COMPUTATION OF STRESSES AND STRAINS FOR THE DESIGN OF FLEXIBLE PAVEMENTS

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The development of improved design methods for flexible pavements requires an analytic tool that is relatively simple and cheap for use in routine design. Currently linear-elastic theory is considered the most satisfactory for this purpose, but the various solutions available are themselves rather inconvenient to use. Tabulated results for three-layer systems require a lengthy interpolation procedure to obtain results for variables other than those tabulated. On the other hand, the powerful and flexible multilayer computer programs require a large, fast computer and can thus be expensive to run. A computer program called "Interpolation" has been developed to carry out interpolation calculations on the three-layer elastic-layered system results tabulated by Jones in 1962. The object of this program is to provide a pavement design tool, considered to be more convenient than either the tables themselves or the complex multilayer computer programs now available. The interpolation procedure is based on fitting a curve to the log-log plot of stress function against each of the dependent variables used by Jones, which specify the system. The results have the same restrictions as Jones' tables, namely that all layers have a value of 0.5 for Poisson's ratio, and results are produced on the centerline of a single wheel load at the two interfaces. From a design point of view, this latter restriction is not likely to be important. The "Bistro" multilayer computer program was used to check the accuracy of results, and this appears satisfactory for design purposes. The Interpolation program has been incorporated in a simplified pavement design program in which an approximate nonlinear analysis may be used if required.

•ONE of the main objects of current highway research is the development of improved design methods for flexible pavements. The need for such research results from the recognition that current methods of design rely heavily on empirical rules that cannot be used with confidence in the heavy-load situations that are likely to exist in the future or under unusual environmental conditions.

The development of a structural design approach, which has been sometimes termed the "rational" approach, aims to reduce empiricism and establish pavement design on a reliable theoretical base. This approach is analogous to that used in other fields of civil engineering design, and it has been outlined in several papers (1, 2, 3).

There are many problems to be solved before the structural design method can be used with confidence for general design work. Many of these problems are associated with the behavior of paving materials (4) and with the correlation of laboratory-determined results and field performance (5).

The availability of high-speed digital computers has helped the development of pavement design during recent years. It has given impetus in particular to the solution of analytic problems concerned with the behavior of layered systems. Linear-elastic analysis (6, 7), viscoelastic analysis (8), and the use of finite-element techniques (9, 10) have all been made possible by the availability of computers.

The application of systems analysis to the problem of pavement design has also arisen because of the increasing use of computers, and several systems and subsystems have been proposed (11, 12).

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The long-term aim of developing improved pavement design procedures may well be toward a complete pavement design computer program dealing with all aspects of the design problem. In the meantime, increasing use is being made of available programs for analyzing pavement structure, whereas the remainder of the design process is carried out manually. To increase interest in this approach and to help in the development of a complete design program, we need to develop an analytic technique that is simple, fast, and accurate.

Currently, the most widely used analytic procedures are those based on linear-elastic theory. The computer programs developed by the Shell (6) and Chevron (7) organizations allow computation of stresses, strains, and deformations in multilayered pavement systems at any depth and radius relative to the applied surface load. There is complete freedom of choice of elastic constants for the layers and geometry of the system. These programs have a capability beyond routine design requirements and are hence mainly of use as research tools. In addition, they require large high-speed computers and are not ideal for building into a complete pavement design program.

The only alternatives available for design computations, where precise accuracy and comprehensive stress distributions are not necessary, are tabulated stress functions for three-layer systems. The most comprehensive of these were produced by Jones (13), but in practice there are a number of restrictions that make their use time-consuming and tedious.

There was a need, therefore, for an analytic procedure that fell between these two where the emphasis was on design use, which implies speed, convenience, and reasonable accuracy. The program "Interpolation" described herein is an attempt to fulfill this need.

ANALYTICAL REQUIREMENTS FOR PAVEMENT DESIGN

The three-layer linear-elastic system (Fig. 1) is thought to be a reasonable approximation of a flexible pavement structure (14). The top layer embraces all asphalt-bound layers, the second layer includes the unbound materials, and the subgrade forms the third, semi-infinite layer.

The maximum stresses and strains are the ones that require computation for design purposes. In a pavement, they generally occur on the centerline of the load and either just above or just below the interfaces.

With the current state of knowledge of material behavior, it is not possible to stipulate the elastic constants needed for analysis with great accuracy. Research has shown that soils and unbound materials behave in a nonlinear-elastic manner when subjected to dynamic loading, though for pavements with a thick asphalt layer the effect of this has been shown to be small (10). A successive approximation procedure has been used to cope with this problem while still using basically linear analysis (15). The finite-element analyses (9, 10) are based on this procedure. It can only be followed, however, if appropriate laboratory tests have been carried out to specify the nonlinearity. Hence, a single value of modulus is generally used for each layer.

Both the powerful multilayer computer programs and the tabulated stress functions for three-layer systems have disadvantages. The former requires access to large high-speed computers, and the computing time involved in solving a particular problem is relatively high. Because of their flexibility, a large number of data cards are required for each system in order to specify elastic constants, loads, geometry, and coordinates of the points where solutions are required.

The tables produced by Jones (13) allow the vertical and radial stresses and strains just above and below the interfaces of a three-layer system to be obtained on the axis of the load (Fig. 1). Poisson's ratio for all layers was taken as 0.5, and stress functions were tabulated for four variables, which were as follows:

$$k_1 = E_1/E_2;\ k_2 = E_2/E_3;\ H = h_1/h_2;\ a_1 = a/h_2$$

where

E_1, E_2, and E_3 = elastic modulus of each layer,
h_1 and h_2 = thickness of the two upper layers, and
a = radius of the loaded area.

The tables were produced for all combinations of four values of k_1 and k_2, seven values of H, and six values of a_1. For each of these, six stress functions were produced, which was the minimum number required to give the stresses and strains at the four positions involved. The stress functions (where σ_r is the horizontal, radial stress) were for a unit contact pressure and are as follows:

1. σ_{z1} = vertical stress at the first interface;
2. $(\sigma_{z1}-\sigma_{r1})$ and $(\sigma_{z1}-\sigma_{r2})$ = stress differences just above and below the first interface;
3. σ_{z2} = vertical stress at the second interface; and
4. $(\sigma_{z2}-\sigma_{r2})$ and $(\sigma_{z2}-\sigma_{r3})$ = stress differences just above and below the second interface.

In solving a particular problem, these tables are fairly satisfactory if the problem fits the tabulated values in terms of the four variables used. Interpolation is a long and not particularly accurate process. This can be shortened by the use of graphs produced in a companion paper by Peattie (16). The computation of strains from the tabulated stresses also requires time and care. Hence, for reasonably quick answers, the problem has to be adjusted to fit the solution, which is clearly unsatisfactory if any accuracy is required. The whole process is, in any event, a manual one that is not in keeping with the idea of a computerized design procedure.

The Interpolation program uses Jones' tables as basic data and carries out interpolation computations to produce stress functions for any reasonable values of the four tabulated variables. It thus overcomes the problem of hand or graphic interpolation and also produces the results in a more useful form. All the vertical and radial stresses and strains just above and below the two interfaces of the three-layer system are printed out for the actual contact pressure required. The input data consist of the thicknesses and elastic moduli of each layer plus the contact pressure and radius of loaded area; therefore, it is not necessary to calculate the four variables used in the tables. The program has been fitted in a simplified design program (17) discussed in a later section.

BASIS OF INTERPOLATION

A study of the charts produced from Jones' tables (13) by Peattie (16) indicates that, when plotted on a log-log base, the variations of stress function with each of the four basic variables are nearly linear. In view of this, it was thought originally that a linear procedure could accurately interpolate the tabulated variables. Early tests with the program indicated that this was not true, and a more accurate procedure was tried. The simplest curve mathematically is the quadratic, so this was used as the basic curve. For linear interpolation, only two points are needed on either side of the value required; for a quadratic curve, at least three points are necessary to specify the curve and to do the interpolation. The basic arrangement is shown in Figure 2, which illustrates a typical single interpolation calculation. In order to cope with curvatures and slopes of various sizes and magnitudes, we chose the circle having a curve of the form

$$x^2 + y^2 = ax + by + c$$

where x represents values of log (k_1, k_2, H, or a_1), y represents values of log (stress function), and a, b, and c are coefficients. The three points are chosen so as to include in their range the required value. Their coordinates are fed into the preceding equation in turn, thus producing three simulataneous equations from which the values of a, b, and c are calculated. The equation of the curve for the chosen points is thus established, and by substituting log (required value) for x the corresponding value of y is computed. This computation includes a procedure for selecting the required root of the equation from the two that are possible.

In general, to compute one of the six tabulated stress functions, we must interpolate at four levels, i.e., for k_1, k_2, H, and a_1. In practice this involves forty interpolation calculations such as the one shown in Figure 2. Hence, to compute all stresses, 240 interpolations are required.

DETAILS OF COMPUTATION

There are six basic stages in Interpolation, and these are shown in Figure 3. Each stage is discussed in the following sections.

Read in Basic Data

Jones' tables include stress functions for a wide range of values of the four basic variables. This extends beyond the practical range of values. In deciding on the basic data for Interpolation, which consists of values from Jones' tables, it was possible to exclude many of his values to reduce the amount of computer store necessary while ensuring that normal practical values were included.

All combinations of the following values of k_1 and k_2 were used: $k_1 = 2$, 20, and 200; and $k_2 = 0.2$, 2, 20, and 200. The combinations of H and a_1 values (totaling 27) used are shown in Figure 4. Hence, for each stress function, there are 324 items of data ($3 \times 4 \times 27$). There are six tabulated stress functions; so the full set of basic data includes 1,944 numbers (6×324).

In the program, the data are identified as A, B, C, D, E, F(I, J, K, L) where A to F are the stress functions tabulated; i.e., σ_{z1} $(\sigma_{z1} - \sigma_{r1})$ and $(\sigma_{z1} - \sigma_{r2})$, σ_{z2} $(\sigma_{z2} - \sigma_{r2})$ and $(\sigma_{z2} - \sigma_{r3})$, and I to L represent the matrix positions for the values of k_1, k_2, H, and a_1. Hence I = 1 or 2 or 3 for k_1 = 2 or 20 or 200, etc. These variables can have the following values: I, 1 to 3; J, 1 to 4; K, 1 to 7; and L, 1 to 6. It can be seen from Figure 4 that not all combinations of these values occur because of the economies effected by omitting some of Jones' basic data, even within the restricted ranges chosen. Subsequently, tests with the completed program have indicated that some additional values could be usefully included, particularly for high values of a_1 at low values of H (Fig. 4). This could be done at the expense of values for $k_2 = 200$. Stress functions for $k_2 = 0.2$ were included at a late stage in the development to accommodate nonlinear analysis (17), which often resulted in values of $k_2 < 2$.

Read in System Details

This short section of the program reads in the specification of the three-layer system including the contact pressure and the radius of the load. The values of k_1, k_2, H, and a_1 are then calculated. Any number of systems may be dealt with in one run on the computer, so the foregoing information is repeated for each system and is preceded by the number of systems.

Calculate Appropriate Gaps

The values of the four basic variables computed in the previous section will in general fall between two tabulated values. The purpose of this part of the program is to find the tabulated values immediately below the one required. This is done by calculating the values of I, J, K, and L for the system. This is done for each of these in turn by using a procedure called "gap."

Check System

Prior to preparing the data cards that specify the systems to be calcualted, a check should be made to ensure that the values of k_1, k_2, H, and a_1 fall within the limits of the program's capability as previously specified. For k_1, this simply means values between 2 and 200 and for k_2 between 0.2 and 200; however, for H and a_1, reference should be made to Figure 4 because there are some gaps in the matrix. In case this check is overlooked, the computer carries it out and indicates by a statement an unacceptable system; i.e., it gives the system number and the message "This system is outside the limits of the program." This check has been included so as to allow the program to continue with other systems, if any, whereas it would fail if calculations were tried with a system outside the limits.

Figure 1. The three-layer system.

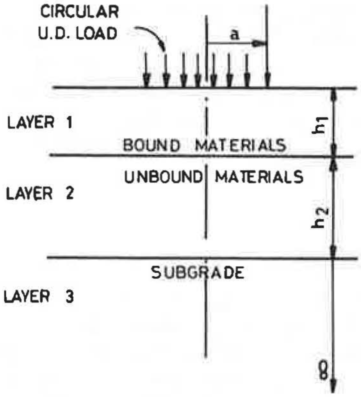


Figure 2. Basis of Interpolation.

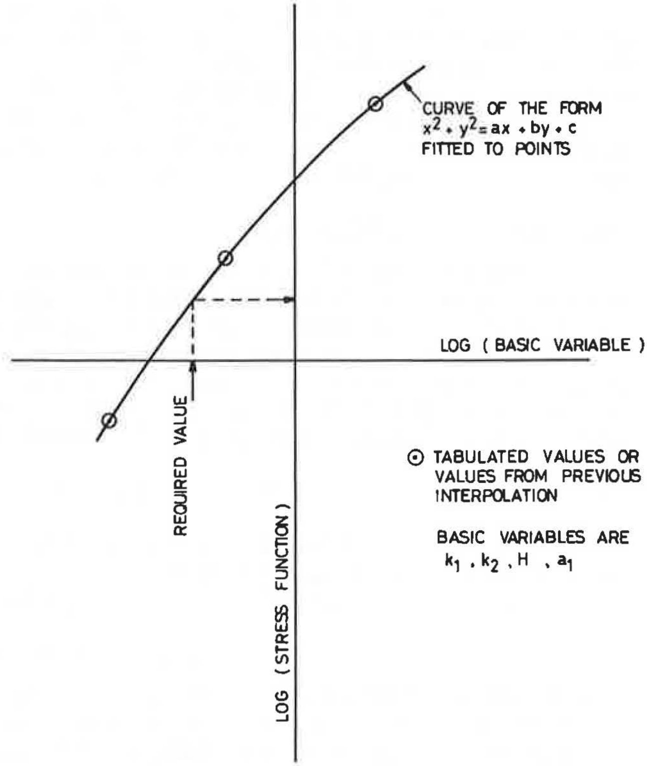


Figure 3. The main stages of program Interpolation.

READ IN BASIC DATA

READ IN SYSTEM DETAILS

CALCULATE APPROPRIATE "GAPS"

CHECK WHETHER SYSTEM IS WITHIN PROGRAM LIMITS

CARRY OUT INTERPOLATION COMPUTATIONS FOR THE 6 STRESS FUNCTIONS

CALCULATE INDIVIDUAL STRESSES AND STRAINS AND PRINT OUT

Interpolation Computations

This is the main part of the program and is carried out by using procedure "inter," which is called in for each of the six stress functions in turn. The individual interpolation calculations required are shown in Figure 5. The first part of "inter" carries out interpolations at the a_1 level of which there are 27. The next part carries out the remaining interpolations at the other three levels. Each successive level uses results from the previous one. Hence interpolation for H uses the results from interpolating for a_1, k_2 from interpolating H and k_1 from k_2.

Calculate and Print Out Results

After the six calls to "inter" are made, the six interpolated values of the stress functions are stored. Before the individual stresses and strains from these values are calculated and printed out, the system specification details are printed. The vertical and radial stresses are easily computed from the stress functions, which are themselves simple functions of these, i.e., $\sigma_{z1}(\sigma_{z1} - \sigma_{r1})$, etc. At this stage the stress for unit contact pressure, which is the basis of Jones' values, is multiplied by the actual contact pressure. Strains are calculated as follows:

$$\epsilon_z = 1/E\,(\sigma_z - \sigma_r) \text{ and } \epsilon_r = (1/2E)\,(\sigma_z - \sigma_r)$$

Since Poisson's ratio is 0.5 and the two horizontal stresses are equal to σ_r, E is the value of modulus for the layer in question.

Typical output is shown in Figure 6 and explained in the Appendix.

ACCURACY AND SPEED OF COMPUTATIONS

The "Bistro" computer program (6) was used as the standard against which to check the accuracy of results obtained from Interpolation. Sixteen systems covering a wide range of the basic variables were computed for this purpose.

The average error was ±2 percent, and the highest individual stress errors were -13 percent and +7 percent. Strain errors were more uniform, not exceeding ±5 percent. The reason for this may be that strains and vertical stresses, which had comparable accuracies, are calculated from just one of the stress functions, whereas radial stresses, which were less accurate, are derived from two such functions. In determining the percentage of errors, stresses less than 1 lbf/in.^{2} were not considered because the results could have been misleading.

A more stringent test of accuracy was carried out by comparing calculations from Interpolation with those from Bistro, wherein the value of Poisson's ratio was not constrained to 0.5. Eight practical structures covering a wide range of conditions were chosen, and the values of Poisson's ratio for each layer were selected from a knowledge of material properties (4). As may be expected, the accuracy was generally poorer, though a slight majority of the results were within ±10 percent of Bistro. Of the stresses and strains that are considered of most importance for design (i.e., vertical stress and strain on the subgrade and horizontal stress and strain at the bottom of the asphalt layer), all values except one were within 10 percent of Bistro. The exception was vertical strain on the subgrade, which was consistently low by an average of 24 percent.

An exact comparison of the computing time required for Bistro and Interpolation is not possible because of the very different natures of the two programs. The length of time for Bistro to produce comparable results at the same four positions as Interpolation depends on the structure geometry and the elastic constants of the layers. In addition, it may depend on the particular computer available. For a range of structures using a KDF9 computer, Interpolation was an average of four times as fast as Bistro.

APPLICATION TO DESIGN PROBLEMS

As a first step toward evolving a computer program for pavement design, a simplified program has been developed incorporating Interpolation as the basic analytic

Figure 4. Combinations of H and a_1 values used in program Interpolation.

H \ a_1	0.1	0.2	0.4	0.8	1.6	3.2
.125	x	x	x			
.25	x	x	x			
.5	x	x	x	x		
1		x	x	x	x	x
2		x	x	x	x	x
4			x	x	x	x
8				x	x	x

For all combinations of:-

k_1 = 2, 20, 200

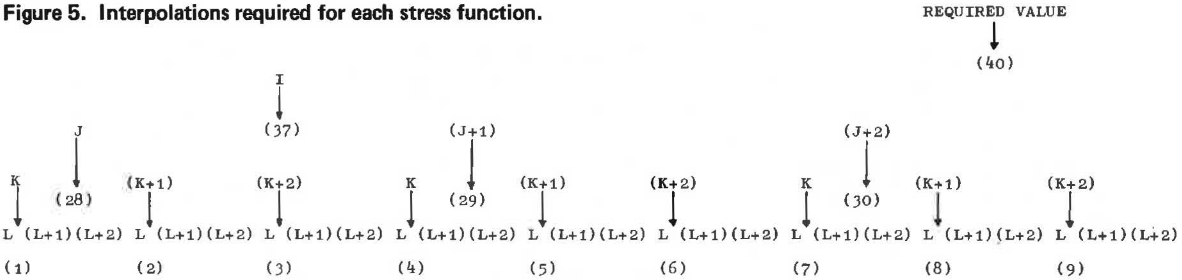
and

k_2 = .2, 2, 20, 200

x Data in program

• Gaps which may be used

Figure 5. Interpolations required for each stress function.



4 LEVELS OF INTERPOLATION, FROM a_1 TO k_1, L, K, J AND I ABOVE

3 POINTS FOR EACH INTERPOLATION, e.g. L, (L+1), (L+2)

40 INTERPOLATIONS IN ALL, NUMBERED ABOVE THUS - (29) IN ORDER

ONLY A THIRD OF THE DETAILS ARE SHOWN ABOVE - THE PATTERN IS REPEATED FOR (I+1) AND (I+2)

Figure 6. Typical output from program Interpolation.

SYSTEM NO. 1

LAYER	MODULUS	THICKNESS
ONE	200000	6.0
TWO	9000	18.0
THREE	3000	

CONTACT PRESSURE 80.0
LOAD RADIUS 6.0

VERSTRS	RADSTRS	RADSTRS	VERSTRS	RADSTRS	RADSTRS	VERSTRN	RADSTRN	VERSTRN	RADSTRN
14.7	-196.4	5.2	2.7	-4.3	0.4	1056	-528	778	-389

procedure (17). This program also incorporates an approximate procedure to deal with nonlinear material behavior such that the final design can be based on either a linear or nonlinear analysis of the structure. The design is based on the following three criteria that are thought to be of importance:

1. Tensile strain at the bottom of the asphalt layer. This is limited to prevent fatigue cracking.
2. Tensile stress at the bottom of the granular layer. This recognizes the fact that the unbound layer can only take a limited amount of tension (18).
3. Vertical strain at the top of the subgrade. This has been suggested (14) as the criterion to prevent excessive permanent deformation of the pavement.

The procedure involves adjustment to layer thicknesses of an initial, estimated structure in order to satisfy the three design criteria. The analysis of adjusted structures during each iteration may be carried out by either the linear or approximate nonlinear procedures.

The simplified nonlinear analysis included in the design program used relations between modulus and stress obtained from laboratory tests while assuming the asphalt layer to be linear-elastic. This procedure is based on the method used by Monismith et al. (15).

DISCUSSION AND SUMMARY

The program Interpolation was developed in an attempt to provide an analytic tool for the structural design of flexible pavements, which was easier to use than those previously available. This was thought necessary if the ideas and research developments taking place in this field are to be extended to use in practice. With the current state of knowledge great accuracy is not considered necessary in analyzing layered systems, since the theory does not model the real situation accurately and the material properties cannot be defined exactly.

Although the direct check of Interpolation against Bistro showed the former to be quite accurate, the more practical check discussed subsequently is the more realistic in terms of design, and in this case the accuracy was less impressive. The reason for this is that the basic data on which Interpolation operates are based on Poisson's ratio of 0.5 throughout the structure. In practice the asphalt layer has values between 0.35 and 0.5, depending on temperature; granular materials have values from 0.25 to 0.4; and cohesive soils have a value of about 0.4 (4). If new basic data were generated by using Bistro (with values of Poisson's ratio of, for example, 0.4, 0.3, and 0.4 for the three layers respectively), the resulting Interpolation values would be more accurate than at present.

A similar interpolation program could be developed to calculate surface deflection based on the tabulated values of Jones and Peattie (19). In this case, the value of Poisson's ratio is 0.35, which is a more realistic average value than the 0.5 used previously by Jones (13).

During the development of Interpolation, consideration was given to inserting chosen values of Poisson's ratio into the calculation of strains from stresses. Because of the horizontal strain compatibility condition built into Jones' original stress calculations, any values of Poisson's ratio other than 0.5 will destroy this compatibility in the final strain results. Though the comparison with Bistro using the practical range of structures was nearly as good as for $\nu = 0.5$, it was felt that the results were rather unrealistic, and this approach was abandoned.

CONCLUSIONS

The data given in this paper support the following conclusions:

1. The computer program Interporlation produces values of stress and strain in a three-layer elastic system, which may be of use for design purpose;
2. It extends the usefulness of Jones' tabulated values by allowing the solution to fit the problem rather than vice versa;

3. It requires approximately a quarter of the computing time taken by the multilayer Bistro program and is more convenient to use;
4. Interpolation can conveniently be fitted into a full pavement design program and can deal approximately with nonlinear behavior;
5. Direct comparison with Bistro indicates that the average accuracy of Interpolation is ±2 percent;
6. When realistic values of Poisson's ratio other than 0.5 are used, the accuracy of Interpolation is poorer than Bistro's, but the majority of stresses and strains are within 10 percent;
7. The accuracy of Interpolation could be improved by generating new basic data that use more realistic values of Poisson's ratio; and
8. A program similar to Interpolation could be developed to calculate surface deflection.

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APPENDIX

INPUT AND OUTPUT DETAILS

Input

The following information (all of which is repeated for each system except for number of systems) is required on the input data cards:

1. Number of systems,
2. Modulus of elasticity for layer No. 1,
3. Modulus of elasticity for layer No. 2,
4. Modulus of elasticity for layer No. 3,
5. Thickness of layer No. 1,
6. Thickness of layer No. 2,
7. Radius of loaded area, and
8. Contact pressure.

The elastic moduli and contact pressure should all be in the same units, and these will be the units of the calculated stresses. The layer thicknesses and radius of loaded area must all be in the same units.

Output

Typical output is shown in Figure 6. Reading from left to right the tabulated results are as follows:

1. Vertical stress at the first interface,
2. Radial stress above the first interface,
3. Radial stress below the first interface,
4. Vertical stress at the second interface,
5. Radial stress above the second interface,
6. Radial stress below the second interface,
7. Vertical strain at the first interface,
8. Radial strain at the first interface,
9. Vertical strain at the second interface, and
10. Radial strain at the second interface.

It should be noted that, at a particular interface, the following effects are equal just above and below the interface:

1. Vertical stress—for equilibrium,
2. Radial strain—for compatibility as the interface is considered "rough," and
3. Vertical strain—because Poisson's ratio is 0.5.