GAP-GRADED AGGREGATES FOR HIGH-STRENGTH QUALITY CONCRETE

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To determine the optimum proportion of materials used in a mix design to achieve a specified quality with least cost is the most important aspect in the production of concrete. Currently available methods for proportioning aggregates are based mostly on previous experience. Even their application to the conventional continuously graded concrete is short of being optimum. It has become essential to seek quantitative information for the optimum proportioning of gap-graded and continuously graded aggregates for various significant mix parameters. In the present work involving 200 mixes with tests carried out according to ASTM standard methods, the results have been analyzed with the objective of revealing optimum mix proportions with respect to significant mix parameters, such as size and quantity of coarse aggregate, size and quantity of the fine aggregate, cement content, and water-cement ratio, at practically the same air content for both gap-graded and continuously graded concretes. The optimum cement contents for both concretes are on the basis of equal maximum size of coarse aggregate and equal compressive strength. The comparison has shown that much less cement is required for concrete with gap-graded aggregates than its continuously graded counterpart with approximately the same workability. Gap grading of the aggregates results in lower requirement of water content, and hence lower water-cement ratio, and permits a higher aggregate-cement ratio to achieve the same workability as its continuously graded counterpart with equal cement content and equal maximum size of coarse aggregate.

•IN the development of concrete technology, a significant step was advanced in 1918 by Abrams (1), who found that the strength of fully compacted concrete solely depended on its water-cement ratio and that aggregate grading was important insofar as it influenced workability required to achieve full compaction. Later research has shown that the influence of other parameters, such as aggregate grading, maximum size of aggregate, surface texture, shape, strength, and other attributes of aggregates, on the compressive strength of concrete cannot be ignored (3, 4). In addition, the parameters of cement content, total water content, and matrix percentage also have considerable influence on the strength of concrete.

Appropriate recognition of these parameters and determination of the relative amounts of constituent materials to be used in a concrete mix to achieve maximum economy and to satisfy the requirements for placement and compaction are of paramount importance in the mix design of concretes with gap-graded and continuously graded aggregates.

Although various investigators $(\underline{1}-\underline{5}, \underline{7}, \underline{12})$ have suggested different methods for proportioning the ingredients of continuously graded concrete, there has been no systematic investigation to determine the optimum proportions of gap-graded versus continuously graded concretes, thereby clearly revealing the quantitative advantages of gap grading.

In 1967-68, Shu-t'ien Li (8, 9, 11) proposed the synthesis of gap-graded shrinkage-compensating concrete. A comprehensive research project was soon initiated by him to verify the hypothesis and to provide working information to concrete technologists

regarding gap-graded versus continuously graded concrete. The results presented here specifically deal with "optimum proportioning of aggregates for high-strength quality concrete."

OBJECTIVES AND SCOPE OF THE STUDY

The main objectives of this investigation are as follows:

1. To determine the optimum amounts, in both gap-graded and continuously graded air-entrained concrete, of basic parameters such as coarse aggregate size and weight, fine aggregate size and weight, water content, cement content, water-cement ratio, and aggregate-cement ratio (for all concretes, an air-entraining agent was added to entrain approximately 5 percent air by volume);

2. To determine the optimum proportioning of gap-graded air-entrained concrete mixtures that are practical from the viewpoints of workability and ability to finish; and

3. To compare optimum-proportioned gap-graded air-entrained concrete with equivalent continuously graded air-entrained concrete, using mixtures having similar workability to the extent that they require similar attention with regard to placement and finishing.

Ranges of variable parameters included in this investigation were water-cement ratios (by weight) of 0.35 to 0.65, aggregate-cement ratios (by weight) of 2.0 to 10.0, maximum sizes of coarse aggregate of $\frac{1}{2}$, $\frac{3}{4}$, 1, and $\frac{1}{2}$ in. (1.27, 1.90, 2.54, and 3.82 cm), and percentages by weight of fine aggregate to coarse aggregate of 28.5, 33.3, 36.5, and 40.0 (for gap-graded concrete only).

MATERIALS

Type I cement was used throughout this investigation. In all mixes, the coarse and fine aggregates were from the same source. Fine aggregate had a water absorption coefficient of 2.04 percent (at 24 hours) and a saturated surface-dry specific gravity of 2.63. Crushed limestone was used as the coarse aggregate. It had an absorption coefficient of 0.58 percent and a saturated surface-dry specific gravity of 2.73.

For all the mixes, a commercially available air-entraining agent in water solution was added to entrain approximately 5 percent air by volume. The required quantity, as recommended by the manufacturer, was mixed with water before it was added to the mixer. The admixture dosage was kept constant, and the air content of only a few mixes was measured. The air contents were generally about 5 percent, ranging from 4 to 7 percent.

The gradings of coarse and fine aggregate are given in Table 1 for gap gradings and in Table 2 for continuous gradings.

SIGNIFICANT PARAMETERS

The proportioning of ingredients is an important phase in the process of manufacturing quality concrete. The nominal amount of air entrainment was kept at 5 percent, and four significant mix parameters were emphasized: size and quantity of coarse aggregate, size and quantity of fine aggregate, cement content, and water-cement ratio. These are the independent variables, whereas the properties of concrete in the plastic and hardened states are the dependent variables.

Among the many possible dependent variables, workability of plastic concrete with respect to placement and compressive strength as a key property of the hardened concrete are important. This is because other attributes of concrete, such as durability, permeability, wear resistance, and tensile strength, are all strongly influenced by compressive strength. For rapid assessment, the 7-day compressive strength was selected as the indicator.

Of the two dependent variables, there has been no quantitative test to measure the workability of fresh concrete according to its ASTM definition, but quantitative determination of the 7-day compressive strength of hardened concrete is a feasibility. Further, workability is directly related to water-cement ratio (13), whereas the

compressive strength varies with the inverse of this ratio. Thus, the 7-day compressive strength of concrete is treated as the more significant dependent variable for the

optimization process.

The aim of optimum proportioning can be boiled down to the design of a concrete mix for specified strength or quality at the least cost of materials. Of all the ingredients for concrete, the cost of cement is the most pronounced factor. An optimum proportioning may require the minimum cement content to satisfy strength or quality requirements. This practical viewpoint has lead to the treating of cement content as the major independent variable.

ANALYSIS OF TEST RESULTS

Optimum Content of Fine Aggregate for Gap-Graded Concrete

To study the influence of fine-aggregate content on 7-day compressive strength of gap-graded concrete, we carried out a pilot program at the beginning of the study. For expediency, a medium maximum-sized coarse aggregate of $\frac{3}{4}$ in. (1.90 cm) was used, but the percentages (by weight) of fine aggregate of the total aggregates were varied. As shown in Figure 1, test results reveal that the optimum content of fine aggregate is nearly 36 percent of total aggregates for all water-cement ratios and aggregate-cement ratios. This fine-aggregate content has been taken for all the other gap-graded concrete mixes involving different maximum-sized coarse aggregates because it has been reported that "when the beakers are filled with one particle size, the void content is constant, regardless of the particle size" (10).

Optimum Mix Proportions for Gap-Graded Concrete and Continuously Graded Concrete

Figures 2 through 5 have been plotted for gap-graded concrete and Figures 6 through 9 for continuously graded concrete to show the influence of cement contents (major independent variable) on 7-day compressive strengths (major dependent variable) for four maximum sizes of coarse aggregate and four water-cement ratios. The curves show a peak at a cement content at which the 7-day compressive strength is maximum. This particular cement content and the corresponding water-cement ratio and the aggregate-cement ratio shown in Figures 10 through 13 are the optimum mix parameters.

The optimum mix proportions for gap-graded and continuously graded concretes and their corresponding 7-day compressive strengths for four maximum sizes of coarse aggregate are given in Tables 3 and 4, which also contain the Vebe Consistometer time

for these mixes.

Based on Figures 2 through 9, the optimum results are given in Tables 3 and 4. These results were used in developing Figures 10 through 17. A few of the results were averaged, when the corresponding optimum cement contents were very close, and the averaged results are shown in Figures 10 through 17. For example, in the case of gap-graded concrete with ½-in. maximum size of coarse aggregate (Table 3), the actual optimum results are given in Table 5. These two results were averaged (Table 5) and plotted in Figures 10 through 14.

The optimum cement contents and their corresponding aggregate-cement ratios for gap-graded versus continuously graded concrete are compared in Figures 10 through 13. They show that the optimum cement content of gap-graded concrete is less than that of continuously graded concrete for equal maximum size of coarse aggregate and 7-day compressive strength. The amount of saving in cement content in gap-graded concrete, when compared with the corresponding continuously graded concrete, varies from 15 to 35 percent. These figures also indicate that gap-graded concrete permits higher aggregate-cement ratios than the continuously graded concrete for equal maximum size of coarse aggregate and optimum cement content.

Figures 14 through 17 show the relation between optimum cement content and its corresponding water-cement ratio for gap-graded and continuously graded concretes for four maximum sizes of coarse aggregate. It can be seen that, for equal optimum cement content, gap-graded concrete requires a lower water-cement ratio than does continuously graded concrete.

Table 1. Gradings of coarse and fine aggregates for gap-graded concrete.

		Percent by Weight for Mix Groups							
Sieve		A49	A35 to	A19 to A34, A68, A71,	A1 to		A67, A70,	A69, A72,	
Passing	Retained	A65		A48		A18	A66	A73, A76	A75, A78
1½ in.	1 in.	63.5	-	-	_	-	_	_	
1 in.	3/4 in.	_	63.5	-	_	_	_	_	
3/4 in.	½ in.	_		63.5	_	71.4	66.7	60.0	
½ in.	3/a in.	_	***	_	63.5	_	-	_	
3/a in.	No. 4	_	-	-	-	_	-	_	
No. 4	No. 8	36.5	-	-	-	-	-	_	
No. 8	No. 16	_	36.5	36.5	-	28.6	33.3	40.0	
No. 16	No. 30	_		-	36.5	-	-	_	

Table 2. Gradings of coarse and fine aggregates for continuously graded concrete.

	Percent by Weight for Mix Groups				
	B47 to	B29 to	B15 to	B1 to	
Retained	B59	B46	B28	B14	
1 in.	25.0	-	2-1	-	
3/4 in.	25.0	22.0	_	-	
½ in.	7.0	22.0	27.5	-	
3/e in.	7.0	14.0	27.5	28.5	
No. 4	12.0	14.0	15.0	41.5	
No. 8	6.0	7.0	7.0	10.0	
No. 16	6.0	6.0	7.0	5.0	
No. 30	5.0	7.0	7.0	5.0	
No. 50	7.0	8.0	9.0	10.0	
	1 in. 3/4 in. 1/2 in. 1/2 in. 3/6 in. No. 4 No. 8 No. 16 No. 30	Retained B47 to B59 1 in. 25.0 3/4 in. 25.0 1/2 in. 7.0 3/6 in. 7.0 No. 4 12.0 No. 8 6.0 No. 16 6.0 No. 30 5.0	Retained B47 to B59 B29 to B46 1 in. 25.0 — 3/4 in. 25.0 22.0 ½ in. 7.0 22.0 3/6 in. 7.0 14.0 No. 4 12.0 14.0 No. 8 6.0 7.0 No. 16 6.0 6.0 No. 30 5.0 7.0	Retained B47 to B59 B29 to B15 to B28 1 in. 25.0 — — 3/4 in. 25.0 22.0 — ½ in. 7.0 22.0 27.5 3/6 in. 7.0 14.0 27.5 No. 4 12.0 14.0 15.0 No. 8 6.0 7.0 7.0 No. 16 6.0 6.0 7.0 No. 30 5.0 7.0 7.0	

Figure 1. Compressive strength and sand content for gap-graded concrete.

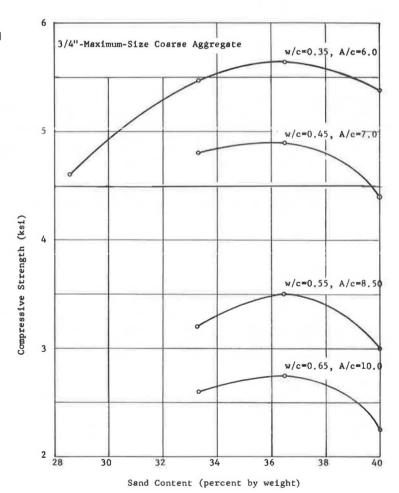


Figure 2. Compressive strength and cement content for water-cement ratios (½-in. maximum-sized aggregate, gap-graded concrete).

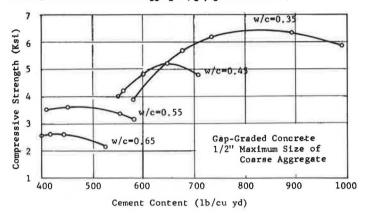


Figure 3. Compressive strength and cement content for water-cement ratios (%-in. maximum-sized aggregate, gap-graded concrete).

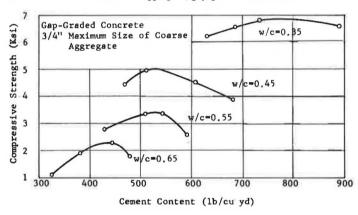


Figure 4. Compressive strength and cement content for water-cement ratios (1-in. maximum-sized aggregate, gap-graded concrete).

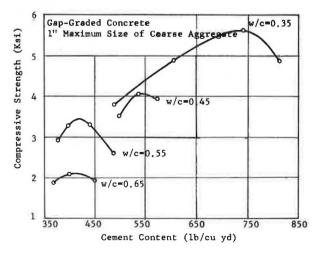


Figure 5. Compressive strength and cement content for water-cement ratios (1½-in. maximum-sized aggregate, gap-graded concrete).

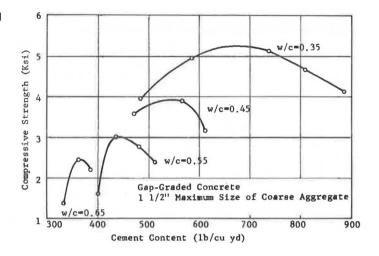


Figure 6. Compressive strength and cement content for water-cement ratios (½-in. maximum-sized aggregate, continuously graded concrete).

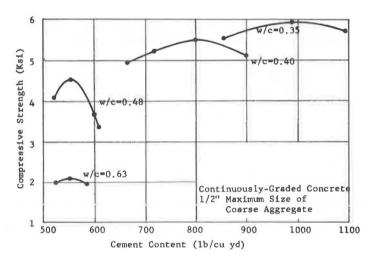


Figure 7. Compressive strength and cement content for water-cement ratios (%-in. maximum-sized aggregate, continuously graded concrete).

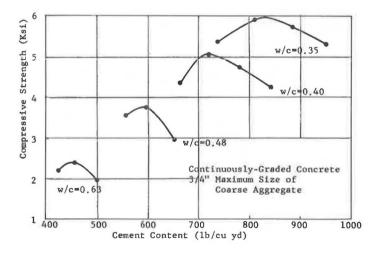


Figure 8. Compressive strength and cement content for water-cement ratios (1-in. maximum-sized aggregate, continuously graded concrete).

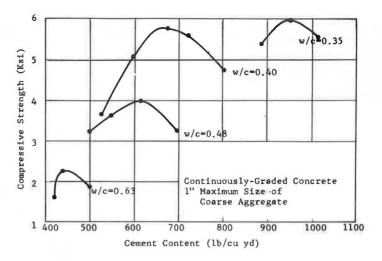


Figure 9. Compressive strength and cement content for water-cement ratios (1½-in. maximum-sized aggregate, continuously graded concrete).

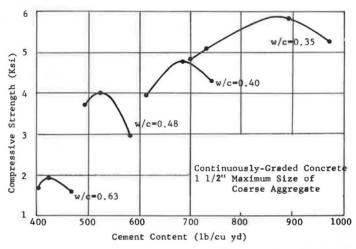


Figure 10. Optimum cement content, compressive strength, and aggregate-cement ratio for gap-graded and continuously graded concrete (½-in. maximum-sized aggregate).

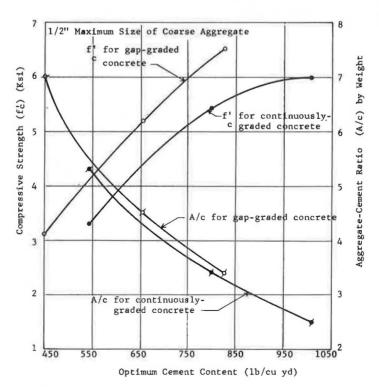


Figure 11. Optimum cement content, compressive strength, and aggregate-cement ratio for gap-graded and continuously graded concrete (%-in. maximum-sized aggregate).

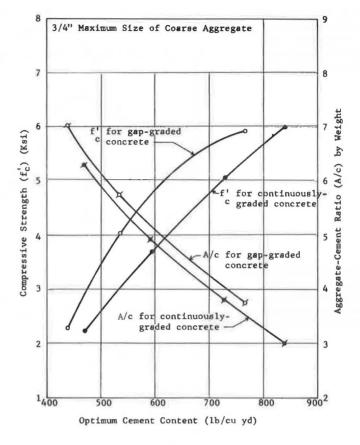


Figure 12. Optimum cement content, compressive strength, and aggregate-cement ratio for gap-graded and continuously graded concrete (1-in. maximum-sized aggregate).

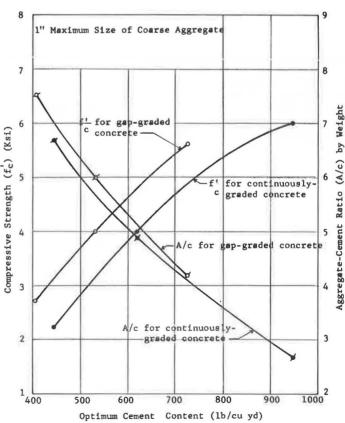


Figure 13. Optimum cement content, compressive strength, and aggregate-cement ratio for gap-graded and continuously graded concrete (1½-in. maximum-sized aggregate).

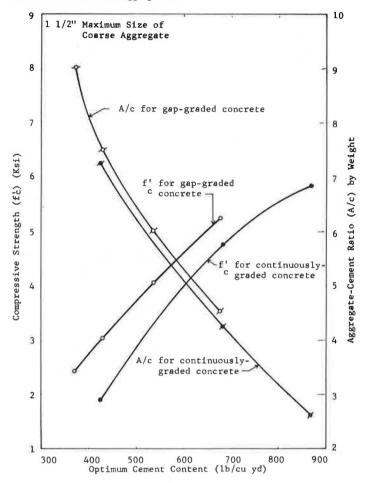


Table 3. Optimum results for gap-graded concrete.

Maximum Size of Coarse Aggregate (in.)	Cement Content (lb/cu yd)	Aggregate- Cement Ratio (by weight)	Water- Cement Ratio (by weight)	7-Day Compressive Strength (lb/in. ²)	Vebe Time (sec)
1/2	825	3.40	0.35	6,500	9
	655	4.50		5, 200	7
	455	7.00	0.55	3,700	5 4
	445	7.00	0.65	2,600	4
3/4	765	3.75	0.35	5,900	9
	530	6.00	0.45	4,950	7
	540	5.50	0.55	3,150	4
	440	7.00	0.65	2,300	2
1	725	4.20	0.35	5,600	8
	535	6.00	Ratio (by weight) (by weight) 3.40 0.35 4.50 0.45 7.00 0.55 7.00 0.65 3.75 0.35 3.00 0.45 5.50 0.55 7.00 0.65 4.20 0.35 3.00 0.45 7.00 0.55 3.00 0.45 7.00 0.55 3.00 0.45 3.00 0.45 3.00 0.45 3.00 0.45 3.00 0.45 3.00 0.45 3.00 0.65 4.50 0.35 3.00 0.65	4,000	8
	415	7.00	0.55	3,400	7
	400	8.00	0.65	2,100	5
11/2	675	4.50	0.35	5,250	12
	540	6.00	0.45	4,000	12
	430	7.50	0.55	3,000	10
	365	9.00	0.65	2,425	4

Table 4. Optimum results for continuously graded concrete.

Maximum Size of Coarse Aggregate (in.)	Cement Content (lb/cu yd)	Aggregate- Cement Ratio (by weight)	Water- Cement Ratio (by weight)	7-Day Compressive Strength (lb/in.2)	Vebe Time (sec)
1/2	1,010	2.50	0.35	6,000	8
	800	3.40	0.40	5,450	7
	560	5.30	0.48	4,550	4
	530	5.30	0.63	2,100	2
3/4	840	3.00	0.35	6,000	15
	730	3.80	0.40	5,050	7
	595	5.00	0.48	3,700	5
	470	6.30	0.63	2,250	4
1	950	2.70	0.35	6,000	7
	675	4.40	0.40	5,750	7
	620	4.90	0.48	4,000	4
	445	7.00	0.63	2,250	4
11/2	870	2.60	0.35	5,850	7
	685	4.30	0.40	4,775	7
	525	6.00	0.48	4,000	6
	425	7.50	0.63	1,000	4

Figure 14. Optimum cement content and water-cement ratio for gap-graded and continuously graded concrete (½-in. maximum-sized aggregate).

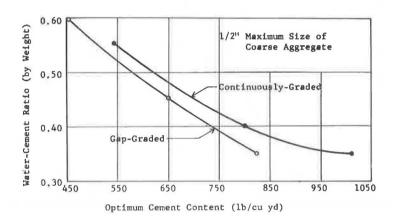


Figure 15. Optimum cement content and water-cement ratio for gap-graded and continuously graded concrete (%-in. maximum-sized aggregate).

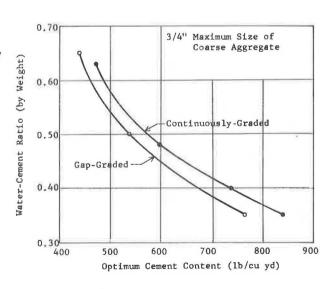


Figure 16. Optimum cement content and water-cement ratio for gap-graded and continuously graded concrete (1-in. maximum-sized aggregate).

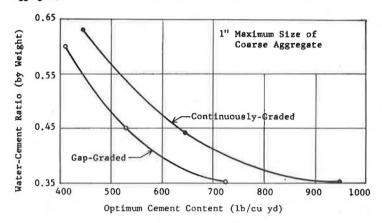


Figure 17. Optimum cement content and water-cement ratio for gap-graded and continuously graded concrete (1½-in. maximum-sized aggregate).

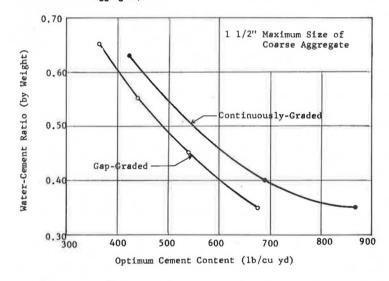


Table 5. Optimum and average results.

Cement Content (lb/cu yd)	Aggregate- Cement Ratio (by weight)	Water- Cement Ratio (by weight)	7-Day Compressive Strength (psi)
455	7.00	0.55	3,700
445	7.00	0.65	2,600
450	7.00	0.60	3,150

The preceding results can be summarized by stating that gap-graded concrete requires lower cement content, lower water-cement ratio (and hence lower water content), and higher aggregate-cement ratio than continuously graded concrete for equal 7-day compressive strength and maximum size of coarse aggregate and workability. These results confirm the hypothesis concerning superior qualities of gap-graded concrete (8, 9, 11).

Vibration Time of Concrete Mixes and Its Relation to Cement Content

Concrete cylinders for all the mixes were made with the aid of internal vibrators, as recommended in ASTM Designation C 192-68. Concrete was filled in the molds in two approximately equal layers, and for each layer the vibrator was inserted at three different equally spaced points. As the vibrator needle was inserted, the time (in seconds) required for the mortar to appear evenly at the top surface around it was recorded as the vibration time, and the vibrator was then withdrawn in such a manner that no air pockets were left in the concrete. The same vibration time was used for all other five insertions. This simple technique not only proved well for achieving full compaction but also provided a measurement for the workability of the mix. The vibration time was higher for the stiffer mixes and lower for the more workable ones.

Vibration times recorded for gap-graded and continuously graded concrete mixes have been plotted against cement contents (Figs. 18 and 19). As would be expected, the vibration time decreases with an increase in the water-cement ratio and cement content. A previous analysis of these results (12, Figs. 12 through 15) indicates that the vibration time required for gap-graded concrete is less than that for continuously graded concrete for all cement contents, water-cement ratios, and maximum sizes of coarse aggregate throughout this investigation. This should mean that gap-graded concrete is more workable and requires less energy for full compaction than does continuously graded concrete having the same mix parameters.

ECONOMICS OF GAP GRADING OF AGGREGATES

The economics of gap grading of the aggregates depends on the locality, the source of the material, and the equipment used for quarry processing. Where there are naturally occurring well-graded aggregates from coarse to fine available, the use of continuous grading may be economical. When naturally occurring well-graded aggregates are not available, the use of continuous grading to fit a certain curve of size variation for maximum density or any other would require size separation, different bins or stockpiles, synthesis, the necessary elaborate making of certain missing intermediate sizes, and even costly precautions to eliminate segregation, all of which will cost more in quarry processing.

Gap grading will constitute a solution where naturally occurring well-graded aggregates are lacking, as is the case in many regions of the world. Savings can occur in those cases when aggregates are naturally gap-graded or where there are excesses of particular sizes that result from a given process. It has been reported (18) that the production of road pavements in India, in many areas, was becoming extremely expensive because the authorities insisted on the use of sand conforming to a British standard specification. This meant that frequently the local fine sand was not acceptable, and hauls of up to 200 miles were necessary to bring in sand of an acceptable grading. The Indian Road Research Laboratory, after studying Stewart's work on gap grading, decided to make its own tests. These showed such promise that, in a short time, standard gradings were discarded and local sands were brought into use, producing completely acceptable concrete to which could be credited the reduced haulage and cement costs.

Apart from demonstrating that a logical approach to the formulation of concrete grading reduced cost and even improved quality, it shows the danger of either a central or local authority introducing codes of practice that specify in detail rather than in principle and thereby prevent the engineer from pursuing fully his profession, the economic use of natural resources.

Figure 18. Cement content and vibration time for water-cement ratios (½-in. maximum-sized aggregate, gap-graded concrete).

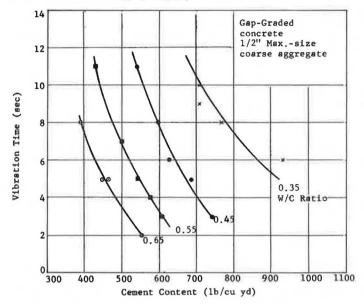
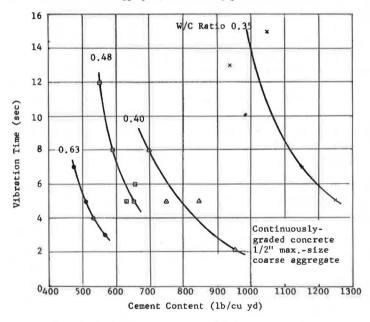


Figure 19. Cement content and vibration time for water-cement ratios (½-in. maximum-sized aggregate, continuously graded concrete).



Aggregate sizes within a narrow range for gap grading can be better accomplished today with modern equipment than several decades ago. They do not need filler sizes, and they are not liable to aggregate segregation. The quarry products of different sizes can all be appropriately used, for instance, in a bridge project: $1\frac{1}{2}$ - to $1\frac{1}{4}$ -in. coarse aggregate can be well used in bridge piers and foundations, $1\frac{1}{4}$ to 1 in. in girders, 1 to $3\frac{1}{4}$ in. in cross beams and stringers, and $3\frac{1}{4}$ to $3\frac{1}{6}$ in. in deck slabs. In a well-planned program there could be no waste.

Even though in some localities gap-graded sand costs more than natural sand, the

saving in cement content can offset this increased sand cost.

Gap grading of the aggregates is imperative for precasting exposed aggregate surfaces, as has been done for more than three decades, to show the exposed aggregate more uniformly and more prominently.

The increasing use of lightweight aggregates that can be easily manufactured in equal maximum sizes will further open a new vista for the application of gap grading

to lightweight aggregate concrete.

Based on the results presented in this paper, an optimum mixture design method for gap-graded concrete, a step-by-step procedure for using this method, and an illustrative numerical example are presented elsewhere (17).

CONCLUSIONS

The work presented here involved 200 concrete mixes having different water-cement ratios, aggregate-cement ratios, and maximum sizes of coarse aggregate for both gap-graded and continuously graded concretes. For all the mixes, 7-day compressive strength of hardened concrete and workability of plastic concrete were tested. The results were analyzed to obtain optimum mix parameters and to compare them using concretes having gap-graded and continuously graded aggregates.

Of the four significant mix parameters (cement content, water-cement ratio, size and quantity of coarse aggregate, and size and quantity of fine aggregate), cement content plays by far the most important role in the optimization for obtaining the most economical concrete mix. The more important properties of plastic and hardened concrete to be considered for optimization are respectively workability and compressive strength, but the latter is more amenable to quantitative determination.

The analysis has borne out the conclusions that follow. However, it should be pointed out that these conclusions are based on the experimental results reported here and are applicable to the types of crushed limestone and sand used in this investigation:

1. Pilot experimental results indicate that, for any specific water-cement ratio, aggregate-cement ratio, and maximum size of coarse aggregate, there is an optimum content of fine aggregate for which the 7-day compressive strength of gap-graded concrete is a maximum, and this fine-aggregate content can be used to arrive at the optimum proportioning.

2. The present test data show that this optimum content of fine aggregate is very nearly equal to 36 percent by weight of total aggregates irrespective of variations in

water-cement ratio and aggregate-cement ratio (Fig. 1).

3. Two-dimensional plots between cement content and 7-day compressive strength for gap-graded and continuously graded concretes of equal water-cement ratio and maximum size of coarse aggregate have shown that, at one particular cement content, the compressive strength reaches its maximum. This cement content and the corresponding water-cement and aggregate-cement ratios are the optimum mix parameters.

4. As expected, optimum cement contents increase with the increase in the 7-day

compressive strength and with the decrease in the water-cement ratio.

5. The optimum cement content of gap-graded concrete is lower than the optimum content for continuously graded concrete of equal 7-day compressive strength and maximum size of coarse aggregate.

6. Quantitatively, for all maximum sizes of coarse aggregates and nominal compressive-strength ranges within this investigation, gap-graded concrete requires 15 to 35 percent lower optimum cement content than its continuously graded counterpart equally proportioned.

- 7. For equal optimum cement content, maximum size of coarse aggregate, and workability, gap-graded concrete requires lower water-cement ratio and hence lower water content than its continuously graded counterpart.
- 8. For equal optimum cement content, maximum size of coarse aggregate, and workability, gap-graded concrete permits a higher aggregate-cement ratio.
- 9. A comparison of the recorded vibration time needed for full compaction of gap-graded concrete with that of the continuously graded has shown that the former is more workable and requires less energy to bring to a homogeneous mass then the continuously graded having the same water-cement ratio, cement content, and maximum size of coarse aggregate.

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