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# MATERIAL PROPERTIES AFFECTING SOIL-STRUCTURE INTERACTION OF UNDERGROUND CONDUITS

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Notwithstanding current limitations of analytic procedures, the greatest error in available techniques for the analysis and design of underground conduits probably lies in the specification of material properties, especially those for the soil surrounding the conduit. However, the mechanical properties of the conduit material and the conditions that exist at the soil-conduit interface may be significant. Included herein is a brief discussion of the material parameters that form a part of the classical procedures for the analysis and design of pipe conduits, and the arguments against their continued long-term use are given. The advent of the high-speed digital computer and the finite-element method have provided the opportunity to handle material properties in a more realistic manner, and soil-conduit problems should be formulated to take this fact into account. Nonhomogeneity resulting from different materials being used for the underlying soils, bedding, side fill, and backfill or embankment can be readily included in the analysis, and incremental approaches allow nonlinear material properties and the actual construction sequence to be incorporated in a piecewise linear manner without too much difficulty; even three-dimensional analyses by numerical methods have recently come into use. Accordingly, appropriate soil properties must be specified to guide the development of increasingly sophisticated analyses and computer programs.

•THE analysis and design of underground conduits are essentially a problem of soil-structure interaction, and the solution of any problem of this type must give full cognizance to the fundamental coupling phenomenon. Interpreted simply, this concept states that the response of the conduit and the behavior of the surrounding soil are not independent but intimately related in some complex manner. In general, the response of a soil-conduit system depends on the characteristics (geometry and stiffness) of the conduit, the characteristics (geometry, order of placing, and mechanical properties) of the adjacent and overlying compacted fill, and the characteristics (compressibility) of the in situ soil under and adjacent to the conduit. Notwithstanding the limitations of analytic procedures, the greatest error in currently available techniques for analysis and design probably lies in the specification of material properties, especially those for the soil surrounding the conduit. However, the mechanical properties of the conduit material and the conditions that exist at the soil-conduit interface may also be very significant. This paper will discuss briefly some of the ideas that can be used to determine appropriate input information for material properties.

## CURRENT DESIGN PROCEDURES

For the most part, buried conduits are constructed of either reinforced concrete or corrugated metal, and the well-known work by Marston and Spangler and their co-workers has exerted a significant influence on virtually all currently used design procedures (5). Generally accepted design methods treat reinforced concrete conduits as

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a rigid structure and metal conduits as a flexible structure, and separate design procedures are available for each. In an effort to account for the soil-structure interaction (or the relative stiffness of the soil and the conduit), these procedure involve a variety of special parameters (such as settlement ratio, modulus of soil reaction, load factor, and projection ratio) that are associated specifically with the buried conduit problem. Although these parameters may achieve their intended goal when used with good engineering judgment within limited ranges of applicability for which experience is available, very often their use cannot be easily extended or generalized. Also of considerable concern is the fact that no techniques are currently available to handle conduits of intermediate stiffness. Despite the limitations outlined, these procedures have served the profession well during the past 50 years, and few, if any, failures can be attributed to the theory itself. As such, this work, including the special parameters that attempt to account for the soil-conduit interaction phenomenon, represents an outstanding example of engineering ingenuity, and the experience gleaned over the years must not be treated lightly. Although good engineering practice dictates that currently used design procedures should not be discarded until better ones have been provided, this same good practice calls for a periodic appraisal of current procedures in the light of recent advances in technology and theoretical developments.

## CONTINUUM APPROACH

Despite the advantages and disadvantages that may be attributed to the Marston-Spangler theories, it seems that the major advances in our knowledge of soil-conduit interaction phenomena do not lie in modifying or improving the existing procedures and the associated material parameters but rather in developing a different approach to the problem. Pursuant to this idea, the most logical approach to the soil-conduit interaction problem lies in treating all components (conduit, underlying soils, bedding, side fill, and overlying soils) as continua, each with its unique material properties. Although the complexities associated with the geometry and material properties of a typical soil-conduit system have in the past either precluded the use of this approach or necessitated relatively crude computational procedures, a very versatile analytic tool has been made available to the profession in recent years by the development of the finite-element method and the advent of the high-speed digital computer. In addition to providing the capability for describing the soil-conduit system as a nonhomogeneous, nonlinear continuum, such a treatment has the following advantages: The coupling or soil-conduit interaction effect is inherently taken into account; input parameters would consist of more fundamental characterizations of the soil and conduit material behavior; conduits of intermediate stiffnesses can be analyzed; and the effects of the construction sequence can be studied. Even three-dimensional analyses have recently been made. Accordingly, the following discussion of material parameters is based on the premise that the finite-element method offers the potential for significant improvement in our ability to analyze complex soil-conduit systems.

## INTERRELATED STEPS IN DESIGN PROCEDURE

Any design procedure consists, either implicitly or explicitly, of a synthesis of the various steps shown in Figure 1; this diagram indicates that the design procedure is intimately related to and dependent on the sampling and testing techniques, the interpretation of the data, and the methods of analysis that are used. Accordingly, a change in the design procedure is likely to bring about changes in one or more of the other steps involved. In particular, use of the finite-element approach leads to a considerable change in currently used design procedures for underground conduits because it requires that the problem be formulated in terms of material properties that are fundamental to continuum mechanics. However, in certain cases these properties have been studied for years, and many soils laboratories are currently equipped to conduct the required tests.

## REQUIREMENTS OF VALID DESIGN PROCEDURE

As shown in Figure 2, three very important components are required to develop a valid design procedure: a mathematical model, material properties, and field verification. The mathematical model, which would probably include a computer program, must be formulated such that it can describe the physical phenomenon under consideration; in the past this component of the overall problem has attracted much attention, and many sophisticated programs have been developed. The applicability of these programs, however, is limited by the assumptions on which they are based and the material properties that are provided as input; very often these programs call for input data that cannot reasonably be provided, and hence the engineering profession obtains little benefit from their use. As has been frequently stated, the results obtained from any computer program are only as good as the input information that is supplied. There is considerable evidence to indicate that in recent years our ability to formulate mathematical models and solve theoretical problems has far outstripped our ability to provide appropriate input information concerning material properties. Finally, field verification of theoretical predications is needed before any analytic procedure can be accepted; however, the high cost of field instrumentation often impedes or totally precludes its use, and the profession is therefore left with no reliable way to assess quantitatively the validity of the combined mathematical model and material properties.

## EMPHASIS OF STUDY

The principal objective of this study is to examine workable approaches that may be taken immediately within the framework of currently available testing techniques to interpret laboratory test results for use in obtaining the solution to a soil-conduit problem. The discussion is intended to be representative, not comprehensive, and the purpose is to survey and compare various methods of testing and interpretation, not to suggest one particular procedure. Although the properties of conduit materials and the interface conditions between various zones are discussed briefly, the characterization of the soil is considered to be of primary importance, and the principal thrust of the presentation is therefore directed toward this end. Emphasis is centered around soil testing procedures that are in common use, and the terminology and techniques of linear elasticity are employed. The observed nonlinear behavior of virtually all soils may be handled conveniently by a piecewise linear model, but no attempt is made to advance more rigorous formulations and interpretations than may be realized within the capabilities of most current laboratory test equipment. For example, consideration of all three principal strains and stresses in a constitutive relation would require the conduct of a true triaxial test; however, except for research purposes, the true triaxial test is far too complex for widespread use at the present time, and a complete variation of the properties in all three directions is therefore not treated herein.

## SOIL PARAMETERS

It is convenient to consider two extremes of soil performance, which represent the behavioral range of engineering soils: an ideal plastic, cohesive clay and an ideal clean, coarse-grained, cohesionless sand. The major difference between these materials lies not primarily in the ultimate shear stress available but rather in their stress-strain-time characteristics for loads sustained over long periods of time. Loads on clay soils will ordinarily cause time-dependent volume decreases, and mobilized shear stresses may relax because of creep. The ideal cohesionless soil usually exhibits a relatively low compressibility under added loads, and it responds with little time delay. Cohesionless soils tend to develop and maintain a specific shear stress where differential movements occur. In general, because of the time-dependent stress-strain characteristics of most clayey soils, stress concentrations dissipate with time; consequently, it is probable that the pressure normal to the conduit wall would tend to approach the overburden pressure with the passage of time. No specific observations are available to demonstrate the degree to which cohesionless soils can permanently sustain loads transferred from the pipe, but it is likely that this relaxation phenomenon does not exist to the same extent. Because most soils used in an actual conduit installation do not fall into either of these extreme categories, it is difficult to predict the effect of time on the response of a buried conduit. The type of soil (and most especially its degree of saturation) and the time of interest in the problem will dictate whether the soil tests to be discussed subsequently should be drained or undrained; however, for sake of brevity, time considerations are not specifically included in this discussion, and the manner in which they are handled is left to the engineering expertise of the designer or researcher.

## Importance of Modulus

As stated previously, the characterization of the soil is probably the most important consideration in a soil-conduit system, and, more specifically, the modulus of the soil is probably the single most important parameter that affects the response of the system. In addition to the interaction between the conduit and the immediately adjacent soil, there is interaction among the various soil zones; this is particularly important when appraising the stiffness of the natural soil relative to the backfill in a trench installation, and it further illustrates the importance of determining the moduli of the soils in the various zones. Accordingly, it follows that considerable attention should be directed toward describing the stress-strain behavior of the soil surrounding the conduit. Unfortunately, the modulus is intimately related to and significantly influenced by Poisson's ratio, and the latter is most difficult to quantify.

## Assumed Isotropy of Modulus

The modulus discussed herein is assumed to be isotropic and dependent on the state of stress in the soil at a point. The assumption of isotropy is very significant, especially in view of the nonlinear behavior of most soils, but currently available testing procedures seem to justify no further refinement at this time. In effect, this means that a change in stress in any given direction at a point causes a change in strain in that direction such that the ratio of the two is a constant that is independent of the existing stress in that direction; hence, this assumption is not strictly compatible with the nonlinear behavior of the soil. However, if this premise is accepted, the problem reduces to one of defining the nature of the modulus and the state of stress on which it depends.

## Nature of Modulus

When conducting a piecewise linear analysis that involves a nonlinear material, it is possible to utilize three different definitions of modulus: the secant modulus (straight line joining the origin and point on the stress-strain curve), the tangent modulus (derivative of the stress-strain curve at a point), and the chord modulus (straight line joining two points on the stress-strain curve). Each has its inherent advantages and disadvantages, which are related in large part to the manner in which the numerical calculations are conducted. In view of the incremental nature of the applied load on a conduit due to the normal construction sequence, the tangent and chord moduli seem to offer some advantages; in such a formulation the problem is solved for any given state of loading, and the stresses within each element are determined. Then, a modulus compatible with each state of stress is assigned to each element, and the response for the next increment of load is determined, after which the procedure is repeated. Although the foregoing procedure can, of course, be followed with the secant modulus, it involves a somewhat less accurate approximation of the actual stress-strain curve.

## Determination of Stress-Strain Response

Three laboratory tests may be conveniently employed to determine the stress-strain response of a soil: the consolidation test, the conventional triaxial test, and the plane strain test. Typical idealized states of stress, as well as typical stress-strain plots and modulus-stress plots, for each of these tests are shown in Figure 3. As can be readily seen, the specification of the three principal stresses in the consolidation test and in the plane strain test requires the assumption of a value for the at-rest pressure coefficient, $K_o$, and Poisson's ratio, $\nu$, respectively; in the triaxial test the intermediate principal stress is considered to be equal to the minor principal stress. All three of these tests represent very special states of stress, and each has deficiencies that have been discussed at length in the literature. In all tests considered herein, the state of stress is assumed to be homogeneous throughout the specimen; this is generally not the case, and serious test errors can be introduced because of rough end platens and sidewall friction. Once again, the methods for handling these conditions, as well as other test errors, must be left to the judgment of the designer or researcher. A casual glance at the drastically different character of the stress-strain plots obtained from these tests suggests that a reconciliation must lie in the interpretation of the test results, and this is indeed the case. If we assume that the same drainage conditions exist for all tests, the reconciliation of the indicated test results may be established by use of Poisson's ratio if the soil is assumed to be linear elastic. Notwithstanding the extensive use of laboratory tests to determine soil properties, there are many engineers who contend that any modulus determined from a laboratory test is subject to serious error, and they advocate the use of a field test (such as a plate bearing test or a borehole pressure meter test) to determine the soil modulus. However, these tests are extremely expensive, and the assumption of linear elastic behavior is usually required to interpret the results; consequently, they will not be discussed herein.

## Relation Between Young's Modulus and Other Moduli

In any finite-element formulation based on a piecewise linear theory, two parameters are required to characterize an isotropic material; these are the modulus of elasticity or Young's modulus, E, and Poisson's ratio, $\nu$. By definition, Young's modulus is the slope of the axial stress-axial strain curve in a uniaxial stress test, and Poisson's ratio is the ratio of the lateral strain to the longitudinal strain for a specimen that is uniaxially stressed in the longitudinal direction. It is significant to note that the definitions of both coefficients include a specification of the state of stress; this fact is often overlooked in the interpretation of much test data. All too often one can find in the literature a case where a conventional triaxial compression test was conducted on a soil, and Young's modulus is defined as $\sigma_1/\epsilon_1$ and Poisson's ratio as $\epsilon_3/\epsilon_1$; both are incorrect, as will be seen later. However, because the ratio $\sigma_1/\epsilon_1$ constitutes a type of modulus that is relatively easy to determine in a consolidation, conventional triaxial, or plane strain test, this parameter will be termed M, $E_T$, and $E_P$ respectively for each of the three tests, and relations between E and each of these moduli will be determined in terms of Poisson's ratio and the state of stress in the test specimen. Although considerable discussion is given to the formulation of analytic expressions for these moduli, graphic representations are also acceptable for use in computer programs. Most of the subsequent discussion will center around the tangent modulus or the chord modulus because these have the ability to approximate most closely the actual stress-strain behavior of the soil. Within the context of a piecewise linear formulation, the reference state for each increment of loading may be taken as either the state of stress that exists after the preceding load increment has been applied or the average of the stress states before and after a given load increment (this latter choice leads to the use of an iterative procedure), and a linear modulus is assumed to govern the response due to the added load increment.

## Determination of Modulus From Consolidation Test

From a consolidation test wherein the lateral strains are held equal to zero, $\sigma_1$ is usually plotted versus $\epsilon_1$, and a so-called constrained modulus, M, is thereby obtained. This modulus would be appropriate for determining the settlement of a large uniformly loaded area such that the lateral deformations are zero or negligible, but it is theoretically not appropriate for use in a two- or three-dimensional problem unless certain modifications, which center around taking into account the Poisson effect, are introduced. Based on the applicability of linear-elastic theory, the first invariant of the stress tensor and the first invariant of the strain tensor can be related by

$$\sigma_1 + \sigma_2 + \sigma_3 = E(\epsilon_1 + \epsilon_2 + \epsilon_3)/(1 - 2\nu) \quad (1)$$

Because $\sigma_2 = \sigma_3 = K_o\sigma_1$ and $\epsilon_2 = \epsilon_3 = 0$ in a consolidation test, Eq. 1 may be written as

$$(1 + 2K_o)\,\sigma_1 = E\epsilon_1/(1 - 2\nu) \qquad (2)$$

which, upon substitution of the relation

$$K_o = \nu/(1 - \nu) \qquad (3)$$

and solution for E/M, becomes

$$E/M = (1 - 2\nu)(1 + \nu)/(1 - \nu) = (1 - \nu - 2\nu^2)/(1 - \nu) \qquad (4)$$

where

$$M = \sigma_1/\epsilon_1 \text{ or } \Delta\sigma_1/\Delta\epsilon_1 \qquad (5)$$

As can be seen in Figure 4a, the variation of E/M as a function of $\nu$ is considerable, and this effect cannot be ignored when applying consolidation test results to multidimensional problems.

One very convenient empirical expression for the constrained modulus, M, has been reported by Janbu (4) as

$$M = m\,p_a\,(\sigma_1/p_a)^{1-n} \qquad (6)$$

where $p_a$ is atmospheric pressure (introduced to maintain dimensional homogeneity), and m and n are termed the modulus number and stress exponent respectively; in effect, these latter two coefficients are empirical parameters to be determined experimentally. However, it is very possible that certain broad correlations, as advanced by Janbu and shown in Figure 5, may be established among these empirical coefficients and certain types of soil or various conditions of a given soil type. As another example, limited data, based on tests by Osterberg (6), have been interpreted by Krizek et al. (5) to suggest that M is a function of dry density and overburden pressure for a large variety of soils. This relation, which is shown in Figure 6, resembles the one proposed by Janbu. Although it admittedly seems to be oversimplified, further study is certainly justified to determine its range of applicability. The combination of Eq. 6 and Eq. 4 gives

$$E = [(1 - \nu - 2\nu^2)/(1 - \nu)]\; m\,p_a\,(\sigma_1/p_a)^{1-n} \qquad (7)$$

Equation 7 deals exclusively with the interpretation of a laboratory consolidation test, and it states specifically that Young's modulus, E, is a nonlinear function of the major principal stress, $\sigma_1$. When used in conjunction with a finite-element model, the modulus within each element for a given condition of loading would depend only on the major principal stress in that element.

This approach might be improved by intuitively extending the Janbu relation, which is developed in terms of $\sigma_1$ only, to an expression in terms of the sum of all three principal stresses, as follows:

$$M = m^*\,p_a\,[(\sigma_1 + \sigma_2 + \sigma_3)/p_a]^{1-n^*} \qquad (8)$$

where m* and n* are empirical coefficients similar to m and n previously described. Substitution of Eq. 8 into Eq. 4 yields

$$E = [(1 - \nu - 2\nu^2)/(1 - \nu)]\; m^*\,p_a\,[(\sigma_1 + \sigma_2 + \sigma_3)/p_a]^{1-n^*} \qquad (9)$$

which is similar in form to Eq. 7 but more general in formulation. As far as the interpretation of the consolidation test is concerned, the extension suggested previously

**Figure 1. Interrelated steps in design procedure.**

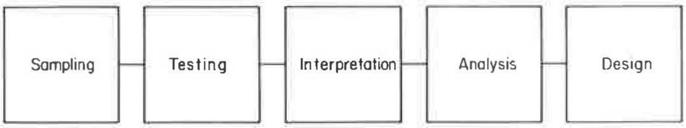



**Figure 2. Requirements of valid design procedure.**

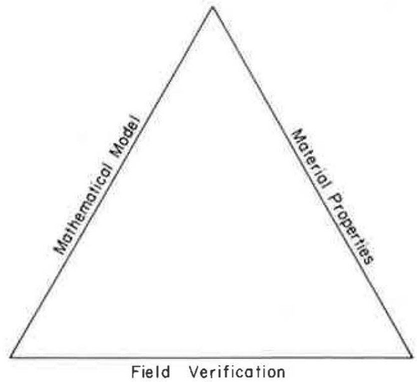



**Figure 3. Typical stress-strain response from various laboratory tests.**

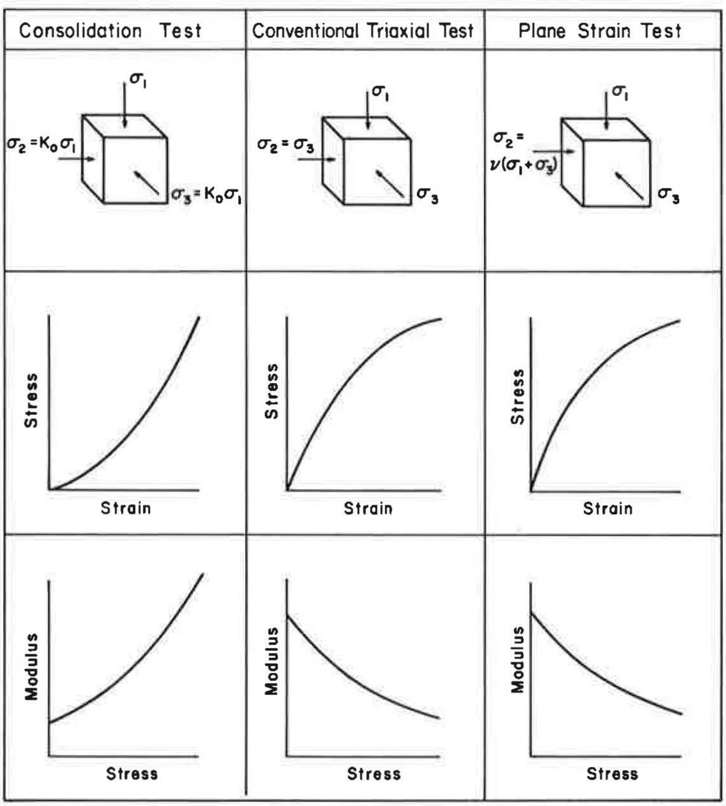
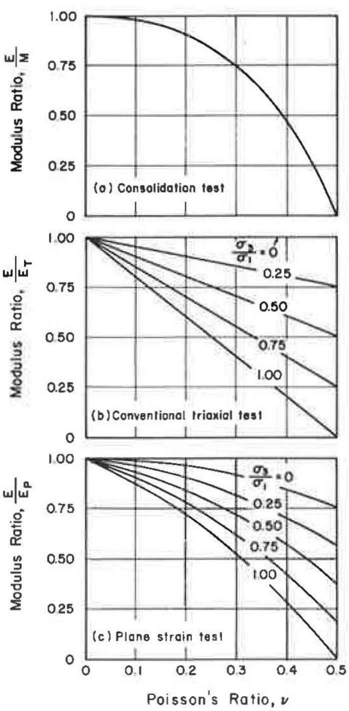



**Figure 4. Modulus ratio as a function of Poisson's ratio and state of stress for various laboratory tests.**



is perfectly admissible; because $\sigma_2 = \sigma_3 = K_o\sigma_1$ in a consolidation test, Eq. 8 may be written as

$$M = m^* (1 + 2K_o)^{1-n^*} p_a(\sigma_1/p_a)^{1-n^*} \quad (10)$$

Another equally good procedure for incorporating stress-strain data into a finite-element model is the use of the graphic representation shown in Figure 7. In this case, one simply utilizes the chord modulus between any two arbitrary values of stress and some representative constant value for Poisson's ratio in this same stress interval to determine a value for Young's modulus, E, which is considered to be applicable within this stress interval. Then, based on the state of stress (either $\sigma_1$ or $\sigma_1 + \sigma_2 + \sigma_3$, depending on the formulation) of each element in the mathematical model for any given loading, an appropriate modulus is assigned to each element for use during the next increment of loading.

Some investigators have simply replaced Young's modulus, E, with the secant constrained modulus, $M_s$, which is associated with the anticipated final state of stress at the elevation of the conduit; in other words, the effect of Poisson's ratio is completely ignored. The apparent successes achieved with this approach indicate that various unknown effects not taken into account tend to offset each other in certain cases. Despite such limited successes, this approach should be used with extreme caution because it is not on a sound theoretical basis. If, on the other hand, field verification is not obtained, there is little justification to continue this theoretically incorrect practice.

Because the nature of a consolidation test precludes the determination of a failure criterion, special provision will have to be made if this condition is approached in a field problem, and this is, in fact, one of the disadvantages of the test. However, most buried conduit problems are generally concerned with permissible deformations rather than catastrophic collapse, and for such conditions it is probably not necessary to be overly concerned with the shear strength of a soil. The use of a consolidation test to determine the soil modulus is particularly attractive in that the test is relatively easy to conduct and most laboratories have the appropriate equipment. It can also be argued (though not technically correct) that the uniaxial strain conditions of the consolidation test are reasonably well duplicated in the free field and for many situations in a radial direction near the conduit.

## Determination of Modulus From Conventional Triaxial Test

In a conventional triaxial test on an unsaturated soil, the major principal stress, $\sigma_1$, the minor principal stress, $\sigma_3$, and the major principal strain, $\epsilon_1$, are either measured or controlled, and results, which exhibit a shape as shown in Figure 3, are usually plotted as $(\sigma_1 - \sigma_3)$ versus $\epsilon_1$. Based on such a plot, a special modulus, termed herein the triaxial modulus, $E_T$, may be defined as

$$E_T = \sigma_1/\epsilon_1 \text{ or } \Delta\sigma_1/\Delta\epsilon_1 \quad (11)$$

This definition ignores the Poisson effect of $\sigma_3$ on $\epsilon_1$, if considered in terms of absolute values, and, if considered in terms of incremental values, it presumes that the Poisson effect of $\sigma_3$ on $\epsilon_1$ is independent of the state of stress within the specimen; actually, however, Poisson's ratio may be expected to vary with the mean stress, $\sigma_m = \frac{1}{3}(\sigma_1 + 2\sigma_3)$, or the shear stress $(\sigma_1 - \sigma_3)$, or both. It should be noted that the often-used ratio of $(\sigma_1 - \sigma_3)$ to $\epsilon_1$ is in effect the ratio of a shear stress to a normal strain; to term this ratio a modulus is inconsistent with most mechanics terminology, and such usage should be avoided. With the assumption of linear elasticity, the constitutive relation

$$\epsilon_1 = (\sigma_1/E) - (\nu/E)(\sigma_2 + \sigma_3) \quad (12)$$

may be combined with the condition that $\sigma_2 = \sigma_3$ and rewritten to give the following relation for Young's modulus, E:

$$E/E_T = [(1 - 2\nu)(\sigma_3/\sigma_1)] \quad (13)$$

where $E_T$ is given by Eq. 11. A plot of Eq. 13 is shown in Figure 4b, and the range of variation is indeed quite substantial.

Provided an acceptable value for $\nu$ can be determined, triaxial test results could be incorporated in the finite-element model as follows. The triaxial modulus, $E_T$, can conveniently be determined from the slope of the $\sigma_1$ versus $\epsilon_1$ curve; then, with a knowledge of the $\sigma_3/\sigma_1$ ratio and an estimate of $\nu$, Young's modulus, E, can be determined from Eq. 13. In the finite-element formulation of the buried conduit problem, an E value consistent with the interpretation of the laboratory test can be selected once the values of $\sigma_1$ and $\sigma_3$ have been determined for a given state of loading.

As an alternative approach that has been demonstrated often in the literature, load-deformation or stress-strain data that exhibit the shape shown in Figure 8a can be very conveniently described by a two-coefficient hyperbola, which for a $(\sigma_1 - \sigma_3)$ versus $\epsilon_1$ plot takes the form

$$\sigma_1 - \sigma_3 = \epsilon_1/(a + b\epsilon_1) \tag{14}$$

where a and b are empirical coefficients to be evaluated by experiment. The limit of Eq. 14 as $\epsilon_1$ approaches infinity yields

$$(\sigma_1 - \sigma_3)_{ult} = 1/b \tag{15}$$

However, because values of $(\sigma_1 - \sigma_3)$ approach $(\sigma_1 - \sigma_3)_{ult}$ asymptotically as $\epsilon_1$ goes to infinity, it may be expected that values of $(\sigma_1 - \sigma_3)$ at failure, or $(\sigma_1 - \sigma_3)_f$, will normally be less than $(\sigma_1 - \sigma_3)_{ult}$, and this will impose an upper bound on the validity of Eq. 14. In general, we have

$$(\sigma_1 - \sigma_3)_f = R\ (\sigma_1 - \sigma_3)_{ult} = R/b \tag{16}$$

where R is another empirical coefficient that usually lies within the range of 0.70 to 0.95 and very often between 0.8 and 0.9. Differentiation of Eq. 14 will yield a tangent modulus, $E_t$, which may be expressed as

$$E_t = d\ \sigma_1/d\epsilon_1 = a/(a + b\epsilon_1)^2 \tag{17}$$

because $\sigma_3$ normally is held constant in a triaxial test, and the evaluation of Eq. 17 where $\epsilon_1$ equals zero will give an initial tangent modulus, $E_i$, which may be written

$$E_i = 1/a \tag{18}$$

In order to avoid potential contradictions in the reference state (zero value) for stress and strain, we should eliminate the strain parameter in Eq. 17; this can readily be accomplished by solving Eq. 14 for $\epsilon_1$ and by substituting the result into Eq. 17 to give

$$E_t = [1 - b(\sigma_1 - \sigma_3)]^2/a \tag{19}$$

Equation 19 is a quite general relation that may readily be used as described previously in conjunction with a mathematical model, and the following procedure is suggested for evaluating the empirical coefficients. When plotted in the conventional form of $(\sigma_1 - \sigma_2)$ versus $\epsilon_1$, a typical set of triaxial test data will usually exhibit the shape shown in Figure 8a. If Eq. 14 is rewritten in terms of transformed variables as

$$\epsilon_1/(\sigma_1 - \sigma_3) = a + b\epsilon_1 \tag{20}$$

the data can be replotted as shown in Figure 8b and described by a straight line. Hence, not only do the coefficients a and b have real physical significance, but they are extremely easy to obtain. The value of R for any given test is obtained simply by multiplying the actual failure value $(\sigma_1 - \sigma_3)_f$ determined in the test by the empirical coefficient b, as shown in Eq. 16.

Figure 5. Typical ranges for Janbu's coefficients.

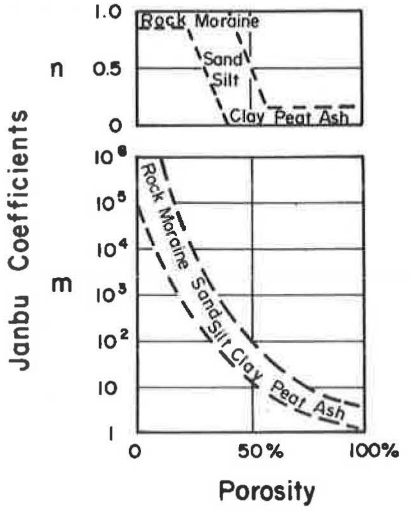



Figure 6. Constrained modulus versus stress level for various dry densities.

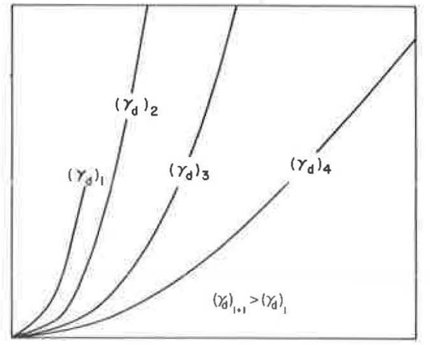



Figure 7. Piecewise linear stress-strain response.

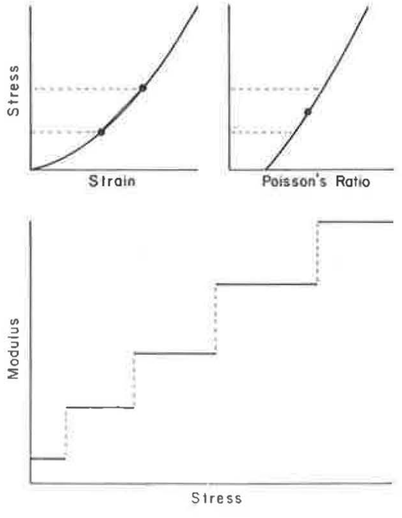
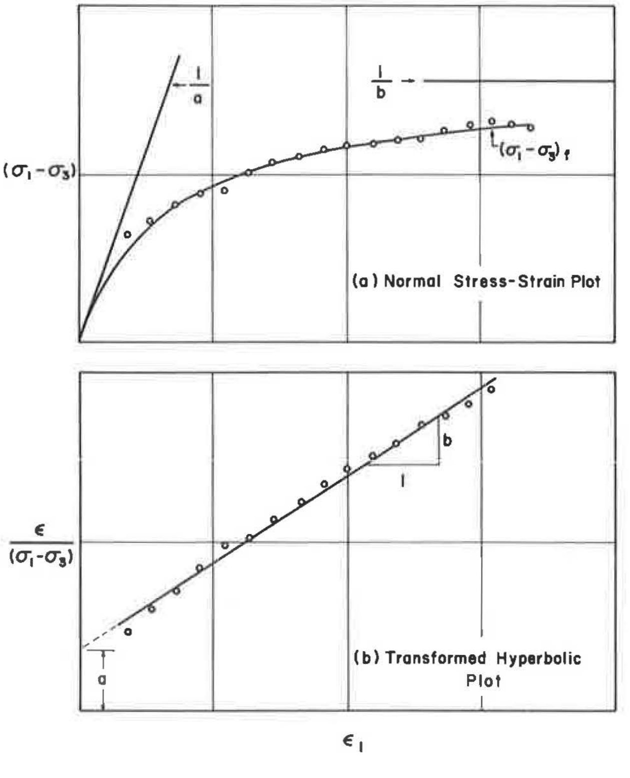



Figure 8. Hyperbolic representation of stress-strain response.



In general, the coefficients a and b will not be constant for different soils, nor will they be constant for the same soil under different test conditions (such as strain rate, confining pressure, and water content). Therefore, this formulation can be generalized by performing a series of tests to determine the functional relation between a and b and the other parameters of interest for a given problem. The test range should cover those conditions that are expected in the field situation. Of particular interest is the dependency of a and b on the state of stress. There is some indication that the initial tangent modulus, $E_i = 1/a$, can be related to the confining pressure, $\sigma_3$, by

$$E_i = 1/a = \alpha\, p_a\, (\sigma_3/p_a)^{\beta} \tag{21}$$

which is a modified form of the relation proposed by Janbu (4), and $(\sigma_1 - \sigma_3)_{ult} = 1/b$ can be replaced by the Mohr-Coulomb failure criterion

$$(\sigma_1 - \sigma_3)_{ult} = 1/b = 1/R\, (\sigma_1 - \sigma_3)_f = (1/R)[2c \cos \phi + (\sigma_1 - \sigma_3) \sin \phi] \tag{22}$$

where c and $\phi$ are empirical coefficients that are determined from a series of triaxial tests. The latter modifications to Eq. 19 have been proposed by Duncan and Chang (2), and the resulting expression has been applied with some success to a variety of soil-structure interaction problems. In view of this success, its application to the buried conduit problem certainly appears justified. Also, this approach allows direct consideration of large strains and a failure condition in the soil (this is in contrast to the approach based on the consolidation test), and most laboratories have the appropriate equipment to conduct the test.

## Determination of Modulus From Plane Strain Test

In a plane strain test, $\epsilon_2$ is held equal to zero, and $\sigma_1$, $\epsilon_1$, and $\sigma_3$ are either controlled or measured; $\sigma_2$ is then calculated by assuming a linear elastic constitutive relation for the soil. Young's modulus, E, can be determined from a plane strain test by combining

$$\epsilon_1 = \sigma_1/E - (\nu/E)\,(\sigma_2 + \sigma_3) \tag{23}$$

and

$$\epsilon_2 = \sigma_2/E - (\nu/E)\,(\sigma_1 + \sigma_3) \tag{24}$$

with the condition of $\epsilon_2 = 0$ to yield

$$E/E_p = [(1 - \nu^2) - (\sigma_3/\sigma_1)\,\nu\,(1 + \nu)] \tag{25}$$

where

$$E_p = \sigma_1/\epsilon_1 \text{ or } \Delta\sigma_1/\Delta\epsilon_1 \tag{26}$$

The strong dependence of $E/E_p$ on $\nu$ and $\sigma_3/\sigma_1$ is shown in Figure 4c. Equation 25 can be incorporated into a finite-element model in the same manner already described, and the appropriate value of the modulus for each element would be selected on the basis of the ratio of $\sigma_3/\sigma_1$. Alternatively, it seems very logical that a hyperbolic formulation similar to that described for the triaxial test could be advanced. Although many buried conduit problems may be considered essentially plane strain in nature, the determination of a modulus by means of a plane strain test has the serious disadvantage that relatively few laboratories are equipped at the present time with plane strain test equipment.

## Comparison of Methods for Determination of Modulus

The soil in the vicinity of an underground conduit is often subjected to considerable confinement, particularly in situations where the horizontal dimensions greatly exceed the vertical height of soil above the conduit. In such cases, the major principal strain at most points throughout the system will usually be much larger than the other principal strains. For example, in a typical embankment installation, the major principal strain in the fill at some distance from the conduit will be primarily vertical, the horizontal strain in the longitudinal direction of the fill will be essentially zero, and the horizontal strain in the lateral direction will probably be very small. Near the conduit, the major principal strain will be predominantly radial, especially where considerable conduit deformation is involved; there may be a slight tensile strain parallel to the centerline of the conduit; and the strain tangent to the conduit wall in a vertical plane will probably be compressive. Although the foregoing reasoning is qualitatively correct, it is difficult to assess intuitively the quantitative relations that are involved. Nevertheless, the predominance of the major principal strain indicates that the response of a soil-conduit system is probably influenced more strongly by dilatational stresses than deviatoric stresses. Therefore, the consolidation test (or uniaxial strain test) may very well provide the most reliable immediate source of input data for soil properties because it is concerned only with volume change characteristics; however, if the conduit deformations are relatively small, as is the case for a concrete pipe, shear deformations in the soil adjacent to the conduit may be important, and the plane strain or triaxial tests may be more appropriate because they involve considerable deviatoric effects. The consolidation test has the very significant advantage that the required equipment is currently available in almost every soils laboratory, whereas triaxial test equipment is less common and plane strain equipment very rare.

## Determination of Poisson's Ratio

As can be seen in Figure 4, the determination of Young's modulus, E, by use of the preceding three tests is strongly dependent on a knowledge of Poisson's ratio, $\nu$. Unfortunately, $\nu$ is a very illusive soil property to obtain, and it has provided a source of frustration for many researchers. As a matter of fact, there is considerable support for the position that one can make an engineering estimate that is as good as or better than any value that can be experimentally determined, and this may indeed be the case. This is largely because of the fact that E, $\nu$, and the state of stress are intimately related, and it is difficult to determine one parameter without a knowledge of the others. This leads to a situation where an error in E causes an error in $\nu$, and vice versa. Although a survey of the literature can provide substantial guidance in the selection of a particular value for $\nu$, there is little quantitative justification to be found. Accordingly, a continual effort must be advanced to improve our understanding of this parameter, and engineering ingenuity must be employed to find better ways of either measuring $\nu$ or offsetting its effect in a mathematical model.

Continuing with an interpretation of test results in terms of linear elastic theory, we may write the following equations for both a conventional test and a plane strain test:

$$E\,\epsilon_1 = \sigma_1 - \nu\,(\sigma_2 + \sigma_3) \tag{27a}$$

$$E\,\epsilon_3 = \sigma_3 - \nu\,(\sigma_1 + \sigma_2) \tag{27b}$$

Multiplication of Eqs. 27a and 27b by $\epsilon_3$ and $\epsilon_1$ respectively, subtraction of the results, and rearrangement lead to

$$\nu = (\sigma_3\epsilon_1 - \sigma_1\epsilon_3)/[\sigma_1\epsilon_1 + \sigma_2(\epsilon_1 - \epsilon_3) - \sigma_3\epsilon_3] \tag{28}$$

For a conventional triaxial test, $\sigma_2 = \sigma_3$, and Eq. 28 reduces to

$$\nu = (\sigma_3\epsilon_1 - \sigma_1\epsilon_3)/[\sigma_1\epsilon_1 + \sigma_3(\epsilon_1 - 2\epsilon_3)] \tag{29}$$

whereas for a plane strain test, $\sigma_2$ equals $\nu(\sigma_1 + \sigma_3)$, as given by Eq. 24, and Eq. 28 becomes

$$A\nu^2 + B\nu + C = 0 \tag{30}$$

where

$$A = (\sigma_1 + \sigma_3)(\epsilon_1 - \epsilon_3) \tag{31a}$$

$$B = \sigma_1\epsilon_1 - \sigma_3\epsilon_3 \tag{31b}$$

and

$$C = \sigma_1\epsilon_3 - \sigma_3\epsilon_1 \tag{31c}$$

Solution of Eq. 30 by using the quadratic formula yields

$$\nu = (-B \pm \sqrt{B^2 - 4AC})/2A \tag{32}$$

Hence, if $\epsilon_3$ is measured in either of the preceding two tests in addition to the conventionally measured $\sigma_1$, $\sigma_3$, and $\epsilon_1$, $\nu$ may be theoretically determined by Eq. 29 for a triaxial test and by Eq. 32 for a plane strain test. However, in view of the limitations imposed by the basic assumptions (such as linear elasticity and stress and strain homogeneity) employed and the probable error associated with the measurement of $\epsilon_3$ (this is related to the assumption of strain homogeneity), it cannot be expected that these equations will yield acceptable results unless extreme care is exercised in the test procedures. Values of $\epsilon_3$ at various stress levels can be obtained either by direct measurement or, for saturated samples, by measurement of the volume change; the latter technique necessarily yields an average value for $\epsilon_3$, whereas the former may very well give a maximum (an erroneous) value if measured at the midheight of the specimen. However, for unsaturated samples direct measurement is the only recourse.

Another possible approach for determining $\nu$ is to utilize the empirical relation (1, 3)

$$K_o = 1 - \sin\varphi \tag{33}$$

in conjunction with Eq. 3 to yield

$$\nu = (1 - \sin\varphi)/(2 - \sin\varphi) \tag{34}$$

where $\varphi$ in this case can be determined for various stress levels prior to failure as well as at failure. Although this approach is rather indirect, Eq. 33 is based on considerable experimental evidence, and the error associated with its use should lie within acceptable limits.

Although not quite so well founded as the hyperbolic stress-strain formulation, there is some evidence to indicate that a hyperbolic equation of the form

$$\epsilon_1 = \epsilon_3/(r + s\epsilon_3) \tag{35}$$

can be used to relate the axial strain, $\epsilon_1$, and the radial strain, $\epsilon_3$, in a conventional triaxial test, where the empirical coefficients r and s are determined in a manner similar to a and b in Eq. 14. Then, analogous to the definition of the tangent modulus, $E_t$, given by Eq. 17, one might define a tangent Poisson's ratio, $\nu_t$, by solving Eq. 35 for $\epsilon_3$ and differentiating with respect to $\epsilon_1$ to obtain

$$\nu_t = d\epsilon_3/d\epsilon_1 = r/(1 - s\epsilon_1)^2 \tag{36}$$

The strain dependence of Eq. 36 can be changed to stress dependence by solving Eq. 14 for $\epsilon_1$ and substituting the result in Eq. 36 to give

$$\nu_t = r[1 - b(\sigma_1 - \sigma_3)]^2/[1 - (b + as)(\sigma_1 - \sigma_3)]^2 \tag{37}$$

The problem now reduces to one of determining the empirical coefficients as functions of the state of stress and the other variables of the soil. From a theoretical point of view, it is noteworthy to point out that the state of stress in a conventional triaxial test is not consistent with that used in the definition of Poisson's ratio; hence, the definition given by Eq. 36 is not strictly correct. Nevertheless, for all practical purposes, and in view of the complexities introduced by the use of Eq. 29, the empirical formulation given by Eq. 37 is probably justified for use in conjunction with Eq. 19. However, such a formulation cannot satisfactorily account for dilatancy effects in soils. If the experimentally determined value for $\nu$ is equal to or greater than one-half, a value such as 0.49 is usually incorporated in the theoretical analysis because values of one-half or greater are incompatible with classical linear elastic theory and the associated finite-element method.

As can readily be appreciated, Poisson's ratio is an important soil parameter, but it is most difficult to quantify. Accordingly, it seems that the best chance for success with this parameter in the near future is to develop and employ a method of analysis that essentially offsets its effect. This is one of the apparent advantages of using the plane strain test to determine soil parameters. Because a plane strain laboratory test essentially models the field situation for many buried conduit installations, it is likely that the effects of Poisson's ratio can be minimized by interpreting the laboratory response in terms of plane strain conditions and by using the result to analyze the field problem.

### Backpacking Materials

Considerable attention must be given to the mechanical properties of backpacking or cushioning materials that may be used immediately adjacent to all or a portion of the conduit. If uncompacted or lightly compacted soils are used, the modulus values (and probably Poisson's ratio) will be strongly affected by the dry density, and laboratory tests (similar to those previously discussed) must be conducted at densities appropriate to the field installation. In addition, care must be taken to ensure that the dimensions of these low-modulus zones in the field are consistent with those used in the finite-element model. If nonsoil backpacking materials are used, special effort must be made to quantify their mechanical properties, and this is often extremely difficult to do. If this task cannot be accomplished with reasonable confidence, a parameter study may be used to assess the relative importance of the value selected. Finally, it is very likely that the time effects on a backpacking material, especially a nonsoil material, cannot be neglected, but the manner in which they should be included is not at all clear.

## INTERFACE CONDITIONS

Within the realm of material properties should be included the conditions that exist along the soil-conduit interface and along the boundaries between different zones of soil. In general, there are three situations that can be handled without too much analytic difficulty: full slip, no slip, and no slip until a prescribed stress has been reached. In the absence of any information to the contrary, it seems most appropriate to utilize the third condition. The upper bound of the prescribed stress would certainly be the shear strength of the soil, and the extent to which the actual stress at the interface differs from this upper bound would depend on the smoothness of the conduit. In general, it seems quite reasonable to use the shear strength of the weaker soil at the interface between different soil zones. For cases where the conduit is very smooth, as with some metal and plastic pipes, the full-slip condition may be more applicable. However, instead of attempting to characterize this condition exactly, one may examine the extreme cases of full slip and no slip (5) for one particular type of problem and thereby evaluate its influence.

## PROPERTIES OF CONDUIT MATERIALS

Although the mechanical properties of the surrounding soil probably exert the greatest influence on the response of a soil-conduit system, the properties of the conduit materials are also extremely important. In most, but not all, cases, however, the properties of the conduit materials exhibit more limited ranges of variation and can be characterized with greater reliability than soils. A few of the key considerations regarding conduit materials will be discussed in the following sections.

## Metal Conduits

In general, steel and aluminum possess relatively well-known mechanical properties, and their determination is not difficult; however, metal pipes have some characteristics that are not readily and reliably evaluated. Among these are their susceptibility to corrosion and abrasion, the effectiveness of protective coatings, the behavior of seams, and the buckling strength. Various empirical equations and statistical correlations are available to handle the durability problem, which indirectly affects the soil-structure interaction problem by altering the pipe wall with time. The stiffness of a seam may significantly influence the response of a conduit, particularly if the seam is less stiff than the rest of the conduit wall or if there is an abrupt change in the cross section of the wall at the seam. Directly related to the behavior of a seam are the characteristics (not only strength, but durability, deformability, and brittleness) of the bolts that are used. For large-diameter conduits, particularly those with relatively shallow cover heights, stability against buckling must be checked. This leads to a relation that involves relative values for the mechanical properties of both the conduit and the surrounding soil as well as the cover height.

## Concrete Conduits

The most important properties that influence the structural response of reinforced concrete conduits are the compressive and tensile moduli of the concrete, the tensile strength of the concrete, the amount and position of the reinforcing steel, and the quality of the bond between the concrete and the steel. The role of the compressive and tensile moduli are self-evident, whereas the tensile strength is important because it is directly related to progressive cracking, which in turn alters the internal stresses and displacements within the pipe wall. As a consequence of progressive cracking, the stiffness of the conduit is continually modified as the load is increased, thereby introducing a decidedly nonlinear response. The amount and position of the reinforcing steel of course controls the stiffness of the conduit, and the associated analyses must be carefully reviewed in light of the final design. Although the amount of steel is relatively easy to determine, accurate placement is largely dependent on fabrication procedures. The quality of the bond between the concrete and the steel is important if the deformation characteristics of the conduit are based on effective bond considerations. Bond failures cause small local slips and some widening of the cracks with an associated increase in the deformation of the conduit.

## Plastic Conduits

The mechanical properties of many plastics used in the manufacture of conduits are not sufficiently well understood, and much additional effort is needed before these properties can be reliably specified for use in a theoretical analysis. In particular, some plastic conduits exhibit significant time-dependent effects, which influence their long-term response. Thus far, however, the use of plastic conduits has been generally restricted to the smaller diameter installations wherein mechanical properties play a considerably lesser role in the overall behavior of the system.

# CONCLUSIONS

Within the scope of the considerations discussed herein, the following conclusions or summary statements can be advanced.

1. The more significant improvements in our ability to analyze soil-conduit interaction problems in the immediate future lies in the use of a continuum approach and a finite-element formulation; however, the successful application of these procedures will depend largely on our ability to provide appropriate input data for material properties.

2. The specification of reliable, quantitative values for material properties is probably the single factor that limits most seriously our ability to predict the mechanical behavior of soil-conduit systems.

3. The particular material properties to be used in the mathematical model, the manner in which these properties are determined, and the methods of analyses and design are intimately related, and a variation in one step will generally precipitate changes in one or more of the other steps.

4. Field verification is absolutely essential before a mathematical model and its associated material properties can be accepted as a valid procedure for analysis.

5. Of all the material properties that affect the interaction of a soil-conduit system, the characteristics of the soil present the greatest difficulty; and, of all the soil properties of interest, the stress-strain behavior is the most important.

6. Within the framework of currently available methods for testing and analysis, the assumption of a stress-dependent, isotropic soil modulus is probably reasonable.

7. Both the stress dependency of the modulus and the incremental nature of the construction sequence in a conduit installation can be conveniently analyzed by use of a piecewise linear formulation.

8. Subject to proper interpretation, which involves consideration of Poisson's ratio and the state of stress, an appropriate soil modulus can be determined from the results of laboratory consolidation tests, conventional triaxial tests, or plane strain tests.

9. Time effects, which are not explicitly considered herein, must be taken into account by conducting the laboratory test at an appropriate strain rate and with appropriate drainage conditions.

10. Dry density significantly affects the soil modulus, and moduli associated with different densities must be determined in the laboratory and assigned to appropriate zones in the field problem.

11. Although Poisson's ratio significantly influences the determination of a soil modulus from any of the suggested tests, this parameter is most difficult to quantify in a reliable manner; the use of linear elastic theory and experimental measurements to back calculate Poisson's ratio leads to rather unreliable results in many cases, and this situation suggests that the exercise of good engineering judgment to estimate Poisson's ratio may be equally useful.

12. The greatest hope for success in dealing with Poisson's ratio in the near future centers around the development of a method of analysis that essentially offsets its effect; this may be possible to some extent in all cases because Poisson's ratio is used both in the interpretation of a laboratory test and in the analysis of the field problem.

13. Insofar as possible, it is desirable to describe the stress-strain behavior of the soil in terms of a three-dimensional formulation, even if only a simplified one- or two-dimensional version is utilized at present because three-dimensional analytic treatments of soil-conduit problems are not too far in the future.

14. The interface conditions between the soil and the conduit and between various soil zones should be specified by a no-slip condition with a limiting shear stress above which slip occurs.

15. The mechanical properties of the conduit materials may exert a significant influence on the behavior of the physical system; particular examples include the properties of the bolts or welds in a metal conduit, the tensile strength of the concrete in a concrete conduit, and the substantial time-dependent response of some plastic conduits.

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