hours, and longer than 9 hours, were used. To facilitate the comparison of estimated statistics with observed or actual statistics, we aggregated the 116-zone area structure into 10 districts. As shown in Figures 4, 5, and 6, total volumes allocated by each model to each of the 10 aggregate districts are generally in close agreement with the observed totals. The index of determination R^2 between estimated and observed values is 0.95 or better for each of the three models.

Similarly, the estimated aggregate disutilities of the parkers in the 10-district structure exhibit a high correlation with the observed values. These disutilities were

Figure 4. Observed and estimated facility totals (model 1).

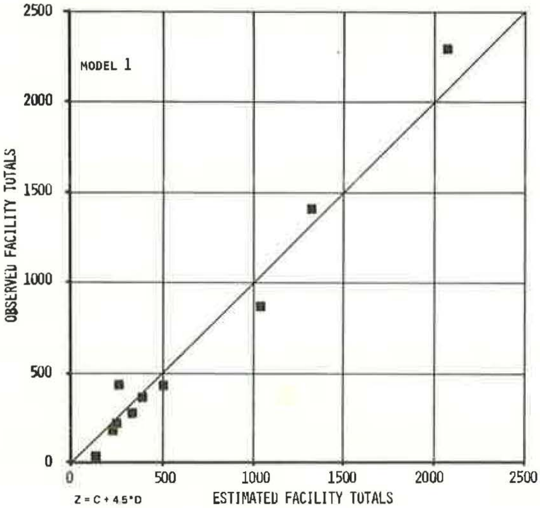


Figure 5. Observed and estimated facility totals (model 2).

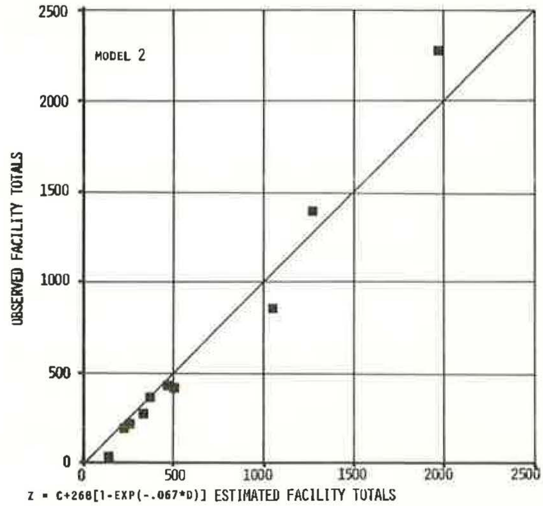


Figure 6. Observed and estimated facility totals (model 3).

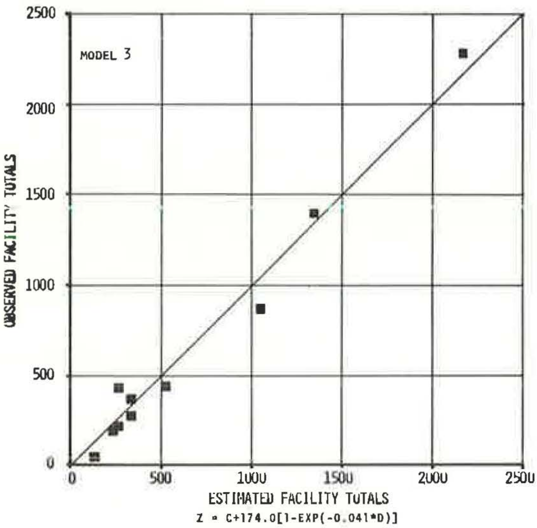
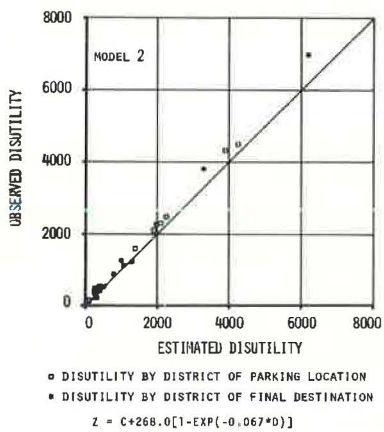


Figure 7. Observed and estimated aggregated disutilities (model 2).



evaluated for each allocated parker by means of the disutility function input to each model. For example, Figure 7 shows the observed versus estimated disutilities at both the facility and final destination levels for model 2.

Finally, it is of interest to compare the interchanges between parking districts and final destination districts. In the 10-district structure, there is a total of 100 such possible interchanges. The observed set of interchanges contains 48 zero cells. Model 2 replicated exactly 47 of these zero cells on a one-to-one basis. Models 1 and 3 replicated 46 zero cells, also on a one-to-one basis. This result is interesting to note inasmuch as it demonstrates the ability of PAM to replicate a parking pattern with reasonable accuracy. The comparison of nonpaired zero cells between actual observations and model estimates are shown for each model in Figure 8 through 10.

For each model, the index of determination R^2 between actual observations and model estimates is relatively high. Specifically, the R^2 values are 0.82, 0.85, and 0.80 for models 1, 2, and 3 respectively.

SUMMARY AND CONCLUSIONS

The present paper develops the underlying assumptions of the parking allocation model and describes the results of its calibration and application in a case study. This version of the model incorporates three of the basic factors influencing parking choice, namely, cost, walking distance, and capacity. Whereas the importance of cost and distance is readily recognized, the importance of capacity should be emphasized in any allocation model.

To negate the importance of capacity would imply in a certain sense that parking supply is always available, which is often not the case. The argument that market forces determine pricing policy is no doubt valid. However, it is when demand exceeds supply that a trader raises his price. Thus, the awareness of the "imbalance" that exists in the real-world cannot be replicated in a simulation model without first noting that capacity is reached and then "raising" the price so as to maintain the parking facility at its peak occupancy. Furthermore, from a modeling standpoint, capacity constraint is certainly a desirable attribute that, everything else being equal, introduces an internal "control mechanism" into the model. Even better, this readily available "mechanism" is neither artificial nor as difficult to define as highway capacity, which is a recognized determinant of route choice in assignment models. Finally, if an allocation model is to become a tool for providing meaningful information to decision-makers, consideration of capacity becomes essential.

The parking allocation model is embedded in a linear programming context in which a disutility concept is used to combine the effect of the trade-offs between cost and distance. It is also a convenient device for incorporating other variables such as those mentioned earlier. Joint or simultaneous minimization of the disutility of parkers is a basic assumption of PAM. It has been noted that PAM is performed as an approximation to individual minimization and that the shorter the time span is within which parkers are grouped, the better this approximation is. In this regard, it is interesting to refer to Figure 7 and observe that, for each final destination district, the total disutility of the parkers allocated by the model is only slightly lower than the corresponding observed disutility. Capacity and demand constraints are easily incorporated in the framework of a linear program. Finally, computational algorithms that are highly efficient are available. (The problem described herein can be solved in approximately 1 min of processing time on an IBM 360/65.)

It should be noted that the results of the model presented in this paper are "uncorrected." Briefly speaking, such corrections or "tuning" of the model results could mainly be performed by examining the final destination-parking facility interchanges in which major discrepancies occur and by adjusting the corresponding disutilities to make the facility more or less attractive, as may be required.

In general, the performance of the parking allocation model in this first operational application is encouraging. It did replicate the distributional pattern of parkers among facilities and final destinations and the facility totals in each district. The close correlations between observed and estimated allocations, shown in Figures 8, 9, and 10 for each of the three models calibrated, suggest that PAM captures the dynamics of the parking process. Based on these results, further careful pilot applications are warranted.

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DISCUSSION

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The authors have developed a rational approach to the problem of parking allocation by embedding PAM in a linear programming context. This is acceptable, assuming that the joint disutility minimization is performed over a relatively short period of time. The discussion that follows aims to add information on the linear programming aspects of the paper and the limitations that possibly can be faced in practice.

Linear programming is a process of optimization where the objective function is linear and the constraints are also linear. Any linear programming problem can be expressed in the form of equality relations among the variables, which are nonnegative. This standard form is as follows:

$$\text{Minimize} \quad z = c_1x_1 + c_2x_2 + c_3x_3 + \ldots + c_nx_n$$

subject to the constraints

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n &= b_2 \\ &\vdots \\ a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n &= b_m \end{aligned} \qquad (1)$$

where

$x_1 \geq 0,\ x_2 \geq 0, \ldots x_n \geq 0$, and
$b_i \geq 0\ (i = 1 \text{ to } m)$.

x_i are the variables, and b_i are the constants.

Generally the problem at hand may not be in standard form as above but may be transformed into one by means of suitable manipulations, introducing slack and surplus variables. For example, if the inequality is of the form

$$a_{k1}x_1 + a_{k2}x_2 + \ldots + a_{kn}x_n \leq b_k \qquad (2)$$

it can be put in the equality form as

$$a_{k1}x_1 + a_{k2}x_2 + \ldots + a_{kn}x_n + x_{n+1} = b_k \tag{3}$$

where x_{n+1} is a nonnegative slack variable.

The authors' formulation of the model is as follows:

$$\text{Minimize} \sum_j \sum_k \sum_q \sum_d Z(j,k,q,d) \times X(j,k,q,d) \tag{4}$$

subject to

$$\sum_j \sum_q \sum_d X(j,k,q,d) \le s(a,k) \quad \text{for each } k \tag{5}$$

$$\sum_k X(j,k,q,d) = T(j,q,d) \quad \text{for each } (j,q,d) \tag{6}$$

where the notations are as defined in their manuscript.

The constraints in Eq. 5 can be transformed in equality form by use of slack variables. For application of linear programming technique s(a,k), the number of spaces available at time period a in facility k should be treated as constants. Hence, theoretically minimization can be performed only over the relatively short period of time during which s(a,k) is reasonably constant. The determination of acceptable variation in s(a,k) is dependent on the sensitivity of the optimal solution to small perturbations in s(a,k).

An important requirement in the linear programming method is that the feasible region formed by the constraint equations be a convex set; i.e., if any two points within the region are connected by a straight line, that line should always lie entirely within the feasible region. This is always true conceptually, even in the higher dimensions, except in the degenerate cases where feasible solutions are nonexistent. In the present linear programming problem of the PAM described by Eqs. 4, 5, and 6, there is a possibility of degeneracy, even though this situation may rarely be faced in practice. For example consider a very simplified representation of constraints in Eqs. 5 and 6 for only two variables, X_1 and X_2. The point brought out can of course be generalized. The constraint equations, Eqs. 5 and 6, take the simplified form

$$X_1 \le S_1$$

$$X_2 \le S_2$$

$$X_1 + X_2 = T \tag{7}$$

The region enclosed by the constraints and the process of minimization of the objective function can be geometrically illustrated as shown in Figure 11.

The region that includes the sets of x_1, x_2 fulfilling the constraints is called the feasible region. In this figure, the straight line AB representing the equality constraint is the feasible region. Any point in AB is a feasible solution, and the optimum occurs either at A or at B. Depending on the values of the constants S_1, S_2, and T in Eq. 7, the graph may take the shape shown in Figure 12 where the feasible region is the line CD and the optimum occurs at C or D. Some values of S_1, S_2, and T, may result in a degenerate case as shown in Figure 13.

Obviously no point can be found that can simultaneously satisfy all the constraints; hence, no feasible region or solution exists. In the model under study, s is the supply available, and T is the total number of parkers. Hence under certain circumstances, when the number of parkers outstrips the supply considerably, there is a possibility of a degenerate case where no feasible solution exists.

A linear programming problem can also face a case of unbounded solution, but in the present model there seems to be no possibility of this occurrence due to the equality constraints in Eq. 6.

Figure 8. Observed and estimated allocation in the 10-district structure (model 1).

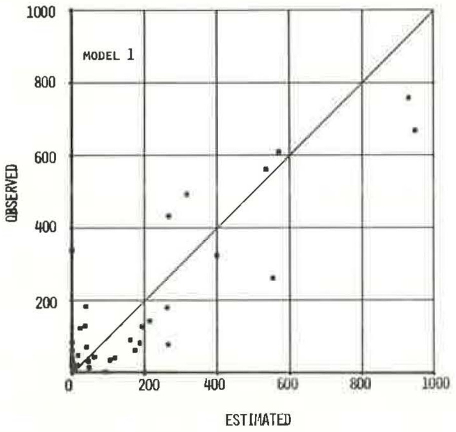


Figure 9. Observed and estimated allocation in the 10-district structure (model 2).

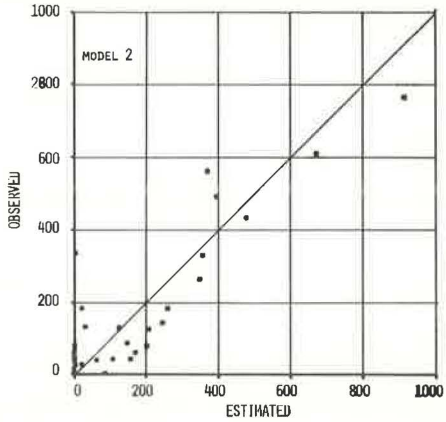


Figure 10. Observed and estimated allocation in the 10-district structure (model 3).

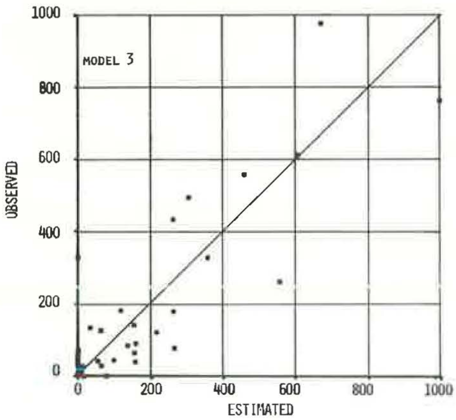


Figure 11. Convex set with feasible region within constraints.

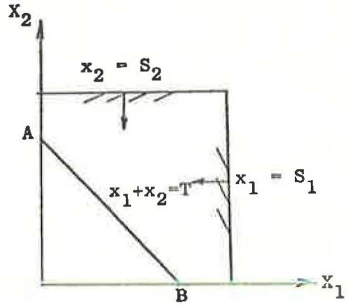


Figure 12. Feasible region partially within constraints.

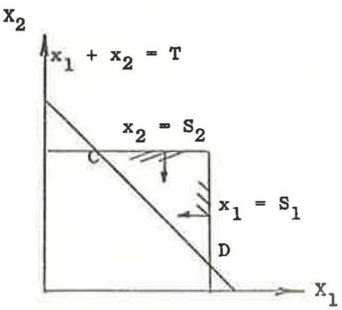
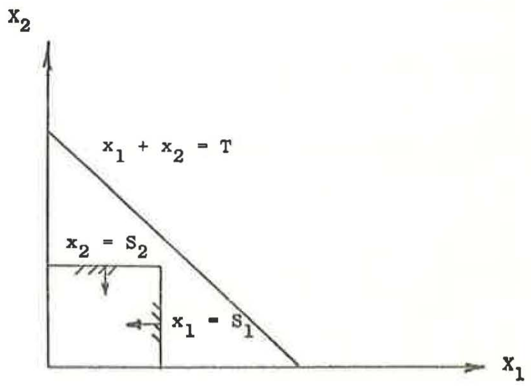


Figure 13. Degenerate case with feasible region outside constraints.



Another observation with respect to the formulation of the objective function is of importance. The authors assume functional forms for f(C, D), where C and D are cost and distance; e.g.,

$$C = \alpha e^{-\beta D} \qquad (\alpha, \beta > 0) \tag{8}$$

Then the contribution of distance to the disutility function is defined by the authors as the sum of the substitutions made between a given distance D* and the ideal distance, namely 0. Thus, the disutility of a facility characterized by C* and D* is expressed as

$$Z + C^* + \alpha(1 - e^{-\beta D^*}) \tag{9}$$

If the functional forms assumed for f(C, D) are indeed a close approximation of the real situation, then, for a facility characterized by D*, the corresponding C* is given as

$$C^* = \alpha e \tag{10}$$

Substitution of Eq. 10 into Eq. 9 yields

$$Z = \alpha e^{-\beta D^*} + \alpha(1 - e^{-\beta D^*}) = \alpha \tag{11}$$

This implies that the disutility is constant for all facilities under consideration. This does not appear to be true as can be seen from Tables 1 and 2, which describe joint distribution of cost and distance. It can be calculated that more than 55 percent of the parkers park closer than 1,500 ft from their final destination, and more than 80 percent park closer than 2,500 ft from their final destination. From this it appears that parkers find distances of more than 2,500 ft to cause more disutility, even at the reduced cost. Hence, the disutility function characterized by Eq. 9 requires some modifications. It is suggested that some form of relative weightage be attached to cost and distance, giving more weightage to less walking distance after 2,000 ft as depicted by the joint cost and distance distribution tables. It is hoped that this modification will add to the work done by the authors and help create a more realistic model.

Harry B. Skinner, Federal Highway Administration, Denver

The information in this paper represents an excellent and innovative approach to the problem of parking allocation. This procedure will surely serve as a basis for an important parking planning tool.

Through PAM the authors treat parking supply and demand as deterministic commodities. The resultant allocation of parking demand to the supply is subject to a fixed facility capacity and a disutility objective function. It would appear that a more appropriate approach would have been to assume a stochastic character to supply and demand because (a) parking supply is, to some extent, a function of demand and demand, to a certain extent, a response to, inter alia, the availability of space; (b) the planning process that generated the input statements of demand is acknowledged to be a less-than-perfect projection of future need; and (c) the allocation output is merely a tool to guide the decision-maker.

The computational effort in treating this problem stochastically would not be significantly increased, and the result would serve as a better tool because it would not be rigidly fixed to a given demand and a given supply. A probabilistic element of the objective function would allow the demand to respond to the supply, and a probabilistic supply constraint would allow the supply to respond to the demand. The resulting allocation would be improved in the following two ways:

1. Establish a supply-demand relation, and
2. Establish a probabilistic demand (rather than merely assign demand deterministically to supply, set the likelihood of accomplishing an occurrence).

This, in turn, should more nearly optimize a system of parking facilities of projected size and location and represent a more valuable tool to the decision-maker.

It may also be beneficial to the decision-maker if a sensitivity analysis were performed on the final parking allocation. The purpose would be to guide the decision-maker in answering such questions as the following: What would be the resultant usage of the facility or system of facilities if a change in pricing policy were implemented? How would a change in location affect usage of a facility?

Also, PAM uses a disutility function that is derived from a cost to park and distance to walk relationship. This is reasonable when considering the accommodation of long-duration work-trip parking. Work-trip parking is an important element of the total parking consideration in any community. For many parking facilities, this is the "bread and butter" of the operation. However, the measure of success of many operations is the accommodation of the short-term parker on a shopping trip, a business call, a professional visit, and the like. Surely, the derivation of a disutility function for nonwork-oriented parking would have to consider parameters other than cost and distance. Experience with the idiosyncrasies of parkers indicates that it may be necessary to take account of such things as self- or attendant-park operations and the character of the neighborhood through which the parker must walk to arrive at his final destination.

For a projected allocation a uniform set of conditions can be assumed. This is not so for current conditions and, therefore, not so for the derivation of the disutility function. For instance, a parking facility on the periphery of a renewed section of the city may be used by parkers having destinations in only half the set of possible destinations in the region of possible influence of the facility. Or a facility one block removed from a renewed area and separated from that area by an economically depressed and despoiled neighborhood will probably demonstrate a different disutility relation than a comparable facility completely surrounded by a renewed and vital setting.

These brief comments are intended only to stimulate consideration of techniques that may make the basic model more useful and should in no way be interpreted as questioning the credibility of the concept or the technique.

George T. Lathrop, Department of City and Regional Planning,
University of North Carolina

In general the authors have presented a potentially useful technique for accomplishing their stated objective: to simulate the choice of a parking facility by a user traveling to a given final destination (within the context of a concentrated travel destination area). They are to be complimented particularly on the simplicity of the model and its assumptions and the success of the simulation, given that simplicity. It goes without saying that one of the self-defeating aspects of many urban simulation models of all types in the past has been the complexity in parameters and mathematics, which have been necessary to make them "work," but which have made them so complicated that they are almost impossible to use (by normal humans).

If I have areas of concern about the model, they might be grouped under two headings: technical concerns and concerns about application.

In the technical area, I would have appreciated a more extensive review of the linkage matches. As the authors note with proper caution, the linear programming algorithm provides a system minimization that is accepted as a reasonable model of the grouped behavior or choices of individual decision-makers. They also note the disparity between the simultaneous behavior assumption of the linear programming format and the sequential nature of the actual process. Clearly, if the objective of the model is to be realized, to even a reasonable extent, there should be some strong correspondence between the choices actually made and the choices simulated or predicted by the simulation.

In the same vein of technical comments, the authors are also quite properly cautious in their claims for applicability of the model. Without replication independent of the data used for calibration, the "utility" of the coefficients of the disutility function must remain unknown. Of course, this question in no way addresses the use of the linear programming model; the specific parameters of the disutility function and the cost-distance aggregations are the application-specific values.

Along those same lines, it is interesting to note that the results of a linear disutility model apparently approach the quality of the results of the more complex functions. Given the nature of the linear programming algorithm itself and the earlier observations on the merits of simplicity, this is most encouraging.

Turning to concerns about application of the model, it must first be noted that, beyond their introductory statements concerning the objective, the authors refrain from suggesting potential uses other than by inference.

I think it is reasonable to raise the question of what might be called the "black-box syndrome." To elaborate, the presumption must be that the model will not be used by the devisors of the model alone. In the long run, assuming that other testing and validation leads to reasonable results, it may be further assumed that the technique (and even perhaps a computer program packaged and distributed for the purpose) will be used by many other persons to accomplish exactly the objective stated: to simulate individual choice. The computer program will be written to accept input of parkers by final destination, parking facilities by capacity and cost, and distance from all destinations to all origins. Output will be a listing of parkers versus capacity and a set of linkages between the final destination and parking locations. Instructions for fitting curves (or straight lines) to minimize variation between the actual and simulated situation will complete the package. Forgotten will be the authors' careful precautions about aggregation of data and grouping of origins and destinations and warnings about simultaneous versus sequential decisions and system versus individual optimization.

My concern about this eventuality should in no way reflect on the authors. They have done a careful job both of developing their technical work and of couching their conclusions about that work in thoughtful, well-chosen, and cautious reservations. My concern is rather with the creation of simulation models and their implementation on the computer, inasmuch as it is apparent that one of the frailties of human nature is to forget what is in the "black box" and to begin to accept the output as infallible. Unfortunately, there does not seem to be much that can be done. Perhaps in this case careful explanation of the assumptions inherent in the model and clear statements of exactly what goes on inside might help. The authors have done this to a large extent. It is up to the distributors and users to continue.

In summary, Ellis, Rassam, and Bennett are to be congratulated for a potentially useful application of a straightforward technique to a nagging problem. The simplicity of the assumptions and procedure will encourage others to both use their particular application and attempt similar applications in the same spirit. We may all look forward to the examination of alternative parking facility location strategies that use the model.

AUTHORS' CLOSURE

The authors would like to thank Sundaram and Feng, Skinner, and Lathrop for taking the time to comment on the paper.

Sundaram and Feng's comments focus on two points: theoretical considerations relative to the transportation problem and interpretation of the disutility function. Regarding the first point, we would like to note that, for lack of space, we deliberately avoided a theoretical discussion of linear programming. We certainly agree with Sundaram and Feng about situations arising in which total parking demand outstrips total supply. In the computer program, requisite cells are introduced if demand and supply are not equal. The second point that they made, namely that disutility remains constant for all facilities, is probably a misinterpretation. In the relationship defining the disutility of a facility characterized by a cost C* and a distance D* to a given destination, namely

$$Z = C^* + \alpha[1 - \exp(-\beta D^*)]$$

C^* and D^* are actually observed values. Specifically, C^* is not derived from the relationship

$$C = \alpha \exp(-\beta D)$$

Thus, we believe that their second point may not be founded.

It should be noted that discontinuities may exist in the disutility function; for example, a parker may be unwilling to accept a walking distance greater than 1,500 ft. Consideration of such discontinuities can be accommodated within the current formulation of PAM. Estimation of the disutilities would require in-depth studies of the attitudes and perceptions of parkers, an alternative to the calibration strategy used in the current study.

We agree with Skinner that the problem could be cast in a stochastic programming framework. His comments regarding the stochastic nature of demand and supply are most appropriate. We envisaged such an approach but wanted to proceed by steps; therefore, we initially cast the problem in a deterministic framework. Also, we feel that the data acquisition and processing necessary to define the stochastic functions, not to mention computer running time, would require significant additional resources.

One of the reasons we chose to cast this problem in a linear programming framework was the relative ease of treating post-optimality problems. It is often possible to analyze the effects of changes in the price and supply vectors without re-solving the problem. For example, the operator of a given garage could determine how much he could raise his price (in the absence of a Phase II Price Board) without losing customers. Parametric programming techniques would be a most useful addition to the current computer package.

We certainly agree with Skinner on the desirability of introducing other variables into the disutility function. The general formulation of the problem does not preclude considering other factors such as waiting time, approach route, or whether a facility is attended or not. However, we wanted to proceed cautiously inasmuch as this was the first operational application of the model. One might also add that consideration of these other factors would require a substantial amount of data, which at present are not easily available. The main difficulty would probably be to find one single data source so that all the factors defining parking choice are compatible, which is not the case when the data base is assembled from secondary sources with all the ensuing definitional and sample size problems.

Lathrop raises two technical issues, namely, the linkage matches and the parameters of the disutility functions. Linkage matches have been one of our primary concerns in testing the model, as shown in Figures 8, 9, and 10. We fully agree with Lathrop that "if the objective of the model is to be realized, to even a reasonable extent, there should be some strong correspondence between the choices actually made and the choices simulated or predicted by the simulation." In this sense, we were particularly encouraged by the near replication of the zero cells (47 and 46 out of 48 on a one-to-one basis). We believe that, by more closely examining the nonzero linkages and by introducing (selectively) more variables into the disutility functions, we might obtain improved linkage matches.

In addressing the issue of the parameters of the disutility functions, Lathrop raises a critical question, the answer to which has eluded us for the time being inasmuch as we have not had the opportunity to test the model in other cities. Ideally, one would seek a set of so-called universal coefficients. However, transportation is a field in which such an optimistic outlook has always been tempered, willingly or not, by reality. Thus, we would rather seek to identify a set of variables and ranges of their associated coefficients that an analyst could easily adapt to a specific situation. In other words, keeping in mind that models, albeit useful tools, are not panaceae, we would attempt to narrow, to a reasonable degree, the options left to a field practitioner.

In conclusion, we would like to state how much we agree with Lathrop about the "black-box" syndrome. Modeling, and especially computer modeling, can easily become misleading whenever the proper caveats are cast aside. Should we conclude by saying that the myth of Icarus has often been forgotten? His "black box," if we dare say, offered great promises until.... He really should have listened to Daedalus' warnings!