DETERMINATION OF YOUNG'S MODULUS FOR BITUMINOUS MATERIALS IN PAVEMENT DESIGN

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The determination of a value of Young's modulus for the bituminous-bound layer of a pavement is an essential part of the use of linear elastic theory in pavement design. Current procedures involve either laboratory testing or the use of the Shell nomographs, neither of which is convenient for routine design calculations. Although it is recognized that temperature is a major factor influencing the in situ modulus of bituminous materials, this paper is concerned primarily with the determination of the other major variable, loading time. Relations were derived for the determination of a representative loading time as a function of vehicle speed and thickness of bituminous layer. This loading time is based on the average pulse time for stresses in the vertical and horizontal directions at various depths in the bituminous layer. Modulus-time-temperature relations for two typical base mixes, of the continuously graded and gap-graded types, were established by modifying Shell nomograph results in the light of a number of experimentally determined values of modulus. A chart was produced from which values of Young's modulus can be determined simply and directly with an accuracy considered adequate for design purposes. An equation was derived that could be used as an alternative to the charts, but its principal purpose is to fit into a computer design program for flexible pavements. The overall objective of this study is to simplify improved design methods with a view to their being more readily acceptable in practice.

•MANY suggestions for improved methods of pavement design using a structural design approach involve analysis of the pavement to determine critical stresses and strains (1, 2, 3). Although various methods of analysis have been used, most require that a value of Young's modulus for the bituminous layer be specified. In structures having a substantial thickness of bituminous-bound material, the characteristics of this layer largely dictate the performance of the pavement. In terms of the analysis, the value of Young's modulus for the bituminous layer has a large influence on the stresses and strains throughout the structure. Hence, a reasonably accurate but straightforward procedure is required for its determination.

Under the dynamic conditions that generally apply to pavement design problems, the bituminous material is assumed to behave elastically. The Young's modulus used in analysis is equated to the stiffness modulus of the material under the applicable conditions of temperature and loading time. When direct measurements are not possible, the most widely used procedure for the determination of stiffness involves using the nomographs developed by the Shell organization (4, 5, 6). The values of stiffness they produce have been shown to correlate reasonably well with other measured values, but their use for practical design is far from convenient. One of the most difficult problems is the specification of a relevant loading time. This arises because of the three-dimensional nature of the structure and of the stresses induced in it by a rolling wheel load.

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The object of this paper is to devise a relatively simple method for determining Young's modulus for the bituminous layers in a pavement that will be of sufficient accuracy for routine design purposes. The results are presented in three different ways so that values of modulus may be obtained from plots or tables or by calculation from a single equation that can easily be incorporated in a computer program.

FACTORS INFLUENCING IN SITU YOUNG'S MODULUS

Although flexible pavement construction normally includes more than one layer of bituminous-bound material, the considerations in this paper refer to a single layer only. This approximation is considered adequate when dealing with pavements incorporating bituminous-bound bases because the influence of a different material in the surfacing can be neglected for design purposes, the base being the main structural layer.

In specifying the modulus to be used for a bituminous layer, consideration has to be given to the two factors that influence its magnitude: temperature and loading time.

Temperature

In general, the layer will be subject to a temperature gradient that will depend on the air temperature and the length of time the air has been at that particular temperature. Measurements made by the Transport and Road Research Laboratory reported by Galloway (7) and Forsgate (8) have shown that the pavement temperature, under equilibrium conditions, differs from that of the air. Galloway's measurements indicated that in summer the average temperature of the base and surfacing was slightly above that of the air and that in winter the structure was somewhat below air temperature. This was partially confirmed by Forsgate, though in some structures studied by him pavement temperatures were always above air temperatures even during the winter. His observations, however, were not made below 2 C. Witczak (9) has presented information on average temperatures for pavements of various thicknesses. He showed that the pavement temperature was always above that of the air except for very thin layers at low temperatures. The precise relation between air temperature and pavement temperature depends on many factors, such as exposure to wind, the influence of rain, and surface texture. However, it would seem that the average annual air temperature is likely to be about the same as that for the bituminous layers in the road. In a particular design exercise, such information could be deduced from local meteorological records. Forsgate has shown that this can be done with reasonable accuracy, and Finn et al. (2) have also recommended this procedure. Although loading time is the main subject of this paper, the importance of the influence of temperature on the behavior of asphalt pavements cannot be overemphasized.

Loading Time

Any point in the bituminous layer is subjected to simultaneous variations of stress and strain in three orthogonal directions under the action of a passing wheel load. The amplitude and length of these pulses depend on the direction of action of the particular stress or strain as well as the depth of the point concerned. Hence, no unique loading time exists for the bituminous material in situ.

The definition of loading time used by Van der Poel (4) in establishing the original Shell nomographs was

$$t = \frac{1}{2\pi f} \tag{1}$$

where t = loading time and f = frequency of the sinusoidally applied stress used in the laboratory tests on which the nomograph stiffness values were based. This same definition is used in this paper.

In attempting to define a loading time in connection with the analysis of full-scale trial sections, Klomp and Niesman (10) established two relations: $t = (\frac{1}{2}\pi) \times 0.45$ V and t = ($\frac{1}{2}\pi$) × 0.3 V for surfacing and bases respectively (where V = vehicle speed in km/hr). These expressions were based on the analysis of measured horizontal strain pulses at various depths in the structures. Some difficulty was encountered in defining exactly the beginning and end of any individual pulse.

This problem was studied further by Hofstra and Valkering (11) in a similar investigation. Measured horizontal strain pulses were again used and the following relation established:

$$t = 0.4d \tag{2}$$

where d = pulse length at half-height.

In both these investigations, the resulting values of Young's modulus, when used in linear elastic analysis of the structures, produced strains that compared favorably with the measured values.

The present aim is to determine a general value of modulus for design use rather than a particular value for computing a particular strain. Hence, the loading time specifications used in these two investigations are not thought to be correct because they are based on only one pulse in one direction, and this does not represent an average loading time for the layer as a whole.

Because the pavement is being subjected to an applied stress regime, the correct loading time is considered to be based on stress pulses rather than strain pulses and should be an average for the three orthogonal stresses: vertical, radial, and tangential. It should also be the mean value for the layer thickness being used because all three pulses increase in length with depth.

DETERMINATION OF LOADING TIME

The development of curves for the determination of pulse loading times to be used in dynamic triaxial testing of paving materials has been described by Barksdale (12). The loading times involved apply only to vertical stresses and for this reason have the same shortcoming as the Shell results discussed in the previous section. However, the method used in developing these curves is based on a combination of theory and experimental observations that form a sound basis for developing the mean loading time required for design purposes.

Barksdale's theoretical analyses indicated that the vertical stress pulse was approximately sinusoidal in shape in the upper part of the pavement. This is the zone of interest in the present investigation and conveniently agrees with the wave shape used in the tests on which the Shell nomographs are based. The definition of loading time used by Barksdale, however, differs from that defined by Van der Poel and is shown in Figure 1. If Barksdale's loading time is t_B, then $f = \frac{1}{2t_B}$, where f = frequency.

Because $f = \frac{1}{2\pi t}$ using Van der Poel's expression,

$$t = \frac{t_B}{\pi} \tag{3}$$

Barksdale took into account inertial and viscous effects in determining his loading times by studying vertical stress pulses measured in the AASHO Road Test. Because of these effects, the length of a particular stress pulse in terms of time will not be proportional to wheel speed.

The theoretical investigation carried out by Barksdale on various structures showed that the vertical stress times were not significantly affected by changes in asphalt stiffness or variations in layer thicknesses. In other words, at any particular depth in a conventional structure, the pulse time is a constant for a given wheel speed.

The curves derived by Barksdale enabled loading time to be determined from depth and vehicle speed and are shown in Figure 2.

In extending this work to deal with stresses in three directions, it has been assumed that the two horizontal stresses will be affected by viscous and inertial effects to the same extent as the vertical stress.

Calculations were performed on two structures in order to determine pulse lengths for vertical, radial, and tangential stresses at various depths. The two structures were chosen to represent constructions with thin and thick bituminous layers, and details are given in Table 1.

The calculations were based on linear elastic layered theory, and the results were obtained using the BISTRO computer program (13). The pulses were produced by plotting each stress against radial distance from the centerline of the single uniformly distributed circular load. This was done at three depths for each structure. The depths were arrived at by dividing the bituminous layer into three equal thicknesses and taking the midpoint of each of these sublayers. The depths were hence one-sixth, one-half, and five-sixths of the thickness of the layer. The object of this was to obtain information that could be used to determine average pulse lengths for the layer as a whole. Examples of some of these pulses are shown in Figure 3. Because of the change from tension to compression, which occurs for horizontal stresses near the center of the layer, the pulses at this depth were not used.

Equivalent pulse lengths were taken off these plots by eye using the same procedure as Barksdale, which involved manually fitting a sine curve to the actual pulse. This procedure avoids the difficulty encountered by Klomp and Neisman (10) of not knowing when the pulse has terminated, because in some cases the stress approaches zero asymptotically.

The next exercise was to establish the relation between the lengths of the two horizontal pulses and that of the vertical pulse. This was done by plotting pulse lengths against depth as shown in Figure 4. A best-fit line was put through the points for each of the three stresses. By taking a mean line for the two horizontal stresses, as shown, it became apparent that the lengths of these pulses were about 1.7 times those of the vertical pulses at any particular depth.

For a given vehicle speed, pulse time is proportional to pulse length. Barksdale's vertical pulse time was denoted by t_B, and the same definition has been used in the preceding analysis. Hence the average pulse time at a particular depth is given by

$$\tfrac{1}{3}(t_B + 1.7t_B + 1.7t_B) \approx 1.5t_B$$

Therefore, the average loading time, using Van der Poel's definition, is

$$t = 1.5t_B/\pi = 0.48t_B$$

Barksdale's corrections for viscous and inertia effects were based on a single AASHO Road Test structure that only contained 125 mm of bituminous-bound material. It was, therefore, necessary to check that the stress pulse depth relations for this structure were similar to those shown in Figure 4 for structures with 300 and 600 mm of bituminous material. Computations similar to those described were carried out, and structural details are given in Table 1. The results are included in Figure 4, and comparison with the other pulse lengths can only be described as fair. Clearly, it would be better if experimental results were available from structures having thicker asphalt layers.

Barksdale's curves were replotted to obtain relations between loading time t and depth for various speeds (Fig. 5). Once the mean loading times were established at various depths for the three stress pulses, it was necessary to determine average loading times for various thicknesses of bituminous layer. This was achieved by changing the abscissas of Figure 5 from depths to layer thicknesses and for each depth taking the average loading time, i.e., the value at half that depth. These modified relations are shown in Figure 5, and it was found that they could be plotted as straight lines.

In converting Barksdale's speeds to the international system of units, odd numbers resulted (Fig. 5). Graphical interpolation on a logarithmic plot enabled round figures to be used (Fig. 6).

Figure 1. Definition of loading time used by Barksdale (12).

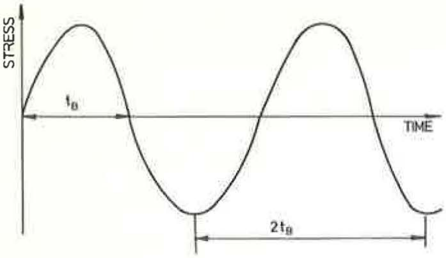


Figure 2. Determination of vertical pulse loading time (after Barksdale, 12).

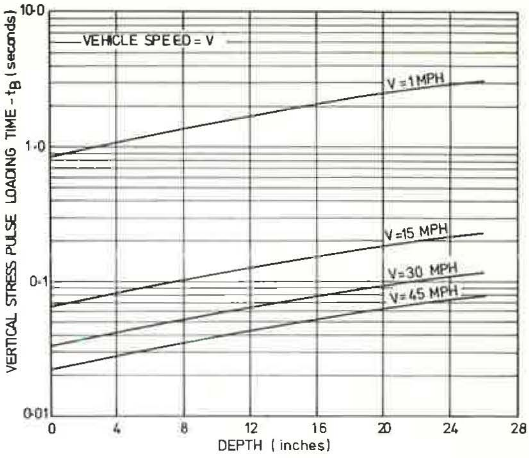
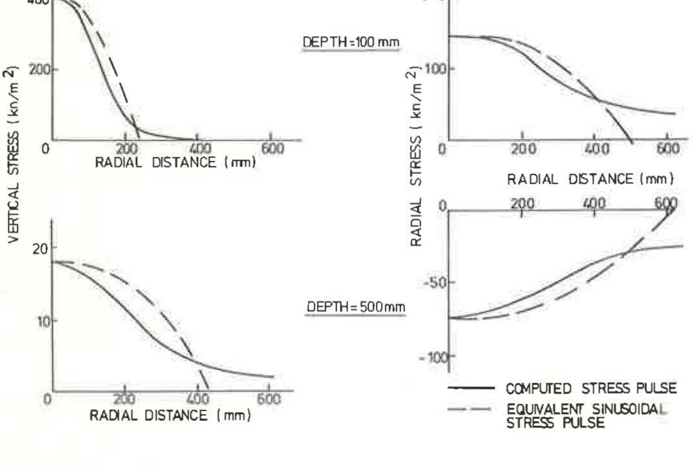


Table 1. Details of structures used for theoretical stress pulse determination.

Layer	Type of Structure	Structure Measure: Modulus $(MN/m)^2$	Poisson's Ratio	Thickness (mm)
Bituminous	Thick	4,000	0.4	600
	Thin	4,000	0.4	300
	AASHO	4,000	0.4	125
Granular base	Thick	–	–	–
	Thin	–	–	–
	AASHO	320	0.3	150
Granular subbase	Thick	125	0.3	150
	Thin	125	0.3	150
	AASHO	125	0.3	300
Subgrade	Thick	50	0.4	–
	Thin	50	0.4	–
	AASHO	50	0.4	–

Note: Load = 40 kN, contact pressure = 500 kN/m^2, and radius of loaded area = 160 mm.

Figure 3. Typical stress pulses in structure with 600-mm bituminous layer.



The procedure suggested by Hofstra and Valkering (11) was used to provide results for the purposes of comparison. The loading times in this case were determined from theoretical, horizontal strain pulses at the lowest of the three depths considered for two of the structures detailed in Table 1, excluding the AASHO structure. The calculations were performed at three speeds: 3, 20, and 80 km/hr. The results shown in Figure 6 indicate similar results to those derived from Barksdale's curves but with somewhat shorter loading times. In use, these would lead to higher values of stiffness than the proposed values, which are on the safe side for design purposes.

MODULUS-TIME-TEMPERATURE RELATIONS

This section describes how relations among modulus, loading time, and temperature were derived for two typical types of mix extensively used in flexible pavement construction. A continuously graded mix with a binder having a penetration between 80 and 100 represents a dense bitumen macadam as used in Great Britain and an asphaltic concrete used in the United States. The other mix is of the gap-graded hot-rolled asphalt type used mainly in Great Britain.

In establishing the modulus-time-temperature relations, many of the published results have been studied and compared with values obtained from the Shell nomographs.

Most experimental results are available for the dense bitumen macadam or asphaltic concrete, and many (14-18) have been incorporated in Figure 7. Details of the various tests and the materials used in each case are given in Table 2.

Nomograph values have been calculated for the mix used by Cooper and Pell (14), which was thought to be typical and for which all the details were available. Lines produced from these nomograph stiffnesses for temperatures of 0, 10, and 20 C are shown in Figure 7.

The experimental results shown in Figure 7 were obtained from a variety of tests including direct compression and several types of flexure. The effect of change in modulus between tension and compression was investigated by Kallas (19), but he only reported significant differences at loading times slower than those considered here. The main shortcoming of the selection of results is that all are based on uniaxial stress conditions, whether direct or bending stress, whereas the in situ situation is three-dimensional. Results from triaxial tests with various confining stresses should provide information nearer to this condition, but preliminary results obtained at the University of Nottingham indicate that the confining stress has a relatively small effect on modulus.

By studying flexure and direct compression test results together, a reasonable representation of the in situ situation is obtained because flexural and compressive stresses may be considered analogous to in situ horizontal and vertical stresses respectively.

The results of compression tests by Snaith et al. (16) on dense bitumen macadam and similar tests by Shook and Kallas (15) on asphaltic concrete compare very well for mixes with the same binder content. The beam tests of Epps and Monismith (18) also yield comparable results as do the cantilever tests reported by Freeme (17), though direct comparison is not possible for these latter tests because they were carried out at shorter loading times. The specimens used by Freeme had very high void contents, but by taking the line plotted from nomograph values as a basis for comparison it can be shown that they are only slightly lower than the other experimental results. However, the results reported by Cooper and Pell (14) are significantly lower than the other experimental results.

In deciding on the relations to be used for design purposes, the various experimental points shown in Figure 7 have been used to suggest modifications to the lines based on nomograph values. The resulting straight-line relations are proposed because they reflect the higher measured values of modulus at longer loading times while erring on the low or conservative side (bearing in mind the design applications).

Figure 8 shows the experimental and nomograph values for rolled asphalt. There are fewer experimental points available for this material (17, 18, 20). The high void content of Freeme's mixes produces significantly lower results in this case than for the dense bitumen macadam.

Figure 4. Variation of pulse lengths with depth for various structures.

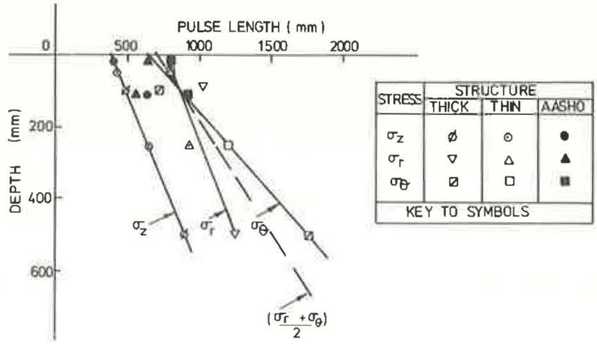


Figure 5. Relation between mean loading time and depth for various vehicle speeds.

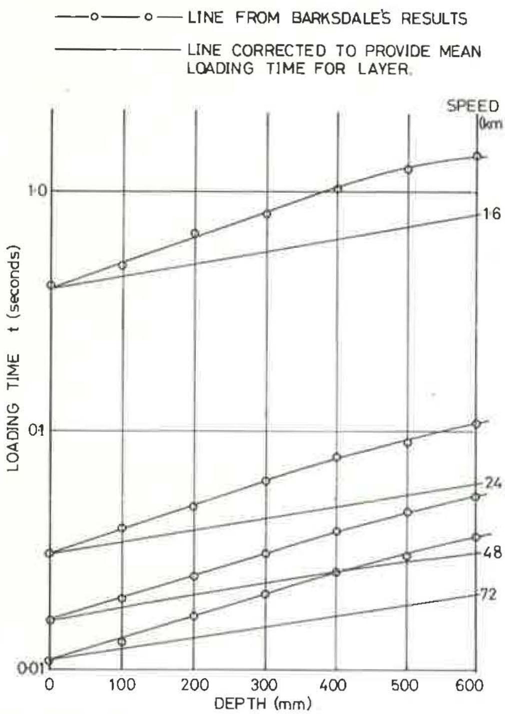


Figure 6. Relation between loading time and thickness of bituminous layer for various vehicle speeds.

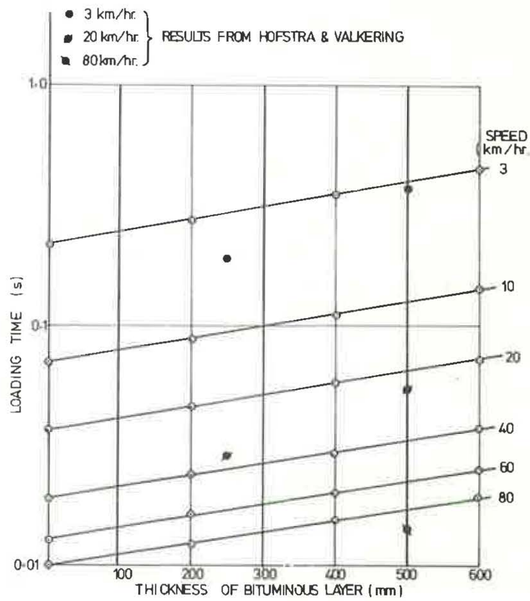
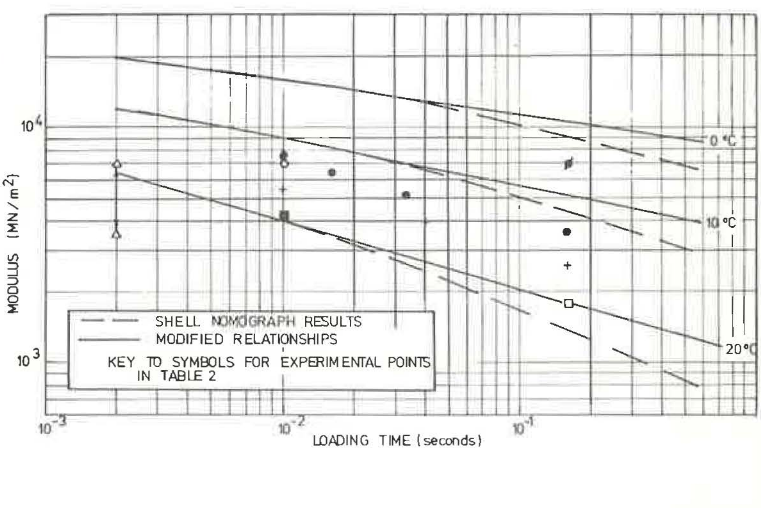


Figure 7. Modulus-time-temperature relation for dense bitumen macadam with 100-penetration binder.



The modified nomograph lines have been established for the rolled asphalt by studying the trends apparent in Figure 7 for the dense bitumen macadam together with the experimental results shown in Figure 8.

An attempt has also been made to carry out the same exercise for a dense bitumen macadam with 200-penetration binder and a dense tar macadam because these materials are also used for base construction in Great Britain. In view of the relatively few experimental results available for these materials, it is difficult to propose any relations with confidence at this time.

GRAPHICAL DETERMINATION OF MODULUS

The previous two sections have described how a representative loading time may be determined from a knowledge of layer thickness and vehicle speed, and this loading time may be used together with the temperature to determine the required Young's modulus.

By combining Figure 6 with Figure 7 or 8, a single chart results from which modulus may be determined directly. This arises because loading time is common to all plots. Figure 9 shows such a combined plot for the dense bitumen macadam with 100-penetration binder.

The advantage of this chart is that it affords the designer a quick method of determining Young's modulus from the design parameters that he is able to specify. At the outset of a design exercise, an estimate has to be made of the thickness of bituminous material. This can be modified subsequently if it is significantly at variance with the final thickness obtained from the design calculations.

Although interpolated values can be estimated from the chart, extrapolation beyond the limits plotted is not likely to be accurate in view of the procedures used to establish the relations.

The dotted line shown in Figure 9 indicates, by way of example, that, for a 275-mm layer at 10 C, the modulus is 7,500 MN/m^2.

EQUATIONS FOR CALCULATION OF MODULUS

All of the plotted lines shown in Figure 9, together with the other modulus-loading time-temperature relations shown in Figure 8, are straight lines on either logarithmic or semilogarithmic scales. It was thus a relatively simple exercise to determine equations to fit all of the plotted lines.

The relation among loading time (t), layer thickness (h), and vehicle speed (V) (Figs. 6 and 9) is as follows:

$$\log_{10}t = 0.5h - 0.2 - 0.94\log_{10}V \tag{4}$$

where t is in seconds, h is in meters, and V is in km/hr.

The expressions derived for the two sets of modulus-time-temperature relations were of the following form:

$$\log_{10}E = \log_{10}(aT^2 - bT + c) - 10^{-4}(dT^2 + eT + f)\log_{10}t \tag{5}$$

where E = Young's modulus in MN/m^2, T = temperature in C, and a, b, c, d, e, and f are constants that depend on the material. When the preceding expressions are combined to eliminate loading time the following equation results:

$$\log_{10}E = \log_{10}(aT^2 - bT + c) - 10^{-4}(dT^2 + eT + f)(0.5h - 0.2 - 0.94\log_{10}V) \tag{6}$$

Values of the constants for the two materials investigated are given in Table 3.

Using this equation, values of modulus have been calculated for a range of values of the three basic variables: temperature, layer thickness, and vehicle speed. Figure 10 shows the results produced using a simple computer program.

One of the main advantages in establishing an equation for the determination of Young's modulus for the bituminous layer of a pavement structure is that it may be incorporated in a computer program to perform the pavement design calculations.

Table 2. Mix details of 100-penetration binder macadams and asphalt concrete and 45-penetration binder rolled asphalt.

Investigator	Material	Type of Test	Binder Content (percent)	Void Content (percent)	Temperature (C)	Symbol Used in Figures
Snaith et al.	Dense bitumen macadam	Compression	4	5	10	ϕ
Cooper and Pell	Dense bitumen macadam	Flexure of cantilever	6	3	10	○
Snaith et al.	Dense bitumen macadam	Compression	4	5	20	●
Epps and Monismith	Asphaltic concrete	Flexure of beam	6	6	20	□
Shook and Kallas	Asphaltic concrete	Compression	4	6	20	◆
Shook and Kallas	Asphaltic concrete	Compression	5	6	20	+
Shook and Kallas	Asphaltic concrete	Compression	6	6	20	■
Freeme	Dense bitumen macadam	Flexure of cantilever	5 to 6	6 to 12	20	△
Taylor	Rolled asphalt	Flexure of cantilever	6	5	0	ϕ
Taylor	Rolled asphalt	Flexure of cantilever	6	5	10	○
Taylor	Rolled asphalt	Flexure of cantilever	6	5	20	ϕ
Freeme	Rolled asphalt	Flexure of cantilever	6	13 to 20	20	△
Epps and Monismith	Rolled asphalt	Flexure of beam	8	6	20	□

Figure 8. Modulus-time-temperature relation for rolled asphalt.

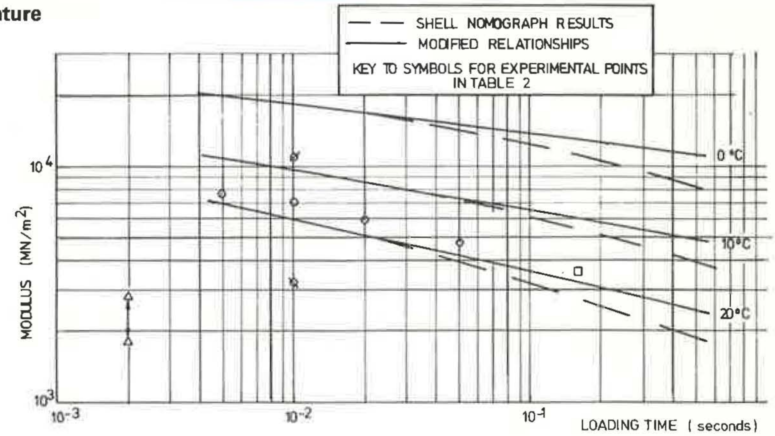


Figure 9. Chart for determination of Young's modulus for dense bitumen macadam with 100-penetration binder.

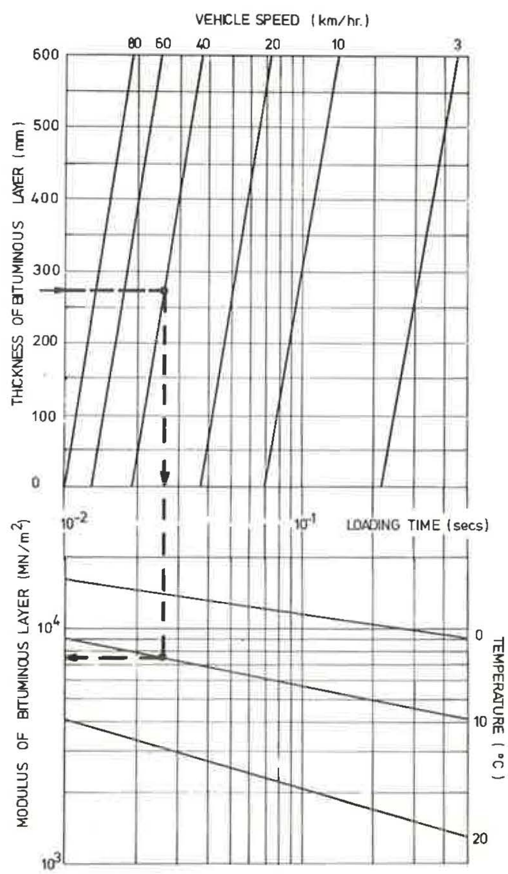


Table 3. Constants for determining Young's modulus (using Eq. 6).

Material	Constant					
	a	b	c	d	e	f
Dense bitumen macadam	10.2	557	8,120	1.8	36.0	1,470
Rolled asphalt	20.2	793	10,360	0	51.5	1,200

Figure 10. Printout of Young's modulus values (computed from Eq. 6).

MATERIAL = D.B.M.100 TEMPERATURE = 15 DEG C ELASTIC MODULI - MN PER SQUARE METRE

V KM PER HOUR

H METRES	3	6	10	15	20	30	40	60	80
0.100	2873	3363	3776	4140	4420	4846	5173	5672	6054
0.125	2853	3339	3750	4112	4389	4812	5137	5632	6013
0.150	2833	3316	3724	4083	4359	4779	5101	5593	5971
0.175	2814	3293	3698	4055	4328	4746	5066	5555	5930
0.200	2794	3270	3673	4027	4298	4713	5031	5516	5888
0.225	2775	3248	3647	3999	4269	4680	4996	5478	5848
0.250	2756	3225	3622	3971	4239	4648	4962	5440	5807
0.275	2736	3203	3597	3944	4210	4616	4927	5402	5767
0.300	2718	3181	3572	3916	4181	4584	4893	5365	5727
0.325	2699	3159	3547	3889	4152	4552	4859	5328	5687
0.350	2680	3137	3523	3862	4123	4520	4826	5291	5648
0.375	2661	3115	3498	3835	4094	4489	4792	5254	5609
0.400	2643	3093	3474	3809	4066	4458	4759	5218	5570
0.425	2625	3072	3450	3782	4038	4427	4726	5182	5531
0.450	2606	3051	3426	3756	4010	4396	4693	5146	5493
0.475	2588	3030	3402	3730	3982	4366	4661	5110	5455
0.500	2571	3009	3379	3704	3954	4336	4628	5075	5417
0.525	2553	2988	3355	3679	3927	4306	4596	5040	5380
0.550	2535	2967	3332	3653	3900	4276	4564	5005	5342
0.575	2517	2947	3309	3628	3873	4246	4533	4970	5305
0.600	2500	2926	3286	3603	3846	4217	4501	4936	5269

Brown (21) has described a computer program called Interpolation, which performs an adequate linear elastic analysis of three-layered structures for design purposes. This program has already been incorporated in a simplified pavement design program. Using the procedure described by Brown and Pell (1) should make it much easier to perform structural design calculations either semimanually or completely by computer.

SUMMARY

In previous publications (1, 21), the author has attempted to present, in a simplified manner, the procedures necessary to design a flexible pavement using the structural design approach. The aim has been to bridge the gap between the very complex procedures being discussed at research level and the empirical rules largely used in practice. The results of the vast amount of research in many countries aimed at developing this improved method of design will only find their way into practice if they are presented in a straightforward manner and in a way that design engineers can use.

The digital computer has become a firmly established tool for civil engineers, and its use is essential to the improvement of pavement design procedures. Nonetheless computing time is expensive, and therefore design calculations need to be as simple as possible yet accurate.

This paper has not considered a number of detailed considerations in order to present something of use to pavement designers. For instance, it is well known that the stiffness of a particular material at a certain temperature and loading time is influenced by mix details such as binder content and filler content. More importantly, the degree of compaction strongly influences stiffness by controlling the void content. The relations presented in this paper are thought to represent the characteristics of the two mixes when well compacted to produce void contents of 6 percent or better.

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