UNCERTAINTY OF SETTLEMENT ANALYSIS FOR OVERCONSOLIDATED CLAYS

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The uncertainty associated with using the Skempton-Bjerrum method for settlement determination in overconsolidated clays is evaluated by means of a probabilistic procedure wherein the usually deterministic parameters are represented by appropriate probability distribution functions. Some of these distributions are determined subjectively and combined with others that are based on measured data to deduce a probability distribution function for the settlement-layer thickness ratio. The uncertainty is characterized in terms of a 90 percent confidence interval, and values are presented graphically for a wide range of parameters. Engineering judgment was used in the selection of the subjectively determined parameter distribution, and the ensuing analysis and interpretation provide the design engineer with a rational and logical procedure whereby the reliability of a given settlement prediction can be assessed. Accordingly, the gross intuitive estimate of uncertainty associated with a conventional deterministic calculation is obviated.

• THE TOTAL settlement of an overconsolidated clay may be arbitrarily divided into (a) immediate settlement, or that settlement which occurs before dissipation of excess pore-water pressure has begun, (b) primary consolidation settlement, or that settlement which occurs while excess pore-water pressure is being dissipated, and (c) secondary consolidation settlement, or that settlement which occurs after excess pore-water pressure has been dissipated. However, the distinction between the abovementioned classifications of settlement becomes vague in a field situation and renders the settlement determination for foundations on overconsolidated clay a very complex problem. Owing to the uncertainty associated with the determination of an undrained stress-strain modulus for soil, the computation of immediate settlement is subject to considerable skepticism; in addition, it is difficult to ascertain the extent to which immediate settlement is influenced by partial dissipation of pore-water pressure during the loading process. Although the prediction of secondary consolidation is also very difficult, such settlement generally constitutes only a minor part of the total settlement of an overconsolidated clay provided a sufficient margin of safety against bearing capacity failure has been employed. Accordingly, the following analysis is restricted to a quantitative evaluation of the uncertainty associated with primary consolidation settlement, as determined by use of the method proposed by Skempton and Bjerrum (6).

NOTATION

The notation used in this paper is defined as follows:

- A = Skempton pore-pressure parameter;
- b = half-width of strip load or radius of circular load;
- C_r = recompression index;

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$\overline{C}_r$ = mean recompression index;
D = layer thickness;
E = error function for pore pressure parameter;
e = void ratio;
$\overline{e}$ = mean void ratio;
F_c = horizontal deformation factor for circular load;
F_s = horizontal deformation factor for strip load;
f = probability distribution function;
G = error function for horizontal deformation;
$\overline{G}$ = mean error function for horizontal deformation;
H = difference between high end of 90 percent confidence interval and $\overline{R}$;
H_R = $H/\overline{R}$;
L = difference between low end of 90 percent confidence interval and $\overline{R}$;
L_R = $L/\overline{R}$;
N = 0.866A + 0.211;
n = number of test values;
OCR = overconsolidation ratio;
p_c = preconsolidation pressure;
p_o = overburden pressure;
Δ_p = pressure increase;
Q = uncertainty factor for C_r - e relation;
R = settlement layer thickness ratio;
$\overline{R}$ = value of R obtained from mean parameter values;
V = coefficient of variation;
Z = $[\overline{C}_r - C_{r(1)}]/[VC_{r(1)}/\sqrt{n}]$;
z = depth below foundation base;
$\alpha = \left(\int_0^D \sigma_3\, dz\right)\Big/\left(\int_0^D \sigma_1\, dz\right)$;
δ = consolidation settlement;
δ' = one-dimensional consolidation settlement;
ξ = standard normal deviate;
μ = settlement reduction factor;
μ_c = settlement reduction factor for circular load;
μ_s = settlement reduction factor for strip load;
σ_1 = vertical stress; and
σ_3 = horizontal stress.

SKEMPTON-BJERRUM PROCEDURE

In order to account for dilatancy in determining settlements of overconsolidated soils, Skempton and Bjerrum (6) proposed a method that utilizes the A pore-pressure parameter, obtained from conventional triaxial tests, and a factor α, determined from the relation

$$\alpha = \left(\int_0^D \sigma_3\, dz\right)\Big/\left(\int_0^D \sigma_1\, dz\right) \tag{1}$$

α depends primarily on the geometry of the system, and tabulated values are readily available (given subsequently in Table 1). Next, a factor μ is given by

$$\mu_c = A + \alpha_c(1 - A) \tag{2}$$

for a circular footing, or

$$\mu_s = N + \alpha_s(1 - N) \tag{3}$$

for a continuous footing, where

$$N = 0.866A + 0.211 \tag{4}$$

is calculated. Then, the settlement δ for the overconsolidated soil is determined by taking the product of μ and the settlement δ', determined on the basis of the conventional, one-dimensional approach, or

$$\delta = \mu\delta' \tag{5}$$

Skempton and Bjerrum have indicated a correlation between the A pore-pressure parameter at working loads and the in situ overconsolidation ratio, and, more recently (1), curves have been presented to facilitate the direct determination of μ from these same parameters.

PROBABILISTIC APPROACH

The accuracy of the above-described Skempton-Bjerrum procedure is evaluated here by use of a probabilistic treatment that is similar to that previously employed (2) to study the one-dimensional consolidation of normally consolidated soils. This treatment consists essentially of representing the independent parameters by probability distribution functions instead of deterministic values in order to derive a probability distribution function for settlement; from this latter function, the uncertainty associated with computed settlement values may be inferred.

Deterministic Formulation

Combination of the probability distribution functions is achieved by Monte Carlo simulation, and, since this process requires formulation of the problem in terms of continuous functions, some curve-fitting of the tabulated relations presented by Skempton and Bjerrum is necessary. Accordingly, it is proposed to replace the discontinuous relation shown in Figure 1 by the empirical equation

$$A = 1.3(0.63)^{OCR} \tag{6}$$

Similarly, the factor α for a uniformly loaded circular area may be expressed by

$$\alpha_c = 1/\exp\{\exp[0.33 - 0.34(2b/D)]\} \tag{7}$$

in which b is the radius of the loaded area. Discussion (4) of the Skempton-Bjerrum work indicates that it is necessary to correct their data for a continuous strip load, and these corrected data can be closely represented by

$$\alpha_s = 1/\exp\{\exp[0.05 - 0.42(2b/D)]\} \tag{8}$$

in which b is the half-width of the strip load. Table 1 compares the previously tabulated values with those calculated from Eqs. 7 and 8.

Although the method does not account for the effect of horizontal strains, Skempton and Bjerrum claim, on the basis of numerical studies, that the error in computed settlements will not exceed approximately 20 percent; this error will be greatest when 2b/D is small and least when 2b/D is large. To account for this error in the formulation developed here, we propose the multiplying factors F_c for a circularly loaded area and F_s for a strip load, where

$$F_c = 1 + 0.01 \exp[2.753 - 0.45(2b/D)] \tag{9}$$

and

$$F_s = 1 + 0.01 \exp[1.654 - 0.45(2b/D)] \tag{10}$$

Typical values for F_c and F_s for various values of 2b/D are as follows:

2b/D	F_c	F_s
0.1	1.150	1.050
1.0	1.100	1.033
10	1.002	1.001

Although entirely empirical, Eqs. 9 and 10 allow the settlement computations to be realistically adjusted to account for the error associated with neglecting horizontal strains, and they form a sound basis for considering this aspect of uncertainty. Equations 2 and 3 may now be modified to

$$\mu_c = [A + \alpha_c(1 - A)] F_c \tag{11}$$

and

$$\mu_s = [N + \alpha_s(1 - N)] F_s \tag{12}$$

and values of μ are shown in Figure 2 as a function of OCR and 2b/D for both circular and strip loads.

The one-dimensional settlement for overconsolidated soils may be determined from

$$\delta'/D = [C_r/(1 + e)] \log [(p_o + \Delta p)/p_o] \tag{13}$$

Equation 13 may be combined with Eq. 5 to obtain the dimensionless settlement-layer thickness ratio, R, given by

$$R = \delta/D = \mu [C_r/(1 + e)] \log [(p_o + \Delta p)/p_o] \tag{14}$$

and the probability distribution function for R can be determined by use of the preceding relations for any given set of conditions.

Application of Monte Carlo Simulation

A convenient means of obtaining the distribution function for R, f(R), is by use of Monte Carlo simulation (7). If each uncertain variable is represented by an appropriate probability distribution function, a simulated sample of the variable may be taken by generating for each a random number and then processing it in accordance with the associated distribution. Combination of the sample values in accordance with the foregoing formulas leads to a sample value for R. If this is repeated a large number of times, we obtain a frequency distribution histogram from which the uncertainty associated with settlement determination may be inferred. An example of such a histogram is shown in Figure 3.

The spread of the histogram is characterized by use of a 90 percent confidence level; this entails the determination of a range that has a 90 percent probability of including the true settlement. As shown in Figure 3, a reference value of R, designated $\overline{R}$, is determined by substituting into the deterministic formulas the mean values of the relevant parameters. From the differences between $\overline{R}$ and the upper and lower 5 percent levels (H and L respectively, as shown), the ratios $H/\overline{R}$ (designated H_R) and $L/\overline{R}$ (designated L_R) are determined.

DISTRIBUTION FUNCTIONS FOR INPUT PARAMETERS

Figure 4 shows a diagrammatic summary of the various parameters that influence the settlement determination for an overconsolidated clay and the manner in which they combine to do so. The variables associated with the system geometry, b and D, are considered to be known without error, and they are represented deterministically. Probability distribution functions are applied to soil properties (C_r and e), to soil stresses (p_o, p_c, and Δp), and to formulation uncertainty.

Soil Properties

For the recompression index, C_r, a previously proposed (3) distribution form is used. Although little supporting evidence in the form of measured data is available for C_r, the physical nature of the phenomenon and its similarity to phenomena for which data are available indicate that both (a) a consistency in the value of the coefficient of variation would exist and (b) the specimen test results would display a normal distribution. Provided these 2 assumptions are satisfied, the distribution for possible values of the true recompression index may be given by

$$f[C_{r(i)}] = \lim_{C_{r(i+1)} \to C_{r(i)}} \left\{ \int_{Z_{i+1}}^{Z_i} f(Z)\, dZ \Big/ [C_{r(i+1)} - C_{r(i)}] \right\} \tag{15}$$

in which

$$Z_i = [\overline{C}_r - C_{r(i)}]/[VC_{r(i)}/\sqrt{n}] \tag{16}$$

and

$$f(Z) = (1/\sqrt{2\pi}) \exp(-0.5\, Z^2) \tag{17}$$

For Monte Carlo simulation, each sample of C_r may be obtained from

$$C_r = \overline{C}_r/[1 - (\xi V/\sqrt{n})] \tag{18}$$

where ξ is the standard normal deviate generated from a normal distribution having a mean of zero and a standard deviation of unity. In this study the value of $V/\sqrt{n}$ is taken as 0.2, which, for example, corresponds to $V = 0.4$ and $n = 4$ or to $V = 0.2$ and $n = 1$.

Determining the probability distribution function for the void ratio, e, gives rise to a complication. Although the uncertainties associated with the other variables are essentially independent of one another and the random variables generated in the simulation process are correspondingly independent, the void ratio is at least partially dependent on the recompression index. However, very little information is available to establish a quantitative relationship between the two, and it is unlikely that a reasonable value of the recompression index could be determined from void ratio alone. Nevertheless, within the same soil mass, there is some indication that a change in the recompression index is related to a change in the void ratio. For example, Schmertmann (5) has suggested the following equation to relate, for a particular soil, any 2 void ratios at the start of rebound (e_{r1} and e_{r2}) to the corresponding recompression indices (C_{r1} and C_{r2}).

$$\log (C_{r1}/C_{r2}) = 2.5 \log [(e_{r2} + 1)/(e_{r1} + 1)] \tag{19}$$

If the inaccuracy associated with this relation is taken into account by a factor Q, which is generated from a normal distribution with a mean of 1.0 and a standard deviation of 0.05, the sample value of the void ratio, e, to be used in the simulation process is given by

$$e = (\overline{e} + 1)(\overline{C}_r/C_r)^{0.4/Q} - 1 \tag{20}$$

in which $\overline{e}$ is the mean initial void ratio obtained from the test specimens, C_r is the sample recompression index generated in accordance with Eq. 18, and $\overline{C}_r$ is the mean recompression index. The value of 0.05 for the standard deviation in the formulation given above was determined by considering the experimental data presented by Schmertmann.

Soil Stresses

Considerable uncertainty is encountered when the stresses associated with settlement determinations are evaluated; and, since there are few data to serve as a guide in establishing the form or spread of probability distribution functions that represent stress uncertainties, a subjective approach is necessary. In addition to the convenience afforded by normal distributions, they appear to offer quite reasonable representations of the phenomena under consideration, as the following examples will indicate. Accordingly, normal distributions with values of 0.1, 0.05, and 0.025 for the coefficients of variation are used for the preconsolidation stress, the overburden stress, and the stress increase respectively. The implications of normal distributions with the indicated coefficients of variation are given in Table 2; for this example, the most likely value of the stress in each case is 1,000 psf.

Formulation Uncertainty

Determination of the A parameter from the table of Skempton and Bjerrum is only approximate, and a graphical indication of the approximation is shown in Figure 1. A degree of uncertainty similar to that indicated by the rectangular blocks is incorporated in the suggested formulation (Eq. 6) by adding an error function, E, which is normally distributed with a mean of 0 and a standard deviation of 0.1; thus, Eq. 6 becomes

$$A = 1.3\ (0.63)^{OCR} + E \tag{21}$$

The effect of this error function on the resulting values of A is shown in Figure 1. Generated values of A are influenced by the distributions associated with E, p_c, and p_o.

Equation 6 provides an empirical means for considering the error associated with lateral strains in the Skempton-Bjerrum approach. Since the uncertainty in the formulation becomes greater as the error increases, one appropriate probabilistic treatment for the case of a uniformly loaded circular area is to generate the random variable F_c from the equation

$$F_c = 1 + G \tag{22}$$

in which G is normally distributed with a mean $\overline{G}$ where $\overline{G} = 0.01 \exp\ [2.753 - 0.45\ (2b/D)]$ in accordance with Eq. 9, and a standard deviation of $0.4\overline{G}$; the large value for the standard deviation is indicative of the considerable degree of uncertainty involved. One implication for the extreme case where $2b/D = 0$ is that there is a 67 percent probability of F_c falling within the range 1.12 to 1.20. This corresponds approximately with the possible error of 20 percent indicated by Skempton and Bjerrum.

ANALYSIS OF UNCERTAINTY

The influence of various parameters on the uncertainty associated with settlement determinations for uniformly loaded circular areas is evaluated by making several series of computations in which many of the parameters are varied over a wide range of values. Subject to the elimination of any case wherein the overconsolidation ratio is less than $(p_o + \Delta p)/p_o$ (that is, the implied settlement extends beyond simple recompression), 81 combinations of the following parameters were utilized to establish distributions, f(R):

Parameter	Various Values Used
$(p_o + \Delta p)/p_o$	1.5, 2, 5, 10
OCR	2, 6, 15
$C_r/(1 + e)$	0.15, 0.04, 0.004
2b/D	0.1, 1, 10

In each case the high and low deviation ratios, H_R and L_R, were determined, and multiple regression techniques were employed to develop the following relations:

$$H_R = 0.96 - 0.001\ \{1.163\ OCR\ [\log\ (2b/D) - 1] + 16.63\ (2b/D)\} \tag{23}$$

and

Figure 1. Relation of parameter A and OCR.

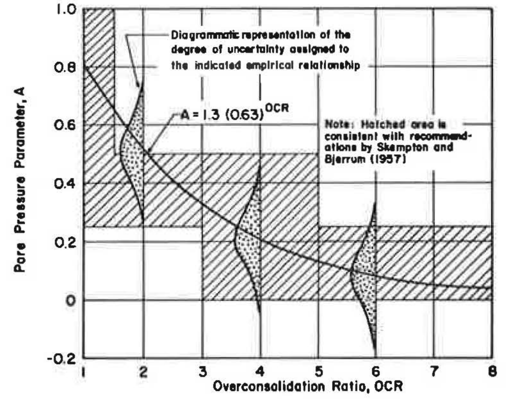

Figure 2. μ determined from OCR and load.

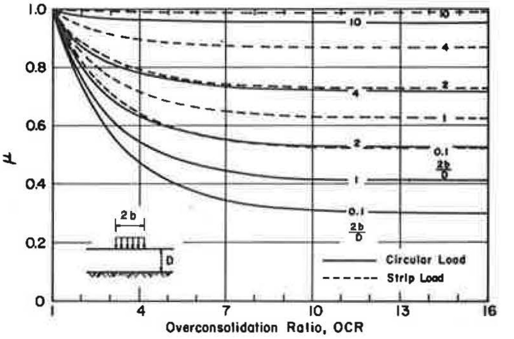

Figure 3. Typical probability distribution of R.

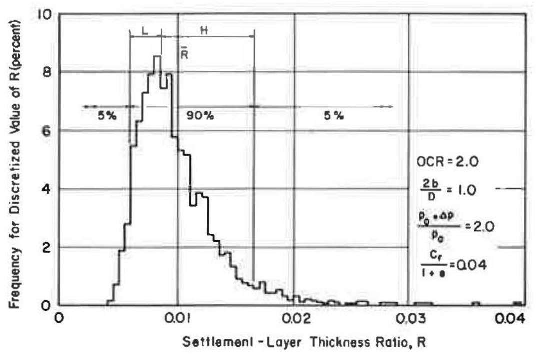

Figure 4. Parameters influencing settlement.

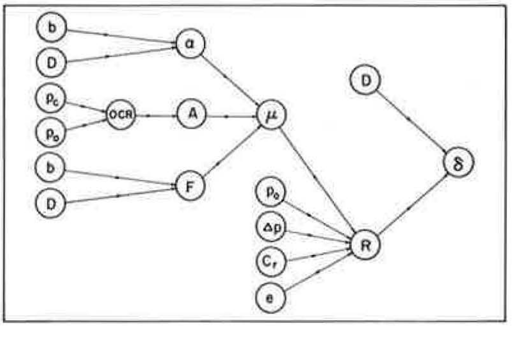

Figure 5. Uncertainty as a function of OCR and depth parameter.

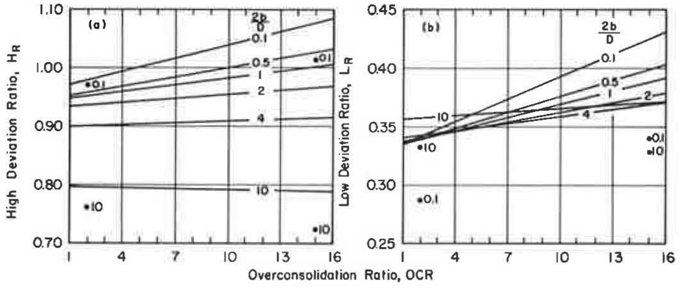

Table 1. Values of a.

2b/D	Circularly Loaded Area: Reported Value (6)	Circularly Loaded Area: Calculated From Eq. 7	Continuous Strip Load: Reported Value (4)	Continuous Strip Load: Calculated From Eq. 8
∞	1.00	1.00	1.00	1.00
4.00	0.67	0.69	0.80	0.81
2.00	0.50	0.49	0.63	0.64
1.00	0.38	0.37	0.53	0.50
0.50	0.30	0.31	0.45	0.43
0.25	0.28	0.28	0.40	0.39
0.10	0.26	0.26	0.36	0.36
0	0.25	0.25	0.25	0.35

Table 2. Probability that indicated stress range includes true value of stress.

Stress Parameter	Coefficient of Variation	67 Percent	95 Percent	99 Percent
Preconsolidation stress	0.1	900 to 1,100	800 to 1,200	742 to 1,258
Overburden stress	0.05	950 to 1,050	900 to 1,100	871 to 1,129
Stress increase	0.025	975 to 1,025	950 to 1,050	935 to 1,065

Note: Stress ranges are in psf.

$$L_R = 0.328 - 0.001 \{0.718\ OCR\ [\log (2b/D) - 1] - 2.78\ (2b/D)\} \qquad (24)$$

which are shown graphically in Figure 5. As implied by Eqs. 23 and 24, the magnitudes of both H_R and L_R are relatively insensitive to variation in $(p_o + \Delta p)/p_o$ and $C_r/(1 + e)$.

Application of the results given above is limited to cases where the expected value of the total stress falls somewhat below the expected value of the preconsolidation stress, since the analysis does not take into account the possibility of virgin consolidation settlement that may occur when these 2 expected stress values are similar. Obviously, for cases where the total stress probability distribution overlaps the preconsolidation stress probability distribution, the settlement will depend on the virgin compression index, C_c, as well as the recompression index, C_r. Therefore, the true deviation ratio, H_R, under these circumstances should be higher than that shown in Figure 5.

Illustrative Problem

Application of the above-described procedure can be best explained in terms of an example problem. Suppose a 10-ft-square foundation rests on a 20-ft-thick layer of overconsolidated clay with an overconsolidation ratio of 8, an average in situ void ratio of 0.5, and a recompression index of 0.03. If the average initial overburden stress is 1,500 psf and the stress increase due to loading is 6,000 psf, determine the probable range of settlement.

From Eq. 13, we obtain

$$\begin{aligned} \delta' &= D[C_r/(1 + e)] \log [(p_o + \Delta p)/p_o] \\ &= (20 \times 12) [0.03/(1 + 0.5)] \log [(1{,}500 + 6{,}000)/1{,}500] \\ &= 3.4 \text{ in.} \end{aligned}$$

From data shown in Figure 1 for OCR = 8 and $2b/D = 0.5$, we get $\mu = 0.4$, which, when used in conjunction with Eq. 5, yields

$$\delta = \mu\delta' = 0.4\ (3.4) = 1.36 \text{ in.}$$

Finally, from data shown in Figure 4 for OCR = 8 and $2b/D = 0.5$, we determine $H_R = 0.99$ and $L_R = 0.37$, which lead to the following relations:

$$(1 + H_R)\delta = (1 + 0.99)\ 1.36 = 2.7$$

and

$$(1 - L_R)\delta = (1 - 0.37)\ 1.36 = 0.9$$

Therefore, there is a 90 percent probability that the consolidation settlement of the foundation will lie within the range from 0.9 to 2.7 in.

Discussion of Problem

For the parameter values utilized, the upper end of the 90 percent probability range varies from 79 to 109 percent above the deterministically computed settlement, while the lower end varies from 33 percent to 44 percent below the deterministically computed settlement. Since the uncertainty indicated by these values is considerable, it is important to discuss some of the factors that may contribute to this situation.

The first point of interest concerns the crudeness of the correlation between the A pore-pressure parameter and the overconsolidation ratio; in fact, Skempton and Bjerrum specifically advise against the use of such a correlation for formal calculations, and they suggest that A be determined by means of a triaxial test. Therefore, in order to study the effect of the 2 different approaches on the uncertainty associated with the end result, the simulation process was repeated with an A value generated from a distribution that is representative of the uncertainty associated with A as determined from a triaxial test. Since few data are available on which a distribution for A may be based, a normal distribution with a standard deviation of 0.05 is subjectively selected; this implies a 67 percent probability that the true value for A lies within ±0.05 of the mean value of A determined from tests or a 95 percent probability that the true value lies within ±0.1. Then, H_R and L_R were determined for overconsolidation ratios of 2 and 15 and 2b/D values of 0.1 and 10; the results are shown in Figure 5. The average reductions in the deviation ratios are 5 percent on the high side and 14 percent on the low side. Since it is unlikely that the distribution given above for A is too broad, it appears that little is gained by determining A from a triaxial test.

It is also interesting to compare the preceding results with those reported (2) for the case in which a similar approach is taken to the simpler problem of one-dimensional consolidation due to a stress in excess of the preconsolidation stress. In the earlier study for cases where the ratio of the total stress to the preconsolidation stress exceeds about 2 (and for $V/\sqrt{n} = 0.2$), the high deviation ratio is in the range of 35 to 40 percent and the low deviation ratio is about 25 percent. However, a considerable increase over these values is not unexpected because of the uncertainties associated with the use of the A pore-pressure parameter and the effect of horizontal displacements. On the other hand, when the ratio of total stress to preconsolidation stress is less than about 2 in the previous study, the uncertainty associated with the settlement computation approaches that associated with foundations on overconsolidated soils; this is due to the greater influence of the less certain preconsolidation stress under these conditions.

CONCLUSION

Probability theory has been used to develop a rational procedure for evaluating the uncertainty associated with the computation of settlement for foundations resting on overconsolidated soils. Although every effort was made to incorporate well-founded and representative expressions in the probabilistic formulation, the lack of measured data in many cases necessitated the use of considerable subjective judgment; however, the developed procedure is readily adaptable to modification if and when appropriate data become available. In the meantime, a quantitative, even though subjective, assessment of the individual aspects of the problem in terms of accepted probabilistic procedures, such as Monte Carlo simulation, seems to provide the best available solution to such problems. This approach is particularly useful when mathematical complexities preclude an intuitive evaluation of uncertainty, as is the case for settlement computations involving overconsolidated soils.

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