STATEWIDE COMPREHENSIVE TRANSPORTATION PLANNING

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•IN 1967 the Governor of Pennsylvania established a Committee for Transportation and charged it with the responsibility of developing a master plan for transportation and preparing an organizational design for a department of transportation that would develop and implement the plan. The Governor's committee presented a design for a Pennsylvania Department of Transportation that was approved by the legislature effective July 1, 1970.

In addition to the organizational design for the department, the committee assisted by various consultants developed a series of related transportation studies and programs for action that could be used and implemented by the new department. These studies included an exhaustive annotated bibliography of material published on transportation in Pennsylvania as well as other relevant sources (1); an inventory of the existing transportation facilities in the state, the capacity of present facilities, and present traffic flows, based on all available information on highways, buses, trucking, railroads, pipeline, urban transportation, waterways, and air (2); a short-range analysis and forecast of intercity passenger and freight demand for the year 1975 based on extrapolation of existing trends and comparisons with present facility capabilities (3); an interim plan consisting of a listing of recommended projects together with a simple technique for evaluating project priorities (4); and a long-range program of comprehensive transportation planning at the state level (5).

This paper describes the elements in a program of statewide comprehensive transportation planning that was recommended for implementation by the department. The paper is in 2 parts. The first part describes the elements of the statewide comprehensive transportation planning process and the specific modeling tasks that were identified, and the second part describes the regional model for forecasting the flow of freight.

ELEMENTS OF STATEWIDE COMPREHENSIVE TRANSPORTATION PLANNING

The process of comprehensive transportation planning at the state level is not unlike the approach developed for urban and regional transportation studies. The planning process is intended to yield information necessary to select from alternative solutions those projects that will direct the state toward achieving regional goals. However, regional transportation systems have impacts on community development and differ in purpose from urban transportation systems. Transportation planning is an element of comprehensive planning at both the state and local levels, and the objectives of the master plan are both to serve movement between points and to reduce or where possible eliminate the need for movement. The statewide transportation plan is a guide to local area development and implementation and should include all modes of transportation. The plan should also be responsive to statewide programs for economic development and recreation.

The statewide comprehensive planning process consists of the identification of the appropriate policy and effect variables, the collection of data, the use of demand models, and the generation and evaluation of alternative systems.

Existing techniques for implementing and expanding these elements were reviewed, and the appropriate methods were selected with recommendations for additional research.

Identification of Policy and Effect Variables

The appropriate span of activity for a comprehensive statewide transportation planning effort is a logical starting point for developing detailed work projects and tasks. Ideally all variables should be included, but in practice the statewide transportation plan will consist of a network of links and nodes each described in terms of capacity, cost, and travel time. Although the statewide master plan is viewed as a corridor network based on relatively few traffic analysis zones, the interaction with local and regional analysis programs is an essential part of the planning program.

The process of transportation plan evaluation is still undergoing refinement and revision. Beginning with economic criteria as the sole measure of effectiveness, transportation plan evaluation has broadened to include cost-effectiveness measures and subjective rating methods that rank order attributes not measurable in economic terms. The evaluation process appropriate at the statewide level is a simple nonformal approach in which the relevant criteria are identified and modified through interaction with appropriate legislative, advisory, and technical committees.

Direct effects of transportation plans include travel cost and time, reliability, comfort, convenience, and safety. Indirect effects include measures of socioeconomic activity such as gross output, employment, population, and income. Both direct and indirect effects are essential information for decision-makers, legislators, lobbyists, and pressure groups whose interests must be understood and who must have the appropriate information on which to base decisions.

Data Collection

The infrastructure studies (2) gave substance to the expected conclusion that existing data are inadequate for implementing a comprehensive transportation planning effort. A key finding is the need for an extensive data collection program that would be used to fill gaps in existing information and to calibrate the forecasting models. The data needs are greatest for freight flows by all modes, passenger flows by bus and rail, and access flow patterns for all modes but highway. To achieve a goal of comprehensiveness in statewide planning requires that data be gathered for all modes and for freight and passenger movements so that the impacts on socioeconomic activity can be considered and travel patterns can be analyzed for the entire journey.

Demand Models to Forecast Statewide Travel

The demand models required to forecast future passenger and freight flows are the econometric model, the freight modal-split model, the passenger demand model, and the network simulation model. These four basic transportation planning tools represent the fundamental mathematical relations for predicting the future distribution and allocation of traffic flows among major nodes in the state. The relation among these models is shown in Figure 1.

The econometric model, to be described at greater length later in the paper, estimates the intensity and distribution of socioeconomic activity based on statewide population projections and the transportation network. The model, which is a form of "input-output," is based on the premise that industrial location and activity are primarily influenced by freight transportation. The output of the model is an origin-destination matrix for commodities and products between each industry.

The freight modal-split model is a multiple regression analysis that allocates the percentage of the total commodity flow between 2 zones to each mode on the basis of shipper characteristics such as reliability, cost, travel time, and frequency of service. The model selected is compatible with the output of the econometric model and consistent with the outputs of the passenger demand model. The "abstract mode" approach is reflected in the variables selected and will allow new modes to be considered, together with existing ones. Because of the relatively large average travel distance for commodities, a fewer number of zones should be required for freight-flow estimation than for passenger-flow estimation.

The passenger demand model is of the "abstract mode" class that uses a single equation to estimate the number of trips between each zone pair by mode. The recommended passenger demand model includes the variables of population, institutional character, travel cost, travel time, number of modes available, frequency of service, and per capita income. An alternative approach that follows the usual trip generation, distribution, assignment, and modal-split procedure was not recommended because a simultaneous model that integrates the trip-making process is theoretically more sound and reduces the number of separate networks and iterations required.

The network assignment model follows conventional practice as developed in the manual published by the U. S. Bureau of Public Roads. The passenger and freight demand volumes by nonhighway modes are assignable directly. Vehicle conversions for passenger and freight flows are allocated to highway links by all-or-nothing assignments with capacity restraints.

Generating Alternative Transportation Systems and Evaluation

The transportation systems that will be developed as the basis for long-range implementation require that a series of plans be produced. The alternatives that are considered can be generated in a variety of ways but will rely on the interaction of the planning staff and the decision-maker. The state of the art of generating transportation supply alternatives is still reliant on judgment and experience. The existing network is a point of reference for modifications of the system, and changes will be based on estimates of travel demands, trip times, and cost that are produced by the demand model simulations. Techniques such as determining demand for travel on a specific mode, assuming that there is a ubiquitous network, can furnish insights concerning the most appropriate corridors and network configurations. The product of the iterative process of generation of alternatives, testing through demand simulations, and evaluation coupled with interaction between technical and policy staffs is a statewide master plan for transportation.

The planning methodology proposed for Pennsylvania was based on approaches that are consistent with contemporary transportation planning practice. The program was developed to permit early implementation of a statewide multimodal transportation plan. The models considered are those that both utilize proven planning practices and suggest new approaches that can be incorporated during the period that the master plan is being developed.

The recommended staging for implementing the program is based on 2 considerations: first, that the existing planning staff will continue and that maximum use should be made of this knowledge and experience and, second, that the creation of a master plan should not wait for new model development but should proceed on the basis of present operational methodologies. Accordingly, those elements concerned with travel demand forecasts and highway planning could be implemented prior to the development of the econometric model. In subsequent years the planning effort could be increased as data are made available.

The report envisioned 10 major tasks for implementing the program during a period of 60 months. The tasks that were included are review of methodological design; synthesis of existing data; design of data collection; collection of data; development of models including link grouping, network simulation, passenger demand, freight modal-split, and econometric; and integration of system.

The following section of the paper describes the econometric model that forms the basis for the socioeconomic inputs and forecasts of freight flow.

FREIGHT MODAL-SPLIT MODEL

The demand for transportation is a derived demand. The flow of commodities exists because production and consumption do not occur at the same time and place.

For any region the character of the commodity flows depends on the characteristics of the producers and consumers of the commodities and on their spatial distribution. Since every producer and consumer will require many commodities from various sources, the flow of these commodities becomes extremely complex, and any change in the socioeconomic character of the region will change the pattern of the flow.

The flow of commodities is accomplished by the transportation system that must be sufficiently versatile to provide the specific service requirements associated with the movement of any commodity. If all commodities were shipped in units of the same shape, size, and weight, there would be considerably less need for versatility in the transportation system. The growing importance of containerization results from the costs involved in maintaining a highly versatile transportation system.

The various modes of transportation employed in the complex movement of commodities are, perhaps, best differentiated in terms of their cost structures in relation to the spectrum of transportation services they provide. And since firms will tend to minimize their costs for any given service, the pattern of modal use throughout a region will tend to correspond to the pattern of commodity flows. In other words, the demand for various types of transportation services will depend on the specific mix of commodities for which the services are required. Thus, any attempt to forecast transportation needs for a region must be based on anticipated flows of different commodities throughout the region, but these flows are directly determined by the socioeconomic structure of the region, that is, by the specific locations of firms that produce and consume goods and by the particular production process in which these firms are engaged. Thus, any serious attempt to forecast transportation needs of a region cannot take the pattern and operational distribution of economic activities as given but must provide a mechanism for anticipating redistributions that will result from changes in final demand and in cost structure, including changes in the transportation system itself. Gravity models, however sophisticated, all take as given the social, economic, and demographic pattern of the region. They have been very useful in short-term predictions of transportation needs, but they are totally inadequate for long-term forecasting.

The ideal transportation forecasting model for a region would present for any given point in time a table, or set of tables, that would show, at a glance, the forecast flow of traffic of each class and by each mode from every point of origin within the region to every destination. In addition, the table should show the extent to which each flow is impeded by the capacity of the transportation system. This ideal model might be difficult to develop, but it must be simple to interpret. In addition, its forecast should be highly reliable.

It was the purpose of the research team that worked on the "framework for transportation planning" to develop a transportation forecasting model for Pennsylvania that satisfied the criteria of the ideal model as defined above. It was recognized at the outset that a reliable model would require reliable data that were not currently available—good freight-movement data are almost nonexistent. Hence, the only data restriction imposed on the model was that it should not be unreasonable to expect that the required data could be collected. The model developed was based on the Leontief type of input-output technique but incorporates certain refinements that enable it to be responsive to changes in price, in freight rates, and in the transportation system itself.

Input-Output Model

The first step in the development of the transportation model is an input-output table that describes not only the shipment of commodities among industries but also their distribution throughout the region and the rest of the world. For this purpose it is necessary to identify shipments by commodity classification and by origin and destination. The table, then, consists of a column and a row assigned to each industry at every node (important center of economic activity) in the region, and additional columns and rows for nodes outside the region. The elements of a column are the flows of commodities from various sources to one industry at a given location. The elements of the corresponding row are the flow of goods from that industry to other industries throughout the state and to the rest of the world. The model requires that technological coefficients be computed for all inputs that are directly related to the various production processes.

Transportation services used by a firm are only incidental to the acquisition of resources and not directly involved in their productive use. There will, therefore, be no rows assigned to this industry although there must be the appropriate columns. However, since the entire input-output table is a detailed description of commodity movements, it may also be regarded as an equally detailed description of the output of the transportation industry.

The fundamental unit in the model is the physical quantity of a commodity that is shipped from one place to another. Let X_{igjh} be the annual amount of commodity i produced in zone g that flows to industry j located in zone h. We assume that there is a one-to-one correspondence between commodity and industry. That is, commodity i is produced by industry i, where i may symbolize agriculture, steel, or glass. If there are n industries, then i and j will have values 1, . . ., n. Similarly, if there are m nodes, g and h will have values 1, . . ., m. The amount X_{igjh}, for the most part, will be given in physical units, e.g., tons, but in certain cases it will be expressed as dollar values.

For any given year (t), the complete set of all $X_{igjh}(t)$ for that year, written X(t), describes completely the flows of all commodities into and out of all nodes in the state. This set of $X_{igjh}(t)$ may be conveniently displayed in a table constructed in the manner shown in Figure 2.

Although the physical quantities of the commodity flows are the prime interest of the transportation model, it is necessary for certain computational purposes to have these flows expressed in dollar terms. Let p_{igjh} be the average price of X_{igjh} at the point of origin, and let f_{igjh} be the average cost per unit of shipping X_{igjh} from the point of origin to its destination. Then, $Y_{igjh} = (p_{igjh} + f_{igjh}) X_{igjh}$ is the total delivered value of the amount of commodity i produced at node g and shipped to industry j at node h. (For purposes of the analysis, it will be convenient to proceed as though all shipment costs pass through the books of the firm at the origin of the shipment and are included specifically as shipment costs in the price paid by the purchasing firm. These assumptions do not affect the equilibrium solutions of the model but simplify the analysis.) For any given year (t), the complete set of all $Y_{igjh}(t)$, written as Y(t), describes the money flows corresponding to the commodity flows into and out of all nodes in the region. A complete table Y(t) may be constructed in a form similar to that shown in Figure 2.

Since we are particularly interested in the pattern of shipment costs involved in the movement of commodities, we define $S_{igjh} = f_{igjh} X_{igjh}(t)$. For any given year (t), the complete set of all shipment costs $S_{igjh}(t)$, written as S(t), describes the freight revenues corresponding to the freight movements throughout the system.

So that the Y(t) table might explicitly show all money flows within the system, it is necessary to add rows that account for strictly monetary transfers, such as taxes, interest payments, and undistributed profits, that are not directly related to the production processes. Various totals and subtotals are defined as follows:

$$X_{igjo} = \sum_{h=1}^{m} X_{igjh}$$

which is the total shipments of industry i at g to industry j everywhere. Totals of Y and S are defined in the same way as the total of X.

The next step in the development of the transportation forecasting model is the formulation of a methodology that will predict how the various inputs and outputs of industries throughout the state will vary with predicted changes in final demand, in cost structures, and in the transportation system itself. The theoretical basis for such a methodology is presented in the following 2 sections. When this methodology is implemented for some given future time period, the predicted commodity flows for that period would be presented in a table of identical form as that shown in Figure 2. For the reasons presented earlier, this table would be interpreted as a detailed description of the demand for transportation throughout the state.

The final stage in the development of the forecasting model is the assignment of the forecast freight flows to various modes and traffic routes. The method by which this is done was described in the first part of this paper. The detailed output of the integrated set of models is a set of tables, each similar to the one shown in Figure 2 in that all commodity flows through the state are shown by origin and destination. In addition, however, the mode and the route used are also given. This degree of detail is necessary in the development of the forecast, but considerable aggregation is necessary for practical purposes. The final output is a table in which commodity flows are aggregated to show expected total freight movements by mode and route from origin to destination throughout the state. In the methodology by which assignment of commodity flows to specific modes and traffic routes is made, considerable attention is given to the capacity of the transportation system that is expected to exist in the period of the forecast. The final tables would, therefore, be constructed to show capacity utilization by mode and route in addition to the actual traffic flows.

From the foregoing description of the output of the forecasting models, it should be apparent that our criteria for an ideal transportation forecasting model would be met in that forecasts are to be presented in readily understood detail of the type necessary for comprehensive statewide transportation planning. It is necessary, however, that there should be considerable confidence in the reliability of the forecasts, and to ensure this requires that a sound theoretical framework be developed and that the model itself be implemented on the basis of adequate and reliable data.

The following 2 sections of this paper outline briefly the way in which a theoretical framework has been developed for making the technological coefficients of the input-output model sensitive to changes in costs of transportation.

Theoretical Framework of Input-Output Model

In the usual Leontief input-output models, the ratios

$$R_{igjh} = Y_{igjh}/Y_{oojh} \tag{1}$$

are formed. (The model usually omits specification of the originating mode g on the assumption that commodity i from all nodes is identical. The ratios then become Y_{igh}/Y_{ojh}. The model here presented does not make the assumption given above for 2 main reasons. The first is that with the aggregation involved in any feasible commodity classification there will be considerable variation in the composition of shipments of the same class from firms at different nodes. The second reason for the specification of the point of origin is that this permits the development of a transportation sensitivity not possible without it.)

Each ratio R_{igjh}, so defined, is called a technological coefficient and is the proportion of total expenditure of firm j at node h that goes to purchase commodity i at node g. If these technological coefficients are assumed to be constant, the input-output model becomes consistent with the assumption that all industries have constant returns to scale. In addition, there is the implication that total expenditures for any commodity are insensitive to price changes. This, in turn, would be consistent with an assumption that industries operate like firms that minimize costs (or maximize profits) and have a very special form of linear homogeneous production functions—the Cobb-Douglas production function—which may be represented by the following equation:

$$X_{jh\infty} = \theta_{jh} \prod_{i=1}^{n} \prod_{g=1}^{m} X_{igjh}^{\alpha_{igjh}} \tag{2}$$

where the assumption is made that

$$\sum_{j=1}^{n} \sum_{h=1}^{m} \alpha_{igjh} = 1 \tag{3}$$

It can be shown that if these assumptions are made with the further assumption that total revenue equals total cost for each industry, then the technological coefficients of the Leontief input-output model at equilibrium are equal to the corresponding exponents of the Cobb-Douglas production function. Thus,

$$R_{igjh} = \alpha_{igjh} = (p_{igjh} + F_{igjh})\, X_{igjh}/Y_{oojh} \qquad (4)$$

Figure 3 shows the solution for a 2-commodity model.

The curve Z represents the various combinations of x_1 and x_2 that yield the output Z according to the given production function. The curve Z' represents a level of output higher than Z. The line joining c/p_1 and c/p_2 represents the combinations of x_1 and x_2 that can be purchased with a given budget C when prices are p_1 and p_2 respectively. (These p's are assumed to contain the freight rates.) The point B shows the optimum combination of x_1 and x_2 for a given budget C and indicates that Z is the highest level of output obtainable with this budget.

The ray OA is an expansion path. It gives the optimum combinations of x_1 and x_2 in the fixed proportion, $x_1/x_2 = (\alpha_1/\alpha_2)\,(p_2/p_1)$, that would be used for all levels of output with the prices held constant. If the price p_2 should fall to p_2', then the expansion path becomes OA$'$. The fall in price p_2 to p_2' leads to no change in the quantity x_1 if total expenditures for all resources (x_1 and x_2) remain fixed at C. It does, however, lead to an increase in the quantity of x_2 from $\alpha_2(c/p_2)$ to $\alpha_2(c/p_2')$ and to an increase in the physical volume of output from Z to Z'.

If, however, the output needed does not rise to Z', then the quantity Z would be most efficiently produced with the combinations of x_1 and x_2 at the point B'' where the curve Z cuts the expansion path A'. At this point the ratios of the costs of the inputs remain as they were before, but actual costs of both inputs are cut by the same proportion. Thus, at whatever level of output the firm operates, the ratios of expenditures on the inputs are constant for efficient production. It may be shown that actual expenditures would be as follows:

$$p_1x_1 = \alpha_1 C$$

$$p_2x_2 = \alpha_2 C$$

That is, α_1 of the budget would be spent for x_1, and α_2 would be spent for x_2. These ratios are exponents of the respective x's in the production function, and, therefore, the production function can be derived from observation of actual expenditures for the different inputs.

The relations discussed above provide a means by which a production function can be used to provide a basis for the input-output model when the assumption cannot be made that all costs are for resources used in actual production.

The production function provides an equilibrium relation between total cost for productive resources and the level of output when the prices of resources are given. Thus, in the Cobb-Douglas 2-industry case presented above it can be shown that

$$C = (Y/p_y) \div \theta\,(\alpha_1/p_1)^{\alpha_1}\,(\alpha_2/p_2)^{\alpha_2} \qquad (5)$$

or

$$Y = [p_y\,\theta\,(\alpha_1/p_1)^{\alpha_1}\,(\alpha_2/p_2)^{\alpha_2}]\,C = kC \qquad (6)$$

If the technological coefficients are redefined as

$$R_{igjh} = Y_{igjh}/C_{jh} \qquad (7)$$

where C_{jh} is the total cost of the resources that appear in the production function, then the input-output model in matrix form may be modified as follows:

$$RC + D = Y = kC \qquad (8)$$

$$(k - R)C = D \qquad (9)$$

where D is final demand and k is a diagonal matrix whose elements are the factors of proportionality such as that shown in Eq. 6 that relate the Y's to the respective C's.

Figure 1. Relation of demand models.

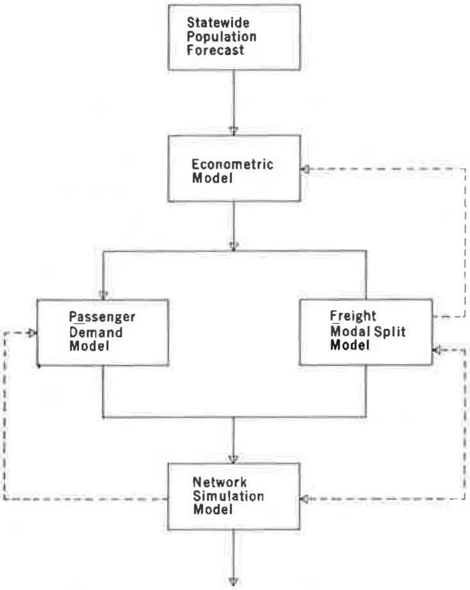

Figure 3. Determination of optimum input combinations.

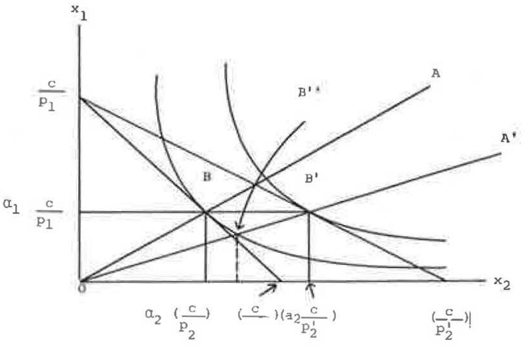

Figure 2. Interindustry internodal flows of commodities and services.

To / From	Industry 1 at Node	Industry j at Node	Industry n at Node	Total
	1 .... m	h	1 ... m	
Industry 1 at node 1 2 . . m				
S. Total				
Industry 2 at node 1 2 . . m				
S. Total				
Industry i at node g		X_{igjh}		X_{igoo}
Industry n at node 1 2 . . m				
S. Total				

The solution to the model given above becomes

$$C = (k - R)^{-1}D \tag{10}$$

on the assumption that the usual precautions are taken to ensure that R is square and nonsingular.

For a forecast D and for assumed prices and freight rates, the vector C may be computed from Eq. 10. It then is possible to construct from C and the set of technological coefficients a new input-output table that will show the flows of commodities resulting from the new D and the assumed prices and freight rates. It will also be possible to construct a revised table of all shipping costs. A comparison of this table with that for the base period will reveal the impact on the transportation industry of the changes incorporated in the forecast D and the assumed prices and freight rates.

Modification of the Model

It has been stated already that the usual input-output model assumes constant technological coefficients, that is, fixed proportions in the ratios of dollar expenditures for the different inputs of each industry. In the previous analysis it was shown how an underlying production function can be used to forecast changes in the proportions of physical quantities of inputs, and hence freight movement, that would result from changes in prices and changes in the cost of transportation. The price sensitivity introduced by this means might be adequate for practical purposes when inputs are aggregated and classified so that no commodity class is a good substitute for another. However, interindustry, interregional commodity movements typically show commodities of the same class but from different regions as inputs of the same industry. In such cases it seems reasonable to suppose that a change in the price differentials among competing sources of the same commodity class will result in more or less considerable change in the money flows as well as in the physical movement of these commodities. The input-output models so far developed have not been able to forecast such shifts. The following modification is sufficiently flexible to do this.

The variables in the model are defined as before. The proposed production function is given by the following formula:

$$X_{jh\infty} = \theta_{jh} \prod_{i=1}^{n} \left[\sum_{g=1}^{m} (\alpha_{igjh} X_{igjh})^{\beta_{ijh}} \right]^{\gamma_{ijh}/\beta_{ijh}} \tag{11}$$

This formula is a combination of the Cobb-Douglas function given above and a "constant elasticity of substitution" production function. The Cobb-Douglas part of the function is seen in the following collapsed form:

$$X_{jh\infty} = \theta_{jh} \prod_{i=1}^{n} X_i^{\gamma_{ijh}} \tag{12}$$

where

$$X_j = \left[\sum_{g=1}^{m} (\alpha_{igjh} S_{igjh})^{\beta_{ijh}} \right]^{1/\beta_{ijh}} \tag{13}$$

The Cobb-Douglas feature of the function affords the limited price and transportation sensitivity previously discussed in connection with that function. But this limitation is confined to only the relations among the different classes of commodities. A considerably greater degree of price and transportation sensitivity is possible among the different sources within any given class of commodities. This additional sensitivity is derived from the constant, but unspecified, elasticity of substitution feature of the function, which is that part given by Eq. 13.

The total effect of the function given is to provide that any given industry will maintain the same ratios for expenditures among commodities of different classes but may vary the shares of the expenditure on any given class that go to different sources. The precise nature of the shifts will become apparent from the following analysis.

It may be shown that if we assume that the sum of the γ's equals 1, then the proposed function is linear and homogeneous.

We now assume, as before, that each industry behaves as a single firm in combining its inputs so as to produce any level of output at minimum cost. The equilibrium solution for any industry is obtained by minimizing

$$C = \sum_{i=1}^{n} \sum_{g=1}^{m} p_{ig} X_{ig} \tag{14}$$

subject to the constraint of the production function

$$X = \left[\theta \prod_{i=1}^{n} \sum_{g=1}^{m} (\alpha_{ig} X_{ig})^{\beta_i} \right]^{\gamma_i/\beta_i} \tag{15}$$

where the subscripts j and h that identify the relevant industry and location have been dropped for simplicity, and p_{ig} includes the freight rate.

It may be shown that the equilibrium solution yields the following results:

$$\sum_{g=1}^{m} p_{ig} X_{ig} \div \sum_{i-1}^{n} \sum_{g=1}^{m} p_{ig} X_{ig} = \gamma_i \tag{16}$$

$$p_{iG} X_{iG} \div \sum_{g=1}^{m} p_{ig} X_{ig} = (\alpha_{iG}/p_{iG})^{\beta_i/(1-\beta_i)} \div \sum_{i=1}^{g} (\alpha_{ig}/p_{ig})^{\beta_i/(1-\beta_i)} \tag{17}$$

The relation in Eq. 16 states that at equilibrium the proportion of total expenditure that goes to commodity i from all nodes is constant and equal to the corresponding γ_i in the production function. This result is expected from the way in which a Cobb-Douglas feature is built into the production function.

The relation in Eq. 17 states that at equilibrium the proportion of expenditure for commodity i that will be spent at node G is a function of all the delivered prices of i from the various nodes. The parameters of the function are the corresponding α's and the β_i of the production function.

If Eq. 17 is modified by multiplying both sides by the corresponding sides of Eq. 16 we get

$$p_{iG} X_{iG} \div \sum_{i=1}^{n} \sum_{g=1}^{m} p_{ig} X_{ig} = \gamma_i \, (\alpha_{iG}/p_{iG})^{\beta_i/(1-\beta_i)} \div \sum_{g=1}^{m} (\alpha_{ig}/p_{ig})^{\beta_i/(1-\beta_i)} \tag{18}$$

The left side of this equation is the technological coefficient R_{ig} of the input-output table. We, therefore, have

$$R_{iG} = \gamma_i \, (\alpha_{iG}/p_{iG})^{\beta_i/(1-\beta_i)} \div \sum_{g=1}^{m} (\alpha_{ig}/p_{ig})^{\beta_i/(1-\beta_i)} \tag{19}$$

It is apparent that the R's are functions of prices and that they may be revised when prices change, provided that the coefficients of the production functions are known. Some progress has been made on the problem of estimating these functions, but more work is needed in this area (6).

It can be shown that the relation between y, the value of the product, and C, the total cost of productive resources, is given by

$$Y = p_y\theta \prod_{i=1}^{n} \left\{ \sum_k \gamma_1{}^{\beta_1} \alpha_{1k}^{\beta_1} \left[(1/p_{1k})(\alpha_{1k}/p_{1k})^{\beta_1/(1-\beta_1)} \div \sum_g (\alpha_{1g}/p_{1g})^{\beta_1/(1-\beta_1)} \right]^{\beta_1} \right\}^{\gamma_1/\beta_1} C \tag{20}$$

$$Y = kC \tag{21}$$

where k is the factor of proportionality in Eq. 20.

By means of the method presented at the end of the previous section, it is possible to adapt the technological coefficients given in Eq. 19 and the k's in Eq. 20 to a flexible input-output model that has considerable potential to reflect the effects of price changes and freight rates as well as changes in final demand.

The model presented in this paper was developed specifically for the analysis of the transportation sector; however, the methodology is readily adaptable to a much wider range of problems and offers much promise as an improved tool for planning.

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