INFLUENCE VALUE GRAPHS FOR CIRCULAR BEARING AREAS

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ABRIDGMENT

Based on the extended use of Boussinesq's theory of elasticity pertaining to vertical stress distribution in a semi-infinite, homogeneous, isotropic, elastic hemispatial medium by surface loading, the author presents for the engineering profession systematic 3-dimensional influence values in graph form for determining vertical, normal stresses at any point in an elastic medium from a uniformly loaded circular bearing area. The influence value graphs are easy to use. Influence values can be picked out directly, and they facilitate a quick determination of vertical stress fields in soil. Thus, these influence values rationalize and expedite effectively the work of soil and foundation design engineers.

•TO MEET the great need for vertical stress distribution influence values in soil from uniformly loaded circular bearing areas and to rationalize and expedite further the work of foundation engineers, the author planned and opened up the necessary analytical material for computer programming and, within the course of his academic activities, prepared and is submitting herewith to the engineering profession corresponding systematic vertical stress distribution influence value graphs shown in Figure 1. A family of isobars is shown underneath the circle in percentage of the contact pressure intensity σ_o (Fig. 2).

These systematically arranged vertical normal stress influence values permit one to determine directly, quickly, and effectively vertical (σ_z) stresses in soil and their distribution and to ascertain stress fields at any point in the soil from uniformly loaded circular bearing areas.

The derivation of the spatial σ_z stress, viz., influence values, is based on extending integration of Boussinesq's basic vertical, spatial stress component equation at any point in the elastic medium for a single, concentrated load (10) to a circular bearing area for a given uniform, nonrigid surface loading distribution σ_o.

EARLY SOLUTIONS

An analytical solution for stress distribution in an elastic, semi-infinite medium from a uniformly loaded circular bearing area on the horizontal boundary surface of the hemispace has been worked out by Lamb (13) and Terzawa (18) by means of Bessel functions. Other authors on this subject are Carothers (2), Jürgenson (12), Love (15; 9, p. 155), Palmer (17), Fröhlich (6; 9, p. 151), Newmark (16), and Fadum (4).

SOME RECENT SOLUTIONS

In 1953, Lorenz and Neumeuer (14) put forward their method of vertical, normal stress calculations from a uniformly loaded circular bearing area for any point inside and outside the circle. Because of the mathematical difficulties involved in solving for the σ_z stress, these authors evaluated the corresponding influence values by means of numerical methods. Their influence values, however, are given for $\nu = 3$ and for the following distances from (0; 0; 0):

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Figure 1. Influence values for vertical, normal σ_z stress at any point in elastic medium from a uniformly loaded circular bearing area.

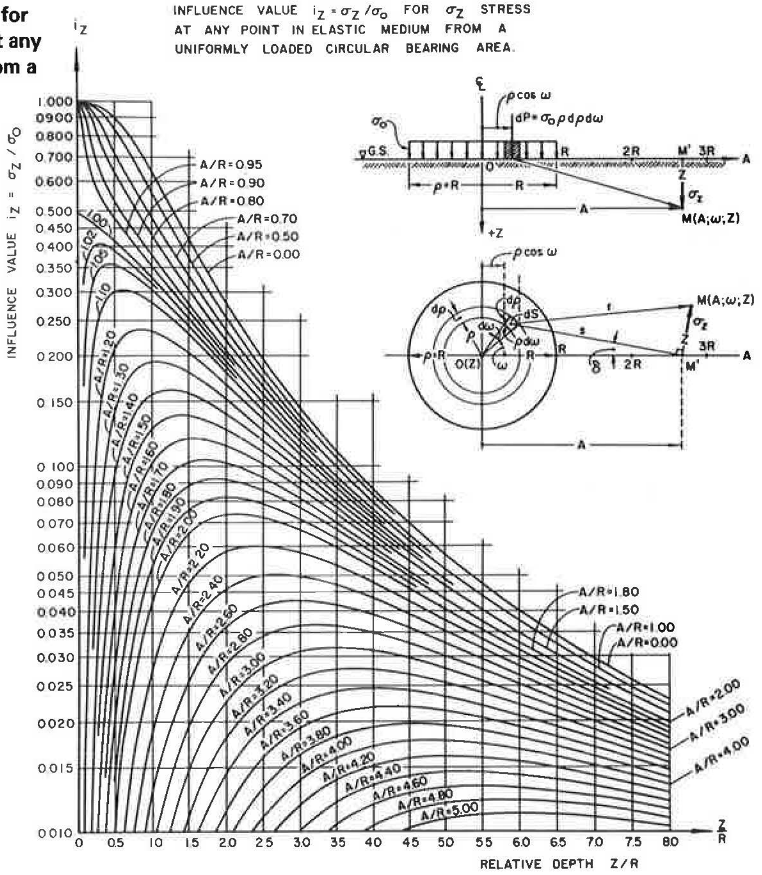
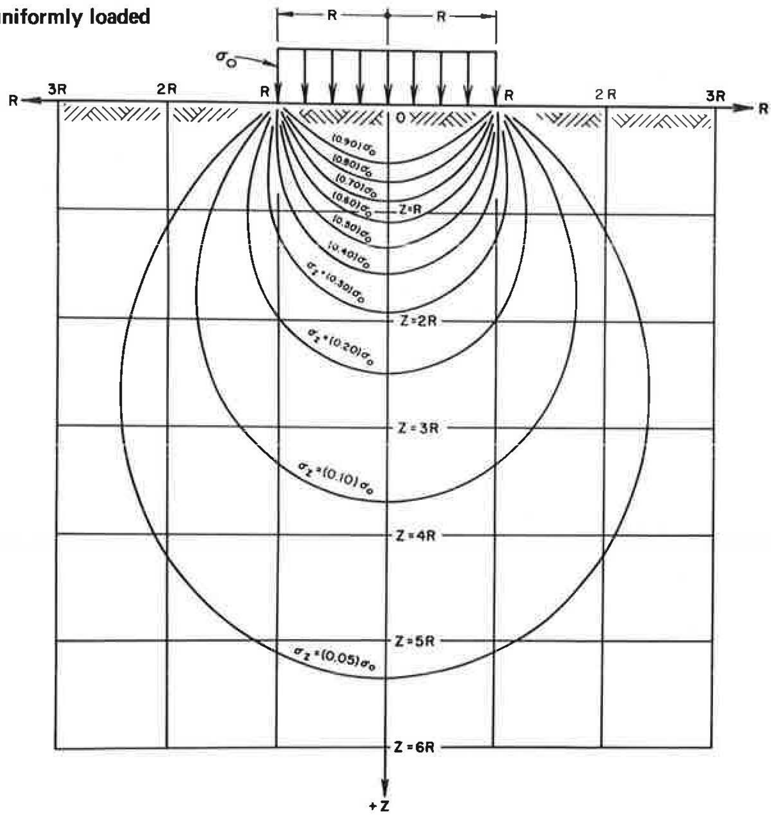


Figure 2. Isobars from a uniformly loaded circular bearing area.



1. For one-half of the radius (r/2);
2. For the periphery of a circle with radius r; and
3. For a point at a distance of 2 radii away from the center of the uniformly loaded circle (9, pp. 153-155). Hence, also, these influence values are for limited use only.

In 1954, Foster and Ahlvin also published charts for stresses from a uniform circular load (5). In this paper, no theoretical basis underlying the preparation of the charts is given.

BASIS FOR PREPARING INFLUENCE VALUES

Derivation

The basis used by this author for calculating the vertical, normal stress σ_z at any point in the elastic medium [say, point M (Fig. 1), whose coordinates are A, $\omega = 0$, and z], brought about by a uniformly loaded circular bearing area S, is the summation of Boussinesq's single, elementary, concentrated loads $dP = \sigma_o\ \rho\ d\omega\ d\rho$ over the entire circular bearing area S (9, p. 155).

$$d\sigma_z = (3/2)(dP/\pi)(z^3/r^5) = (3/2)(\sigma_o/\pi)\left[z^3/\sqrt{(s^2+z^2)^5}\right]\rho\, d\rho\, d\omega \qquad (1)$$

where

σ_o = intensity of uniformly distributed pressure over circle;
z = depth coordinate of point M below base level of circular bearing area;
$r = \sqrt{s^2 + z^2}$; (2)
= radius-vector from dS (viz., dP) to point M (Fig. 1);
$s = \sqrt{A^2 + \rho^2 - 2\,A\,\rho\cos\omega}$; (3)
= horizontal distance from dP (dS) to point M′ (trigonometric cosine law);
A = horizontal, radial distance of point M, viz., point M′, from z-axis;
ω = amplitude for ρ; and
ρ = variable radius for elementary load dP over elementary surface area dS of circle.

Substituting Eq. 3 into Eq. 1 and indicating integration of Eq. 1 over the entire circular area, we obtain an expression for the σ_z stress sought for any point in the elastic medium as

$$\sigma_z = (3/2)(\sigma_o/\pi)z^3 \int_0^R \int_0^{2\pi} (\rho\, d\rho\, d\omega)/[(A^2 + z^2 + \rho^2 - 2\,A\,\rho\cos\omega)^{5/2}] \qquad (4)$$

where R = radius of circle.

Solution

The integral from $\omega = 0$ to $\omega = 2\pi$ has been indicated already by Love (15) and by Lorenz and Neumeuer (14; 9, pp. 156-159)

Although the problem of finding the σ_z from a loaded circular area involves a uniformly distributed load σ_o, the computation of the σ_z stress is quite involved. The integral as given by Eq. 4 cannot be solved in a closed form. Its complete solution goes through elliptic integrals (15) or by numerical evaluation (14). The analysis necessary for evaluating the integral in Eq. 4 is rather long and involved. However, an analytical solution to Eq. 4 (due to Egorov, 3) for calculating the σ_z stress at any point in the hemispatial, elastic medium from uniformly loaded circular bearing areas involving elliptic integrals, as cited by Harr and Lovell (7) and by Harr (8), is given here by the author in terms of dimensionless parameters A/R and Z/R (Fig. 1) as

$$\sigma_z = \sigma_0 \left(N - \frac{(Z/R)}{\pi\sqrt{(Z/R)^2 [1 + (A/R)]^2}} \left\{ \frac{(Z/R)^2 - 1 + (A/R)^2}{(Z/R)^2 + [1 - (A/R)]^2} E(k) + \frac{1 - (A/R)}{1 + (A/R)} \Pi_0(k, p) \right\} \right) \tag{5}$$

where

A/R and Z/R = dimensionless parameters for relative distance and relative depth respectively;

$E(k)$ and $\Pi_0(k, p)$ = Legendre's complete elliptic integrals of the second and third kind respectively (1);

$k = \sqrt{[4(A/R)]/\{(Z/R)^2 + [1 + (A/R)]^2\}}$ = modulus of elliptic integral; and

$p = -[4(A/R)]/[1 + (A/R)]^2$ = parameter of elliptic integral.

Equation 5 comprehends a general solution in general terms in 1 equation for 3 special cases characterized by the quantity N, namely:

Point M	A/R	N
On central Z-axis	0 or < 1	1.0
Inside of circle	< 1	1.0
Periphery of circle	1	0.5
Outside of circle	>1	0.0

These 3 N-values (1.0, 0.5, and 0.0) result after the first integration of Eq. 4 with respect to ρ.

The nature of the computer printout (in capital letters) necessitates the use of capital letters in this development for programming of the influence values $i_z = \sigma_z/\sigma_0$.

INFLUENCE VALUE GRAPHS

Use of Graphs

The influence value curves $i_z = \sigma_z/\sigma_0$ resulting from Eq. 5 are shown in Figure 1. The numerical value of the vertical σ_z stress is obtained by simply multiplying the corresponding influence value i_z with the given uniformly distributed surface loading σ_0 over a circular bearing area as

$$\sigma = i_z \sigma_0 \tag{6}$$

For example, if A/R = 2.2, Z/R = 2.5, and σ_0 = 2.0 kg/cm^2, the influence value (Fig. 1) is i_z = 0.050, and the vertical stress σ_z is $\sigma = i_z \sigma_0$ = (0.050) (2.0) = 0.10 (kg/cm^2). Figure 2 shows isobars from a uniformly loaded circular bearing area.

Checking of Influence Values

These influence values were checked by comparing them with those already available in the technical literature. For example, the influence values along the vertical centerline (Z-axis) of the circle presented here were compared with those as published by Fadum (4). The comparison revealed a complete agreement in the $i_z = \sigma_z/\sigma_0$ values.

Further, a comparison was made of the author's and some of the few influence values given by Love (15). Again, a good agreement with the author's i_z-values was observed. The results independently arrived at by Love (15) and Carothers (2) are also in general in good agreement according to Palmer (17), thus, in a way, giving a tie-in check with Carothers' information. Also, the author's values [obtained by rigorous mathematical analysis (Eqs. 4 and 5)] are in reasonably good agreement with those determined by Lorenz and Neumeuer (14), obtained by the approximate method of numerical analysis using Simpson's rule (11). Thus, there is fairly good reason to believe that the influence values as prepared and presented here by the author are plausible ones to use.

CONCLUSION

The essential feature of these tables and graphs is that they give influence values directly and immediately for the vertical σ_z stress for any size of circular bearing area, for any depth, at any point in the hemispace of the elastic medium, and for any contact pressure σ_o —not merely along the centerline under the center of the circle or half-radius, along the periphery of the circle, or at points distant 2r from the vertical z-axis of the uniformly loaded circular bearing area.

The reader will surely appreciate the analytical effort that went into the opening up of Eq. 5 for computation and preparation of these influence value tables and graphs now so easy to use.

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