REINFORCING REQUIREMENTS FOR CONCRETE BEAMS WITH LARGE WEB OPENINGS

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The finite element technique was used to assess the effects of large web openings in the bent cap region of concrete box girder highway bridges. Bent caps were modeled that contained openings ranging in size from 0.21 to 0.625 of the member depth. The shape and location of the opening were varied. Sixty simply supported beams were also analyzed to provide the basis for a working stress design procedure suggested for use in determining the reinforcing requirements in the vicinity of the opening. The analyses showed that a design based on a single rectangular opening would be satisfactory as long as adjacent openings were separated by at least half the depth of the member. The Vierendeel method was found to be acceptable for designing the reinforcing in the chord regions of the opening, whereas special curves were developed to permit the design of the reinforcing needed to resist the stress concentrations at the corners. Twelve laboratory specimens were designed in accordance with the suggested design procedure. Subsequent testing indicated that the reinforcing provided around the opening adequately strengthened this portion of the member so that the load-carrying capacity was governed by the behavior of the solid part of the beam. AUTHOR

•INCREASINGLY, beams and girders are being designed with large web openings to provide passage for service conduits that, for either aesthetic reasons or headroom problems, cannot be suspended below the girder. Such openings alter the stress distribution significantly and usually require special reinforcing around their periphery. In highway construction this technique has been employed to place pipelines, electrical conduits, and drainage systems inside concrete box girder bridges. The cellular nature of the box girder highway bridge is particularly well suited for carrying service conduits; however, to provide an unobstructed pathway through the bridge requires that large holes be built into the bent caps (Fig. 1).

This paper presents the results of a study aimed at developing a suitable working stress design procedure for determining the reinforcing requirements around the web openings used in concrete box girder highway bridge bent caps. The finite element method was used to obtain the elastic stress distribution around various sizes, shapes, and locations of these holes, and a limited experimental program was undertaken to test the suitability of a proposed design procedure.

The analytical technique often used to determine the elastic stress distribution around holes in beams evolves from elasticity theory that uses conformal mapping techniques (1, 2, 3). In general the solutions apply to members where the opening is small in comparison to the face of the beam in which the opening is placed. Further, when this method is used, it is usually difficult to treat complicated boundary conditions and oddly shaped holes.

A different but common analytical method used to determine the stresses in the vicinity of the opening is the Vierendeel truss technique. This technique has been used

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Figure 1. Cutaway view of box girder highway bridge.

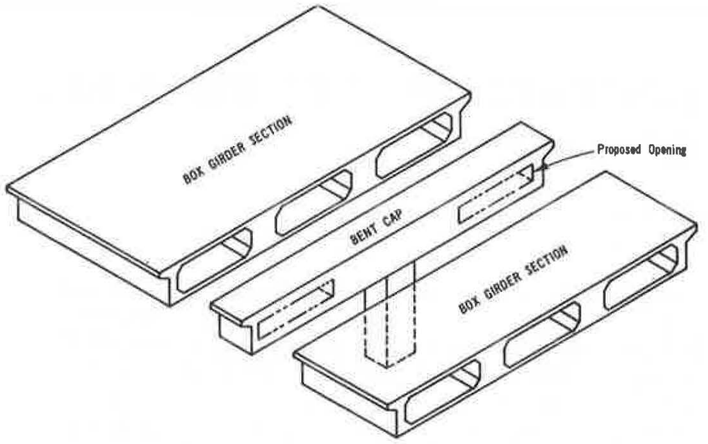
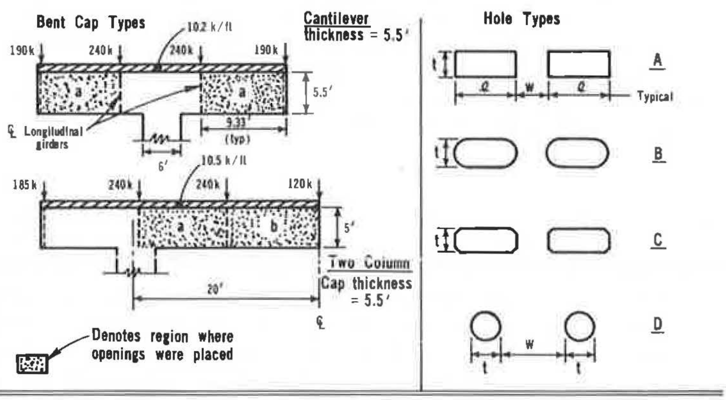


Figure 2. Cross section and dimensions of cantilever and two-column box girder highway bridge.

TABLE 1 BENT CAPS ANALYZED



Mark	Bent Type	Location of Opening (s)	No. Opening per bay	Hole Type	ℓ (ft)	W (ft)	t (ft)
B-0	Cantilever	None	—	—	—	—	—
B-1		a	1	A	4.0	0	2.0
B-2		a	2	A	2.0	2.75	2.0
B-3		a	2	A	2.0	1.0	2.0
B-4		a	1	A	6.0	0	2.5
B-5		a	2	A	3.0	1.0	2.5
B-6		a	1	C	6.0	0	2.5
B-7		a	1	B	6.0	0	2.5
B-8		a	2	D	–	2.0	2.25
C-1	Cantilever	a	1	A	7.0	0	0.5
C-8	Cantilever	a	1	A	7.0	0	4.5
D-0	Two Column	None	–	–	–	–	–
D-1		a	1	A	6.0	0	3.0
D-2		b	1	A	6.0	0	3.0
D-3		a, b	1	A	6.0	0	3.0
D-4		a	1	C	6.0	0	3.0
D-5		b	1	C	6.0	0	3.0
D-6		a, b	1	C	6.0	0	3.0
D-7		a	1	B	6.0	0	3.0
D-8		b	1	B	6.0	0	3.0
D-9		a, b	1	B	6.0	0	3.0
D-10		a	1	A	4.0	0	2.0
D-11		b	1	A	4.0	0	2.0
D-12		a, b	1	A	4.0	0	2.0
D-13		a	2	D	–	2.75	2.0
D-14		b	2	D	–	2.75	2.0
D-15		a, b	2	D	–	2.75	2.0
D-16		a	1	D	–	0	3.0
D-17		b	1	D	–	0	3.0
D-18	Two Column	a, b	1	D	–	0	3.0

Varies in 0.5-ft. intervals

to proportion reinforcements surrounding the openings in steel beams (5) and has been compared to a classical elasticity solution for both round and rectangular openings in steel beams (3). In his study, Bower (4) found that the Vierendeel analysis did not predict the stress concentrations that occurred at the corners of the holes, whereas the elasticity analysis did. Further investigation showed, however, that the simpler Vierendeel analysis was adequate for most design problems in steel inasmuch as local yielding of the material at the corners was permissible.

Previous experimental data on both steel and concrete members clearly indicate that, if properly reinforced, the region of the beam containing the opening does not prevent the member from supporting the same loads as when the hole was not present (4 through 13). Many of the tests were performed on beams subjected to pure flexure or flexure and small shear forces. It was generally concluded that a beam loaded to provide pure flexure in the region of an unreinforced central opening is as strong as a similar beam without a hole. However, under combined shear and flexure, the presence of an unreinforced opening often causes a reduction in beam strength. In a study of the effects of the interaction between two or more holes, it was concluded that adjacent, identical circular openings did not reduce steel I-beam capacities for the spacings tested, but identical, adjacent rectangular openings had a substantial effect when their spacing was less than one-half their depth (13).

The use of the finite element technique permitted an analysis to be made of the member where the following parameters were varied:

1. Magnitude of the shear and moment at the opening,
2. Size and shape of the opening, and
3. Influence of adjacent openings.

A subsequent review of these results was used to suggest a working stress design method to proportion the reinforcing around the web opening.

METHOD OF ANALYSIS

The various bent cap configurations were modeled on an IBM 7044 digital computer using a plane stress finite element computer program developed by E. L. Wilson (14). The material properties were assumed to be linearly elastic, homogeneous, and isotropic. The original program was slightly modified to provide for automatic finite element mesh generation. A contour plotting program (15) was used to generate contours of maximum principal tensile stresses. This was a convenient way to survey the entire stress field and find areas of particularly high stress.

Two groups of structures were analyzed. The first group consisted of 35 bent caps subjected to the HS20-44 live load as specified by AASHO and the state of California (16). The most severe loading condition was a standard lane load plus a concentrated load rider for shear. The bent caps were supported with either a single column at midspan (cantilever type) or two columns spaced 40 ft apart (Fig. 2). In both cases the loads and openings were symmetrically arranged. The overall geometry and loads for the cantilever cap were based on the example found in Chapter 6 of the California Division of Highways Manual of Bridge Design Practice, 2nd Edition. The loads used in the two-column bent cap were derived from a three-span continuous box girder structure having equal spans of 90 ft.

In practice the bent caps are often subjected to torsional effects due to asymmetrical loadings on spans adjacent to the cap. In the present study, such torsional effects are not included.

The second group of structures analyzed consisted of 60 simply supported beams containing one or two openings. Concentrated loads were applied at specified locations to produce desired moments and shears at the mid-length of the opening. All beams were the same overall size (11 ft long and 2 ft deep) and contained 2-ft long openings. Of the 60 beams, 44 were simply supported on 10-ft centers, whereas the remaining 16 were supported on $8\frac{1}{2}$-ft centers with a 2-ft cantilever (Fig. 3). The variables considered are as follows:

1. Ratio of beam depth h to hole depth t,
2. Shear-to-moment ratio M/V at the mid-length of the opening, and
3. Interaction of adjacent openings.

All openings were located at mid-depth of the beams.

ANALYTICAL RESULTS

Region Affected by the Opening

The principal tensile stress contours for representative cantilever bent cap configurations are shown in Figure 4. Comparison of the principal tensile stress patterns in the solid member (Fig. 4a) to similar members containing large web openings illustrates the vast alteration of the stress distribution. First, note that inflection points occur approximately at the midspan of the chords, particularly for those members with a single rectangular opening. These inflection points are key factors in the design of the chords where a Vierendeel analysis is used. Second, for all but the circular openings, stress concentration patterns were evident at each corner of the hole. As reported by Nasser, Acavalos, and Daniel (8) and predicted by the Vierendeel analysis, corners along a common diagonal have the same stress sign. For example, in the cantilever bent cap shown in Figure 4b the stresses in corners A and C are tensile, whereas those in B and D are compressive.

An examination of the principal tensile stress contours and the computer output showed that the disturbance caused by the addition of square, rectangular, and nearly rectangular openings was limited to a small region around the opening. For a bent cap of depth h, the stress approximately 0.46h from the edge of the hole was found to be essentially the same as in the solid member. Similar results were obtained from the analyses of the simply supported beams.

Effect of Size and Shape of the Opening

The effect of various hole sizes was studied by increasing the depth of a 7-ft long rectangular hole in a cantilever bent cap from 0.5 to 4.5 ft in 0.5-ft increments (Fig. 2). The resulting principal tensile stress contours showed that, as the depth of the hole increased relative to the height of the bent cap, the stress concentrations at the corners and the stresses in the chords also increased. The same effect has been reported by others for different geometries and loading conditions (1).

To study the effect that shape of the opening had on the magnitude and distribution of the stress concentrations around the corners of the opening, we analyzed members with rectangular, square, or circular holes. Additionally, analyses were done on rectangular openings with small corner fillets and openings having an oval shape. Holes of these shapes were selected because they are representative of those currently being placed in the web regions of flexural members.

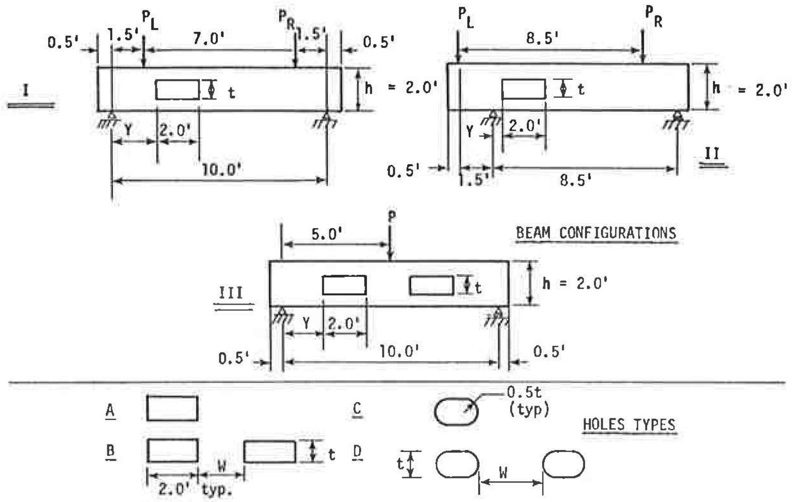
Analyses showed that the stress concentrations for the rectangular holes were not significantly different from the rectangular openings with filleted corners or those with rounded ends. For example, one may compare the stress contours in Figures 4b, 4d, and 4e. Not only did the circular openings have smaller stress concentrations, but also the distance affected by the opening was smaller than the same size rectangular openings. Compare Figures 4c and 4f.

The results suggest that a design procedure based on rectangular openings will be satisfactory for the usual shapes of holes and that the size of the opening, as represented by the depth, is a significant variable.

Effect of Adjacent Openings

The large spacing between the longitudinal girders of the bridge makes it possible to consider locating either one large opening or several smaller ones in the bent cap. To study the latter situation, we placed two holes in a member and varied the distance between them. Figure 5 shows how the stress distribution around a single hole, L, changed when a second hole, R, was introduced and moved closer.

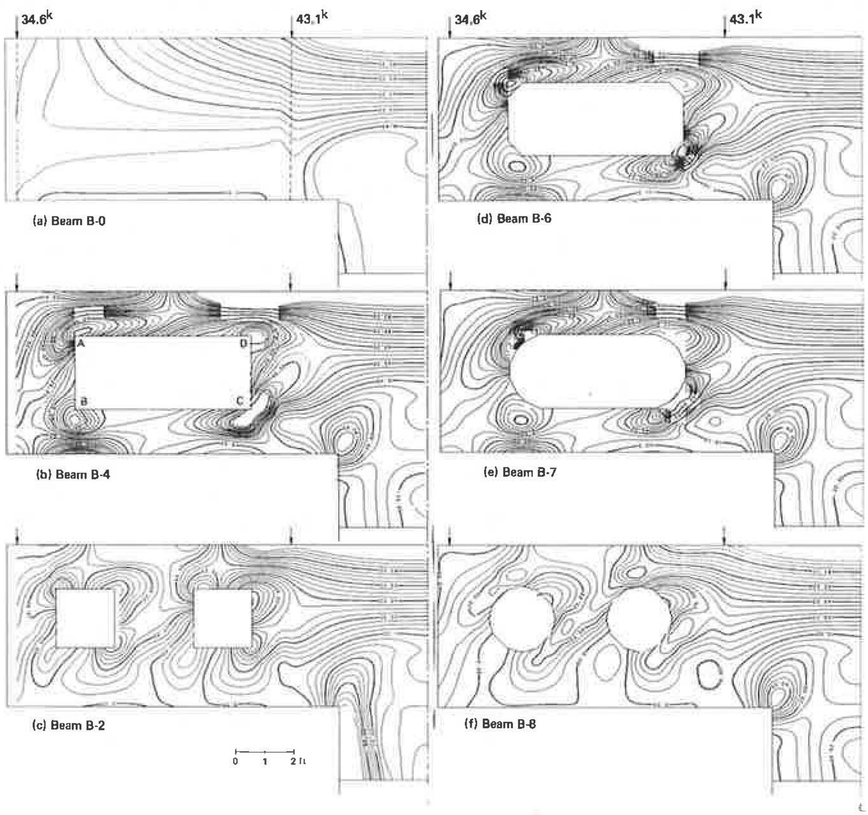
Figure 3. Beam configurations and hole types.



NOTES:

1. t/h varies from 0.21 to 0.625.
2. M/Vh varies from 0.1 to 6.48, obtained by using different values of P, P_L and P_R.
3. W varies from 0.5' to 2.0'.
4. Y varies from 0.5' to 4.5'.
5. See reference 17 for additional details.

Figure 4. Principal tension stress contours.



When hole R was farther than 0.5h away from hole L, the magnitude and distribution of the stresses around the corners of hole L changed very little from their values when hole R was not present. When hole R was closer than 0.5h the stress distribution around hole L changed significantly, particularly on the side closest to hole R. Similar results have been reported for experimental tests on steel wide-flanged sections (13).

Studies of the effects of multiple web openings in steel beams have shown that the resulting interaction equations for the shear and moment in the web post separating closely spaced holes are quite complex (7). Nevertheless, the equations were found to be conservative, and the single-hole analysis was found to be sufficient for the prediction of the failure load. Similar results for concrete members are not available, but it is clear that, when the holes are close (less than 0.5h separating them), special attention must be given to the design of the web post.

In the development that follows, it is assumed that adjacent holes are separated by at least 0.5h so that the design may be based on a single-hole analysis. In most situations, a single opening of the desired size can be more easily designed and fabricated than two separate openings.

Comparison of Finite Element and Vierendeel Solutions

The Vierendeel method is most often used to design the chord regions of beams containing large openings. In this study, the finite element and Vierendeel solutions were compared to determine whether the Vierendeel method could be used satisfactorily for different load and geometry variations. Fourteen of the simply supported beams were used in this case, and the resultant forces and moments occurring at the ends of the chords were determined by each method. Concentrated loads were applied in varying magnitudes to produce different moment-to-shear ratios at the middle of the opening.

A classical Vierendeel analysis assumes inflection points at the midspan of the upper and lower chords. The total shear acting at the vertical plane through the inflection points is assumed to be distributed to the two chords in proportion to their respective areas. Moments at the ends of the chords are determined from the combined effects of the external moment at the midspan of the chords and the product of the shears and the half-chord lengths (4).

Empirical evidence indicates that the assumption of the midspan inflection point location may be significantly in error. Experiments by Nasser, Acavalos, and Daniel (8) demonstrated that the point of inflection might occur in locations between the chord midspan and 40 percent of the chord length away from that point.

The finite element analysis confirmed this variation in inflection point location as a function of load and size of opening. Figure 6 shows these indicated variations in location as functions of the ratio of hole depth to beam depth t/h, with M/Vh ratio parametric. It is seen that the inflection point moves away from the midspan of the chord with decreasing t/h and also with decreasing M/Vh ratios.

Tests were made of two variations that departed from the classical assumptions of the Vierendeel analysis, and the resulting stresses were compared with those from tests that used those assumptions and with values from the finite element analysis. In the first variation, the moment at the end of the chord was determined from the combined effects of the external moment at the midspan of the chord and the product of the midspan shear and the distance from the end of the chord to the point of inflection established by the finite element analysis. In the second variation, the moment at the end of the chord was determined from the combined effects of the external moment at the inflection point established by the finite element analysis and the product of the shear at the point and the distance from that point to the end of the chord.

Table 1 gives the results where all values have been normalized by dividing by the corresponding values from the finite element solution. It is seen that the values for axial force and the moment on the right end of the chord are close to the finite element solution for all variations of the Vierendeel technique. The moment at the left end is generally quite low for the two variations of the Vierendeel analysis, whereas the classical Vierendeel analysis gives results much closer to the elastic analysis.

Figure 5. Principal tension stress contours for single-hole and two-hole beams.

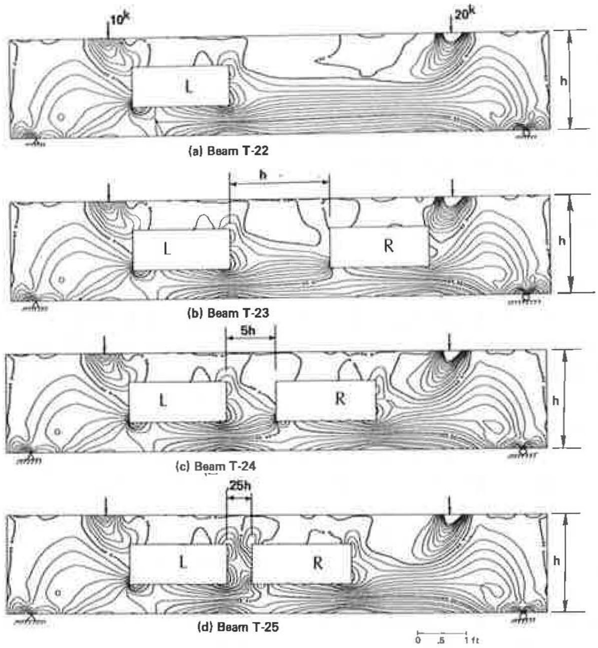


Figure 6. Position of point of inflection in chord.

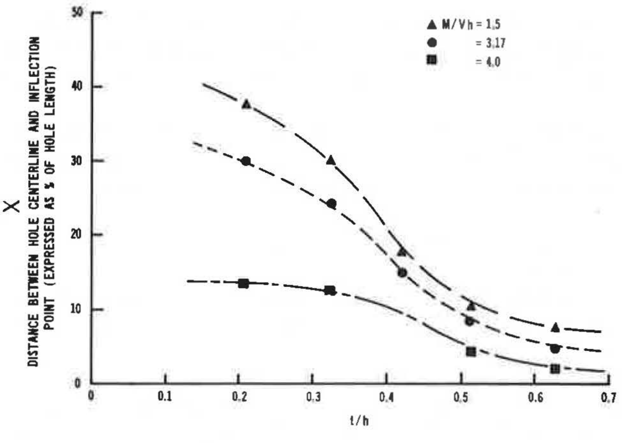


Table 1. Beam chord end forces normalized by their respective finite element values.

Beam	$\frac{M}{Vh}$	$\frac{t}{h}$	Finite Element			Classical Vierendeel			Vierendeel, Variation 1			Vierendeel, Variation 2		
			F	M_R	M_L	F	M_R	M_L	F	M_R	M_L	F	M_R	M_L
T-11	0.098	0.417	1.0	1.0	1.0	1.11	0.90	0.88	1.11	0.90	0.88	1.11	0.90	0.88
T-22	6.33	0.417	1.0	1.0	1.0	1.00	0.70	0.94	1.00	0.97	0.16	0.96	0.96	0.18
T-26	4.0	0.417	1.0	1.0	1.0	1.04	0.81	1.03	1.04	1.05	0.54	1.05	1.05	0.56
T-30	3.17	0.417	1.0	1.0	1.0	1.06	0.86	1.07	1.06	1.09	0.66	1.01	1.09	0.67
T-34	4.0	0.625	1.0	1.0	1.0	1.05	1.07	1.09	1.05	1.07	0.78	1.03	1.07	0.78
T-35	4.0	0.521	1.0	1.0	1.0	1.04	0.87	0.91	1.04	1.03	0.69	1.02	1.03	0.69
T-36	4.0	0.313	1.0	1.0	1.0	1.04	0.76	1.32	1.04	1.11	0.03	0.96	1.09	0.09
T-42	3.17	0.625	1.0	1.0	1.0	1.07	0.98	0.99	1.07	0.98	0.88	1.05	0.97	0.89
T-43	3.17	0.521	1.0	1.0	1.0	1.09	0.94	0.99	1.08	1.10	0.80	1.06	1.09	0.80
T-44	3.17	0.313	1.0	1.0	1.0	1.06	0.81	1.26	1.06	1.13	0.38	0.98	1.11	0.41
T-57	1.5	0.625	1.0	1.0	1.0	0.97	1.02	1.06	0.97	1.06	0.96	0.96	1.06	0.96
T-58	1.5	0.521	1.0	1.0	1.0	0.90	0.90	0.93	0.90	0.98	0.84	0.87	0.98	0.84
T-59	1.5	0.313	1.0	1.0	1.0	0.92	1.17	1.04	0.92	1.44	0.73	0.84	1.43	0.74
T-60	1.5	0.208	1.0	1.0	1.0	0.89	0.77	1.15	0.89	0.96	0.75	0.81	0.94	0.77

Note: F = axial force; M_L = moment, left end; and M_R = moment, right end.

The results given in Table 1 indicate that there is no significant advantage of using either of the two variations of the Vierendeel technique to ascertain the moments and axial forces at the ends of the chords. Thus, once these values from the usual Vierendeel technique have been obtained, the reinforcing for the chords can be determined using standard design techniques.

Effect of Shear and Bending Moment on the Design of Corner Reinforcing

The magnitudes of the shear and bending moment at the opening are known to affect the design of the chords and alter the stress concentration at the corners. To investigate the latter situation, we used finite element solutions of the simply supported beams to develop curves showing the variation of net resultant tensile force T_n occurring at the corners as a function of the shear and bending moment present at the mid-length of the opening. For the corner on the compressive side of the beam, T_n was obtained along a 45-deg plane extending from the corner of the opening to the surface of the member. Stresses from the finite element solution were resolved normal to this plane, after which a regression analysis by least squares was performed to establish a third-order polynomial fitting these stresses. Functions were integrated within the limits of the tensile stresses to assess a resultant force at each tensile corner. The plane chosen in this instance was based on experimental results showing that cracking around the corners occurs at approximately 45 deg.

For corners on the tensile side of the beam, T_n was obtained as the difference between the resultant tensile force with and without the hole being present. These force resultants were designated T_{an} and T_{bn} respectively. This procedure was necessary in order to reflect the fact that the reinforcing provided in the solid portion of the beam would support a significant portion of the tensile force and that any special corner reinforcing would only be required to resist the stresses caused by the stress concentration.

T_{an} was obtained from the integration of the finite element stress distribution resolved normal to a 45-deg plane; T_{bn} was obtained from an integration of the usual bending stresses ($\sigma = M'y/I$) and the shearing stresses ($\tau_{xy} = V'Q/I_b b$) in the solid member, which were resolved to normal stresses along the same plane. Here M' is the bending moment at the edge of the opening, V' is the shear at the edge of the opening, and I_b is the moment of inertia of the gross beam cross section. The finite element and beam theory stress distributions were integrated from the corner of the hole to the point where their stresses were equal, and the difference was taken to obtain T_n (Fig. 7b).

Curves showing the variation of the net tensile stress resultant as a function of moment M and shear V occurring at the middle of the opening were plotted for five hole sizes: $t/h = 0.21$, 0.312, 0.417, 0.52, and 0.625. Figure 7 shows the variation of T_n for the tensile corners on the compression and tension sides of the beam. Note that, for a given value of M/Vh, as t/h increases T_n/V also increases. The variation reflects the fact that the stress concentration becomes larger as the size of the hole increases.

These curves may be used to design the reinforcing required to resist the increased stresses due to the stress concentration. For example, if one desires to use special corner reinforcing in the form of bars close to the corners sloping at 45 deg (a commonly used technique), the bars can be proportioned to carry all of T_n. If, instead, vertical bars are to be used close to the side of the opening, they can be proportioned to carry $0.707T_n$ with the horizontal component carried by the reinforcing in the chord.

SUGGESTED DESIGN PROCEDURE TO PROPORTION THE REINFORCING AROUND RECTANGULAR OPENINGS LOCATED AT MID-DEPTH

Based on the previous analytical results, the suggested procedure for designing the reinforcing around a single rectangular web opening is as follows (working stress design techniques to be used):

1. Design the reinforcing for the main sections of the member, where there is no opening, in the usual manner.
2. Determine the amount and distribution of reinforcing in the chord regions of the opening by using the classical Vierendeel analysis. Often this results in reinforcing the chords with both tension and compression reinforcing as well as stirrups.
3. Obtain a value of T_n/V from Figure 7b for the size of opening desired (t/h) and the value of M/Vh occurring at the mid-length of the opening.
4. Calculate T_n from the value of T_n/V.
5. Provide special corner reinforcing to resist the net tensile force T_n.

This procedure, when used to provide reinforcing around all of the corners, means that all corners will be reinforced to resist the forces in the most highly stressed corner (the tensile corner on the tension side of the beam). If the tensile corner on the compressive side of the beam were desired to have a different amount of reinforcing, the curves of Figure 7a would then be used. From a practical point of view, it is often easier to provide the same reinforcing around all corners and limit mistakes that can occur during construction. Additionally, however, the laboratory tests to be described later showed that the measured stresses in the compressive side of the member were sometimes larger than those predicted by using Figure 7a.

EXPERIMENTAL STUDY

The suggested design procedure was based on the results of an analysis that includes many assumptions about the behavior of the materials and that does not fully model many other variables (i.e., cracking). To test the suitability of the design procedure, we conducted a limited number of laboratory tests on beams designed on the basis of the suggested method. The purpose was to determine whether the reinforcing around the hole was sufficient to force the failure to occur in the solid portion of the beam before it occurred around the opening.

Tests were done on 12 simply supported reinforced concrete beams arranged in a series of four groups having M/Vh ratios ranging from 0.0 to 5.4. Each group of beams consisted of from one to four beams that were identical with the exception of the manner in which the corners were reinforced. The beams were all 13 ft long, 20 in. deep, 8 in. wide and, with one exception, simply supported on a 12.0-ft span. The remaining beam was supported on a 9.0-ft span with a 3.0-ft cantilever. All but one beam had a 10-in. high by 24-in. long web opening. (Details of the beams are found in Fig. 8.) The concrete used in the beams was a seven-sack mix with $^3/_8$-in. maximum size aggregate and a mean 21-day compressive strength of 4,200 psi. Curing was done under wet burlap for 14 days, and all tests were conducted between 14 and 21 days after casting.

Three types of special corner reinforcing were used: (a) bars placed 45 deg to the horizontal and proportioned to resist the entire tensile force resultant; (b) vertical bars placed close to the ends of the hole and proportioned to resist the vertical component of the tensile force resultant; and (c) a combination of vertical bars and 45-deg bars, each proportioned to take one-half the total tensile force resultant.

The corner reinforcing was designed to resist the total tensile force resultant occurring at the corner instead of the net tensile force resultant as outlined in the design procedure. This was done because the curves showing the variation of the net tensile force were not developed at the time of the design and fabrication of the specimens. The total tensile force resultant T was obtained from an integration of the finite element stresses resolved on a 45-deg plane extending from the edge of the opening to the surface of the member. A graph of T versus M/Vh was obtained for the specimen hole size of t/h = 0.417 (Fig. 9). Thus, in step 4 of the suggested design procedure, Figure 9 was used to determine the tensile force resisted by the corner reinforcing. It was later found that providing corner reinforcing to resist the total tensile force instead of the net tensile force, as suggested, overreinforced the corners substantially.

Strain measurements were obtained from resistance strain gauges attached to selected reinforcing bars in five of the beams. Most of the instrumentation was on the bars surrounding the opening. Deflections of the beams were measured at midspan with a mechanical scale.

Figure 7. Effect of hole size on net tensile force resultant in corners.

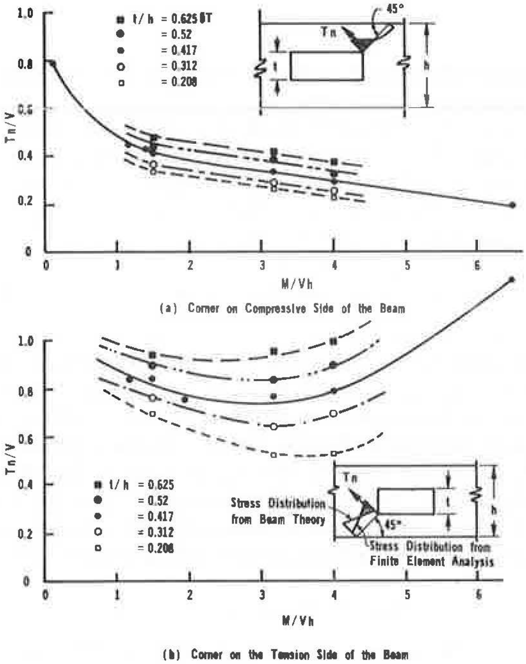
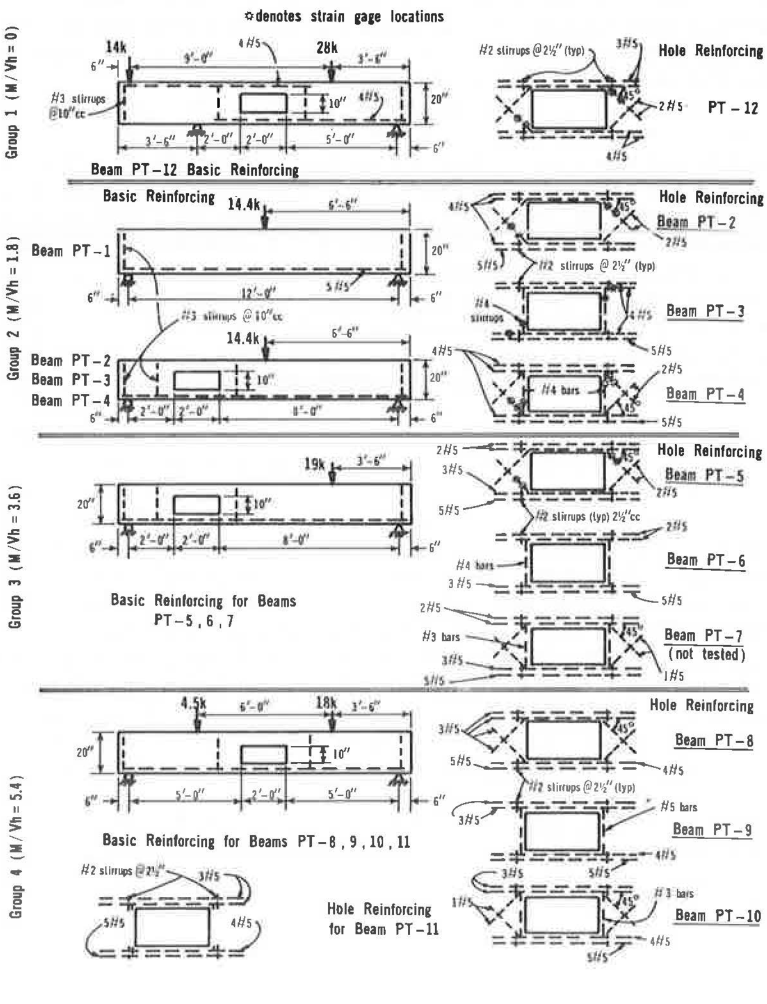


Figure 8. Beam details.



The load was applied by one or two hydraulic rams (50 and 100 kip) controlled by a closed-loop testing machine (Fig. 10). The number of loads and their relative proportions were adjusted to produce a desired M/Vh ratio at the mid-length of the opening. Each load was increased from zero to the maximum value in proportion to the original design load ratio.

Test Results

The results showed that, of the nine members tested that contained the special corner reinforcing, seven failed due to distress in the main portion of the beam. Figure 11, a photograph taken during the test of beam PT-4, shows the typical type of cracking pattern that occurred. The two beams that failed as a result of cracks originating in the corners were found to be improperly detailed. These two members had the 45-deg bars located so that cracks initiated at the corners were able to bypass the reinforcing. Evidence of this type of behavior is shown in Figure 12 for beam PT-2.

The chord regions of the opening were found to be adequately reinforced inasmuch as no subsequent failures occurred in this region. There were flexural cracks in the tensile chord at loads approaching ultimate, but they were well controlled by the reinforcing designed by the Vierendeel method.

A measure of the forces present in the corner portion of the openings was obtained from the strain gauges mounted on the special corner reinforcing in several of the beams. Figure 13 is typical of these forces as recorded in the test and as predicted using the suggested design procedure. For the lower levels of loading, the predicted and observed values of the corner tensile force were substantially the same in the upper right corner (tensile corner in the compression side of the beam). However, at this corner, as the load increased, the observed value exceeded the predicted value, which was obtained from Figure 7a. The predicted value obtained for the lower left corner (tensile corner on the tension side of the member) was always greater than the observed value. In this instance the predicted value was obtained from Figure 7b as suggested in the design procedure. The fact that this latter predicted value was greater than the observed value is significant inasmuch as it is suggested that all corners be designed and reinforced in accordance with the data provided from this region of the opening.

SUMMARY AND CONCLUSIONS

The finite element technique was used to provide elastic stress analyses of beams with large, centrally placed web openings, where the loads, geometry of the beams, and configurations of the web openings were varied. The analyses showed that, for working stress design purposes, a standard Vierendeel analysis was satisfactory for the determination of axial forces and moments on which the reinforcing requirements of the chords are based. It was observed that, as long as the adjacent openings were no closer than half the depth of the member, a single-hole analysis was satisfactory and that for most cases the design based on a rectangular hole was sufficient. The laboratory tests indicated that vertical bars, diagonal bars placed at a 45-deg angle, or a combination of both can adequately restrain corner cracking.

The reinforcing needed to resist the stress concentrations was obtained from the curves developed that gave the net tensile force in the corners. It is worth noting that, for the corner on the tension side of the beam, as the shear V tends toward zero (i.e., $M/Vh \to \infty$) the amount of reinforcing required becomes very large (T_n/V increases as M/Vh increases). This is shown in Figure 7b. This particular result is inconsistent with the results of others (13) who have noted that, for openings in a zero-shear region of a beam, the stress concentrations at the corners of the opening are small and that the effect of the opening on the strength of the member is minimal.

It is felt that the inconsistency occurring in this case is due to the fact that the design curves shown in Figure 7b were based on a resolution of the forces along a 45-deg plane. For situations where the shear is reasonably high, this direction is close to the direction of the principal tensile stresses around the corner and, hence, describes the effect of the stress concentration in an adequate manner. On the other hand, when the shear is small, the principal tensile stresses are not so oriented, and the resolution of forces

Figure 9. Variation of the total tensile force resultant.

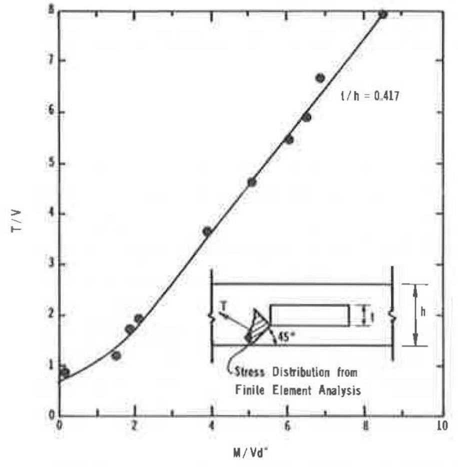


Figure 10. Test configuration.

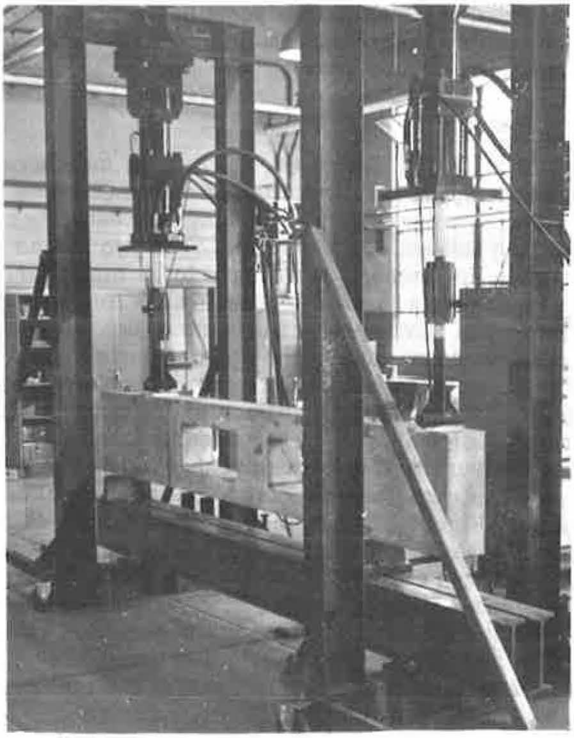


Figure 11. Testing of beam PT-4, nearly at maximum load.

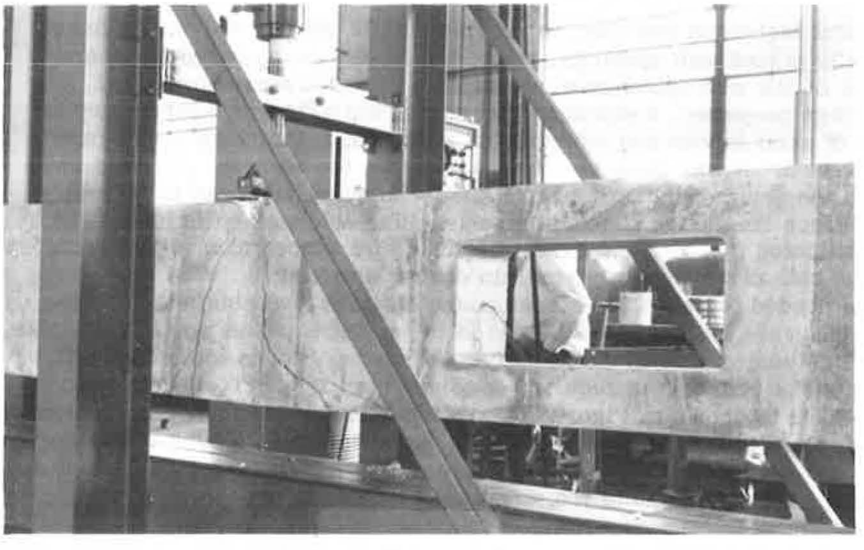


Figure 12. Testing of beam PT-2.

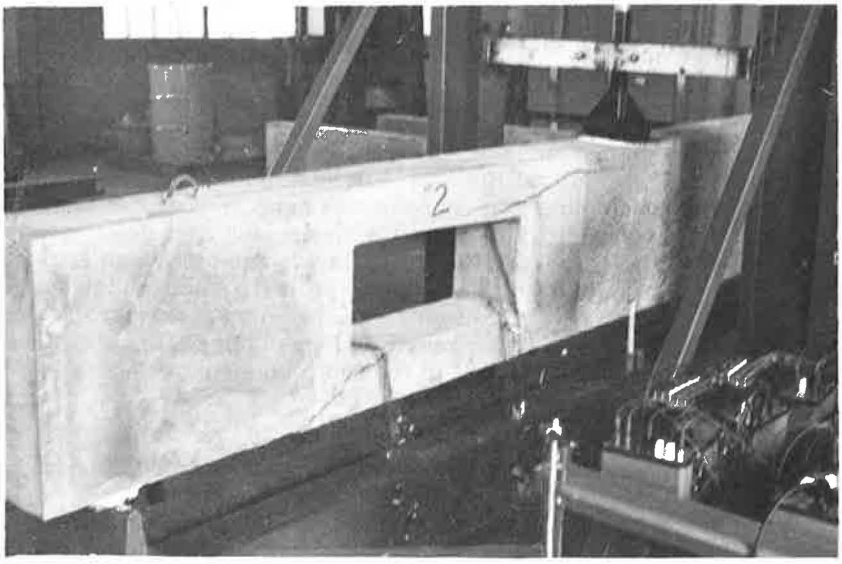
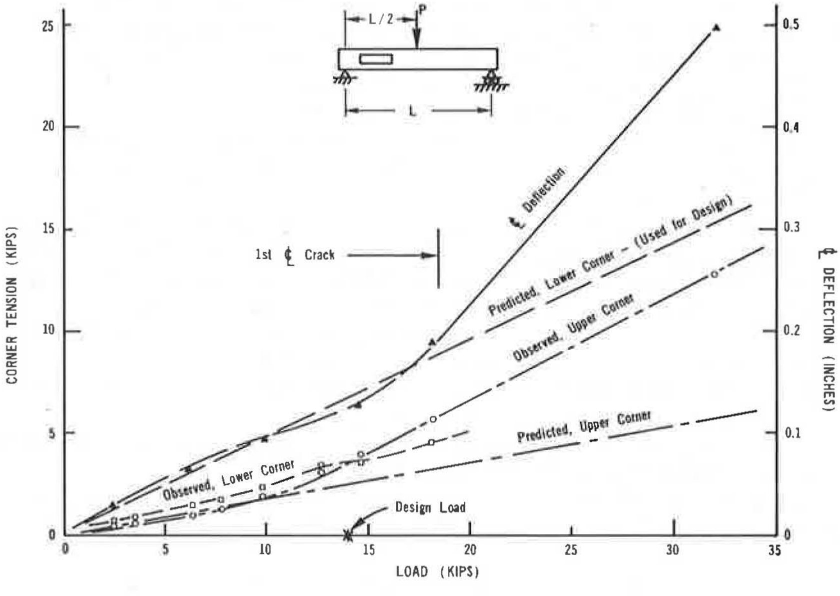


Figure 13. Corner bar tension forces, beam PT-3.



along a 45-deg line does not correctly indicate the effect of the stress concentration. It is felt, for this reason, that the data shown in Figure 7b for values of M/Vh up to approximately 4 will give suitable estimates of the amount of special corner reinforcing. For values in excess of M/Vh $\simeq$ 4, the design curves will be conservative.

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REFERENCES

1. Savin, G. N. Stress Concentrations Around Holes. Pergamon Press, London, 1961.
2. Heller, S. R., Brock, J. S., and Bart, R. The Stresses Around a Rectangular Opening With Rounded Corners in a Beam Subjected to Bending With Shear. David Taylor Model Basin, Rept. 1311, March 1959.
3. Bower, J. E. Elastic Stresses Around Holes in Wide-Flange Beams. Jour. Structural Div., Proc. ASCE, Vol. 92, No. ST2, Proc. Paper 4773, April 1966.
4. Bower, J. E. Experimental Stresses in Wide-Flange Beams With Holes. Jour. Structural Div., Proc. ASCE, Vol. 92, No. ST5, Oct. 1966, pp. 167-186.
5. Segner, E. P. Reinforcement Requirements for Girder Web Openings. Jour. Structural Div., Proc. ASCE, Vol. 90, No. ST3, June 1964, pp. 147-164.
6. Bower, J. E. Ultimate Strength of Beams With Rectangular Holes. Jour. Structural Div., Proc. ASCE, June 1968, pp. 1315-1337.
7. Bower, J. E. Analysis and Experimental Verification of Steel Beams With Unreinforced Web Openings. In Design of Beams With Web Openings, Univ. of Wisconsin Institute, Milwaukee, Jan. 22-23, 1970.
8. Nasser, K. W., Acavalos, A., and Daniel, H. R. Behavior and Design of Large Openings in Reinforced Beams. ACI Jour., Vol. 64, No. 1, Jan. 1967, pp. 25-33.
9. Ragan, H. S., and Warwaruk, J. Tee Members With Large Web Openings. Prestressed Concrete Institute Jour., Vol. 12, No. 4, Aug. 1967, pp. 52-65.
10. Carpenter, J. E., and Hanson, N. W. Tests of Reinforced Concrete Wall Beams With Large Web Openings. ACI Jour., Sept. 1969, pp. 756-766.
11. Hanson , J. M. Square Openings in Webs of Continuous Joists. Portland Cement Assn., Res. and Development Bull., RD001.01D, 1969.
12. Lorentsen, M. Holes in Reinforced Concrete Girders. Byggmastaren, Stockholm, Vol. 41, No. 7, July 1962; Trans. by Portland Cement Assn.
13. Redwood, R. G., and McCutcheon, J. E. Beam Tests With Unreinforced Web Openings. Jour. Structural Div., Proc. ASCE, Vol. 94, No. ST1, Jan. 1968, pp. 1-16.
14. Wilson, E. L. Finite Element Analysis of Two-Dimensional Structures. Univ. of California, Berkeley, PhD thesis, 1963.
15. Calcomp General Purpose Contouring Program. California Computer Products, Inc., Anaheim, 1968.
16. Bridge Planning and Design Manual. California Department of Public Works, Sacramento, Vol. 1, Section 2.
17. Ramey, M. R. Vierendeel Bent Caps. Civil Engineering Dept., Univ. of California, Davis, Rept. 72-1, July 1972.

DISCUSSION

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The authors have presented an interesting paper that demonstrates the applicability of the Vierendeel method of analysis to openings in reinforced concrete bent caps. Their paper also provides analytical confirmation of experimental data (18) showing that multiple openings separated by posts with a width equal to more than half the depth of the member do not reduce ultimate strength.

However, the authors' conclusion that special corner reinforcing is needed to resist stress concentrations may be fallacious and is not supported by the experimental evidence in the paper.

In the first place, openings formed in concrete beams often develop shrinkage cracks along their sides and particularly in their corners. The presence of these random shrinkage cracks will significantly alter the stresses computed by the authors' elastic analysis. Cracking due to stress will also generally occur below the working stress design load, further limiting the applicability of the analysis.

In the second place, columns, piers, spandrels, and other members may be subjected to force systems similar to those in the chords above or below an opening. These members frequently have a reentrant corner where they frame into another member, and it is standard practice to design these members without regard to the effect of stress concentrations. There is no evidence that the strength of these members is reduced by the reentrant corner.

Experimental studies by the writer (11) have indicated that the first prominent cracking observed at an opening can be satisfactorily related to tensile stresses computed from forces obtained from a Vierendeel analysis, without regard to stress concentration. Accordingly, the writer contends that an opening reinforced with adequate vertical stirrups along its side will cause the beam to behave as if two subbeams were bridging the opening and that the strength of this system depends only on the ultimate strength of the subbeams. Of course, the behavior of the subbeams will depend on their reinforcement.

The authors point to their 12 test beams and indicate that the special corner reinforcement provided adequately strengthened the members. However, the nine beams with corner reinforcement either failed in the main portion of the beam or were improperly detailed. Therefore, it cannot be concluded that this reinforcement was necessary. The only evidence intended to support the authors' contention that corner reinforcement is needed is provided by Figure 13. However, the example shown in Figure 13 is for a beam that contains horizontal and vertical bars along the sides of the opening, and the relationship presented between the observed and the predicted stresses is certainly not convincing.

Of the three remaining beams, one did not contain openings (PT-1), one was not tested (PT-7), and the other did not contain vertical reinforcement along the sides of the opening (PT-11). The writer inquires about the behavior of PT-1 and PT-11 and requests a comparison of the maximum moment at failure in these two beams with the others in the test program.

In the remainder of this discussion, the writer would like to comment on several other points in the paper. The authors indicated that, in a classical Vierendeel analysis, the total shear acting at the vertical plane through the inflection points at the midlength of the upper and lower chords is assumed to be distributed to two chords in proportion to their respective areas, and they note that their analysis and test results support this method of analysis. However, the writer would like to point out that the authors' experimental and analytical program was based on an opening located at mid-depth of a rectangular member. This is a special case in which, at least until cracking, the distribution of shear is independent of the sectional properties of the chords. When the opening is not at mid-depth, or the member is not rectangular, the distribution of shear will be related to the span-to-depth ratio of the chords and, after cracking, to the reinforcement in the chords. For low span-to-depth ratios, the shear distribution will depend on the areas of the top and the bottom chord, and, for high span-to-depth ratios, the shear distribution will depend on the flexural stiffnesses of the chords.

The authors indicate that Figure 6 shows variation in location of the inflection position as a function of the ratio of the hole depth to beam depth. Is this ratio the same for both the top and bottom chords above and below the opening?

From the description of the manner in which the force T_n was obtained on the tensile side of a beam with an opening, the writer gained the impression that T_{an} was computed from normal stresses acting on a 45-deg plane through the corner, whereas T_{bn} was computed by resolving stresses on a vertical plane through the corner to an angle of 45 deg. The writer would like to see a more rigorous explanation of this approach. Furthermore, the curves for T_n shown in Figure 7 appear to be independent of the horizontal dimension of the hole. Is this actually the case, or are these curves restricted to the specified 2-ft length of hole investigated by the authors? The writer notes that the authors have used these curves in their recommended design procedure, which does not contain any restriction about the horizontal length of the opening.

References

18. Hanson, J. M., Corley, W. G., and Hognestad, E. Evaluation of Structural Concrete Members Penetrated by Service Systems. Bureau of Standards, Spec. Publ. 361, Vol. 1, Feb. 1972, pp. 545-556.

AUTHORS' CLOSURE

The writers appreciate the in-depth discussion by Hanson. He noted that, in tests of his own on joints containing rectangular openings, reinforcing close to the opening would be sufficient to prevent cracking around the corners. In his discussion he points out that an "adequately" designed vertical stirrup would suffice for reinforcing the corners of the opening. The writers believe that the word "adequately" is a key term here because few studies show the manner by which the design engineer determines how much reinforcing is "adequate." In fact, in the reference cited by Hanson it is merely stated that a No. 3 stirrup placed close to the side of the opening was used to reinforce the corners, and no mention was made of the design procedure used to proportion this stirrup. We feel that an important contribution of this paper is a method by which the design engineer can determine what is adequate.

The basis of the development rests on using an elastic analysis of the member that indicates that there are stress concentrations at the corners of the opening. It is recognized that the analysis shows larger stress concentration values than are no doubt present; however, tests on concrete elements containing variously shaped openings clearly show that such increased stresses do exist (1). In the proposed design method the resultant of these stress concentrations is used as a measure of the reinforcing required at the opening. This technique is similar to that used to determine the reinforcing requirements in the end zones of prestressed concrete beams to prevent tensile splitting.

Hanson raised some other points that the writers would briefly like to comment on. As noted in his discussion, most of the beams used in the limited experimental program failed in the main portion of the beam. This was as desired inasmuch as we did not want the opening to weaken the member. Tests by others, including Hanson, showed that an unreinforced opening in a high shear region weakens the beam and that some corner reinforcing is required. Beam PT-12 (Fig. 12) shows significant corner cracking in a member with a rectangular opening in a high shear region and the corner reinforcing improperly placed. It is easy to see that a similar result might be obtained if the corner reinforcing were absent.

The usual assumptions of the Vierendeel analysis pertaining to the shear distribution in the chord members was found to be valid based on the elastic finite element analyses reported in the paper as long as the opening was centrally placed with respect to the depth of the member. It is clearly stated that no other hole placements were considered. The writers are aware that other parameters influence the shear distribution between the chords when the openings are not centrally located and make reference to a brief discussion of this point (17).

Although the finite element analyses showed that the location of the points of contraflexure in both chords varied with the loading and the size of the opening, this fact was shown to be of little consequence in developing the suggested working stress design procedure and was not used.

The method described in the paper to estimate the net tensile force around the corner of the opening on the tensile side of the member considered the fact that the 45-deg line on which the stresses were resolved extended into the solid portion of the beam. Recognizing that there usually is tensile reinforcing provided to carry stresses that are present when the opening is absent, additional stresses introduced by the opening must be carried by the corner reinforcing. In the paper the stresses from the usual beam theory and those obtained from the finite element analyses were each resolved normal to a 45-deg line originating from the corner of the opening and extending to the surface of the beam. The net tensile force T_n was obtained as the integral of the difference between these stresses, with the integration being done from the corner of the opening to the intersection of the stress distributions plotted along the 45-deg line. Figure 7b shows the region taken as T_n.

It was correctly noted that the curves showing T_n are not a function of the length of the opening. It is our judgment that one of the more important variables in the analysis is the length-to-depth ratio of the chords of the opening rather than the absolute length of the opening. T_n was thus presented as a function of the depth of the opening relative to the depth of the beam, which, in effect, reflected a variation of length-to-depth changes in the chord members.

Hanson cited some important experimental work done by others, and a survey of the literature (20) will show that limited research has been done on this subject of web openings in concrete flexural members. Further studies dealing with both the theoretical and experimental aspects of the problem are obviously needed.

Reference

20. Imbert, I. D. C. The Effect of Holes on Tensile Deformations in Plain Concrete. Highway Research Record 324, 1970, pp. 54-65.