A CAPACITY ANALYSIS TECHNIQUE FOR HIGHWAY JUNCTIONS

Joseph A. Wattleworth, University of Florida; and
Jerry W. Ingram, Florida Department of Transportation

ABRIDGMENT

• LINEAR PROGRAMMING is a general form of applied mathematics in which a linear function (called the objective function) of a set of variables is maximized or minimized (depending on the nature of the problem) subject to a set of linear constraints. The procedure used to solve the linear programming problem will determine the values of the variables that maximize or minimize the objective function and that do not violate any of the linear constraints.

To supplement this verbal description, consider the following mathematical statement of the general linear programming model. The objective function takes the form

$$W_1V_1 + W_2V_2 + \ldots + W_nV_n$$

where V_i is a set of variables for which the values are sought and W_i is the value weights of each of the variables. This is the linear function that is to be either maximized or minimized. Next, a set of constraints is developed that define the numerical range from which the values of the variables are taken. An example of such a constraint is

$$A_{i1}V_1 + A_{i2}V_2 + \ldots + A_{in}V_n \ \{<, =, >\} \ C_i$$

where C_i is a constant and the A values are the value weights of each of the variables. The general form of the linear programming problem can be stated as follows:

$$\text{Maximize or minimize } W_1V_1 + W_2V_2 + \ldots + W_nV_n$$

subject to

$$\begin{aligned}
A_{11}V_1 + A_{12}V_2 + \ldots + A_{1n}V_n \ \{<, =, >\} \ C_1 \\
A_{21}V_1 + A_{22}V_2 + \ldots + A_{2n}V_n \ \{<, =, >\} \ C_2 \\
\vdots \\
A_{m1}V_1 + A_{m2}V_2 + \ldots + A_{mn}V_n \ \{<, =, >\} \ C_m
\end{aligned}$$

This is a brief discussion of a rather complex form of mathematics. However, because of the widespread applications of linear programming, sufficient documentation is easily available.

INTERCHANGE CAPACITY ANALYSIS MODEL

General Approach

A linear programming model provides a rather effective means of determining the capacity of any type of interchange, regardless of the complexity of the geometric configuration. The capacity is defined as a maximum volume capable of entering the interchange without causing the capacity of any geometric element of the interchange to be exceeded. Because the capacity is the maximum number of vehicles that can use the interchange, the objective function of the model is to be maximized subject to the geometric characteristics of the interchange and vehicular distribution through the interchange.

Development of the Objective Function

The total number of vehicular movements that may occur at an interchange is a function of the number of approaches. The function is

$$x = n (n - 1)$$

where x is the maximum number of possible movements and n is the number of approaches. Therefore, for a four-leg interchange, with all movements permitted, there are 12 possible movements. Thus, the variables for the objective function represent the volume of each of the 12 movements approaching the interchange. Each possible movement must be explicitly defined. Because the volume entering the interchange is to be maximized, the objective function for a four-leg interchange capacity problem is

$$\text{Maximize } V_1 + V_2 + V_3 + V_4 + \ldots + V_{12}$$

or

$$\max \sum_{i=1}^{12} V_i$$

Because each variable in the objective function represents an individual movement, a weight value of one is associated with each variable. To determine the maximum volume capable of entering the interchange requires that this function be maximized subject to various geometric and vehicular distribution constraints.

Development of the Constraints

There are two types of constraints that must be met. The first set of constraints relates to the capacity of each of the interchange elements. Each of these constraints is related to a particular geometric element of the interchange and states that the volume using the element must not exceed the capacity (or service volume) of the geometric element. The specific values of the capacity constraints actually define the geometric characteristics of the interchange to be analyzed. The number of approach lanes, the number of lanes for a particular movement, and the signalization must be defined with respect to their individual capacities. Data shown in Figure 1 indicate that, if movements V_3 (right-turn volume from the west) and V_7 (left-turn volume from the east) must both use the same entrance ramp that has a capacity C_3, the following constraint must hold true:

$$V_3 + V_7 \leq C_3 = f \text{ (number of lanes)}$$

where C_3 is the capacity of the entrance ramp. Another example of physical constraint is

$$V_1 + C_2 + V_3 \leq C_{12} = f \text{ (number of lanes)}$$

where C_{12} is the capacity of the west approach.

This constraint must not be violated because these three movements enter the interchange from the west approach. If signalization is to be considered, its capacity is to be defined in terms of a critical lane volume concept (1). This will be elaborated on later.

The second set of constraints is developed to define the distribution of the movements through the interchange. This set of constraints ensures that the distribution of traffic among the various movements entering the interchange will be the appropriate distribution. If V_1 is 20 percent of the total volume entering the interchange and V_2 is 10 percent of the total, V_1 will always be twice V_2, and the following relationship is true:

$$V_1 - 2V_2 = 0$$

All possible movements can be interrelated in this manner. Therefore, it is quite obvious that accurate count data or accurate estimates of future distribution must be available to develop these constraints.

Period of Analysis

The analysis can be conducted on a peak-hour basis or a 24-hour basis. However, it is recommended that a peak-hour analysis be used for the following two reasons:

1. Peak-hour capacity must be adequate for the interchange to operate efficiently during these periods; and
2. The afternoon peak-period traffic patterns generally differ from the morning peak patterns.

FORMULATION OF THE MODEL FOR A DIAMOND INTERCHANGE CONFIGURATION

The following is the formulation of the linear programming model for a conventional diamond interchange configuration, with signalization (Fig. 1).

Development of the Objective Function

For this particular interchange all 12 possible movements are permitted. These are defined as shown in Figure 1. Therefore, the objective function is $V_1 + V_2 + V_3 + V_4 + V_5 + V_6 + V_7 + V_8 + V_9 + V_{10} + V_{11} + V_{12}$, which is to be maximized.

Development of Constraint Equation

Figure 1 shows the identification of the physical constraints of the interchange. A capacity constraint is placed on each major geometric element. The next step is to develop the physical constraint equations shown later. Basically each constraint states that, for a particular geometric element of the interchange, the volume of the movements using the element is less than or equal to its capacity.

$$\begin{aligned}
V_1 + V_9 &\le C_1 \\
V_{10} + V_{12} &\le C_2 \\
V_3 + V_7 &\le C_3 \\
V_4 + V_6 &\le C_4 \\
V_3 + V_7 + V_{11} &\le C_5 \\
V_1 + V_5 + V_9 &\le C_6 \\
V_2 + V_6 + V_{10} &\le C_7 \\
V_4 + C_8 + V_{12} &\le C_8 \\
\text{critical lane volumes} &\le C_9 \\
V_7 + V_8 + V_9 &\le C_{10} \\
V_{10} + V_{11} + V_{12} &\le C_{11} \\
V_1 + V_2 + V_3 &\le C_{12} \\
V_4 + V_5 + V_6 &\le C_{13}
\end{aligned}$$

The capacity C_9 is the capacity of the sum of the critical lane volumes (1) passing through the two traffic signals. To illustrate this procedure, consider the following example. Movements V_1 and V_2 enter the signalization from the west and are equally distributed over two approach lanes (such might be the case when V_1 and V_2 are relatively low). Therefore, half of V_1 and half of V_2 make up the critical lane volume from the west (Fig. 2). From the two-lane east approach, V_7 and V_8 pass through the signalization in separate lanes, but, because V_7 is greater than V_8 (for the example considered), V_7 is the critical lane volume for the east approach. The one-lane north approach to the signalization is used only by movement V_{10}; therefore, V_{10} is a critical lane volume on this approach. The only movement that enters the signalization by the one-lane south approach is V_4, and it must also be considered a critical lane volume. For this particular example the constraint equation for the signalization (C_9) is

$$0.5V_1 + 0.5V_2 + V_4 + V_{10} + V_7 \leq C_9$$

If two lanes were provided for the V_7 movement, that is, a double left turn for the east to south movement, the critical lane volume on the east approach would be $0.5V_7$ if $0.5V_7$ were greater than V_8. However, if V_8 were greater than $0.5V_7$, then V_8 would be the critical lane volume for the east approach.

It can be seen at this point that the physical characteristics of the entire interchange are modeled by the selection of the 13 capacity values (C_1 through C_{13}) and by the signalization capacity constraint coefficients.

Data given in Table 1 illustrate the development of the interrelationship constraints. From these data the 11 interrelationship constraints are developed and are presented as follows:

$$\begin{array}{rcl} 13.7V_1 - V_2 = C_{14} = 0 & \quad & 7.5V_1 - V_8 = C_{20} = 0 \\ 15.3V_1 - V_3 = C_{15} = 0 & & V_1 - V_9 = C_{21} = 0 \\ 4.3V_1 - V_4 = C_{16} = 0 & & 3.0V_1 - V_{10} = C_{22} = 0 \\ 48.4V_1 - V_5 = C_{17} = 0 & & 32.4V_1 - V_{11} = C_{23} = 0 \\ 10.8V_1 - V_6 = C_{18} = 0 & & 3.0V_1 - V_{12} = C_{24} = 0 \\ 26.5V_1 - V_7 = C_{19} = 0 & & \end{array}$$

As discussed previously, either a peak-hour or a 24-hour analysis may be conducted. If separate analyses of the morning and afternoon peak periods are conducted, two sets of interrelationship constraints must be developed, one set defining the morning peak-period vehicular distribution and the other defining the afternoon peak-period distribution, and two linear programming models are to be solved independently. One model will have the morning peak-period vehicular distribution, and the second model will have the afternoon distribution.

Assignment of Capacity Values

Thus, with the objective function defined and the constraint equations developed, only assignment of specific capacity values to the physical constraint equations remains before the linear programming problem is solved. The hourly capacity values are a function of the level of service at which analysis is to be conducted. For level of service E, as defined by the 1965 Highway Capacity Manual (2), the following hourly capacities are applicable:

Facility	Design	Capacity
Arterial street (minor)	Two lanes, one direction	3,000 vphg
	Three lanes, one direction	4,500 vphg
Freeway	Two lanes, one direction	4,000 vph
	Three lanes, one direction	6,000 vph
Ramp	One lane	1,500 vph
	Two lanes	3,000 vph

The capacities given for arterial streets (measured in vehicles per hour of green time) cannot be obtained if interference from nearby signalized intersections exists. If such is the case, some provision must be made to reflect these conditions.

Establishing an hourly capacity of the signalization is done with respect to the critical lane volume concept (1).

Figure 1. Identification of movement and physical constraints for a conventional diamond interchange.

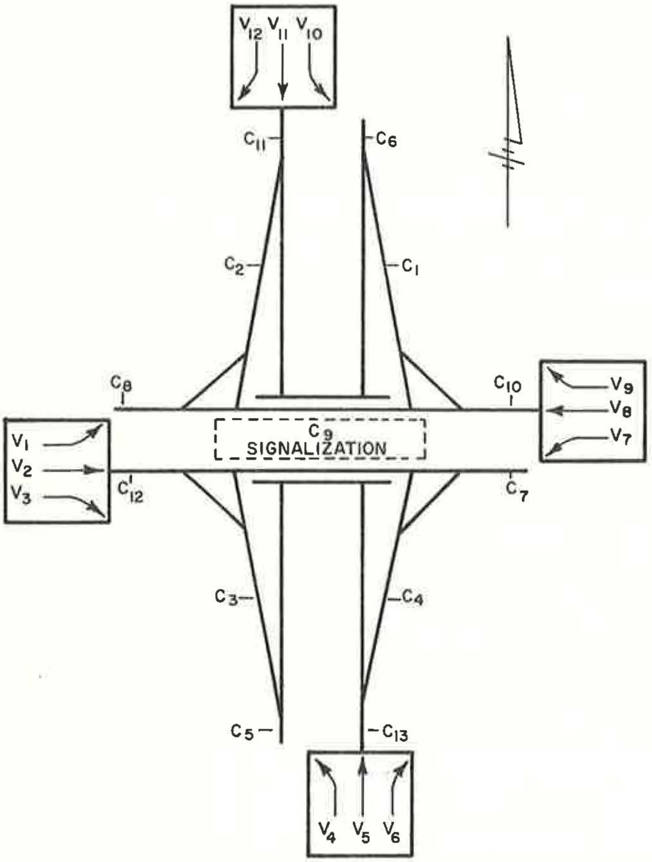

Figure 2. Lane use for determining critical lane volumes.

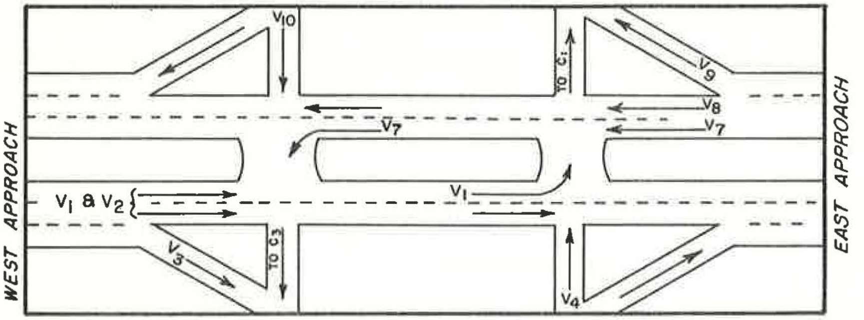

Table 1. Peak-hour distribution of entering traffic.

Movement	Percentage of Total Volume Entering Interchange	Movement	Percentage of Total Volume Entering Interchange
V_1	0.6	V_7	15.9
V_2	8.2	V_8	4.5
V_3	9.2	V_9	0.6
V_4	2.6	V_{10}	1.8
V_5	29.0	V_{11}	19.4
V_6	6.4	V_{12}	1.8

Solution of the Model

The linear programming model for the interchange is now complete, and the next step is to solve the problem. The simplex method is a mathematical procedure for solving linear programming problems. It can best be described as a technique of matrix algebra used to obtain the optimum values for the objective function. However, because of the widespread application of linear programming, numerous computer programs are available for solving such problems. IBM's Mathematical Programming System (MPS) was used for this study.

INTERPRETATION OF RESULTS

The purpose of the physical and interrelationship constraints is to define the area of possible solutions that will maximize the objective function. At least one of the physical constraints will be the critical constraint, and in the optimal solution that constraint becomes an equality. It is possible for two or three physical constraints to become critical simultaneously. The solution will indicate the value of the objective function, the values of each of the vehicular movements, the critical physical constraint or constraints, and the unused capacity of the physical constraints that are not critical.

The value of the objective function is interpreted as the maximum total volume that may be accommodated by the interchange configuration before congestion begins to develop. The critical element, that element on which the volume equals the element's capacity, is also identified. This is interpreted as the element at which congestion will first develop and is, therefore, the segment that limits the overall capacity of the interchange. The critical element could be any of the 13 elements shown in Figure 1. By identifying the critical element or elements, the designer can direct his attention to the needed areas of improvement and can modify these elements to increase their capacities. Thus, the linear programming model can aid the designer in developing new interchange configurations that provide higher capacities than the one currently under analysis.

CONCLUSIONS

The linear programming model provides the highway designer with an effective mathematical tool for the evaluation of the operational characteristics of an interchange subject to basic configuration, physical features, and traffic patterns.

With this ability, the designer can more effectively consider the problems of vehicular interactions and peak-period congestion within the framework of an interchange design sequence.

Wattleworth and Ingram (3) cover a case study in which the linear programming model was applied.

ACKNOWLEDGMENT

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