CALCULATION OF THE ELASTIC MODULI OF A TWO-LAYER PAVEMENT SYSTEM FROM MEASURED SURFACE DEFLECTIONS

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This report gives the theoretical background and a description of a new computer program, ELASTIC MODULUS II, which is capable of converting deflections measured by the Dynaflect on the surface of a highway pavement-subgrade (two-layer elastic) system to the elastic moduli of the pavement and subgrade. Included in this paper are the solutions of several typical problems. The computer program may eventually become a part of a comprehensive flexible pavement design system now being implemented by the Texas Highway Department.

•IN the early 1960s, following publication of the AASHO Road Test findings (1, 2), the perennial attempt to find a general solution to the flexible pavement structural design problem received a new impetus. On an unprecedented scale there became available masses of data interrelating axle load, accumulated number of axle applications, structural design, and pavement performance.

Shortly thereafter an interim design guide based on the road test findings was written by an AASHO committee for trial use by the member states, and special studies of pavement performance were initiated by several states, including Texas, to find—in their own environments—how the design guide and/or other road test data could best be used locally.

The Texas study, conducted by the Texas Transportation Institute in cooperation with the Texas Highway Department over a period of several years, led to two general conclusions: (a) the most logical way to reduce the AASHO Road Test flexible pavement performance data to a form useful in Texas was to relate these data to the surface deflection (Benkelman beam) data accumulated at the road test and (b) the resulting deflection-performance relation had to be incorporated into an optimizing system capable of producing an array of lowest cost alternate designs, subject to the funds available, other practical constraints, and pavement performance demands inposed on the system by the engineer using it. The cost for each design (or, more accurately, the design strategy) was to include not only the initial construction cost but also all other costs (including the public of traffic delays forced by periodic overlay construction) that could be predicted.

A computer program satisfying the preceding requirements was created at Texas Transportation Institute and turned over to the Texas Highway Department in 1968 (3, 4, 5). At that time, a new research project, involving not only researchers at Texas Transportation Institute and the Texas Highway Department but also those at the Center for Highway Research at Texas University, was initiated for the purpose of carrying on the work of testing and improving the design system, implementing it, and extending it to include rigid pavements (6, 7, 8).

Sponsored by Committee on Strength and Deformation Characteristics of Pavement Sections.

STRUCTURAL SUBSYSTEM

At the core of the flexible pavement system is the empirically derived deflection-performance relation (or subsystem) previously mentioned. It was decided that the reliability of the overall system might be improved if this subsystem were replaced by one based primarily on linear-elastic theory. The use of such a subsystem demands, among other things, that the in situ moduli of typical pavement materials be determined from rapid, economical, nondestructive tests performed on existing roads in the general area where a new pavement is to be constructed. This paper describes a method and a computer program, ELASTIC MODULUS II, for finding such moduli from Dynaflect data in the case of a simple, two-layer system, that is, a pavement where all of the material above a presumably homogeneous subgrade is predominately one material, such as the pavement shown in Figure 1.

The Loading Device (Dynaflect)

Through two steel wheels the trailer-mounted Dynaflect exerts two vertical loads, separated by 20 in. and varying sinusoidally in phase at 8 Hz (Fig. 1). The total load, exerted by rotating weights, varies from 500 lb upward to 500 lb downward. The upward thrust is overcome by the deadweight of the trailer so that the load wheels are always in contact with the pavement. The load-pavement contact areas are small and are considered to be points, rather than areas, in order to simplify the mathematics.

From the symmetry of Figure 1 it can be seen that one load of 1,000 lb can be substituted for the two loads shown, without affecting the vertical motion at points along the line of sensors. For this reason, in what follows only one point load, P, of 1,000 lb, will be considered to be acting on the surface of the pavement.

Notation

Following is a list of the mathematical symbols used in this paper:

- P = vertical force acting at a point in the horizontal surface of a two-layer elastic half-space;
- h = thickness of upper layer;
- E_1 = Young's modulus of upper layer;
- E_2 = Young's modulus of lower layer;
- w = the vertical displacement of a point in the surface;
- r, z = cylindrical coordinates (the tangential coordinate, θ, does not appear because only one load is used as explained, and the resulting vertical deflections are symmetrical about the z-axis);
- m = a parameter;
- x = mr/h;
- $J_0(x)$ = Bessel function of the first kind and zero order with argument x;
- V = a function of m and N (Eqs. 1 and 2); and
- N = a function of E_1 and E_2 (Eq. 2a).

The load P acts downward at the points $r = 0$ and $z = 0$. Positive z is measured downward.

Development of a Deflection Equation

A vertical load, P (Fig. 2), is applied at the point, 0, in the horizontal plane surface of a two-layer elastic system. The point of load application is the origin of cylindrical coordinates, r and z. Positive values of z are measured vertically downward.

The thickness of the upper layer is h, and its elastic modulus is E_1. The thickness of the lower layer is infinite, and its elastic modulus is E_2. Poisson's ratio for both layers is taken as one-half.

It can be shown from Burmister's early work in elastic layered systems (9) that the deflection, w, of a surface point at the horizontal distance, r, from the point, $\overline{0}$, is related to the constants, h, E_1, and E_2 by the equation

$$\frac{4\pi E_1}{3P} wr = \int_{x=0}^{\infty} V \times Jo(x)dx \tag{1}$$

where

$$x = mr/h, \tag{1a}$$

m = a parameter,

$$V = \frac{1 + 4Nme^{-2m} - N^2e^{-4m}}{1 - 2N(1 + 2m^2)e^{-2m} + N^2e^{-4m}}, \text{ and} \tag{2}$$

$$N = \frac{1 - E_2/E_1}{1 + E_2/E_1} = \frac{E_1 - E_2}{E_1 + E_2} \tag{2a}$$

An Approximation of the Deflection Equation

The integration indicated in Eq. 1 must be performed by numerical means. This task is made easier by taking advantage of the fact that (a) as x varies from zero to infinity in the integration process, m varies over the same range, and r and h are held constant; (b) as m varies from zero to infinity, the function V varies monotonically from E_1/E_2 to 1.0; and (c) for practical ranges of the ratio E_2/E_1, V approaches its limiting value of 1.0 at surprisingly low values of m. For example, it was found (Table 1) that, if m is set equal to 10 and E_2/E_1 is restricted to a range of 0 to 1,000, then $V = 1.0 \pm 0.000001$. Thus, we conclude that, for practical purposes, when m is in the range of 0 to 10, V is given by Eq. 2, and, when m is in a range of 10 to infinity, V = 1. This approximation can be expressed algebraically as follows:

$$\int_{x=0}^{\infty} V \times Jo(x)dx \approx \int_{x=0}^{10r/h} V \times Jo(x)dx + \int_{x=10r/h}^{\infty} Jo(x)dx \tag{3}$$

The second integral on the right side of Eq. 3 is equivalent to the difference of two integrals, as indicated by

$$\int_{x=10r/h}^{\infty} Jo(x)dx = \int_{x=0}^{\infty} Jo(x)dx - \int_{x=0}^{10r/h} Jo(x)dx = 1 - \int_{x=0}^{10r/h} Jo(x)dx \tag{4}$$

By making the obvious substitution from Eq. 4 in Eq. 3 we have

$$\int_{x=0}^{\infty} V \times Jo(x)dx \approx \int_{x=0}^{10r/h} V \times Jo(x)dx + 1 - \int_{x=0}^{10r/h} Jo(x)dx, \text{ or}$$

or

$$\int_{x=0}^{\infty} V \times Jo(x)dx \approx 1 + \int_{x=0}^{10r/h} (V - 1)\, Jo(x)dx$$

Comparing the last approximation with Eq. 1, we arrive at the approximation

$$\frac{4\pi E_1}{3P} wr \approx 1 + \int_{x=0}^{10r/h} (V - 1)\, Jo(x)dx \tag{5}$$

where all symbols are as previously defined.

It is of interest to note from Eq. 2 that V = 1 when $E_2 = E_1$ (that is, when the layered system of Fig. 1 degenerates into a homogeneous elastic half-space) and that for this case Eq. 5 reduces to

Figure 1. Relative position of Dynaflect loads and sensors.

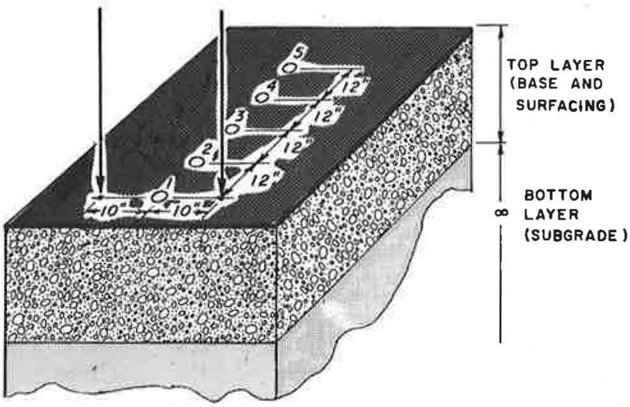

Figure 2. Two-layer elastic system loaded at a point on the surface.

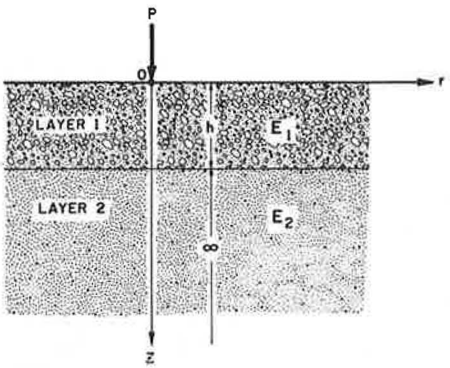

Table 1. Values of the function, V, corresponding to selected values of parameter m and modular ratio E_2/E_1.

	E_2/E_1							
m	0	0.001	0.01	0.1	1	10	100	1,000
0.0	Infinite	1,000	100	10	1	0.1	0.01	0.001
0.1	6,012	855.6	98.14	9.967	1	0.1006	0.01065	0.001655
0.5	50.49	47.94	32.98	8.056	1	0.1542	0.06727	0.05854
1.0	7.382	7.363	6.826	4.112	1	0.3250	0.2491	0.2414
3.0	1.137	1.137	1.134	1.110	1	0.9058	0.8888	0.8869
5.0	1.006	1.006	1.005	1.005	1	0.9955	0.9946	0.9945
10.0	1.000001	1.000001	1.000001	1.000001	1	0.9999993	0.9999991	0.9999991
Infinite	1	1	1	1	1	1	1	1

$$\frac{4\pi E_1}{3P} wr \approx 1$$

The correct equation for this case, according to Timoshenko (10), is

$$\frac{4\pi E_1}{3P} wr = 1$$

Thus, for the homogeneous case Eq. 5 becomes exact.

NUMERICAL INTEGRATION OF DEFLECTION EQUATION

To use Eq. 5, we had to employ some form of numerical integration process for evaluating the integral in that equation. The method known as Simpson's rule was selected (11). This procedure required that a small but finite increment, Δx, be chosen and that the integrand be calculated at $x = 0$, $x = \Delta x$, and $x = 2\Delta x$ over the specified range of integration. The smaller the value assigned to Δx, the greater would be the accuracy of the result; on the other hand, the larger the value of Δx, the less would be the required computer time. Thus a compromise between computer time and accuracy had to be made.

Because the integral of Eq. 5 is the product of the factor $V - 1$, which is a function of m and N, and $J_o(x)$—which is a function of $x = mr/h$ (Eq. 1a)—two safeguards against inaccurate results had to be incorporated into the program:

1. Δm had to be small enough to ensure a sufficiently accurate numerical representation of the function V, and
2. Δx had to be small enough to ensure an accurate numerical representation of the function $J_o(x)$.

After some study of the numerical values of V given in Table 1 and of the values of $J_o(x)$ available from numerous sources (12), the following rules were incorporated into the computer program for solving Eq. 5:

1. In the range $m = 0$ to $m = 3$, $\Delta m \leq 0.01$ (in FORTRAN, DELM1 .LE. XK1.).
2. In the range $m = 3$ to $m = 10$, $\Delta m \leq 0.10$ (in FORTRAN, DELM2 .LE. XK2.).
3. In the entire range of x from 0 r/h to 10 r/h, not less than 61 values of $J_o(x)$ are computed as x increases from any value $x = c$ to the value $x = c + 3$. This also ensures that the number of values of $J_o(x)$ computed between successive zeroes of that alternating function exceeds 61 (in FORTRAN, XNO = 61).

Because Δx and Δm are interdependent according to Eq. 1a, that is,

$$\Delta x = \Delta m \, r/h \tag{1b}$$

the computer program had to ensure that the rules previously given were consistent with Eq. 1b. The accuracy of the solutions obtained (or the computer time used) can be changed by altering the values assigned to the FORTRAN variables XK1, XK2, and XNO.

As an example of how Eq. 5 is used in ELASTIC MODULUS II to find pavement and subgrade moduli, consider the following conditions.

Let us assume that w_1 has been measured on the surface of a pavement structure at the distance r_1 from either Dynaflect load and w_3 at the distance r_3. The thickness, h, of the pavement is known.

Now let F represent the function on the right side of Eq. 5. We may then write two equations:

$$\frac{4\pi E_1}{3P} w_1 r_1 \approx F(E_2/E_1,\ r_1/h) \tag{6a}$$

$$\frac{4\pi E_1}{3P} w_3 r_3 \approx F(E_2/E_1,\ r_3/h) \tag{6b}$$

By dividing Eq. 6a by Eq. 6b, we obtain

$$\frac{w_1 r_1}{w_3 r_3} = \frac{F(E_2/E_1,\ r_1/h)}{F(E_2/E_1,\ r_3/h)} \tag{7}$$

where E_2/E_1 is the only unknown.

By a convergent process of trial and error, a value of E_2/E_1 usually can be found that satisfies Eq. 7 to the desired degree of accuracy. After this has been done, E_1 is calculated from Eq. 6a, and finally E_2 is found from the relation

$$E_2 = E_1 \left(\frac{E_2}{E_1}\right)$$

Accuracy Check

As was mentioned previously, a point load was substituted in ELASTIC MODULUS II for the area loads exerted by the Dynaflect. To check the effect of this assumption on accuracy, as well as the effect of the approximations described previously, we used the following procedure.

The contact area of each load wheel was measured approximately by inserting light-sensitive paper between each wheel and the pavement, running the Dynaflect for a short time in strong sunlight, and removing the paper and measuring the unexposed areas.

From these measurements it was concluded that each 500-lb load could be represented by a uniform pressure of 80 psi acting on a circular area with a radius of 1.41 in. Furthermore, because of the symmetry of the load-geophone configuration, it was reasoned that the effect of both loads could be represented by a pressure of 160 psi acting on one circular area of the radius given previously (1.41 in.).

The surface deflections w_1 and w_3 (Fig. 1), occurring at the distances $r = 10$ in. and $r = \sqrt{10^2 + 24^2} = 26$ in. from the center of the circle, could then be calculated from the program BISTRO, written by Koninklijke/Shell-Laboratorium, Amsterdam, and compared with deflections obtained by the program ELASTIC MODULUS II modified slightly to receive as inputs E_1, E_2, h, and r and to print out w_1 and w_3.

The two programs were compared as described previously over a range of the ratio E_1/E_2 from 0.1 to 1,000 and a range of the thickness h from 5 to 40 in. The results are given in Table 2 in the same manner that Dynaflect deflections are recorded, that is, in mils to two decimal places.

The table shows near-perfect agreement in the range $1 \le E_1/E_2 \le 1{,}000$, for which the pavement is stiffer than the subgrade. On the other hand, with the subgrade much stiffer than the pavement ($E_1/E_2 = 0.1$, Table 2), the agreement was not as good. In addition, upheavals occurred, as indicated by the negative signs of some of the deflections. In these cases, the deflected surface is very irregular, and Dynaflect data from such a pavement would be difficult to interpret because this device is not equipped to distinguish phase differences between load and geophone.

Because most pavements of the type shown in Figure 1 are obviously intended to be stiffer than their subgrades, and, in view of the fact that irregular basin shapes are seldom encountered in practice, it is concluded from the data given in Table 2 that ELASTIC MODULUS II represents the theory of elasticity with sufficient accuracy to accomplish the purpose for which it was designed.

NON-UNIQUE SOLUTIONS

To investigate the possibility that the use of the program could lead to more than one solution (that is, to more than one value of the ratio E_1/E_2) or, perhaps, to no solution at all in some cases, ELASTIC MODULUS II was modified slightly to receive as inputs selected values of E_1/E_2 and the layer thickness h and to compute the corresponding ratio w_1r_1/w_3r_3 (Eq. 7). The results of these computations were plotted as contours of the layer thickness (Fig. 3). The range of input data was limited to the largest range that might be expected from field deflection tests made on real highways of the type shown in Figure 1.

Figure 3. Contours of pavement thickness plotted as a function of the ratios E_1/E_2 and $w_1 r_1/w_3 r_3$.

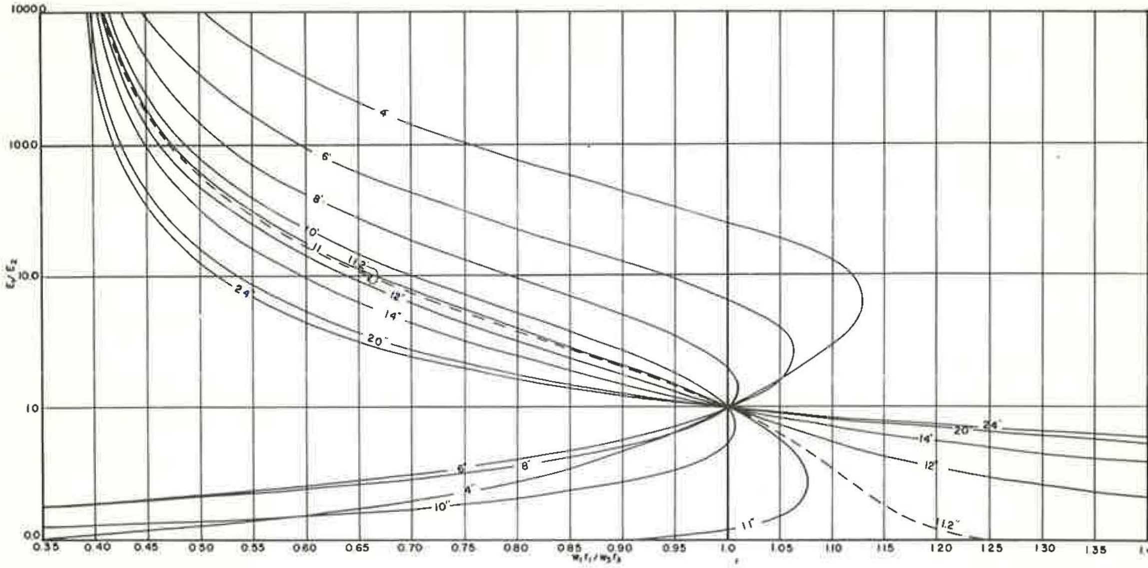

Table 2. Comparison of ELASTIC MODULUS II and BISTRO.

Young's Modulus of Upper Layer (psi)	Young's Modulus of Lower Layer (psi)	E_1/E_2	Thickness (in.)	Computed Deflections (mil) w_1 ELASTIC MODULUS II	w_1 BISTRO	w_3 ELASTIC MODULUS II	w_3 BISTRO
10,000,000	10,000	1,000	5	0.99	0.99	0.81	0.81
			10	0.52	0.52	0.48	0.48
			20	0.26	0.26	0.26	0.26
			40	0.13	0.13	0.13	0.13
1,000,000	10,000	100	5	1.86	1.85	1.09	1.09
			10	1.07	1.07	0.84	0.84
			20	0.57	0.57	0.51	0.51
			40	0.30	0.30	0.28	0.28
100,000	10,000	10	5	2.65	2.65	0.98	0.98
			10	1.94	1.93	1.06	1.06
			20	1.20	1.20	0.86	0.86
			40	0.74	0.74	0.56	0.56
10,000	10,000	1	5	2.39	2.39	0.92	0.92
			10	2.39	2.39	0.92	0.92
			20	2.39	2.39	0.92	0.92
			40	2.39	2.39	0.92	0.92
1,000	10,000	0.1	5	-0.01	-0.04	0.80	0.80
			10	-0.15	-0.06	0.35	0.35
			20	7.45	7.52	0.42	0.42
			40	14.90	14.90	1.60	1.60

Note: For ELASTIC MODULUS II, there is a point load of 1,000 lb. For BISTRO, circular loaded area has a radius of 1.41 in., a pressure of 160 psi, and a load of 1,000 lb. For both programs, vertical deflection was computed at the points r = 10 in. (z = 0) and r = 26 in. (z = 0).

To facilitate interpretation, Figure 3 has been divided into four quadrants. For example, by referring to quadrants I and II (Fig. 3) it can be seen that, if the measured inputs to ELASTIC MODULUS II satisfy the inequalities, $w_1r_1/w_3r_3 > 1$ and $h \geq 11.2$ in., a unique solution satisfying the inequality $E_1/E_2 < 1$ exists, and in this case the program finds and prints the two moduli. If, on the other hand, $w_1r_1/w_3r_3 > 1$ (as before) but $h < 11.2$ in., the possibility of two solutions exists; also there may be no solution if the measured ratio w_1r_1/w_3r_3 is sufficiently great. In this case, i.e., $w_1r_1/w_3r_3 > 1$ and $h < 11.2$ in., the program abandons the search for a solution and prints the message NO UNIQUE SOLUTION.

By examining quadrants III and IV, it can be concluded that, if $w_1r_1/w_3r_3 < 1$ and $h \geq 11.2$ in., a unique solution satisfying the inequality $E_1/E_2 > 1$ exists. In this case the program finds the solution and prints the two moduli. On the other hand, if $w_1r_1/w_3r_3 < 1$ as before but $h < 11.2$ in., there are two possible solutions: one in quadrant III for $E_1/E_2 > 1$ and another in quadrant IV for $E_1/E_2 < 1$. Of these two solutions, the one in quadrant III, representing a pavement whose elastic modulus is greater than that of the subgrade, is the more probable; therefore, the program seeks out the quadrant III solution, prints the corresponding moduli, and ignores the quadrant IV solution.

The information deduced from Figure 3 and used in the control of the program ELASTIC MODULUS II is summarized in Table 3.

EXAMPLES OF SOLUTIONS OBTAINED BY ELASTIC MODULUS II

In May 1968, Dynaflect deflections were measured at 10 points in the outer wheelpath on each of several 500-ft sections of highways in the vicinity of College Station, Texas. Some of these data, including thicknesses obtained by coring at five points in each section, were used as inputs to the computer program discussed here for the purpose of illustrating its use in obtaining the elastic moduli of pavements and subgrades. The results are summarized in Tables 4 and 5, and an example of the computer printout, in the standard format of the program, is shown in Figure 4. In the computer printout, the readings of each of the five geophones at each test station are given, although only the deflections w_1 and w_3 were actually used in estimating the moduli E_1 and E_2.

Tables 4 and 5 are arranged in descending order of the magnitude of the average modulus of pavement and subgrade respectively. In comparing these two tables it is of interest to note that the within-section variability of the pavement modulus, as indicated by the standard deviation, is generally greater, both in absolute value and in relation to the section average, than that of the subgrade. In addition, it is apparent that the range of E_1 (14,000 to 314,100 psi) is much greater than the range of E_2 (11,100 to 19,100 psi). It is also noteworthy that the pavement of section 12 (Table 4) had an average modulus (14,900 psi) of approximately the same magnitude as that of its subgrade (14,000 psi).

The low pavement modulus found for section 12 may be due to the relatively poor quality of the major component of the pavement, a sandstone that, according to local engineers, has in some cases performed poorly. In any event the surfacing of this section had been overlaid (because of map cracking) shortly before it was tested in 1968; it again developed severe map cracking that required sealing in 1970. The seal coat failed to arrest the progress of surface deterioration, and the section was again overlaid with 1 in. of hot-mix asphaltic concrete in 1971. In short, the contrast between the stiffness of the surfacing material and that of the base seems to be at the root of the trouble in this section.

The greater within-section uniformity of the calculated foundation modulus may be due in part to the necessary assumption of infinite foundation depth. This assumption would seem to lead to the association of relatively large changes in surface deflections with relatively small changes in foundation modulus.

The last column in Tables 4 and 5 can be explained by referring to Figure 4, which shows that replicate measurements, designated A and B, were taken at each of the numbered stations. The stations were selected at intervals of 100 ft, and the replicate measurements at each station were taken at points separated by only 10 ft. The rep-

Table 3. ELASTIC MODULUS II information summary.

Measured Input Data				
w_1r_1/w_3r_3	Thickness (in.)	Unique Solution	Layer Having the Greater Modulus	Program Printout
>1	>11.2	Yes	Subgrade	Subgrade and pavement moduli
>1	<11.2	No	May be either	NO UNIQUE SOLUTION[a]
<1	>11.2	Yes	Pavement	Subgrade and pavement moduli
<1	<11.2	No	May be either, but the more probable of two possible solutions is selected	Subgrade and pavement moduli for solution having $E_1/E_2 > 1$

[a]When the experimental data $w_1r_1/w_3r_3 > 1$ and h is < 11.2 in., some cases can arise for which no solution at all is possible.

Table 4. Average pavement modulus for flexible pavement sections.

Test Section	Pavement Materials and Thicknesses: Surfacing	Base	Pavement Thickness: Average Value (in.)	Standard Deviation	Pavement Modulus: Number of Solutions[a]	Average Value (psi)	Standard Deviation (psi)	Replication Error (psi)
15	1.2-in. asphalt concrete	14.0-in. cement-stabilized limestone	15.2	1.2	10	314,100	75,200	9,800
4	0.5-in. seal coat	7.5-in. asphalt-stabilized gravel	8.0	0.4	4	110,500	90,400	3,300
16	1.0-in. asphalt concrete	6.5-in. asphalt emulsion stabilized gravel	7.5	0.4	10	109,300	19,700	3,600
17	0.5-in. seal coat	7.8-in. iron ore gravel	8.3	0.7	10	81,900	47,700	7,400
5	0.5-in. seal coat	11.5-in. lime-stabilized sandstone	12.0	2.8	10	23,800	15,400	4,200
3	0.5-in. seal coat	12.0-in. red sandy gravel	12.5	1.0	10	23,700	11,600	2,300
12	3.7-in. asphalt concrete	16.2-in. sandstone	19.9	0.5	10	14,900	3,300	1,200

Note: Deflection measurements were made May 21, 1968, near College Station, Texas.

[a]Measurements were made at 10 locations in each section. Fewer than 10 solutions occur in cases where $w_1r_1/w_3r_3 > 1$ and h < 11.2 in., as given in Table 3.

Table 5. Average subgrade modulus for flexible pavement sections.

Test Section	Thickness Investigated (in.)	Subgrade Material: Description	Formation	Subgrade Modulus: Number of Solutions[a]	Average Value (psi)	Standard Deviation (psi)	Replication Error (psi)
15	32	Red sandy clay, some gravel	Stone City	10	19,100	790	390
3	23	Sand over clay	Spiller sandstone, member of Cook Mountain formation	10	19,000	1,300	370
5	24	Tan sandy clay	Caddell	10	14,800	1,600	540
12	22	Black stiff clay	Lagarto	10	14,000	980	250
4	25	Gray sandy clay	Spiller sandstone, member of Cook Mountain formation	4	11,800	1,270	320
17	21	Gray sandy clay	Spiller sandstone, member of Cook Mountain formation	10	11,400	1,200	210
16	18	Brown clay	Alluvium deposit of Brazos River	10	11,100	530	120

Note: Deflection measurements were made May 21, 1968, near College Station, Texas.

[a]Measurements were made at 10 locations in each section. Fewer than 10 solutions occur in cases where $w_1r_2/w_3r_3 > 1$ and h < 11.2 in., as given in Table 3.

lication error at any station is defined as one-half the difference between the two measurements, whereas the replication error for the entire section is obtained by squaring and adding the station replication errors, dividing by the number of stations involved, and taking the square root of the result.

By noting the differences between the replication errors and the standard deviations given in Tables 4 and 5, we can compare the variability encountered in a distance of 10 ft with the variability found over a distance 50 times greater. With one exception, the standard deviation of a section was larger than its replication error by a factor ranging from 2 to 8. In the one exception (section 4 in Table 4), only four solutions could be found, and these were at two stations differing greatly in the calculated base modulus: In this instance, the standard deviation was 27 times greater than the replication error.

ADJUSTMENT OF MODULI FOR PRACTICAL USE IN PAVEMENT DESIGN

As previously noted, the elastic moduli estimated by the computer program are based on deflections produced and measured by the Dynaflect system. Correlation studies of Dynaflect deflections with those produced by a 9,000-lb dual-tired wheel load and measured by means of the Benkelman beam on highways in Illinois and Minnesota in 1967 (13) indicated that the 9,000-lb wheel load deflection could, with reasonable accuracy, be estimated from the Dynaflect deflection, w_1, by multiplying w_1 by 20.

But the peak-to-peak load of the Dynaflect is 1,000 lb; thus, one would expect that the multiplying factor would be about 9 rather than 20, as was found by actual field experience.

Various explanations could be advanced to explain this discrepancy. However, they would not alter the fact, brought out by the correlation study, that, if one desires to use the values of E_1 and E_2 found from Dynaflect deflections to calculate the static, rebound deflection of a linear-elastic-layered system acted on by a 9,000-lb wheel load, then he should approximately halve these moduli before using them in his calculations.

CONCLUSIONS

The following conclusions are supported by the study results:

1. The accuracy of ELASTIC MODULUS II was tested against the widely used program BISTRO with favorable results (Table 2).
2. Within-section variability of the upper (base and surfacing) layer was greater, both in absolute value and in relation to the section average, than that of the subgrade (Tables 4 and 5).
3. With one exception, where the volume of data was insufficient to permit valid comparisons, the variability of a base (or subgrade) modulus encountered in a distance of 10 ft within a section ranged from $\frac{1}{8}$ to $\frac{1}{2}$ of the variability found over the full 500-ft length of the section. In the one exception (out of 14 comparisons) the ratio was 1 to 27 (compare last two columns of Tables 4 and 5).

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DISCUSSION

H. Y. Fang and T. J. Hirst, Lehigh University

The authors have presented an interesting method for computing the elastic moduli of a two-layer pavement-subgrade system based on Dynaflect surface deflections. The writers wish to provide additional information concerning a similar investigation undertaken by Lehigh University.

A simple relation, developed by Fang and Schaub (14) and based on the work of Boussinesq and Burmister, showed that the modulus of subgrade section K_E, and Young's modulus of elasticity of the subgrade E_s, can be computed from Benkelman beam surface deflections if the thickness of the pavement is known. The relations among surface deflection, Young's modulus of elasticity, and modulus of subgrade reaction (based on a 30-in. diameter rigid plate) are as follows:

$$E_s = 0.477 \frac{P\beta}{\delta} \tag{8}$$

$$K_E = 0.027 \frac{P\beta}{\delta} \tag{9}$$

where

β = coefficient of deflection that is a function of pavement thickness,
δ = pavement surface deflection determined by Benkelman beam, and
K_E, E_S, P = as previously defined.

Good agreement between theoretical and experimental data from the AASHO Road Test Loop 1 was observed.

Further studies of the relation between both Benkelman beam and Dynaflect surface deflections and the elastic properties of the pavement and subgrade have been con-

Figure 4. Computer printout for section 16.

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                         TEXAS HIGHWAY DEPARTMENT

                       DISTRICT 17 - DESIGN SECTION

          DYNAFLECT DEFLECTIONS AND CALCULATED ELASTIC MODULI

                     THIS PROGRAM WAS RUN - 07/15/71

                        DIST.        COUNTY
                         17          BRAZOS

         CONT.   SECT.   JOB   HIGHWAY    DATE      DYNAFLECT
         1560      1      1    FM 1687   5-21-68        1

                              PAV. THICK. =  7.50 INCHES

       ASPHALT SURFACING     1.00     ASPH EMUL STAB GRAVL  6.50

       BROWN CLAY SUBGRADE    0.0

STATION   W1    W2    W3    W4    W5    SCI  ** ES  ** ** EP  **  REMARKS

 1 - A  2.160 1.500 0.960 0.660 0.520 0.660    10900.    86100.
 1 - B  2.130 1.530 0.960 0.650 0.510 0.600    10900.    93000.
 2 - A  1.920 1.410 0.930 0.640 0.490 0.510    11500.   140500.
 2 - B  1.860 1.350 0.900 0.630 0.500 0.510    11800.   144300.
 3 - A  2.040 1.470 0.930 0.630 0.490 0.570    11300.   102300.
 3 - B  2.070 1.500 0.960 0.650 0.500 0.570    11000.   109200.
 4 - A  2.220 1.620 1.020 0.670 0.490 0.600    10300.    97000.
 4 - B  2.220 1.590 1.020 0.650 0.490 0.630    10300.    97000.
 5 - A  1.980 1.380 0.900 0.610 0.470 0.600    11700.   103800.
 5 - B  1.980 1.440 0.930 0.610 0.460 0.540    11400.   120100.

AVERAGES 2.058 1.479 0.951 0.640 0.492 0.579   11110.   109330.
STANDARD DEVIATION                    0.049      528.    19723.
NUMBER OF POINTS IN AVERAGE =            10       10       10

    W1    DEFLECTION AT GEOPHONE 1
    W2    DEFLECTION AT GEOPHONE 2
    W3    DEFLECTION AT GEOPHONE 3
    W4    DEFLECTION AT GEOPHONE 4
    W5    DEFLECTION AT GEOPHONE 5
    SCI   SURFACE CURVATURE INDEX ( W1 MINUS W2)
    ES    ELASTIC MODULUS OF THE SUBGRADE FROM W1 AND W3
    EP    ELASTIC MODULUS OF THE PAVEMENT FROM W1 AND W3
```

Figure 5. Benkelman beam and Dynaflect values.

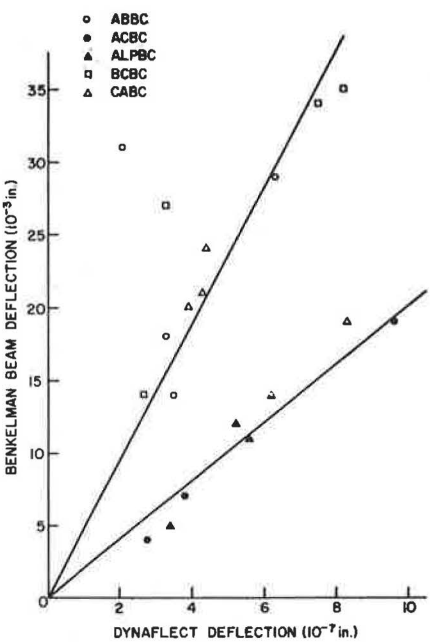
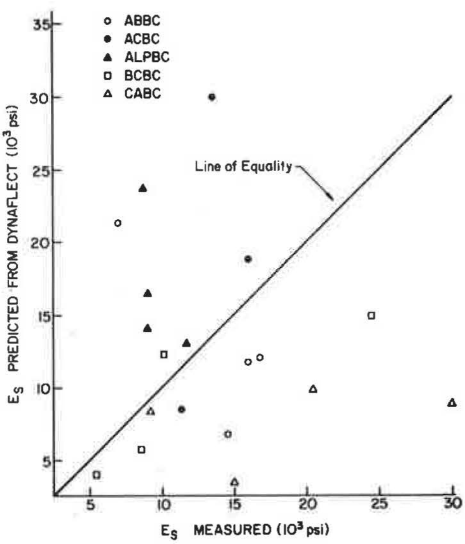


Figure 6. Predicted versus measured subgrade modulus.



ducted on various pavement sections in southeastern Pennsylvania (15). Of particular interest has been the observation that the relation between Benkelman beam and Dynaflect deflections is base-type dependent. Figure 5 shows a comparison of Benkelman beam and Dynaflect deflections for 19 different in-service flexible pavements. The variables in this investigation were base type [aggregate bituminous (ABBC), aggregate cement (ACBC), aggregate-lime-pozzolan (ALPBC), bituminous concrete (BCBC), crushed aggregate (CABC)] and subgrade support. As may be seen from Figure 5, although some scatter is evident, the data occur in two distinct groups. One group contains crushed aggregate and asphalt-stabilized bases, whereas the other group consists of lime and cement-stabilized base materials. Least-squares orthogonal regressions on the data yielded the following linear equations:

For lime- and cement-stabilized bases

$$\Delta_1 = 4.85 \times 10^4 \, \Delta_2 \tag{10}$$

For crushed aggregate and asphalt-stabilized bases

$$\Delta_1 = 2.02 \times 10^4 \, \Delta_2 \tag{11}$$

where

Δ_1 = Benkelman beam deflection (in.), and
Δ_2 = Dynaflect deflection (in.).

By utilizing the correlations shown in Figure 5 and Eqs. 10 and 11 in conjunction with Eqs. 8 and 9, a Young's modulus of elasticity of the subgrade may be computed from the Dynaflect deflections. Figure 6 shows a comparison between the computed modulus and that measured on the subgrade (30-in. diameter plate load test). Again, scatter in the data is noticeable, which may be attributed to many causes including the assumptions of the analysis and the inherent difficulties associated with large-diameter field-plate loading tests. However, in spite of the scatter, the influence of base type on the computed results is evident. Generally, subgrade moduli predicted for pavements with lime- or cement-treated bases are higher than those for asphalt and crushed aggregate bases. Similar results were obtained for the modulus of subgrade reaction.

Determination of subgrade properties by conventional plate loading tests is a most time-consuming and expensive procedure. Alternate means of establishing the strength and deformation properties, such as those utilizing Benkelman beam and Dynaflect deflections, are badly needed. Their use is to be encouraged; however, care must be taken to ensure that the influence of the various pavement components is properly recognized.

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