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Soil-Structure Interaction: A Symposium

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Subject Areas

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63 Mechanics (Earth Mass)

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FOREWORD

Since the introduction of corrugated metal pipe in the late 1890s, there has been a continuing interest in proper pipe usage for maintenance purposes. Early methods of design were truly "rule of thumb" and were developed through "failure criteria." Refinements of these methods resulted in a set of systems that were very effective in dealing with pipe problems. However, the building of the Interstate Highway System brought with it a need to develop new approaches that would incorporate the latest soil-structure interaction theories and be applicable to the larger structures, higher embankments, and new materials. Better inspection and construction control are considered requisite to the utilization of more refined theories.

Practice in the design and construction of culverts continues to improve, but, because many organizations have not been able to keep up with or apply the latest theories, the Highway Research Board Committee on Subsurface Soil-Structure Interaction organized a symposium to assess the state of the art and to delineate problems needing further attention. This is considered the first step in an effort to provide an application bulletin. Committee Chairman Ernest T. Selig, in his paper, reviews the symposium and sets forth some of the needs as established by the committee, one of which is to "define and categorize all of the important failure criteria for design, and indicate expected safety factors in current practice."

This RECORD will be valuable to those concerned with the proper design and construction of culverts because it brings together the latest theories along with discussions of recent experimental work and construction practices. A historical review sets the stage for presentations describing material properties, balanced design, finite-element analysis, structural design practices, and design charts.

In addition to the seven symposium papers, there are discussions of buckling of cylinders and of the historic Iowa deflection formula.

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SUBSURFACE SOIL-STRUCTURE INTERACTION: A SYNOPSIS

Ernest T. Selig, State University of New York at Buffalo

•THE loads imposed on a structure buried in soil depend on the stiffness properties of both the structure and the surrounding soil. This results in a statically indeterminate problem in which the pressure of the soil on the structure produces deflections that in turn determine the pressure. This soil-structure interaction is a subject that has been of technical interest for many decades, and some of the basic concepts and design methods in use today were initiated in the early 1900s. More recently, as a result of an increase in the cost and the importance of buried structures, new information has been accumulated through research, analysis, and testing. However, much of this knowledge has not yet been adapted to design practice, and there are still many questions to be resolved.

The Highway Research Board Committee on Subsurface Soil-Structure Interaction, which has responsibility for this general topic, has set as its immediate goal a compilation of the essence of current knowledge with the purpose of improving design procedures. Although ultimately the publication of an applications manual may result, the necessary first step is an assessment of the state of the art for the purpose of establishing the known facts, the areas of accomplishment, the subjects of uncertainty, and the problems needing further research.

From the outset of the subcommittee effort, the scope and complexity of the subject caused difficulty in establishing the best method of separating the subject into topics for review. Historically, the design of underground structures has been subdivided into a few general categories based on flexibility, configuration, and size. For example, a structure would be classified as an arch or circular or box culvert based on shape, and the design procedures would be classified as rigid or flexible. The former usually represents corrugated steel pipe, and the latter represents reinforced concrete pipe. Each of these subdivisions had empirical design methods associated with it. One of the obvious limitations of this approach is that structures do not necessarily fit precisely into one of the categories. Furthermore, the existing transition from one group to another is not considered, and the limits of applicability are not clearly defined.

Current design should continue to involve available methods that are backed up by field experience when these methods are applicable. At the same time, however, new approaches are needed that incorporate the fundamental system parameters and that are more suitable to larger structures, greater loads, new materials, and better installation techniques.

At a symposium sponsored by the Committee on Subsurface Soil-Structure Interaction the state of the art was reviewed and past accomplishments and future needs were discussed. The subject was subdivided into the following categories:

1. Historical development: major past efforts and their chronological relation;
2. Material properties: basic properties of the structure and surrounding medium and their influence on performance;
3. Experimental studies: laboratory and field experience that aids in understanding or improving design procedures;
4. Analytic methods: finite element and other procedures for predicting performance; and
5. Design philosophy: methods of design used in practice and their advantages and limitations.

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The paper by Linger briefly traces the history of the major accomplishments on the topic of soil-structure interaction. The earliest work concerned conventional conduit design and had as its focal point the contributions of Marston and Spangler at Iowa State University. Most of the recent analytic and experimental contributions have come from sponsored research dealing with protective construction. In the literature, frequent mention is made of the term "arching," and numerous explanations of the arching phenomenon are given. However, confusion and disagreement still exist as to the meaning and cause of arching.

Arching should be considered as the transfer of load to or away from buried structures as a result of the difference in stiffness properties of the structure, with its adjacent encompassing material, and the surrounding expanse of soil. The stress distribution around the structure is therefore different from that which would exist in the same region of soil if the structure were not present. This latter condition is sometimes referred to as the free field. The paper by Allgood and Takahashi defines arching A as

$$A = 1 - (p_i/p_v)$$

where p_i is the vertical pressure on the structure at the crown, and p_v is the free-field vertical stress at the elevation of the crown. If the deformation characteristics of the structure are the same as those of the soil, then $p_i = p_v$ and $A = 0$; i.e., no change in the state of stress occurs because of the presence of the structure. If the structure is not as stiff as the soil it replaces, then $p_i < p_v$ and $A > 0$, i.e., the arching is positive. Conversely, if the structure is stiffer than the soil, then $p_i > p_v$ and $A < 0$; i.e., the arching is negative. If the structure is surrounded by a zone of material that differs from the free-field soil, the same concept applies as long as the structural unit is taken to be the structure together with the zone of material. For example, a rigid concrete structure encompassed in a layer of polyurethane foam or loose soil may have positive arching rather than negative because the composite system is not as stiff as the free-field soil.

In order to provide a quantitative definition of flexible and rigid structures, we must consider both the properties of the soil and the properties of the structure. Allgood and Takahashi recommend for circular culverts the use of the nondimensional term $M_s D^3/EI$, where M_s is the secant modulus of the soil in one-dimensional compression, D is the pipe diameter, E is the modulus of elasticity of the structure, and I is the moment of inertia per unit length of the pipe wall. The classification proposed is as follows:

1. Flexible: $M_s D^3/EI > 10^4$,
2. Intermediate: $10^1 \leq M_s D^3/EI \leq 10^4$, and
3. Stiff: $M_s D^3/EI \leq 10^1$.

Structures in the stiff category will experience negative arching, whereas structures in the flexible category will experience positive arching. The transition occurs in the intermediate category for which the most common design methods are least applicable.

The properties of the structure and the surrounding medium must be considered in any rational design. Determination of the important structural parameters is relatively straightforward, and accepted procedures are available for obtaining numerical values with sufficient accuracy. In contrast, measurement of the appropriate parameters for the surrounding medium is much more difficult. This is partly a result of the inherent complexity of soil stress-strain relations, but it also results from a need to incorporate the influence of bedding conditions and variations caused by construction procedures. Most current design methods treat the system properties, particularly those associated with the soil, by grouping them into several broad categories or by using empirical parameters selected by experience and judgment. Few researchers rely on testing to obtain quantitative values. However, the newest computer methods use rational material properties that are more easily defined, and procedures are being prepared for tests to provide direct determination of these properties.

A thorough discussion of the topic of material properties in relation to soil-structure interaction is provided in the paper by Krizek and Kay. In addition, Parmelee and

Corotis review the parameters required in the commonly used Iowa deflection formula for flexible pipe design, indicating the empirical nature of these parameters and the difficulty in relating them to measurable soil properties.

During the past two decades, a variety of laboratory model studies have been conducted as part of research to better understand soil-structure interaction and to improve on the theories. These studies have been very valuable in determining the key parameters and in demonstrating their influence. Model tests are also useful for comparing the effect of new sets of conditions with those for which previous experience exists. Examples are uncommon loading situations, new culvert shapes, or the alteration in load on one culvert by an adjacent one in multiple installations. Model studies on the other hand have serious limitations in quantitative prediction of full-scale performance because of the difficulty in modeling field conditions. For example, the loading is often caused by soil weight, which is not easily scaled, along with depth and stiffness, and details in the construction process such as buildup of backfill in thin layers are hard to represent correctly.

Field observations of buried structure performance are badly needed to prove new theories and refine existing ones. However, few suitable data are available, and the cost of obtaining needed information is substantial.

Papers by Nielson and Statish and by Nielson describe some of the past experimental work on culverts. The major omissions in these papers are the results of studies in protective construction research and studies using other experimental techniques, such as photoelastic models, to investigate soil-structure interaction.

One of the major problems in developing a suitable analytic method for design of buried structures is the difficulty in defining failure. For example, failure may be based on either local or general buckling, seam or bolt rupture, substantial cracking of concrete, or deflections sufficient to cause surface subsidance. The approach taken to analyze for buckling failure, for example, is given in the paper by Chelapati and Allgood.

The elasticity theory has been useful in providing some general trends, but the more versatile finite-element analysis provides the most comprehensive analytic tool available for predicting load distribution on buried structures. For example, nonlinear soil behavior, bedding details, slippage between the soil and the structure, and any desired geometric shape can be analyzed. The paper by Allgood and Takahashi shows the benefits of this method in relation to other methods.

By using the finite-element method we can carry out an analysis of a buried structure to any degree of detail desired. Of course, the greater is the refinement, the greater is the cost. The limiting constraint then is the ability to define the real conditions, particularly the soil properties and bedding conditions, in order to properly simulate them analytically. It is feasible now to establish package computer programs that can analyze common culvert situations at an economical cost.

Design practice in New York State is described in a paper by Butler to illustrate the manner and degree to which past research has been applied. Not only must structural design be considered to resist the soil loads, but handling qualities during construction and durability to withstand adverse environmental conditions must also be incorporated into the design factors. Recommendations for further research to improve design methods are also suggested by Butler.

Following the symposium, a general discussion was held to review the achievements and to establish the steps that should be taken to apply the new information to design practice. The following tasks were identified as the most important steps to be accomplished under the direction of the subcommittee:

1. Define the basic terminology such as arching, backpacking, and bedding;
2. Determine the key parameters and groups of parameters that determine the performance of the structure and the soil, giving recommended standard symbols for their designation;
3. Define and categorize all of the important failure criteria for design, and indicate expected safety factors in current practice;
4. Outline important aspects of installation techniques, and indicate the desired inspection procedures to ensure satisfactory results;

5. Define the requirements for suitable backfill;
6. Describe the important material properties for the soil and the structure, and indicate how they should be measured;
7. List the requirements for field tests to verify design concepts;
8. Recommend steps to be taken for application of research results to design practice; and
9. Prepare educational plans for dissemination of available information.

The plan of action is to complete many of the aforementioned tasks through committee effort, drawing on available information. Needed research and more extensive effort that may be required to develop design procedures will be recommended.

Based on the presentations at the symposium, it may be concluded that (a) information exists from past research, and more is being generated that should be incorporated into design practice; (b) current methods for design of small culverts must be modified or replaced by methods that can accommodate large culverts and new structural materials with proper economy; and (c) agreement is needed on the best methods to describe and measure the relevant properties of the structure and the surrounding media. Further activity directed to the accomplishment of these tasks will be valuable to the profession.

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HISTORICAL DEVELOPMENT OF THE SOIL-STRUCTURE INTERACTION PROBLEM

Don A. Linger, University of Notre Dame

The term "soil-structure interaction" refers to the general phenomena involved in the behavior of buried structures as a result of the properties of both the structure and the surrounding medium in response to loading imposed on this system. This subject has been of technical interest for many decades, and some of the basic concepts and design methods in use today were initiated in the early 1900s. More recently, as the cost and importance of buried structures have increased, new information has been accumulated through research, analysis, and testing. However, much of this knowledge has still to be adapted to design practice. This paper traces the historical development of the subject of soil-structure interaction.

• THE subject of earth pressure and its application in engineering design have been discussed since the time of Rankine and Coulomb. Since that time considerable effort has been expended in the determination of loads on retainment and underground structures. The term "soil-structure interaction" is used because of the indeterminate effects of the interaction between a structure and the soil. This indeterminancy is the result of the distribution and magnitude of earth pressure varying with the amount and type of deflection of the structure. The phenomenon of an earth pressure that is related to soil deformation was recognized by Rankine and is referred to as the active and the passive Rankine state in the analysis of horizontal earth pressures. It is, of course, obvious that the general phenomenon is not adequately defined by this definition. The soil properties and condition, the structural geometry and rigidity, and the characteristics of the loading all affect the magnitude and distribution of earth pressure on a structure. All of these characteristics are combined into the very complicated, indeterminate problem of soil-structure interaction.

Until recently the design of buried structures was based primarily on the loading produced by the overburden material on the structure, with only the shallow buried structure receiving any significant live load. The advent of nuclear weapons and the resulting need for protective structures brought a new dimension to the study of loads on buried structures with loadings that are orders of magnitude greater than earlier loadings. Almost simultaneously, the development of the Interstate highway program began requiring more and larger highway culverts with greater fill heights and culvert loadings than ever before.

The increase in highway construction and the national defense requirements renewed the interest in underground structures. This interest has resulted in large-scale research and development projects directed at the problem of soil-structure interaction. It has also made us aware of the shortcomings and unknowns in the design of underground structures.

Most important, however, this increased research effort has resulted in a corresponding increase in the level of knowledge on the subject. Moreover, the subject has received so much attention that it is difficult to keep abreast of the technical advances. As a result of these great strides in research, development, and design knowledge of

Sponsored by Committee on Subsurface Soil-Structure Interaction.

soil-structure interaction, it is important to occasionally review the status of what we know, or think we know, about the subject. The objective of this symposium is to review the state of the art of soil-structure interaction knowledge in order to stimulate current research and development on this subject.

The soil-structure interaction symposium has been broadly divided into the subjects that generally provide the topical areas for design and research. Each of these broadly defined subjects has been discussed in detail by the other authors contributing to this symposium. However, it is the purpose of this paper to present a comprehensive coverage of the historical background on the subject of the design of underground structures, a subject often referred to as the soil-structure interaction problem.

For convenience, the subject has been divided into two major areas: the development of concepts in classical culvert design and the development of phenomenological concepts in the response of buried structures. These two subject areas are intimately related because both deal with the same subject. However, this division allows the reader to follow the development of soil-structure interaction with a clearer perspective of the research and development efforts.

The first area, classical culvert design concepts, traces the development of an approach that has attained a level of acceptance that is characteristic of traditional earth pressure theories. The improvements and refinements in this approach are significant and have formed the basis for the design of buried conduit. These theories are still applicable and are used currently in design.

The second area, phenomenological concepts, traces the various studies that have made significant developments in the understanding of the soil-structure interaction problem. Many of these studies have been milestones in the understanding of the phenomenon, but the application of the results of these studies is sporadic and often lost in the confusion of technical advances.

The references discussed in this paper are not inclusive. An extensive bibliography concerning the period 1900 to 1968 was compiled by Krizek, Parmelee, Kay, and El-naggar (1).

DEVELOPMENT OF CONCEPTS

Not enough is known about soil-structure interaction to predict with any degree of accuracy the ultimate load-carrying capacity of buried structures. In the design of underground structures, the loading is usually based on empirical relations that are not fully understood. If the loading on the underground structure is determined from classical earth pressure theory, large variations can be expected between the actual and the theoretical loadings. These variations can result from the underground structure deflecting more than the adjacent soil and thereby causing a reduction in the pressure transmitted to the structure with a corresponding increase in the pressure carried by the adjacent medium. Conversely, under load, the structure may not deform as much as the adjacent soil, and the resulting redistribution can produce an increase in load on the structure and a decrease in the pressure carried by the adjacent medium. These two opposite conditions are similar to the active and passive earth pressure conditions defined by Rankine more than 100 years ago. The difference in the conditions is determined by the direction of the soil stress produced by the soil movement along some slippage plane. Because of the elegance of the classical earth pressure theory, it is understandable that the earliest approaches to the loading of buried structures should take a form similar to the Rankine earth pressure theory. The two opposite conditions of soil-structure interaction loading are characterized by the soil-structure systems shown in Figures 1 and 2.

The amount of pressure redistribution is very difficult to quantify and depends on the degree to which the relative deflection along the shearing plane has mobilized the soil shear strength. From this it is apparent that identifying the location of the shearing plane and the amount and type of stresses induced along the shearing plane is an important part of defining the problem. .

In the case of a large underground structure deflecting under load, the soil at the center of the roof span of the structure displaces with respect to the soil over the supports and also with respect to the adjacent soil in which it is buried. Because of the differential deflection of the various parts of the structure and the relative flexibility of the soil and the buried structure, the soil-structure interaction phenomenon will occur as a redistribution of pressure among various segments of the structure in addition to the redistribution of load from the structure to the adjacent soil. This simplification of a very complicated problem is shown in Figure 3.

Classical Concepts

Marston was the first to recognize that the loading on an underground structure is dependent on the interaction of the structure and the surrounding soil. In 1913 he published the Marston theory on soil loads on drainage pipes (2). This theory was based on a prism of soil whose movement developed the forces shown in Figure 4 as it imposed a load on the underground structure. This theory clearly took into account the relative deflection of the pipe and the settlement of the soil. However, the design of the buried pipe was based on vertical loads only and was only applicable in the design of buried rigid pipes such as clay tile or concrete pipes.

The earliest development of flexible conduit design criteria was based on empirical equations developed by using the results of the 1926 American Railway Engineering Association investigation (3). Tables were developed for the necessary pipe thickness and diameter for various heights of fill. The design tables were based on the assumption that failure occurred when the pipe deflection reached 20 percent of the diameter. For design, the deflection was limited to 5 percent, thus providing a safety factor of 4. It is interesting to note that no attempt was made to correlate the load-carrying capacity with soil characteristics, and therefore there was little evidence of any understanding of soil-structure interaction. It is also interesting to note that the original fill height versus pipe requirement table was the forerunner of the gauge tables commonly used today.

As highway construction increased during the 1930s, the use of larger and more costly drainage structures also increased. The need for a more rational concept for the design of flexible pipes was observed by Spangler, a former student of Marston. Consequently in 1941, Spangler (4) published his Iowa formula for predicting the deflection of buried flexible pipe. Spangler introduced the first well-defined soil-structure interaction concept (Fig. 5). This concept recognized that a passive type of soil pressure is developed by the horizontal expansion of the pipe, which allowed the pipe to carry more load with less deflection than in the unrestrained condition. Moreover, he proposed that the deflection might be used as a basis for determining the magnitude of the horizontal pressure developed on the sides of the pipe. He defined the proportionality constant between the pipe deflection and the developed pressure and proposed limiting values for use in design. This method was the first procedure that required an evaluation of the necessary soil properties for application in design.

In 1960, White and Layer (5) proposed the ring compression theory for the design of flexible buried pipes as shown in Figure 6. This theory assumes that the ring deflection of the structure is negligible and that the failure occurs by the crushing of the pipe walls. Model tests were conducted separately by Meyerhof (6) and Watkins (7) to evaluate the ring compression theory. The results of these studies showed that failure could result from an additional parameter, that of the buckling of the culvert wall (Fig. 7).

Further studies by Watkins (8), Meyerhof and Baikie (6), and Meyerhof and Fisher (10) resulted in the further refinement of structural response in terms of the deflection, crushing, and buckling aspects of the buried structure. However, in some of these studies, loosely defined soil terms such as "good backfill," "compressible soil," and "plastic soil" appear in the description of formulas and coefficients.

It is apparent that by the middle 1960s extensive studies had defined the problem and isolated the important parameters, but a complete definition of the parameters did not exist. In 1967, in an attempt to further clarify the interaction of the soil characteristics and the deflection of the structure, Nielson presented a theory for determining loads on buried conduit by an arching analysis (11). The proposed method used an adaptation of the Spangler deflection equation, but, despite the apparent good agreement with the experimental data studied, little use has been made of this novel diversion from the classical Marston procedure. The proposed arching condition is shown in Figure 8.

An interesting aspect of the research and design procedures is the way in which generally accepted methods treat either rigid structures or flexible structures, with adequate procedures being available for each. However, only limited research has been directed toward development of a comprehensive design procedure covering the full range of pipe stiffnesses.

Phenomenological Concepts

Even though soil-structure interaction phenomena are still not completely understood, it was recognized during the early studies that the overall compressibility of the structure relative to the soil it replaces is important. Terzaghi treated this phenomenon in considerable detail in his trapdoor tests (12). This was one of the first papers to comprehensively evaluate the stress distribution on a structure in a fully buried condition. Terzaghi discusses the fundamental assumptions of the researchers who contributed to an understanding of the problem (13): Engesser (1882), Bierbaumer (1913), Coquot (1934), and Vollmy (1937). The principal contribution of these studies was to delineate the formation of the soil surface along which the soil arching stresses were mobilized (Fig. 9).

One of the next major milestones was a paper by Whitman, which reported on the results of a buried dome study in which the soil was simulated by a uniformly placed granular backfill (14). These tests were a part of a program that set an example for many of the tests that followed in the study of buried structure responses.

The requirements for buried structure design criteria resulted in a unique conceptual approach developed and presented by Newmark and Haltiwanger (15). This publication advanced many new ideas and provided the impetus for much of the research that followed.

In 1964, the state of the art of soil-structure interaction was reviewed at a symposium held at the University of Arizona (16). The participants at this symposium discussed in detail the various aspects of the phenomenon. A paper by Triandafilidis et al. (17) delineated the important variables of soil-structure interaction in a series of tests performed on vertical cylindrical and disk structures designed to separate arching stresses from sidewall friction effects. The results of this study provided the necessary quantitative data to enable researchers to make an analytical relation between structure stiffness and medium stiffness and the load on the structure.

At this symposium, Luscher and Hoeg (18) presented the results of a study that discussed the uncertainty in the lateral pressures acting at the sliding surface. Considerable attention was given by these authors to the assigned values of the at-rest and active pressure coefficients used by other investigators. The results of a study by Donnellan (19) were reported, which demonstrated the effect of depth of burial on the load-carrying capacity of a cylinder. Donnellan's study defined the shallow and deep burial conditions and the effect of burial depth on the deflection behavior of rigid and flexible cylinders. An example of these results is shown in Figure 10.

Additional studies reported at this symposium by Selig (20) defined the methodology for the measurement of soil pressures and deformations with great accuracy. This was an important step forward in the research on soil-structure interaction. It was this aspect that implied that soil tests could be used to evaluate and define the necessary properties for the design of soil-structure systems. Researchers were quick to begin studies of the effect of soil characteristics on the response of buried structures. A notable example of this effort was one of the studies of Allgood (21). This research presented a method for determining deflections and critical buckling loads based on the one-dimensional confined compression modulus of the soil. From this research, it was apparent that we had a handle on the problem of the interaction of the structure and the soil. The next obvious step was to develop the means of modifying the pressure on the structure by modifying the characteristics of the surrounding soil. This approach had been tried by Spangler (22) with considerable success but without any quantified design criteria.

Figure 1. Active soil pressure condition.

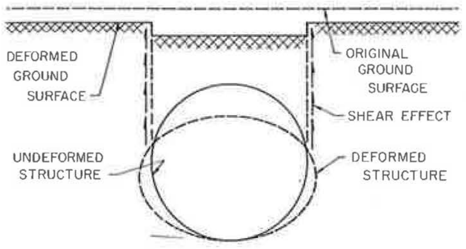


Figure 2. Passive soil pressure condition.

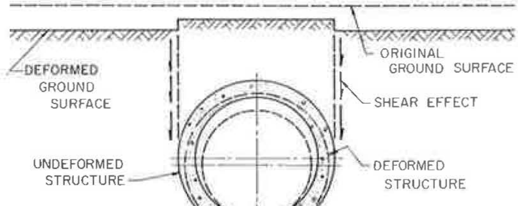


Figure 3. Soil pressure redistribution.

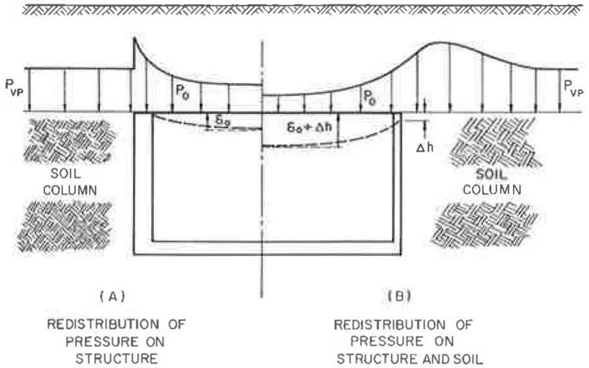


Figure 4. Marston theory of soil loads on rigid buried pipes.

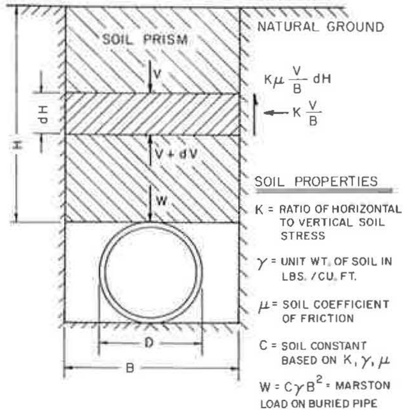


Figure 5. Loading assumptions for the Iowa formula.

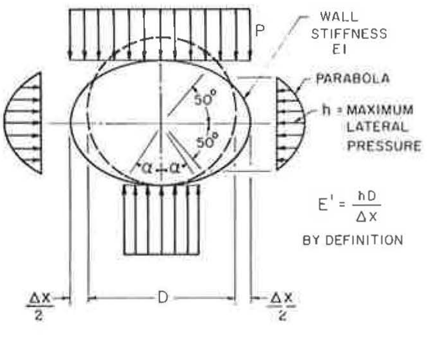


Figure 6. Ring compression theory for design of flexible pipe embedded in dense soil.

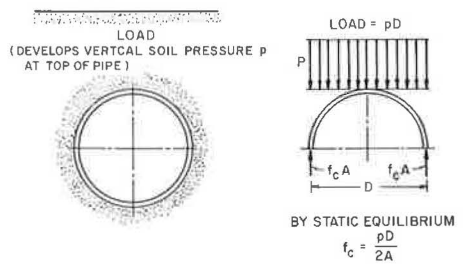


Figure 7. Ring buckling curves for buried flexible pipes.

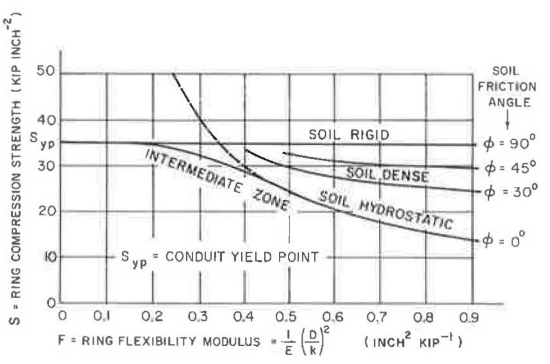


Figure 8. Free-body diagram for arching analysis.

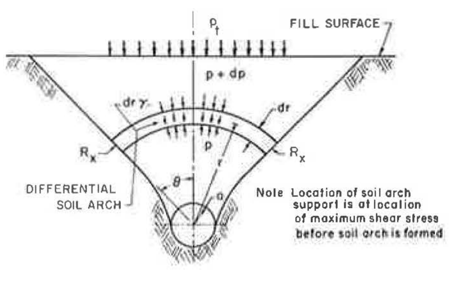


Figure 9. Formation of soil arching stresses.

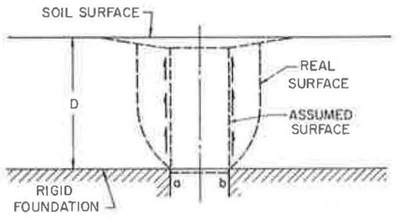


Figure 10. Normalized radial displacement of horizontal cylindrical structure (diameter/thickness = 114).

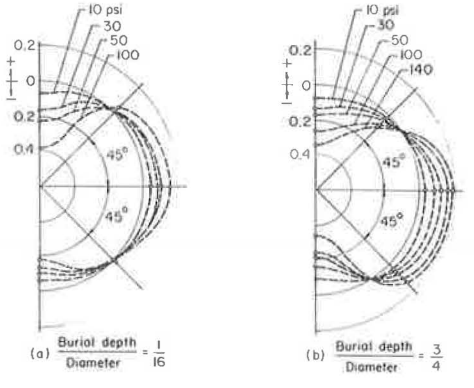


Figure 11. Effective density profile for standard backfill.

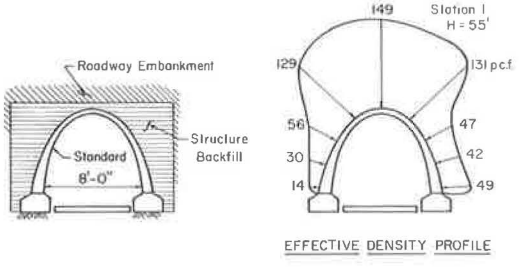


Figure 12. Effective density profile for uncompacted backfill over culvert.

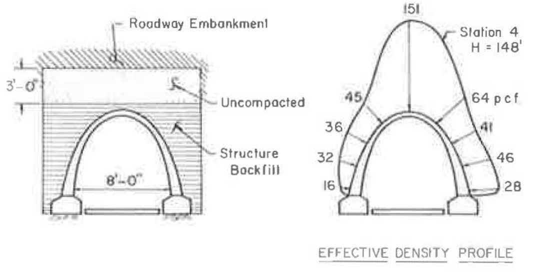
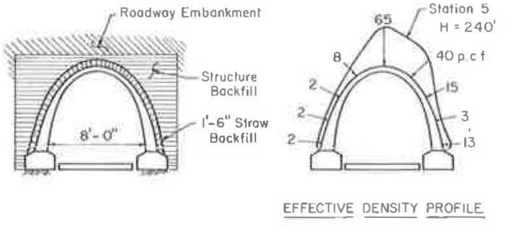


Figure 13. Effective density profile for straw backfill adjacent to culvert.



After the state of the art was advanced further, this procedure was explored once again by the California Division of Highways in an extensive research program that included the measurement of soil pressures on several large full-scale culverts with various conditions of the surrounding media. These results were reported by Davis and Bacher (23) and present an interesting characterization of the changes that occurred during the development of further insight into the soil-structure interaction phenomenon. The changes produced in the soil-structure interface pressure in this study by using the soft "backpacking" material are evident in Figures 11, 12, and 13.

SUMMARY

The problem of soil-structure interaction is illustrated by the design requirements for culverts to carry tremendous overburden fill heights and for complex buried structure systems to resist large surface loadings. The problem is further complicated by the scarcity of failures attributable to design shortcomings and the difficulty in evaluating a failure when it does occur.

Current design practice is based largely on work conducted in the 1920s and 1930s, and, despite the success of these practices, they are empirical in nature and depend heavily on experience and engineering judgment. However, recent refinements in these procedures have made possible much more daring uses of soil-structure interaction concepts.

Design engineers now have enough confidence to construct soil-structure systems in which the soil pressures are controlled by the backfilling techniques or the backfilling materials. This concept seems to have great potential, but the irony of this "breakthrough" is that it was first presented by Spangler and Marston as a result of their first tests on buried conduit, and it was called the "imperfect ditch method."

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13-29

MATERIAL PROPERTIES AFFECTING SOIL-STRUCTURE INTERACTION OF UNDERGROUND CONDUITS

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Notwithstanding current limitations of analytic procedures, the greatest error in available techniques for the analysis and design of underground conduits probably lies in the specification of material properties, especially those for the soil surrounding the conduit. However, the mechanical properties of the conduit material and the conditions that exist at the soil-conduit interface may be significant. Included herein is a brief discussion of the material parameters that form a part of the classical procedures for the analysis and design of pipe conduits, and the arguments against their continued long-term use are given. The advent of the high-speed digital computer and the finite-element method have provided the opportunity to handle material properties in a more realistic manner, and soil-conduit problems should be formulated to take this fact into account. Nonhomogeneity resulting from different materials being used for the underlying soils, bedding, side fill, and backfill or embankment can be readily included in the analysis, and incremental approaches allow nonlinear material properties and the actual construction sequence to be incorporated in a piecewise linear manner without too much difficulty; even three-dimensional analyses by numerical methods have recently come into use. Accordingly, appropriate soil properties must be specified to guide the development of increasingly sophisticated analyses and computer programs.

•THE analysis and design of underground conduits are essentially a problem of soil-structure interaction, and the solution of any problem of this type must give full cognizance to the fundamental coupling phenomenon. Interpreted simply, this concept states that the response of the conduit and the behavior of the surrounding soil are not independent but intimately related in some complex manner. In general, the response of a soil-conduit system depends on the characteristics (geometry and stiffness) of the conduit, the characteristics (geometry, order of placing, and mechanical properties) of the adjacent and overlying compacted fill, and the characteristics (compressibility) of the in situ soil under and adjacent to the conduit. Notwithstanding the limitations of analytic procedures, the greatest error in currently available techniques for analysis and design probably lies in the specification of material properties, especially those for the soil surrounding the conduit. However, the mechanical properties of the conduit material and the conditions that exist at the soil-conduit interface may also be very significant. This paper will discuss briefly some of the ideas that can be used to determine appropriate input information for material properties.

CURRENT DESIGN PROCEDURES

For the most part, buried conduits are constructed of either reinforced concrete or corrugated metal, and the well-known work by Marston and Spangler and their co-workers has exerted a significant influence on virtually all currently used design procedures (5). Generally accepted design methods treat reinforced concrete conduits as

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a rigid structure and metal conduits as a flexible structure, and separate design procedures are available for each. In an effort to account for the soil-structure interaction (or the relative stiffness of the soil and the conduit), these procedure involve a variety of special parameters (such as settlement ratio, modulus of soil reaction, load factor, and projection ratio) that are associated specifically with the buried conduit problem. Although these parameters may achieve their intended goal when used with good engineering judgment within limited ranges of applicability for which experience is available, very often their use cannot be easily extended or generalized. Also of considerable concern is the fact that no techniques are currently available to handle conduits of intermediate stiffness. Despite the limitations outlined, these procedures have served the profession well during the past 50 years, and few, if any, failures can be attributed to the theory itself. As such, this work, including the special parameters that attempt to account for the soil-conduit interaction phenomenon, represents an outstanding example of engineering ingenuity, and the experience gleaned over the years must not be treated lightly. Although good engineering practice dictates that currently used design procedures should not be discarded until better ones have been provided, this same good practice calls for a periodic appraisal of current procedures in the light of recent advances in technology and theoretical developments.

CONTINUUM APPROACH

Despite the advantages and disadvantages that may be attributed to the Marston-Spangler theories, it seems that the major advances in our knowledge of soil-conduit interaction phenomena do not lie in modifying or improving the existing procedures and the associated material parameters but rather in developing a different approach to the problem. Pursuant to this idea, the most logical approach to the soil-conduit interaction problem lies in treating all components (conduit, underlying soils, bedding, side fill, and overlying soils) as continua, each with its unique material properties. Although the complexities associated with the geometry and material properties of a typical soil-conduit system have in the past either precluded the use of this approach or necessitated relatively crude computational procedures, a very versatile analytic tool has been made available to the profession in recent years by the development of the finite-element method and the advent of the high-speed digital computer. In addition to providing the capability for describing the soil-conduit system as a nonhomogeneous, nonlinear continuum, such a treatment has the following advantages: The coupling or soil-conduit interaction effect is inherently taken into account; input parameters would consist of more fundamental characterizations of the soil and conduit material behavior; conduits of intermediate stiffnesses can be analyzed; and the effects of the construction sequence can be studied. Even three-dimensional analyses have recently been made. Accordingly, the following discussion of material parameters is based on the premise that the finite-element method offers the potential for significant improvement in our ability to analyze complex soil-conduit systems.

INTERRELATED STEPS IN DESIGN PROCEDURE

Any design procedure consists, either implicitly or explicitly, of a synthesis of the various steps shown in Figure 1; this diagram indicates that the design procedure is intimately related to and dependent on the sampling and testing techniques, the interpretation of the data, and the methods of analysis that are used. Accordingly, a change in the design procedure is likely to bring about changes in one or more of the other steps involved. In particular, use of the finite-element approach leads to a considerable change in currently used design procedures for underground conduits because it requires that the problem be formulated in terms of material properties that are fundamental to continuum mechanics. However, in certain cases these properties have been studied for years, and many soils laboratories are currently equipped to conduct the required tests.

REQUIREMENTS OF VALID DESIGN PROCEDURE

As shown in Figure 2, three very important components are required to develop a valid design procedure: a mathematical model, material properties, and field verification. The mathematical model, which would probably include a computer program, must be formulated such that it can describe the physical phenomenon under consideration; in the past this component of the overall problem has attracted much attention, and many sophisticated programs have been developed. The applicability of these programs, however, is limited by the assumptions on which they are based and the material properties that are provided as input; very often these programs call for input data that cannot reasonably be provided, and hence the engineering profession obtains little benefit from their use. As has been frequently stated, the results obtained from any computer program are only as good as the input information that is supplied. There is considerable evidence to indicate that in recent years our ability to formulate mathematical models and solve theoretical problems has far outstripped our ability to provide appropriate input information concerning material properties. Finally, field verification of theoretical predications is needed before any analytic procedure can be accepted; however, the high cost of field instrumentation often impedes or totally precludes its use, and the profession is therefore left with no reliable way to assess quantitatively the validity of the combined mathematical model and material properties.

EMPHASIS OF STUDY

The principal objective of this study is to examine workable approaches that may be taken immediately within the framework of currently available testing techniques to interpret laboratory test results for use in obtaining the solution to a soil-conduit problem. The discussion is intended to be representative, not comprehensive, and the purpose is to survey and compare various methods of testing and interpretation, not to suggest one particular procedure. Although the properties of conduit materials and the interface conditions between various zones are discussed briefly, the characterization of the soil is considered to be of primary importance, and the principal thrust of the presentation is therefore directed toward this end. Emphasis is centered around soil testing procedures that are in common use, and the terminology and techniques of linear elasticity are employed. The observed nonlinear behavior of virtually all soils may be handled conveniently by a piecewise linear model, but no attempt is made to advance more rigorous formulations and interpretations than may be realized within the capabilities of most current laboratory test equipment. For example, consideration of all three principal strains and stresses in a constitutive relation would require the conduct of a true triaxial test; however, except for research purposes, the true triaxial test is far too complex for widespread use at the present time, and a complete variation of the properties in all three directions is therefore not treated herein.

SOIL PARAMETERS

It is convenient to consider two extremes of soil performance, which represent the behavioral range of engineering soils: an ideal plastic, cohesive clay and an ideal clean, coarse-grained, cohesionless sand. The major difference between these materials lies not primarily in the ultimate shear stress available but rather in their stress-strain-time characteristics for loads sustained over long periods of time. Loads on clay soils will ordinarily cause time-dependent volume decreases, and mobilized shear stresses may relax because of creep. The ideal cohesionless soil usually exhibits a relatively low compressibility under added loads, and it responds with little time delay. Cohesionless soils tend to develop and maintain a specific shear stress where differential movements occur. In general, because of the time-dependent stress-strain characteristics of most clayey soils, stress concentrations dissipate with time; consequently, it is probable that the pressure normal to the conduit wall would tend to approach the overburden pressure with the passage of time. No specific observations are available to demonstrate the degree to which cohesionless soils can permanently sustain loads transferred from the pipe, but it is likely that this relaxation phenomenon does not exist to the same extent. Because most soils used in an actual conduit installation do not fall into either of these extreme categories, it is difficult to predict the effect of time on the response of a buried conduit. The type of soil (and most especially its degree of saturation) and the time of interest in the problem will dictate whether the soil tests to be discussed subsequently should be drained or undrained; however, for sake of brevity, time considerations are not specifically included in this discussion, and the manner in which they are handled is left to the engineering expertise of the designer or researcher.

Importance of Modulus

As stated previously, the characterization of the soil is probably the most important consideration in a soil-conduit system, and, more specifically, the modulus of the soil is probably the single most important parameter that affects the response of the system. In addition to the interaction between the conduit and the immediately adjacent soil, there is interaction among the various soil zones; this is particularly important when appraising the stiffness of the natural soil relative to the backfill in a trench installation, and it further illustrates the importance of determining the moduli of the soils in the various zones. Accordingly, it follows that considerable attention should be directed toward describing the stress-strain behavior of the soil surrounding the conduit. Unfortunately, the modulus is intimately related to and significantly influenced by Poisson's ratio, and the latter is most difficult to quantify.

Assumed Isotropy of Modulus

The modulus discussed herein is assumed to be isotropic and dependent on the state of stress in the soil at a point. The assumption of isotropy is very significant, especially in view of the nonlinear behavior of most soils, but currently available testing procedures seem to justify no further refinement at this time. In effect, this means that a change in stress in any given direction at a point causes a change in strain in that direction such that the ratio of the two is a constant that is independent of the existing stress in that direction; hence, this assumption is not strictly compatible with the nonlinear behavior of the soil. However, if this premise is accepted, the problem reduces to one of defining the nature of the modulus and the state of stress on which it depends.

Nature of Modulus

When conducting a piecewise linear analysis that involves a nonlinear material, it is possible to utilize three different definitions of modulus: the secant modulus (straight line joining the origin and point on the stress-strain curve), the tangent modulus (derivative of the stress-strain curve at a point), and the chord modulus (straight line joining two points on the stress-strain curve). Each has its inherent advantages and disadvantages, which are related in large part to the manner in which the numerical calculations are conducted. In view of the incremental nature of the applied load on a conduit due to the normal construction sequence, the tangent and chord moduli seem to offer some advantages; in such a formulation the problem is solved for any given state of loading, and the stresses within each element are determined. Then, a modulus compatible with each state of stress is assigned to each element, and the response for the next increment of load is determined, after which the procedure is repeated. Although the foregoing procedure can, of course, be followed with the secant modulus, it involves a somewhat less accurate approximation of the actual stress-strain curve.

Determination of Stress-Strain Response

Three laboratory tests may be conveniently employed to determine the stress-strain response of a soil: the consolidation test, the conventional triaxial test, and the plane strain test. Typical idealized states of stress, as well as typical stress-strain plots and modulus-stress plots, for each of these tests are shown in Figure 3. As can be readily seen, the specification of the three principal stresses in the consolidation test and in the plane strain test requires the assumption of a value for the at-rest pressure coefficient, K_o, and Poisson's ratio, ν, respectively; in the triaxial test the intermediate principal stress is considered to be equal to the minor principal stress. All three of these tests represent very special states of stress, and each has deficiencies that have been discussed at length in the literature. In all tests considered herein, the state of stress is assumed to be homogeneous throughout the specimen; this is generally not the case, and serious test errors can be introduced because of rough end platens and sidewall friction. Once again, the methods for handling these conditions, as well as other test errors, must be left to the judgment of the designer or researcher. A casual glance at the drastically different character of the stress-strain plots obtained from these tests suggests that a reconciliation must lie in the interpretation of the test results, and this is indeed the case. If we assume that the same drainage conditions exist for all tests, the reconciliation of the indicated test results may be established by use of Poisson's ratio if the soil is assumed to be linear elastic. Notwithstanding the extensive use of laboratory tests to determine soil properties, there are many engineers who contend that any modulus determined from a laboratory test is subject to serious error, and they advocate the use of a field test (such as a plate bearing test or a borehole pressure meter test) to determine the soil modulus. However, these tests are extremely expensive, and the assumption of linear elastic behavior is usually required to interpret the results; consequently, they will not be discussed herein.

Relation Between Young's Modulus and Other Moduli

In any finite-element formulation based on a piecewise linear theory, two parameters are required to characterize an isotropic material; these are the modulus of elasticity or Young's modulus, E, and Poisson's ratio, ν. By definition, Young's modulus is the slope of the axial stress-axial strain curve in a uniaxial stress test, and Poisson's ratio is the ratio of the lateral strain to the longitudinal strain for a specimen that is uniaxially stressed in the longitudinal direction. It is significant to note that the definitions of both coefficients include a specification of the state of stress; this fact is often overlooked in the interpretation of much test data. All too often one can find in the literature a case where a conventional triaxial compression test was conducted on a soil, and Young's modulus is defined as σ_1/ϵ_1 and Poisson's ratio as ϵ_3/ϵ_1; both are incorrect, as will be seen later. However, because the ratio σ_1/ϵ_1 constitutes a type of modulus that is relatively easy to determine in a consolidation, conventional triaxial, or plane strain test, this parameter will be termed M, E_T, and E_P respectively for each of the three tests, and relations between E and each of these moduli will be determined in terms of Poisson's ratio and the state of stress in the test specimen. Although considerable discussion is given to the formulation of analytic expressions for these moduli, graphic representations are also acceptable for use in computer programs. Most of the subsequent discussion will center around the tangent modulus or the chord modulus because these have the ability to approximate most closely the actual stress-strain behavior of the soil. Within the context of a piecewise linear formulation, the reference state for each increment of loading may be taken as either the state of stress that exists after the preceding load increment has been applied or the average of the stress states before and after a given load increment (this latter choice leads to the use of an iterative procedure), and a linear modulus is assumed to govern the response due to the added load increment.

Determination of Modulus From Consolidation Test

From a consolidation test wherein the lateral strains are held equal to zero, σ_1 is usually plotted versus ϵ_1, and a so-called constrained modulus, M, is thereby obtained. This modulus would be appropriate for determining the settlement of a large uniformly loaded area such that the lateral deformations are zero or negligible, but it is theoretically not appropriate for use in a two- or three-dimensional problem unless certain modifications, which center around taking into account the Poisson effect, are introduced. Based on the applicability of linear-elastic theory, the first invariant of the stress tensor and the first invariant of the strain tensor can be related by

$$\sigma_1 + \sigma_2 + \sigma_3 = E(\epsilon_1 + \epsilon_2 + \epsilon_3)/(1 - 2\nu) \tag{1}$$

Because $\sigma_2 = \sigma_3 = K_o\sigma_1$ and $\epsilon_2 = \epsilon_3 = 0$ in a consolidation test, Eq. 1 may be written as

$$(1 + 2K_o)\,\sigma_1 = E\epsilon_1/(1 - 2\nu) \tag{2}$$

which, upon substitution of the relation

$$K_o = \nu/(1 - \nu) \tag{3}$$

and solution for E/M, becomes

$$E/M = (1 - 2\nu)(1 + \nu)/(1 - \nu) = (1 - \nu - 2\nu^2)/(1 - \nu) \tag{4}$$

where

$$M = \sigma_1/\epsilon_1 \text{ or } \Delta\sigma_1/\Delta\epsilon_1 \tag{5}$$

As can be seen in Figure 4a, the variation of E/M as a function of ν is considerable, and this effect cannot be ignored when applying consolidation test results to multidimensional problems.

One very convenient empirical expression for the constrained modulus, M, has been reported by Janbu (4) as

$$M = m\,p_a\,(\sigma_1/p_a)^{1-n} \tag{6}$$

where p_a is atmospheric pressure (introduced to maintain dimensional homogeneity), and m and n are termed the modulus number and stress exponent respectively; in effect, these latter two coefficients are empirical parameters to be determined experimentally. However, it is very possible that certain broad correlations, as advanced by Janbu and shown in Figure 5, may be established among these empirical coefficients and certain types of soil or various conditions of a given soil type. As another example, limited data, based on tests by Osterberg (6), have been interpreted by Krizek et al. (5) to suggest that M is a function of dry density and overburden pressure for a large variety of soils. This relation, which is shown in Figure 6, resembles the one proposed by Janbu. Although it admittedly seems to be oversimplified, further study is certainly justified to determine its range of applicability. The combination of Eq. 6 and Eq. 4 gives

$$E = [(1 - \nu - 2\nu^2)/(1 - \nu)]\; m\,p_a\,(\sigma_1/p_a)^{1-n} \tag{7}$$

Equation 7 deals exclusively with the interpretation of a laboratory consolidation test, and it states specifically that Young's modulus, E, is a nonlinear function of the major principal stress, σ_1. When used in conjunction with a finite-element model, the modulus within each element for a given condition of loading would depend only on the major principal stress in that element.

This approach might be improved by intuitively extending the Janbu relation, which is developed in terms of σ_1 only, to an expression in terms of the sum of all three principal stresses, as follows:

$$M = m^*\; p_a\,[(\sigma_1 + \sigma_2 + \sigma_3)/p_a]^{1-n^*} \tag{8}$$

where m* and n* are empirical coefficients similar to m and n previously described. Substitution of Eq. 8 into Eq. 4 yields

$$E = [(1 - \nu - 2\nu^2)/(1 - \nu)]\; m^*\,p_a\,[(\sigma_1 + \sigma_2 + \sigma_3)/p_a]^{1-n^*} \tag{9}$$

which is similar in form to Eq. 7 but more general in formulation. As far as the interpretation of the consolidation test is concerned, the extension suggested previously

Figure 1. Interrelated steps in design procedure.

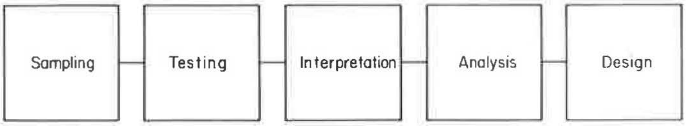


Figure 2. Requirements of valid design procedure.

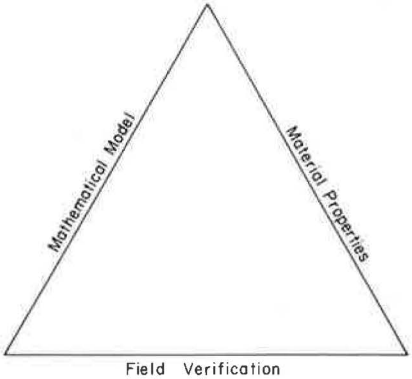


Figure 3. Typical stress-strain response from various laboratory tests.

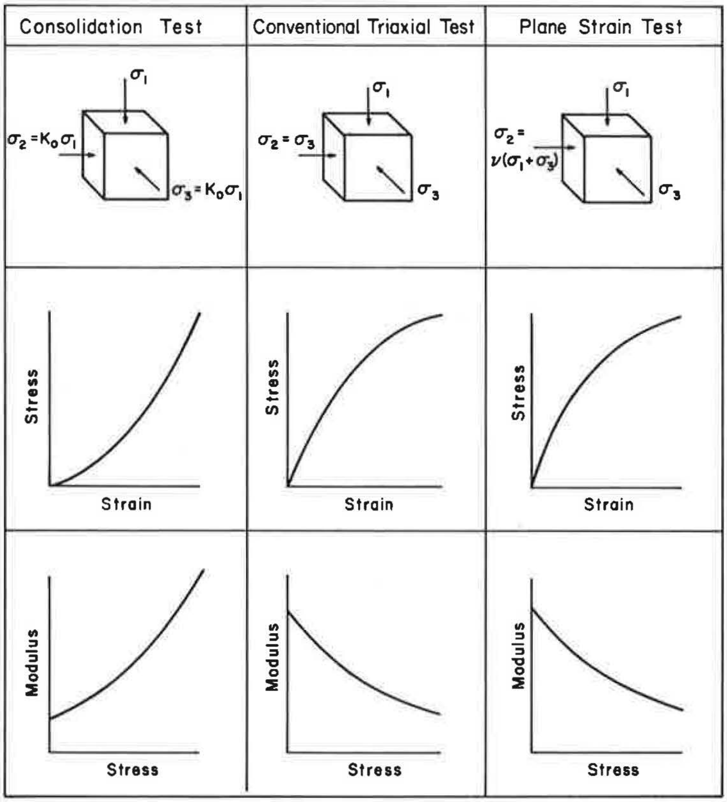
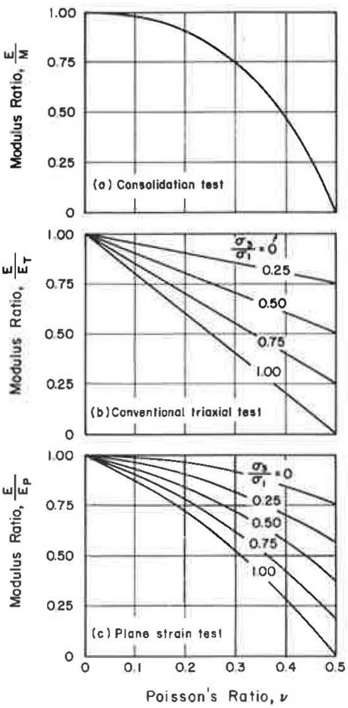


Figure 4. Modulus ratio as a function of Poisson's ratio and state of stress for various laboratory tests.



is perfectly admissible; because $\sigma_2 = \sigma_3 = K_o\sigma_1$ in a consolidation test, Eq. 8 may be written as

$$M = m^* (1 + 2K_o)^{1-n^*} p_a(\sigma_1/p_a)^{1-n^*} \tag{10}$$

Another equally good procedure for incorporating stress-strain data into a finite-element model is the use of the graphic representation shown in Figure 7. In this case, one simply utilizes the chord modulus between any two arbitrary values of stress and some representative constant value for Poisson's ratio in this same stress interval to determine a value for Young's modulus, E, which is considered to be applicable within this stress interval. Then, based on the state of stress (either σ_1 or $\sigma_1 + \sigma_2 + \sigma_3$, depending on the formulation) of each element in the mathematical model for any given loading, an appropriate modulus is assigned to each element for use during the next increment of loading.

Some investigators have simply replaced Young's modulus, E, with the secant constrained modulus, M_s, which is associated with the anticipated final state of stress at the elevation of the conduit; in other words, the effect of Poisson's ratio is completely ignored. The apparent successes achieved with this approach indicate that various unknown effects not taken into account tend to offset each other in certain cases. Despite such limited successes, this approach should be used with extreme caution because it is not on a sound theoretical basis. If, on the other hand, field verification is not obtained, there is little justification to continue this theoretically incorrect practice.

Because the nature of a consolidation test precludes the determination of a failure criterion, special provision will have to be made if this condition is approached in a field problem, and this is, in fact, one of the disadvantages of the test. However, most buried conduit problems are generally concerned with permissible deformations rather than catastrophic collapse, and for such conditions it is probably not necessary to be overly concerned with the shear strength of a soil. The use of a consolidation test to determine the soil modulus is particularly attractive in that the test is relatively easy to conduct and most laboratories have the appropriate equipment. It can also be argued (though not technically correct) that the uniaxial strain conditions of the consolidation test are reasonably well duplicated in the free field and for many situations in a radial direction near the conduit.

Determination of Modulus From Conventional Triaxial Test

In a conventional triaxial test on an unsaturated soil, the major principal stress, σ_1, the minor principal stress, σ_3, and the major principal strain, ϵ_1, are either measured or controlled, and results, which exhibit a shape as shown in Figure 3, are usually plotted as $(\sigma_1 - \sigma_3)$ versus ϵ_1. Based on such a plot, a special modulus, termed herein the triaxial modulus, E_T, may be defined as

$$E_T = \sigma_1/\epsilon_1 \text{ or } \Delta\sigma_1/\Delta\epsilon_1 \tag{11}$$

This definition ignores the Poisson effect of σ_3 on ϵ_1, if considered in terms of absolute values, and, if considered in terms of incremental values, it presumes that the Poisson effect of σ_3 on ϵ_1 is independent of the state of stress within the specimen; actually, however, Poisson's ratio may be expected to vary with the mean stress, $\sigma_m = \frac{1}{3}(\sigma_1 + 2\sigma_3)$, or the shear stress $(\sigma_1 - \sigma_3)$, or both. It should be noted that the often-used ratio of $(\sigma_1 - \sigma_3)$ to ϵ_1 is in effect the ratio of a shear stress to a normal strain; to term this ratio a modulus is inconsistent with most mechanics terminology, and such usage should be avoided. With the assumption of linear elasticity, the constitutive relation

$$\epsilon_1 = (\sigma_1/E) - (\nu/E)(\sigma_2 + \sigma_3) \tag{12}$$

may be combined with the condition that $\sigma_2 = \sigma_3$ and rewritten to give the following relation for Young's modulus, E:

$$E/E_T = [(1 - 2\nu)(\sigma_3/\sigma_1)] \tag{13}$$

where E_T is given by Eq. 11. A plot of Eq. 13 is shown in Figure 4b, and the range of variation is indeed quite substantial.

Provided an acceptable value for ν can be determined, triaxial test results could be incorporated in the finite-element model as follows. The triaxial modulus, E_T, can conveniently be determined from the slope of the σ_1 versus ϵ_1 curve; then, with a knowledge of the σ_3/σ_1 ratio and an estimate of ν, Young's modulus, E, can be determined from Eq. 13. In the finite-element formulation of the buried conduit problem, an E value consistent with the interpretation of the laboratory test can be selected once the values of σ_1 and σ_3 have been determined for a given state of loading.

As an alternative approach that has been demonstrated often in the literature, load-deformation or stress-strain data that exhibit the shape shown in Figure 8a can be very conveniently described by a two-coefficient hyperbola, which for a $(\sigma_1 - \sigma_3)$ versus ϵ_1 plot takes the form

$$\sigma_1 - \sigma_3 = \epsilon_1/(a + b\epsilon_1) \tag{14}$$

where a and b are empirical coefficients to be evaluated by experiment. The limit of Eq. 14 as ϵ_1 approaches infinity yields

$$(\sigma_1 - \sigma_3)_{ult} = 1/b \tag{15}$$

However, because values of $(\sigma_1 - \sigma_3)$ approach $(\sigma_1 - \sigma_3)_{ult}$ asymptotically as ϵ_1 goes to infinity, it may be expected that values of $(\sigma_1 - \sigma_3)$ at failure, or $(\sigma_1 - \sigma_3)_f$, will normally be less than $(\sigma_1 - \sigma_3)_{ult}$, and this will impose an upper bound on the validity of Eq. 14. In general, we have

$$(\sigma_1 - \sigma_3)_f = R\ (\sigma_1 - \sigma_3)_{ult} = R/b \tag{16}$$

where R is another empirical coefficient that usually lies within the range of 0.70 to 0.95 and very often between 0.8 and 0.9. Differentiation of Eq. 14 will yield a tangent modulus, E_t, which may be expressed as

$$E_t = d\ \sigma_1/d\epsilon_1 = a/(a + b\epsilon_1)^2 \tag{17}$$

because σ_3 normally is held constant in a triaxial test, and the evaluation of Eq. 17 where ϵ_1 equals zero will give an initial tangent modulus, E_i, which may be written

$$E_i = 1/a \tag{18}$$

In order to avoid potential contradictions in the reference state (zero value) for stress and strain, we should eliminate the strain parameter in Eq. 17; this can readily be accomplished by solving Eq. 14 for ϵ_1 and by substituting the result into Eq. 17 to give

$$E_t = [1 - b(\sigma_1 - \sigma_3)]^2/a \tag{19}$$

Equation 19 is a quite general relation that may readily be used as described previously in conjunction with a mathematical model, and the following procedure is suggested for evaluating the empirical coefficients. When plotted in the conventional form of $(\sigma_1 - \sigma_2)$ versus ϵ_1, a typical set of triaxial test data will usually exhibit the shape shown in Figure 8a. If Eq. 14 is rewritten in terms of transformed variables as

$$\epsilon_1/(\sigma_1 - \sigma_3) = a + b\epsilon_1 \tag{20}$$

the data can be replotted as shown in Figure 8b and described by a straight line. Hence, not only do the coefficients a and b have real physical significance, but they are extremely easy to obtain. The value of R for any given test is obtained simply by multiplying the actual failure value $(\sigma_1 - \sigma_3)_f$ determined in the test by the empirical coefficient b, as shown in Eq. 16.

Figure 5. Typical ranges for Janbu's coefficients.

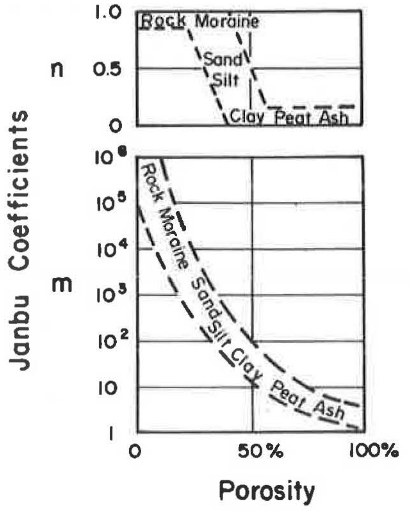


Figure 6. Constrained modulus versus stress level for various dry densities.

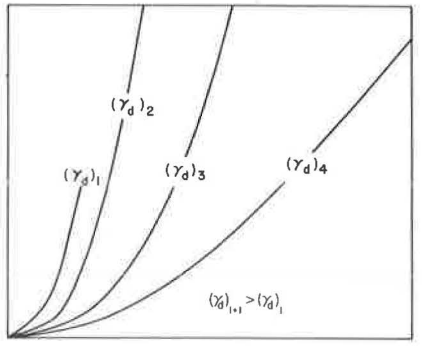


Figure 7. Piecewise linear stress-strain response.

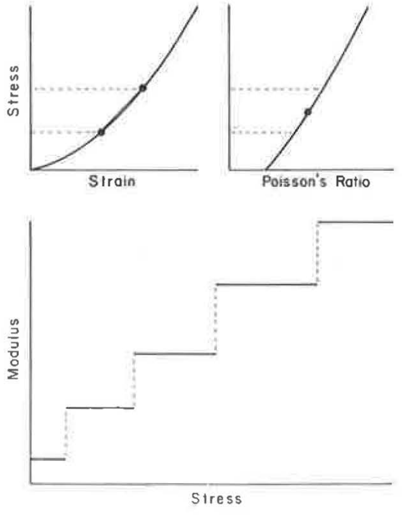
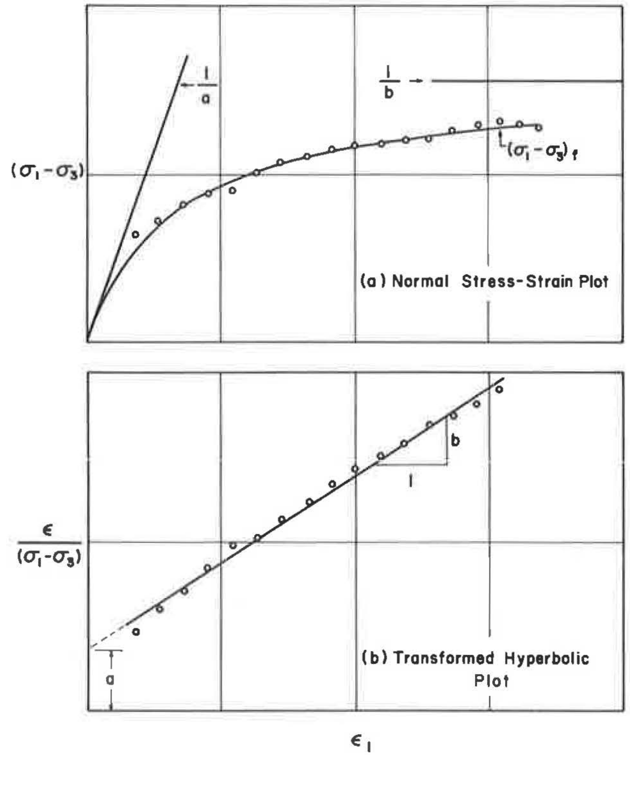


Figure 8. Hyperbolic representation of stress-strain response.



In general, the coefficients a and b will not be constant for different soils, nor will they be constant for the same soil under different test conditions (such as strain rate, confining pressure, and water content). Therefore, this formulation can be generalized by performing a series of tests to determine the functional relation between a and b and the other parameters of interest for a given problem. The test range should cover those conditions that are expected in the field situation. Of particular interest is the dependency of a and b on the state of stress. There is some indication that the initial tangent modulus, $E_i = 1/a$, can be related to the confining pressure, σ_3, by

$$E_i = 1/a = \alpha\, p_a\, (\sigma_3/p_a)^{\beta} \tag{21}$$

which is a modified form of the relation proposed by Janbu (4), and $(\sigma_1 - \sigma_3)_{ult} = 1/b$ can be replaced by the Mohr-Coulomb failure criterion

$$(\sigma_1 - \sigma_3)_{ult} = 1/b = 1/R\ (\sigma_1 - \sigma_3)_f = (1/R)[2c \cos \phi + (\sigma_1 - \sigma_3) \sin \phi] \tag{22}$$

where c and ϕ are empirical coefficients that are determined from a series of triaxial tests. The latter modifications to Eq. 19 have been proposed by Duncan and Chang (2), and the resulting expression has been applied with some success to a variety of soil-structure interaction problems. In view of this success, its application to the buried conduit problem certainly appears justified. Also, this approach allows direct consideration of large strains and a failure condition in the soil (this is in contrast to the approach based on the consolidation test), and most laboratories have the appropriate equipment to conduct the test.

Determination of Modulus From Plane Strain Test

In a plane strain test, ϵ_2 is held equal to zero, and σ_1, ϵ_1, and σ_3 are either controlled or measured; σ_2 is then calculated by assuming a linear elastic constitutive relation for the soil. Young's modulus, E, can be determined from a plane strain test by combining

$$\epsilon_1 = \sigma_1/E - (\nu/E)\ (\sigma_2 + \sigma_3) \tag{23}$$

and

$$\epsilon_2 = \sigma_2/E - (\nu/E)\ (\sigma_1 + \sigma_3) \tag{24}$$

with the condition of $\epsilon_2 = 0$ to yield

$$E/E_p = [(1 - \nu^2) - (\sigma_3/\sigma_1)\ \nu\ (1 + \nu)] \tag{25}$$

where

$$E_p = \sigma_1/\epsilon_1 \text{ or } \Delta\sigma_1/\Delta\epsilon_1 \tag{26}$$

The strong dependence of E/E_p on ν and σ_3/σ_1 is shown in Figure 4c. Equation 25 can be incorporated into a finite-element model in the same manner already described, and the appropriate value of the modulus for each element would be selected on the basis of the ratio of σ_3/σ_1. Alternatively, it seems very logical that a hyperbolic formulation similar to that described for the triaxial test could be advanced. Although many buried conduit problems may be considered essentially plane strain in nature, the determination of a modulus by means of a plane strain test has the serious disadvantage that relatively few laboratories are equipped at the present time with plane strain test equipment.

Comparison of Methods for Determination of Modulus

The soil in the vicinity of an underground conduit is often subjected to considerable confinement, particularly in situations where the horizontal dimensions greatly exceed the vertical height of soil above the conduit. In such cases, the major principal strain at most points throughout the system will usually be much larger than the other principal strains. For example, in a typical embankment installation, the major principal strain in the fill at some distance from the conduit will be primarily vertical, the horizontal strain in the longitudinal direction of the fill will be essentially zero, and the horizontal strain in the lateral direction will probably be very small. Near the conduit, the major principal strain will be predominantly radial, especially where considerable conduit deformation is involved; there may be a slight tensile strain parallel to the centerline of the conduit; and the strain tangent to the conduit wall in a vertical plane will probably be compressive. Although the foregoing reasoning is qualitatively correct, it is difficult to assess intuitively the quantitative relations that are involved. Nevertheless, the predominance of the major principal strain indicates that the response of a soil-conduit system is probably influenced more strongly by dilatational stresses than deviatoric stresses. Therefore, the consolidation test (or uniaxial strain test) may very well provide the most reliable immediate source of input data for soil properties because it is concerned only with volume change characteristics; however, if the conduit deformations are relatively small, as is the case for a concrete pipe, shear deformations in the soil adjacent to the conduit may be important, and the plane strain or triaxial tests may be more appropriate because they involve considerable deviatoric effects. The consolidation test has the very significant advantage that the required equipment is currently available in almost every soils laboratory, whereas triaxial test equipment is less common and plane strain equipment very rare.

Determination of Poisson's Ratio

As can be seen in Figure 4, the determination of Young's modulus, E, by use of the preceding three tests is strongly dependent on a knowledge of Poisson's ratio, ν. Unfortunately, ν is a very illusive soil property to obtain, and it has provided a source of frustration for many researchers. As a matter of fact, there is considerable support for the position that one can make an engineering estimate that is as good as or better than any value that can be experimentally determined, and this may indeed be the case. This is largely because of the fact that E, ν, and the state of stress are intimately related, and it is difficult to determine one parameter without a knowledge of the others. This leads to a situation where an error in E causes an error in ν, and vice versa. Although a survey of the literature can provide substantial guidance in the selection of a particular value for ν, there is little quantitative justification to be found. Accordingly, a continual effort must be advanced to improve our understanding of this parameter, and engineering ingenuity must be employed to find better ways of either measuring ν or offsetting its effect in a mathematical model.

Continuing with an interpretation of test results in terms of linear elastic theory, we may write the following equations for both a conventional test and a plane strain test:

$$E\,\epsilon_1 = \sigma_1 - \nu\,(\sigma_2 + \sigma_3) \tag{27a}$$

$$E\,\epsilon_3 = \sigma_3 - \nu\,(\sigma_1 + \sigma_2) \tag{27b}$$

Multiplication of Eqs. 27a and 27b by ϵ_3 and ϵ_1 respectively, subtraction of the results, and rearrangement lead to

$$\nu = (\sigma_3\epsilon_1 - \sigma_1\epsilon_3)/[\sigma_1\epsilon_1 + \sigma_2(\epsilon_1 - \epsilon_3) - \sigma_3\epsilon_3] \tag{28}$$

For a conventional triaxial test, $\sigma_2 = \sigma_3$, and Eq. 28 reduces to

$$\nu = (\sigma_3\epsilon_1 - \sigma_1\epsilon_3)/[\sigma_1\epsilon_1 + \sigma_3(\epsilon_1 - 2\epsilon_3)] \tag{29}$$

whereas for a plane strain test, σ_2 equals $\nu(\sigma_1 + \sigma_3)$, as given by Eq. 24, and Eq. 28 becomes

$$A\nu^2 + B\nu + C = 0 \tag{30}$$

where

$$A = (\sigma_1 + \sigma_3)(\epsilon_1 - \epsilon_3) \tag{31a}$$

$$B = \sigma_1\epsilon_1 - \sigma_3\epsilon_3 \tag{31b}$$

and

$$C = \sigma_1\epsilon_3 - \sigma_3\epsilon_1 \tag{31c}$$

Solution of Eq. 30 by using the quadratic formula yields

$$\nu = (-B \pm \sqrt{B^2 - 4AC})/2A \tag{32}$$

Hence, if ϵ_3 is measured in either of the preceding two tests in addition to the conventionally measured σ_1, σ_3, and ϵ_1, ν may be theoretically determined by Eq. 29 for a triaxial test and by Eq. 32 for a plane strain test. However, in view of the limitations imposed by the basic assumptions (such as linear elasticity and stress and strain homogeneity) employed and the probable error associated with the measurement of ϵ_3 (this is related to the assumption of strain homogeneity), it cannot be expected that these equations will yield acceptable results unless extreme care is exercised in the test procedures. Values of ϵ_3 at various stress levels can be obtained either by direct measurement or, for saturated samples, by measurement of the volume change; the latter technique necessarily yields an average value for ϵ_3, whereas the former may very well give a maximum (an erroneous) value if measured at the midheight of the specimen. However, for unsaturated samples direct measurement is the only recourse.

Another possible approach for determining ν is to utilize the empirical relation (1, 3)

$$K_o = 1 - \sin \varphi \tag{33}$$

in conjunction with Eq. 3 to yield

$$\nu = (1 - \sin \varphi)/(2 - \sin \varphi) \tag{34}$$

where φ in this case can be determined for various stress levels prior to failure as well as at failure. Although this approach is rather indirect, Eq. 33 is based on considerable experimental evidence, and the error associated with its use should lie within acceptable limits.

Although not quite so well founded as the hyperbolic stress-strain formulation, there is some evidence to indicate that a hyperbolic equation of the form

$$\epsilon_1 = \epsilon_3/(r + s\epsilon_3) \tag{35}$$

can be used to relate the axial strain, ϵ_1, and the radial strain, ϵ_3, in a conventional triaxial test, where the empirical coefficients r and s are determined in a manner similar to a and b in Eq. 14. Then, analogous to the definition of the tangent modulus, E_t, given by Eq. 17, one might define a tangent Poisson's ratio, ν_t, by solving Eq. 35 for ϵ_3 and differentiating with respect to ϵ_1 to obtain

$$\nu_t = d\epsilon_3/d\epsilon_1 = r/(1 - s\epsilon_1)^2 \tag{36}$$

The strain dependence of Eq. 36 can be changed to stress dependence by solving Eq. 14 for ϵ_1 and substituting the result in Eq. 36 to give

$$\nu_t = r[1 - b(\sigma_1 - \sigma_3)]^2/[1 - (b + as)(\sigma_1 - \sigma_3)]^2 \tag{37}$$

The problem now reduces to one of determining the empirical coefficients as functions of the state of stress and the other variables of the soil. From a theoretical point of view, it is noteworthy to point out that the state of stress in a conventional triaxial test is not consistent with that used in the definition of Poisson's ratio; hence, the definition given by Eq. 36 is not strictly correct. Nevertheless, for all practical purposes, and in view of the complexities introduced by the use of Eq. 29, the empirical formulation given by Eq. 37 is probably justified for use in conjunction with Eq. 19. However, such a formulation cannot satisfactorily account for dilatancy effects in soils. If the experimentally determined value for ν is equal to or greater than one-half, a value such as 0.49 is usually incorporated in the theoretical analysis because values of one-half or greater are incompatible with classical linear elastic theory and the associated finite-element method.

As can readily be appreciated, Poisson's ratio is an important soil parameter, but it is most difficult to quantify. Accordingly, it seems that the best chance for success with this parameter in the near future is to develop and employ a method of analysis that essentially offsets its effect. This is one of the apparent advantages of using the plane strain test to determine soil parameters. Because a plane strain laboratory test essentially models the field situation for many buried conduit installations, it is likely that the effects of Poisson's ratio can be minimized by interpreting the laboratory response in terms of plane strain conditions and by using the result to analyze the field problem.

Backpacking Materials

Considerable attention must be given to the mechanical properties of backpacking or cushioning materials that may be used immediately adjacent to all or a portion of the conduit. If uncompacted or lightly compacted soils are used, the modulus values (and probably Poisson's ratio) will be strongly affected by the dry density, and laboratory tests (similar to those previously discussed) must be conducted at densities appropriate to the field installation. In addition, care must be taken to ensure that the dimensions of these low-modulus zones in the field are consistent with those used in the finite-element model. If nonsoil backpacking materials are used, special effort must be made to quantify their mechanical properties, and this is often extremely difficult to do. If this task cannot be accomplished with reasonable confidence, a parameter study may be used to assess the relative importance of the value selected. Finally, it is very likely that the time effects on a backpacking material, especially a nonsoil material, cannot be neglected, but the manner in which they should be included is not at all clear.

INTERFACE CONDITIONS

Within the realm of material properties should be included the conditions that exist along the soil-conduit interface and along the boundaries between different zones of soil. In general, there are three situations that can be handled without too much analytic difficulty: full slip, no slip, and no slip until a prescribed stress has been reached. In the absence of any information to the contrary, it seems most appropriate to utilize the third condition. The upper bound of the prescribed stress would certainly be the shear strength of the soil, and the extent to which the actual stress at the interface differs from this upper bound would depend on the smoothness of the conduit. In general, it seems quite reasonable to use the shear strength of the weaker soil at the interface between different soil zones. For cases where the conduit is very smooth, as with some metal and plastic pipes, the full-slip condition may be more applicable. However, instead of attempting to characterize this condition exactly, one may examine the extreme cases of full slip and no slip (5) for one particular type of problem and thereby evaluate its influence.

PROPERTIES OF CONDUIT MATERIALS

Although the mechanical properties of the surrounding soil probably exert the greatest influence on the response of a soil-conduit system, the properties of the conduit materials are also extremely important. In most, but not all, cases, however, the properties of the conduit materials exhibit more limited ranges of variation and can be characterized with greater reliability than soils. A few of the key considerations regarding conduit materials will be discussed in the following sections.

Metal Conduits

In general, steel and aluminum possess relatively well-known mechanical properties, and their determination is not difficult; however, metal pipes have some characteristics that are not readily and reliably evaluated. Among these are their susceptibility to corrosion and abrasion, the effectiveness of protective coatings, the behavior of seams, and the buckling strength. Various empirical equations and statistical correlations are available to handle the durability problem, which indirectly affects the soil-structure interaction problem by altering the pipe wall with time. The stiffness of a seam may significantly influence the response of a conduit, particularly if the seam is less stiff than the rest of the conduit wall or if there is an abrupt change in the cross section of the wall at the seam. Directly related to the behavior of a seam are the characteristics (not only strength, but durability, deformability, and brittleness) of the bolts that are used. For large-diameter conduits, particularly those with relatively shallow cover heights, stability against buckling must be checked. This leads to a relation that involves relative values for the mechanical properties of both the conduit and the surrounding soil as well as the cover height.

Concrete Conduits

The most important properties that influence the structural response of reinforced concrete conduits are the compressive and tensile moduli of the concrete, the tensile strength of the concrete, the amount and position of the reinforcing steel, and the quality of the bond between the concrete and the steel. The role of the compressive and tensile moduli are self-evident, whereas the tensile strength is important because it is directly related to progressive cracking, which in turn alters the internal stresses and displacements within the pipe wall. As a consequence of progressive cracking, the stiffness of the conduit is continually modified as the load is increased, thereby introducing a decidedly nonlinear response. The amount and position of the reinforcing steel of course controls the stiffness of the conduit, and the associated analyses must be carefully reviewed in light of the final design. Although the amount of steel is relatively easy to determine, accurate placement is largely dependent on fabrication procedures. The quality of the bond between the concrete and the steel is important if the deformation characteristics of the conduit are based on effective bond considerations. Bond failures cause small local slips and some widening of the cracks with an associated increase in the deformation of the conduit.

Plastic Conduits

The mechanical properties of many plastics used in the manufacture of conduits are not sufficiently well understood, and much additional effort is needed before these properties can be reliably specified for use in a theoretical analysis. In particular, some plastic conduits exhibit significant time-dependent effects, which influence their long-term response. Thus far, however, the use of plastic conduits has been generally restricted to the smaller diameter installations wherein mechanical properties play a considerably lesser role in the overall behavior of the system.

CONCLUSIONS

Within the scope of the considerations discussed herein, the following conclusions or summary statements can be advanced.

1. The more significant improvements in our ability to analyze soil-conduit interaction problems in the immediate future lies in the use of a continuum approach and a finite-element formulation; however, the successful application of these procedures will depend largely on our ability to provide appropriate input data for material properties.

2. The specification of reliable, quantitative values for material properties is probably the single factor that limits most seriously our ability to predict the mechanical behavior of soil-conduit systems.

3. The particular material properties to be used in the mathematical model, the manner in which these properties are determined, and the methods of analyses and design are intimately related, and a variation in one step will generally precipitate changes in one or more of the other steps.

4. Field verification is absolutely essential before a mathematical model and its associated material properties can be accepted as a valid procedure for analysis.

5. Of all the material properties that affect the interaction of a soil-conduit system, the characteristics of the soil present the greatest difficulty; and, of all the soil properties of interest, the stress-strain behavior is the most important.

6. Within the framework of currently available methods for testing and analysis, the assumption of a stress-dependent, isotropic soil modulus is probably reasonable.

7. Both the stress dependency of the modulus and the incremental nature of the construction sequence in a conduit installation can be conveniently analyzed by use of a piecewise linear formulation.

8. Subject to proper interpretation, which involves consideration of Poisson's ratio and the state of stress, an appropriate soil modulus can be determined from the results of laboratory consolidation tests, conventional triaxial tests, or plane strain tests.

9. Time effects, which are not explicitly considered herein, must be taken into account by conducting the laboratory test at an appropriate strain rate and with appropriate drainage conditions.

10. Dry density significantly affects the soil modulus, and moduli associated with different densities must be determined in the laboratory and assigned to appropriate zones in the field problem.

11. Although Poisson's ratio significantly influences the determination of a soil modulus from any of the suggested tests, this parameter is most difficult to quantify in a reliable manner; the use of linear elastic theory and experimental measurements to back calculate Poisson's ratio leads to rather unreliable results in many cases, and this situation suggests that the exercise of good engineering judgment to estimate Poisson's ratio may be equally useful.

12. The greatest hope for success in dealing with Poisson's ratio in the near future centers around the development of a method of analysis that essentially offsets its effect; this may be possible to some extent in all cases because Poisson's ratio is used both in the interpretation of a laboratory test and in the analysis of the field problem.

13. Insofar as possible, it is desirable to describe the stress-strain behavior of the soil in terms of a three-dimensional formulation, even if only a simplified one- or two-dimensional version is utilized at present because three-dimensional analytic treatments of soil-conduit problems are not too far in the future.

14. The interface conditions between the soil and the conduit and between various soil zones should be specified by a no-slip condition with a limiting shear stress above which slip occurs.

15. The mechanical properties of the conduit materials may exert a significant influence on the behavior of the physical system; particular examples include the properties of the bolts or welds in a metal conduit, the tensile strength of the concrete in a concrete conduit, and the substantial time-dependent response of some plastic conduits.

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EXPERIMENTAL STUDIES IN SOIL-STRUCTURE INTERACTION

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This paper contains a literature review of certain experimental studies that pertain to soil-structure interaction. It is divided into two parts: model studies and field or full-scale testing. The requirements that govern model studies are reviewed briefly, and examples of applications are presented. A brief presentation is made of several studies that have used model analysis. They cover topics such as effects of soil moisture and density on culvert deflection, effects of differential soil compaction on culvert stresses, imperfect ditch method of construction, stresses on multiple pipe installations, pressure distribution on pipe, and soil properties. The field study portion presents field studies of the imperfect ditch method of construction, full-scale failure tests, and certain Canadian large pipe tests. A circular culvert design method that takes into account most of the significant variables that affect culvert performance is also presented.

•IN reviewing the literature on soil-structure interaction, I found that many researchers had solved buried structure problems by using a field or model study. In spite of all the work that has been done, the field of culvert design is still relatively new. There is a great deal that is not known about the performance of culverts in various situations. Some design tools that have been available for many years have been applied on an experiential basis only. Some of these will be discussed later in this paper.

Only those studies that have been done in the past few years will be presented here. Many topics that are of considerable importance are omitted because of space limitations.

For purposes of discussion, experimental studies are divided into two groups: model studies and field or full-scale tests.

MODEL STUDIES

Model tests have been successfully used to investigate the performance of full-scale culverts. Other soil mechanics problems that can be treated as plane strain problems, such as slope stability, have also been successfully modeled.

Models may be used to solve complex problems that cannot be readily evaluated by using analytic means. The use of models in obtaining solutions related to performance of underground structures is most attractive because of the complexity of the problems involved. Time and money often prohibit the use of full-scale tests to investigate the effect of different variables on a system. Because of the ease with which models can be fabricated and tested, they yield much more information for a given amount of time and money than do full-scale tests. Full-scale tests are useful in verifying results obtained from model tests.

In order to establish reliable similitude requirements for a given model system, we must define all variables that influence the phenomena. Defining the variables involved requires that an investigator have considerable experience within the area of investigation. As experience is gained in an area, model analysis can be applied with confidence to the solution obtained.

SIMILITUDE REQUIREMENTS

Before a brief review of similitude requirements is presented, it might be helpful to give an example of how similitude and model analysis works.

Sponsored by Committee on Subsurface Soil-Structure Interaction.

Most people interested in culvert design are familiar with the Iowa formula; therefore, it will be used to illustrate the modeling concept as follows:

$$\Delta X = K W_c r^3/(EI + 0.061 E'r^3) \tag{1}$$

where ΔX is the change in horizontal diameter; K is a bedding constant; W_c is the load acting on the culvert and is equal to $W_c = pD$; P is the average vertical pressure acting on conduit; E' is the modulus of soil reaction; r is the radius of the culvert = D/2; E is the modulus of elasticity of the culvert wall; and I is the moment of inertia of the pipe wall per unit length.

A discussion on the use of the Iowa formula and modulus of soil reaction is included later in this report.

If pD is substituted for W_c and both sides of Eq. 1 are divided by D, we derive

$$\Delta X/D = K P r^3/(EI + 0.061 E'r^3) \tag{2}$$

If the numerator and denominator on the right side of the equal sign in Eq. 2 are divided by EI,

$$\Delta X/D = K(Pr^3/EI)/[1 + (0.061 \times E'r^3/EI)] \tag{3}$$

or

$$\pi_1 = K \pi_2/(1 + 0.061 \pi_3) \tag{4}$$

Consider the two following pipes:

1. No. 1 (model): 4-in. diameter; modulus of elasticity, 10×10^6 psi (aluminum); wall thickness (smooth wall), 0.050 in.; moment of inertia = $t^3/12 = 1.0416 \times 10^{-5}$ in.^{4}/in.; $D^3/EI = 0.61443$.
2. No. 2 (prototype): 10-ft diameter; modulus of elasticity, 30×10^6 psi (steel); 8 gauge, 6- × 2-in. corrugation; moment of inertia = 1.15 in.^{4}/ft = 0.0958 in.^{4}/in.; D^3/EI = 0.60106.

Assume that both culverts are embedded in a soil with a modulus of soil reaction of 5,000 psi and that they are subjected to an average vertical pressure of 100 psi. Then, in the case of No. 1, $pD^3/EI = 61.443$ and $E'D^3/EI = 3{,}072.15$. In the case of No. 2, $pD^3/EI = 60.106$ and $E'D^3/EI = 3{,}005.30$.

By substituting the value for each pipe into Eq. 3 and by letting K = 0.083, we get the following:

No. 1

$$\Delta X/D = 0.083\ (61.443)/[1 + (0.061 \times 3{,}072.15)]$$

$$\Delta X/D = 0.02706 = 2.706 \text{ percent}$$

No. 2

$$\Delta X/D = 0.083\ (60.106)/[1 + (0.061 \times 3{,}005.30)]$$

$$\Delta X/D = 0.02706 = 2.706 \text{ percent}$$

Thus Spangler's deflection equation predicts exactly the same percentage of deflection for both the 4-in. pipe and the 10-in. pipe. This is the principle on which model analysis is based. If the individual pi-terms are the same, the results, regardless of size, will also be the same.

The properties of the soil and pipe that govern soil-structure interaction under static load have been defined, so models can be used to predict soil-structure interaction performance.

In most soil-structure interaction problems, the form of the equation is not known. If it were known, there would be little need to use model analysis. Therefore, one must select the primary independent variables that influence the phenomenon and place them in dimensionless pi-terms.

SIMILITUDE REQUIREMENTS FOR DEFLECTIONS UNDER HIGH FILLS

To illustrate the use of model analysis for a specific soil-structure interaction problem, we will use an example for determining culvert deflection under a fill that is high in comparison with the diameter of the culvert. The culvert is assumed to be long enough such that end effects can be neglected and that maximum stress in the culvert wall is below the yield point stress. The primary independent variables and dimensions involved are as follows:

Primary Independent Variable	Dimension
Culvert diameter, D	L
Pipe wall stiffness, EI	$FL^{-2} \times L^4/L = FL$
Constrained modulus of elasticity of the soil or modulus of soil reaction, M_s	FL^{-2}
Pressure applied at the level of the pipe because of the fill or other load above the pipe, P	FL^{-2}
Deflection of some point in the culvert, ΔX (horizontal diameter used)	L

It can be shown that these are all the variables that have a significant influence on the amount of deflection that a culvert will experience. It may be argued that such things as soil density, water content, and other soil properties must be included. The influence of these variables is included in the constrained modulus of elasticity of the soil. The constrained modulus is a function of soil density, water content, plasticity, soil type, grain-size distribution, and all other soil variables plus boundary conditions.

According to the Buckingham pi-theorem, there must be three pi-terms (five primary independent variables minus two dimensions). These pi-terms may be formulated as follows:

$$\pi_1 = \Delta X/D$$

$$\pi_2 = PD^3/EI$$

$$\pi_3 = M_s D^3/EI$$

or

$$\pi_1 = f(\pi_2, \pi_3)$$

If model analysis is to be used to predict the deflection of a full-scale pipe, all the pi-terms must be the same for the model as for the prototype structure, as previously illustrated in the example, or

$$\pi_{1_m} = \pi_{1p}$$

$$\pi_{2_m} = \pi_{2p}$$

$$\pi_{3_m} = \pi_{3p}$$

where m refers to the model and p refers to the prototype or actual structure.

The model must be designed so that each pi-term for the model is the same as for the prototype structure. Sometimes deviations are necessary, but then distorted modeling techniques are encountered that complicate the analysis and will not be considered here. A more complete treatment of soil modeling is given elsewhere (19).

Distorted modeling methods are discussed in various books on engineering similitude. The modeling of soils by using distorted models may lead to unexpected problems because of the nonlinear nature of the soil culvert system.

SIMILITUDE REQUIREMENTS FOR WALL BUCKLING

Similitude requirements for wall buckling are not nearly as easy to satisfy as those for deflection and pressure. The moment of inertia, area of the pipe wall, and the yield point stress of the culvert material must be included. To get an exact model, not a distorted one, is very difficult because the corrugations of a metal culvert preclude the use of plane wall pipe as models. In deflection measurements, the area of the pipe wall has been shown to make very little difference if moment of inertia is constant. This can be seen by the elasticity solution of the problem and from model tests. However, in buckling problems, the area of the pipe wall or the radius of gyration of the pipe wall must be included. Because of the difficulty in obtaining an exact model for a corrugated metal culvert, model studies of wall buckling have not been very rewarding.

MODEL ANALYSIS OF DYNAMIC LOADING

Model analysis of dynamic loading becomes even more difficult in that the inertial properties of the soil-culvert system must also be modeled. When this is attempted, the density of the soil as well as the elastic properties must be modeled. A few investigators have modeled specific situations, but most of these studies have been associated with blast-shelter construction and will not be presented here.

EFFECTS OF SOIL DENSITY AND MOISTURE ON CULVERT DEFLECTION

As examples of what can be done with models, a few results will be presented. Figure 1 shows the results of a study to determine the effects of initial density of clay soil on the deflection that a culvert undergoes (1). As can be seen, the soil density has a considerable effect on the deflection. Figure 2 shows the effects that initial density of sand has on the deflection of the culvert. A decrease in density shows a marked increase in deflection for both sands and clays. Figure 3 shows the influence initial moisture content has on the deflection of the culvert in clay soils. As the moisture content increases, the deflection increases very rapidly. At a load of 50 psi, for example, the deflection at 20 percent moisture is approximately 35 times the deflection at 10.3 percent moisture. This is why density alone (even including grain-size distribution and Atterberg limits) is a poor indicator of soil stiffness in cohesive soils.

EFFECTS OF BACKFILL DENSITY ON STRESSES IN PIPE

In 1964, Watkins (2) presented the idea of differential soil density to release stresses in the pipe and pressures on the pipe. The soil around the pipe is compacted to a high degree of density, and the soil next to the pipe is left in a rather loose state to form a cushion around the pipe. Data were not presented at that time. Figure 4 shows the method proposed by Watkins for achieving the differential density.

Figure 5 shows the results obtained from two different studies (3) that used models. Both studies were made on the same 4-in. diameter 6061-T6 aluminum model. Both studies used the same soil (Ottawa sand) compacted to the same density in the same simulator. The only difference was in the manner in which the models were placed. The soil in Stankowski's study was placed before the model, and then the model was "jacked" into position. Insertion of the model appeared to compact the soil adjacent to the model to a slightly higher density; however, no measurements of density were made after model placement. The soil in Mohammed's study was "rained" around the model pipe. As soil particles fell, those hitting the very edge of the model were deflected away from it. The density of soil adjacent to the pipe was believed to be lower than the soil mass. Therefore, density of the soil immediately adjacent to the model in Stankowski's study is assumed to be higher than in Mohammed's study. As can be seen, there is a significant difference in stress because of the method of soil and culvert placement. The difference in soil density and pipe stress shown points out that a very

Figure 1. Variation in horizontal deflection in clay.

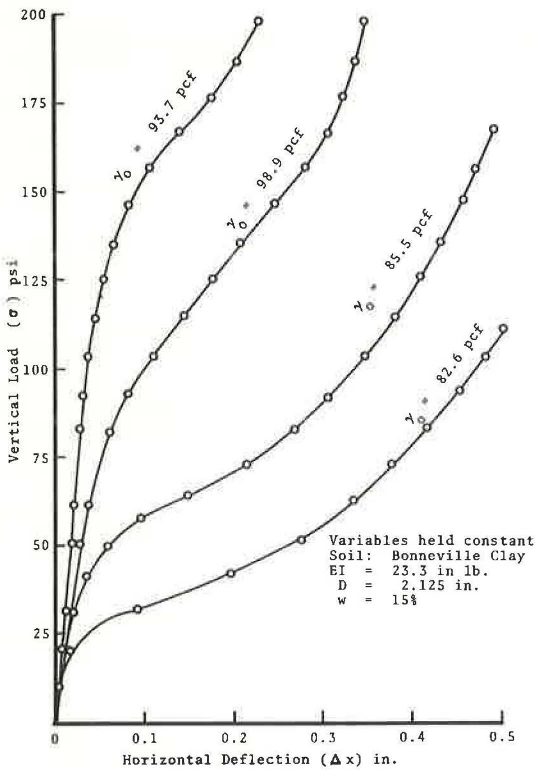


Figure 2. Variation in horizontal deflection in sand.

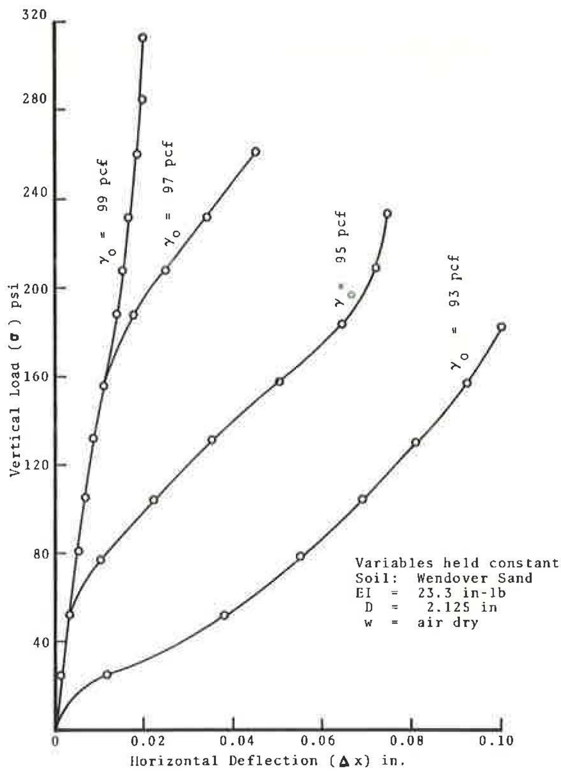


Figure 3. Effect of moisture on horizontal deflection.

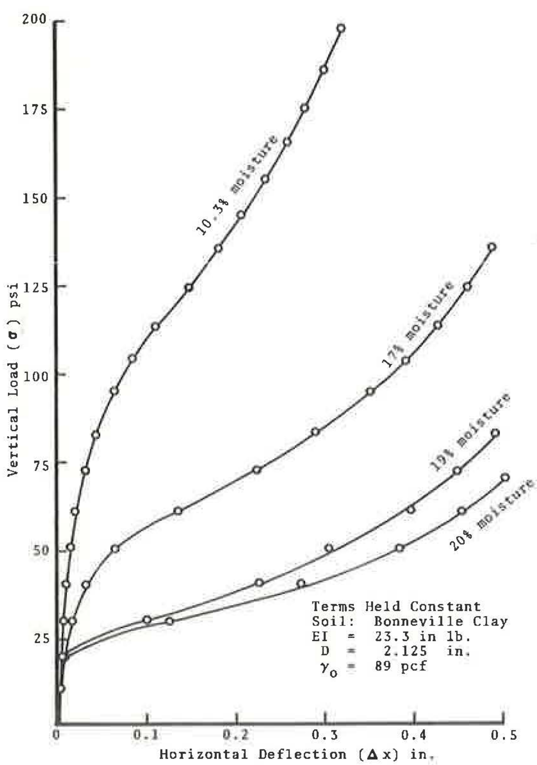
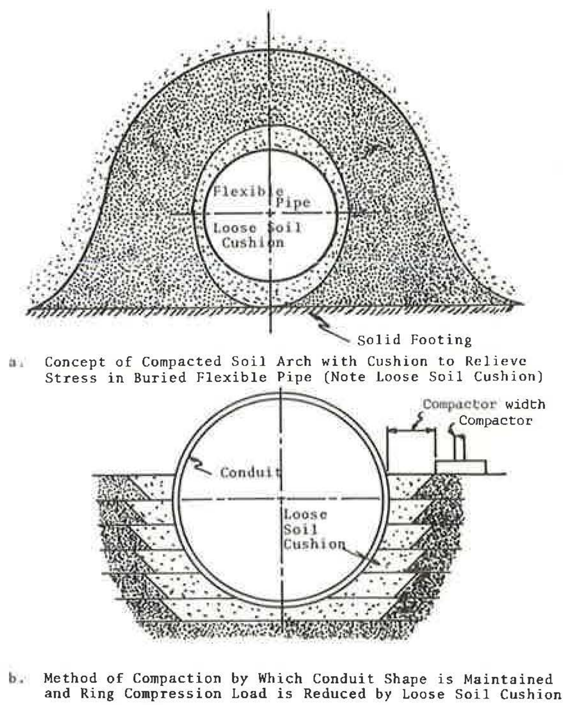


Figure 4. Watkins' method for reducing pressure on buried circular conduits.



easy and economical method of "backpacking" may be achieved by making use of differential soil density; however, much more study is required.

IMPERFECT DITCH METHOD OF CONSTRUCTION

The imperfect ditch method of construction was one of the earliest methods used to reduce the loads exerted on an underground conduit. Figure 6 shows the usual method of installing the conduit by this method. The culverts that have been installed by using this method have used a compressible layer that is the same width as the culvert and placed almost directly above the culvert. Figure 7 shows the results of a study (3) made to determine the effect of the width and height of the compressible layer above the culvert on the stresses in the pipe wall.

The upper limit stress is the average of the absolute value of the stress plus two standard deviations. In this model study, the upper limit pipe wall stress in almost all cases was higher for an installation with a compressible layer than it was for no compressible layer at all. The reason for this is that the pipe tended to deflect upward into the compressible layer instead of flattening into an approximate elliptical shape. Figure 8 shows a summary of the deflections obtained.

The model was a 6061-T6 circular aluminum tube, 4 in. in diameter. The compressible layer was a common household sponge, with a thickness of $^5/_8$ in. and widths of 4, 5, 6, and 8 in.

The data presented in Figure 7 are not intended to discredit the validity of the imperfect ditch method of construction. The figure does, however, serve as a warning that, if the imperfect ditch is not properly designed, stress and deflection conditions can be more severe in a pipe installed by this method than in one installed by normal procedures. The compressibility of the sponge used in the model study was much higher than most material used in actual field construction. The higher compressibility of the imperfect ditch in the model would induce higher stresses because of the vertical elongation that may not be experienced in most field structures.

Several other studies, in which different compressibilities of the compressible layer have been used, have shown that the imperfect ditch is effective in reducing loads on the structure.

MULTIPLE PIPE INSTALLATIONS

Model analysis can also be applied in other ways. Figure 9 (4) shows a photoelastic model that was used to determine the stress concentration caused by multiple culverts. Similitude requirements can be easily established for such studies. Figure 10 shows a plot of the radial pressure-applied pressure versus spacing between the model culverts. The numbers in parentheses are the values obtained for single culverts. This type of soil-structure interaction model is extremely sensitive to "fit" of the culvert model in the plastic. Figure 11 shows a plot of shear stress-applied pressure versus model spacing. Solutions such as those shown in Figures 10 and 11 are plane stress solutions, whereas most model solutions are probably closer to a plane strain solution. The difference is usually insignificant when compared with the accuracy with which such variables as soil properties can be determined.

Figure 12 shows a comparison (1) between stresses and deflections obtained from a model analysis. The values were obtained from the elasticity solution of Burns and Richard. Results shown in Figure 13 were obtained by the technique used to get the data shown in Figure 12. It shows the results of a study to determine the pressure concentration (radial pressure and applied pressure) caused by multiple pipes.

The pressure acting on the outside of the pipe, the stress in the pipe wall, and the deflection of the pipe were obtained by fitting a Fourier series to a general loading distribution as shown in Figure 14. The Fourier series coefficients were determined from measured strains and displacements on the inside of the pipe. No pressure transducers were placed on the outside of the model to disturb the pressure distribution.

Currently, most culverts are designed to carry considerably more load than is actually imposed on them. The designer should not worry about the strength of multiple installation culverts. Even with the increased stress induced by multiple installations,

Figure 5. Effect of placement method on circumferential stress.

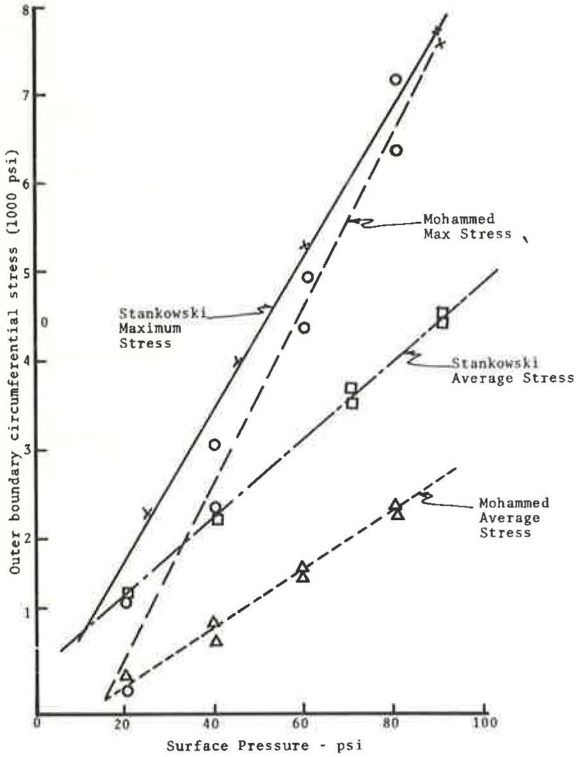


Figure 6. Imperfect ditch construction.

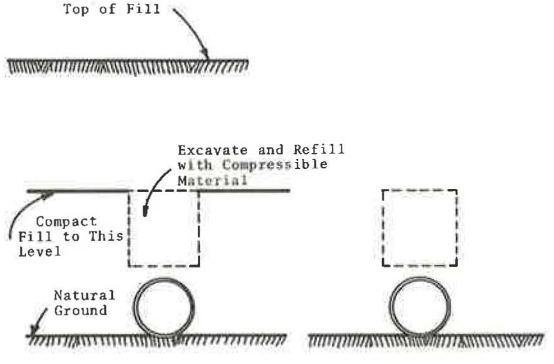


Figure 7. Variation of circumferential stress with compressible layer parameters.

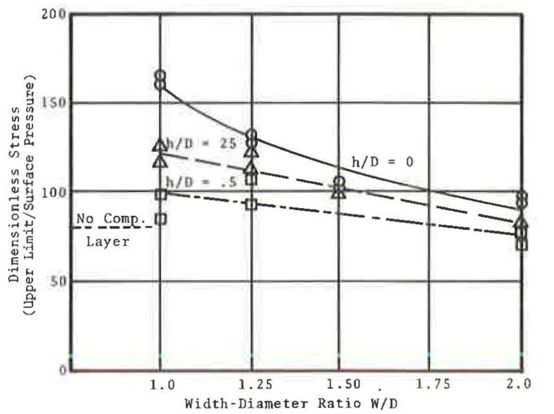


Figure 8. Effect of location of imperfect ditch on deflection of horizontal and vertical diameters (surface pressure = 60 psi).

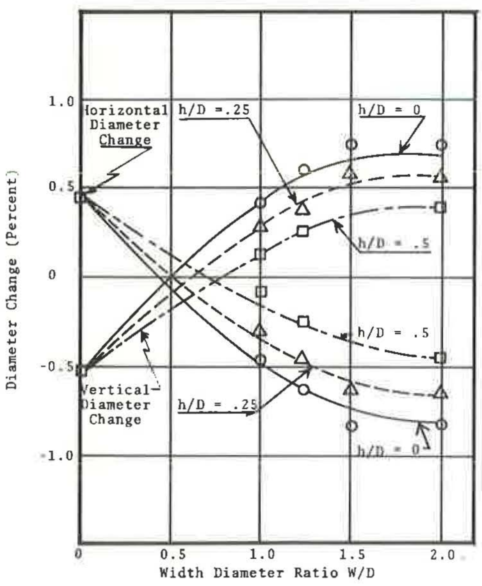


Figure 9. Photoelastic model.

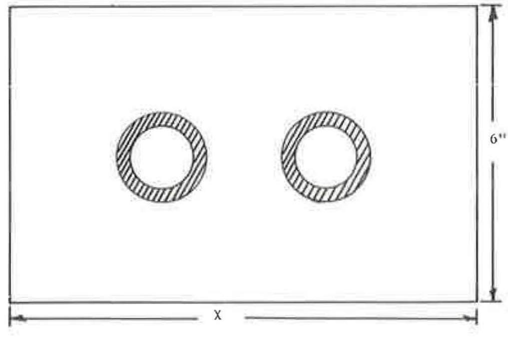


Figure 10. Experimental radial interface pressure concentration factors.

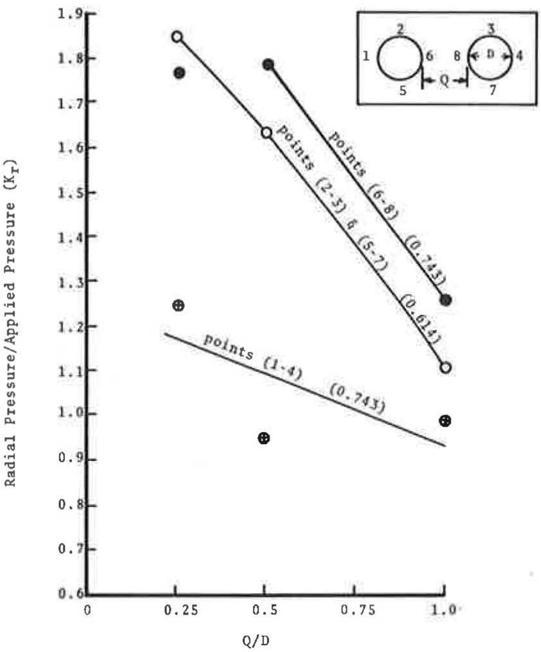


Figure 11. Experimental shear stress concentration factors.

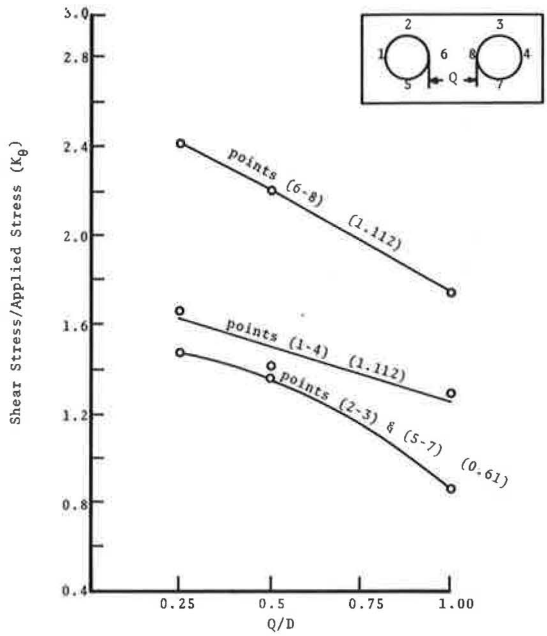


Figure 12. Comparison of predicted and recorded horizontal diameter changes for the single cylinder system.

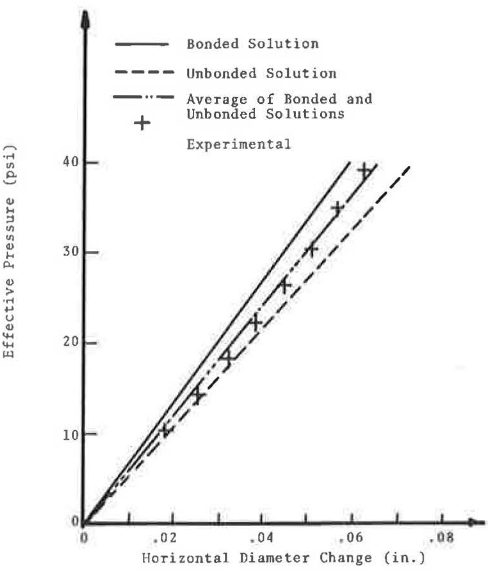
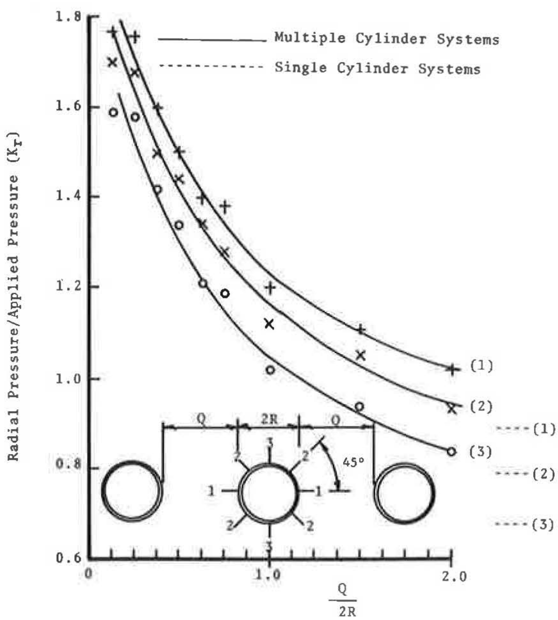


Figure 13. Experimental radial interface pressure concentration factors.



most culverts designed by current methods will carry the load with an adequate factor of safety. However, as research and development continue and more refined and accurate solutions of the loads on pipe become available, the increased stresses due to multiple installations will have to be considered.

PRESSURE DISTRIBUTION ON PIPE

In 1941, Spangler (5) presented the Iowa equation for determining the deflection of a flexible pipe. The pressure distribution used by Spangler was arrived at by measuring the radial pressure on the pipe with friction ribbons. The steel ribbons were placed on the pipe before the fill was placed. After completion of the fill, the ribbons were pulled out from under the fill, and the pressure was assumed to be related to the coefficient of friction and the force necessary to pull the ribbon out from under the fill. The ribbons were calibrated in the laboratory before they were installed on the structure; however, before measurements are made, many things can change the calibration.

It is highly probable that the friction ribbons measured only the radial component of pressure acting on the pipe. The shear component was neglected. This is a common mistake with most field measurements that use some type of pressure transducer on the outside of the pipe. Only the radial pressures are measured.

The pressure distribution used by Spangler is shown in Figure 15. This pressure distribution is adequate for low-density soils that have a high value of Poisson's ratio; however, for high-density granular soils with a low value of Poisson's ratio, the pressure distributions appear to change somewhat. These conclusions are based on a theory presented by Burns and Richard, which involves elastic media. Figures 16 and 17 show the pressure distribution obtained for two different conditions. Figure 16 shows the pressure distribution for a low-density soil (low value of constrained modulus) and a high value of Poisson's ratio. This condition would represent the low-density clay that Spangler did most of his work on. For this case, the pressure distribution obtained from the elastic theory is approximately the same as the pressure distribution measured by Spangler. Figure 17 shows the pressure distribution obtained for a high-density soil (high value of constrained modulus) and a low value of Poisson's ratio. This condition represents a well-compacted granular soil.

To determine whether the pressure distribution acting on the pipe changes for granular soils, we compared data obtained by Stankowski (6) with the data shown in Figure 17. Figure 18 shows the results obtained. They show that the pressure distribution does change from that used by Spangler. The major change is in the horizontal pressure distribution. Spangler's pressure distribution has the horizontal pressure acting only over a 100-deg section of the culvert. The measured pressures shown in Figure 18 act over essentially the entire 180 deg. The measured pressures, therefore, present more resistance to horizontal movement of the pipe than that used in Spangler's pressure distribution. The measured vertical pressure also has a dip in the pressure distribution at the center and at the edge of the pipe. This would reduce the magnitude of the vertical load that is exerted on the pipe. This dip in pressure seems to become more pronounced as the constrained modulus of elasticity of the soil gets higher.

If the modulus of the soil reaction E' is determined by using the calculated values from the theory of elasticity for pressure and deflection at the horizontal diameter, the difference in pressure distribution makes the Iowa formula (Eq. 1) predict too much deflection for soils with a high value of constrained modulus and low value of Poisson's ratio. For example, the Iowa formula predicts approximately 40 percent more deflection than Burns and Richard's (full slippage) elastic theory for a soil with a high constrained modulus (10,000 psi) and low Poisson (0.1).

On the basis of the information presented here, it cannot be said that the Iowa formula should be modified for dense granular soils. It does suggest, however, that more study is needed in this area. Field studies that are made should be instrumented so that shear stresses, as well as radial pressures acting on the pipe, can be measured in order for the necessary verification to be obtained.

Figure 14. General loading components.

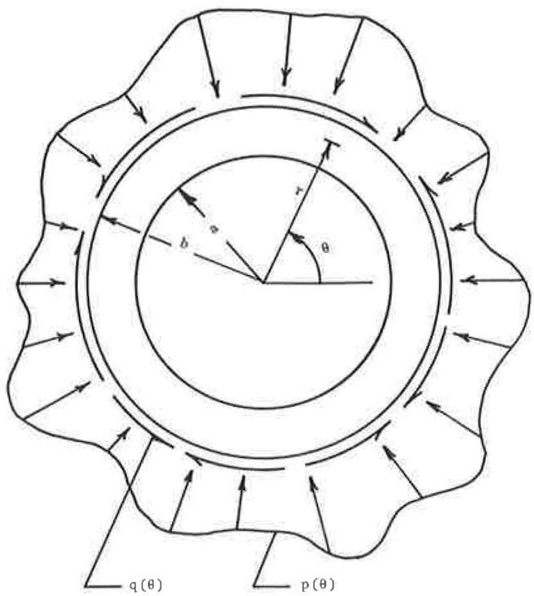


Figure 15. Assumed distribution of pressure on flexible culvert pipe.

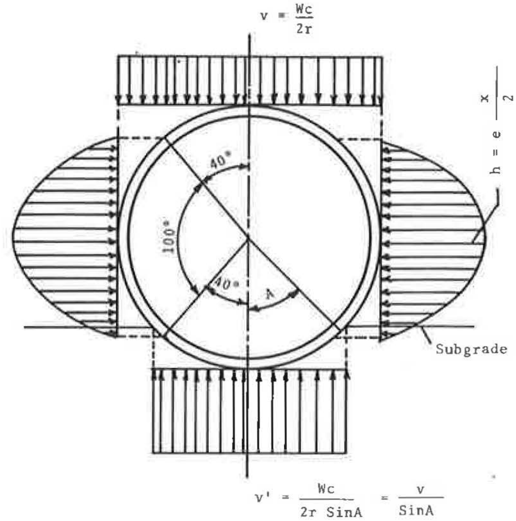


Figure 16. Pressure distribution for a low-density soil.

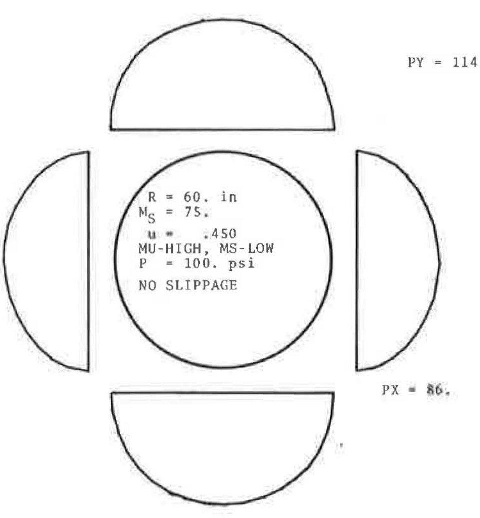
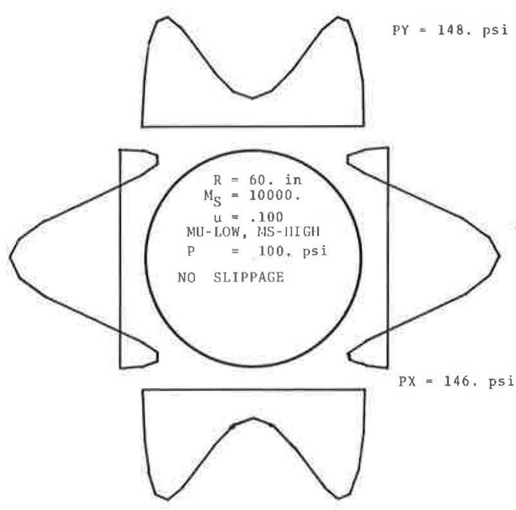


Figure 17. Pressure distribution for a high-density soil.



SOIL PROPERTIES DETERMINATION FROM MODEL STUDIES

The author, in another report (7), established the relation between the modulus of soil reaction and the constrained modulus of elasticity. In establishing this approximate relation, he used the bonded shell equation of Burns and Richard's solution (8) of a pipe embedded in an elastic medium. He concluded that the modulus of soil reaction can be approximated by

$$E' = 1.5\ M_s \qquad (5)$$

If the unbonded shell solution of Burns and Richard had been used, it would have been shown that

$$E' = 0.7\ M_s \qquad (6)$$

It has been shown by Stankowski (6) and Nielson and Statish (9), who used model studies, that the actual modulus of soil reaction of soil is between the predicted modulus of soil reaction by the bonded shell and the unbonded shell solution. Figure 19 shows the results of a confined compression test used to determine the constrained modulus of elasticity. Figure 20 shows how the modulus of soil reaction varies with pressure for this same soil at the same initial density. If an average value of the modulus of soil reaction is used, it can be shown that

$$E' = 0.8\ M_s \qquad (7)$$

Because the modulus of soil reaction is directly associated with Spangler's pressure distribution (Fig. 15), any question concerning the validity of the pressure distribution on the pipe will also be directly applicable to the modulus of soil reaction. If the pressure distribution acting on the pipe is different from that used by Spangler, modification of the modulus of soil reaction may also be needed.

SPECIAL PROBLEMS

Model analysis has been applied to many soil-structure interaction problems. Watkins has applied model studies to determine the minimum necessary height of cover over a conduit for the safe crossing of construction equipment (10), determination of pressures on culverts under stockpiles (2), and determination of the movement of soil around a pipe (11). Linger (12) has applied model studies to determine how the flexibility of a flat-top buried structure affects the redistribution of pressure on the structure.

FIELD OR FULL-SCALE TESTS

Many investigators have made attempts at field studies to determine or verify certain phenomena. Some of these studies have been associated with determining the pressure on the culvert and the corresponding displacement. One variable almost invariably neglected is the shear stress acting on the culvert. Measurements of shear stress are difficult to obtain on an actual field installation, whereas radial pressures acting on the culvert are relatively easy to obtain. A significant part of the load is neglected if only the radial pressures are measured.

Other studies have been made to determine the maximum load that a culvert can carry and to determine how large a culvert can be installed without failure.

One of the earliest applications of the imperfect ditch method of construction reported in the literature (13) involved a 48-in. concrete sewer pipe. Twenty years after the construction of the sewer, the height of an embankment above a ¼-mile section was increased from 60 to 78 ft without making any changes in the pipe.

Davis and Bacher (15) reported on California's culvert research program in which field studies and observations were made on several culverts. Different backfill designs were employed including an imperfect ditch construction and a variation therefrom in which a compressible layer of baled straw surrounded the culvert.

In one case in which a layer of baled straw surmounted a rigid culvert, horizontal and vertical pressures were found to be much less than in another in which no compressible layer was used. When baled straw was placed above a flexible conduit, the profile of the average induced pressure per foot of fill showed super-hydrostatic pressure bulbs at the invert. Effective densities at the crown, sides, and midpoints of the lower quadrants were about one-half that of the embankment. With increasing fill heights, maximum effective densities were observed to decrease and minimal densities to increase "so that some tendency toward a more uniform distribution is indicated." The case where a layer of baled straw surrounded the conduit "showed the most promise, inasmuch as the lateral pressures were almost negligible and vertical (effective) densities were about half that of the embankment."

An 18.5-ft diameter structural plate culvert under 83 ft of cover was reconstructed using the imperfect ditch method (16). The rebuilt culvert and the fill were instrumented such that pressure and deformation measurements could be taken. The behavior of the rebuilt culvert was observed to be significantly different from that predicted by the Marston-Spangler theory. Some of the most important conclusions of this study were as follows:

1. The predicted average vertical pressure, by Marston's theory, at the straw level was almost double the average measured pressure.
2. The shear-plane method may be used between the straw level and the conduit level to predict the average vertical pressure at the top of the conduit.
3. The maximum average vertical pressure on the conduit was approximately one-half of the overburden pressure. This is comparable to the effective densities reported by Davis and Bacher (15).
4. Lateral pressure on the side of the conduit was found to be substantially larger than the average vertical pressure.
5. A reduction in both the horizontal and vertical diameters was observed; however, at certain locations it appeared that the culvert may have deflected upward into the compressible layer.

FULL-SCALE TESTS FOR DETERMINATION OF FAILURE

Watkins and Moser (17) conducted tests on full-size pipe to determine the failure mechanism of pipes. Figure 21 shows a cross section of the test cell used. Spangler stated that the test cell does not represent field conditions and that the data obtained are a function of the test cell as well as the soil and pipe properties. Figure 22 shows some of the data obtained by Watkins and Moser, which are plotted in dimensionless form. If the three points at very low deflections and a PD^3/EI of approximately 400 are neglected, most of the data appear to plot approximately as a smooth curve.

The effect that the test cell would have would be to increase the stress in the σ_1 direction shown in Figure 21. The increase in σ_1 should have the effect of increasing the elastic properties (modulus of elasticity) of the soil. There seems to be a compensating effect in that the length of the test section of the cell was quite short in comparison to an actual field installation. This short section would have the effect of reducing the σ_2 stresses, which would have a decreasing effect on the elastic properties of the soil (modulus of elasticity).

For pipe deflections of less than about 5 percent, the change in elastic properties of the soil causes only small changes in the pressure at which failure occurred (approximately 15 percent change in pressure from failure and no deflection to failure at 5 percent deflection). If the cell boundaries caused as much as 100 percent change in the constrained modulus of the soil, the pressures at which failure occurred in the cell would still be approximately the same as the failure pressure in actual field installation. Placement method, soil friction plus cohesion, moment of inertia, and area of the pipe wall will probably shift the incipient failure line in Figure 22 up or down.

Because of the compensating effect of the stresses on the constrained modulus and the low sensitivity of buckling to the constrained modulus, it is believed that the data for buckling, at least at low deflections, are as good as any that are available.

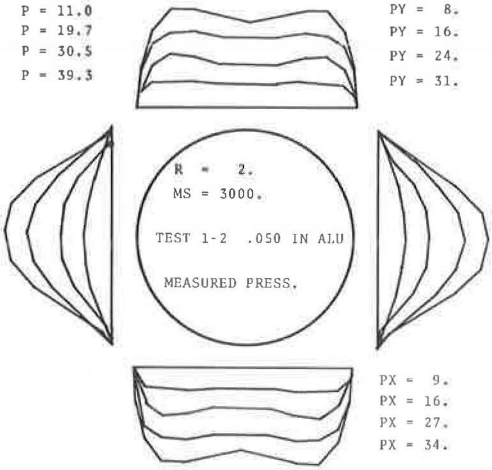


Figure 18. Horizontal and vertical pressures on model culvert.

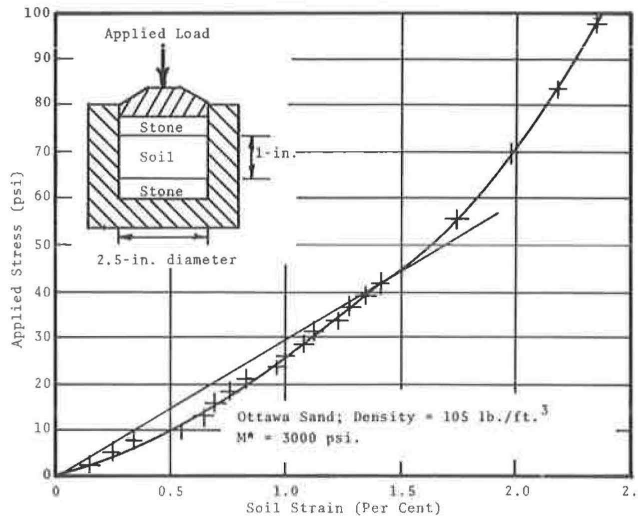


Figure 19. Confined stress-strain curve.

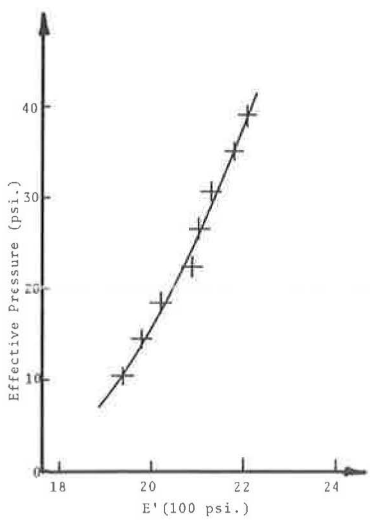


Figure 20. Variation of the modulus of horizontal soil reaction.

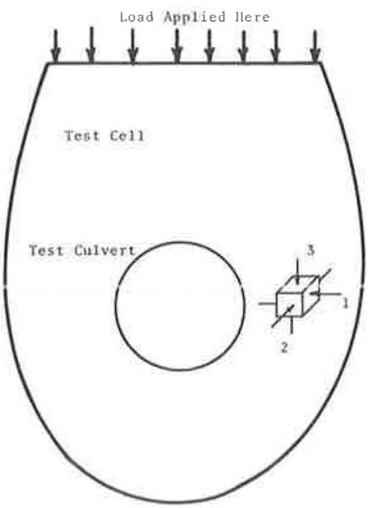


Figure 21. Watkins and Moser's test cell.

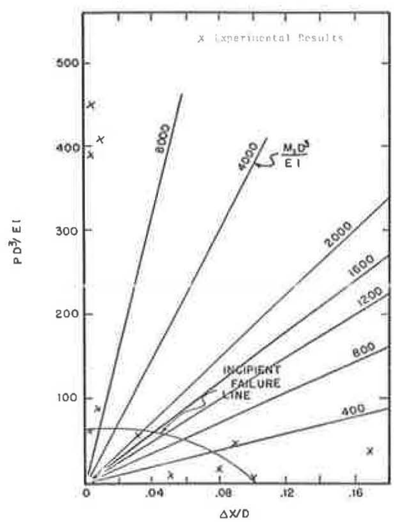


Figure 22. Load deflection obtained from data by Watkins and Moser.

Figure 23. Large culverts in British Columbia (furnished by Chriss Fisher, Armco Corporation).

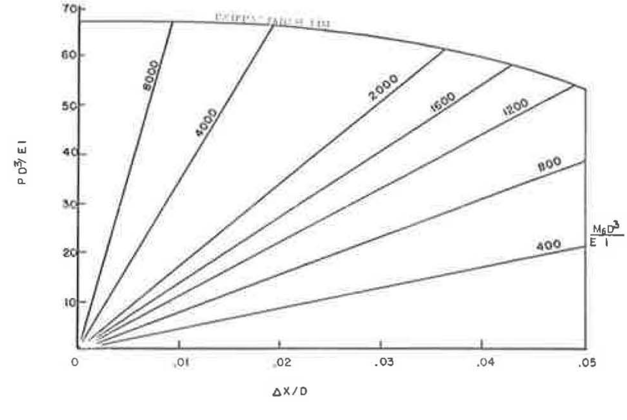


Figure 24. Design chart for circular conduits.

LARGE-SPAN TESTS

Fisher (18) has installed several culverts with 40-ft spans. There have been no failures, and the measured deflections are usually less than 1 in. The secret of the success of these large spans may be in the installation procedure. Fisher used what he calls a thrust beam along the side of the culvert. Figure 23 shows the installation of one of these large culverts.

PLASTIC PIPE

The Bureau of Reclamation in Denver, Colorado, is conducting field experiments on plastic pipe. Because these studies are not complete at this time, no data are available yet.

DESIGN OF CIRCULAR CULVERTS

The way in which Watkins and Moser's (17) large-scale test data are plotted in Figure 22 forms the basis for a possible design procedure. The M^*D^3/EI lines shown in Figure 22 are determined from the elastic theory for analysis of pipe presented by Burns and Richard (8). Most of Watkins and Moser's data plot very close to calculated M^*D^3/EI values from the elastic theory (9). Watkins and Moser's data can be used to delineate an upper boundary. The resulting design chart is shown in Figure 24. As better failure data become available, which take into account all soil and pipe properties, the upper boundary can be adjusted accordingly. Future work may show that one upper boundary is not sufficient to account for all variables. Examples for the use of the design chart can be found elsewhere (9).

SUMMARY

In summary, a great deal of work has been directed toward a better understanding of soil-structure interaction phenomena. Much more research is needed, but an organized approach needs to be made. At the current time, many individuals are conducting research directed toward a better understanding of the performance of underground structures. Much of this research would have been far more valuable if only a few more dollars had been spent for proper instrumentation.

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BALANCED DESIGN AND FINITE-ELEMENT ANALYSIS OF CULVERTS

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This paper outlines a method for treating culverts in embankments. Design is accomplished with approximate relations based on empirical determination of arching and maximum induced moment. Relations are provided that permit determining the factor of safety against failure and collapse in the various possible modes. This permits the adjustment of designs to achieve a desired balance in these modes against failure and collapse. Discussion is provided regarding finite-element analysis of resulting designs. A three-dimensional linear finite-element solution is provided for the design example. It is shown that linear solutions predict deflections reasonably well but that they overestimate thrusts and moments. The area of utility of the approach as compared to other methods is discussed.

• A BODY of knowledge germane to the culvert problem has resulted from efforts to develop design and analysis methods for buried shelters in order to resist the effects of nuclear weapons. This paper adapts that information to the design of culverts. The principal goal is to provide a method of solution for culvert problems excluding the class of small culverts where handling and durability govern the design.

A classification of conduits, including a distinction of those governed by handling and durability, may be found elsewhere (1). NCHRP Report 116 (1) reviews the older design methods and delineates their limitations and deficiencies. Also, it proposes two new methods: one based on foundation settlement considerations and the other based on elastic theory relations (2, 3).

An approximate approach that circumvents some of the limitations of the elastic theory has been formulated elsewhere (4). The use of this method is limited to fully buried cylinders under high loads. It utilizes the elastic theory and an empirical arching relation.

Among the contributions from weapons effects work is the development of relations for modeling earth materials (5, 6). Some of these relations have been incorporated in nonlinear finite-element programs that permit far better analysis of soil-structure systems than has been possible in the past (7, 8, 9).

The approach recommended here is to use the approximate method to obtain approximate designs and then to use the finite-element method to analyze the resulting designs. The importance of the soil and of the relative stiffness in controlling system behavior is emphasized. It is control of these independent parameters that permits efficient designs.

The following notation is used in this paper:

A_o = maximum active arching,
A_s = area of section at springing,
b = width of base of embankment,
c = distance from midplane to extreme fiber of plate,
$\bar{C}$ = constant that depends on the mode of buckling,
$\tilde{C}$ = constant that depends on the Poisson's ratio of steel,

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$C_{xx}, C_{xy}, \ldots C_{ys}$ = material property constants,
C_1, C_2, C_3, C_4, C_5 = constants to account for different bedding conditions and flow characteristics of the soil,
D = cylinder mean diameter,
d_e = depth to the plane of equal settlement,
E = Young's modulus of elasticity,
E' = modulus of soil reaction,
$f(\ldots)$ = function of several variables,
G_{xy} = shear modulus,
h = height of embankment above the invert,
I = moment of inertia,
N = thrust at the spring line,
P = surface pressure,
$p_{1_{cr}}$ = transitional buckling stress,
t_e = equivalent thickness of culvert,
Δx = horizontal diametral extension,
Δy = vertical diametral extension,
ϵ_c = unit vertical strain of culvert across diameter,
ϵ_s = unit vertical strain in the soil,
ϕ = angle of friction,
γ = unit weight of soil,
σ_{allow} = allowable stress,
σ_y = yield stress, and
Ω = arching coefficient.

BASIC CONSIDERATIONS

At the outset, it is worthwhile to establish a few basic definitions and to review the dominant knowns and unknowns of the culvert problem. For present purposes, failure will connote the occurrence of visible distress as indicated by wall crushing or excessive cracking; buckling; plastic deformation (other than local); separation or rupture of seams and joints; or excessive deflection that impairs the functional performance or psychological acceptance of the installation. This definition requires the establishment of suitable design criteria (1, 4). Designs should be evaluated in terms of their failure and collapse loads.

Dimensional analysis and experiments on buried cylinders in a uniform granular soil field (10) show that to a first approximation

$$\frac{y}{D} = f\left(\frac{p_i}{M_s}, \frac{L}{D}, \frac{M_s}{EI/D^3}, \frac{d_o}{D}\right) \tag{1}$$

in which y = radial deflection of the crown, D = cylinder mean diameter, p_i = interface load at the crown, M_s = effective secant confined compression modulus at a stress equal to the applied load, L = cylinder length, EI = cylinder wall stiffness, and d_o = depth of cover over the crown.

For culverts, the effect of L/D is usually negligible, and the behavior can be expressed by the remaining four nondimensional pi-terms. Equation 1 may also be deduced from the elasticity theory on neglecting the influence of Poisson's ratio.

From past analyses, tests, and experience, we know the following:

1. EI/D^3 strongly influences the magnitude and distribution of the load on the culvert;
2. Soil stiffness, M_s, and variations of the soil field in the vicinity of the culvert (bedding and backfill) strongly influence behavior;
3. Time effects are usually small except for clayey soils (with clayey soils, time effects result in a shift in load to the pipe);
4. The possibility of shear failure in the soil in most circumstances is slight until the onset of a culvert failure;
5. Live loads due to traffic are significant only for depths of cover of less than about 5 ft;

6. Extensional flexibility does not affect the behavior or design of steel, aluminum, or reinforced concrete culverts; and

7. Use of the constrained (one-dimensional compression) modulus is advantageous because it is relatable to dry density, vane shear strength, and certain other useful indexes of field soil conditions and because it is a standard, widely used laboratory test.

Principal uncertainties or deficiencies in knowledge of culverts are attributable to insufficient data on the influence of variations in the adjacent field around the pipe—particularly the manner in which behavior is influenced by bedding, backfilling, and backpacking; deficiency of measurements on actual pressure distribution, especially in regard to interface shear; and inadequate criteria and definitions relating to failure and factor of safety. Also, analytic methods available in the past were inadequate for designing culverts under high fills or conduits with shallow overburden. Within limits, the elastic theory is suitable as a basis for design.

A principal weakness of the elastic theory is that it does not account for arching as it occurs in soils. According to the elastic theory, the minimum thrust possible at the spring line in any culvert is equal to pr; but it is known from numerous experiments that the thrust is usually less than pr (where r = D/2) (10, 11, 12, 13). A comparison of thrust calculated from an empirical arching equation (4) and from the elastic theory shows a 20+ percent divergence. From these results, it is clear that thrust cannot be determined directly by the elastic theory. Moments from the elastic theory are also in error, especially at high loads.

Elastic theory is adequate for determining deflection if (a) the boundary conditions correspond to those in the theory, (b) the soil field surrounding the culvert is uniform, (c) M_s is taken as the secant modulus to the stress-strain diagram at a stress corresponding to the applied load, (d) the shear stresses do not exceed the shear strength of the soil, and (e) time effects are negligible. Conditions (a) and (b) often are not met in practice; consequently, adjustments to the applied load and the effective modulus may be necessary. Space does not permit an elaboration of how such adjustments are made; however, if one recognizes that behavior is governed primarily by the relative strain in the soil with respect to the average strain over the height of the inclusion, the method of adjustment is fairly obvious.

Other considerations that aid in converging on a sensible design approach are as follows:

1. Deflection and wall-crushing are the most likely failure modes where existing seams are properly designed.

2. Moments are more variable than deflections and thrusts.

3. Deflections are the most predictable quantity and are the easiest to measure in a completed installation. However, deflection is not the best criterion for the design of a flexible culvert because, for such culverts, deflection has a weak dependence on culvert stiffness. By elimination, then, wall strength and soil stiffness are the appropriate principal bases for culvert designs where handling and durability do not govern.

Ideally, one would like to achieve an equal probability of survival in all possible failure modes. This means that, for a "balanced design," a higher factor of safety is required against buckling than against wall-crushing and that a higher factor of safety is required against wall-crushing than against deflection failure. Also, one might logically have separate factors of safety against failure and against collapse.

A final item of background is desirable regarding the use of the elastic theory versus the Iowa formula (1, 14). Actually, the Iowa formula conforms closely to the elastic theory as may be seen by writing the deflection equations in a similar form (with $\Delta x \cong \Delta y$) as follows:

Elastic theory (full slip):

$$(\Delta y/D)/(p/M_s) = \left(C_1\{[M_s/(EI/D^3)] + 1\}\right)/\left(C_2\{[M_s/(EI/D^3)] + 9\}\right) \tag{2}$$

Iowa formula:

$$(\Delta y/D)/(p/M_s) = \{C_3[M_s/(EI/D^3)]\}/(C_4\{[M_s/(EI/D^3)] + C_5\}) \qquad (3)$$

These two equations differ appreciably only for $M_s/(EI/D^3) \gtrsim 10$. For $M_s/(EI/D^3) > 10^3$, the two equations give essentially identical results.

In its usual form, the Iowa formula contains a so-called modulus of soil reaction, E′, and coefficients to account for different bedding conditions and flow characteristics of the soil. By letting $M_s/(EI/D^3)$ go to infinity in Eqs. 2 and 3, it may be shown that $M_s \approx 1.1$ to $1.5E'$. For a value of the bedding coefficient of 0.1, $M_s = 1.22E'$. Use of the bedding coefficient is an indirect means of accounting for different arching conditions. It is desirable to determine arching directly and to use M_s instead of E′. These features and the foregoing requirements and reasoning are reflected in the design procedure that follows.

Because the main deficiency in our knowledge concerns the load-carrying capability of the soil, a few words of clarification are in order.

SOIL PARAMETERS

The term "constitutive properties" (or laws) is used in continuum mechanics to mean the set of equations that define the stress-strain properties of the media of concern. Rather refined constitutive relations have been developed to represent earth materials (5, 6), and some of these have been incorporated in the computer programs that will be discussed later.

Fortunately, the required constitutive relations for static loading are relatively simple because unloading is not usually involved. For approximate calculations, the following are required: an average value of Poisson's ratio, ν_s; sometimes the cohesion, c; the coefficient of lateral earth pressure, K_o; the angle of friction, ϕ; and the effective secant modulus at a stress equal to the applied load, M_s. One should keep in mind that, for embankments with large height-to-width ratios, the effective value of K_o may be less than in fully buried installations. Of the preceding properties, the secant modulus is of dominant importance.

Soil elements at different locations throughout an embankment are subject to different confining stresses; thus it would be expected that the effective modulus would vary throughout the cross section. For a properly designed and compacted embankment, the effective modulus in the bottom central region should be nearly equal to the confined compression modulus. The effective modulus at a given depth should correspond to the overburden stress at that depth, except near the sides. Near the sides the effective soil modulus decreases, but so does the load; consequently, the conditions at the midbottom region may be presumed to control the design. (Stress conditions near the ends are discussed later in the paper.) These comments are predicated on the assumption that the width of the embankment is large ($\lesssim$ 10D) compared to the diameter of the pipe. If this is not the case, the effective soil modulus will have to be appropriately reduced.

Designers of culverts should appreciate the wide range of values that M_s may have depending principally on the soil type, the placement methodology, and the applied stress. M_s may vary from essentially zero for saturated clays to several hundred thousand for granular (locking) materials under high stresses. One should also be aware of the large variability in M_s attributable to placement. As an example, in laboratory tests (11) where care was taken to replicate placement of dry sand in a test bin by using the sand-fall method, variation of M_s from the mean value was ±20 percent; certainly a greater variation must be expected in field installations.

Because of the wide variability in soil properties, the use of an unduly refined design procedure does not seem appropriate. The need is for a design method based on the proper criteria that contain the principal parameters in correct relation to one another.

DESIGN PROCEDURE

A proposed design procedure for culverts in embankments is given by means of an example. An example employed by others (1) is used so that comparisons can be made.

We are required to select a section for a 60-in. diameter steel culvert under a 20-ft embankment of soil weighing 120 pcf. The basic steps in the design are as follows:

1. Determine the vertical stress at midheight, p_s, and the vertical stress at the elevation of the crown, p_v.

Dead load: $p_s = [h - (D/2)]\gamma = [20 - (5/2)](120/144) = 14.6$ psi

$p_v = (h - D)\gamma = (20 - 5)(120/144) = 12.5$ psi

Live load: 0

2. Determine M_s corresponding to the stress at midheight of the culvert from confined compression test results. For carefully placed granular fills compacted to at least 85 percent AASHO T-99, one may use $M_s = 1{,}000\, p_s^{0.8}$ if no test results are available (15). In the present example, use $M_s = 1.22\,E' = 1.22 \times 700 = 854$ psi to conform to the comparison design (1).

3. Estimate the arching A and calculate the thrust from

$$N = p_v(1 - A)(D/2) \tag{4}$$

The relations $A = 0.2 - 0.2[1 - (d_o/D)]^2$ for $d_o/D \le 1.0$ and $A = 0.2$ for $d_o/D > 1.0$ may be used if better bases for the estimate do not exist (16). Bedding angle, projection ratio, and certain other factors will influence A:

$$N = 12.5\,(1 - 0.2)\,(60/2) = 300 \text{ lb/in.}$$

4. Calculate the stiffness required by the handling criterion $D^2/EI \le 0.0433$:

$$EI = 60^2/0.0433 = 83{,}200 \text{ lb-in.}^2\text{/in.}$$

Thus, $M_s/(EI/D^3) = (854 \times 60^3)/83{,}200 = 2{,}220$.

5. Determine the moment using Figure 1 (experimental curve) with

$$p_1 = p_v\,(1 - A) = 12.5\,(1 - 0.2) = 10 \text{ psi}$$

$$M = 0.005\, p_1\, D^2 = (0.005 \times 10.0)60^2 = 180 \text{ in.-lb/in.}$$

6. Determine the equivalent flat plate thickness required to resist the thrust and moment based on a factor of safety of 2 against yielding of the total section. Use the approximate relation

$$\sigma_{allow} = \sigma_y/FS = (N/A_s) \pm (Mc/I)$$

where $A_s = t_e \times 1$ and $I = t_e^3/12$. Substituting, $33{,}000/2 = 300/t_e + (180 \times 6)/t_e^2$ then

$$t_e = 0.265 \text{ in.}$$

7. The corresponding stiffness is

$$EI = [(29 \times 10^6)\,0.265^3]/12 = 44{,}950 \text{ lb-in.}^2\text{/in.}$$

which is less than 83,200 lb-in.²/in.; therefore, handling governs. Use a 12-gauge plate with 2⅔- by ½-in. corrugation (17). Spangler chose the same corrugation but a 10-gauge plate (1). Calculations are given as follows for both gauges:

$$\left.\frac{EI}{D^3}\right|_{12} = \frac{(29 \times 10^6)0.00343}{60^3} = 0.461 \;;\; \left.\frac{EI}{D^3}\right|_{10} = 0.604$$

8. Determine $M_s/(EI/D^3)$ and find the corresponding value of $(\Delta y/D)/(p_v/M_s) \cong \epsilon_c/\tilde{C}\,\epsilon_s$ using Figure 2 with $\tilde{C} = (1 + \nu_s)(1 - 2\nu_s)/(1 - \nu_s)$:

$$\left.\frac{M_s}{EI/D^3}\right|_{12} = \frac{854}{0.461} = 1{,}854 \;;\quad \left.\frac{M_s}{EI/D^3}\right|_{10} = 1{,}413$$

$$\left.\frac{\Delta y/D}{p_v/M_s}\right|_{12} \cong 2.46 \quad;\quad \left.\frac{\Delta y/D}{p_v/M_s}\right|_{10} = 2.44$$

9. Calculate the depth to the plane of equal settlement (4) or use $d_e = D$ for $h \geq 2D$; then, determine the arching coefficient

$$\Omega = (2d_e/D)\,[(\epsilon_c/\epsilon_s) - 1]$$

and find the arching using Figure 3

$$\Omega_{12} = (2 \times 60)/60\ (1.83 - 1) = 1.66$$

$$A = 0.2 = A_{assumed}$$

therefore OK. When the predicted and calculated arching values do not agree, iterate as necessary to bring the two values into agreement.

10. Determine the conformance with design criteria and the factor of safety for the various possible modes of collapse. For deflection, from step 8,

$$\Delta x_{12} \cong \Delta y_{12} = 2.46(p_s/M_s)D = 2.46(14.6/854)60 = 2.52 \text{ in.}$$

$$\Delta y_{10} = 2.38 \text{ in.}$$

$$\Delta x/D = (2.52/60)100 = 4.2 \text{ percent} < 5 \text{ percent}$$

therefore OK.

$$FS\,\Big|^{12}_{caving} = 0.20/(\Delta x/D) = 0.20/(1.88/60) = 6.4$$

An alternate would be to calculate the buckling load corresponding to the second mode and use $FS\,|_{caving} \approx p_{1_{cr}(2)}/p_1$. For wall crushing, by limit-equilibrium of the soil block above the culvert,

$$FS\,|_{wall} = [2(1 - A)(2cd_e + \sigma_y t_e)/\sigma_y t_e(D - 2d_e K_o \tan\phi)]$$

For granular soils, $c = 0$, $d_e = D/2K_o$, and $A_o \approx \tan\phi$

$$FS\,|_{wall} = [2(1 - A)/(1 - A_o)] = [2(1 - 0.3)/(1 - 0.87)] = 10.8$$

For seam strength, load = 300 x 12 = 3,600 lb/ft and capacity (17) = 23,400 lb/ft

$$FS\,|_{seam} = 23{,}400/3{,}600 = 6.5$$

For transitional buckling (4),

$$p_{1_{cr}} = \overline{C}\sqrt{M_s(EI/D^3)}$$

where

$\overline{C} = 6\sqrt{\tilde{B}\tilde{C}}$;
$\tilde{B} = 0.75$ for $\nu_s = 0.3$, $D/b < 0.2$; and
$\tilde{C} = (1 + \nu_s)(1 - 2\nu_s)/(1 - \nu_s)$.

For $\nu_s = 0.33$, $\tilde{C} = 0.742$, and $\overline{C} = 4.5$

$$p_{1_{cr}} = 4.5\sqrt{854 \times 0.461} = 89 \text{ psi}$$

$$FS\Big|_{buckling} = 89/10 = 8.9$$

11. Determine longitudinal deflection and tension, bending, and durability requirements in the manner suggested elsewhere (1). Other methods yield essentially the same design (1). For large diameters, where handling no longer governs, resulting designs are different.

One could argue that the minimum factor of safety of 6.4 is excessive for many installations; however, reducing the factor of safety for such small culverts is not possible unless special handling provisions are instituted. The actual factor of safety in an installation is probably greater than 6 because the value of M_s used (854 psi) is low for granular fills if reasonably good construction controls are maintained.

Principal advantages of the proposed method are that it permits treatment of embankment-culvert systems of all materials, depths of cover, and sizes; incorporates a rational method for accounting for arching; considers all potential modes of failure and collapse and enables achievement of a "balanced" design where the factors of safety in the different modes are in a desired ratio to each other; incorporates the means of accounting for moments; and enables the logical design of systems with backpacking (4).

The principal deficiency of the proposed method is that resulting designs for large-diameter culverts are strongly influenced by induced moments that are subject to relatively large variations. Also, the method involves the use of empirical relations with constants that are not as yet defined for cohesive soils.

Other deficiencies of the proposed design method are almost too obvious to mention. Clearly, test data on large pipes and conduits are needed for Figure 1. No tests on prototype culverts with the required measurements are known to exist although instrumentation exists to obtain the needed quantities. Empirical exponents are also desired for cohesive soils (tests are being planned to obtain the data). The method can be used with reduced accuracy without the data by assuming a conservative value of arching. In spite of these limitations, the method represents an improvement over prior ones because (a) it accounts for all principal variables, (b) all possible modes of failure are considered, and (c) the factor of safety in all modes can be estimated and, based on these values, adjustments can be made to the design to achieve a desired "balance" among them.

Important designs developed with the preceding methodology can be analyzed by using the finite-element method as is indicated in the following section.

FINITE-ELEMENT ANALYSIS

Once a culvert is selected, two- or three-dimensional finite-element analyses can be performed on the system. One may obtain two-dimensional linear or nonlinear solutions that incorporate either small- or large-deformation theory (8, 18, 19, 20). Also, the linear small-deformation code has been modified to permit accounting for interface slip and boundary separation (21). Some of the referenced codes have been modified to account for the presence of initial stresses and for dynamic loading. Examples of the use of some of these two-dimensional codes can be found elsewhere (19, 21, 22).

If other than stresses and deformations on a transverse section are desired, a three-dimensional solution is required (9). The remainder of this paper is devoted to presentation of a linear three-dimensional solution of the problem employed in the example design (with 10-gauge plate) of the previous section. The geometry and material properties used are given in Figures 4a and 4b respectively. Only one-quarter of the soil-culvert system was analyzed because of symmetry. Note that different moduli are used for the different layers because of differing dead load stresses at different depths. The elastic modulus at the culvert corresponds to the M_s used in the design example.

One thousand, five hundred and sixty-eight 8-node hexahedron solid elements were used to represent the soil, and 84 shell elements were employed to model the culvert. The culvert was represented by a plate of equivalent thickness (0.387 in.) to properly model the stiffness of its transverse section. No correction was made to account for corrugations; thus, the longitudinal stresses in the cylinder would be expected to be greater than those where circumferential corrugated plate is used. Stresses in all elements and deflections at all node points were obtained as output data; however, space limitations permit visual display of only a small portion of these data.

Vertical soil stress contours in the Y-Z plane and in the X-Y plane are shown in Figures 5 and 6 respectively. As may be observed, the interface stress at the crown is about 15 psi as compared to 10 psi predicted with the arching relation in the example design. At the invert, the normal stress is about 16 psi. The horizontal stress at the spring line is about 15 psi. It is interesting to note from Figure 6 that the vertical soil stress is greater at about 1 to 2 ft above the culvert than it is at the crown.

Horizontal stress contours in the Y-Z plane are shown in Figure 7. Perhaps the most significant aspect of the horizontal stress is the rapid dispersal of the stress concentration adjacent to the spring line.

Stresses and forces in the culvert are shown in Figures 8 and 9. The contour plots in Figure 8 show the longitudinal and circumferential stresses in the extrados of one-half of the developed longitudinal section. At the center spring line the circumferential stress in the extrados is about 11,100 psi.

The peak longitudinal stress is about 18,000 psi for the modeled plate; however, it would be less for a longitudinally corrugated culvert.

Forces and deflections on the transverse sections at midlength and one-quarter of the total culvert length from one end are shown in Figure 9. It is interesting to note that the thrust at the spring line is about double the thrust at the crown and invert. Moments at the crown and spring line are about equal in amplitude but of opposite sign.

Horizontal diametral expansion at the center section is 1.27 in., and the vertical diametral shortening was 1.35 in. The corresponding vertical deflection determined in the design was 2.38 in. The deflection by the Iowa formula, excluding deflection lag, was 2.14 in. Absolute displacement of the invert at the center section was 4.77 in. This is the amount of camber that should be provided initially to ensure that the longitudinal axis of the culvert is straight when the embankment is completed.

In the example used, design deflections are in approximate agreement with values from the finite-element solution. Peak thrusts and moments are about a factor of 2 larger than in the design. The reason for this is that the elastic theory does not properly account for the arching in granular soils. Two-dimensional elastic theory indicates that, for $M_s/(EI/D^3) = 1,850$, the thrust at the spring line is 1.32 p_r. Experimental data and the arching theory give the thrust as 0.8 p_r. The latter value is considered correct for granular soils; however, elastic theory results may be more nearly correct for other than granular soils. Of course, knowledge of stress distribution from elastic finite-element solutions is useful. Codes with constitutive relations that properly model soil behavior must be used to obtain correct amplitudes.

Incidentally, stresses and deflections are expected to be lower than the values from the example for installations in granular soils. The reason is that the modulus of properly compacted granular fill would be an order of magnitude greater than that used. The low value was used to permit comparison of the results with existing designs for the same problem.

Although the example design and analysis were for a steel culvert, the same methodology is applicable to concrete culverts. The principal difficulty in the design and analysis of concrete cylinders is that the effective section modulus changes with load as fine cracks develop around the perimeter. As a consequence, concrete cylinders are not as "rigid" as is often presumed.

SUMMARY

This paper presents the bases for improved design and analysis methods for culverts. The design method illustrated is the only known method that permits accounting for arching and moment as an integral part of the design procedure. This is unimportant for small metal conduits under moderate fill heights because handling and durability usually govern designs under those circumstances. For culverts of cementitious materials and for all culverts under high loads, induced moments become important,

Figure 1. Moments in buried cylinders.

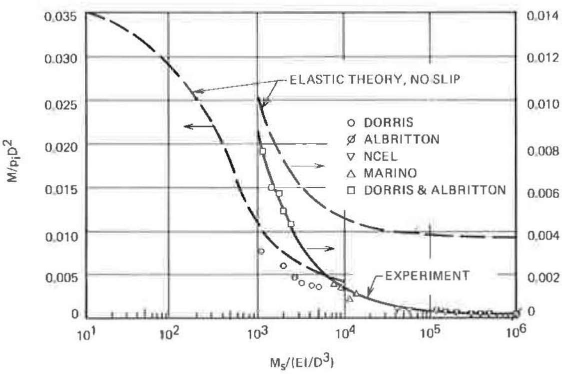


Figure 2. Deflections of buried cylinders (corresponding to average of full- and no-slip cases from the elastic theory and υ = 0.3).

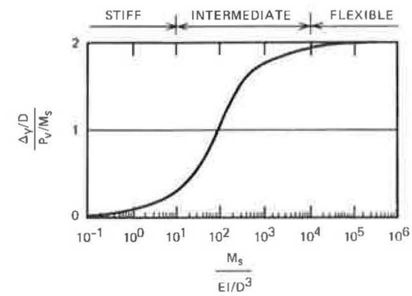


Figure 3. Plot for determining arching over culverts in granular soil.

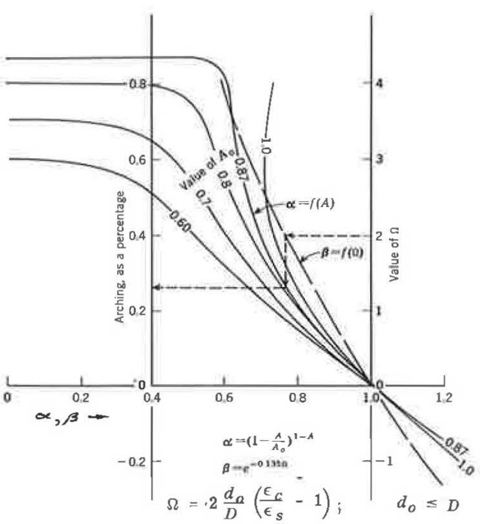


Figure 4. Soil-culvert system model.

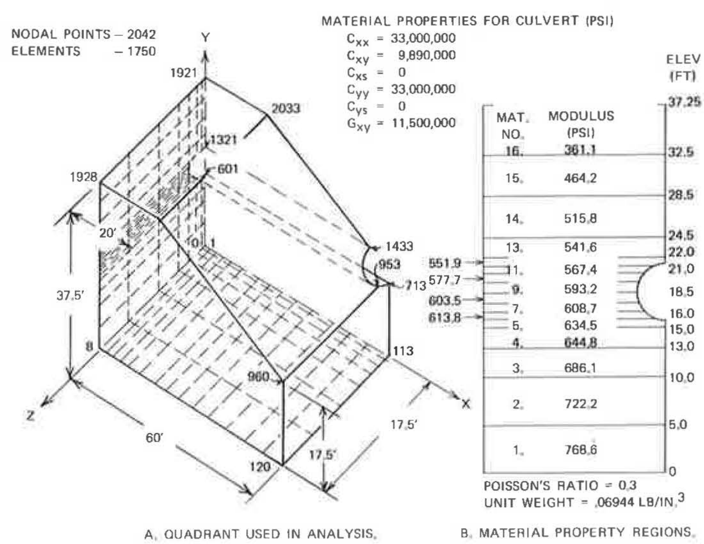


Figure 5. Vertical soil stress contours (Y-Z plane).

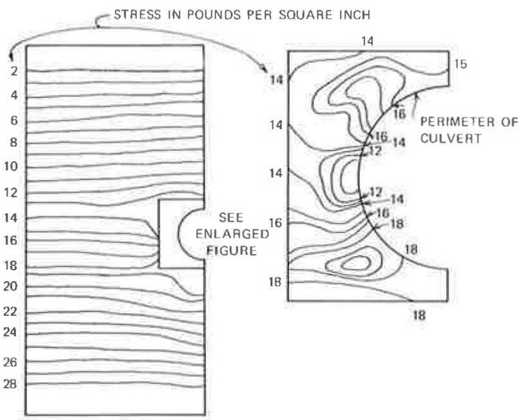


Figure 6. Vertical soil stress contours (X-Y plane).

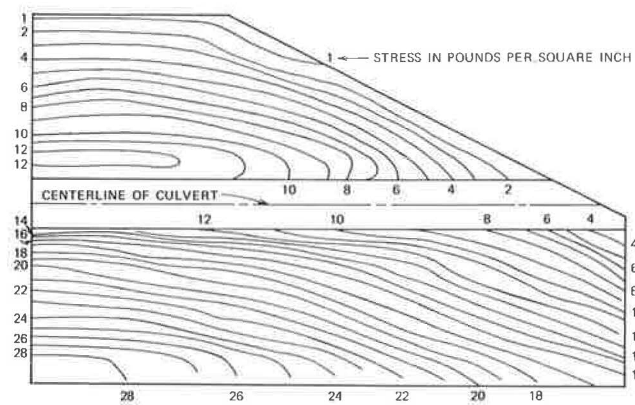


Figure 7. Horizontal soil stress contours (Y-Z plane).

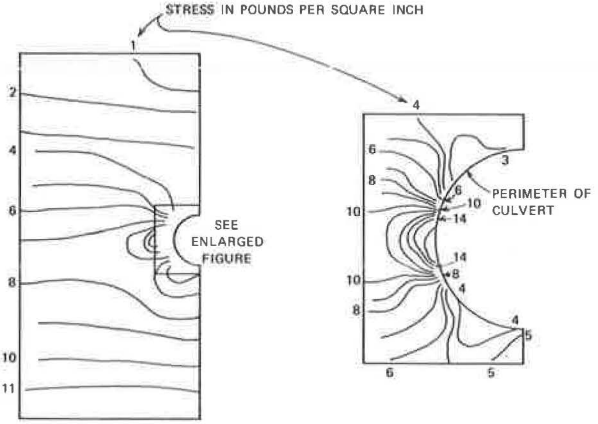


Figure 8. Stress contours on developed longitudinal half-section.

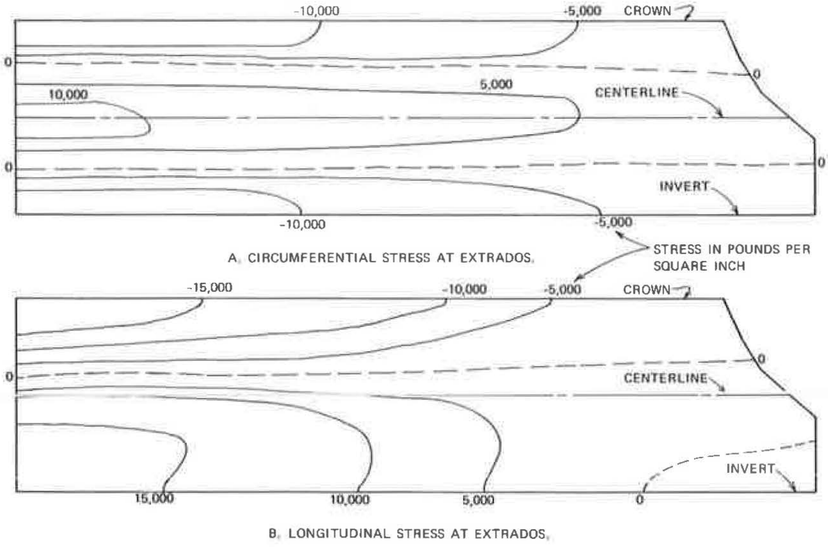
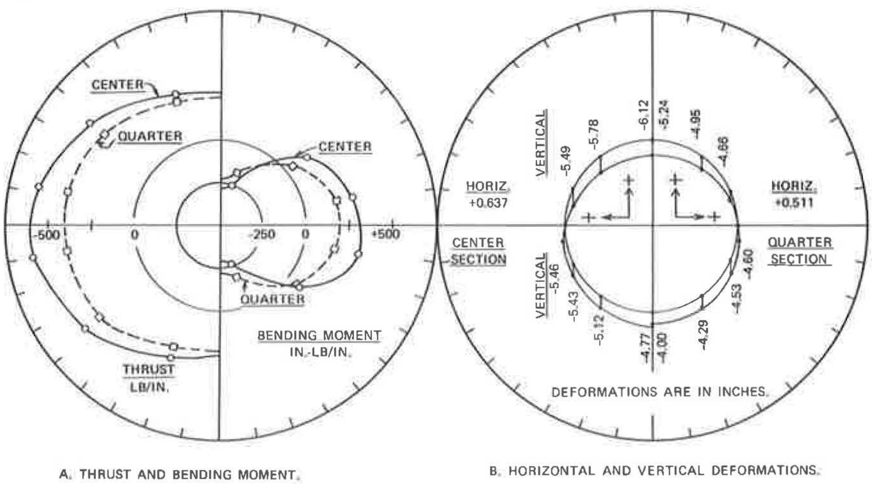


Figure 9. Forces and deflections on transverse section.



and economics require the use of arching. It is in these circumstances that the method given is useful.

Important installations warrant detailed analysis, and the finite-element method is useful for this purpose. A variety of two-dimensional codes may be used, including one that accounts for interface slip and boundary separation. Linear three-dimensional solutions, as illustrated in the text, may also be obtained; however, the magnitudes of the resulting thrusts and moments will be markedly larger than those in actual installations in granular soils.

Key points in the design of culverts are as follows:

1. If adequate construction procedures are followed, handling and durability considerations govern the required section properties of metal culverts less than about 5 ft in diameter under fills or equivalent loads less than about 30 ft in height (1).
2. For larger diameters or equivalent fill heights, good design and economics require consideration of moment and arching. With the larger diameters, achieving soil control via quantitative measurements becomes essential.

The methods outlined here, together with the use of backpacking in accordance with the relations found elsewhere (4), should permit more accurate and efficient designs of culvert systems than has been possible heretofore.

ACKNOWLEDGMENT

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57-66

STRUCTURAL DESIGN PRACTICE OF PIPE CULVERTS

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•A DISCUSSION of culvert design practice must include methods of correlating design with actual construction procedures. Also pertinent are adaptations or extensions of the most widely used design procedures (including durability), which satisfy the practical needs of both the designer and the installation.

This paper discusses these items, and reference is made to the methods used by a large design and construction agency, the New York State Department of Transportation. Various aspects of the New York approach may be useful and can be adapted and utilized by any size organization if desired.

A great deal of research on culvert design has been done and is continuing. This paper suggests that the results of significant theoretical studies be reduced to practical terms so that they can be used routinely by all types of designers.

The structural design procedures of most organizations are based on the Marston-Spangler formulas for rigid pipe and on adaptations of the "ring compression theory" for flexible pipes. These methods have been thoroughly discussed, analyzed, and evaluated in innumerable studies and will not be repeated here.

The results of all evaluations, however, generally support the same conclusion—that current design procedures are satisfactory when properly applied. Proper application must, of course, include engineering judgment, provision for durability, and consistent, appropriate installation practices.

The most significant of the shortcomings commonly ascribed to the foregoing design methods are as follows:

1. Interactions of the soil-structure system are not properly considered;
2. Some of the input parameters are difficult to ascertain and must generally be assumed;
3. Results are usually conservative; and
4. Their applicability to extremely large structures under very shallow and extremely high fills is questionable.

Of all these criticisms, only the last one is not easily satisfied or provided for with a satisfactory degree of confidence in current design practice. It is reasonable to assume that the other shortcomings are sufficiently overcome such that practical results for routine installations are obtained. It is acknowledged, however, that the resultant safety factor for structural criteria is usually greater than required. The savings in cost that would accrue with the use of better-known formula input parameters with either existing or new design methods, however, would not always be especially significant for routine cases. For example, the minimum gauge of flexible pipe required to satisfy strength criteria after installation is often inadequate to also meet handling and durability requirements.

Durability is not included as an integral part of the structural design process for flexible structures. An initial attempt has been made in New York State to overcome this shortcoming and is described later.

The number of designs made each year by any large agency such as the New York State Department of Transportation is often staggering. Furthermore, design selections are made by perhaps hundreds of individuals within a state (or within a large design organization) who have limited expertise in soil mechanics and conduit structural

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analysis. Consequently, the need exists for designs and installation details to be standardized for the most commonly used shapes of rigid and flexible pipes for heights of cover up to about 100 ft. This concept is followed in varying degrees by many states. The approach used in New York is, therefore, not new in all respects but is believed to be quite comprehensive, easy to use, and successful in meeting the continuing challenge of designing large numbers of culverts.

The section that follows discusses this approach and the factors involved in establishing uniform design and installation standards. It should not be assumed from this presentation that current methods are considered to be entirely satisfactory or not in need of improvement or change. A case is being made, however, for the workability of current design methods when coordinated with construction practice and experience.

DESIGN APPROACH

The selection of a rigid or flexible pipe is not always made objectively. Sometimes it is based on the personal preference of the designer. The most recent procedure now being implemented in New York, however, is intended to be more objective; it is based on anticipated performance and cost. This design selection process first involves a determination of the required cross section for rigid and flexible pipes (including hydraulic factors). Structural analysis then follows, which must satisfy site requirements for fill height, foundation conditions, and durability considerations caused by the structure's environment.

This procedure often requires that a design cost analysis be made for both rigid and flexible pipes for many individual installations. Consequently, the desirability of easy-to-use standardized designs becomes even more apparent.

Rigid Pipe

In order to reduce the design process for routine conditions to its simplest form, i.e., a fill-height table, we first established a minimum number of practical installation conditions that cover all designs normally encountered in the field.

On New York State highway projects, for example, only two types of bedding are considered to be consistently attainable in the field when using reasonable procedures with varying qualities of inspection: class C or ordinary bedding and class A or concrete cradle bedding.

Only two design analysis loading conditions, positive projecting and imperfect trench, are routinely needed for each bedding condition to cover all methods of pipe installation commonly used on New York State projects. The positive projecting case covers the embankment installation as well as the trench and negative projecting conditions because the latter two types have been shown to require a trench width that is too wide for a trench or negative projecting loading. This minimum trench width is based on a 2-ft clearance between the pipe and the inside face of the trench plus the width of a sheeting section, which is always required for safety with trenches 5 ft or more in depth. Sheeting is seldom pulled incrementally as desired, and therefore the width of the sheeting section must be considered as a part of the trench width for load analysis. With the design analysis loading and bedding conditions thus established, the input parameters for the Marston-Spangler formulas can be selected or assumed as required.

In the approach followed by New York State for reinforced concrete pipe, a safety factor of 1 on the first crack strength and a soil unit weight of 125 pcf are used. The variation in settlement ratio and load factor has been shown to have a limited effect on the design results, providing reasonably representative constants are selected for these parameters. Height of fill-gauge tables for all field installation conditions such as embankment, trench (wide trench condition), and imperfect trench are therefore readily established by assuming values for settlement ratio and load factor and by making other appropriate inputs into the load analysis formulas. The designer, using these tables, needs only to select the "pipe strength-bedding type-installation method" combination that most economically satisfies the field condition. New York State's current allowable fill-height tables for reinforced concrete pipe are shown in Figure 1. It is noted that, in preparing these tables, design assumptions were modified and fill heights were

Figure 1. Fill-height design tables for reinforced concrete pipe.

INSTALLATION METHODS

Method D
(Imperfect Trench)

MAXIMUM HEIGHT OF FILL (H) IN FEET OVER TOP OF PIPE						
PIPE DIAMETER D (inches)	ORDINARY BEDDING			CONCRETE CRADLE BEDDING		
	14R CLASS III	14 CLASS IV	14X CLASS V	14R CLASS III	14 CLASS IV	14X CLASS V
12	34	51	77	60	88	110
15	34	51	76	60	88	110
18	34	51	76	60	88	110
21	34	51	76	59	88	113
24	34	50	76	59	88	113
27	34	50	76	59	88	113
30	34	50	76	59	87	113
33	34	50	76	59	87	113
36	34	50	75	59	87	113
42	34	50	75	58	87	113
48	33	50	75	58	87	113
54	33	50	74	58	86	113
60	33	50	74	58	86	113
66	33	50	74	58	86	113
72	32	50	74	58	86	113
78	32	48	74	57	86	113
84	32	48	74	57	86	113
90	32	48	74	57	86	113
96	31	48	74	57	85	113
102	30	48	74	57	85	113
108	30	48	74	57	85	113
114	29	48	73	57	85	113
120	28	48	73	57	85	113
126	26	48	73	57	85	113
132	26	48	72	56	84	113
138	26	48	72	55	84	113
144	26	47	72	55	84	113

Method A (Sheeted Trench)
Method B (Open Excavation)
Methods C-1&C-2 (Embankment)

MAXIMUM HEIGHT OF FILL (H) IN FEET OVER TOP OF PIPE								
PIPE DIAMETER D (inches)	ORDINARY BEDDING				CONCRETE CRADLE BEDDING			
	14N CLASS II	14R CLASS III	14 CLASS IV	14X CLASS V	14N CLASS II	14R CLASS III	14 CLASS IV	14X CLASS V
12	9	13	19	28	18	24	35	54
15	9	13	19	28	18	24	35	54
18	9	13	19	28	18	24	35	54
21	9	13	19	28	18	24	35	54
24	9	13	19	28	18	24	35	54
27	10	13	19	28	18	24	35	54
30	10	13	19	28	18	24	35	54
33	10	13	19	28	18	24	35	54
36	10	13	19	28	18	24	35	54
42	10	13	19	28	18	24	35	54
48	10	13	19	28	18	24	35	54
54	10	13	19	28	18	24	35	54
60	10	13	19	28	18	24	35	54
66	11	13	19	28	18	24	35	54
72	11	14	19	28	18	24	35	54
78	11	14	19	28	18	24	35	54
84	11	14	19	28	18	24	35	54
90	11	15	19	28	18	24	35	54
96	11	15	19	28	17	24	35	54
102	11	15	20	28	17	24	35	54
108	11	15	20	28	17	24	35	54
114		15	21	28		24	35	54
120		15	21	28		24	35	54
126		15	21	28		24	35	54
132		16	21	28		24	35	54
138		16	21	28		24	35	54
144		16	22	29		24	35	54

Notes:

1. H-20 live loading effects are accounted for in the above tables with a minimum 2 ft. cover.

2. All design selections are field constructed in accordance with the corresponding Installation Methods and Bedding Details on the "Standard Drawing" except where modified by special requirements.

reduced where initially calculated allowable fill heights greatly exceeded 100 ft to ensure that these installations are given special consideration.

Although it is acknowledged that many excellent manuals (1) and guidelines exist for the application of the Marston-Spangler formulas, few if any are intended to be directly related to the actual construction practice of a given agency.

The key aspects involved in coordinating construction practice and design include standards for bedding, backfill materials, compaction requirements, minimum temporary cover and final cover, and construction procedures. Again, using New York State as an example, the coordination of construction and design is accomplished on a standard drawing that automatically becomes part of the plans and specifications whenever rigid pipe is used. Bedding details are shown that include additional procedures to be used by the field engineer in preparing the foundation. For example, rock foundations are required to be undercut below invert to as much as 75 percent of the pipe diameter, and temporarily unstable wet soil is replaced during the bedding process. Installation details are also shown for the trench and imperfect trench methods and two conditions of embankment installation: one for placing the pipe before filling and the other for building the fill to partial height and excavating a trench for pipe placement. All pipes after bedding, whether in cut or fill, are backfilled with a specified type of granular material placed to minimum established limits and compacted to between 95 and 100 percent of the maximum density as determined by AASHO T-99 Method C.

Durability design for rigid pipes must not be completely overlooked although it is not so significant a consideration as for flexible pipes. A brief but informative guideline on this subject for concrete pipe is given elsewhere (2).

Flexible Pipe

The most widely used design methods in current use are presented elsewhere (3, 4, 5). Each presentation is essentially the same and requires that the culvert be designed to meet seam strength, buckling, deflection, and flexibility criteria for the materials and corrugation configuration being used. Height-of-fill-gauge tables are given in some of the previously mentioned references, and similar ones have been prepared by many design agencies, which also incorporate their experience.

The most frequently discussed problem concerning the use of the design method is the inability to satisfactorily analyze the deflection aspect. This phase of the analysis relies largely on the determination or assumption of the modulus of soil reaction, E′, for use in the Iowa formula developed by Spangler. Various approaches for determining a reasonable value of E′ are summarized elsewhere (2). NCHRP Report 116 (2) should be referred to when a deflection estimate is of specific interest.

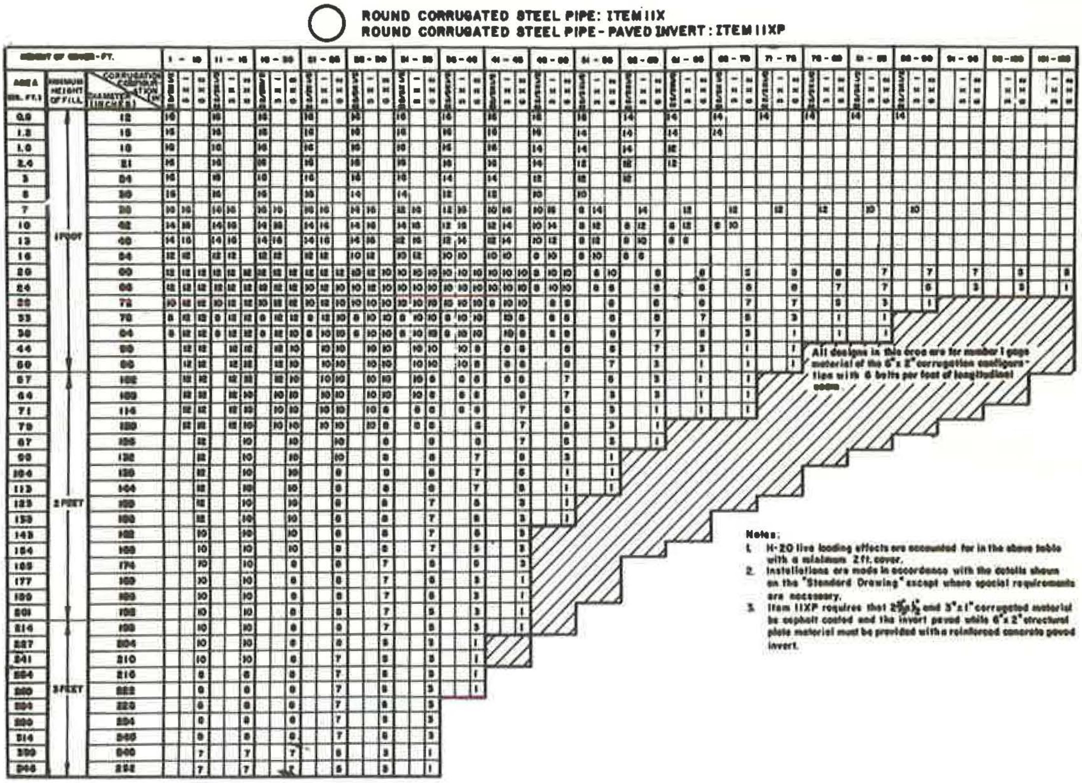
Extensive experience in New York highway work has shown that deflection of flexible pipes has been insignificant, i.e., generally much less than the usual arbitrary 5 percent criterion. This very favorable deflection performance is related to the compressibility of the surrounding backfill and therefore is attributable to the installation methods used. This has led to a recent decision to use round pipe exclusively for routine designs (except of course when pipe arches or other shapes are desired) and to eliminate elongation and strutting as a design consideration and construction requirement. This modified design approach will not result in any practical sacrifice to the value that E′ might attain if elongated pipe were to be used in place of round pipe. More economical designs should be realized although any resultant savings may not be reflected directly in a contractor's bid prices for this work.

The design for durability is not so easily accounted for as was deflection. Use of a New York State report (6) has been recommended for corrosion analysis. Durability design in New York, however, now includes considerations for abrasion, flow conditions, and service category as well as corrosion. In addition, the New York State corrosion study has been continuing, and based on more data, refinements, and new findings, the original report (6) is now used only as a reference for the corrosion part of durability design.

Attempts have been made to evaluate the corrosion aspect for a particular site by a series of field tests. Unfortunately, no completely satisfactory field test program has yet been developed that can be correlated to all of the factors that affect durability. New York State is currently initiating an interim qualitative approach to durability that relies on experience and past performance until more refined methods are developed.

Durability design for corrugated steel pipe (using the New York State method) provides for the use of plain galvanized material where feasible. It also requires, however, the coating and paving of inverts where indicated by using a rating system test applied to the proposed installation. Height-of-fill-gauge tables have been prepared for all corrugations of plain galvanized material by using the aforementioned design methods, but the tabulated gauges also contain an additional built-in consideration for durability. More specifically, each gauge that is listed must provide a minimum safety factor of 1 for seam strength and buckling at the end of a 40-year design life, assuming a uniform rate of metal loss over this period. The gauge table for round steel pipe is shown in Figure 2. Although the basis for durability assumptions is beyond the scope of this presentation, it may be of interest to know that rates of metal loss used were 1 mil per year for diameters up through 48 in. and 2 mils per year for larger sizes. These rates correspond to a statistical confidence level of about 85 percent. The remaining aspect of durability design for steel pipe requires that a determination be made for the need to completely asphalt coat and pave the invert of $2^2/_3$- by $1/_2$-in. and 3- by 1-in. corrugated material or to install a reinforced concrete paved invert in structural plate (6- by 2-in.) pipes. It should be noted that experience in New York State has shown that asphalt coating alone is not significantly beneficial, and its single use is not recommeded. When a pipe with an asphalt coating and a paved invert or a reinforced concrete paved invert for plate is placed in an aggressive environment, the statistical rate of metal loss is equal to or lower than the rate for plain galvanized material in a nonaggressive environment.

Figure 2. Fill-height design table for round corrugated steel pipe.



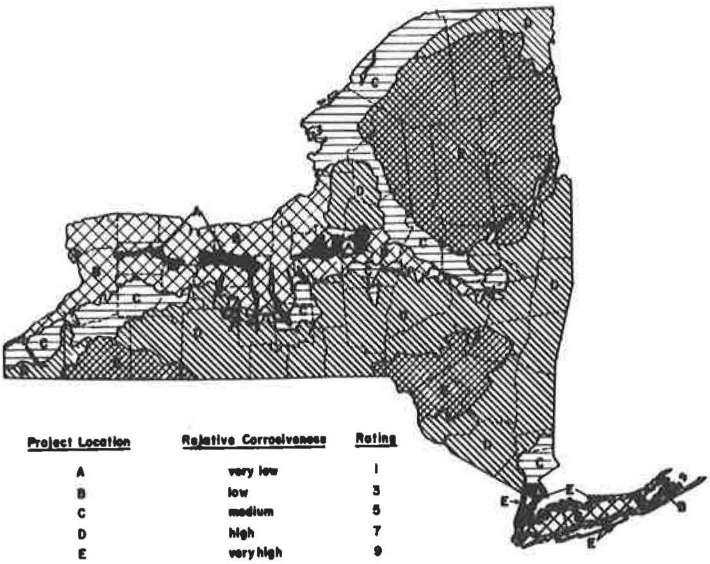
A rating system (durability index) was devised to determine the need to treat steel culverts. The system is intended to supplement experience where it exists and provide a rational substitute where none is available. The durability index represents the sum of the individual numerical ratings assigned to each of the following categories.

1. Surface water corrosiveness rating: The location of the project site on a reference map determines the relative corrosiveness rating, i.e., 1, 3, 5, 7, and 9. The numerical ratings, which are assigned to geographic areas of the state (Fig. 3), are in general accord with available field data and the relative influence of contributing corrosive factors in these areas such as soil types, water hardness, and pH. The designer is cautioned at this stage, however, that abnormally high corrosion rates related to unique local surface water conditions, such as heavy agricultural and industrial water or mine tailings, may necessitate the use of a different pipe material.

2. Abrasiveness rating: This rating category, which receives about half the emphasis of the corrosion category, describes potentially abrasive and nonabrasive bed loads and relates them to the gradient for the numerical rating. An abrasive or nonabrasive bed load with a gradient 2 percent or less has a rating of 1. An abrasive bed load with a gradient between 2 and 4 percent has a rating of 2, whereas the same bed load with a steeper gradient has a rating of 5.

3. Flow rating: This category attempts to recognize the effect of frequency of flow for various applications of pipe usage and is based largely on judgment. Ratings for three conditions of flow and usage are as follows: (a) "highly intermittent" with a rating of 1 applies generally to storm and side drains that are not subject to long-standing water; (b) "intermittent" has a rating of 2 and generally applies to cross culverts carrying streams or small flows that seasonally go dry; and (c) "continuous" with a 3 rating usually applies to cross culverts with stream flows or any long-standing water condition.

Figure 3. Guide map for corrosiveness rating.



4. Service rating: The relative importance of installations is given some recognition in this rating section, again based on judgment. Side drains and driveway pipes are assigned a rating of 1. Cross culverts have a 2 rating.

The durability index is obtained by simply adding the preceding ratings that apply to the site. A durability index of less than 14 indicates that plain galvanized material can be used, whereas a higher rating indicates the need for coatings and invert paving. Extensive experience in a project area, as stated earlier, would possibly modify the decision indicated by the durability index.

The durability design aspects for aluminum material are different from those for steel. The New York height-of-fill-gauge table covers 2⅔- by ½-in. and 9- by 2½-in. corrugation material and has no built-in allowance for metal loss. Currently, asphalt coating and paved invert treatment are only to be considered for the extremes of pH commonly mentioned (6) or where a high abrasion potential exists. This interim approach, which is based on much more limited data, does not necessarily imply that we are not concerned with the durability of aluminum. This is especially true because a recent finding at a few sites in New York has revealed soil-side corrosion at the crown. This phenomenon is thus far unexplained.

A problem that frequently confronts designers in considering the use of steel pipe is the type of corrugation configuration to specify. There is an appreciable overlap of sizes in the different corrugations, and the desirability of one over the other is usually not apparent. This problem is overcome by using the gauge table arrangement shown in Figure 2 and by allowing the contractor to supply any of the structural equivalents for a given size where alternates exist. Economies could be expected with this procedure. Supplying alternates by this method would not be so simple, however, where hydraulic analysis for a given installation reveals the need for different pipe sizes for each corrugation.

Installation requirements and procedures for flexible pipe are also presented on a standard drawing that is automatically made part of the plans. This drawing shows only ordinary bedding including treatment of an unstable soil foundation and rock undercut as well as a trench (sheeted or open cut) and two embankment methods of installation similar to those for rigid pipe. Backfill and compaction criteria are also similar to rigid pipe standards.

Nonroutine Designs

Large pipe culverts of commonly used cross sections under very shallow fills, although designed routinely from allowable fill-height tables, should not be treated as routine installations. Their satisfactory performance under heights of cover between 2 and about 6 ft is credited in large measure to the installation procedures used rather than the design methods, which are even less applicable to this condition. Every known failure experienced on New York highway projects has been attributable to heavy construction loads on pipes with inadequate cover and other substandard installation practices. For this reason, structures in this design situation should be installed in a very closely controlled manner with special provisions made to protect the pipe from construction loads. Warning the contractor is not necessarily sufficient, and consideration should therefore be given to providing temporary ramping details, detours, or other possible protective measures on the plans.

Special shapes, heavier live loads, or large sizes not covered on the fill-height tables of course require individual attention. The most representative of the alternate design procedures available for this situation should be used either for the design or as a check of the design by the more routine methods. Another publication (2) discusses alternate design methods and provides a basis with references for approaching this problem.

Pipes of all sizes and cross sections under very high fills not provided for on standard design tables are occasionally required. Rigid pipes of standard wall-strength design for this case are seldom used because of questions concerning their ability to resist high induced wall stresses and their compatibility with other devices such as the imperfect trench and concrete cradle under very high fills. Davis, Bacher, and Obermuller (7), for example, noted serious rupture of a concrete pipe on a 60-deg cradle at a stage where adjacent sections installed under the same fill height, but without a cradle, were not nearly so distressed. In addition, concern over possible detrimental long-term redistribution of loading effects with the imperfect trench method suggests limitations on its use. Consequently, these cases require a thoroughly analyzed approach, including a possible special design of the pipe itself as well as special installation details. The requirements of a special design for rigid pipe, however, are often too complex and unfamiliar to the average designer, and an alternate such as a box culvert (which is usually overdesigned) or a flexible pipe installation is often selected.

A number of large flexible pipes under very high fills (up to about 140 ft) have been used without incident in New York State. These have been generally designed or reviewed on a project-by-project basis by a central unit that follows the same procedures for structural design as were discussed previously with added emphasis given to soil mechanics considerations.

Special designs should employ engineering judgment in evaluating the anticipated performance of the structure. For example, the construction of a 22-ft span semicircular structural plate arch with abutments founded on rock beneath about 60 ft of fill was recently analyzed. The design analysis provided marginally satisfactory results, but there was concern as to whether a semicircular structure restrained at the abutments could deflect laterally and mobilize adequate passive soil resistance. This was resolved by using either of two alternates, a horseshoe-shaped arch or a round pipe on a prepared bed.

Adverse foundation conditions that present stability and settlement problems for a proposed culvert must of course be resolved prior to installation if problems affecting cost and performance as well as failures are to be avoided. The evaluation and resolution of these problems are based on an analysis of subsurface conditions, which should be part of any important structure design. Such problems are varied and range from removal and backfill of unstable soils to settlement analysis predictions for otherwise stable foundations. Settlement analysis provides the basis for camber recommendations and reveals the possible need for special joints for rigid types where definite tendencies exist for the fill to spread. Standard-type joints used with rigid pipes on steep gradients beneath high fills are also subject to opening, and this possibility should be reviewed. All of the foundation problems that may be encountered cannot be summarized here but can be identified if reviewed by a soils engineer.

Existing pipe culverts are frequently crossed by new highway embankments, which presents a most perplexing problem to the designer. Most of these situations occur with existing reinforced concrete sewer lines, and frequently insufficient data are obtainable on the pipe strength class, bedding type, installation method, and backfill materials, especially if the installation is not recent. This type of analysis, therefore, most often involves many assumptions—beginning with a determination of how much cover could be supported with the known or conservatively assumed pipe strength with a positive projecting (or wide trench) condition and comparing this to the new height of cover. The analysis can go anywhere from this point, but some of the directions it should take are as follows.

1. A determination of the condition of the existing pipe is necessary to see if it is worth saving or to provide a basis for adjusting the assumed or known strength of the pipe. An internal inspection by trained personnel or a television survey may be sufficient if the pipe cannot be occupied.
2. If the approximate analysis indicates that the pipe probably cannot support the new load, some of the possible alternatives, short of replacing the line, include the use of lightweight fill such as expanded shale that has a density of about 70 pcf or cinders at 80 pcf, the imperfect trench used successfully by Spangler for this condition (8), a combination of one or more of the preceding plus lowering the gradeline if feasible, and a protective structural relieving platform (which is usually not economical).

If these or other methods do not provide the desired degree of confidence for an existing pipe in good condition and if the cost of replacement is significant, it may be worthwhile to use load-reducing methods and proceed as planned but monitor the immediate and extended performance of the pipe and assume responsibility for repairs or replacement as necessary. A failure means different things to different people; but in the case of a structurally sound concrete pipe handled in this way, the worst that could be expected to occur where the computed overstress is not unreasonable would be some vertical deformation, crack patterns, and spalling of concrete with possibly some steel exposed. Therefore, repairs would be only a fraction of the total replacement cost. Caution should be exercised before adopting this approach because other factors such as settlement and a reduced diameter, if repairs become necessary, affect the hydraulic capacity. In addition, a period of responsibility must be assumed. This approach, however, has been used successfully on at least one New York State project with large savings and possibly on others.

RESEARCH OBJECTIVES

Current Methods

An attempt has been made here to illustrate the utilization and workability of accepted design methods, particularly when coordinated with construction practice. Deficiencies associated with these design methods were also discussed, and improvements in this area would be of value to the practicing designer. The most notable of these are believed to be the following.

1. A substitute for the 3-edge bearing test that relates more to field conditions would be very desirable and is long overdue.
2. A review and evaluation of current and possibly new bedding methods should be undertaken for rigid pipe. This should include an appraisal of any limitations of concrete cradle bedding as well as consideration of possible benefits from the use of yielding types. The results of a study on this topic should include use criteria that, if of value, can be economically and routinely incorporated into design.
3. A similar review and evaluation of the imperfect trench method is believed to be important because it is so widely used. Its apparent success has been established, but a more uniform methodology is needed to define the best materials to be used, their locations and limits with respect to the pipe, and their possible limitations. Even though this method is usually successful, a few installations in New York exhibited cracking patterns that, although not considered serious, were generally objectionable.

4. Backpacking studies have shown this to be a promising device for both reducing stress and making its distribution more uniform around a structure. A methodology is required for design, materials, and construction practices; it should be incorporated into current design methods, or a new one should be used.

5. It appears that available research results provide an adequate basis for correlating deflection design and field performance, either by recognizing that the E′ values existing in the field are much higher than those suggested for use in design manuals (4, 5) or by using other design methods available. An opinion is offered, therefore, that too much attention and concern are frequently given to the subject of deflection for properly installed flexible pipes.

6. A need exists to improve load analysis procedures for large pipes under very shallow fills where live loads are the principal forces to be resisted. As stated previously, current design methods do not adequately account for this situation, and now that structures are becoming ever larger, these design deficiencies are even more critical.

7. The design of large flexible structures under very high fills (i.e., greater than about 100 ft) is generally accomplished without problems although refinements that would give the vertical load more accurately are desirable.

Rigid pipes under very high fills are not so easily designed because the nonuniform peripheral pressures developed cause large induced wall bending moments that must be resisted. Although backpacking and possibly different bedding methods might assist in overcoming this problem, it would appear desirable to develop a "flexible" reinforced concrete pipe. Prestressed concrete pipe might also offer a greater range of large sizes as well as design advantages under very high fills, and its wider use is suggested where economical.

New Methods

The need for new or improved methods of design has not been generated by a rash of failures. The failures that have occurred are generally attributable to poor installation practices. Advances are desired and necessary, therefore, to obtain better methods that include soil-structure interaction concepts. This objective is well stated elsewhere (2): "The ultimate objective of any culvert design procedure is to predict stresses and deformations at any part of the system at any point in time for a given fill height...." The remainder of this statement details the desired aspects of the theory in a very thorough manner. An added recommendation, however, is that this new method, to whatever extent achieved, be adaptable or reducible to relatively simple terms, graphs, tables, or computer solutions so that standard designs can be established for routine installation situations. The widespread acceptance and adoption of any such new method would thereby be ensured.

New design methods must also place greater emphasis on durability. New York State is continuing its durability studies toward the objective that the design methods described here will become more quantitative, based on the development of a reliable field test program.

ACKNOWLEDGMENTS

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DESIGN OF CIRCULAR SOIL-CULVERT SYSTEMS

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In predicting the performance of a soil-conduit system, soil properties must be included in the design procedure. It is suggested that the relation between the constrained modulus of elasticity and the dry density of a soil and the solution from theory of elasticity can be used to develop the desired design procedure for circular flexible pipe. The design chart is developed by using the solutions derived from the theory of elasticity and from the relation between constrained modulus of elasticity and dry density. The upper boundary of the design chart was approximated from the full-scale experimental results on corrugation pipe as reported by Watkins and Moser (9). The ring deflection of 5 percent was used as a limiting deformation of pipe. It is shown that the actual experimental results of Watkins and Moser (9) fall in the range of values of a dimensionless parameter. Two examples are presented to illustrate the use of the design chart.

• THE development of a design procedure that includes soil properties is a problem of considerable importance in the design of soil-conduit systems. During the past 60 years, experience tables and rational procedures for the structural design of culverts have been developed. For circular pipes, two major rational procedures are commonly used: One is concerned with limiting deformation, and the other is concerned with limiting the compressive load in the conduit wall. The proposed design procedure considers both deflection and loads.

Watkins and Moser (9), using full-scale corrugated steel pipes, have made experimental studies in which they evaluated the effect of soil properties on the soil-conduit system.

A solution based on the theory of elasticity has been developed by Burns and Richard (7) for predicting the soil-structure phenomenon.

The object of this paper is to develop a design procedure by using (a) the solutions from the theory of elasticity to predict the performance of the soil-conduit system and (b) the experimental results of Watkins and Moser (9) to develop upper limits on the design. Soil properties must be included in the design procedure for a flexible soil-conduit system.

LITERATURE REVIEW

The design and the installation of a flexible pipe are based on empirical and experimental data. Marston (1), Spangler (2), and White and Layer (6) have attempted to develop theories for the soil-culvert system.

Marston (1) presented his load theory in 1913. The choice of an abstract parameter, settlement ratio, is important because it includes the effects of the deformation of the fill, culvert, and natural ground under the culvert.

The pressure distribution, as shown in Figure 1, was used by Spangler in 1942 to derive the Iowa formula, by which the horizontal deflection of a flexible conduit can be computed. The pressure at the horizontal diameter is given by

Sponsored by Committee on Subsurface Soil-Structure Interaction.

$$P_h = E'(\Delta X/D) \quad (1)$$

where

P_h = maximum unit pressure at the extreme horizontal diameter,
E' = modulus of soil reaction of the side fill material,
ΔX = horizontal deflection of pipe, and
D = normal diameter of the pipe.

The Iowa formula is

$$\Delta X = D_1 [KW_c R^3/(EI + 0.061\ E'R^3)] \quad (2)$$

in which

ΔX = horizontal deflection of the conduit in inches;
D_1 = deflection lag factor—suggested values normally range from 1.25 to 1.50 for design purposes;
K = bedding constant whose values depend on the bedding angle (0.083 for 90-deg bedding);
W_c = vertical load per unit length of the pipe in pounds per linear inch;
R = mean radius of the pipe in inches;
E = Young's modulus of elasticity of pipe in pounds per square inch; and
I = moment of inertia of pipe wall.

ELASTICITY SOLUTION

The plane strain solution for an elastic cylindrical shell in an infinite linearly elastic medium subjected to a uniformly distributed overburden pressure was presented by Burns and Richard (7). Equations are presented (Fig. 2 shows the notation) for the following two cases:

1. In the bonded shell case, the shear stress exists at the interface between the shell and the medium. For the bonded shell, the interaction pressures are

$$P = P_o[B(1 - a_o^+) - C(1 - 3a_2^+ - 4b_2^+) \cos 2\theta] \quad (3)$$

The shell displacement is

$$U = \tfrac{1}{2}(P_oR/M_s) \{[1 + (B/C)a_o^+] - [1 + a_2^+ + (2/B)b_2^+] \cos 2\theta\} \quad (4)$$

The shell thrust and moment are

$$N_\theta = P_oR[B(1 - a_o^+) + C(1 + a_2^+) \cos 2\theta] \quad (5)$$

and

$$M_\theta = P_oR^2 [(CUF/6VF)(1 - a_o^+) + \tfrac{1}{2}C(1 - a_2^+ - 2b_2^+) \cos 2\theta] \quad (6)$$

2. In the unbonded shell case, shear stress is zero at the interface between shell and medium. For the unbonded shell, the interaction pressures are

$$P_R = P_o[B(1 - \bar{a}_o) - C(1 + 3\bar{a}_2 - 4\bar{b}_2) \cos 2\theta] \quad (7)$$

The shell displacement is

$$U = \tfrac{1}{2}(P_oR/M_s) \{1 + [(B/C)\bar{a}_o] - [1 - \bar{a}_2 + (2/B)\bar{b}_2] \cos 2\theta\} \quad (8)$$

and the shell thrust and moment are

$$N_\theta = P_oR[B(1 - \bar{a}_o) + \tfrac{1}{3}C(1 + 3\bar{a}_2 - 4\bar{b}_2) \cos 2\theta] \quad (9)$$

and

$$M_\theta = P_o R^2 [(CUF/6VF)(1 - \bar{a}_o) + \tfrac{1}{3}C(1 + 3\bar{a}_2 - 4\bar{b}_2) \cos 2\theta] \quad (10)$$

where the soil parameters are

M_s = constrained modulus of elasticity, FL^{-2}; i.e., $M_s = [E^*(1 - \mu^*)/(1 + \mu^*)(1 - 2\mu^*)]$;
K^* = lateral pressure coefficient (-); i.e., $K^* = \mu^*/(1 - \mu^*)$;
E^* = Young's modulus of elasticity, FL^{-2}; and
μ^* = Poisson's ratio (-).

The shell parameters (culvert) are as follows:

R = mean radius, L;
D = diameter, L;
E_1 = plane strain Young's modulus, FL^{-2};
A = area of shell wall per unit longitudinal length, L; and
I = moment of inertia of shell wall per unit longitudinal length, L^3.

The load parameters are as follows:

B = $\tfrac{1}{2}(1 + K^*)$, uniform pressure component;
C = $\tfrac{1}{2}(1 - K^*)$, variational pressure component;
UF = $M_s DB/E_1 A$, relative extensional flexibility; and
VF = $M_s D^3 2C/48E_1 I$, relative bending flexibility.

The dimensionless constants for the bonded shell are as follows:

$a_o^+ = (UF - 1)/[UF + (B/C)]$,
$a_o^+ = [C(1 - UF)VF - (C/B)UF + 2B]/\{(1 + B)VF + C[VF + (1/B)]UF + 2(1 + C)\}$,
and
$b_o^+ = [(B + CUF)VF - 2B]/\{(1 + B)VF + C[VF + (1/B)]UF + 2(1 + C)\}$.

The dimensionless constants for the unbonded shells are as follows:

$\bar{a}_o = (UF - 1)/[UF + (B/C)]$,
$\bar{a}_2 = [(2VF - 1) + (1/B)]/[(2VF - 1) + (3/B)]$, and
$\bar{b}_2 = (2VF - 1)/[(2VF - 1) + (3/B)]$.

Watkins and Moser (9), using corrugated steel pipes, performed interrelationship studies between the properties of the pipe and the soil. The studies evaluated the significant factors affecting the soil-culvert system. A test cell was constructed to test a 5-ft diameter pipe. Hydraulic cylinders were used for loading. The results from the studies indicated that the soil compression and pipe wall-crushing strength were the important factors. Soil density was the main factor in predicting the soil modulus and also proved to be the most important factor in predicting the performance of buried corrugated steel pipes.

Constrained Modulus of Elasticity

Nielson (8) established the relation between the modulus of soil reaction and constrained modulus of elasticity $[E' = f(M_s)]$. In establishing this approximate relation, he used the bonded shell equations of Burns and Richard's solution (7). Nielson concluded that modulus of soil reaction can be approximated by

$$E' = 1.5\, M_s \quad (11)$$

The relations for the unbonded equations of Burns and Richard's solution (7) are as follows.

Using Eqs. 7 and 8 at $\theta = 0$ deg

$$P_h = P_o[B(1 - \bar{a}_o) - C(1 + 3\bar{a}_2 - 4\bar{b}_2)] \quad (12)$$

and

$$\Delta X/2 = (P_oR/2M_s)\ \{[1 + (B/C)\bar{a}_o] - [(1 - \bar{a}_2) + (2/B)\bar{b}_2]\} \tag{13}$$

Substituting Eqs. 12 and 13 into Eq. 1 gives

$$E' = 2M_s[B(1 - \bar{a}_o) - C(1 + 3\bar{a}_2 - 4\bar{b}_2)]/\{[1 + (B/C)\bar{a}_o] - [1 - a_2 + (2/B)\bar{b}_2]\} \tag{14}$$

We used a computer to evaluate Eq. 14 for modulus of soil reaction. It can be concluded that the modulus of soil reaction for the full slippage case can be approximated by

$$E' = 0.70\ M_s \tag{15}$$

It was shown by Stankowski and Nielson (10), who used a model, that the actual modulus of soil reaction of a soil-culvert system is between the predicted modulus of soil reaction by bonded shell and the unbonded shell equations of Burns and Richard's (7) solution. Therefore, the relation between the modulus of soil reaction and the constrained modulus of elasticity is assumed as

$$E' = 0.8\ M_s \tag{16}$$

Constrained Modulus Versus Dry Density

Hsieh (11) has determined the relation between the modulus of soil reaction and dry density. He determined the modulus of soil reaction by using the Modpares device (4). The experimental data of Hsieh (11) are given in Table 1 for soils at optimum moisture content.

The modulus of soil reaction is related to the constrained modulus of elasticity by Eq. 16.

Watkins and Moser (7) reported their results in graphic form, relating percentage of ring deflection to vertical soil pressure for a particular gauge, diameter, average density, and percentage of standard density (using AASHO T 99 compaction test).

To obtain M_s values for the experimental results of Watkins and Moser (9), we developed the relation of the constrained modulus and the ratio of dry density to the dry density as determined by using the AASHO T 99 compaction test $[M_s = f(\gamma d/\gamma 99)]$. We used the values given in Table 1. The constrained modulus was obtained by applying Eq. 16 to the data given in Table 1 and is shown in Figure 3.

Figure 3 also shows the correlation used to obtain a constrained modulus from Watkins and Moser's soil density information. The relation of constrained modulus of elasticity and ratio of dry density to the dry density as obtained by using the AASHO T 99 compaction test may be significantly in error as shown by the scatter of data. Changes in soil moisture content are not considered in this correlation. Other means of determining M_s, such as the confined compression test, are superior to this method.

Similitude Analysis

The deflection of a flexible conduit in a fill that is long in relation to the diameter of the pipe can be considered to be a function of the following primary quantities if the pipe wall stress is below the yield point.

Primary Quantity	Basic Dimension
ΔX = increase in horizontal diameter	L (length)
D = original conduit diameter	L
EI = conduit wall stiffness per unit length of conduit	FL (force × length)
P = vertical pressure on conduit	FL^{-2}
M_s = constrained modulus of elasticity	FL^{-2}

There are five primary quantities and two dimensions; therefore, according to Buckingham's pi-theorem, the number of pi-terms should be 5 - 2 = 3.

These three pi-terms may be formulated as follows:

1. $\pi_1 = \Delta X/D$, deflection term;
2. $\pi_2 = PD^3/EI$, load term; and
3. $\pi_3 = M_s D^3/EI$, similar to soil term.

The pi-terms can be arranged into a functional equation form as follows:

$$\pi_1 = f(\pi_2, \pi_3)$$

Stankowski and Nielson (10) showed, by experimental analytic studies of underground structural cylinders, that the horizontal diameter change of the conduit can be approximated by taking the average value of the bonded and unbonded equations, Eqs. 4 and 8 respectively, as predicted by Burns and Richard's solution (7).

The load-deflection diagram (Fig. 4) is obtained by plotting the load term versus the ratio of average value of ΔX, Eqs. 4 and 8, divided by the original diameter. The dimensionless soil parameter is also plotted on the load-deflection diagram. The value of Poisson's ratio of 0.25 is used for soil (5), and a Poisson's ratio of 0.33 is used for the conduit material.

The experimental results of Watkins and Moser (9) at failure (Tables 2 and 3) show varied failure condition. The failure criterion is considered as a percentage of deflection where one of the following conditions was first noticed: buckling of the pipe, wall-crushing, or inverting of bottom of the pipe. The test data at failure are plotted on the load-deflection diagram.

It can be seen from Figure 4 and Tables 2 and 3 that Watkins and Moser's experimental data, at a failure condition, are generally within range of the $M_s D^3/EI$ value calculated from the theory of elasticity (once allowances are made for the error involved in the determination of M_s). The buckling data show some scattering. The incipient failure line shown in Figure 4 is an approximate boundary that delineates the failure zone of the conduit. The seam failure condition of the conduit in Test 19 falls in the "safe" zone.

The design chart (Fig. 5) is developed from the load-deflection diagram by limiting the deformation of the conduit to 5 percent of the original diameter. The incipient failure line as shown in Figure 4 is used as the upper limit of the design chart. Figure 5 can be used for designing the soil-conduit system. The upper boundary of the design chart may have to be adjusted as more failure information becomes available.

DESIGN PROCEDURE

The general form of a design procedure is as follows:

1. Select the diameter of the conduit according to the drainage requirement.
2. Determine the constrained modulus of elasticity by using an appropriate soil test. If no other alternative is available, find the appropriate ratio of dry density of soil to the dry density obtained by using AASHO T 99 standard compaction test and apply it to Figure 3.
3. Select trial gauge of the pipe.
4. Determine the load parameter PD^3/EI and the soil parameter $M_s D^3/EI$. Values for D^3/EI are given in Tables 4 and 5 for different corrugations and gauges.
5. $M_s D^3/EI$ and PD^3/EI values should be in the range of the design chart (Fig. 5). If these values are out of range, then change section or gauge until it falls within the range of the chart.
6. Enter in the design chart the PD^3/EI and $M_s D^3/EI$ values, and find the percentage of ring deflection.
7. Check the ring deflection with allowable ring deflection (5 percent).
8. If the ring deflection is less, select the appropriate gauge or repeat with a lighter gauge if feasible.
9. If the ring deflection is greater, revise the pipe gauge and repeat steps 3, 4, 5, 6, 7, 8, and 9.

Figure 1. Pressure pattern around flexible pipe.

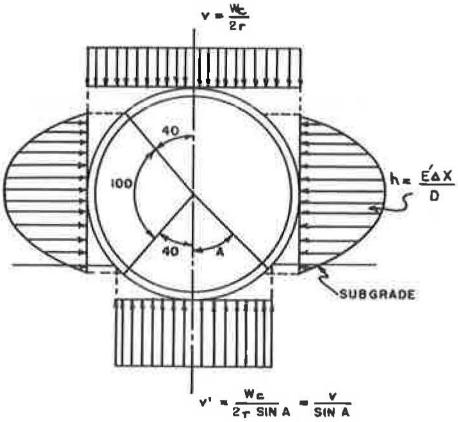

Figure 2. Solution notation for buried pipe.

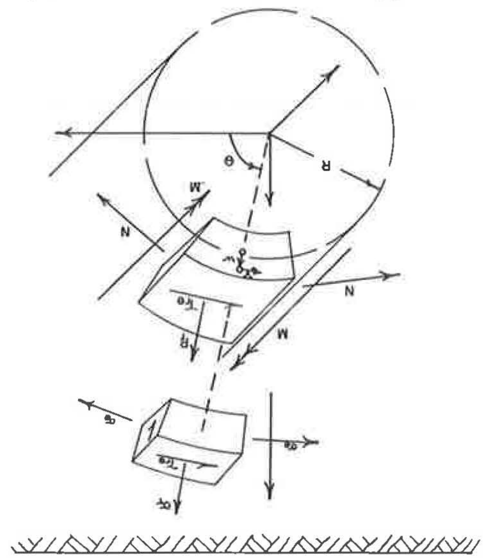

Table 1. Properties of soil samples.

Gradation	γ_{180} (lb/ft^3)	γ_{99} (lb/ft^3)	γ_d (lb/ft^3)	E'_s (lb/in.^{2})
Sand	118.2	112.3	112.30[a]	2,105.4
			115.25[b]	14,041.5
			95.50[c]	1,474.3
Mix 1	134.5	132.1	132.10[a]	14,287.4
			133.30[b]	15,418.9
			112.30[c]	1,711.1
Mix 2	131.2	127.6	127.60[a]	12,313.1
			129.40[b]	14,389.3
			108.40[c]	1,866.3
Mix 3	132.5	128.0	128.00[a]	16,953.6
			130.25[b]	24,157.4
			108.00[b]	1,883.1
Mix 4	126.7	117.5	117.50[a]	1,827.5
			122.10[b]	10,993.5
			98.10[b]	815.4
Mix 5	133.0	123.9	123.90[a]	3,766.0
			128.45[b]	9,734.5
			105.00[c]	1,212.1
Mix 6	130.5	121.8	121.80[a]	3,070.6
			126.15[b]	5,273.1
			103.30[c]	984.1
Mix 7	127.4	122.4	122.40[a]	2,475.9
			124.90[b]	13,215.6
			104.00[c]	1,179.0
Mix 8	126.9	120.2	120.20[a]	15,015.6
			123.55[b]	28,678.1
			102.20[c]	1,160.2

[a] γ_{99}. [b] $(\gamma_{180} + \gamma_{99})/2$. [c] $0.85\ \gamma_{99}$.

Figure 3. Correlation of M_s and percentage of standard compaction.

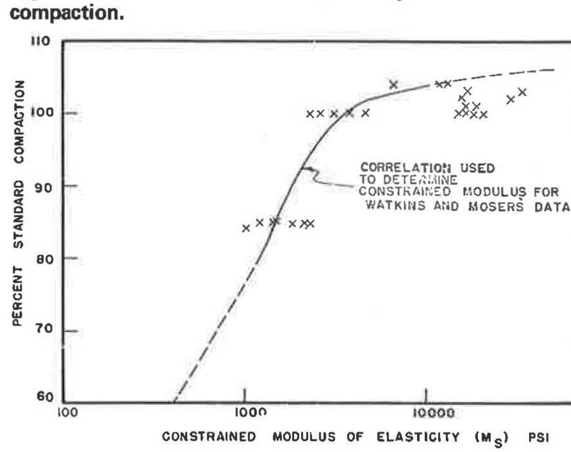

Figure 4. Data plot on dimensionless chart.

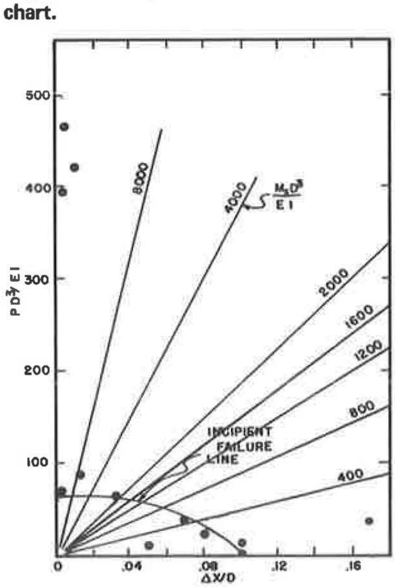

Table 2. Experimental results obtained by Watkins and Moser.

Test Number	Diameter (in.)	Vertical Pressure (lb/ft^2)	Ring Deflection (percent)	Dry Density (lb/ft^3)	Standard Density (percent)
6	48.0	8,400.0	8.0	84.6	73.0
12	60.0	8,600.0	3.2	94.0	77.0
17	60.0	6,000.0	17.2	84.6	69.0
18	48.0	14,000.0	7.0	91.2	75.0
19	36.0	13,600.0	5.0	97.3	80.0
21	36.0	4,200.0	10.0	74.9	61.0
52	60.0	12,800.0	1.20	101.7	83.0
61	36.0	14,000.0	1.40	120.1	93.1
62	60.0	14,000.0	0.60	129.0	106.0
63	48.0	4,200.0	0.20	119.7	98.0
68	60.0	12,000.0	0.40	118.4	97.0

Table 3. Data obtained by Watkins and Moser.

Test Number	Diameter of Gauge (in.)	Corrugation (in.)	Type of Failure	PD^3/EI	M_sD^3/EI
6	16	3 by 1	Buckling	25.7	361
12	18	3 by 1	Wall-crushing	64.6	1,135
17	16	3 by 1	Buckling	35.8	585
18	16	3 by 1	Inversion in top	42.8	405
19	16	3 by 1	Buckling, seam failure	17.5	2,530
21	18	3 by 1	Sides buckling	6.81	118
22	16	3 by 1	Sides buckling	3.61	98
52	18	$2\frac{2}{3}$ by $\frac{1}{2}$	Sharp inversion, wall-crushing	414.0	6,760
61	16	$2\frac{2}{3}$ by $\frac{1}{2}$	Inversion rippling	78.2	1,820
62	18	$2\frac{2}{3}$ by $\frac{1}{2}$	Wall-crushing	453.0	201,285
63	18	$2\frac{2}{3}$ by $\frac{1}{2}$	Bottom inverting	69.5	7,370
68	18	$2\frac{2}{3}$ by $\frac{1}{2}$	Bottom inversion	388.0	13,420

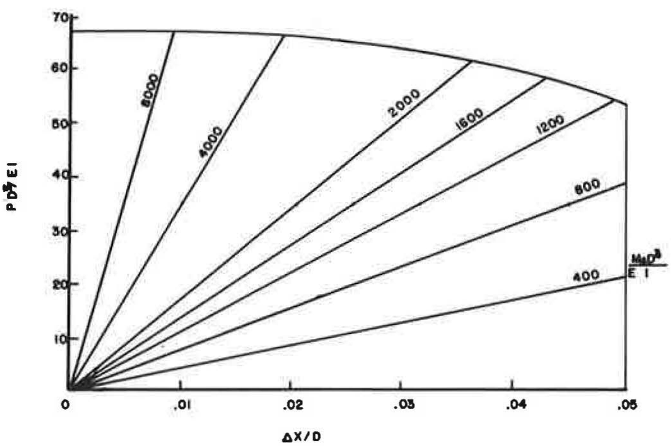

Figure 5. Proposed design chart for circular corrugated steel culverts.

Table 4. Values of D^3/EI (in.^{2}/lb) for $2\frac{2}{3}$- by $3\frac{1}{2}$-in. corrugations.

Gauge	Diameter (in.) 36	48	54	60	66	72	78	84
20	1.430	3.390	4.826	6.621	8.812	11.440	14.546	18.167
18	1.072	2.542	3.620	4.970	6.610	8.589	10.909	13.625
16	0.850	2.016	2.870	3.937	5.241	6.804	8.650	10.804
14	0.672	1.594	2.270	3.114	4.145	5.381	6.842	8.546
12	0.470	1.113	1.585	2.175	2.894	3.758	4.778	5.967
10	0.355	0.8412	1.198	1.643	2.187	2.839	3.610	4.508
8	0.281	0.6661	0.9484	1.301	1.732	2.248	2.858	3.570

Note: E = 29 x 10^6 psi; moment of inertia values taken from AISI Handbook, 1967.

Table 5. Values of D^3/EI (in.^{2}/lb) for 3- by 1-in. corrugations.

Gauge	Diameter (in.) 36	48	54	60	66	72	78	84
20	0.3123	0.7405	1.054	1.446	1.925	2.499	3.177	3.969
18	0.2334	0.5534	0.7879	1.081	1.438	1.868	2.874	2.966
16	0.1858	0.4404	0.6271	0.8602	1.145	1.486	1.900	2.360
14	0.1478	0.3504	0.4989	0.6844	0.9109	1.183	1.504	1.878
12	0.1040	0.2467	0.3512	0.4818	0.6413	0.8326	1.058	1.322
10	0.0797	0.1890	0.2691	0.3691	0.4914	0.6379	0.8111	1.013
8	0.0641	0.1520	0.2165	0.2969	0.3952	0.5131	0.6524	0.8148

Note: E = 29 x 10^6 psi; moment of inertia values taken from AISI Handbook, 1967.

To illustrate this design procedure and the use of Figures 3 and 5, we present two examples.

In example 1, the following are given: 48-in. diameter pipe, $2\frac{2}{3}$- by $\frac{1}{2}$-in. corrugation, height of cover is 10 ft, dry density of soil = 110.4 pcf, and dry density of soil using AASHO T 99 compaction test = 120.0 pcf.

The following steps are taken to solve example 1.

1. To find the constrained modulus of elasticity, first determine the ratio of dry density soil to dry density obtained by using AASHO T 99 compaction test:

$$\gamma d/\gamma 99 = 110.4/120.0 = 0.920$$

Then apply it to Figure 3:

$$M_s = 2{,}000 \text{ lb/in.}^2$$

2. Try a 12-gauge pipe.
3. Derive from Table 4 the following:

$$\begin{aligned} D^3/EI &= 1.113 \text{ in.}^2/\text{lb}, \\ M_sD^3/EI &= 2{,}000 \times 1.113 = 2{,}226, \\ P &= \gamma H, \text{ and} \\ PD^3/EI &= (110.4 \text{ lb/ft}^3 \times 10 \text{ ft})/(144 \text{ in.}^2/\text{ft}^2) \times 1.113 = 8.05. \end{aligned}$$

4. Derive from Figure 5 the following: $\Delta X/D = 0.5$ percent. Change in horizontal diameter is derived as follows: $\Delta X = 48 \times 0.005 = 0.24$ in. With a factor of safety of 2.0, $PD^3/EI = 17.10$.
5. Derive from Figure 5 the following: $\Delta X/D = 1.8$ percent. Change in horizontal diameter is derived as follows: $\Delta X = 0.01 \times 48 = 0.48$ in. With a factor of safety of 1.0, the ring deflection is 0.24, and, with a factor of safety of 2.0, the ring deflection is 0.48. Ring deflection is less than allowable; therefore, 12-gauge pipe is overdesigned. Try a 14-gauge pipe as the ring deflection; the PD^3/EI value for a 12-gauge pipe is well less than the allowable deflection.

In example 2, the following are given: 60-in. diameter pipe, 30 ft of fill, dry density of soil = 106.0 pcf, and dry density of soil using AASHO T 99 compaction test = 110.0 pcf.

The following steps are taken to solve example 2.

1. To find the constrained modulus of elasticity, determine the appropriate ratio:

$$\gamma d/\gamma 99 = 106.0/116.0 = 0.962$$

Then apply it to Figure 3:

$$M_s = 2{,}500 \text{ lb/in.}^2$$

2. Try a 14-gauge pipe with $2\frac{2}{3}$- by $\frac{1}{2}$-in. corrugation.
3. Derive from Table 4 the following:

$$\begin{aligned} D^3/EI &= 3.114 \text{ in.}^2/\text{lb}, \\ M_sD^3/EI &= 2{,}500 \times 3.114 = 7{,}800, \\ P &= \gamma H, \text{ and} \\ PD^3/EI &= (106.0 \text{ lb/ft} \times 30 \text{ ft})/(144 \text{ in.}^2/\text{ft}^2) \times 3.114 = 68. \end{aligned}$$

4. Derive from Figure 5 the following: $\Delta X/D = 1$ percent. The PD^3/EI value is on the upper limit of the design chart; therefore, try a 12-gauge pipe, which would provide an additional factor of safety. The ring deflection is less than the allowable 5 percent. Therefore, a 14-gauge pipe ($2\frac{2}{3}$- by $\frac{1}{2}$-in. corrugation) will carry the load with a factor of safety of about 1.

CONCLUSIONS

The design of soil-culvert systems is concerned basically with predicting the ultimate percentage of conduit deflection and the factor of safety against conduit wall failure. The soil property must be included in the design chart.

The following conclusions can be drawn from the design chart and the relation between constrained modulus and soil density.

1. The relation between constrained modulus of elasticity and soil density indicates that, if the soil has a high dry unit weight, then the constrained modulus is high. If the soil has a low dry unit weight, then the constrained modulus is low.
2. The load deflection diagram indicates that, if M_sD^3/EI is high, then the ultimate deflection of conduit is low, but the load-carrying capacity of the soil-conduit system is high. If M_sD^3/EI is low, the ultimate deflection is high, but the load-carrying capacity of the soil-conduit system is low.
3. A comparison of Watkins and Moser's (9) results with the design chart (Fig. 5) and the error in evaluating the constrained modulus from $\gamma d/\gamma 99$ indicate that an appropriate factor of safety is required.

RECOMMENDATIONS

In developing the design procedure, it is found that additional studies in several areas should prove valuable. They are as follows:

1. The design procedure as developed in this paper should be verified on field installations.
2. Investigation of the relation of the constrained modulus to other standard soil tests is necessary. This should include well-known tests such as the California bearing ratio test and the Hveem stabilometer test.
3. The experimental results of Watkins and Moser (9), as plotted on the load deflection diagram, indicate that the incipient failure might vary with the radius of gyration of the pipe. The relation between radius of gyration and the failure line should be investigated.

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BUCKLING OF CYLINDERS IN A CONFINING MEDIUM

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A large-deflection theory is presented for defining the critical radial pressures that result in the buckling of elastically supported homogeneous cylinders with simple ends. Solutions are given for radial support around the extrados and for radial support only on the outward acting lobes of the circumferential waves that form on loading. It is shown that the difference between the energy load and the Euler load is a maximum for cylinders with no radial support and that the energy load approaches the Euler load as the length-to-radius ratio and the foundation coefficient decrease and as the radius-to-thickness ratio increases. Critical pressures are greater for cylinders supported around the entire extrados than for cylinders supported only on the outward displacing lobes. Practical utility of the theory is found in its application to conduits and cylindrical protective structures buried in soil fields. For such applications, the equation defining the lower bound of buckling is modified to permit one to account for the influence of the surface boundary and to include soil parameters that are readily obtainable in the laboratory. A few tests were performed to check the theory. The results of these tests and other available data agree reasonably well with the theory.

•SOLUTIONS to the problem of defining the strength of conduits, tunnel liners, and cylindrical shelters buried in soil depend on the ability of engineers to predict buckling loads. The general purpose of this paper is to provide the needed solutions. Specific objectives were to define and compare the Euler and energy loads, study the effect of all-around support as compared to support only on outward-deflecting lobes, determine the influence of the dominant parameter on buckling resistance, and compare the results of the elastic theory solutions with experimental results. These goals were pursued to provide improved criteria and methods for the design of buried cylinders.

The configuration analyzed (Fig. 1) consists of a radially supported cylinder with a bonded interface and simple ends. The cylinder is subjected to uniform radial pressure. Two types of radial support are considered: elastic support around the complete perimeter and elastic support only on the outward-deflecting portions of the lobes that form during loading. Parallel solutions are developed from the theory for both of these cases.

In studying the buckling of shells, von Karman and Tsein (1) showed that some systems have states of stable equilibrium at loads less than the Euler load. These states of equilibrium are separated by "energy barriers" such that work must be expended to pass from the unbuckled to the buckled configuration. Such work might be supplied by perturbations in the load or by other means; consequently, the critical load may be less than the Euler load as shown in Figure 2.

Subsequent to the work of von Karman and Tsein, Friedrichs (2) pointed out that at state B′ (Fig. 2) the potential energy, V, is greater than at B; consequently, buckling is not likely at a load less than p_m where the potential energy in the two states is equal. The latter condition is indicated qualitatively by points D and D′ (Fig. 2). The load p_m will be referred to as the transitional buckling load or the energy load.

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A number of investigators, the earliest of whom was Link (3), have developed plane strain solutions for elastically supported cylinders by using the classical small-deformation theory. Later, Luscher endeavored to fit such relations to data from tests of buried cylinders (4).

Langhaar and Boresi (5) derived an expression for the energy load for cylinders subjected to hydrostatic loading and found that the energy to cause a transition from the unbuckled to the buckled state is small and could easily be supplied by accidental shocks. Their work was subsequently compared with the results of experiments by Kirstein and Wenk (6) who observed both elastic snap-through and recovery.

Forrestal and Herrmann (7) showed that the Poisson's ratio of a confining medium has less than a 15 percent effect on the critical load but that the presence or absence of interface shear has a relatively large effect for confining media with low Poisson's ratios.

In studies of other geometries, Gjelsvik and Bodner (8) found that the energy load is relatively insensitive to the assumption (number of terms in the series) for the deflected shape.

In the following sections, the Euler and energy loads are investigated by modifying the methodology of Langhaar and Boresi to include terms that contain the strain energy of the support. The solutions are compared with the results of experiments on cylinders buried in sand.

DEVELOPMENT OF BUCKLING EQUATIONS

The general procedure for the theoretical development is to express the strain energy and the potential energy in terms of displacement by using the large-deflection theory. Then the various energy components are summed to obtain the increment of total potential energy, which is set to zero to obtain the critical pressures.

FORMULATION OF DISPLACEMENT RELATIONS

Rectangular and cylindrical coordinates for a cylinder are shown in Figure 1b, where the origin of coordinates is taken at the center of the section at midlength. Let u, v, and w represent the axial, circumferential, and radial displacements respectively of the point P due to buckling. The components of strain may be expressed in terms of these displacement components and the coordinates as (5)

$$\left.\begin{aligned} \epsilon_x &= \epsilon_x^{(o)} + u_x + \tfrac{1}{2} v_x^2 + \tfrac{1}{2} w_x^2 \\ \epsilon_\theta &= \epsilon_\theta^{(o)} + [(v_\theta + w)/a] + \tfrac{1}{2} [(v - w_\theta)/a]^2 \\ \gamma_{x\theta} &= (u_\theta/a) + (v_x - w_x)\,[(v - w_\theta)/a] \end{aligned}\right\} \tag{1}$$

In these expressions, the subscripts on displacements denote partial derivatives, ϵ_x is the axial strain, ϵ_θ is the circumferential strain, and $\gamma_{x\theta}$ is the shearing strain. The strains at incipient buckling are denoted by $\epsilon_x^{(o)}$ and $\epsilon_\theta^{(o)}$.

Because u, v, and w are unknown, the shape after buckling has to be assumed so that we can compute the changes in total potential energy. They are assumed to be represented by the terms containing up to $3n\theta$ of the Fourier series as

$$\left.\begin{aligned} u &= u_0 + u_1 \cos n\theta\ u_2 \cos 2n\theta + u_3 \cos 3n\theta \\ v &= v_1 \sin n\theta + v_2 \sin 2n\theta + v_3 \sin 3n\theta \\ w &= w_0 + w_1 \cos n\theta + w_2 \cos 2n\theta + w_3 \cos 3n\theta \end{aligned}\right\} \tag{2}$$

where u_i, v_i, and w_i are functions of x alone and n denotes the number of complete waves around the perimeter.

If we also assume that the incremental circumferential strain, $\Delta\epsilon_\theta$, is equal to zero, Eq. 1 yields

$$\Delta\epsilon_\theta = \epsilon_\theta - \epsilon_\theta^{(o)} = [(v_\theta + w)/a] + \frac{1}{2}[(v - w_\theta)/a]^2 = 0 \tag{3}$$

By substituting the expressions for v and w in Eq. 2 in Eq. 3 and by regrouping terms, we can show that all the coefficients v_i and w_i can be expressed in terms of w_1 alone as follows:

$$\left.\begin{aligned} v_1 &= -(w_1/n) \\ v_2 &= n[n - (1/n)]^2 w_1^2/[2a(4n^2 - 1)] \\ v_3 &= 0 \end{aligned}\right\} \tag{4}$$

$$\left.\begin{aligned} w_0 &= -\{[n - (1/n)]^2 w_1^2\}/4a \\ w_1 &= w_1 \\ w_2 &= -\{[n - (1/n)]^2 w_1^2\}/[4a(4n^2 - 1)] \\ w_3 &= 0 \end{aligned}\right\} \tag{5}$$

On the basis of experimental data, it is assumed that

$$w_1/a = [W_o \cos(\pi x/L)]/[n - (1/n)] \tag{6}$$

where W_o is a constant, n is the number of circumferential waves around the perimeter, and L is the length of the cylinder. These displacement relations permit one to express the components of the total potential energy in terms of W_o^2 as exemplified in the following section.

Strain Energy Due to Radial Support

All-Around Support—The change in strain energy due to radial support is given by

$$\Delta U_s = 2a \int_0^{\frac{L}{2}} \int_0^{2\pi} (k_z w^2/2)\, d\theta dx \tag{7}$$

where k_z is the coefficient of soil reaction in lb/in.²/in. or other consistent units.

Substitution of Eqs. 2 and 6 in Eq. 7, integration, and elimination of dimensions by EahL give

$$\Delta U_s/EahL = k_z a/[E(h/a)^3]\,(h/a)^2\,(d_1 W_o^2 + d_2 W_o^4) \tag{8}$$

where

$d_1 = \pi n^2/[4(n^2 - 1)^2]$, and
$d_2 = [3\pi(32n^4 - 16n^2 + 3)]/[256(4n^2 - 1)^2]$

Outward Lobe Support—In deriving Eq. 8, it was assumed that the radial support is effective for both positive and negative values of the displacement w, the positive direction of w indicating the outward radial displacement.

With radial support only for positive values of the radial displacement, strain energy is stored in regions of positive displacement, w, and is zero for portions with negative displacement. It is assumed that the strain energy stored in the media is equal to the strain energies because the components of the displacement w are imposed separately. Thus, recognizing that w_3 is zero by using Eq. 5, we may write the strain energy as

$$\Delta\overline{U}_s = k_z a^3 \int_0^{\frac{L}{2}} \left\{ \int_0^{2\pi} (w_0/a)^2 \, d\theta + n \int_0^{\frac{2\pi}{2n}} [(w_1/a)\cos\ n\theta]^2 \, d\theta \right.$$

$$\left. + \ 2n \int_{\frac{2\pi}{8n}}^{\frac{6\pi}{8n}} [(w_2/a)\cos 2n\theta]^2 \, d\theta \right\} dx \tag{9}$$

The first term does not contribute to $\Delta\overline{U}_s$ because w_0 is negative; therefore,

$$\Delta\overline{U}_s = k_z a^3 \int_0^{\frac{L}{2}} [(w_1/a)^2 (\pi/2) + (w_2/a)^2 (\pi/2)] \, dx \tag{10}$$

If we use Eqs. 4 and 6 (eliminating dimensions by EahL) and simplify, the change in strain energy is

$$\Delta\overline{U}_s/EahL = k_z a/[E(h/a)^3] (h/a)^2 (\overline{d}_1 W_o^2 + \overline{d}_2 W_o^4) \tag{11}$$

where

$\overline{d}_1 = \pi n^2/[8(n^2 - 1)^2]$, and
$\overline{d}_2 = 3\pi/[512(4n^2 - 1)^2]$.

Other Components of Total Potential Energy

Expressions for the strain energy components, ΔU, due to membrane stresses and bending stresses and the increment of potential energy, $\Delta\Omega_p$, due to external pressure (as developed by Langhaar and Boresi, 5) are given below. ΔU due to membrane stresses is as follows:

$$\Delta U_a/EahL = b_1 W_o^2 + b_2 W_o^4 + b_3 W_o^6 \tag{12}$$

where

$b_1 = f_1 + K_6 r^2$,
$b_2 = f_2 - (K_7 - f_3) r^2 + K_1 r^4 - \varphi_1$,
$b_3 = f_4 + (K_8 - f_5) r^2 + (K_3 + f_6) r^4 - \varphi_2 - g$, and
$r = a/L$

The f_i's, φ_i's, and g_i's are functions of the ratio a/L and n, and the K_i's are functions of n only. Expressions defining these functions are given in the Appendix. ΔU due to bending stresses is as follows:

$$\Delta U_b/EahL = (h^2/a^2) W_o^2 (C_1 + C_2 W_o^2 + C_3 W_o^4) \tag{13}$$

where

$C_1 = \pi/[48(1 - \nu^2)] \left(n^2 + 2\pi^2 r^2 \{1 + [\nu/(n^2 - 1)]\} + [\pi^4 n^2 r^4/(n^2 - 1)^2]\right)$,

$C_2 = \pi/[256(1 - \nu^2)] \left(n^2 + \pi^2 r^2 \{1 + [\nu/(n^2 - 1)]\} + \frac{2}{3}\pi^4 r^4 \{2 + [1/(4n^2 - 1)^2]\}\right)$, and

$C_3 = \pi/[12{,}288(1 - \nu^2)] \left(5n^2 + 2\pi^2 r^2 \{5 - 17\nu + [8\nu/(4n^2 - 1)]\}\right)$.

$\Delta\Omega_p$ due to external pressure is as follows:

$$\Delta\Omega_p/EahL = p/[E(h/a)^3] (h/a)^2 (-a_1 W_o^2 + a_2 W_o^4) \tag{14}$$

where

$a_1 = \pi n^2/[4(n^2 - 1)]$, and

$a_2 = (3\pi/128)\left(1 - \{1/[2(4n^2 - 1)]\}\right)$.

Increment of Total Potential Energy

The increment of total potential energy, ΔV, which results from buckling, is given by

$$\Delta V = \Delta U_m + \Delta U_b + \Delta U_s + \Delta\Omega_p \tag{15}$$

Substituting Eqs. 12, 13, 8, and 14 in Eq. 15 yields ΔV for the case of all-around support as

$$\Delta V/EahL = (B_1 - \bar{A}a_1t^2)\,W_o^2 + (B_2 + \bar{A}a_2t^2)\,W_o^4 + B_3W_o^6 \tag{16}$$

where

$\bar{A} = p/[E(h/a)^3]$,
$\bar{D} = k_z a/[E(h/a)^3]$,
$t = h/a$,
$B_1 = b_1 + C_1t^2 + \bar{D}d_1t^2$,
$B_2 = b_2 + C_2t^2 + \bar{D}d_2t^2$, and
$B_3 = b_3 + C_3t^2$.

For the case of support on the outward acting lobes only, Eqs. 12, 13, 11, and 14 in Eq. 15 yield the same relation as Eq. 16 except that d_1 and d_2 are replaced by $\bar{d}_1$ and $\bar{d}_2$ as given in Eq. 11.

Euler and Energy Loads

According to Tsein's criterion, the increment of total potential energy from the unbuckled to the buckled configuration is equal to zero, that is,

$$\Delta V = 0 \tag{17}$$

Therefore, by setting Eq. 16 to zero, the expression for the critical pressure coefficient, $\bar{A} = p_{cr}/E(h/a)^3$, is

$$\bar{A} = (B_1 + B_2W_o^2 + B_3W_o^4)/(a_1t^2 + a_2t^2W_o^2) \tag{18}$$

Minimizing Eq. 18 with respect to W_o^2 gives

$$(a_2W_o^4/2a_1) - W_o^2 - \tfrac{1}{2}\,[(B_2/B_3) + (a_2B_1/a_1B_3)] = 0 \tag{19}$$

The smallest positive value of W_o^2 from Eq. 19 substituted in Eq. 18 gives the energy load. Note that the root W_o^2 of Eq. 19 must be positive. If the roots are either negative or imaginary, the energy load does not exist. In some cases, a positive root gives an energy load greater than the corresponding Euler load. For such cases, the energy load and Euler load merge, and the phenomenon of snap-through does not occur.

The Euler load can be obtained simply by setting $W_o^2 = 0$ in Eq. 18. Thus,

$$\bar{A}^{Euler} = B_1/a_1t^2 \tag{20}$$

METHOD OF SOLUTION

Uniform Surrounding Field

Equation 18 is solved by calculating the critical pressure for a given set of independent variables corresponding to several values of $\bar{A}$ (9). The minimum value is the true critical pressure. A computer code was written to obtain solutions for $\bar{A}$ for permutations of the length-to-radius ratio, L/a, thickness-to-radius ratio, h/a, and foundation coefficient, $\overline{D}$. All calculations were based on a Poisson's ratio, ν, of 0.33. Plots of the computer output are shown in Figures 3 and 4.

Differences between the Euler load and the energy load (Figs. 3 and 4) are largest for small L/a, large a/h, and small $\overline{D}$. The difference is as large as 25 percent in some cases. The energy load approaches the Euler load as L/a increases, a/h decreases, and $\overline{D}$ increases. With short, thin cylinders, the Euler load at small values of $\overline{D}$ is nearly the same for both types of support considered. The same is true for the energy load. At large values of $\overline{D}$, type of support is much more important as is evident from the following considerations. For infinitely long cylinders, it can be shown that the Euler and the energy loads are identical and that for all-around support they may be expressed by the relation

$$\overline{A}^{EL} = \alpha + \beta\overline{D} \tag{21}$$

where

$\overline{A} = p_{cr}/[E(h/a)^3]$,
$\overline{D} = k_z a/[E(h/a)^3]$,
$\alpha = (n^2 - 1)/[12(1 - \nu^2)]$, and
$\beta = 1/(n^2 - 1)$.

The lower bound for the critical pressure is

$$\overline{A}^{EL} = 2\sqrt{\alpha\beta\overline{D}} = 0.6116\sqrt{\overline{D}} \quad \text{for } \nu = 0.33 \tag{22}$$

Likewise, for infinitely long cylinders with lobar support (support only on the outward-deflecting lobes)

$$\overline{A}^{LO} = \alpha + 0.5\,\beta\overline{D} \tag{23}$$

The lower bound for lobar support is given by

$$\overline{A}^{LO} = \sqrt{2\alpha\beta\overline{D}} = 0.4325\sqrt{\overline{D}} \quad \text{for } \nu = 0.33 \tag{24}$$

From the ratio of Eqs. 22 and 24, the influence of type of soil support is

$$p_{cr}^{EL}/p_{cr}^{LO} = \sqrt{2} \tag{25}$$

or about 40 percent difference.

The foundation coefficient, k_z, in Eq. 22 can be expressed in terms of the one-dimensional (confined) compression modulus by using the theory for soil-surrounded tubes (4). For concentrically surrounded tubes, the modulus of elastic support, $k_s = k_z a$, may be expressed in terms of the modulus of elasticity of the soil, E_s, by the relation plotted in Figure 5. Further, E_s is related to the confined compression modulus, M_s, by the expression

$$E_s = \widetilde{C}M_s \tag{26}$$

where $\widetilde{C} = [(1 + \nu_s)(1 - 2\nu_s)]/(1 - \nu_s)$.

From the theory of a soil-surrounded tube (4), it has been shown that the ratio of k_s to E_s is

$$\widetilde{B} = k_s/E_s = [1 - (a/a_o)^2]/\{(1 + \nu_s)\,[1 + (a/a_o)^2\,(1 - 2\nu_s)]\} \tag{27}$$

where a and a_o are radii as shown in Figure 5.

It follows that

$$k_z a = \widetilde{B}\widetilde{C}M_s \tag{28}$$

Figure 1. System geometry.

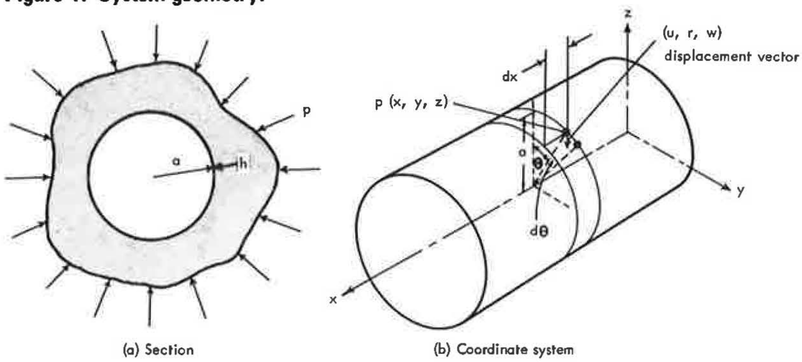


Figure 2. Pressure, energy, and displacement interrelations.

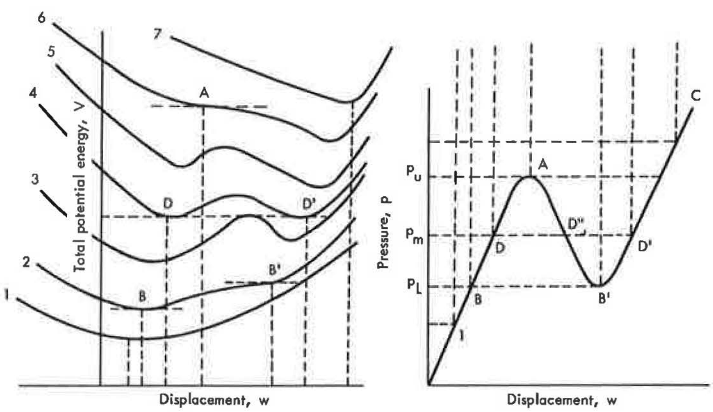


Figure 3. Variation of critical pressure with $\overline{D}$ for elastic support.

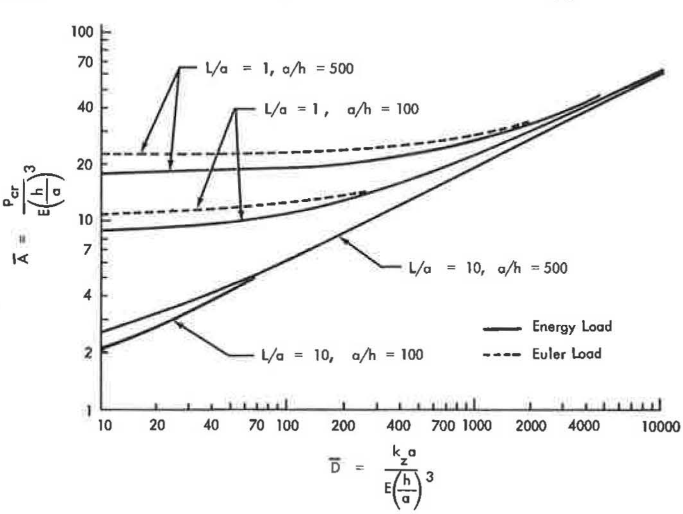


Figure 4. Variation of critical pressure with $\overline{D}$ for lobar support.

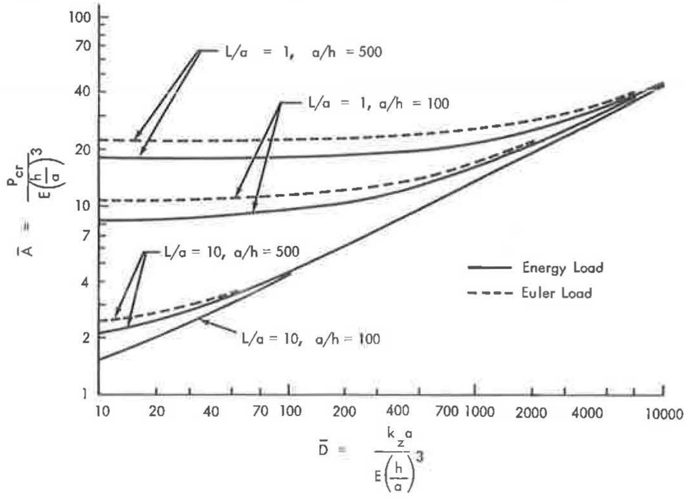


Figure 5. Modulus of soil support for elastic ring (4).

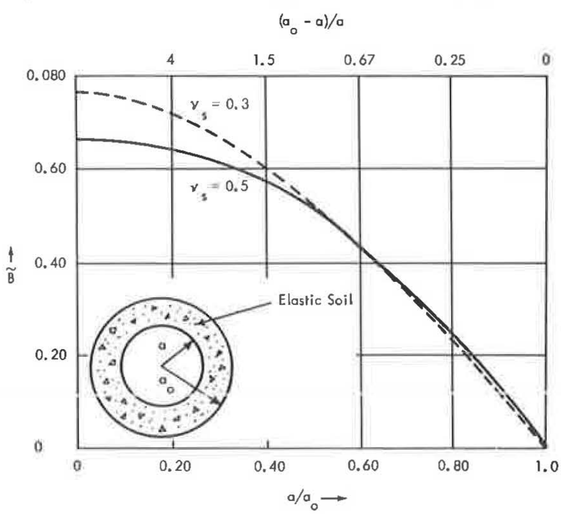
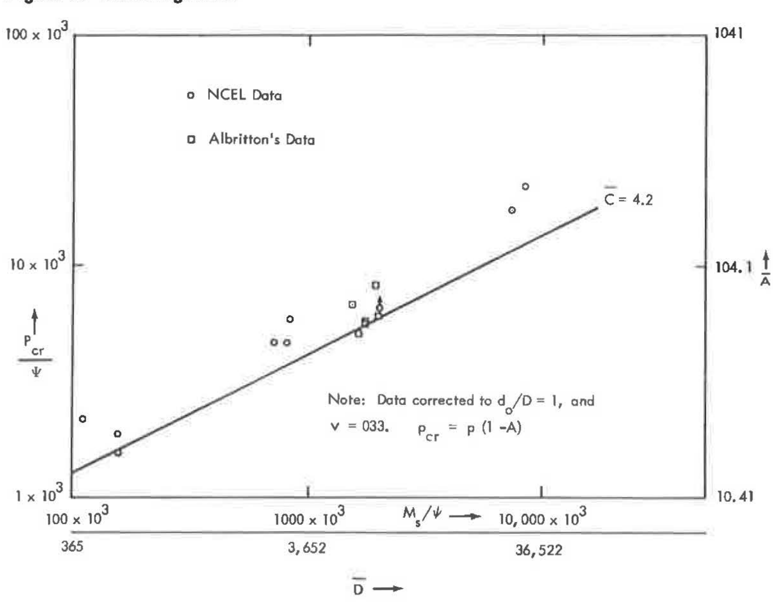


Figure 6. Buckling data.



Substituting Eq. 27 in Eq. 22 gives

$$p_{cr}/\Psi = \overline{c}\sqrt{M_s/\Psi} \tag{29}$$

where

$\overline{c} = 6\sqrt{\tilde{B}\tilde{C}}$,
$\Psi = EI/D^3$,
D = mean diameter of cylinder, and
EI = stiffness of section.

In the strict mathematical sense, Eq. 29 is correct only for concentrically surrounded tubes. It is approximately correct, however, for cylinders buried beneath the earth's surface, which are loaded by the soil cover and possibly a uniform surface loading.

THEORY VERSUS EXPERIMENT

To facilitate a valid comparison of theory and experiment, we performed tests on thin metal cylinders in a segmented soil tank where the confined compression modulus, M_s, and the at-rest coefficient of lateral earth pressure, k, could be determined as an integral part of each test (10). All cylinders were 5 in. in diameter, 21 in. long, and either 6, 12, or 18 mils thick. Sand type and density were varied to get a wide range of confined compression moduli. A dry, rounded sand and a sharp-grained sand were used, each at three different densities. Measurements included surface pressure, cylinder deflections, and strains in the soil, tank, and cylinder. Details of the experiment may be found elsewhere (10).

The experimental data are plotted together with Eq. 29 in Figure 6. The buckling load was taken as $p_{cr} = p_f(1 - A)$ where p_f is the surface pressure at failure, and the arching was calculated from the corresponding strains at the haunches. The soil modulus used was the secant modulus at the failure load. All data plotted were corrected to one-diameter depth of cover.

Representative data from Albritton's tests (11) (SDA1, SDA2, SE1, SE2, SF1, and SF2) are also plotted in Figure 6. In these data, the moduli were from confined compression data. No other data are known to the authors in which all the necessary parameters are available. As may be seen, however, the Naval Civil Engineering Laboratory and the Albritton data agree reasonably well with the theory. There is, of course, the expected spread common to buckling and soil test data.

In general, the experimental buckling loads are greater than those from Eq. 29. This is considered to be due to greater compaction of the soil in the vicinity of the cylinder than at corresponding depths in the free-field. As a consequence, elastic support gives a better approximation of actual buckling loads than does lobar support.

Soil moduli back-calculated from deflections were in close agreement with measured values. This adds to confidence in the validity of the experimental data.

Predicting the surface and overburden pressure to cause buckling of a buried cylinder involves a determination of the arching over the structure (12). The percentage of the applied load that reaches the interface may vary by a factor of 20 or more depending on the relative stiffness of the inclusion and the confining soil.

CONCLUSIONS

The theoretical analysis and the comparison of the theory with test data substantiate the following conclusions:

1. The energy load and the Euler load are significantly different only for conditions (low length-to-radii ratios, low values of the foundation coefficient, and large radii-to-thickness ratios) that are not commonly encountered in practice. The theory agrees reasonably well with applicable experimental results.
2. The critical load with elastic support around the total perimeter is considered the best model of actual buried cylinders.
3. The buckling resistance of long buried cylinders is principally dependent on the bending flexibility, the secant value of the confined compression modulus, and Poisson's

ratio of the confining media as defined by Eq. 29. Arching is extremely important in governing the percentage of the applied load that reaches the interface.

4. The theory is considered adequate for obtaining conservative estimates of the elastic buckling load of buried cylinders.

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APPENDIX

EXPRESSIONS FOR VARIOUS TERMS IN THE THEORETICAL DEVELOPMENT

$$K_1 = \frac{3\pi^5}{256(1 - \nu^2)(n^2 - 1)^2}$$

$$K_2 = \frac{51\pi^5}{65536(1 - \nu^2)(4n^2 - 1)^2}$$

$$K_3 = \frac{\pi^5\left[\frac{5n^2}{(4n^2 - 1)^2} + \frac{7n^2 - 1}{4n^2 - 1} + 2n^2\right]}{256(1 - \nu^2)(n^2 - 1)^2}$$

$$K_4 = \frac{\pi^2(n^2 + 1)}{8(n^2 - 1)^2}$$

$$K_5 = \frac{\pi^2}{128}\left[2 + \frac{4n^2 + 1}{(4n^2 - 1)^2}\right]$$

$$K_6 = \frac{\pi^3}{8(1 + \nu)(n^2 - 1)^2}$$

$$K_7 = \frac{\pi^3}{32(1 + \nu)}\left[\frac{-n^2}{(4n^2 - 1)^2} + \frac{2n^2 - 1}{2(n^2 - 1)(4n^2 - 1)} + \frac{3n^2 - 4}{4(n^2 - 1)^2}\right]$$

$$K_8 = \frac{\pi^3}{512(1 + \nu)}\left[\frac{1}{(4n^2 - 1)^2} - \frac{2}{(4n^2 - 1)} + 2\right]$$

$$K_9 = \frac{\pi^3 n(8n^2 + 1)}{16(n^2 - 1)(4n^2 - 1)}$$

$$K_{10} = \frac{\pi(1 - \nu)n(8n^2 - 3)}{32(4n^2 - 1)}$$

$$K_{11} = \frac{(1 - \nu)\pi n}{2(n^2 - 1)}$$

$$K_{12} = \frac{\pi(1 - \nu)n^2(2n^2 + 1)}{4(n^2 - 1)(4n^2 - 1)}$$

$$K_{13} = \frac{\pi^3}{4(n^2 - 1)}$$

$$K_{18} = \frac{\pi^3}{8(4n^2 - 1)}$$

$$\xi = \frac{n}{2r}\sqrt{\frac{1 - \nu}{2}}$$

$$\alpha = \pi^2 + 4\xi^2$$

$$\beta = 9\pi^2 + 4\xi^2$$

$$\gamma = 4\pi^2 + 4\xi^2$$

$$\delta = \pi^2 + 36\xi^2$$

$$f_1 = \frac{-\pi K_{11}^2}{4(1 - \nu^2)\alpha}$$

$$f_2 = \frac{\pi K_{10} K_{11}}{2(1 - \nu^2)\alpha}$$

$$f_3 = \frac{-\pi K_9 K_{11}}{2(1 - \nu^2)\alpha}$$

$$f_4 = \frac{-\pi K_{10}^2}{4(1 - \nu^2)} \left[\frac{1}{\alpha} + \frac{1}{\beta}\right]$$

$$f_5 = \frac{\pi K_9 K_{10}}{2(1 - \nu^2)} \left[-\frac{1}{\alpha} + \frac{3}{\beta}\right]$$

$$f_6 = \frac{-\pi K_9^2}{4(1 - \nu^2)} \left[\frac{1}{\alpha} + \frac{9}{\beta}\right]$$

$$\varphi_1 = \frac{\pi(K_{12} + K_{13} r^2)^2}{16(1 - \nu^2)\alpha} + \frac{\pi^3 \tanh 2\xi}{16(1 - \nu^2)\xi} \left[\frac{\pi r^2}{2(n^2 - 1)} - \frac{K_{12} + K_{13} r^2}{\alpha}\right]^2$$

$$\varphi_2 = \frac{\pi^3 K_{18} r^2 \tanh 2\xi}{4(1 - \nu^2)\gamma\xi} \left[\frac{\pi r^2}{2(n^2 - 1)} - \frac{K_{12} + K_{13} r^2}{\alpha}\right]$$

$$g = \frac{\pi}{4(1 - \nu^2)} \left[\frac{(K_{14} r^2 + K_{15})^2}{\delta} + \frac{(3K_{14} r^2 - K_{15})^2}{9\alpha}\right]$$

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THE IOWA DEFLECTION FORMULA: AN APPRAISAL

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The Iowa deflection formula is based on the assumption that the supporting strength of buried corrugated metal pipe installations arises through the lateral reactive soil pressures induced at the sides of the pipe. Since its introduction more than 30 years ago, the Iowa deflection formula has served as a basic criterion for the design of buried flexible pipe systems. The formula is based on an assumed distribution of loading around the pipe, and contains three parameters that are empirical in nature. The present study examines the significance and possible variation of these parameters in the deflection equation. Limited field studies have indicated that realistic variations in these parameters can lead to an almost threefold change in design requirements. One of the most influential parameters in the formula is the modulus E′. Values of E′ as determined from the measured response of full-scale installations exhibit a thirtyfold variation. Yet these data form the basis for establishing recommended values of E′ for design use. Extrapolation of the observed deflections from field tests shows that in most cases use of the suggested design criteria yields unconservative fill heights. The present study shows no strong correlation between E′ and percentage of standard Proctor density for the soil adjacent to the pipe, the pipe diameter, or the ratio of fill height to pipe diameter. A statistical analysis compares observed values of E′ with common probability distributions, and a log-normal distribution is fitted. Probabilities associated with various ranges of E′ confirm that there is no rational basis for recommending a design value that is much greater than the median, or central, value.

•THE use of corrugated metal pipe was originally confined to small culverts in which the size rarely exceeded 36 in. in diameter. The height of fill over the early culverts was usually very low, too low in many cases to protect the pipe from damage by traffic loads. As their use developed, corrugated metal pipes were made in larger diameters, and the heights of the fills constructed over them were increased markedly.

During these early days, no successful attempt was made to develop a rational means for designing this type of structure according to the principles of mechanics. Rather, reliance was placed almost wholly on service experience and intuition in the determination of safe heights of fill.

The predominant source of supporting strength for a flexible pipe is the lateral pressure of the soil at the sides of the pipe. The pipe itself has relatively little bending strength, and a large part of its ability to support vertical loads must be derived from the passive pressures induced as the sides move outward against the soil. The ability of a flexible pipe to deform readily and thus utilize the passive pressure of the earth on each side of the pipe is its principal distinguishing structural characteristic and accounts for the fact that such a relatively lightweight pipe can support earth fills of considerable height. Because so much of the total supporting strength depends on the sidefill material, any attempt to analyze the structural behavior of this type of pipe under a fill must consider the soil at the sides to be an integral part of the structural system.

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IOWA DEFLECTION FORMULA

During experiments on circular flexible pipe at Iowa State College, Spangler observed that, as soil loads are applied, the flexible ring is flattened down into an approximately elliptical cross section with a decrease in vertical diameter Δy and an increase in horizontal diameter Δx. In the now classic report (11) on the design of flexible pipe design, Spangler noted, "Preliminary observations had led to the hypothesis that, as a fill is built over a flexible culvert, the horizontal pressure on the pipe at any point bears a nearly constant relationship to the horizontal movement of the point. This constant ratio has been called the modulus of passive pressure, e, of the fill material and may be expressed as units of pressure per unit of movement."

The loci of horizontal movement of all points within the top and bottom 40 deg of the pipe ring are assumed to be small, and for mathematical convenience a simple parabolic curve embracing only the middle 100 deg of the semicircle is used to define the distribution of horizontal pressure.

Next, it was necessary to make some assumptions concerning the distribution of pressure around the remaining portions of the pipe. The Marston load theory (6) was used to evaluate the total vertical load on the pipe, W_c. For flexible pipe, Spangler assumed that this total load is uniformly distributed across the breadth of the conduit.

The vertical reaction on the bottom of the pipe is equal to the vertical load, W_c, and is distributed uniformly over the width of bedding of the pipe. The half-width of this bedding and vertical reaction is defined in terms of the angle α as measured from the vertical centerline of the conduit.

The distribution of pressures around a flexible pipe under an earth fill, determined in accordance with this hypothesis, is shown in Figure 1.

Once this fill hypothesis was established, it was a simple matter to develop mathematical expressions for the moments, thrusts, shears, and deflections of the pipe. The resulting expression for the horizontal deflection of a flexible pipe is known as the Iowa formula:

$$\Delta x = K\, W_c\, R^3/(EI + 0.061\, e\, R^4) \tag{1}$$

in which R is the radius of the pipe, E is the modulus of elasticity of the pipe material, I is the moment of inertia of the cross section of a 1-in. length of pipe wall, and K is a dimensionless bedding constant that results from the necessary mathematical manipulations and is a function of the bedding angle α.

Equation 1 gives a value of deflection that may be called the immediate deflection. For the deflection that will develop over a long period of time, the equation must be multiplied by a deflection lag factor, D_l.

In 1957, Watkins and Spangler (18) examined the Iowa formula dimensionally. They had hoped to design some small-scale model tests to measure the soil modulus, e. To their surprise, however, they found that e could not possibly be a property of the soil because its dimensions were not those of a true modulus. The dimensionally correct modulus is

$$E' = e\, R \tag{2}$$

and is known as the modulus of soil reaction.

Consequently, the modified Iowa formula is given as

$$\Delta x = D_l\, K\, W_c\, R^3/(EI + 0.061\, E'\, R^3) \tag{3}$$

If the Iowa formula is rearranged, the following terms can be introduced to describe three separate factors that affect the pipe deflection: load factor = $D_l\, K\, W_c$; ring stiffness factor = EI/R^3; and soil stiffness factor = $0.061\, E'$. Thus, the Iowa formula equation (4) can be represented as

$$\Delta x = \frac{\text{load factor}}{(\text{ring stiffness factor} + \text{soil stiffness factor})} \tag{4}$$

In the following sections, the range of variation and influence of each of these factors will be examined in detail.

Ring Stiffness Factor

The ring stiffness factor is a function of the modulus of elasticity of the pipe material and the moment of inertia of the particular corrugation configuration and the radius of the pipe. The variation of this factor with respect to different corrugations and radii is shown in Figure 2. In most cases, the ring stiffness factor has very little influence on the deflection of a particular pipe because the soil stiffness factor is much larger.

Load Factor

The load factor incorporates the parameters that have to do with the magnitude and distribution of the soil pressures on a buried pipe. The pipe deflection is directly proportional to the load factor, and yet less is known about its components than any other value in the Iowa formula.

In a discussion of the deflection lag factor, D_1, Spangler (11) has stated, "The deflection lag factors observed in the experiments ranged from 1.38 to 1.46, and in no instance was equilibrium completely attained. Therefore, 1.5 is suggested as a conservative value for design use for standard corrugated-pipe culverts installed without strutting or predeforming."

In the second edition of his textbook (14), Spangler states, "The deflection lag factor cannot be less than unity and has been observed to range upward toward a value of 2.0. A normal range of values from 1.25 to 1.50 is suggested for design purposes."

The load on the pipe depends on whether the pipe is installed in a trench condition or in an embankment condition; however, in an attempt to obtain information of direct use to the design engineer, we may "simplify" the Iowa formula by assuming that the settlement ratio is zero. Then the total load on the pipe can be expressed as

$$W_c = H \gamma B_o \tag{5}$$

It is now possible to substitute this value in the Iowa formula and solve for the fill height. The deflection can be expressed as a percentage of the nominal diameter (i.e., let $P = \Delta x/B_o$), and the equation can be expressed as

$$H = 144\ P/(D_1\ K\ \gamma)\ [(EI/R^3) + 0.061\ E'] \tag{6}$$

in which the constant 144 was introduced for dimensional consistency to yield a field height in feet.

Equation 6 provides a basis for making a parametric study of the various factors influencing the height of fill as predicted by using the Iowa formula. From this equation it can be seen that the design fill height is inversely proportional to the value of the deflection lag factor. Thus, any variation of the lag factor can have a very significant effect on the design height of fill, and care should be exercised in selecting an appropriate value for this parameter. In addition, it should be noted that a change in the bedding constant, K, over its range of values can modify the design fill height by as much as 30 percent.

Soil Stiffness Factor

Spangler regards E′ as a semi-empirical constant and has stated that the properties of the soil that influence this factor are somewhat obscure although qualitatively it is certain that texture and density characteristics are of prime importance.

The influence of the soil stiffness factor can be illustrated by means of Eq. 6. It will be assumed that K = 0.100. For purposes of comparison three design criteria will be established: design criterion I assumes E′ = 700 psi and D_1 = 1.50; design criterion II assumes E′ = 1,400 psi and D_1 = 1.50; and design criterion III assumes E′ = 1,400 psi and D_1 = 1.25.

The variation of fill height for design criterion I is shown as the lowest curve in Figure 3. The effect of doubling the soil stiffness factor is reflected by the middle curve in Figure 3 (i.e., design criterion II). The corresponding change in fill height is not linear but is dependent on the value of the ring stiffness factor.

The variation of fill height for design criterion III is shown as the highest curve in Figure 3. It should be noted that the only distinction between this curve and the curve for design criterion II is a change in the value of D_l from 1.50 to 1.25. The modified fill height as reflected by this change is of significance but is not as large as the variation produced by doubling the value of E′.

For an R^3/EI value of 50, the ratio of fill heights for design criterion III and design criterion I is 2.01. This difference is due to the changes in the D_l and E′ parameters. If the bedding coefficient is also included in the variation, an even greater difference in fill heights is obtained. Assuming the bedding angle α to be 0 deg for design criterion I yields a design fill height of 22.84 ft; assuming a value of $\alpha = 90$ deg for design criterion III yields a design fill height of 60.21 ft. The resulting ratio of fill heights is 2.64. Thus, an almost threefold variation of design fill heights is possible, depending on which set of "realistic" values are assigned to the various parameters in the Iowa deflection formula.

The values of the parameters for design criterion I are equivalent to those suggested by the Bureau of Public Roads in 1966 (17), whereas the values of the parameters for design criterion III are equivalent to the values recommended by the Federal Highway Administration in 1970 (20).

ANALYSIS OF TEST DATA

From the foregoing discussion it can be seen that the most influential parameter in the Iowa deflection formula is the modulus of soil reaction E′. Unfortunately, this modulus is not a fundamental soil property that can be evaluated by means of a standard laboratory test. Researchers (13, 19) have attempted to determine E′ by direct laboratory measurements but without success. Recently, several investigators (1, 5, 7, 8) attempted to correlate E′ with basic soil properties, but these correlations have not yet been widely tested.

Consequently, the design engineer must rely on past experience and "recommended values" of E′ for design purposes. Spangler has suggested that E′ be considered as a semi-empirical constant whose values may best be determined by observation and measurements on actual culvert installations. Recently, Spangler published a table showing values of E′ for various kinds of soil in various states of compaction as deduced from 18 actual culvert installations (15). These data are given in Table 1. This limited study reveals that the modulus E′ varies over a wide range, from as little as 234 psi to as much as 7,980 psi, a 34-fold variation.

These data are from two experimental programs and three actual installations. The original sources of the information are as follows:

Item Number	Test Dates	Date Report Published	Reference Number
1 to 10	1927 to 1936	1938	10, 11
11 to 15	1924 to 1926	1929	2, 12
16	1952	1956	3
17	1952	1956	16
18	1966	1969	9

Thus 83 percent of the data are from test installations that were constructed more than 35 years ago. Spangler (15) has stated that these data are the result of his efforts to accumulate information on values of E′ and to correlate them with soil properties.

For purposes of later identification, the culverts have been divided into three groups. Group I includes items 1 through 10 and consists of the pipe in the Iowa State College tests. Group II includes items 11 through 15 and consists of the pipe in the North Carolina tests. Group III consists of the three large-diameter installations having high fill heights. Additional data from the original test results are given in the first five columns and the eighth column of Table 2.

The reported values of e and E′ for items 1 through 17 (as shown in the last two columns of Table 1) were evaluated by means of the basic definitions. The test data used for these calculations consisted of the measured deflection and the side pressure, h, which were measured for items 1 through 10 and estimated for items 11 through 17.

As an alternative approach, the modulus can be evaluated from the Iowa deflection formula. Thus, if Eq. 3 is rewritten in the form

$$E' = 16.39\,[(D_1\,K\,W_c/\Delta x) - (EI/R^3)] \tag{7}$$

and assumptions are made concerning values of D_1, K, and W_c, then E′ can be evaluated on the basis of the measured values of the deflection and the ring stiffness factor.

It has been suggested that the modulus of passive resistance and/or the modulus of soil reaction is dependent on the density of the soil adjacent to the pipe. To check this hypothesis, we made a linear regression analysis of the e and E′ data given in Table 1 with the percentage of standard Proctor density data for group I pipe given in Table 2. The prediction equations, as obtained from the linear regression analysis, are given in Table 3 together with the correlation coefficient for each case. From Table 3 it can be seen that there is a higher degree of correlation between e and PPD than there is between E′ and PPD. This analysis cannot be considered as conclusive because only 10 data points are available for consideration.

Regression analyses were also made to explore the dependence of the moduli e and E′ on the parameter D and the nondimensional ratio H/D. By considering these parameters, it was possible to analyze various combinations of the 18 items given in Tables 1 and 2. The resulting prediction equations and the associated correlation coefficients are given in Table 3.

The results given in Table 3 do not establish a strong enough dependence of E′ on PPD, D, or H/D to warrant the use of any of these equations for design purposes. For example, if the data for item 17 are considered, the prediction equations at the bottom of each group in Table 3 yield the following values:

Parameter Considered	E′ Value
Percentage of Proctor density = 100	916 psi
Diameter = 84 in.	2,432 psi
Ratio of H/D = 19.6	2,834 psi

All of these values fall short of the value of 7,980 psi as given in Table 1.

The measured horizontal and vertical deflections for 16 pipe culverts are given in Table 2 together with the ratio of these deflections. The mean value of the ratio $\Delta_{vert.}/\Delta_{horiz.}$ is 1.13, and the standard deviation is 0.18. Thus, it can be seen that the vertical and horizontal deflections are not equal, and the vertical deflection is generally the larger.

All of the test pipe in groups I and II (items 1 through 15) were studied prior to the introduction of the E′ concept, and the "exact" values of e as reported in the original publications (2, 11) are given in Table 2. Using these values of e, we calculated the theoretical horizontal deflection of the pipe by means of the Iowa formula as given by Eq. 1. These values are given in Table 2 as $(\Delta_h)_{calc.}$, together with the ratio of this deflection to the measured horizontal deflection. The mean value of this ratio is 0.96, and the standard deviation is 0.38, indicating a fairly wide scatter of the data.

As an alternative to evaluating E′ by means of the basic defining equation, we explored the possibility of utilizing more of the test data. This was accomplished by assuming $D_1 = 1.00$ and $K = 0.100$ and by rewriting Eq. 7 in the form

$$E'' = 16.39\,[0.20\,W_c/(\Delta_{horiz.} + \Delta_{vert.}) - (EI/R^3)] \tag{8}$$

The measured values of the ring stiffness factor, W_c, $\Delta_{horiz.}$, and $\Delta_{vert.}$ as reported in the test results were utilized to evaluate this alternative form of the modulus. The

Table 1. Values of E′ for 18 flexible pipe culverts.

Item	Group	Pipe Diameter (in.)	Fill Height (ft)	Fill Height/ Pipe Diameter (ft/ft)	Side Pressure	Deflection	Load	Type of Soil	Backfill	e (psi/in.)	E′ (psi)
1	I	42	15	4.28	M	M	–	Loam topsoil	U	14	294
2	I	42	16	4.56	M	M	–	Well-graded gravel	U	32	672
3	I	36	15	5.00	M	M	–	Sandy clay loam	U	13	234
4	I	42	15	4.28	M	M	–	Sandy clay loam	U	15	315
5	I	48	15	3.74	M	M	–	Sandy clay loam	U	14	336
6	I	60	15	3.00	M	M	–	Sandy clay loam	U	12	360
7	I	36	15	5.00	M	M	–	Sandy clay loam	T	28	502
8	I	42	15	4.28	M	M	–	Sandy clay loam	T	25	525
9	I	48	15	3.74	M	M	–	Sandy clay loam	T	29	696
10	I	60	15	3.00	M	M	–	Sandy clay loam	T	26	780
11	II	20	12	7.20	E	M	–	Sand	NA	35	350
12	II	21	12	6.86	E	M	–	Sand	NA	82	861
13	II	30	12	4.80	E	M	–	Sand	NA	25	375
14	II	30	12	4.80	E	M	–	Sand	NA	80	1,200
15	II	31.5	12	4.58	E	M	–	Sand	NA	56	882
16	III	66	170	30.90	E	M	–	Clayey sandy silt	C	40	1,320
17	III	84	137	19.60	E	M	–	Crushed sandstone	C	190	7,980
18	III	216	83	4.60	–	M	M	Graded crushed gravel	C	58	6,300

Note: e = modulus of passive resistance, E′ = modulus of soil reaction, M = measured, U = untamped, T = tamped, E = estimated, NA = not available, and C = compacted.

Table 2. Summary and analysis of test data for 16 flexible pipe culverts.

Item	Data From Test Results: Percentage of Standard Proctor Density	r_{sd}	W_o (lb/in.)	Measured Deflections: $\Delta_{horiz.}$ (in.)	$\Delta_{vert.}$ (in.)	$\Delta_{vert.}/\Delta_{horiz.}$	e (psi/in.)	Horizontal Deflection Calculated Using e Value: $(\Delta_h)_{calc.}$ (in.)	$(\Delta_h)_{calc.}/(\Delta_h)_{meas.}$	E″ (based on Δ_{avg}) (psi)	E″/E′ (E′ from Table 1)
1	87.7	–	443	1.38	1.43	1.04	14.0	1.23	0.89	226	0.77
2	93.2	–	443	0.75	0.80	1.07	32.0	0.78	1.04	699	1.04
3	86.8	-0.38	300	1.27	1.21	0.95	12.93	1.17	0.92	241	1.03
4	89.2	-0.26	375	1.39	1.31	0.94	14.82	1.41	1.01	332	1.05
5	91.4	-0.38	408	1.62	1.56	0.96	14.39	1.56	0.96	338	1.01
6	89.0	-0.67	508	1.79	1.82	1.02	11.60	1.72	0.96	402	1.12
7	93.5	-0.18[a]	342	0.67	0.71	1.06	28.30	0.83	1.24	657	1.31
8	91.5	-0.14[a]	425	0.77	0.84	1.09	24.67	0.94	1.22	747	1.42
9	92.6	-0.17[a]	458	0.86	1.02	1.19	29.30	0.97	1.13	716	1.03
10	90.5	-0.76[a]	483	1.01	1.26	1.25	25.71	0.87	0.86	640	0.82
11	–	–	98	0.48	0.543	1.13	33.3	0.479	1.00	296	0.85
12	–	–	173	0.212	0.233	1.10	63.2	0.176	0.83	313	0.36
13	–	–	162	0.631	0.812	1.29	26.3	0.711	1.13	352	0.94
14	–	–	233	0.234	0.261	1.12	70.0	0.255	1.09	1,030	0.86
15	–	–	224	0.322	0.379	1.18	50.3	0.324	1.01	673	0.76
18	–	–	9,800	1.4	2.4	1.71	–	–	–	8,366	1.33

Note: Item 11 was a smooth iron pipe with t = 0.076 in. Item 13 was a smooth iron pipe with t = 0.109 in. Item 14 was a steel tube with t = 0.349 in. All others were corrugated metal pipes.

[a]Adjusted value.

Table 3. Prediction equations.

Group	Characteristic	Value	Correlation Coefficient
Percentage of Proctor density[a]			
I	e	–	0.718
I	E′	–	0.557
Pipe diameter[b]			
I	c_1, c_2	25.0, -0.09	0.086
I	C_1', C_2'	96.4, 8.2	0.363
I, II combined	c_1, c_2	66.0, -0.09	0.572
I, II combined	C_1', C_2'	955.0, -10.1	0.431
I, II, III combined	C_1', C_2'	-550.0, 35.5	0.721
Fill height-pipe diameter ratio[c]			
I	c_3, c_4	14.4, 1.56	0.120
I	C_3', C_4'	810.7, -83.1	0.305
I, II combined	c_3, c_4	-4.8, 7.6	0.468
I, II combined	C_3', C_4'	492.0, 14.6	0.061
I, II, III combined	C_3', C_4'	521.0, 118.0	0.385

[a]$e = -240.5 + 2.886$ and $E' = -3{,}790 + 47.06$.
[b]$e = c_1 + c_2(D)$ and $E' = C_1' + C_2'(D)$.
[c]$e = c_3 + c_4(H/D)$ and $E' = C_3' + C_4'(H/D)$.

resulting values are given in Table 2 together with the ratio of this modulus to the value of E′ as reported by Spangler and given in Table 1. The mean value of this ratio is 0.98, and the standard deviation is 0.25, indicating that approximately two-thirds of the data lie within the range of 0.73 to 1.23.

The quantity W_c as used in Eq. 8 was that value reported in the test results and given in Table 2. In order to investigate the validity of the design assumption that the settlement ratio is zero, we evaluated the weight of the total prism of soil above the culvert by means of Eq. 5 (listed as the quantity $\overline{W}_c$). The values of W_c are compared with the $\overline{W}_c$ values. Perfect correspondence does not exist between these two quantities; however, the data are very well correlated (the correlation coefficient is equal to 0.953), and a regression analysis yields the equation

$$\overline{W}_c = 10 + 1.32\ W_c \tag{9}$$

The mean value of the ratio $\overline{W}_c/W_c$ is 1.38, and the standard deviation is 0.20.

From the foregoing it can be seen that the value of the modulus of soil reaction as evaluated from test results is very sensitive to the method of calculation. In addition, the assumed value of the bedding coefficient will have a large influence on the result.

The design fill height as predicted by Eq. 6 and the values of the parameters as given by design criterion I ($K = 0.100$, $E' = 700$ psi, and $D_1 = 1.50$) and design criterion III ($K = 0.100$, $E' = 1{,}400$ psi, and $D_1 = 1.25$) are shown in Figure 4, and it can be observed that only seven of the points for design criterion I lie in the conservative region of the figure. A linear regression analysis was made of both sets of data and yielded the following results:

1. Design criterion I:

$$H_{design} = 11.4 + 0.47\ H_{extrapolated} \tag{10}$$

The correlation coefficient is 0.67.

2. Design criterion III:

$$H_{design} = 33.4 + 0.52\ H_{extrapolated} \tag{11}$$

The correlation coefficient is 0.66.

Thus, within the range of values considered, there is a fairly good correlation of the data; however, both criteria tend to yield unconservative results when compared to the observed behavior of actual culvert installations. Consequently, it appears that a more complete definition is required for the various parameters used in the Iowa deflection formula.

STATISTICAL EVALUATION OF E′

There are really insufficient data to do a truly meaningful statistical evaluation of the data concerning E′ as given in Table 1. However, because no other data are available, the most one can do is to provide a statistical analysis of the 18 observed values.

The first case is group I, the second is group II, the third is groups I and II combined, and the fourth is groups I, II, and III combined. The first three cases indicate a fairly symmetric clustering of the data around the 300- to 600-psi range but with a slight tendency for the number of observations to decrease more gradually on the higher side than on the lower side. The histogram in such a case is said to be skewed to the right. The fourth case produces a much greater skewness. This skewness leads one to suspect that the normal, or Gaussian, distribution, which is symmetric, will not represent all four cases well. Three common probability distributions that exhibit skewness toward higher values are the Rayleigh, exponential, and log-normal distributions.

A useful statistical aid is a graphic display of the observed "cumulative" made by plotting the data on a special graph paper. In Figure 5, the observed "cumulative" for

Figure 1. Pressure pattern around flexible pipe (10).

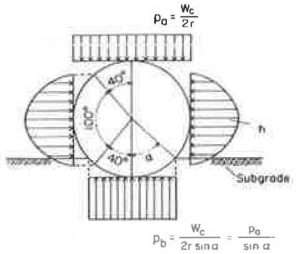


Figure 2. Equivalence of corrugation configurations and R^3/EI values.

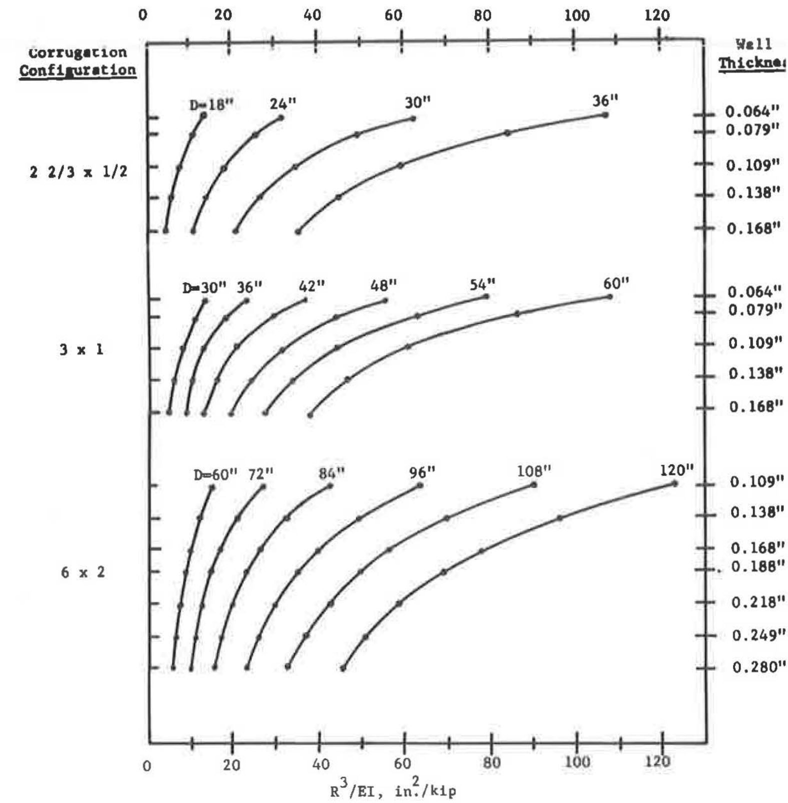


Figure 3. Fill height as a function of assumed criteria.

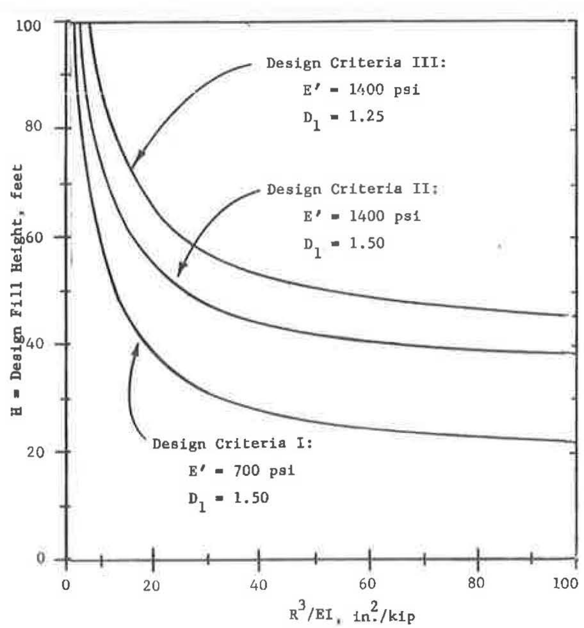
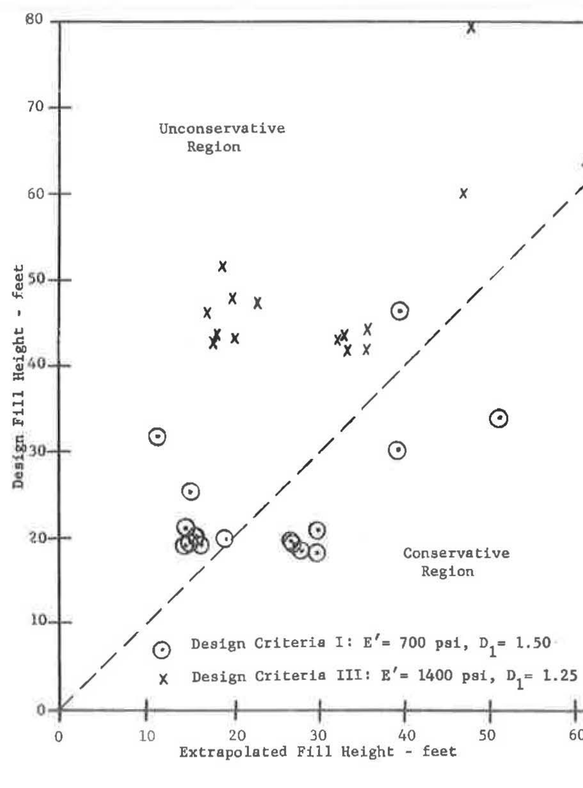


Figure 4. Fill heights for 5 percent long-time deflection.



groups I, II, and III combined is graphed by using four different types of probability scales: normal, Rayleigh, exponential, and log-normal. The more colinear are the observed points on a particular scale, the better is the assumption that the data follow that particular distribution.

It is observed that graphing the observed "cumulatives" on a log-normal probability scale produces a result significantly closer to a straight line than with the other distributions. It should be noted that there are not sufficient data to expect exact fit for any proposed distribution.

Some important statistics of the data are given in Table 4. The observed mean, m, is found from the data as

$$m = (1/n) \sum_{i=1}^{n} E_i' \tag{12}$$

where E_i' is one observed value of E' and n is equal to the number of observed values.

The observed standard deviation, σ, a measure of dispersion in the observed data, is found as

$$\sigma = \left\{(1/n) \sum_{i=1}^{n} [(E_i' - m)^2]\right\}^{1/2} \tag{13}$$

The observed coefficient of variation, V, is simply σ/m. The observed median, $\check{m}$, is the central value of the observed data.

Using the observed mean and standard deviation, we fitted a log-normal distribution to the data for all four groupings of the data. From this fitted distribution, the model-calculated median and standard deviation of the logarithm of E' were found. The appropriate formulas are

$$\check{m}^* = m \exp [(-1/2)\, \sigma_{Ln}^{*2}] \tag{14}$$

$$\sigma_{Ln}^* = [\mathrm{Ln}\, (V^2 + 1)]^{1/2} \tag{15}$$

If we consider the number of observations, the observed and model-calculated medians given in Table 4 agree rather well.

In order to substantiate the adoption of the log-normal distribution, we performed a chi-square test on the observed data. This test is a measure of the difference between the observed values of the histogram and the values predicted by the assumed distribution. The difference, d_1, is compared with a table value of the chi-square distribution for a given level of significance. If d_1 is less, the deviations are not considered significant (at the given level), and the assumed distribution may be accepted.

Because of the limited amount of data at hand for this problem, the results of a chi-square test must be used cautiously. The test was carried out at the common 10 percent significance level. The results are as follows: group I, d_1 = 1.64 and chi-square = 2.7; group II, d_1 = 0.66 and chi-square = 4.6; groups I and II, d_1 = 0.97 and chi-square = 4.6; groups I, II, and III, d_1 = 70.31 and chi-square = 12.0. The chi-square value has been found as a function of the number of degrees of freedom in the deviations.

For groups I and II separately and I and II combined, the deviations are well below the chi-square level, and the log-normal distribution is acceptable. For groups I, II, and III combined, however, the chi-square test indicates rejection of the assumed distribution. Data often fail a chi-square test because of the uncertainty surrounding the tail areas of a distribution. This is especially true when insufficient data are available. In this case, a graphic Kolmogorov-Smirnov test can be performed. Such a test defines an acceptable region, at a given significance level, for observed data surrounding the assumed distribution. It is based on the fact that the maximum difference between the "cumulative" of the histogram and the "cumulative" of the assumed distribution is a variable whose distribution is independent of the assumed distribution. Figure 6 shows

Figure 5. Probability "cumulatives" for groups I, II, and III.

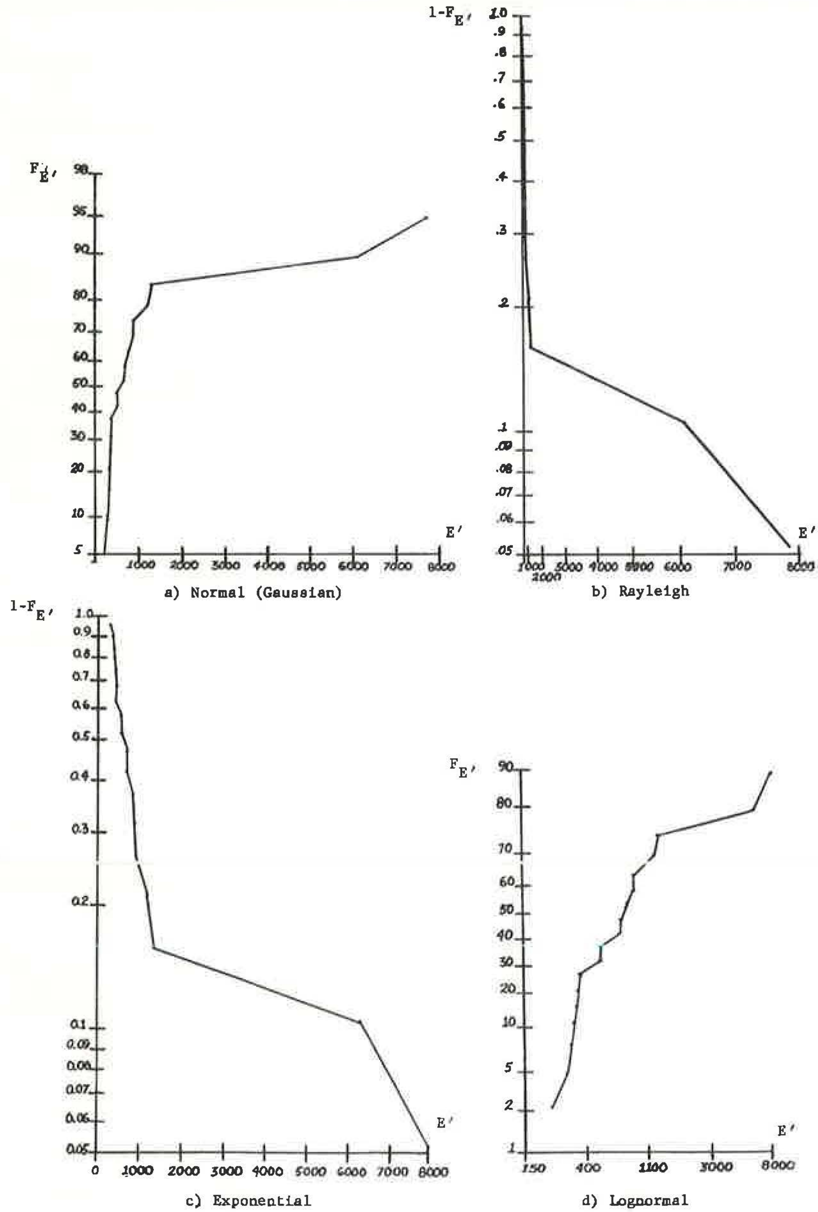


Table 4. Observed and calculated statistical parameters for E′.

Group	Observed Mean (psi)	Observed Standard Deviation (psi)	Observed Coefficient of Variation	Model Calculated Standard Deviation of Log	Observed Median (psi)	Calculated Median (psi)
I	471.4	182.42	0.3870	0.3736	368	439.63
II	733.6	326.04	0.4444	0.4246	861	670.37
I, II combined	558.8	269.99	0.4832	0.4581	502	503.15
I, II, III combined	1,332.33	2,094.15	1.5718	1.1155	600	715.18

Figure 6. Kolmogorov-Smirnov test for groups I, II, and III.

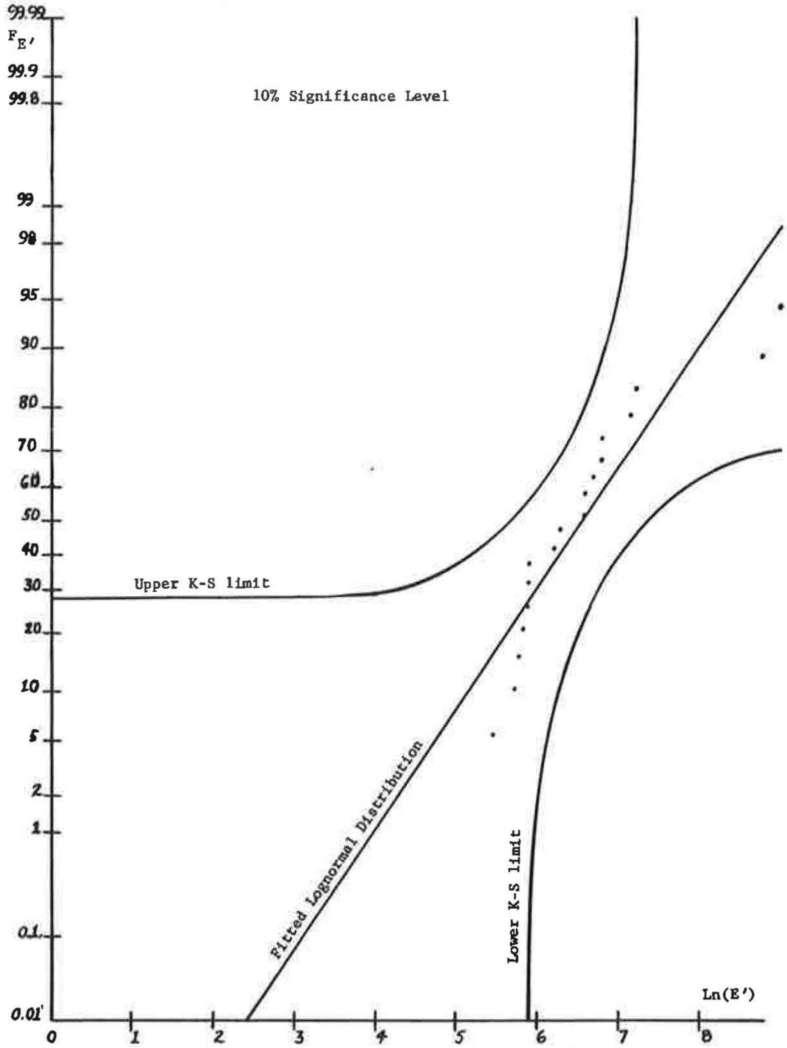


Figure 7. Probability ranges for data in groups I and II.

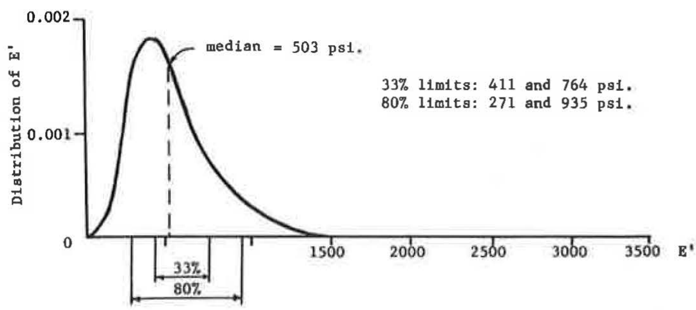
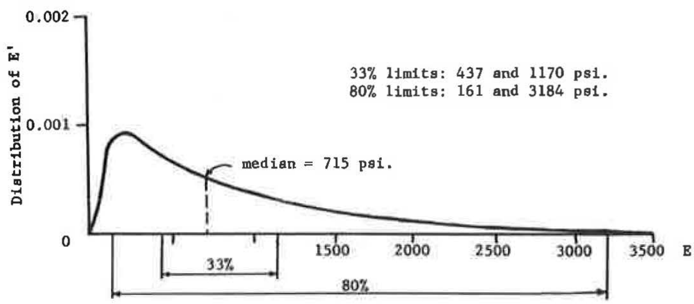


Figure 8. Probability ranges for data in groups I, II, and III.



a Kolmogorov-Smirnov test on groups I, II, and III combined at the 10 percent significance level. Because all of the observed data points are contained between the upper and lower Kolmogorov-Smirnov limits, the fitted log-normal distribution may be accepted at the 10 percent significance level.

It is desirable to determine ranges of probable levels for E′. To do this, the log-normal distribution was used with the parameters given in Table 4. A standardized normal variable, u, was determined from the general log-normal variable, E′, by the following relation:

$$u = (1/\sigma_{Ln})\ \mathrm{Ln}\ (E'/\check{m}) \tag{16}$$

Design levels were chosen from the student t distribution. This distribution is similar to the normal, but it reflects the uncertainty associated with having only a finite number of data observations. Both 80 percent and 33 percent ranges were found. The results are shown in Figures 7 and 8 respectively for groups I and II combined and groups I, II, and III combined.

The 80 percent range is defined as the range in which there is an 80 percent probability that an observed value will lie. This means there is a 20 percent probability that an observed value will lie outside of the range. The limits of this range have been chosen so that there is a 10 percent probability that an observed value will be below the range and a 10 percent probability that an observed value will be above the range. Notice that such a definition produces unsymmetric limits with respect to the median. This is because of the skewed nature of the log-normal distribution.

Based on the information shown in Figures 7 and 8, it can once again be observed that the wide spread of the data as given in Table 1 makes it extremely difficult to justify the use of E′ values in excess of the median values. Thus, from a statistical point of view, it is essential that additional data be recorded from full-scale culvert installations in order to obtain a statistically meaningful and realistic value of the modulus E′ for design purposes.

CONCLUSIONS

Since its introduction more than 30 years ago, the Iowa deflection formula has been utilized as a fundamental design criterion for corrugated metal pipe. The Iowa formula is based on an assumed distribution of loading around the pipe, including the total vertical load, W_c, as predicted by the Marston load theory. In addition, the Iowa formula contains three parameters, D_l, K, and E′, which are empirical quantities. To date, no extensive study has been made to establish a sound basis for estimating or evaluating these very important parameters. Realistic variations of these parameters can yield an almost threefold variation in the design requirement for a particular installation of pipe.

Spangler has collected data concerning experimentally determined values of E′; however, as given in Table 1, these 18 values vary over a very wide range. The modulus of soil reaction E′ is not a basic material property whose value can be determined from field or laboratory tests. The values of E′, which have been suggested for design use, are based on empirical considerations, and this study has shown that no strong correlation exists between this modulus and the parameters of the soil-culvert systems whose properties are given in Tables 1 and 2. A statistical study of these data confirms the fact that there is no rational basis for assigning any value for E′ that is greater than the median value.

This study has shown that the Iowa deflection formula per se can be an effective design tool. However, there are insufficient data available in the open literature to assist the designer in making realistic and rational decisions concerning the establishment of appropriate and reliable values of the parameters r_{sd}, W_c, D_l, K, and E′. Until such time that sufficient quantities of additional field data can be obtained and analyzed in a statistical sense, the designer should exercise extreme caution and discretion in assigning values to all of these parameters in the Iowa deflection formula.

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DISCUSSION

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The paper by Parmelee and Corotis is a very welcome addition to the literature that deals with the structural design and performance of flexible conduits under earth fill loads. The authors have reviewed the development of the Iowa deflection formula and have pointed out the empirical and semi-empirical nature of the deflection lag factor, D_1, and the modulus of soil reaction, E'. They have skillfully applied methods and principles of the science of statistics to a determination of the adequacy of currently available data on which to base selections of workable values of these parameters. The writer is in complete agreement with the authors' conclusion that there is insufficient knowledge available at this time to enable a designer to select realistic and economical values of the needed parameters.

At the time the deflection formula was developed, experimental evidence clearly indicated the important influence of the modulus of soil reaction, but the range of the experiments was relatively narrow. Since then, applications of the equation to actual situations has revealed that E' appears to vary over a very wide range, from as little as 234 psi to as much as 8,000 psi. These facts indicate and the authors' statistical analyses confirm the need for a massive program of field measurements to establish a working body of data for use in this area of design. The paper provides a valuable background and guidance material for any future study directed toward fulfilling this need.

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