A LOCATION PRINCIPLE OF URBAN TERMINAL DISTRIBUTION

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The study was an attempt to determine the optimum spatial pattern of passenger transportation terminals in urban areas with the objective of maximizing terminal accessibility. The concepts of terminal-destination environment used by this study were introduced to manifest the basis of analysis and the generality of the resultant techniques. Mathematics was used to evaluate alternative geometries of terminal location patterns and to investigate the possible effect of location displacement on the service efficiency of terminals. Also, a search procedure provided through computer programs was developed to help establish the optimum location to best serve a particular area. Study results indicate that the optimum location consists of the following: terminal location pattern that forms diamond-shaped service areas centered at intersection with the major axis of the pattern being parallel to the urban streets; border lines of the service areas that are parallel to street block diagonals; and the service area diagonal that is an integer multiple of block sides. The theoretical solution developed by this study appears to be applicable to the transportation mode that has definite locations of users' access in urban areas, particularly transportation planning for new towns or planning of new transportation system for existing cities.

•WITH today's population explosion and increasing travel needs, the urban transportation problem has become serious enough to justify a great deal of effort to determine means for improving urban mobility. Traffic is not, in itself, generated for the sole purpose of movement; instead, it is the result of one's desire to reach a destination. Therefore, the service effectiveness of a mode of transportation, to a great extent, lies in terminal accessibility, particularly for the relatively short urban trip. When travelers must choose among available modes of travel, that system providing poorly located terminals is at a competitive disadvantage. However, most urban transportation terminal facilities in use today have not been located in a systematic manner. It is therefore becoming increasingly important to consider a total system approach in designing transportation media and to include a rational selection of terminal location systems.

A properly designed location system avoids duplication and waste and makes possible the maximization of service effectiveness. Location theorists consistently make use of the concept of regional hierarchy for service area planning. The basic argument supporting the use of central-place ideas is that system access points arranged in a hierarchy provide an effective way of articulating distribution to the area served. By definition, the optimum location system minimizes the average access distance of all users in the service area, and the central-place principle is well suited for application to transportation terminal locations. Under the assumption of an unbounded plane with equal access in all directions, regional scientists suggest that the triangular arrangement of service sites, and hexagonal market areas, represents the optimum (2, 4, 15). This result has been derived by the assumption of the straight-line access distance of customers to and from service sites. However, locating transportation terminals in urban areas by this geometrical arrangement does not appear to be applicable because most urban street systems follow a gridiron pattern (17). In this respect, an analysis of the terminal location pattern based on the rectilinear access distance would be of significance in providing urban travelers with the most effective transportation service.

STUDY OBJECTIVES

The study was intended to define the optimum spatial arrangement of transportation terminal locations for passengers in urban areas. The basic criterion on which optimum characteristics were determined was that of maximizing terminal accessibility. The method first identified urban environmental variables to realistically provide valid assistance in actual application. It is hoped that this method would provide valuable insight in locating such transportation terminals as automobile parking facilities, bus stops, and taxi stands. The theoretical apparoch may also provide assistance in solving other location problems of service facilities in urban areas.

TERMINAL-DESTINATION ENVIRONMENT

The terminal distribution system must be analyzed in terms of the competitive environment within which it operates. It is therefore essential to make explicit some environmental variables and the underlying assumptions considered by this study to help determine the generality of the resulting solutions. All possible efforts have been exerted to produce a theoretical solution that is most representative of a real situation.

Although many environmental variables affect the terminal location of a particular transportation mode, accurate estimation of demand for the mode is the most crucial part of the optimization consideration. The level and distribution of demand depend on population density and other socioeconomic characteristics in the area under consideration. The demand for a specific transportation mode among other alternatives is determined by the relative service performance of the mode in question. Assuming that terminal accessibility is an important factor affecting public choice of mode, few people are willing to walk farther distances to use a given mode of transportation. In other words, other available modes will be more competitive at farther distance if remaining performance factors are equal.

Theodorson (17) found that the grid pattern is characteristic of American cities with few exceptions. Therefore, the rectilinear walking path along the gridiron street network is typical for access to an urban terminal. For a mode of transportation to provide adequate service to a city requires a certain number of terminals so that all points in the city are accessible with reasonable rectilinear walking distances for all customers. Each terminal has associated with it an area that includes a number of destination points served. The imaginary boundaries of a terminal service area are determined by the maximum acceptable walking distance. Because numerous factors are considered in determining an acceptable walking distance for individual passengers to reach a terminal, it is difficult to choose a simple definition for all circumstances.

The cost of walking is a subjective value placed on the walking time and the discomfort and inconvenience of walking. The value of time depends on many factors such as the person, the circumstance, the amount of available time, and the time productivity. There have been some attempts to determine the value of travelers' walking time by statistical methods. More recent work (14) has been based on the theory relating parking cost profiles to commuters' marginal valuing of time and comfort. It has been found that commuters are willing to spend about 12 cents per minute, or 15 cents per $^1/_{16}$-mile block, to save walking time between parking places and work. This 12 cents per minute includes two cost components: value of time and value of comfort. A typical commuter would like to pay about 8 cents per minute at the margin to avoid the discomfort and inconvenience of walking. This figure is significant because commuters appear to be willing to pay a high price to save walking time and thus are very responsive to choosing a mode of travel in accordance with terminal accessibility. For these obvious reasons, determining the optimum terminal location for reducing access walking distance should play an important role in planning the transportation terminal system.

In attempting to resolve the complexities of the terminal-destination environment, we based this analysis on the following important assumptions:

1. The passenger's walking distance to and from terminals is along rectilinear paths corresponding to the gridiron street pattern inasmuch as square or rectangular block configurations are predominant in urban areas.

2. All passengers tend to minimize their walking distance by using the terminal nearest their destinations. This assumption is supported by the economic and psychological make-up of travelers.

3. The number of travelers willing to use a mode of transportation decreases with increased distance between the point of destination and the terminal location.

4. A terminal location is considered to be an area located anywhere along the outer periphery of a city block but not within the confines of a block.

TERMINAL LOCATION PATTERNS

Because the terminal access time is a major criterion in evaluating the service provided by the transportation system, the optimum pattern of terminal locations will be that providing the least access distance for most terminal users within service areas. It first appears that this criterion, minimizing the access distance based solely on the cost of the access walking time, tends to favor a high-density terminal network with a small service area. Ideally, each terminal user would have his own terminal at his point of destination. Practical limitations, however, prevent optimization on such a basis because it is not economically and physically feasible to provide a very large number of terminals to serve a city area.

A complete definition of the necessary conditions that the service area of terminals must fulfill is essential to describing the optimum terminal distribution system. If a single terminal is to be provided, it will serve an area bounded only by the maximum distance people are willing to walk. Because this study was ultimately concerned with the determination of a terminal network to provide service to a larger area or an entire city, a geometry consisting of multiple areas served by individual terminals is formed. Figure 1 shows a multiterminal structure superimposed on the Philadelphia CBD for illustrative purposes.

Many geometrical patterns could be used for the location distribution of terminals, but most of these were ruled out by simple reasoning. To provide complete service coverage for an area would limit the geometrical pattern of terminals to those that neither deprive any portion of the area of service nor create overlapping service areas. In other words, all adjacent service areas must border each other. In this analysis, it was proved that the only possible geometries satisfying this unique requirement are regular polygons, the square, the diamond, the triangle, and the hexagon, as shown in Figure 2.

Single Terminal Considerations

After the limited number of possible geometric configurations has been described, it is necessary to compare the relative values to users of average access distances within each service area in order to determine the optimum geometry. The most accessible location pattern would minimize users' travel (walking) distance along rectilinear routes between the terminal and the destination point. For an analytical treatment, it was assumed that there are an infinite number of demand points within the service area and that all people in the service area, regardless of walking distance, will be willing to use the terminal for travel. Based on an equivalent area for all four geometries, the following function was applied to compute the average walking distance $\bar{D}$:

$$\bar{D} = \iint_A (1/A)\ (x + y)\ dx\ dy$$

Figure 3 shows the diamond configuration. Using the terminal spacing for the square

Figure 1. Philadelphia CBD multiterminal structure.

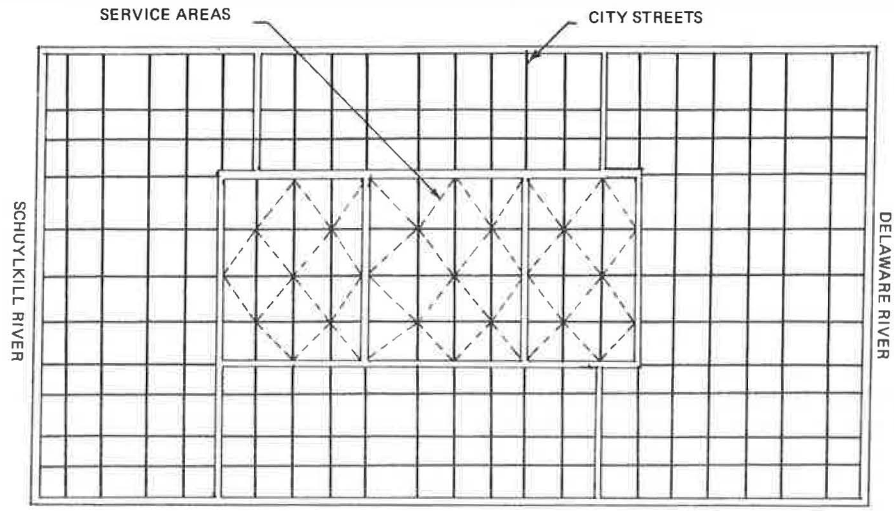
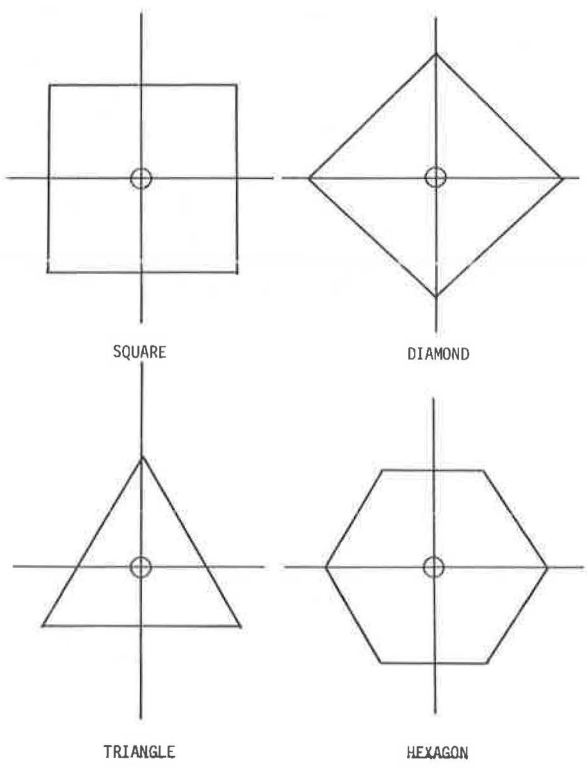


Figure 2. Feasible service area geometries.



pattern S, the resultant, normalized values of the average walking distance for the four geometries are given as follows:

Service Area Geometry	Average Walking Distance
Diamond	0.47 S
Triangle	0.48 S
Square	0.50 S
Hexagon	0.51 S

This comparison indicates that the diamond-shaped service area is superior to other configurations because it provides a minimum average rectilinear distance. The second best is the triangle shape, which approximates the efficiency of the diamond shape and could be used for cases in which natural boundaries constrain service area shape. It first appears that the difference in the average walking distances among various geometries, ranging from 2 to 6 percent, may not be manifested. However, these differences should be greatly magnified by the quantity of daily users over the entire life cycle of the terminal.

This theoretical treatment developed important principles that are applied to the practical conditions. Hakimi (12) has shown that the absolute median of a graph, a rectangular network with weights attached to each branch and vertex, must be at a vertex of the graph. An intuitive application of this theorem to a terminal location would substitute the demand at each point in a city for the arbitrary weights used by Hakimi. In this context, it would appear that the optimum location of a terminal unit is at the intersection that minimizes the absolute median function. Combining the result derived by this study with the absolute median concept developed by Hakimi indicates that, for a city block-street system, a terminal should be located at the center of an intersection and should serve a number of separate destination points (building entrances) within a diamond-shaped service area; access to and from the terminal would be along a rectilinear walking path. Each building has only some finite number of entrances through which a traveler must pass to reach his actual destination. Minimizing walking distances to these building entrances has the same effect as minimizing the entire route to the actual destinations within.

The theoretical analysis indicated that the optimum geometry of service areas should be the symmetrical diamond shape, which may be superimposed on uniformly square street-blocks. Practically, the configuration of the city block may not always be uniformly square shaped. To extend the optimum characteristics of the diamond-shaped service area requires, therefore, that the relative efficiency of service areas that form a nonsymmetric diamond be examined. For example, for a terminal network superimposed on rectangular city blocks, the geometry of individual service areas would become elongated diamonds as shown in Figure 4. This geometrical deformation is due to an important constraint; that is, the distance from the center of the service area (or the terminal point for uniformity in sense) to the vertexes of the diamond shape must be an integer multiple of block sides. This constraint may be proved by assuming a supposedly nonfeasible terminal-to-vertex distance of a noninteger multiple of block sides, say, one and a half blocks. The adjacent service area must border the test area and must be of equal size for the uniform demand case. The terminal for the adjacent service area therefore would be specified at one and a half blocks directly below (or above) the vertex at the middle of the block. This location at the center of the block violates the rule that a terminal must lie along the outer periphery of a block. This assumption appears to be mandatory because of the extra cost associated with locating a terminal within a block and the need to provide walking routes to all block sides.

The average and total walking distances to an individual terminal in an elongated-diamond service area can be expressed in the following functions:

$$A = 2\ell L$$

Figure 3. Diamond-shaped service area.

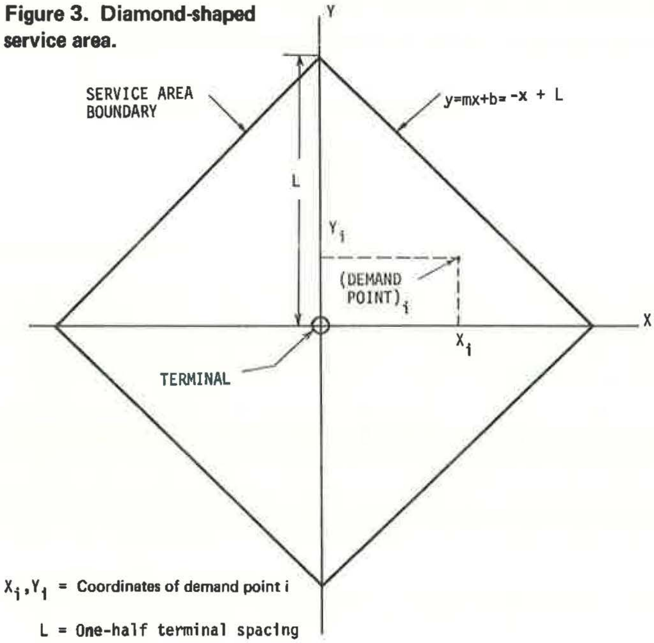
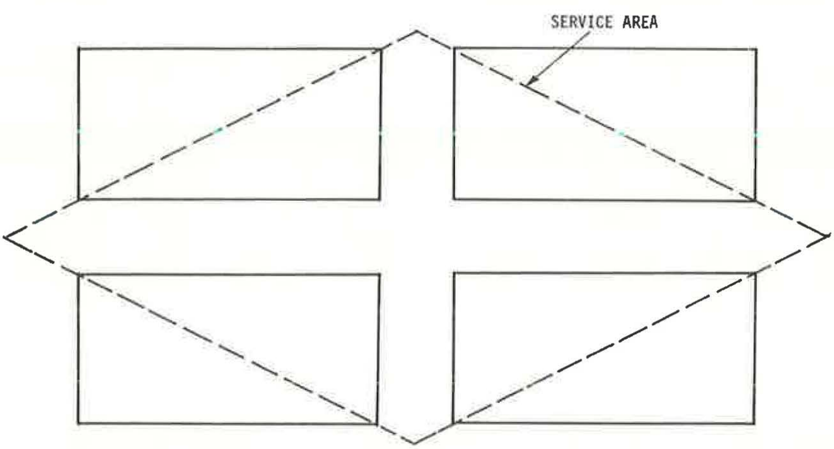


Figure 4. Elongated diamond-shaped service area superimposed on rectangular city blocks.



$$D_t = 4 \int_0^L \int_0^{(\ell - \ell x/L)} (x + y)\, dx\, dy = \tfrac{2}{3}\, \ell L(L + \ell)$$

$$\bar{D} = (1/A)D_t = \tfrac{1}{3}\,(\ell + L)$$

where

A = size of service area,
D_t = total walking distance, and
$\bar{D}$ = average walking distance.

The comparative efficiencies of the elongated diamonds and the symmetric diamond are graphically shown in Figure 5. Because the difference in efficiency varies with the degree of elongation, an actual block configuration will determine the relative efficiency of various diamond-shaped service areas.

Multiple Terminal Considerations

As discussed previously, a truly optimum transportation mode must provide some level of service to all points within a city; therefore, adjoining service areas must border each other so that the certain area or the entire city is completely serviced (Fig. 1). For the network to operate at maximum efficiency, it is important that service areas be easily discernible and terminals be found easily, even by the inexperienced user. Uniformity of terminal service area boundaries that do not cut across a block side was assumed to contribute to providing these optimum characteristics.

If demand within an entire area served were uniform, a single service area size would prove optimum, and the entire area would be best serviced by a set of equally sized service areas. This is proved by contradiction; there would be nothing to require a different service area size. If demand within the total served area were not uniform, two possibilities exist. First, the area could still be covered with equally sized service areas, or the service area size could be determined on a unit basis so that each area would optimally service the demand peculiar to the area it covers. Second, because customers will most often use the terminal nearest their destinations, the practical service area boundaries will not coincide with those specified by analysis, and optimum characteristics will not be retained. The perpendicular bisector of the (imaginary) line connecting any two terminal locations will establish the service area boundaries between the two terminals. The dotted lines in Figure 6 show this perpendicular bisector effect. Extending this concept across the entire city area served indicates that all service areas should be of equal size. This may force a non-optimum service area size on a particular city section, so a large number of comparisons between different, feasible service area sizes must be made before local improvements can be made.

If demand unexpectedly and radically changes at some future time, an auxiliary terminal could be provided for somewhat improved service without disrupting the entire network. For instance, a large department store attracting many retail customers might be built on a vacant lot near the point where an auxiliary terminal could be located. Central-place theory would indicate that an additional terminal could be located at a point central to all neighboring terminals. However, because of the perpendicular bisector effect, the service area of the auxiliary terminal would become a non-optimally shaped, square pattern.

SERVICE AREA SIZES

Previous studies (16, 22) indicate that the trip density is constant for a constant type of generating area. Accepting the assumption that demand within an area of each section of a city is nearly uniform, we see that there is a unique optimum service area size

Figure 5. Examination of elongated diamond service areas.

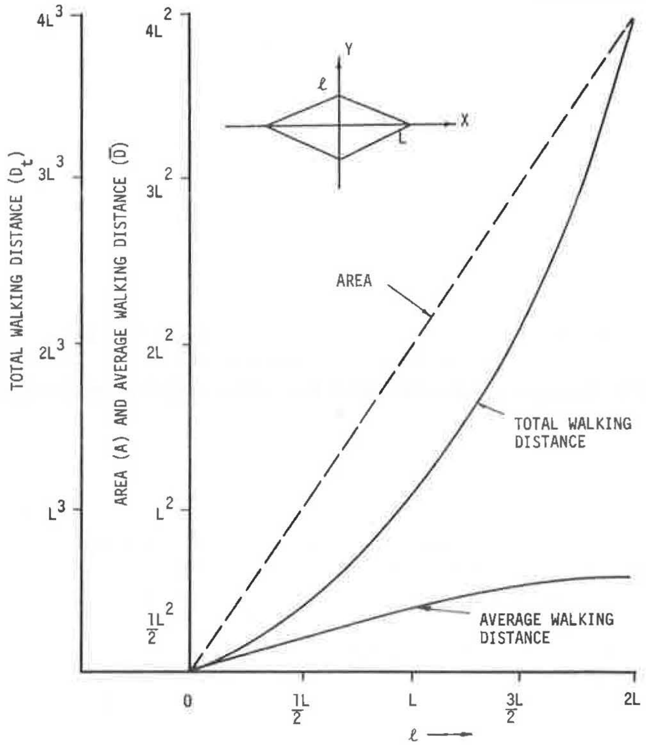
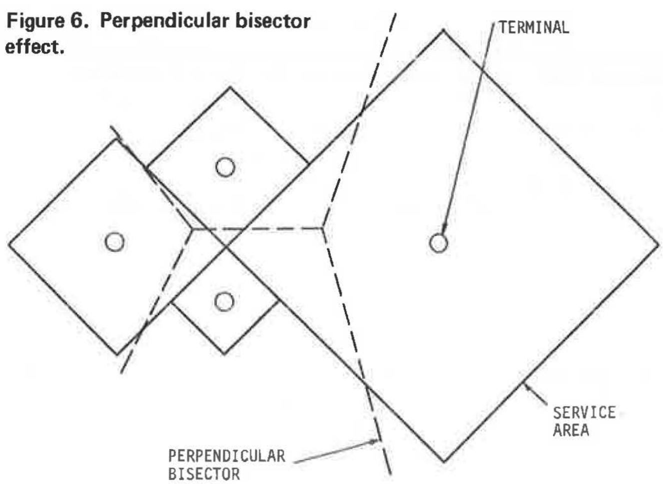


Figure 6. Perpendicular bisector effect.



within each section and that further improvements cannot be obtained by replacing that with equivalent service areas within the section. If the service areas of two bordering localities are of different sizes, the bordering service areas must be analyzed in detail to determine the best geometry of transition between the two service area sizes.

As stated before, the unique optimum size for each area requires that the distance from the center of the service area to the vertexes of the service area be only one block, two blocks, three blocks, and so forth. The upper limit of feasible size is about 1,250 ft (6, 16) inasmuch as this was assumed as the maximum desirable walking distance under the normal condition. Scaling a map of the central part of Philadelphia, for example, and performing the necessary calculations showed the average block length to be approximately 407 ft (19). For this case, sizes larger than three blocks are not feasible. Comparison with the maps of other cities indicated that this size can be considered an upper limit in a great many cases. Indeed, a two-block upper limit would be used in many areas.

One other feasible possibility exists. If the distance from the center of the service area to the vertexes were one-half block, all of the constraints mentioned would be met. However, this would call for a terminal at every intersection in a city. This possibility was not considered further because it was assumed that land for this excessive number of terminal facilities would not be available in a practical application. Therefore, only three feasible service area sizes were considered in this study: size 1, size 2, and size 3 corresponding to the number of blocks between the center of a service area and a vertex.

Several practical advantages are gained by this constraint. If service areas were of various sizes and boundaries were cut block sides at various spots as shown in Figure 7, it would be difficult for a user to determine which service area included his point of destination. Incorrect decisions would cause increased walking cost and disturb the optimum capacity of terminals. Terminals may also be easier for the inexperienced traveler to find under constraints that yield some uniformity.

A size 1 service area provides service to half of each of four blocks, or a total of eight block sides. A size 2 service area encompasses a total of 32 block sides, and a size 3, a total of 72 block sides. Thus, the total walking distance increases exponentially with the size of service area, so actual values will not increase as rapidly. Figure 8 shows the total walking distance as related to service area size.

EFFECT OF TERMINAL DISPLACEMENT

As indicated previously, the center of the service area should be located at an intersection. The distance between the service area center and the vertexes of the diamond, therefore, must be an integer multiple of block sides. Figure 9 shows the major advantages of such requirements, which are listed in the following:

1. Terminal A provides minimum average walking distances for all users. The cross-hatched areas show that, for the service area with terminal B, some demand points can be reached only by walking longer distances outside the defined service area.
2. Terminal A provides more easily discernible service area boundaries, inasmuch as boundaries do not intersect block sides.
3. Terminal A provides better safety features and less walking time by reducing the number of street crossings.

Although locating the terminal at an intersection is desirable, congested traffic and land availability at urban intersections usually limit an intersection location for a terminal. In this respect, a slight displacement of a terminal from the intersection is often necessary to be practical. For the uniform demand case, the terminal may be a small distance from the center of the service area (the intersection) without changing the average and total walking distances. The service area geometry, however, must remain centered at the intersection. This fact is proved analytically and shown in Figure 10. The permissible displacement of individual terminals from the intersection center would provide great flexibility in determining a terminal location because it would be at any of four corners or along any of eight block sides near the intersection.

Figure 7. Nonfeasible service area size.

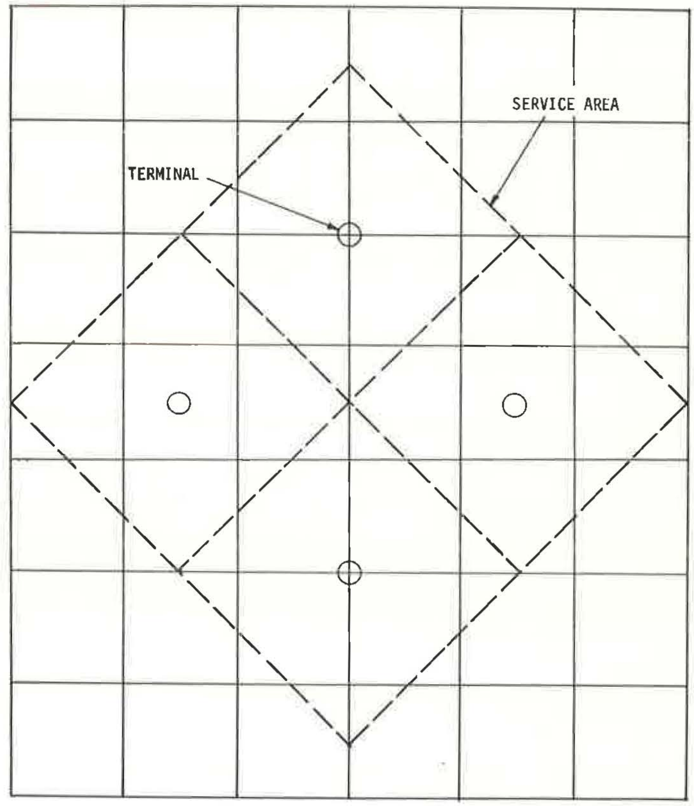


Figure 8. Total walking distance by service area.

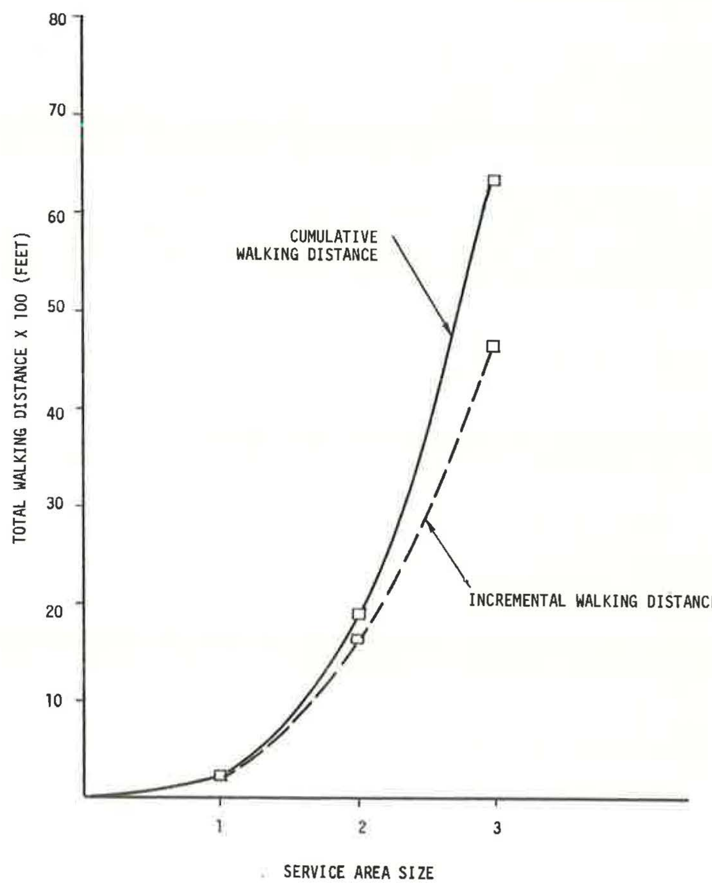


Figure 9. Effect of terminal displacement.

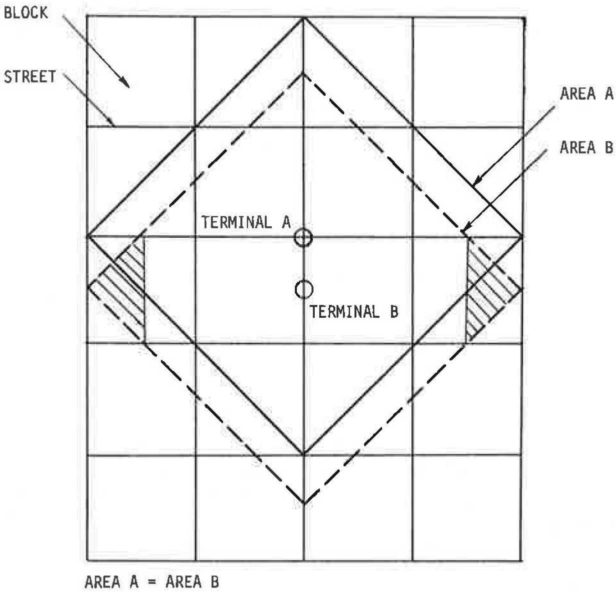
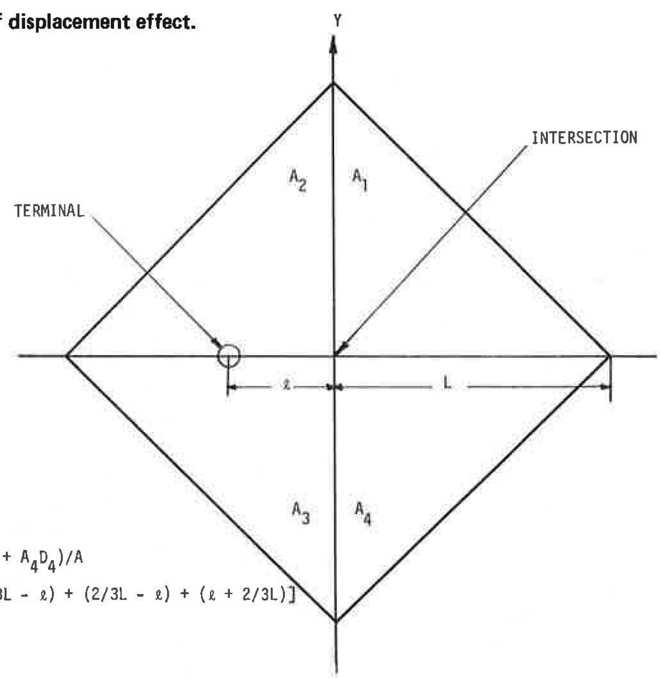


Figure 10. Analysis of displacement effect.



$$\bar{D} = (A_1D_1 + A_2D_2 + A_3D_3 + A_4D_4)/A$$

$$= \frac{1}{4}\left[(\ell + 2/3L) + (2/3L - \ell) + (2/3L - \ell) + (\ell + 2/3L)\right]$$

$$= 0.67L$$

$$= 0.47S$$

The site where land is available at lowest cost and is most compatible with the traffic flow in the area should be selected. If a terminal is located near an intersection, the entrance should be as far away from the intersection as possible. This criterion minimizes the adverse effect of terminal traffic (vehicle and pedestrian) on intersection capacity.

A reduction in users' walking distances may be gained by locating the terminal closer to higher demand points under non-uniform demand cases. Major displacements of the terminal, however, may affect the service area boundaries (it was assumed that people will always use the nearest terminal in a network of terminals). This assumption led to a determination of the optimum terminal location under the non-uniform demand case.

If demand within the defined service area of a terminal is non-uniform, the optimum terminal location is not necessarily at the center of the service area (the intersection). To determine the optimum location for a terminal with the objective of minimizing the walking distance, we used the Golden Section Search procedure (20). Because a small displacement of a terminal from the intersection was shown to shorten the average walking distances and because the terminal is constrained to lie along the outer periphery of blocks, determination of the optimum terminal location can be accomplished by two undimensional Golden Section Searches. One search determined the best location parallel to the X axis along the street that corresponds to the diagonal parallel to the Y axis. The search was limited in all directions to within the arbitrarily selected 200 ft from the intersection; and the objective function, user walking distance, was minimized. The minimum walking distances for the location in two directions were compared, and the better of the two was selected. This location was at the center of the street, and the best side of the street was determined to be the side favored by the search along the other axis direction. A computer program was developed to control this optimization. Results were found to be consistent with the general hypothesis that the unique optimum location of the terminal must be found in accordance with the demand distribution within the service area. In this case, the quantity and direction of terminal displacement toward points of greater demand can be effectively governed to achieve the optimum characteristics.

SUMMARY AND CONCLUSIONS

To be successful, a transportation mode must go beyond providing satisfactory service from terminal to terminal. A total system approach is called for in which consideration is given to the cost of convenience associated with the actual location of the terminal. This study attempted to quantify some of these critical relationships in terminal location. Although a number of diverse theories, such as the classical central-place principle, have been employed to determine the optimum location pattern for service centers, this study represents a departure from previous work in two distinctive ways: Gridiron routes corresponding to urban street patterns are assumed to prevail inasmuch as this configuration is found in the majority of American cities; and the terminal location system is optimized from the viewpoint of minimizing user access distance, which, in turn, maximizes the urban mobility.

A terminal in a network of terminals should privide service to all points within a bounded area. In this study, terminal location was constrained to lie along the outer periphery of block sides, and users were assumed to patronize the terminal nearest their destination. The configuration of the optimum service area was shown to have several important characteristics: The service area must be diamond shaped with major axes paralleling the gridiron street pattern in urban areas; the service area must be centered at an intersection; the distance between the service area center and vertexes must be an integer number of block sides; and the border lines of the service area should be parallel to street block diagonals.

For the multiple service areas of a terminal network that provides some level of service to all points within an urban area, curtailment of the possible sets of service areas was shown to be possible. The best set of service areas of uniform size may be determined for a section of uniform demand characteristics. However, the economical consideration of the terminal location pattern must be weighted with the present land availability and the traffic flow conditions for practical applications.

The method developed by this study appears to show considerable promise in helping to solve the location problems of transportation terminals of existing cities as well as those of new towns. However, a complete set of actual data describing user characteristics, the specified service level of the transportation mode in question, and the physical layout of the concerned area should be collected to test the proposed method. If a city is served by a number of transportation modes, the integration of all terminal locations for all modes is extremely essential to optimize the system operations. Therefore, further study is necessary not only to test the applicability of the derived solution method by practical applications but also to extend the analysis to a more comprehensive scale in consideration of the terminal interfaces among multimodes in urban areas.

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