# USE OF RANDOM-SEARCH TECHNIQUE TO OBTAIN OPTIMAL LAND USE PLAN DESIGN 


#### Abstract

Kumares C. Sinha and Jerome T. Adamski, Marquette University; and Kenneth J. Schlager, Venture Computer Systems, Inc.

This paper deals with the problem of land use plan design. An efficient land use plan design model can provide a ready tool to achieve the optimal future planfor an urban area or a regionthrough the satisfaction of the design constraints and at an optimum of public and private costs. After the basic features of a land use plan design model are discussed, the difficulties associated with the existing model procedure are examined. The use of a simple procedure based on a random-search technique is then evaluated. The validity of the random technique is established through a series of small-scale, controlled experiments with hypothetical areas. The controlled-experiment procedure is also used to estimate the planeffectiveness parameters involving the random method.


-IN RECENT years considerable interest has been directed toward the formulation of mathematical models to describe the process of land use planning and design. Under the general heading of the land use model, there are 2 distinct types of models: design model and simulation model. A land use simulation model is an attempt to describe the process of supply and demand of land for various activity uses in an area over a space of time under a set of public and private decisions. Such a model tends to trace the future land use pattern that will evolve from the existing pattern under certain given conditions. On the other hand, the concept of a land use design model is to create an ideal land use design for an area at some future year-an ideal plan that will minimize the total cost as well as satisfy the community development objectives and design standards. In other words, a land use simulation model attempts to predict what the land use pattern will be, and the design model attempts to depict what the land use pattern should be. To be precise, the fundamental distinction between a land use simulation model and a land use design model is essentially the functional distinction between the positivistic and normative models.

## BACKGROUND INFORMATION

## Basic Features of a Land Use Design Model

Most of the land use models developed in the past few years are simulation models and are positivistic in nature (1, 4, 7). These models are concerned with the problem of land use forecasting and are used to design only in a trial-and-error fashion. Little attempt has been made, however, to develop a normative model that will provide a target plan for an area. The present study involves the development of a land use design model that will offer a ready tool to achieve the ideal future plan for an urban area or a region at optimum of combined public and private costs. Such an optimal design is effected through the satisfaction of the constraints imposed by a series of predetermined design standards for the elements of the proposed plan. In other words, that plan is chosen from a series of alternative plans that best satisfies both cost and design constraints. The usefulness of such a model is evident from the fact that it will enable practicing planners and engineers to arrive at a desired pattern of future

Sponsored by Committee on Transportation Systems Design.
land use through a more systematic and expeditious process than that offered by the conventional approach.

A land use design model can be viewed as a design tool for the land use pattern of an area. The total available land is quantified by dividing the plan area into a number of cells and by specifying the size and location of each cell in the area. The design demand is established by the land area required by each of the discrete land use activities or elements, such as residential neighborhoods, schools, hospitals, and parks. These discrete land use elements are termed modules, and the entire land use system of an area is expressed in terms of a set of modules. The basic operation of a land use design model consists of the placement of given modules in the specified cells of the plan area. In the following paragraphs, the functions and definitions of some of the key elements of the model are discussed.

## Modules

In the land use plan design process, the modules are the basic building blocks. A module, as it is used in the model, is a discrete design unit expressed as a physical entity, for example, a single-family residential area, a neighborhood commercial area, an industrial area, or a recreational area. Each module unit is expressed in terms of space required for the primary land use activity as well as the secondary service areas necessary for the proper functioning of the activity concerned. For example, a module for a neighborhood commercial center might have as the primary area a building site for the stores and as supporting areas a parking lot, truck-loading facilities, warehouses, internal vehicular circulation space, pedestrian malls, open space and landscaped places, ingress-egress zones, and arterial and collector street rights-of-way. In this way, the entire land use activity of an area can be decomposed into a set of discrete, self-sufficient modular units of uniform functional characteristics. The number of modules that may be located in the area under consideration will depend on the space requirements for various land use activities at the design year. These space requirements are obtained by translating the forecast information of socioeconomic variables such as population and employment on the basis of the design standards that are established by the planning agency of the area under consideration.

## Cells

After the module types are defined and the numbers and sizes are established, the plan area is divided into a number of cells-spatial units that have more or less uniform characteristics, such as soil and topography or natural or man-made boundaries. Although the shape of these cells can have almost any pattern, there is a limitation on the size; the smallest cell should be large enough in area to accommodate at least one of the largest modules. The delineation of the cells, therefore, depends on the specification of the modules and their sizes. Because the size of the modules depends on the type of plan design, the size of the cells is also influenced by the level of planning; therefore, the cells for a regional plan would obviously be much larger than the cells for a community plan.

## Plan Constraints

After the modules are defined and the cells are delineated, it is necessary to identify the constraints that are associated with the land use plan of the area. These constraints form an essential part of the plan design process, for as a whole they control the feasibility of a plan. They are derived from the general planning objectives and, consequently, from the specified design standards. As model input, these constraints are expressed in mathematical form, either by a binary standard or in terms of quantifiable distances. As an example of a binary standard, a particular constraint might be so established that a specific module can or cannot be placed in certain soil types. On the other hand, other constraints might be of a nature that a specific module must be within or at least a certain measurable distance from other module types, and so on.

In general, these constraints can be classified into 2 major groups: cell-module constraints and intermodule constraints. The cell-module constraints are those that exclude location of specific modules in certain cells consisting of incompatible soil. This type of constraint can also be used to preserve some cell areas for the exclusive employment of specific land uses. For example, the cells of a regional corridor that consists of wildlife habitat, wetlands, forests, and woodlands can be excluded from the module-placement process. The intermodule constraints are specified to ensure spatial accessibility and compatibility among the module units, and they are expressed in terms of spatial distances between modules. For example, a constraint can be established specifying that a school must be within a 2 -mile radius from the neighboring residential areas, or a sewage treatment plant must be placed at a distance of at least 1 mile from the closest residential areas.

## Site and Linkage Costs

The public and private costs associated with the placement of all land use activities can be broadly divided into 2 basic categories: site development cost, which includes the construction and maintenance costs of the module elements, and linkage cost, which consists of construction, maintenance, and operation costs of facilities such as transportation routes, water and sewer lines, and connections for other public utilities between a pair of module units. The site costs are computed on the basis of the soil type and the type of module unit, and they are expressed as dollars per acre of module size. The linkage costs are dependent on the types of module unit to be linked and on the comparative sizes of these units. The linkage requirements for any pair of module types are determined, and construction and maintenance costs per unit distance of linkage as well as the vehicle operation and road-user costs are calculated. The cost values represent present worth values of all cash flows for an interest rate of 6 percent considered during a period of 20 years.

## EXISTING PROCEDURE OF MODULE PLACEMENT

The original attempt to develop a design model for a land use plan of an area was initiated at the Southeastern Wisconsin Regional Planning Commission (3, 5, 6). Although the work of the commission firmly established the potential value of a land use design model, the present form of the model is not adequate to develop realistically an ideal land use pattern for an area. The most serious objection to this model is the technique used in the spatial placement of the activity modules.

In order to locate a module in a cell, the design area is successively divided in half, and module elements are assigned to either of the 2 halves of the partition so as to minimize the combined site and linkage costs in the selected partition. The evaluation of minimum costs is made by means of a hill-climbing procedure, and such an evaluation continues until no improved partition can be obtained by shifting a unit element from one half of the partition to the other half. The entire sequence of partitioning is repeated again and again within each of the halves of the preceding scanning process until all the module elements are assigned to cells in the last partition. In this process of module placement, no module once located in one half of a partition can ever be reassigned to the other half in a later scanning.

The technique, which has been utilized in this model, of set decomposition in a series of binary partitions has an inherent shortcoming in its failure to account for the possibility that a particular module element might have been better placed in a different topographic area after the initial partitioning had placed it earlier in a less desirable area. To remedy such errors, called holistic, the provision of higher value partitions in model operations has been suggested (5). However, such a modification might not be too advantageous for the following possible reasons:

1. Although this modification might be expected to cut the holistic errors to some extent, it would not eliminate the errors completely. For example, in a 3 -way or 4 -way partitioning process, the possibility of a particular module element being trapped in a certain cell would obviously be less than that in a 2 -way or 3 -way operation respectively; however, it would still be subject to a certain degree of holistic errors.
2. Even if it were at all possible to establish a level of multi-way partitioning, which would reduce the holistic errors to an insignificant degree, the incorporation of such an improvement in model operation would result in an excessively large number of computations, and this in turn, would increase the computer time to an impractically large amount.

## RANDOM-SEARCH TECHNIQUE

The most ideal model operation would be an exhaustive search to develop a series of experimental plans by placing each of the modules in each of the cells and sequentially evaluating the respective costs in order to arrive at an optimal design. Such an operation is practically impossible in a complex system with a large number of cells and modules. However, a probabilistic procedure can be adopted to eliminate the large number of trials required in such an exhaustive search. The random-search technique, as discussed by Brooks (2), can be modified and used in the module-placement process. In this probabilistic procedure, a set of experimental plans is developed through the combination of module-cell arrangement designed in a random fashion. The estimate of the best plan is simply that experimental plan where the random assignment of the module-cell combinations produces the lowest total cost and best satisfies the design constraints.

In applying this technique, one can assume that there is an optimal zone of modulecell combinations, which contains a number of best alternative plans. This optimal zone can be defined a priori by establishing the level of plan accuracy that can be assumed as the proportion of optimal zone plans in the entire space of possible experimental plans. Another element that has to be decided a priori is the probability of "success," or the probability that at least one of the experimental plans is contained in the optimal zone. This probability can be expressed as

$$
S=1-(1-a)^{n}
$$

where
$S$ = probability of success of the experiment,
$\mathrm{a}=$ level of plan accuracy, or the ratio of optimal zone to total number of possible experimental plans,
$1-\mathrm{a}=$ probability that a trial plan will fail to be made inside the optimal zone, and
$\mathrm{n}=$ number of trials required to obtain the best plan with the plan accuracy a and the probability of success $S$.
Solving the above equation for $n$, we get

$$
\mathrm{n}=\log (1-\mathrm{S}) / \log (1-\mathrm{a}) \quad\left\{\begin{array}{l}
0 \leq \mathrm{S} \leq 1 \\
0 \leq \mathrm{a} \leq 1
\end{array}\right.
$$

By predetermining the values of $S$ and $a$, we can obtain the value of $n$ from the equation given above. Table 1 gives the respective values of $n$ for corresponding values of a and S. Even if the optimal zone is assumed to be relatively too small or, in other words, if the number of possible alternative best plans is too small compared to the total number of possible experimental plans, the number of trials required to achieve the best plan with a very high probability of success is not more than 919 . Furthermore, the number of trials required to achieve a certain level of plan accuracy as well as probability of success does not depend on the number of module-cell combinations. Therefore, such a technique can be conveniently applied to the land use design problem of a comparatively large area or region.

## MODEL ALGORITHM

The operation of the random-placement method as applied to a land use plan design model is briefly discussed in the following steps:

1. Input information is fed to include cell data, site and linkage costs, constraint schedule, and plan effectiveness parameters a and S.
2. A module type is chosen through the use of a uniformly distributed randomnumber generator. If the module chosen is one that has been used before, more random numbers are generated until an acceptable one is found.
3. A cell is then chosen for this module again through the use of a uniformly distributed random-number generator. If the cell chosen is already occupied with a module, another number is generated until one is found that is unoccupied.
4. Before a chosen module is placed in a chosen cell, a check is made to test whether such a placement violates the site and design constraints. The scanning continues until all the modules are assigned to the cells. At this point one experimental plan has been developed.
5. Site costs for the individual modules are computed, and the total site cost of the plan is obtained.
6. Linkage costs for all required links between the different module types are calculated, and the results are totaled. The total cost of the experimental plan is the combined total site cost and total linkage cost.
7. The entire procedure is repeated for as many trials as necessary to obtain the desired plan accuracy and the probability of success that are both specified as input information.
8. During the iterations, the minimum total cost and the module-cell arrangement that gives this cost is stored as running data. At the completion of the trials, the optimal plan and its cost are printed out.

## VALIDATION OF THE PROPOSED TECHNIQUE

Controlled experiments were conducted to test the validity of the random-model algorithm. The results obtained from the random algorithm were compared with the results generated by an algorithm based on the exhaustive-search technique. This was accomplished by considering a number of hypothetical study areas consisting of 10 to 15 cells and a total land area ranging from 1,600 to 2,400 acres. Because of the limitation imposed by the number of iterations required for an exhaustive search (101 or $3,628,800$ iterations or number of possible plans for 10 cells and 10 modules, excluding the repetitions of a plan), the study area was not made any larger. Each cell contained a combination of soil types, and no 2 cells had the same soil combination. The experiments were conducted in the following 3 steps:

1. Optimal plan based on site and linkage costs;
2. Optimal plan based on all costs and only positive intermodule constraints; and
3. Optimal plan based on all costs and both positive and negative intermodule constraints.

In each step, experiments were made with various given cell-module combinations. The two parameters a and $S$ involving the plan effectiveness in a random-search technique were also varied over a range. Furthermore, trials were made with the same cell-module combinations and plan effectiveness parameters but with different random-number seed values to initialize the random-number generators.

Both site and linkage costs for the given soil and module types were prepared on the basis of data obtained from the Southeastern Wisconsin Regional Planning Commission. The module definitions and the types of intermodule constraints were kept the same as those used in the existing model. This was done in order to coordinate the present research with the commission's ongoing work on the problem of land use plan design.

In general, the probability obtained experimentally of a given plan falling within the predetermined optimal zone was observed to be greater than the theoretical value. This would give an overall indication that the random procedure of module placement can be used with a good degree of success. Apart from the testing of the validity of the random technique, the controlled-experiment procedure was made also to estimate the optimal values of the parameters involving the plan effectiveness. A more detailed description of the experiments and their results is given in the following paragraphs.

## Experiment 1: Random Placement With No Constraints

In this series of experiments, the total cost of any one plan is equal to the summation of site and linkage costs. Because each cell was assigned with its own individual soil characteristics, the site cost, which is soil-dependent, was different for each module placement. The cost data were derived from the data used by the Southeastern Wisconsin Regional Planning Commission for the village of Germantown, Wisconsin (5). The study area consisted of 10 cells, each one having an area of 160 acres. Each cell was made 0.5 mile square to aid in defining the locations of the cell centroids, for all linkages are measured from centroid to centroid. Only 5 module types were included in this set of experiments in order to limit the number of iterations in exhaustive search. The module types and their space requirements are as follows:

| Type | Number |  | Acres |
| :--- | :---: | ---: | ---: |
|  |  |  | 126.10 |
| Low-density residential |  |  | 62.70 |
| Medium-density residential |  |  | 64.40 |
| Neighborhood park |  |  | 64.00 |
| Neighborhood commercial center | 4 |  | 20.00 |
| Secondary school | 5 |  |  |

Both the site and linkage costs are given in Table 2. For site costs, the rows are the different cell numbers and the columns are the different module types. Each element of the matrix gives the cost of placing that particular module type in that particular cell. For linkage costs, both the rows and the columns are the various module types. In this case, any one element in the matrix gives the aggregated cost per mile of linking any one module type to another. These linkage costs take into account the cost for connecting module types not only by roads or highways but also by water, sewer, gas, electric, and telephone lines.

Double counting was eliminated in the process of linking module types by a linkagesatisfaction matrix that was set up. This matrix is of the same size as the linkage cost matrix, and it is operated by a simple binary code where 0 means that a particular linkage is not satisfied and 1 means that the linkage is satisfied. The linkage-satisfaction matrix is initialized with all 0 's, and it simply becomes a matter of providing linkages until all the elements in the matrix become 1.

The exhaustive-search algorithm was then used to develop all possible experimental plans. Inasmuch as there were 10 cells and 5 modules, the number of combinatorial, nonrepetitive plans was $10 \times 9 \times 8 \times 7 \times 6$ or 30,240 plans. As a part of the output, the total plan costs were printed in descending order. The output data were used to define the optimal zone as well as to obtain the associated plan costs in order to check the results obtained from the random-search algorithm.

Todiscount any experimental error, we ran the random algorithm several times with different values of a and S. The results obtained from these runs are given in Table 3. These results seem to indicate that the random procedure provides an effective technique when no constraints are assigned with the module-placement process; all the best plans obtained from the random-model runs are well within the specified optimal zone. Therefore, the next step was to determine whether constraints put on the module placement would have any effect on the performance of the technique.

## Experiment 2: Random Placement With Positive Constraints

In this experiment, the same set of modules and the same study area as used in experiment 1 were considered, and only positive distance constraints were added to the program. By a positive distance constraint is meant that certain types of modules cannot be farther apart than a specified distance. If, in any experimental plan, the modules are placed farther apart, then the plan is considered infeasible.

The exhaustive-search algorithm was run, and both the feasible and infeasible plan costs were printed out in descending order. The number of feasible plans resulting from the run was 20,880 , while the number of infeasible plans was 9,360 . This gave a feasibility of 69.05 percent.

Table 1. Plans required to obtain at least 1 plan in optimal zone by random-placement method.

|  | S |  |  |  |  |  |  |  |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | :---: |
|  |  |  |  |  |  |  |  |  |
|  | 0.80 | 0.85 | 0.90 | 0.925 | 0.95 | 0.975 | 0.99 |  |
| 0.1000 | 16 | 18 | 22 | 25 | 29 | 35 | 44 |  |
| 0.0750 | 21 | 24 | 30 | 33 | 38 | 47 | 59 |  |
| 0.0500 | 32 | 37 | 45 | 50 | 59 | 72 | 90 |  |
| 0.0375 | 42 | 50 | 60 | 68 | 78 | 97 | 120 |  |
| 0.0250 | 64 | 75 | 91 | 102 | 119 | 146 | 182 |  |
| 0.0125 | 128 | 151 | 183 | 206 | 238 | 293 | 366 |  |
| 0.0100 | 161 | 189 | 230 | 258 | 299 | 367 | 459 |  |
| 0.0075 | 214 | 252 | 306 | 344 | 398 | 490 | 612 |  |
| 0.0050 | 322 | 378 | 460 | 517 | 598 | 736 | 919 |  |

Table 2. Input data for experiments.

| Item | Cell | Experiment 1 and 2 Modules |  |  |  |  | Experiment 3 Modules |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 |
| Site development cost | 1 | 859,337 | 1,098,970 | 124,538 | 166,663 | 50,000 | 859,837 | 1,098,970 | 402,204 | 166,663 |
|  | 2 | 753,013 | 1,002,898 | 92,871 | 496,912 | 50,000 | 753,014 | 1,002,899 | 302,619 | 496,912 |
|  | 3 | 646,190 | 906,827 | 63,204 | 827,160 | 50,000 | 646,190 | 906,827 | 203,034 | 827,160 |
|  | 4 | 570,050 | 846,836 | 60,128 | 773,276 | 50,000 | 570,050 | 846,835 | 142,533 | 773,275 |
|  | 5 | 493,910 | 786,844 | 75,016 | 719,390 | 50,000 | 493,910 | 786,844 | 182,032 | 719,390 |
|  | 6 | 522,857 | 827,096 | 55,112 | 703,690 | 50,000 | 522,857 | 827,096 | 176,204 | 703,690 |
|  | 7 | 584,524 | 866,962 | 59,158 | 765,425 | 50,000 | 584,523 | 866,962 | 189,619 | 765,425 |
|  | 8 | 560,819 | 808,316 | 59,348 | 754,155 | 50,000 | 560,819 | 808,216 | 189,633 | 754,155 |
|  | 9 | 475,448 | 709,804 | 55,492 | 681,150 | 50,000 | 475,448 | 709,804 | 176,232 | 681,150 |
|  | 10 | 503,273 | 802,468 | 59,824 | 758,950 | 50,000 | 508,273 | 802,468 | 191,704 | 788,950 |
|  | 11 |  |  |  |  |  | 577,231 | 854,648 | 197,069 | 793,055 |
|  | 12 |  |  |  |  |  | 667,643 | 904,387 | 289,218 | 433,907 |
|  | 13 |  |  |  |  |  | 684,055 | 950,719 | 296,654 | 462,807 |
|  | 14 |  |  |  |  |  | 676,874 | 942,907 | 292,113 | 443,027 |
|  | 15 |  |  |  |  |  | 499,153 | 768,450 | 176,218 | 692,420 |
| Linkage cost | $1{ }^{\text {a }}$ | 0 | 0 | 54,800 | 109,600 | 13,700 | 0 | 0 | 50,000 | 109,600 |
|  | $2^{\text {a }}$ | 0 | 0 | 54,800 | 109,600 | 13,700 | 0 | 0 | 50,000 | 109,600 |
|  | $3^{\text {a }}$ | 54,800 | 54,800 | 0 | 0 | 0 | 50,000 | 50,000 | 0 | 50,000 |
|  | $4^{8}$ | 109,600 | 109,600 | 0 | 0 | 0 | 109,600 | 109,600 | 50,000 | 0 |
|  | $5{ }^{\text {a }}$ | 137,000 | 137,000 | 0 | 0 | 0 |  |  |  |  |
| Intermodule distance constraints | $1^{2}$ | 10.00 | 10.00 | 1.75 | 1.75 | 4.00 | 10.00 | 10.00 | -0.70 | 2.00 |
|  | $2^{\text {a }}$ | 10.00 | 10.00 | 1.75 | 1.75 | 4.00 | 10.00 | 10.00 | -0.70 | 2.00 |
|  | $3^{\text {a }}$ | 1.75 | 1.75 | 10.00 | 10.00 | 10.00 | -0.70 | -0.70 | 10.00 | -0.70 |
|  | $4^{\text {a }}$ | 1.75 | 1.75 | 10.00 | 10.00 | 10.00 | 2.00 | 2.00 | -0.70 | 10.00 |
|  | $5^{\text {a }}$ | 4.00 | 4.00 | 10.00 | 10.00 | 10.00 |  |  |  |  |

${ }^{\text {a }}$ Modules.

Table 3. Results of random placement.

| Constraints | a | S | Number <br> of <br> Trials | Optimal Zone | Rank Number of Selected Plan |  |  |  |  | Feasible <br> Plans (percent) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  | Test 1 | Test 2 | Test 3 | Test 4 | Test 5 |  |
| None | 0.005 | 0.99 | 918 | $\leq 151$ | 54 | 13 | 15 | 4 | 11 |  |
|  | 0.005 | 0.95 | 597 | $\leq 151$ | 28 | 20 | 10 | 46 | 109 |  |
|  | 0.005 | 0.90 | 459 | $\leq 151$ | 26 | 7 | 1 | 82 | 5 |  |
|  | 0.005 | 0.80 | 321 | $\leq 151$ | 31 | 87 | 2 | 5 | 15 |  |
|  | 0.01 | 0.99 | 458 | $\leq 302$ | 14 | 14 | 31 | 215 | 1 |  |
|  | 0.01 | 0.95 | 298 | $\leq 302$ | 82 | 94 | 10 | 44 | 46 |  |
|  | 0.01 | 0.90 | 229 | $\leq 302$ | 35 | 120 | 137 | 223 | 164 |  |
|  | 0.01 | 0.80 | 160 | $\leq 302$ | 129 | 196 | 36 | 275 | 40 |  |
| Positive (method 1) | 0.005 | 0.99 | 1,165 | $\leq 104$ | 14 | 3 | 6 | 7 | 21 | 78.83 |
|  | 0.005 | 0.95 | 765 | $\leq 104$ | 41 | 15 | 6 | 101 | 46 | 78.10 |
|  | 0.005 | 0.90 | 588 | $\leq 104$ | 16 | 8 | 34 | 1 | 2 | 78.14 |
|  | 0.005 | 0.80 | 401 | $\leq 104$ | 141 | 31 | 52 | 8 | 22 | 80.12 |
|  | 0.01 | 0.99 | 584 | $\leq 208$ | 23 | 49 | 61 | 14 | 15 | 78.49 |
|  | 0.01 | 0.95 | 379 | $\leq 208$ | 10 | 56 | 23 | 41 | 9 | 78.69 |
|  | 0.01 | 0.90 | 294 | $\leq 208$ | 35 | 95 | 36 | 28 | 72 | 78.00 |
|  | 0.01 | 0.80 | 201 | $\leq 208$ | 18 | 201 | 25 | 205 | 14 | 79.72 |
| Positive (method 2) | 0.004 | 0.99 | 1,178 | $\leq 85$ | 87 | 6 | 25 |  |  | 78.22 |
|  | 0.004 | 0.95 | 768 | $\leq 85$ | 5 | 67 | 14 |  |  | 78.16 |
|  | 0.004 | 0.90 | 588 | $\leq 85$ | 4 | 37 | 39 |  |  | 78.75 |
|  | 0.004 | 0.80 | 411 | $\leq 85$ | 16 | 52 | 36 |  |  | 78.51 |
|  | 0.008 | 0.99 | 586 | $\leq 175$ | 111 | 62 | 5 |  |  | 77.26 |
|  | 0.008 | 0.95 | 374 | $\leq 175$ | 77 | 142 | 31 |  |  | 79.63 |
|  | 0.008 | 0.90 | 296 | $\leq 175$ | 38 | 271 | 135 |  |  | 77.70 |
|  | 0.008 | 0.08 | 205 | \$175 | 73 | 84 | 141 |  |  | 79.03 |
| Positive and negative | 0.005 | 0.99 | 2,516 | s68 | 41 |  |  |  |  | 36.49 |
|  | 0.005 | 0.99 | 2,574 | $\leq 68$ | 2 |  |  |  |  | 35.66 |
|  | 0.005 | 0.99 | 2,530 | $\leq 68$ | 54 |  |  |  |  | 36.28 |
|  | 0.005 | 0.99 | 2,556 | $\leq 68$ | 32 |  |  |  |  | 35.92 |
|  | 0.005 | 0.99 | 2,368 | $\leq 68$ | 4 |  |  |  |  | 38.77 |

The random algorithm was then run a number of times for various values of a and $S$. The problem of infeasibility of a plan was handled in 2 different ways. In the first case, only the feasible plans were included in the universe of possible experimental plans. An a-value and an S-value were chosen, and an $n$ was calculated from the formula. This $n$ was then the number of feasible plans that must be generated in order to obtain 1 experimental plan that falls within the optimal zone a of the feasible plans. In this procedure, it was not important how many total plans were generated as long as the number of feasible plans equaled $n$. The results obtained by running the random algorithm using this approach are given in Table 3. Based on the optimal zones as defined by the exhaustive search, the plans selected are well within the desired subregions in almost all cases.

Although the first method of handling the infeasible plans is the most direct and accurate approach, it would require comparatively long computer time as the complexity of the system would increase. Therefore, it was decided to test an approximate approach in dealing with the infeasible plans. In the second method, the infeasible as well as the feasible plans were included in the universe of total plan designs. Accordingly, an adjustment of the a-value, or optimal zone, was necessary and was accomplished in the following manner:

$$
a^{*}=(a)(P f)
$$

where
$\mathrm{a}^{*}=$ probability of obtaining an optimal feasible plan, a = plan accuracy, and
Pf = probability that a plan is feasible.
The original formula was then modified as

$$
S=1-\left(1-a^{*}\right)^{n}
$$

or

$$
n=\log (1-S) / \log \left(1-a^{*}\right)
$$

The occurrence of feasible plans was recorded for some initial iterations, and at some point in the program the percentage of feasible plans Pf was calculated. Because the same level of plan accuracy was desired, that is, an optimal zone of some small percentage of the feasible plans, the original a-value read in as input data was then multiplied by the percentage of feasible plans. The resulting a*-value along with the given probability $S$ was put back into the formula, and a new $n$ or number of experimental plans was calculated. The inclusion of plan constraints has the effect of increasing the number of experimental plans necessary to achieve a given level of plan accuracy. For example, if the specified values of a and $S$ are 0.05 and 0.90 respectively, then the number of experimental plans is 45 ; whereas, if the plan constraints are considered and the percentage of feasible plans Pf is observed to be 0.10 , then the number of experimental plans increases to 460 from 45 to achieve the same plan accuracy.

The results as obtained from the second method are also given in Table 3. The results would again indicate the effectiveness of the random procedure; the rank of the best plan is within the specified optimal zone in almost all trials. Comparing the number of trials required by this approach and those required by the first method shows that they do not vary considerably. However, there is a shortcoming inherent in this method of handling the infeasibility of plans. Because, in this case, the universe of total plans included both feasible and infeasible plans, the modified optimal zone also could contain, by chance, some plans that are not feasible. Furthermore, a comparison of the actual percentage of feasibility with the corresponding percentage values, as obtained in the trials conducted by this method, indicated that these values are on the average 9.36 percent higher than the value calculated by the exhaustive search.

This would mean that the adjusted value of a is larger than it should be, and, therefore, the required trials calculated would be less than they actually should be.

The percentage values of feasible plans as generated by the first method were also computed to check with the results of the second method. In this case, as in the second, the percentage values of feasibility were higher than the actual percentage. Having on the average a value of 9.71 percent higher than the known value would again seem to indicate that a running average from the random procedure would not provide a very reliable estimate of the percentage of feasible plans. It is also felt that this method would decrease in its reliability as the percentage of feasible plans would decrease.

Experiment 3: Random Placement With Positive and Negative Constraints

In this set of experiments a negative distance constraint was added to the constraint schedule. A negative distance constraint means that a certain module type, in this case a sewage-treatment plant, cannot be closer than a specified distance to any of the other module types.

The study area was increased to 15 cells in order to accommodate the negative distance constraint because too small a study area would cause a great percentage of infeasible plans. But the increase in number of cells necessitated a decrease in the number of module types to be considered so that the computer time to conduct an exhaustive search would not be excessive. Accordingly, the number of module types was reduced to 4 . Therefore, the number of combinatorial, nonrepetitive plans is $15 \times$ $14 \times 13 \times 12$ or 32,760 . The module types and their space requirements are as follows:

| Type |
| :--- |
| Low-density residential |
| Medium-density residential |
| Sewage-treatment plant |
| Neighborhood commercial center |


| Number | Acres |
| :---: | :---: |
| 1 | 126.10 |
| 2 | 62.70 |
| 3 | 45.00 |
| 4 | 64.00 |

An exhaustive search of all possible plans showed that there were 13,664 feasible plans and 19,096 infeasible plans. The percentage of feasibility was therefore 41.71 percent. The random algorithm was run only 5 times with the same values for a and S for each run. The number of trials was calculated on the basis of the feasible plans only. The results of these runs are given in Table 3. In each case, the selected plan fell within the desired optimal zone. Although the number of iterations necessary to generaie the required number of feasible plans was high, more than 2,500 in most cases, this number is still small compared with the 32,760 possible plans that would have to be checked if an exhaustive search were to be made.

The percentage of feasibility was also computed for each run. Comparing these values with the known percentage value of 41.71 percent shows that on the average these values are 5.09 percent lower than the known value.

## ESTIMATION OF PLAN-EFFECTIVENESS PARAMETERS

When the random method is used, the 2 parameters involving the plan effectiveness must be decided beforehand. These 2 parameters are the desired level of plan accuracy, a, and the probability of success of achieving an optimal plan, S.

Because a land use plan design process is a complex system that entails a large number of factors, an absolute lowest cost plan may not be attainable. Moreover, it can be reasonably assumed that in the total universe of experimental plans there exists a subregion, consisting of a number of plans whose total costs are close to the lowest cost, where the differences in plan costs are not so significant. Accordingly, a satisfactory solution to the problem of optimization can be well achieved if an experimental plan from this subregion is attained. In the random procedure, the relative size of this subregion of good response with respect to the total universe is defined as the plan accuracy. On the other hand, the definition of success used in the procedure is to ob-
tain an experimentally selected plan that falls within the desirable subregion, and the probability of success is the desired level of assurance that a planner would like to have of achieving an optimal land use plan.

To estimate the values of a and $S$ that might be reasonably used in a land use plan design mode, we conducted experiments with various cell-module combinations over a range of values for each of the 2 parameters. The soil characteristics of each cell were also varied to provide different cost values for placing the various module types in different cells. Furthermore, the random-number generators were initialized at the beginning of each trial with different random-number seed values.

First, an exhaustive search procedure was adopted in placing modules in cells, and plan ranks and their costs were obtained in descending order. The algorithm was run several times with changing soil characteristics. The results were then plotted to develop the cost curves. Some of the typical cost curves as obtained from trials with 4 modules and 8 cells are shown in Figure 1. Similar cost curves were also developed for several higher order cell-module combinations, but they are not shown here because of the enormousness of their sizes. It was consistently observed, however, that the assumption of the existence of a low-cost region in the universe of experimental plans is reasonable. Such a subregion or optimal zone is characterized by a valley at the end of the cost curves. This valley region, where the plan costs approximate the lowest cost, is shown in Figure 2, which shows the plan costs of 152 best plans out of a possible 30,400 experimental plans in a set of trials with 6 modules and 10 cells. Each curve shown in Figure 2 represents a trial with different soil characteristics and consequently with different site costs.

It was observed that the significant breaks in cost curves take place at points causing low-cost regions to include, on the average, about 5 to 10 percent of the total plans. Therefore, assumption of a plan accuracy value within a range of 0.5 to 1.0 percent would be very reasonable. Lower values would be desirable for plan design of small areas, such as in site planning or for those situations where cost estimates are more precise and accurate and where a comparatively small difference in plan cost could affect the planning decisions. On the other hand, higher values of plan accuracy would be reasonable for the design problem of large areas, such as a region where the model input data cannot be so precise and the planning decisions are dependent on maryfactors.

In the next set of trials, results from the random algorithm were compared with the exhaustive search output to obtain a distribution of best plan rank. In each trial 50 runs were made for each a and $S$ combination, and the rank of the best plan obtained from a random run was noted. The ranking was done on the basis of the exhaustive search output. The results obtained from the trials made with 4 modules and 10 cells are given in Table 4, where the distribution of best plan rank is presented for various combinations of a- and S-values. The best plan rank numbers for only 95 th percentile and lower values are shown because for higher percentile values the best plan rank numbers, in some cases, fell beyond the optimal zone. The percentile value indicates the probability that the best plan obtained from a particular random run will be at least the $x$ th lowest cost plan. For example, the probability that the best plan generated by a random run with a-value of 0.005 and $S$-value of 0.99 will be the fourth lowest cost plan or better is 0.80 .

It may be observed from the entries given in Table 4 that, for a given value of plan accuracy, the best plan rank number seems to become better as the S-value for the probability of plan success is made higher. However, although the $S$-value of 0.9 gave significantly better results than the $S$-value of 0.8 , the results did not improve appreciably as the $S$-value was increased to values higher than 0.9 . This trend was consistent for all the percentile values of best plan rank number distribution. Therefore, it would seem to be reasonable to assume that a range of 0.85 to 0.95 is an appropriate range of values for $S$. Because an increase in the $S$-value would mean an increase in number of iterations, the $S$-value could be taken as 0.9 in most cases without affecting the plan results. However, higher values can be considered in such cases where cellmodule combinations are not large, and lower values would be reasonable for a high order of cell-module combinations.

Figure 1. Costs of plans with 4 modules and 8 cells.


Figure 2. Optimal zones of costs for plans with $\mathbf{6}$ modules and $\mathbf{1 0}$ cells.


Table 4. Distribution of best plan rank.

|  | Percentile |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | ---: | ---: | ---: | ---: | :---: |
| a | S | 0.25 | 0.50 | 0.60 | 0.70 | 0.80 | 0.90 | 0.95 |  |
| $0.005^{\mathrm{a}}$ | 0.99 | 1 | 2 | 3 | 3 | 4 | 7 | 9 |  |
|  | 0.95 | 1 | 3 | 3 | 6 | 9 | 11 | 14 |  |
|  | 0.90 | 3 | 5 | 6 | 8 | 12 | 16 | 20 |  |
|  | 0.80 | 3 | 5 | 8 | 11 | 17 | 22 | 25 |  |
| $0.01^{\circ}$ | 0.99 | 3 | 5 | 6 | 8 | 12 | 16 | 20 |  |
|  | 0.95 | 3 | 5 | 8 | 11 | 17 | 22 | 25 |  |
|  | 0.90 | 3 | 9 | 12 | 14 | 18 | 25 | 33 |  |
|  | 0.80 | 4 | 16 | 21 | 23 | 28 | 37 | 38 |  |
| $0.025^{\circ}$ | 0.99 | 5 | 10 | 13 | 16 | 23 | 30 | 36 |  |
|  | 0.95 | 7 | 24 | 27 | 35 | 50 | 80 | 100 |  |
|  | 0.90 | 6 | 21 | 27 | 38 | 58 | 90 | 97 |  |
|  | 0.80 | 11 | 38 | 47 | 57 | 70 | 94 | 120 |  |

[^0]
## CONCLUSIONS

A land use plan design nolds immense potentiality in the field of land use planning. The model can be used to design a set of ideal plans for a series of forecast years ranging from 5 to 30 , each design being developed independent of others and based only on the initial conditions and the forecast requirements. The series of land use plan designs derived from the model will then display the most economic and efficient land use pattern that can be obtained at a particular design year. This, in turn, will aid in making decisions concerning the development of public and private policies regarding the use of land in a systematic and efficient way.

The model provides a normative tool rather than an evaluation process to arrive at an optimal land use plan design. While considering the role of the plan constraints in shaping the final solution, one should make a distinction between values and a design to satisfy values. The search procedure used in the model attempts to create a design, and the plan constraints are simply the limitations imposed on such a design. The model does assume a set of values that is reflected in a set of plan constraints. Because values are subjective, there can be no optimal set of values. The model is therefore a normative one given a set of values. The same statement can be made of any normative model.

The basic use of the proposed model is in preparing a target plan at any level of land use planning, such as neighborhood, community (town or village), metropolitan, regional, state, or even national level. Although at any of these levels the general structure of the model algorithm remains the same, the nature of the input data and plan constraints would be different from one level to another. The type of input data currently available makes it possible to apply the design model only at community, metropolitan, and regional levels. Furthermore, the model can be well utilized in capital works programming in a time-simulation framework. By running a series of design model runs on a 5 -year time increment starting back from the target year, the proper sequence of capital works programming can be determined. The greatest impact of the proposed model on metropolitan plan-making will probably be in establishing a standard or norm against which all proposed plans can be evaluated. Another application relates to measuring the cost of any suggested plan design constraints.

The use of a simple procedure for improving the operation of the model based on the random-search technique has been discussed here. The validity of the random technique was established through a series of controlled experiments where the results obtained from the random model algorithm were compared with the results generated by an algorithm based on an exhaustive-search technique. The study clearly indicated that the random method of module placement can be used with a good degree of success. Apart from the testing of the validity of the random technique, the controlled experiment procedure was also used to determine the appropriate values of the parameters involving the plan effectiveness.

It was the intent of the paper to evaluate the merit of the random-search technique as a useful tool in the preparation of a land use plan design. This has been established through the result of small-scale controlled experiments with hypothetical areas. However, the random algorithm is currently being applied in several real-world problems of land use plan design at different levels of planning in the southeastern Wisconsin region.

## ACKNOWLEDGMENT

The study was in part sponsored by the National Science Foundation, and this support is gratefully acknowledged. The necessary data were obtained from the Southeastern Wisconsin Regional Planning Commission, whose cooperation is highly appreciated.

REFERENCES

1. Arad, B. A. Urban Performance Model. Planning Research Corp., McLean, Va., Feb. 1970.
2. Brooks, S. A Discussion of Random Methods for Seeking Maxima. Jour. of Operations Research Society of America, Vol. 7, 1958.
3. Schlager, K. J. A Land Use Plan Design Model. Jour. of American Institute of Planners, May 1965.
4. A Mathematical Approach to Urban Design. Southeastern Wisconsin Regional Planning Commission, Waukesha, Tech. Rept. 3, 1966.
5. A Land Use Plan Design Model-Model Development. Southeastern Wisconsin Regional Planning Commission, Waukesha, Vol. 1, Tech. Rept. 8, 1968.
6. A Land Use Plan Design Model-Model Test. Southeastern Wisconsin Regional Planning Commission, Waukesha, Vol. 2, Tech. Rept. 8, 1969.
7. Empiric Land Use Forecasting Model. Final Rept., Traffic Research Corp., Washington, D. C., Feb. 1967.

[^0]:    ${ }^{2}$ Optimal zone, 25 lowest cost plans.
    ${ }^{\text {b }}$ Optimal zone, 50 lowest cost plans.
    ${ }^{\text {cop }}$ Optimal zone, 125 lowest cost plans.

