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# FOREWORD

Mineral aggregates constitute roughly 75 percent of the absolute volume of concrete, and substantial investment is made annually to ensure that this important constituent has suitable grading (i.e., particle-size composition). Challenges to accepted practice regarding grading are periodically made, and this group of papers aims to update the knowledge of such practices for both the practitioner and the researcher in concrete technology.

Three papers consider the rationale of gap grading of concrete aggregates, namely, those of Lees, Li and Stewart, and Li and Ramakrishnan. Lees considers continuous gradings as well as gap-graded aggregates and extends a proposed proportioning method to "textured" pavement surfaces. Measurements of locked-wheel friction resistance are reported for the latter surfaces. The Li and Stewart paper provides field experiences in England and elsewhere on use of gap-graded aggregates. The Li-Ramakrishnan studies are confined primarily to laboratory investigations.

Stark and Klieger present substantial evidence that, where certain nondurable aggregates cause D-cracking of concrete pavements in the north-central United States, such cracking can often be eliminated or greatly postponed by reducing the maximum particle size in the paving mixture. Also, laboratory procedures for distinguishing between aggregate sources contributing to D-cracking and those of satisfactory record are detailed.

Imbert's paper concerns the concrete-making properties of aggregates, some of which are of unusual grading and mineralogy, available on three islands in the eastern Caribbean. With suitable adjustments of mix proportions, acceptable concrete has been made employing these aggregates.

Lee and Popovics provide information in separate papers on several methods used to compute blending proportions of two or more aggregates to achieve a desired final grading. These methods range from simple graphical solutions to more complicated exact analytical procedures.

In the paper by Popovics concerning the internal structure of concrete, it is proposed that much can be learned by measuring the mortar layer intercepts between coarse aggregate particles in the hardened concrete. A minimum mortar thickness is needed and is influenced by, among other things, the sand grading.

The paper by Marais, Otte, and Bloy considers the influence of grading on drying shrinkage and tensile properties of "lean-mix concrete" or what would more probably be called cement-treated base materials.

Baker and Scholer present laboratory data assessing the changes in certain concrete properties resulting from fluctuations in aggregate grading found to be typical of those encountered in actual construction operations.

Papers in this Record were prepared for an international symposium held during the Board's fifty-second annual meeting. There was representation from Great Britain, Trinidad, South Africa, and the United States.

—Frank Legg

# RATIONAL DESIGN OF CONTINUOUS AND INTERMITTENT AGGREGATE GRADINGS FOR CONCRETE

Geoffrey Lees, University of Birmingham, England

The author has earlier presented an improved method for the design of aggregate gradings and demonstrated its applicability to the design of dense asphaltic compositions (1). Subsequently, further work has been done to confirm that both continuously graded and gap-graded bituminous mixes can be satisfactorily designed according to the method and to show that, for highway and airfield pavement surfacing purposes, certain principles governing skid resistance over a practical range of speeds can be incorporated into the design method. The current study describes the application of the method to the design of concrete mixes. For gap or intermittent gradings, the relevance of studies of interparticle voids is demonstrated with respect to critical ratios of entrance and occupation (measured for typical aggregates) and a newly defined term, the critical ratio of dilation. The influence of gradings, designed according to this rational method, on concrete workability and strength is discussed. Reference is also made to the design of pavement concrete mixes to meet the different requirements of skid resistance for low-speed and high-speed conditions on roads and airfields.

•THE properties of concrete are highly dependent on the structural arrangement of included aggregate particles, and the literature contains many references to the study of aggregate gradings. However, comparatively few of the methods that have been proposed for selection or design of aggregate gradings have considered adequately the packing properties of the aggregate particles. Rather, these have assumed that the optimum grading follows some simple mathematical law or law of past experience, irrespective of variations of shape from source to source, or with variation in crushing plant or method, or from size to size within the same source and irrespective of the several other factors that affect the structural packing of particles. Other methods of grading design that have considered some of these factors have neglected others or have treated them as factors to be dealt with in isolation, ignoring their interaction.

In the present method, a simple experimental procedure has been developed for assessing the packing properties of intended component aggregates in the laboratory, from the results of which may be determined the optimum percentage of fine material to obtain maximum density in a two-component mix. It has been subsequently verified that this approach can be extended into the field of multicomponent mixes, enabling maximum-density gradings of both the intermittent and continuous types to be derived. These gradings, in contrast to those of a more empirical nature (2-6), take into account any change in aggregate packing properties that may occur at any size level.

In addition to the important effects on packing of particle shape, the method recognizes the importance of any lubricating or adhesive coatings, surface static effects on small particles, degree and type of compactive effort, and the effect of external boundaries including external form and minimum thickness of any section and of internal boundaries such as reinforcement of various degrees of congestion.

The complexity of the interactions of these factors makes it impossible to derive reliable porosity values (upon which the design calculations will be based) by way of correction factors for the variables concerned. Accordingly, it is considered that all assessments of porosity should be made in containers that reproduce as nearly as practicable the critical minimum dimension(s), external form and attitude, and internal boundary effects of the proposed concrete member. Representative samples of the actual aggregates to be used are employed in the laboratory tests and under a compactive effort varied in accordance with the anticipated effort on site.

Four factors determine the porosity of a system of packed aggregate particles: the mode of packing, the size distribution, the shape (in which is included surface texture), and the shape distribution. Other factors have been mentioned in the literature, but all exert their influence ultimately via one of these four main factors.

## AGGREGATE GRADING DESIGN

The factors that have been mentioned previously as being of major importance in influencing the packing and porosity of aggregate particles have in the proposed method been recognized as those that should be considered in the design of aggregate gradings.

It has further been recognized that, for this design purpose, it is unnecessary to undertake detailed and time-consuming measurements of influencing parameters such as particle shape, surface texture, specific surface, area-volume ratio of container, surface area of reinforcement, and compaction energy and that the total and mutually interacting effects of these parameters can be most simply taken into account by simulative laboratory tests in which measurements are taken of the mass porosity for each considered size component. The mass porosity is the measured porosity in a system of packed aggregate particles, excluding any internal porosity of the particles.

Justification for this approach has been given in an earlier paper (1) in which it has been shown by reference to theoretical and practical studies of packing of two-component systems that the maximum density is achieved at an optimum percentage of the fine component, this optimum being a function of the porosities of the individual separate components and the size ratio between the components; i.e.,

$$\begin{array}{l} \text{Optimum} \\ \text{percentage of} \\ \text{fines (by} \\ \text{volume)} \end{array} = f\left\{\begin{array}{llll} \begin{array}{l}\text{particle shape and}\\ \text{surface texture,}\\ \text{surface charge,}\\ \text{lubricating and}\\ \text{adhesive coatings}\end{array} & \begin{array}{l}\text{container wall}\\ \text{and internal}\\ \text{boundary}\\ \text{effects}\end{array} & \begin{array}{l}\text{compactive}\\ \text{effort}\end{array} & \text{size ratio}\end{array}\right.$$

$$= f\left\{\underbrace{\text{particle friction} \quad \begin{array}{l}\text{boundary}\\ \text{effects}\end{array} \quad \begin{array}{l}\text{compactive}\\ \text{effort}\end{array}}_{\begin{array}{l}\text{measured porosities of fine and coarse components}\\ \text{expressed as}\end{array}} \quad \text{size ratio}\right.$$

$$= f\left\{\overbrace{P_{av} \qquad\qquad P_{diff}} \qquad\qquad \text{size ratio}\right.$$

where $P_{av}$ = average porosity of separate coarse and fine components, $P_{diff}$ = difference between porosity values of coarse and fine components, and where differences are positive if the coarse component has the higher porosity and negative if the fine component has the higher porosity.

Figure 1 shows how, for two-component systems, the optimum percentage of fine material for minimum porosity in the mix varies significantly with average particle shape, differences in particle shape, compactive effort, and boundary wall effects.

The upper points (F and C) on the vertical axes of the graphs of Figure 1 represent the specific void content (void ratio) and porosity values for hypothetical cases of packing of aggregates of certain generalized shape categories under different compactive efforts and in different size containers.

Lines are drawn from the specific void content value of any considered component to the partial specific void content of the other component plotted on the opposite axis [following the methods of Furnas (7, 8) and Powers (9)]. The partial specific void content of a component may be defined as the contribution made by it to the total void content of the mix, assuming that no dilation of the other component is caused by its presence. This assumption is only valid for the theoretical case of size ratio equals zero, where, for example, the fine particles can be considered to be infinitely small. Thus, the partial specific void content of any coarse aggregate ($C_p$) equals zero because coarse aggregate particles can, under this assumption of no dilation of the other component, be added to a system of fine aggregate without any addition of voids to the mix; i.e., such particles add their own volume, but no more than their own volume, to the system.

For the fine aggregate (again when size ratio equals zero), it is possible to add small particles, up to a certain proportion, to a system of coarse aggregate particles without increasing the bulk volume of the system. In other words, these particles reduce the void content of the system. The partial specific void content of the fine aggregate ($F_p$) is therefore negative (-1), each added particle deducting its own volume from the total void content.

The lines drawn as indicated from C to $F_p$ and from F to $C_p$ (Fig. 1) indicate the drop in voids that would occur by adding increasing quantities of one component to the other. Beyond the point of intersection O, from whichever direction it is approached, the opposite trend takes over. The point of intersection therefore represents the lowest possible void content for the combined fine and coarse aggregates blended at their optimum proportions.

For mixes of finite size ratio, i.e., between 0 and 1, the presence of one component will always dilate the other. Thus, no void contents lower than the construction lines drawn can exist, and the total range of void contents for all possible blends lies within the shaded triangle bounded by the straight line (size ratio equals one) and the upper portions of the size-ratio-equals-zero lines (i.e., FO and OC).

For the theoretical case of size ratio equals zero, Figures 1a, 1b, 1c, and 1d show that the optimum proportions for minimum voids are dependent on the shape characteristics of the aggregate. For example, if both fine and coarse aggregates are of high sphericity, high roundness particles packing individually to low porosities of, say, 30 percent, then a low percentage of fines (23.1) is indicated (Fig. 1a).

However, if both components are angular, packing to high voids values of, say, 60 percent, then the optimum percentage of fines is higher, i.e., 37.5 (Fig. 1b). Still more extreme differences in the optimum proportions are noticed if aggregates of contrasting shape properties are combined, e.g., 47 percent fines for an angular coarse aggregate combined with a rounded fine aggregate (Fig. 1c) contrasting with 21 percent fines for a rounded coarse aggregate combined with an angular fine aggregate (Fig. 1d).

Figures 1e and 1f show further that, even if the aggregate type is not changed, a difference in compactive effort itself produces a difference in the optimum proportion, from 28.6 to 33.3 percent fines in the case illustrated.

The further effect of boundaries on packing is shown in Figures 1g and 1h, which show that, even if aggregate type and compactive effort are held constant, a change in the dimensions of the section to be filled also leads to a change in the optimum proportions, from 28.6 to 40.0 percent fines in the case shown. It has been hypothesized in this case that, although both fine and coarse aggregates would be affected by boundary walls so as to increase their porosity, the coarse aggregate would be more affected by a reduction in the size of the container than the fine aggregate (10, 11).

Although all effects in Figure 1 have been shown by reference to the theoretical case of size ratio equals zero, it has been established that, for real systems with finite size ratio, the same trends are apparent but that the value of the optimum is affected by the size ratio.

In the light of this analysis, the approach in the present study has been to consider that a common law might exist for the proportioning of particles in a two-component system, based on the size ratio and on the measured porosities of each of the components but irrespective of the means by which the porosities were produced.

In application of the method to the design of two-component and multicomponent systems, the following procedure is adopted:

1. Representative samples of the aggregates to be used are obtained by accepted sampling procedures. The sizes to be tested are chosen as those that are to be used

in the mix in question. Thus, for a continuous grading, all sizes from maximum to minimum are tested in the same arbitrarily divided subgroups as those from which the final mix will be made up. If the mix is to be intermittently graded, only the selected sizes will be subject to test.

2. Tests are performed to determine the mass porosity (i.e., excluding internal porosity) of the aggregate components. These tests are performed under conditions that simulate as closely as possible the field case with respect to container dimensions and compactive effort.

A container is made up in a laboratory, which simulates the field section, i.e., vertical slab, horizontal slab, column, etc., as closely as practical, with or without reinforcement as designed. The dimensions of the container will normally include the minimum dimension(s) of the field section. Thus, in the case of design for a slab, the minimum dimension of the container will be the slab thickness. The remaining dimensions are not critical in this case. In practice, it is considered satisfactory if the remaining dimensions are at least two to three times the minimum thickness.

The compactive effort applied to the aggregate is determined on the basis of the anticipated effort on site. Heavy compaction with a vibrating plate is, for example, simulated in the laboratory by vibration of the aggregate components to a maximum density condition on a vibrating table. The maximum density condition is assessed as that at which no further decrease in volume can be observed with time or with change in amplitude of vibration. Hand placing with no vibration would be simulated by dropping the aggregate at random into the container from a scoop held approximately 2 in. above the aggregate level and applying no further effort.

3. For each of the components, the mean equivalent spherical diameter (ESD) is calculated [ESD = $\sqrt[3]{(6V)/\pi}$, where V = average volume per particle, calculated from the average weight per particle and the known bulk specific gravity, taking not less than 500 particles]. The ESD for particles too small to be counted out is obtained by extrapolation from a series of determinations on larger aggregate from the same source.

4. The calculated values of porosity and equivalent spherical diameter (columns 2 and 3, Tables 1 and 2) are input to a computer program that has been prepared for the design method (12), from which the output is the data in columns 4 through 12 of these tables. The output gives the design grading as proportionate volumes of the component sizes (column 8). The completion of the design by conversion to proportionate weight (column 13, Table 1; column 14, Table 2) percentage of weight (column 14, Table 1; column 15, Table 2), and summation percentage of weight (column 16, Table 2) are separately performed by reference to specific gravity differences among component aggregates.

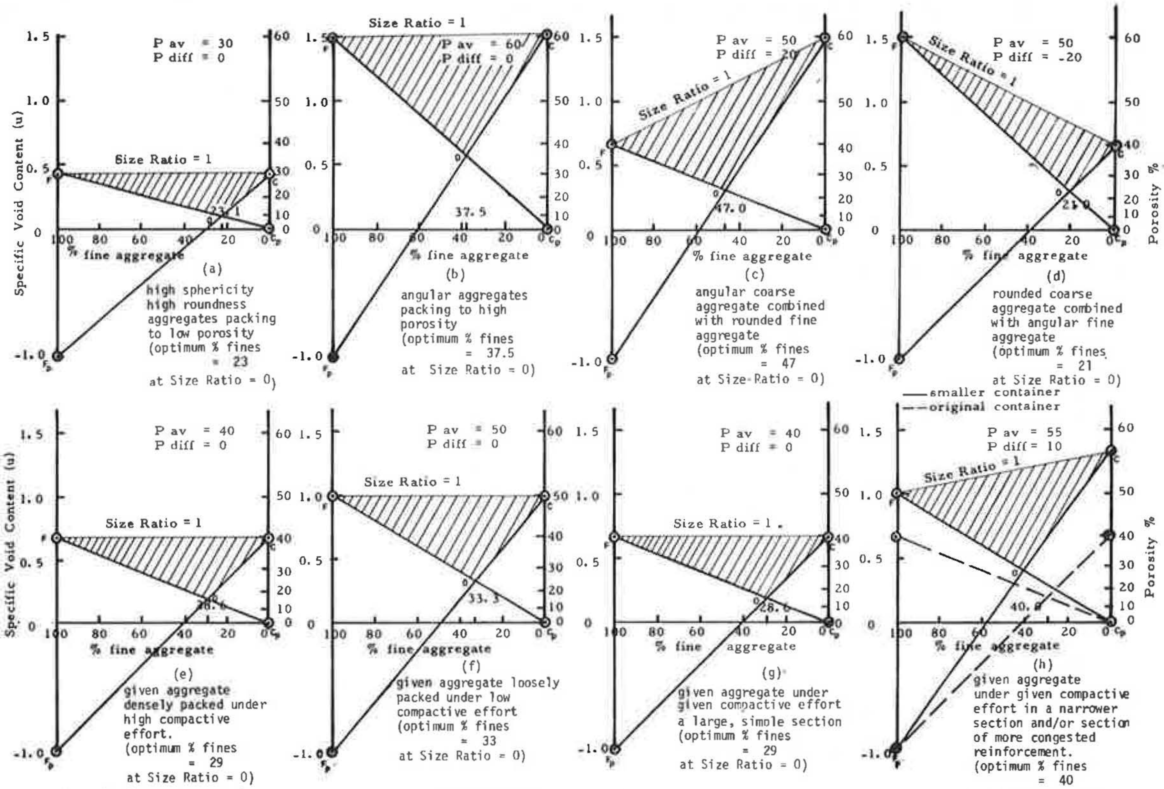
It should further be noted that, if the aggregate has been tested according to quarry sizes (i.e., with its attendant oversize and undersize material), the design percentage of weights will be relevant to the quarry sizes. A grading curve for the mix is plotted by correcting for oversize and undersize and calculating the summation percentage on this basis.

Table 3 gives the application of the method to two- and three-component mixes of a wide variety of particle shape combinations. Predicted porosities for the proposed method at the two-component stage and the final three-component stage are generally close to the measured value with a similar order of accuracy. Mixes with up to 15 components have similarly shown good agreement between predicted and measured porosities (1).

## Design of Continuous Gradings

In this type of grading system, there is by definition some representative portion of each size between the selected maximum and minimum sizes (and normally continuous with the size of cement particles in the case of concrete). It has sometimes been taken that a continuous system will be of the form of $p = (d/D)^n \times 100$, where p = percentage finer than size d, d = the particular sieve size considered, D = the maximum size, and n = an exponent (generally in the range of 0.2 to 0.5). Also, much of the literature has been devoted to discussion of the merits of the various values of n.

**Figure 1. Effect of particle shape, compactive effort, and boundary on optimum percentage of fine aggregate for maximum density in two-component mixes.**




Note: Specific void content = e/(i - e) where e = unit voids in a system of packed aggregate particles.

**Table 1. Computation for continuous grading for concrete (mix C, Table 5).**

| Sieve Size (1) | Porosity (percent) (2) | Equivalent Spherical Diameter (mm) (3) | Porosity Avg[a] (percent) (4) | Porosity Difference[b] (percent) (5) | Size Ratio[c] (6) | Optimum Percentage of Fine[d] (7) | Proportional Volume[e] (8) |
|---|---|---|---|---|---|---|---|
| ½ in. to ⅜ in. (crushed rock) | 46.5 | 11.24 | — | — | — | — | 50.0 |
| ⅜ in. to ¼ in. | 45.9 | 7.51 | 46.2 | +0.6 | 0.66 | 50.0 | 50.0 |
| ¼ in. to No. 7 | 45.0 | 4.73 | 44.8 | -0.3 | 0.5253 | 40.9 | 69.2 |
| 3/16 in. to No. 200 (concrete-sand) | 28.1 | 0.414 | 35.3 | +14.4 | 0.073 | 41.6 | 120.0 |

| Sieve Size (1) | Cumulative Volume[f] (9) | Mean Equivalent Spherical Diameter[g] (mm) (10) | Relative Contraction[h] (11) | Mix Porosity (percent)[i] (12) | Proportional Weight (13) | Percentage of Weight (quarry sizes) (14) |
|---|---|---|---|---|---|---|
| ½ in. to ⅜ in. (crushed rock) | 50.0 | 11.24 | — | 46.5 | 50.0 | 17.7 |
| ⅜ in. to ¼ in. | 100.0 | 9.00 | 0.048 | 44.7 | 50.0 | 17.7 |
| ¼ in. to No. 7 | 169.2 | 5.729 | 0.088 | 42.5 | 69.2 | 24.5 |
| 3/16 in. to No. 200 (concrete-sand) | 289.2 | 0.91 | 0.450 | 20.8 | 113.0 | 40.0 |

[a] Average of b(line n)/i(line n - 1).
[b] i(line n - 1) - b(line n).
[c] c(line n)/j(line n - 1).
[d] Derived from reference 1 or computer program.
[e] Proportional volume = [i(line n - 1) x g(line n)]/[100 - g(line n)].
[f] Sum of column 8.
[g] Mean ESD = $1\Big/\left[\sum_{i=1}^{n}(p_i/d_i)\right]$ where $p_1, p_2, \ldots p_n$ are the proportions by volume of particles of diameter $d_1, d_2, \ldots d_n$ respectively for any number of components.
[h] Derived from reference 1 or computer program.
[i] Mix porosity = a - relative contraction $[a - (p_{coarse} \times p_{fine}/100)]$ where $a = p_{coarse}$ or $p_{fine}$, whichever is the lower; relative contraction = column 11; $p_{coarse}$ = column 1 (line n - 1); and $p_{fine}$ = cloumn b (line n).

**Table 2. Design calculations for four-component gap-graded concrete.**

| Sieve Size (1) | Porosity (percent) (2) | Equivalent Spherical Diameter (mm) (3) | Porosity Avg (percent) (4) | Porosity Difference (percent) (5) | Size Ratio (6) | Optimum Percentage of Fine (7) | Proportional Volume (8) | Cumulative Volume (9) |
|---|---|---|---|---|---|---|---|---|
| 1/2 in. to 3/8 in. (basalt) | 45.7 | 10.06 | – | – | – | – | 48.35 | 48.35 |
| No. 7 to No. 14 (sand) | 32.6 | 1.73 | 39.1 | +13.1 | 0.172 | 51.65 | 51.65 | 100.0 |
| No. 36 to No. 52 | 37.0 | 0.3651 | 33.52 | -6.96 | 0.127 | 15.96 | 18.97 | 118.97 |
| No. 150 to No. 200 | 37.5 | 0.09164 | 29.43 | -16.06 | 0.067 | 10.7 | 14.4 | 133.37 |

| Sieve Size (1) | Mean Equivalent Spherical Diameter (mm) (10) | Relative Contraction (11) | Calculated Mix Porosity (percent) (12) | Actual Mix Porosity (percent) (13) | Proportional Weight (14) | Percentage of Weight (15) | Summation (percent) (16) | Size Ratio[a] (adjacent) (17) |
|---|---|---|---|---|---|---|---|---|
| 1/2 in. to 3/8 in. (basalt) | 10.06 | – | 45.7 | – | 48.35 | 37.5 | 100.0 | – |
| No. 7 to No. 14 (sand) | 2.88 | 0.145 | 30.0 | 28.5 | 47.9 | 37.2 | 62.5 | 0.172 |
| No. 36 to No. 52 | 1.37 | 0.455 | 21.4 | 21.1 | 18.55 | 14.4 | 25.3 | 0.211 |
| No. 150 to No. 200 | 0.544 | 0.47 | 15.9 | 16.7 | 14.05 | 10.9 | 10.9 | 0.251 |

Note: Further explanation of columns 4 through 12 is given in Table 1, footnotes a through i.
[a]c(line n - 1)/c(line n).

**Table 3. Composition and predicted porosities.**

| Mix | Material | British Standard Sieve Size | Recommended Proportions (percent) | Predicted Porosity (percent) | Measured Porosity (percent) |
|---|---|---|---|---|---|
| G1 | Rounded gravel | 3/4 in. to 1/2 in. | 60.3 | | |
| | Crushed gravel | 1/4 in. to 3/16 in. | 28.4 | 31.0 (2 components) | 30.5 |
| | Leighton buzzard sand | No. 14 to No. 25 | 11.3 | 19.8 (3 components) | 21.9 |
| B1 | Basalt (equidimensional) | 1 1/2 in. to 1 in. | 34.0 | | |
| | Crushed gravel | 5/16 in. to 1/4 in. | 46.9 | 30.0 (2 components) | 32.1 |
| | Leighton buzzard sand | No. 7 to No. 14 | 19.2 | 23.0 (3 components) | 24.5 |
| B2 | Basalt (disks) | 1 1/2 in. to 1 in. | 35.9 | | |
| | Crushed gravel | 1/4 in. to 3/16 in. | 46.7 | 30.3 (2 components) | 32.2 |
| | Leighton buzzard sand | No. 14 to No. 25 | 17.5 | 20.7 (3 components) | 23.5 |
| B3 | Basalt (blades) | 1 1/2 in. to 1 in. | 17.7 | | |
| | Crushed gravel | 5/16 in. to 1/4 in. | 49.9 | 32.0 (2 components) | 34.8 |
| | Leighton buzzard sand | No. 7 to No. 14 | 32.3 | 24.5 (3 components) | 24.4 |
| B4 | Basalt (rods) | 1 1/2 in. to 1 in. | 31.3 | | |
| | Crushed gravel | 5/16 in. to 1/4 in. | 51.8 | 30.0 (2 components) | 31.8 |
| | Leighton buzzard sand | No. 7 to No. 14 | 17.0 | 24.0 (3 components) | 24.6 |
| B5 | Basalt (equidimensional) | 1/2 in. to 3/8 in. | 45.3 | | |
| | Crushed basalt | 3/16 in. to No. 5 | 30.3 | 34.9 (2 components) | 33.8 |
| | Leighton buzzard sand | No. 14 to No. 25 | 24.4 | 22.8 (3 components) | 23.9 |
| B6 | Basalt (rods) | 1/2 in. to 3/8 in. | 42.5 | | |
| | Crushed gravel | 3/16 in. to No. 5 | 36.2 | 34.5 (2 components) | 33.1 |
| | Leighton buzzard sand | No. 14 to No. 25 | 21.4 | 21.9 (3 components) | 24.7 |
| B7 | Basalt (equidimensional) | 1 in. to 3/4 in. | 57.5 | | |
| | Crushed gravel | 3/16 in. to No. 5 | 30.9 | 29.2 (2 components) | 30.6 |
| | Leighton buzzard sand | No. 14 to No. 25 | 11.6 | 21.7 (3 components) | 23.9 |
| B8 | Basalt (disks) | 1 in. to 3/4 in. | 37.8 | | |
| | Crushed gravel | 3/16 in. to No. 5 | 40.0 | 32.1 (2 components) | 33.0 |
| | Leighton buzzard sand | No. 14 to No. 25 | 21.3 | 22.8 (3 components) | 23.5 |
| B9 | Basalt (blades) | 1 in. to 3/4 in. | 31.4 | | |
| | Crushed gravel | 3/16 in. to No. 5 | 44.4 | 32.1 (2 components) | 33.7 |
| | Leighton buzzard sand | No. 14 to No. 25 | 24.3 | 22.8 (3 components) | 24.6 |
| B10 | Basalt (rods) | 1 in. to 1/4 in. | 35.6 | | |
| | Crushed gravel | 1/4 in. to 3/16 in. | 44.3 | 31.0 (2 components) | 32.8 |
| | Leighton buzzard sand | No. 14 to No. 25 | 20.1 | 20.8 (3 components) | 22.7 |

However, the author proposes that a more rational approach is to adjust the proportion of any constituent in accordance with its own packing properties. This is the effect of the proposed design method in that the calculations result in steeper portions of the grading curve (hence allowing greater quantities) for components that have good packing properties, i.e., tend to pack to low porosities, and less steep portions (hence limiting the content) for components with poor packing properties.

Thus, for example, the fallacy is revealed of introducing a quantity of crushed gravel in order to "correct" a grading that deviates from some supposed ideal "type" grading by absence of some middle sizes, without paying attention to the packing properties and effects of the added constituent.

The "type" gradings given in Road Note 4 (5) and similar standards are subject to criticisms similar to those that follow unquestioningly some oversimplified mathematical law because these gradings do not take account of the properties of the aggregates that are to comprise them. Further, as has been stated by Hughes (13), "the object of good mix design is to utilize the available material as economically as possible so as to obtain a hardened concrete of the required minimum quality, and any arbitrary preference for particular aggregate gradings tends to defeat this object." Although "type" gradings have the advantage that they are easy to apply and give generally reliable concretes in practice, they tend to favor richer mixes and to lead to the rejection of aggregates that do not conform to the grading but that would be capable of being used successfully in a mix designed to suit their special characteristics.

An example of a computation for a continuous grading by the proposed method is given in Table 1, and a section of a concrete mix made to this grading is shown in Figure 2.

The method adopted for the completion of the design of a concrete mix from the designed aggregate grading is described following the section on gap gradings.

## Design of Gap Gradings

Gap gradings are defined as systems of aggregate in which certain size components are missing. The component(s) may be excluded either by circumstance or by choice.

The design method proposed may be utilized in either case. If circumstance (i.e., availability of aggregate sizes) dictates the absent size(s), the method involves no change from that employed for continuous gradings. If with the object of minimizing the porosity of the grading a choice of sizes is made, it is suggested that this choice be based on consideration of three properties of the void system within the packed aggregate, namely, the critical ratios of occupation, entrance, and dilation. The first two of these terms were originally defined by Fraser (14) in terms of the diameters of the largest spheres that could respectively (a) occupy the void spaces in the structure and (b) pass along the "throats" between adjacent cells. These diameters were expressed as ratios to the diameters of the host particles. Fraser computed ratios of entrance and occupation for the loosest (cubical), densest (rhombohedral), and intermediate packings of spherical particles. Lees (15) described the application of these concepts to systems of typical aggregate particles, i.e., to loosely and densely packed gravels and crushed rock of various shape categories (Fig. 3), and determined the critical ratios for such systems by means of a void impregnation and dissection technique. Measurements by this technique indicated that, for a wide variety of particle shapes, these values for loose packing are in the region of 0.22 to 0.29 for the critical ratio of entrance and 0.33 to 0.44 for the critical ratio of occupation.

In application of the study of void characteristics to the design of gap gradings, it is the author's view that the optimum size ratio to be employed in selection of sizes should lie between the critical ratios of entrance and occupation for loose packing of typical nonspherical aggregates.

The reasons for choosing these conditions are as follows (it is assumed throughout that maximum and minimum sizes are fixed):

1. Loose packing—The densest packing of one component cannot exist in the presence of other components; i.e., dilation of one component by the other will always occur in cases of finite size ratio. The dense packing case is therefore irrelevant where two or more aggregate size groups are combined.

2. Typical concrete aggregates—The critical ratios for packings of spherical particles are of academic interest only and have no relevance to the packing of typical concrete aggregates. Compare the critical ratios for cubical and rhombohedral packing of spheres with those for loose and dense packing of typical aggregates given in Table 4.

3. Lower limit, critical ratio of entrance—If a very low size ratio is employed, voids within the coarser aggregate of a considered pair will be occupied by groups of very small particles plus their attendant voids. With a higher size ratio, opportunity is provided for comparatively larger particles to occupy a proportion of these sites, replacing solid plus voids with solid matter. A greater number of component sizes are therefore included than when the size ratio is small, and hence the porosity of the aggregate mass, using optimum proportions of the components, is lower than can be achieved with gradings based on smaller size ratios.

4. Upper limit, critical ratio of occupation—Size ratios greater than the critical ratio of occupation imply that the coarser aggregate particles of the considered pair are so far separated as to have largely lost contact with one another even when there is only one particle of the finer size present in each void. With still higher ratios, this dispersal effect intensifies, the number of components becomes large, and in the limit the grading becomes continuous. In this condition, the advantage of increasing the number of components is offset by the tendency to disperse the larger particles.

5. Size ratio between the limits of critical ratios of entrance and occupation—Use of a size ratio between these limits acts to prevent segregation of the fine particles from the coarse during normal handling of the bulk concrete because the fine particles would be too large to filter out from the voids within the framework of coarser particles.

## Critical Ratio of Dilation

It was mentioned previously that the particles of the coarser component of any pair would begin to lose contact with each other should the size ratio with the fine component exceed the critical ratio of occupation, even if only one such fine particle existed in each coarse aggregate void. Clearly, even at or below the occupation size, similar dispersal of the coarse component would occur if more than one fine aggregate particle per void was in place.

These considerations led to the concept of the critical ratio of dilation, which is defined as the size ratio at which the fine aggregate dilates the coarse aggregate to the state of its loosest packing when both components are combined at their optimum proportions, at the compactive effort designed to produce the densest state of packing. The use of aggregates to this size ratio aims at ensuring that the coarser particles are not dilated away from each other beyond the point of mutual contact in a fully compacted mix. In doing so, it recognizes that the choice of size ratio should not be made in isolation but only in consideration of the proportions in which the components will be mixed.

The concept of the critical ratio of dilation (CrD) is shown in Figure 4. For size ratios greater than CrD, the dilation of the coarse aggregate by the fine would be to porosities greater than the maximum porosity, and thus the coarse aggregate particles would begin to lose contact within the mix. For low size ratios less than CrD, the coarse aggregate is dilated to a state between its own densest and loosest packings. This in itself is satisfactory for two-component mixes, but in the context of multicomponent mixes too low a size ratio may lead to some of the disadvantages mentioned previously, such as an insufficient reduction of porosity in the total mix and segregation of fine aggregate and coarse aggregate.

An analysis of experimental data obtained in the laboratory showed that, for a wide variety of shape combinations, the critical ratio of dilation lay in the region of 0.23. This value, it will be noted, lies within the range of critical ratios of entrance and occupation previously noted (0.22 to 0.44), confirming the conclusion drawn from studies of interparticle voids that, for rational gap-grading design, the size ratios chosen should preferably lie within this range, preferably toward the lower end of the range. Availability of aggregate sizes is such that it is not always possible to satisfy the criterion of size ratio very closely, and it is not suggested that the choice is critical. A number of gap-grading designs have now been made up according to the proposed method utilizing size ratios as near as possible to 0.23. Table 2 gives one example of these gradings, and Figure 5 shows a section of the concrete made to this grading.

Brief comment may be made on other methods of design of gap gradings for concrete that has been developed by other workers. Bate and Stewart (16) and Stewart (17) point to the desirability of avoiding honeycombed zones in a concrete, which might arise out of an inability of the fine particles to filter into the voids between the coarse aggregate. Such an aim leads to their adoption of the "admittance size," a synonym for the entrance size, as the basis of design. These authors refer to the critical ratio of entrance of 0.154 calculated by Fraser for rhombohedral packing of spheres, but Stewart states that in practice the admittance size "will be found to be 0.125D, where D = diameter of coarse aggregate." This represents a smaller size ratio than that recommended previously. In this approach the fine aggregate is generally considered as one component and the coarse aggregate as another, i.e., a two-component system.

Vallette (18, 19) recommended size ratios in the order of 0.20 to 0.33 for multicomponent gap-graded mixes with the stated aim that each size of particle shall be able to fit into the voids of the next larger size. A close similarity is noted between these values and those proposed in this paper. Bahrner (20) commented on Vallette concrete as having the appearance of "a heap of stones when it leaves the mixer. If, however, it is properly proportioned and placed in the forms, it will be found that after vibration (all Vallette concrete has be be vibrated) it will yield a very smooth and uniform surface." This has also been the author's experience in gap gradings designed to the method proposed in this paper.

Bahrner proposed a modification of Vallette concrete that would allow the fine aggregate to be used as one undivided aggregate, in view of the difficulties that would sometimes be found in obtaining the closely defined single sizes of fine aggregate proposed by Vallette. This he referred to as skeleton concrete (a return to a two-component system). It is to be noted that a 10 percent cement mortar surplus was recommended over that which would be calculated from the void-filling criterion adopted for design purposes. For too low a cement mortar surplus, in this case 0.5 percent, Bahrner observed that the mix lacked cohesion and was subject to segregation.

## Concrete Mixes With Designed Aggregate Gradings

An aggregate grading (gap or continuous) designed by the method here proposed may be included in a concrete mix by any of a wide variety of published methods of concrete mix design (including methods of trial mix formulation). However, in a further stage of development of the present design process, the method adopted for the completion of the design of a concrete mix from the designed aggregate grading has been as follows:

1. Select water-cement ratio on the basis of required level of compressive strength (5, Fig. 1; 21). A control ratio appropriate to the anticipated level of supervision on site will also be used to determine the required level of compressive strength, as in normal practice (5, 21).
2. From the calculated specific gravity of the unhydrated cement paste at the selected water-cement ratio, determine the weight percentage of paste to fill the voids in the designed grading as previously computed.
3. Hence, determine the cement content, water content, and aggregate content (as percentage by weight of total mix).

The following is an example of this method as used to compute a four-component gap-graded concrete (Table 2):

1. Selected water-cement ratio = 0.4;
2. Specific gravity of ordinary portland cement = 3.15;
3. Bulk specific gravity of mixed aggregate = 2.725;
4. Voids in mixed aggregate = 15.9 percent (in 2.5-in. deep mold, with maximum compaction by vibration table);

5. Specific gravity of water-cement ratio = 0.4, and specific gravity of unhydrated cement paste = [(28.5 x 1) + (71.5 x 3.15)]/100 = 2.54;
6. Hence, 15.9 percent (volume) = (15.9 x 2.54)/[(15.9 x 2.54) + (84.1 x 2.725)] = 14.98 percent (by weight); and
7. Therefore, cement content = (14.98 x 71.5)/100 = 10.71 percent (by weight), water content = (14.98 x 28.5)/100 = 4.27 percent (by weight), and aggregate content = 85.02 percent (by weight) (aggregate-cement ratio $\simeq$ 8:1).

The characteristics of this mix were as follows: compacting factor 0.84; Vebe time 25 sec; compressive strength (7-day) 5,530 psi.

In the case of air-entrained mixes, adjustments can be made in the calculation of cement content and water content to allow for the volume of air required; e.g., if the total required entrapped plus entrained air = 4 percent, then, in the case of the preceding mix, the following holds:

1. Voids to be filled by cement paste = 15.9 - 4 percent = 11.9 percent (by volume);
2. 11.9 percent volume = 11.18 percent wt (as calculated previously);
3. Therefore cement content = (11.18 x 71.5)/100 = 7.99 percent (by weight); and
4. Water content = (11.18 x 28.5)/100 = 3.19 percent (by weight) and aggregate content = 88.82 percent (by weight) (aggregate-cement ratio $\simeq$ 11:1).

Workability—It will be noted that the mixes proportioned as described have been designed with no apparent reference to workability. It is recalled, however, that the measurements of porosity on which the design is based are carried out with as close a simulation as possible to anticipated compactive effort and to the external and internal geometry of the site section. The aggregate grading and the aggregate, cement, and water contents are thus all determined with reference to factors that are recognized as imposing different requirements for workability. It follows from the normal trend of porosity values measured in these tests that low compactive efforts, narrow or complex sections (including narrow pipes for pumped concrete), sections with congested reinforcement, etc., all of which demand high workability mixes, will, by their influence on the design factors, tend to increase the content of the finer aggregate components and to decrease the aggregate-cement ratio, compared with mixes designed for high compactive effort and simple, massive, and unreinforced sections. Maximum aggregate size is adjusted to the site section geometry in the proposed method, as in all recognized mix design methods.

The proposed method determines the aggregate grading and the mix proportions from measurements that are related to the ability of the aggregate particles to pack together under given conditions of boundary interference and effort. The criterion of workability is thus included as an intrinsic part of the initial design procedure of aggregate grading and selection of mix proportions in contrast to the more usual assumption that the grading would be a prior fixed parameter and that the workability requirement would be sought as a final stage in the mix design process by adjustment of the aggregate-cement ratio.

In the mixes designed according to this method, a check on workability has been made by use of the compacting factor and Vebe tests (Tables 5 and 6).

Strength and Economic Factors—The object of the proposed design method is to take account of the particular packing properties of the component aggregates, therefore allowing a more effective and more economic use of available aggregates including those of nonstandard shape and grading characteristics, and to relate these in terms of density and workability to the anticipated site compactive effort and site geometry. In an early trial it has been confirmed that leaner mixes of equivalent strength can be designed by the proposed method than by following a standard mix formulation with the same aggregates. For example, mix C (proposed design method, Tables 1 and 5) and mix S (standard mix, 5, 6) were both designed to give a 28-day strength of 4,160 psi (to satisfy the strength requirement of 4,060 psi for pavement quality concrete, 22). Assuming good site supervision, a 75 percent control factor was used. Thus the designed strength became 5,550 psi, giving a water-cement ratio of 0.6 (5). A high compactive effort requiring only a very low workability mix was assumed in both cases.

**Figure 2. Continuously graded concrete (mix C).**

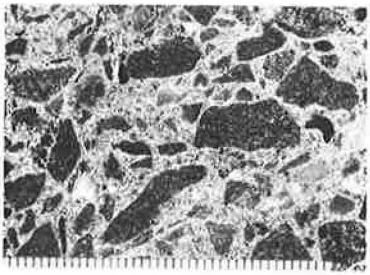
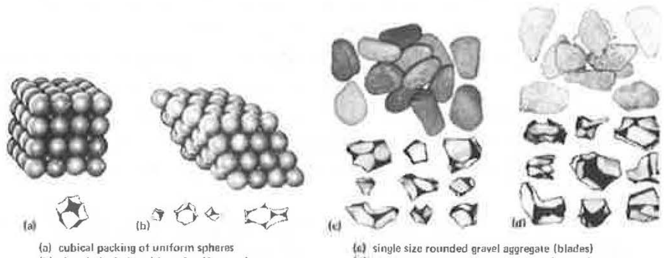


**Figure 3. Interparticle voids.**



(a) cubical packing of uniform spheres
(b) rhombohedral packing of uniform spheres
(c) single size rounded gravel aggregate (blades)
(d) single size crushed basalt aggregate (blades)

**Table 4. Critical ratio of entrance and critical ratio of occupation values.**

| Aggregate | Loose Packing | | Dense Packing | |
|---|---|---|---|---|
| | CrE | CrO | CrE | CrO |
| Spheres | 0.414 | 0.732 | 0.154 | 0.225, 0.414 |
| Rounded gravel (average) | 0.24 | 0.37 | 0.21 | 0.32 |
| Crushed igneous rock (average) | 0.28 | 0.41 | 0.23 | 0.40 |

**Figure 4. Critical ratio of dilation.**

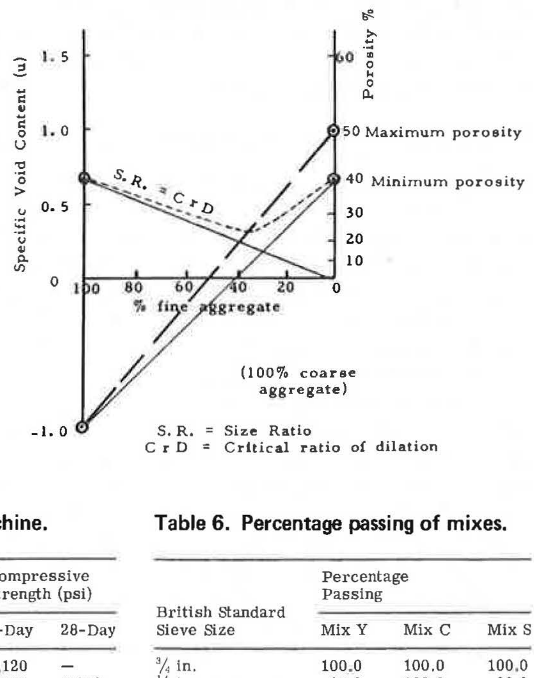



**Table 5. Mixes tested on variable-speed internal drum machine.**

| Mix | Texture | Water-Cement Ratio | Aggregate-Cement Ratio | Compacting Factor | Vebe Time (sec) | Compressive Strength (psi) | |
|---|---|---|---|---|---|---|---|
| | | | | | | 7-Day | 28-Day |
| Y[a] | Open | 0.6 | 7.2 | 0.80 | 21 | 3,120 | — |
| C[a] | Closed | 0.6 | 8.5 | 0.73 | 30 | 4,550 | 6,150 |
| S[b] | Closed | 0.6 | 6.3 | 0.84 | 4 | 3,260 | 5,460 |

[a] Lee's design method used. [b] Road Note 4 (5) method used.

**Table 6. Percentage passing of mixes.**

| British Standard Sieve Size | Percentage Passing | | |
|---|---|---|---|
| | Mix Y | Mix C | Mix S |
| 3/4 in. | 100.0 | 100.0 | 100.0 |
| 1/2 in. | 95.8 | 100.0 | 80.0 |
| 3/8 in. | 89.6 | 81.5 | 55.0 |
| 1/4 in. | 81.2 | 60.1 | 43.0 |
| 3/16 in. | 49.7 | 34.7 | 35.0 |
| No. 7 | 22.9 | 27.3 | 28.0 |
| No. 25 | 9.9 | 20.4 | 14.0 |
| No. 100 | 2.3 | 3.5 | 2.3 |
| No. 200 | 0.7 | 0.6 | 0.4 |

In the proposed design method mix, this was related to the very high compactive effort utilized in obtaining the design factors of aggregate porosity. In the standard mix, the aggregate-cement ratio for the given water-cement ratio was read off the column for "very low workability" (5).

The aggregates (a Precambrian graywacke and a concreting sand of glaciofluvial origin) were assumed to be in an air-dry condition, and the appropriate adjustment in water-cement ratio was made to allow for water absorption (6).

Both mixes were air-entrained for frost resistance, allowing an air content of 4.5 percent, by addition of an air-entraining agent at the rate of 1 cc of agent to 2 kg of cement.

Mix C slightly surpassed the design strength of 5,550 psi, with 6,150 psi at 28 days, with an aggregate-cement ratio of 8.5. The standard mix, mix S, achieved slightly under the design strength, i.e., 5,460 psi with an aggregate-cement ratio of 6.3.

The results indicate that satisfactory results can be achieved with leaner, and hence more economical, mixes by the proposed method.

As in most cases where a rational approach to mix design has been recommended, it is recognized that any complexity in the method may dictate that it is uneconomic to use that method for small jobs. The suggestion is made that the present method is most applicable where comparatively large amounts of concrete are to be placed and where site conditions will remain uniform with respect to aggregate type, section dimensions, reinforcement, etc. for a reasonable period.

## Laboratory Studies of Skid-Resistant Concrete

It has elsewhere been shown (23) that the grading design method can be used to design bituminous mixes of controlled void content whose special feature is that they have a dense, impermeable internal structure but at the same time possess a surface with a system of intercommunicating channels of high drainage efficiency. The anomaly is explained by the special characteristics of voids at a boundary. The boundary in this case is the upper compacted surface. It is suggested that the same requirements exist and the same design philosophy can be applied in concrete as in bituminous surfacings.

Attention has also been drawn by Holmes, Lees, and Williams (24) to the different bulk water drainage requirements of low-speed and high-speed sites, and the suggestion is made that, where circumstances are appropriate, surfaces should ideally be designed to suit the special skid resistance requirements of the highway or airfield environment. It is in this context that the present design method offers scope for design of a range of surface macrotextures to suit particular highway and traffic circumstances. The importance of producing and preserving a suitable level of microtexture for all categories of speed has also been stressed by these authors.

Valuable work has been done by members of the British Road Research Laboratory (25-28) and others on the influence on skid resistance of cut, flailed, and formed grooves and of fine and coarse aggregate types. The present section describes an initial study that is in progress on the design of concrete mixes of high skid resistance properties, as an alternative to grooving treatments or for areas in which such treatments are not available.

Two contrasting types of concrete surfacing have been designed in accordance with the proposed method, namely, an open-surface texture mix (mix Y)(Fig. 6) and a close-surface texture mix (mix C). These two mixes and the mix designed to standard composition (mix S)(6) have been compared, all at a water-cement ratio of 0.6, for cube-crushing strength and also for skid resistance properties on the variable-speed internal drum machine (Fig. 7). This machine, designed by Williams and Lees (29), is, so far as these authors are aware, the only laboratory machine in existence on which it is possible to carry out with a tire wear-polishing cycles and rolling and locked-wheel friction tests on any chosen tire compound-road material combination on the same apparatus. Its main features are as follows:

1. It can be used for studies of complete concrete and bituminous surfacing materials;
2. The wear-polishing procedure can be varied by use of different water flows, types of abrasive, and tire slip-angle conditions;

3. The wear on the surface (e.g., under the action of normal and spiked tires) and on the tire tread can be determined;
4. The tire-road friction can be measured in both the peak rolling and the locked-wheel slide conditions; and
5. These friction coefficients can be obtained over a range of speeds from 0 to 70 mph and under various water-depth, slip-angle, inflation-pressure, and load conditions.

Some properties of the concrete mixes studied in this investigation are given in Tables 5 and 6. The coarse aggregate used for all mixes was a Precambrian gritrock of high polishing resistance. The fine aggregate used in mix Y was the crushed fines of this rock and in mix C and mix S was a concreting sand (Zone 2, British Standard 882)(30). In all tests the sample surface was wetted with a thin film of water, which was maintained at constant thickness by adjustment of the rate of water flow relative to the varying drum speed. Tires used were all plain-treaded cross-ply tires with a standard tread compound. Separate groups of tires were used for the wear-polishing cycles and for the braking tests to ensure that the results of the latter were not affected by tire wear.

A contact pressure of 32.8 psi, realistic in comparison with that of an average private automobile, was achieved with a 30-psi tire inflation pressure and a 140-lb wheel load.

Wear-polish cycles were carried out at a speed of 40 mph at a slip angle of 6 deg in wet conditions.

At an early stage in the test, following an initial calibration stage of some 50,000 revolutions, braking tests were performed at speeds of 20, 30, 40, 50, and 60 mph at 0-deg slip angle. Twelve tests were performed at each speed, and the peak and locked-wheel friction values were averaged.

Figure 8 shows the performance in locked-wheel friction of the 3 mixes over this range of speeds.

Confirmation of the frequently observed trend in road tests for the friction of close-textured surfaces to decrease more rapidly with speed than that of open-textured surfaces is clearly seen; e.g., the open-textured surface mix Y maintains a friction torque value of 4.70 m-kg at 60 mph (≡4.70 coefficient of friction) compared with that of 2.54 m-kg (≡2.54 coefficient of friction) for the standard close-textured surface mix S. These mixes had virtually identical friction values (5.73 m-kg ≡5.73 coefficient of friction) at 20 mph.

Further braking tests have been performed at intervals of 50,000 revolutions of wear-polish cycles. The braking tests in this series have been performed at 30 mph. No change in the order of merit has been observed up to 400,000 revolutions. Further tests and analysis of results are in progress.

## CONCLUSIONS

A new method for the design of aggregate gradings has been applied to the design of continuously graded and gap-graded concretes. For normal dense concretes, the grading and cement content are calculated on the basis of simple laboratory tests for aggregate packing relative to the anticipated compactive effort on site and the geometric complexities of the site section, i.e., to factors that control the aggregate structural arrangement and the workability requirement of the mix. Concrete mixes made by using this method in the laboratory have been shown to possess high strength for economical compositions.

Concrete mixes can also be designed that have a controlled void content above the minimum possible. The application of this facility to the design of open-surface texture pavement quality concretes, which have good skid resistance at high speeds, is under investigation. Results to date show that these mixes possess good frictional properties that tend not to fall off as rapidly at high speeds as normal, dense, untextured concrete surfaces.

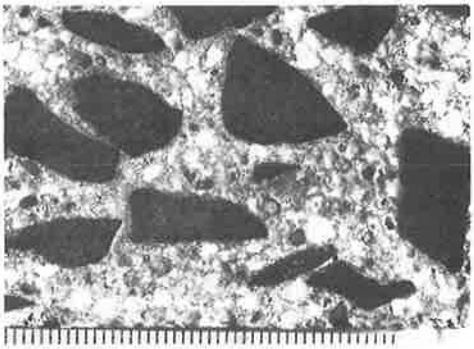


Figure 5. Four-component gap-graded concrete.

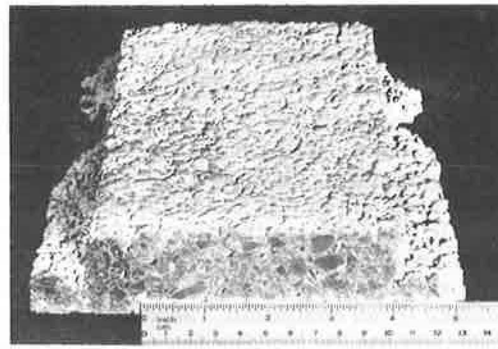


Figure 6. Surface texture and internal structure of mix Y.

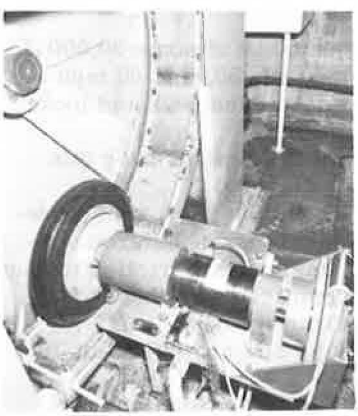


Figure 7. Variable-speed internal drum machine.

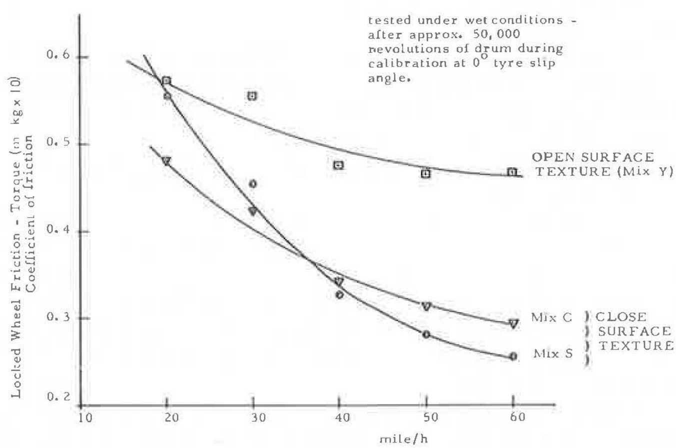



Figure 8. Friction versus speed on concrete surfaces.

## ACKNOWLEDGMENTS

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# GAP-GRADED AGGREGATES FOR HIGH-STRENGTH QUALITY CONCRETE

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To determine the optimum proportion of materials used in a mix design to achieve a specified quality with least cost is the most important aspect in the production of concrete. Currently available methods for proportioning aggregates are based mostly on previous experience. Even their application to the conventional continuously graded concrete is short of being optimum. It has become essential to seek quantitative information for the optimum proportioning of gap-graded and continuously graded aggregates for various significant mix parameters. In the present work involving 200 mixes with tests carried out according to ASTM standard methods, the results have been analyzed with the objective of revealing optimum mix proportions with respect to significant mix parameters, such as size and quantity of coarse aggregate, size and quantity of the fine aggregate, cement content, and water-cement ratio, at practically the same air content for both gap-graded and continuously graded concretes. The optimum cement contents for both concretes are on the basis of equal maximum size of coarse aggregate and equal compressive strength. The comparison has shown that much less cement is required for concrete with gap-graded aggregates than its continuously graded counterpart with approximately the same workability. Gap grading of the aggregates results in lower requirement of water content, and hence lower water-cement ratio, and permits a higher aggregate-cement ratio to achieve the same workability as its continuously graded counterpart with equal cement content and equal maximum size of coarse aggregate.

•IN the development of concrete technology, a significant step was advanced in 1918 by Abrams (1), who found that the strength of fully compacted concrete solely depended on its water-cement ratio and that aggregate grading was important insofar as it influenced workability required to achieve full compaction. Later research has shown that the influence of other parameters, such as aggregate grading, maximum size of aggregate, surface texture, shape, strength, and other attributes of aggregates, on the compressive strength of concrete cannot be ignored (3, 4). In addition, the parameters of cement content, total water content, and matrix percentage also have considerable influence on the strength of concrete.

Appropriate recognition of these parameters and determination of the relative amounts of constituent materials to be used in a concrete mix to achieve maximum economy and to satisfy the requirements for placement and compaction are of paramount importance in the mix design of concretes with gap-graded and continuously graded aggregates.

Although various investigators (1-5, 7, 12) have suggested different methods for proportioning the ingredients of continuously graded concrete, there has been no systematic investigation to determine the optimum proportions of gap-graded versus continuously graded concretes, thereby clearly revealing the quantitative advantages of gap grading.

In 1967-68, Shu-t'ien Li (8, 9, 11) proposed the synthesis of gap-graded shrinkage-compensating concrete. A comprehensive research project was soon initiated by him to verify the hypothesis and to provide working information to concrete technologists regarding gap-graded versus continuously graded concrete. The results presented here specifically deal with "optimum proportioning of aggregates for high-strength quality concrete."

## OBJECTIVES AND SCOPE OF THE STUDY

The main objectives of this investigation are as follows:

1. To determine the optimum amounts, in both gap-graded and continuously graded air-entrained concrete, of basic parameters such as coarse aggregate size and weight, fine aggregate size and weight, water content, cement content, water-cement ratio, and aggregate-cement ratio (for all concretes, an air-entraining agent was added to entrain approximately 5 percent air by volume);
2. To determine the optimum proportioning of gap-graded air-entrained concrete mixtures that are practical from the viewpoints of workability and ability to finish; and
3. To compare optimum-proportioned gap-graded air-entrained concrete with equivalent continuously graded air-entrained concrete, using mixtures having similar workability to the extent that they require similar attention with regard to placement and finishing.

Ranges of variable parameters included in this investigation were water-cement ratios (by weight) of 0.35 to 0.65, aggregate-cement ratios (by weight) of 2.0 to 10.0, maximum sizes of coarse aggregate of ½, ¾, 1, and 1½ in. (1.27, 1.90, 2.54, and 3.82 cm), and percentages by weight of fine aggregate to coarse aggregate of 28.5, 33.3, 36.5, and 40.0 (for gap-graded concrete only).

## MATERIALS

Type I cement was used throughout this investigation. In all mixes, the coarse and fine aggregates were from the same source. Fine aggregate had a water absorption coefficient of 2.04 percent (at 24 hours) and a saturated surface-dry specific gravity of 2.63. Crushed limestone was used as the coarse aggregate. It had an absorption coefficient of 0.58 percent and a saturated surface-dry specific gravity of 2.73.

For all the mixes, a commercially available air-entraining agent in water solution was added to entrain approximately 5 percent air by volume. The required quantity, as recommended by the manufacturer, was mixed with water before it was added to the mixer. The admixture dosage was kept constant, and the air content of only a few mixes was measured. The air contents were generally about 5 percent, ranging from 4 to 7 percent.

The gradings of coarse and fine aggregate are given in Table 1 for gap gradings and in Table 2 for continuous gradings.

## SIGNIFICANT PARAMETERS

The proportioning of ingredients is an important phase in the process of manufacturing quality concrete. The nominal amount of air entrainment was kept at 5 percent, and four significant mix parameters were emphasized: size and quantity of coarse aggregate, size and quantity of fine aggregate, cement content, and water-cement ratio. These are the independent variables, whereas the properties of concrete in the plastic and hardened states are the dependent variables.

Among the many possible dependent variables, workability of plastic concrete with respect to placement and compressive strength as a key property of the hardened concrete are important. This is because other attributes of concrete, such as durability, permeability, wear resistance, and tensile strength, are all strongly influenced by compressive strength. For rapid assessment, the 7-day compressive strength was selected as the indicator.

Of the two dependent variables, there has been no quantitative test to measure the workability of fresh concrete according to its ASTM definition, but quantitative determination of the 7-day compressive strength of hardened concrete is a feasibility. Further, workability is directly related to water-cement ratio (13), whereas the compressive strength varies with the inverse of this ratio. Thus, the 7-day compressive strength of concrete is treated as the more significant dependent variable for the optimization process.

The aim of optimum proportioning can be boiled down to the design of a concrete mix for specified strength or quality at the least cost of materials. Of all the ingredients for concrete, the cost of cement is the most pronounced factor. An optimum proportioning may require the minimum cement content to satisfy strength or quality requirements. This practical viewpoint has lead to the treating of cement content as the major independent variable.

## ANALYSIS OF TEST RESULTS

### Optimum Content of Fine Aggregate for Gap-Graded Concrete

To study the influence of fine-aggregate content on 7-day compressive strength of gap-graded concrete, we carried out a pilot program at the beginning of the study. For expediency, a medium maximum-sized coarse aggregate of ¾ in. (1.90 cm) was used, but the percentages (by weight) of fine aggregate of the total aggregates were varied. As shown in Figure 1, test results reveal that the optimum content of fine aggregate is nearly 36 percent of total aggregates for all water-cement ratios and aggregate-cement ratios. This fine-aggregate content has been taken for all the other gap-graded concrete mixes involving different maximum-sized coarse aggregates because it has been reported that "when the beakers are filled with one particle size, the void content is constant, regardless of the particle size" (10).

### Optimum Mix Proportions for Gap-Graded Concrete and Continuously Graded Concrete

Figures 2 through 5 have been plotted for gap-graded concrete and Figures 6 through 9 for continuously graded concrete to show the influence of cement contents (major independent variable) on 7-day compressive strengths (major dependent variable) for four maximum sizes of coarse aggregate and four water-cement ratios. The curves show a peak at a cement content at which the 7-day compressive strength is maximum. This particular cement content and the corresponding water-cement ratio and the aggregate-cement ratio shown in Figures 10 through 13 are the optimum mix parameters.

The optimum mix proportions for gap-graded and continuously graded concretes and their corresponding 7-day compressive strengths for four maximum sizes of coarse aggregate are given in Tables 3 and 4, which also contain the Vebe Consistometer time for these mixes.

Based on Figures 2 through 9, the optimum results are given in Tables 3 and 4. These results were used in developing Figures 10 through 17. A few of the results were averaged, when the corresponding optimum cement contents were very close, and the averaged results are shown in Figures 10 through 17. For example, in the case of gap-graded concrete with ½-in. maximum size of coarse aggregate (Table 3), the actual optimum results are given in Table 5. These two results were averaged (Table 5) and plotted in Figures 10 through 14.

The optimum cement contents and their corresponding aggregate-cement ratios for gap-graded versus continuously graded concrete are compared in Figures 10 through 13. They show that the optimum cement content of gap-graded concrete is less than that of continuously graded concrete for equal maximum size of coarse aggregate and 7-day compressive strength. The amount of saving in cement content in gap-graded concrete, when compared with the corresponding continuously graded concrete, varies from 15 to 35 percent. These figures also indicate that gap-graded concrete permits higher aggregate-cement ratios than the continuously graded concrete for equal maximum size of coarse aggregate and optimum cement content.

Figures 14 through 17 show the relation between optimum cement content and its corresponding water-cement ratio for gap-graded and continuously graded concretes for four maximum sizes of coarse aggregate. It can be seen that, for equal optimum cement content, gap-graded concrete requires a lower water-cement ratio than does continuously graded concrete.

**Table 1. Gradings of coarse and fine aggregates for gap-graded concrete.**

| Sieve | | Percent by Weight for Mix Groups | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Passing | Retained | A49 to A65 | A35 to A48 | A19 to A34, A68, A71, A74, A77 | A1 to A18 | A66 | A67, A70, A73, A76 | A69, A72, A75, A78 |
| 1½ in. | 1 in. | 63.5 | – | – | – | – | – | – |
| 1 in. | ¾ in. | – | 63.5 | – | – | – | – | – |
| ¾ in. | ½ in. | – | – | 63.5 | – | 71.4 | 66.7 | 60.0 |
| ½ in. | ⅜ in. | – | – | – | 63.5 | – | – | – |
| ⅜ in. | No. 4 | – | – | – | – | – | – | – |
| No. 4 | No. 8 | 36.5 | – | – | – | – | – | – |
| No. 8 | No. 16 | – | 36.5 | 36.5 | – | 28.6 | 33.3 | 40.0 |
| No. 16 | No. 30 | – | – | – | 36.5 | – | – | – |

**Table 2. Gradings of coarse and fine aggregates for continuously graded concrete.**

| Sieve | | Percent by Weight for Mix Groups | | | |
|---|---|---|---|---|---|
| Passing | Retained | B47 to B59 | B29 to B46 | B15 to B28 | B1 to B14 |
| 1½ in. | 1 in. | 25.0 | – | – | – |
| 1 in. | ¾ in. | 25.0 | 22.0 | – | – |
| ¾ in. | ½ in. | 7.0 | 22.0 | 27.5 | – |
| ½ in. | ⅜ in. | 7.0 | 14.0 | 27.5 | 28.5 |
| ⅜ in. | No. 4 | 12.0 | 14.0 | 15.0 | 41.5 |
| No. 4 | No. 8 | 6.0 | 7.0 | 7.0 | 10.0 |
| No. 8 | No. 16 | 6.0 | 6.0 | 7.0 | 5.0 |
| No. 16 | No. 30 | 5.0 | 7.0 | 7.0 | 5.0 |
| No. 30 | No. 50 | 7.0 | 8.0 | 9.0 | 10.0 |

**Figure 1. Compressive strength and sand content for gap-graded concrete.**

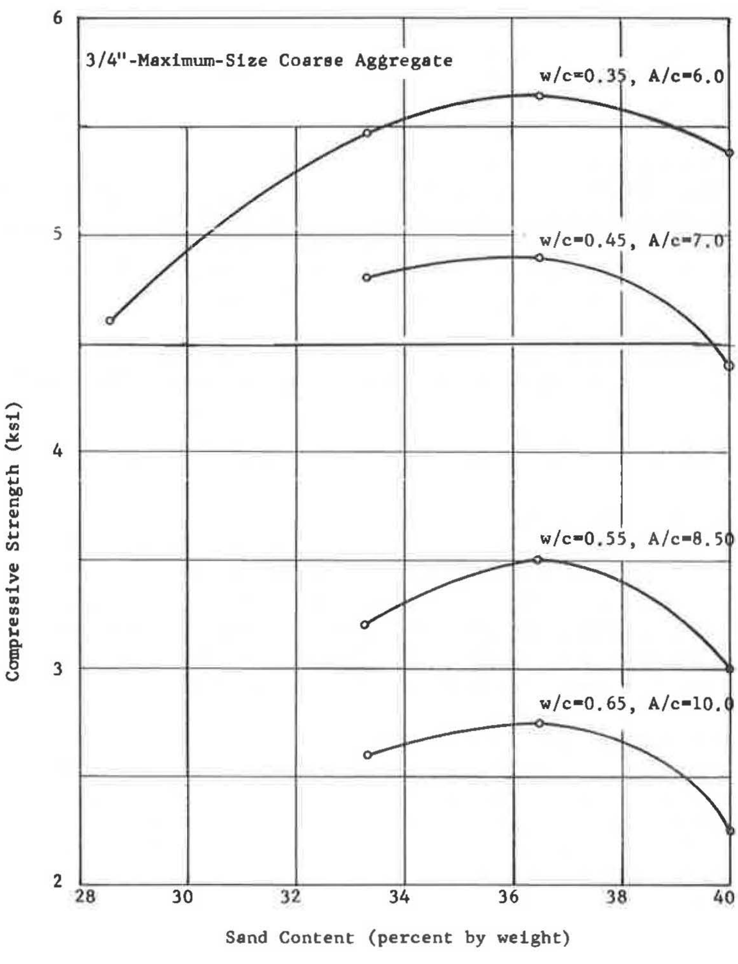


**Figure 2. Compressive strength and cement content for water-cement ratios (½-in. maximum-sized aggregate, gap-graded concrete).**

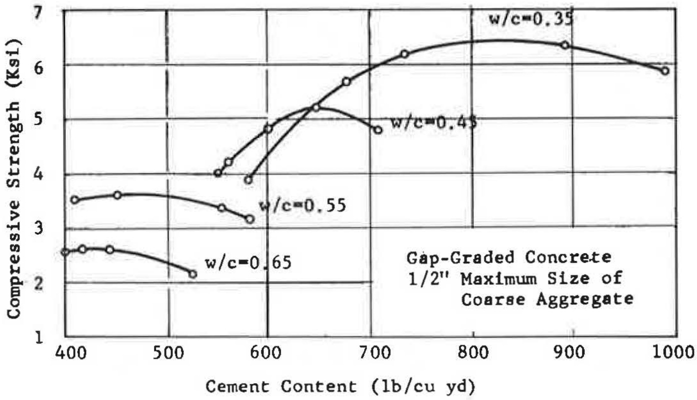



**Figure 3. Compressive strength and cement content for water-cement ratios (¾-in. maximum-sized aggregate, gap-graded concrete).**

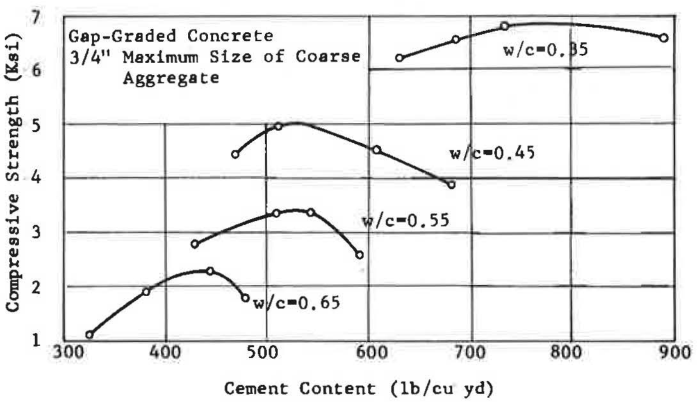



**Figure 4. Compressive strength and cement content for water-cement ratios (1-in. maximum-sized aggregate, gap-graded concrete).**

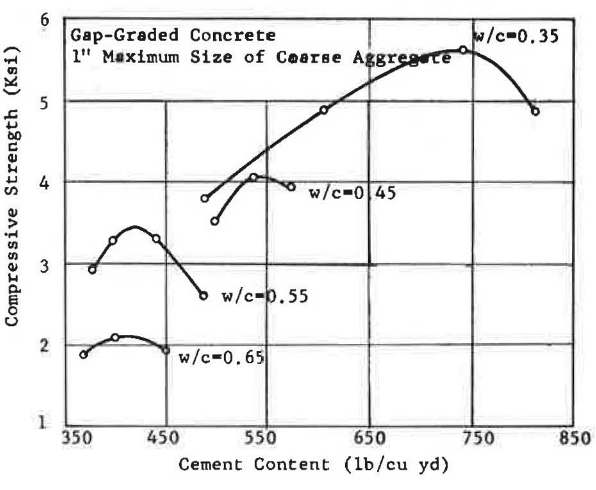


Figure 5. Compressive strength and cement content for water-cement ratios (1½-in. maximum-sized aggregate, gap-graded concrete).

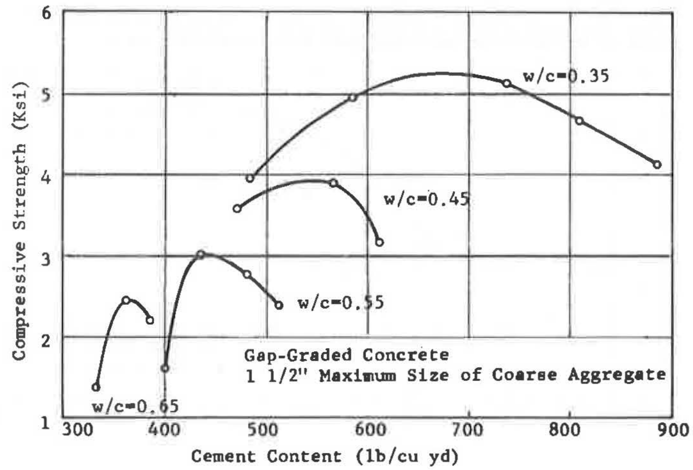



Figure 6. Compressive strength and cement content for water-cement ratios (½-in. maximum-sized aggregate, continuously graded concrete).

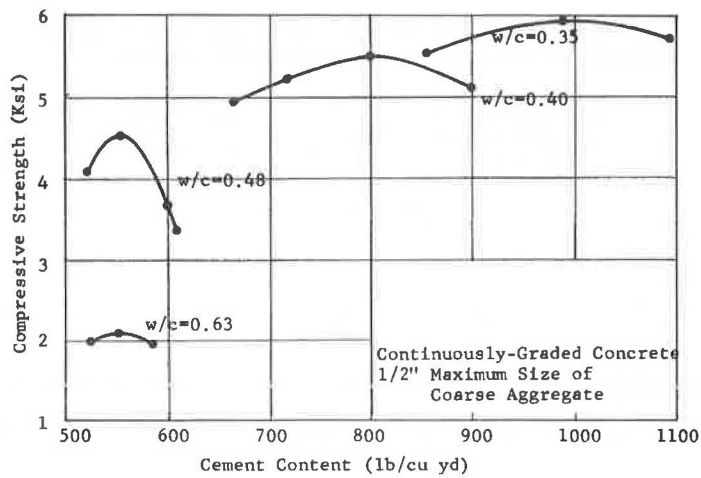
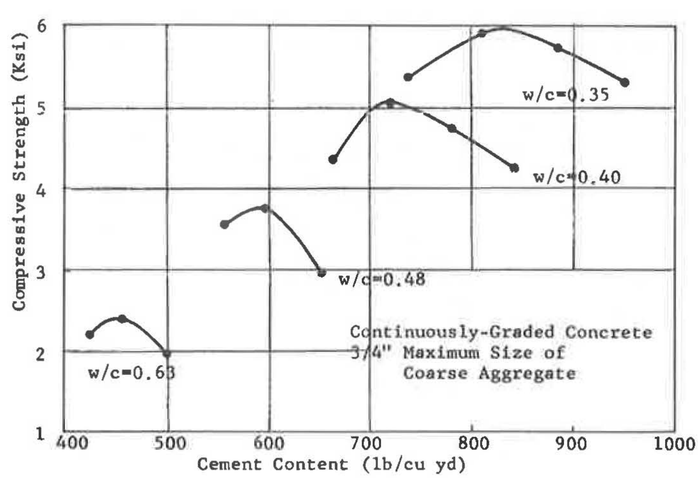



Figure 7. Compressive strength and cement content for water-cement ratios (¾-in. maximum-sized aggregate, continuously graded concrete).



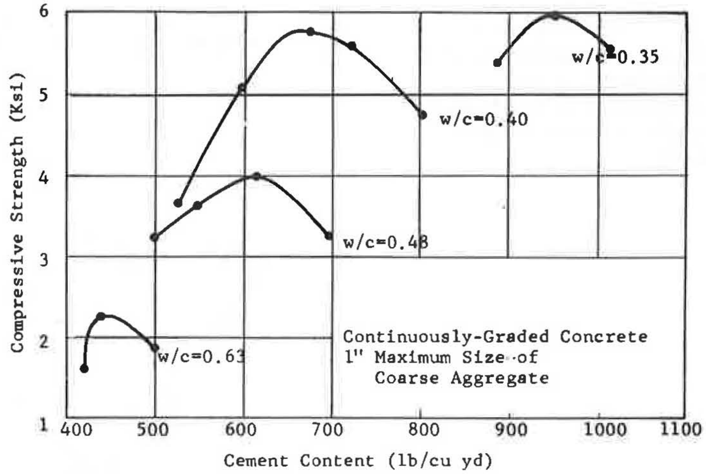



**Figure 8. Compressive strength and cement content for water-cement ratios (1-in. maximum-sized aggregate, continuously graded concrete).**

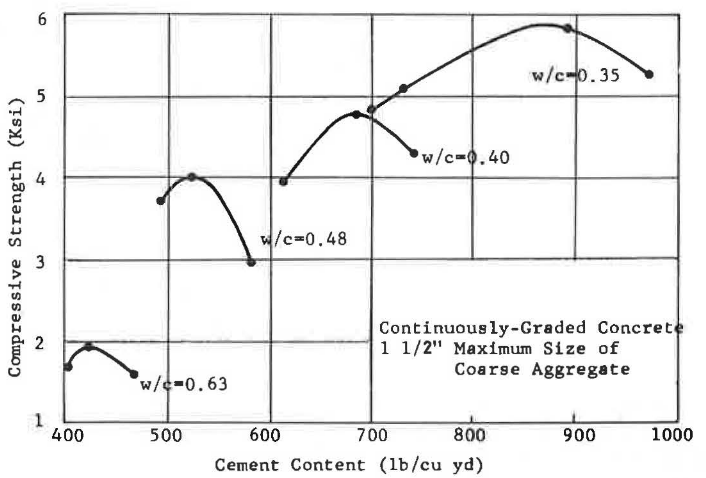



**Figure 9. Compressive strength and cement content for water-cement ratios (1½-in. maximum-sized aggregate, continuously graded concrete).**

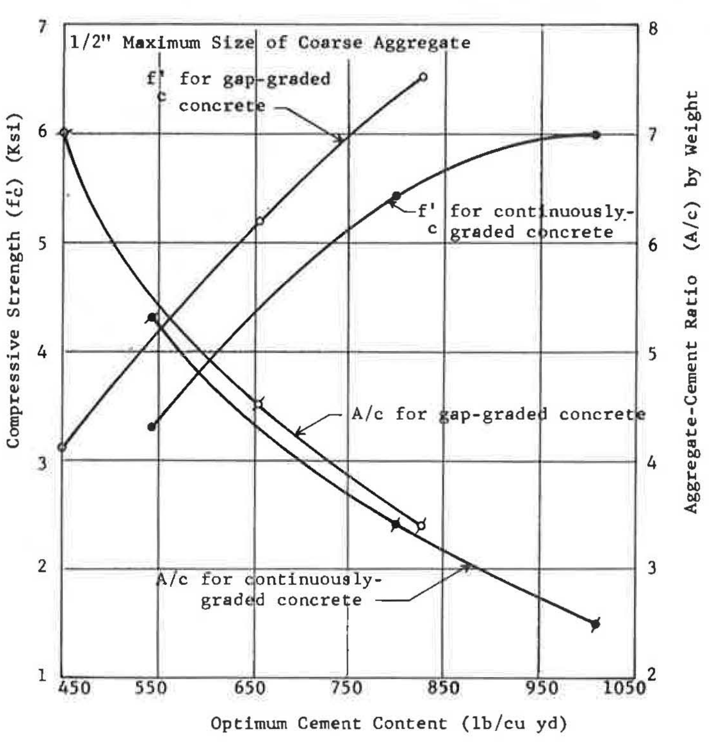



**Figure 10. Optimum cement content, compressive strength, and aggregate-cement ratio for gap-graded and continuously graded concrete (½-in. maximum-sized aggregate).**

**Figure 11. Optimum cement content, compressive strength, and aggregate-cement ratio for gap-graded and continuously graded concrete (¾-in. maximum-sized aggregate).**

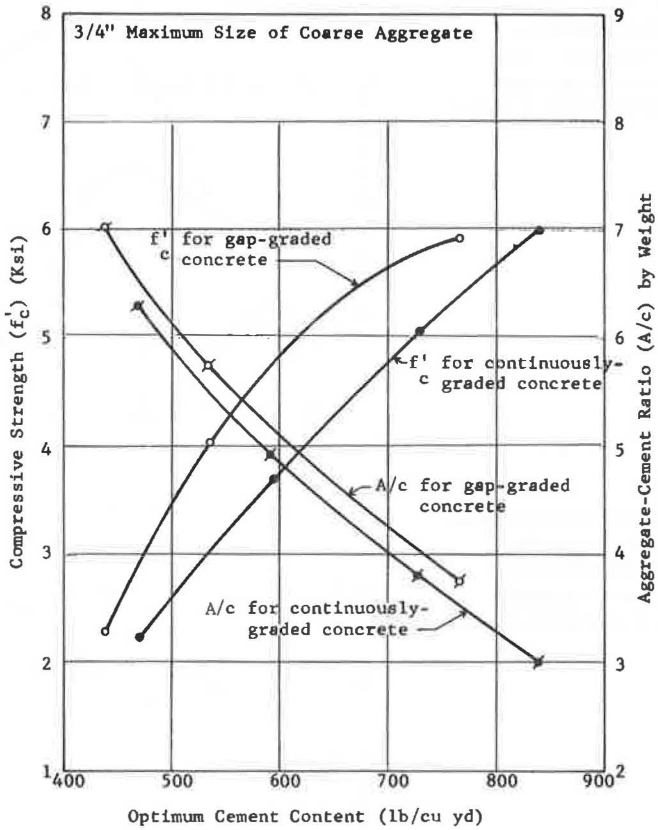



**Figure 12. Optimum cement content, compressive strength, and aggregate-cement ratio for gap-graded and continuously graded concrete (1-in. maximum-sized aggregate).**

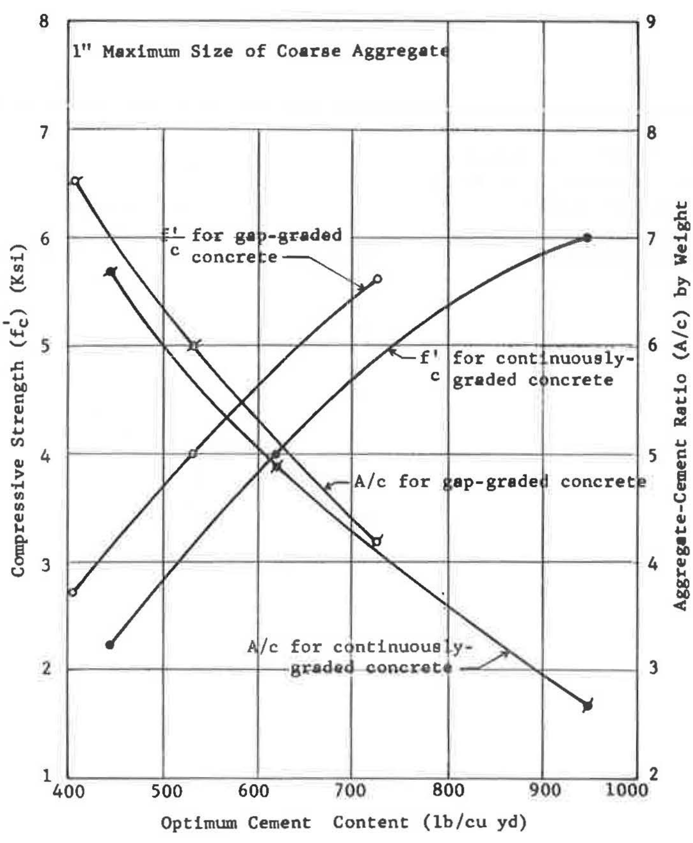


**Figure 13. Optimum cement content, compressive strength, and aggregate-cement ratio for gap-graded and continuously graded concrete (1½-in. maximum-sized aggregate).**

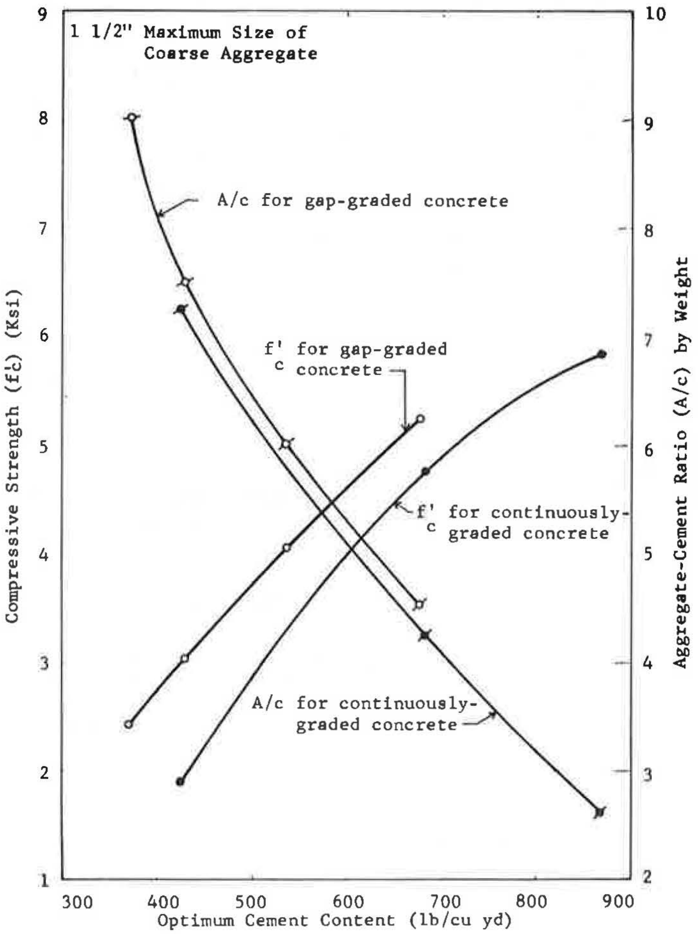



**Table 3. Optimum results for gap-graded concrete.**

| Maximum Size of Coarse Aggregate (in.) | Cement Content (lb/cu yd) | Aggregate-Cement Ratio (by weight) | Water-Cement Ratio (by weight) | 7-Day Compressive Strength ($lb/in.^2$) | Vebe Time (sec) |
|---|---|---|---|---|---|
| ½ | 825 | 3.40 | 0.35 | 6,500 | 9 |
| | 655 | 4.50 | 0.45 | 5,200 | 7 |
| | 455 | 7.00 | 0.55 | 3,700 | 5 |
| | 445 | 7.00 | 0.65 | 2,600 | 4 |
| ¾ | 765 | 3.75 | 0.35 | 5,900 | 9 |
| | 530 | 6.00 | 0.45 | 4,950 | 7 |
| | 540 | 5.50 | 0.55 | 3,150 | 4 |
| | 440 | 7.00 | 0.65 | 2,300 | 2 |
| 1 | 725 | 4.20 | 0.35 | 5,600 | 8 |
| | 535 | 6.00 | 0.45 | 4,000 | 8 |
| | 415 | 7.00 | 0.55 | 3,400 | 7 |
| | 400 | 8.00 | 0.65 | 2,100 | 5 |
| 1½ | 675 | 4.50 | 0.35 | 5,250 | 12 |
| | 540 | 6.00 | 0.45 | 4,000 | 12 |
| | 430 | 7.50 | 0.55 | 3,000 | 10 |
| | 365 | 9.00 | 0.65 | 2,425 | 4 |

**Table 4. Optimum results for continuously graded concrete.**

| Maximum Size of Coarse Aggregate (in.) | Cement Content (lb/cu yd) | Aggregate-Cement Ratio (by weight) | Water-Cement Ratio (by weight) | 7-Day Compressive Strength (lb/in.$^2$) | Vebe Time (sec) |
|---|---|---|---|---|---|
| ½ | 1,010 | 2.50 | 0.35 | 6,000 | 8 |
| | 800 | 3.40 | 0.40 | 5,450 | 7 |
| | 560 | 5.30 | 0.48 | 4,550 | 4 |
| | 530 | 5.30 | 0.63 | 2,100 | 2 |
| ¾ | 840 | 3.00 | 0.35 | 6,000 | 15 |
| | 730 | 3.80 | 0.40 | 5,050 | 7 |
| | 595 | 5.00 | 0.48 | 3,700 | 5 |
| | 470 | 6.30 | 0.63 | 2,250 | 4 |
| 1 | 950 | 2.70 | 0.35 | 6,000 | 7 |
| | 675 | 4.40 | 0.40 | 5,750 | 7 |
| | 620 | 4.90 | 0.48 | 4,000 | 4 |
| | 445 | 7.00 | 0.63 | 2,250 | 4 |
| 1½ | 870 | 2.60 | 0.35 | 5,850 | 7 |
| | 685 | 4.30 | 0.40 | 4,775 | 7 |
| | 525 | 6.00 | 0.48 | 4,000 | 6 |
| | 425 | 7.50 | 0.63 | 1,000 | 4 |

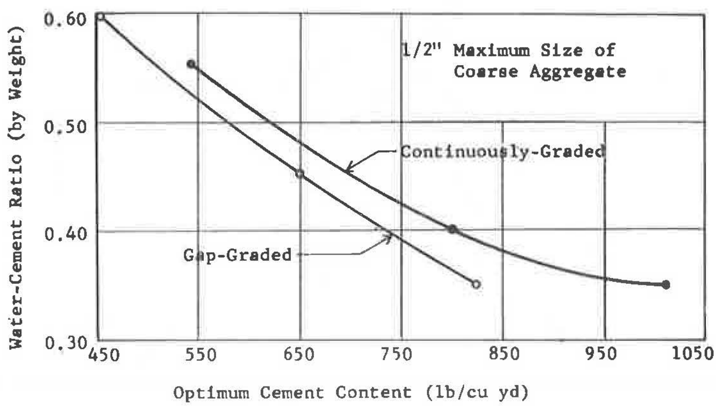



**Figure 14. Optimum cement content and water-cement ratio for gap-graded and continuously graded concrete (½-in. maximum-sized aggregate).**

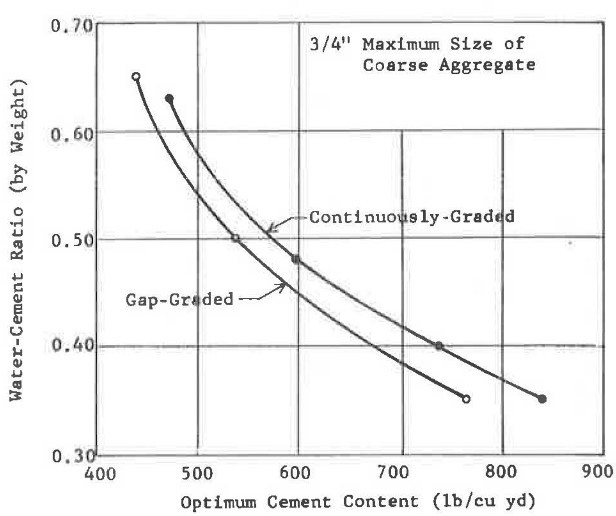



**Figure 15. Optimum cement content and water-cement ratio for gap-graded and continuously graded concrete (¾-in. maximum-sized aggregate).**

**Figure 16. Optimum cement content and water-cement ratio for gap-graded and continuously graded concrete (1-in. maximum-sized aggregate).**

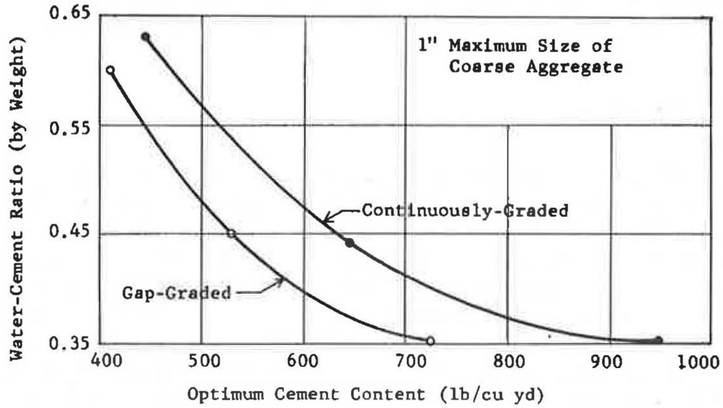



**Figure 17. Optimum cement content and water-cement ratio for gap-graded and continuously graded concrete (1½-in. maximum-sized aggregate).**

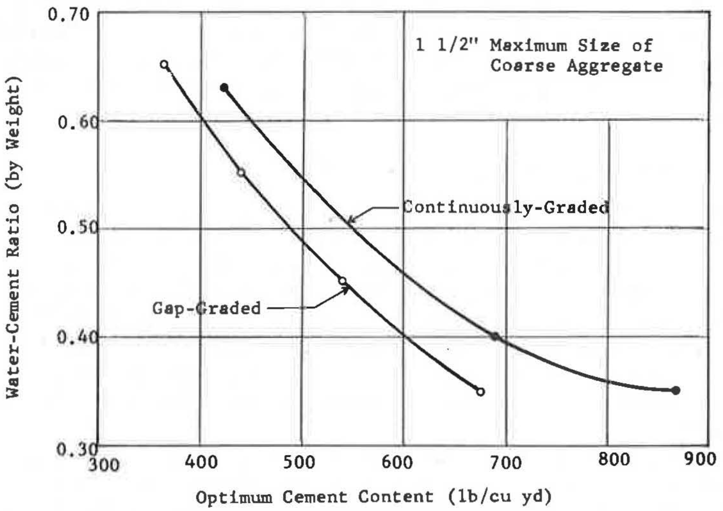



**Table 5. Optimum and average results.**

| Cement Content (lb/cu yd) | Aggregate-Cement Ratio (by weight) | Water-Cement Ratio (by weight) | 7-Day Compressive Strength (psi) |
|---|---|---|---|
| 455 | 7.00 | 0.55 | 3,700 |
| 445 | 7.00 | 0.65 | 2,600 |
| 450 | 7.00 | 0.60 | 3,150 |

The preceding results can be summarized by stating that gap-graded concrete requires lower cement content, lower water-cement ratio (and hence lower water content), and higher aggregate-cement ratio than continuously graded concrete for equal 7-day compressive strength and maximum size of coarse aggregate and workability. These results confirm the hypothesis concerning superior qualities of gap-graded concrete (8, 9, 11).

## Vibration Time of Concrete Mixes and Its Relation to Cement Content

Concrete cylinders for all the mixes were made with the aid of internal vibrators, as recommended in ASTM Designation C 192-68. Concrete was filled in the molds in two approximately equal layers, and for each layer the vibrator was inserted at three different equally spaced points. As the vibrator needle was inserted, the time (in seconds) required for the mortar to appear evenly at the top surface around it was recorded as the vibration time, and the vibrator was then withdrawn in such a manner that no air pockets were left in the concrete. The same vibration time was used for all other five insertions. This simple technique not only proved well for achieving full compaction but also provided a measurement for the workability of the mix. The vibration time was higher for the stiffer mixes and lower for the more workable ones.

Vibration times recorded for gap-graded and continuously graded concrete mixes have been plotted against cement contents (Figs. 18 and 19). As would be expected, the vibration time decreases with an increase in the water-cement ratio and cement content. A previous analysis of these results (12, Figs. 12 through 15) indicates that the vibration time required for gap-graded concrete is less than that for continuously graded concrete for all cement contents, water-cement ratios, and maximum sizes of coarse aggregate throughout this investigation. This should mean that gap-graded concrete is more workable and requires less energy for full compaction than does continuously graded concrete having the same mix parameters.

## ECONOMICS OF GAP GRADING OF AGGREGATES

The economics of gap grading of the aggregates depends on the locality, the source of the material, and the equipment used for quarry processing. Where there are naturally occurring well-graded aggregates from coarse to fine available, the use of continuous grading may be economical. When naturally occurring well-graded aggregates are not available, the use of continuous grading to fit a certain curve of size variation for maximum density or any other would require size separation, different bins or stockpiles, synthesis, the necessary elaborate making of certain missing intermediate sizes, and even costly precautions to eliminate segregation, all of which will cost more in quarry processing.

Gap grading will constitute a solution where naturally occurring well-graded aggregates are lacking, as is the case in many regions of the world. Savings can occur in those cases when aggregates are naturally gap-graded or where there are excesses of particular sizes that result from a given process. It has been reported (18) that the production of road pavements in India, in many areas, was becoming extremely expensive because the authorities insisted on the use of sand conforming to a British standard specification. This meant that frequently the local fine sand was not acceptable, and hauls of up to 200 miles were necessary to bring in sand of an acceptable grading. The Indian Road Research Laboratory, after studying Stewart's work on gap grading, decided to make its own tests. These showed such promise that, in a short time, standard gradings were discarded and local sands were brought into use, producing completely acceptable concrete to which could be credited the reduced haulage and cement costs.

Apart from demonstrating that a logical approach to the formulation of concrete grading reduced cost and even improved quality, it shows the danger of either a central or local authority introducing codes of practice that specify in detail rather than in principle and thereby prevent the engineer from pursuing fully his profession, the economic use of natural resources.

Figure 18. Cement content and vibration time for water-cement ratios (½-in. maximum-sized aggregate, gap-graded concrete).

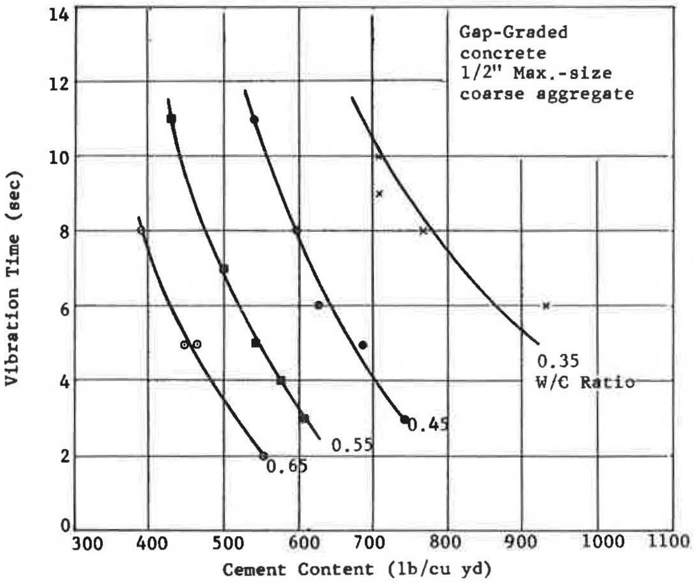
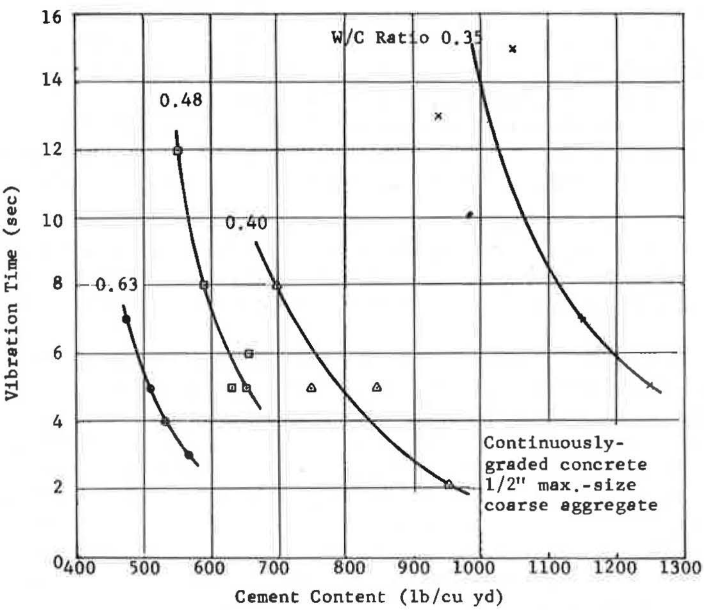



Figure 19. Cement content and vibration time for water-cement ratios (½-in. maximum-sized aggregate, continuously graded concrete).



Aggregate sizes within a narrow range for gap grading can be better accomplished today with modern equipment than several decades ago. They do not need filler sizes, and they are not liable to aggregate segregation. The quarry products of different sizes can all be appropriately used, for instance, in a bridge project: $1\frac{1}{2}$- to $1\frac{1}{4}$-in. coarse aggregate can be well used in bridge piers and foundations, $1\frac{1}{4}$ to 1 in. in girders, 1 to $\frac{3}{4}$ in. in cross beams and stringers, and $\frac{3}{4}$ to $\frac{3}{8}$ in. in deck slabs. In a well-planned program there could be no waste.

Even though in some localities gap-graded sand costs more than natural sand, the saving in cement content can offset this increased sand cost.

Gap grading of the aggregates is imperative for precasting exposed aggregate surfaces, as has been done for more than three decades, to show the exposed aggregate more uniformly and more prominently.

The increasing use of lightweight aggregates that can be easily manufactured in equal maximum sizes will further open a new vista for the application of gap grading to lightweight aggregate concrete.

Based on the results presented in this paper, an optimum mixture design method for gap-graded concrete, a step-by-step procedure for using this method, and an illustrative numerical example are presented elsewhere (17).

## CONCLUSIONS

The work presented here involved 200 concrete mixes having different water-cement ratios, aggregate-cement ratios, and maximum sizes of coarse aggregate for both gap-graded and continuously graded concretes. For all the mixes, 7-day compressive strength of hardened concrete and workability of plastic concrete were tested. The results were analyzed to obtain optimum mix parameters and to compare them using concretes having gap-graded and continuously graded aggregates.

Of the four significant mix parameters (cement content, water-cement ratio, size and quantity of coarse aggregate, and size and quantity of fine aggregate), cement content plays by far the most important role in the optimization for obtaining the most economical concrete mix. The more important properties of plastic and hardened concrete to be considered for optimization are respectively workability and compressive strength, but the latter is more amenable to quantitative determination.

The analysis has borne out the conclusions that follow. However, it should be pointed out that these conclusions are based on the experimental results reported here and are applicable to the types of crushed limestone and sand used in this investigation:

1. Pilot experimental results indicate that, for any specific water-cement ratio, aggregate-cement ratio, and maximum size of coarse aggregate, there is an optimum content of fine aggregate for which the 7-day compressive strength of gap-graded concrete is a maximum, and this fine-aggregate content can be used to arrive at the optimum proportioning.
2. The present test data show that this optimum content of fine aggregate is very nearly equal to 36 percent by weight of total aggregates irrespective of variations in water-cement ratio and aggregate-cement ratio (Fig. 1).
3. Two-dimensional plots between cement content and 7-day compressive strength for gap-graded and continuously graded concretes of equal water-cement ratio and maximum size of coarse aggregate have shown that, at one particular cement content, the compressive strength reaches its maximum. This cement content and the corresponding water-cement and aggregate-cement ratios are the optimum mix parameters.
4. As expected, optimum cement contents increase with the increase in the 7-day compressive strength and with the decrease in the water-cement ratio.
5. The optimum cement content of gap-graded concrete is lower than the optimum content for continuously graded concrete of equal 7-day compressive strength and maximum size of coarse aggregate.
6. Quantitatively, for all maximum sizes of coarse aggregates and nominal compressive-strength ranges within this investigation, gap-graded concrete requires 15 to 35 percent lower optimum cement content than its continuously graded counterpart equally proportioned.

7. For equal optimum cement content, maximum size of coarse aggregate, and workability, gap-graded concrete requires lower water-cement ratio and hence lower water content than its continuously graded counterpart.

8. For equal optimum cement content, maximum size of coarse aggregate, and workability, gap-graded concrete permits a higher aggregate-cement ratio.

9. A comparison of the recorded vibration time needed for full compaction of gap-graded concrete with that of the continuously graded has shown that the former is more workable and requires less energy to bring to a homogeneous mass then the continuously graded having the same water-cement ratio, cement content, and maximum size of coarse aggregate.

## ACKNOWLEDGMENTS

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# EFFECT OF MAXIMUM SIZE OF COARSE AGGREGATE ON D-CRACKING IN CONCRETE PAVEMENTS

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Field and laboratory observations have indicated that D-cracking is caused by freeze-thaw failures in certain types of coarse aggregate particles. In areas where durable aggregates are not available, it has been found that the rate of development of D-cracking can be reduced by decreasing the maximum particle size. These observations were extended during a laboratory investigation that was carried out to find a test procedure that would distinguish between durable coarse aggregates and those that cause D-cracking and provide an indication of the benefits to be derived by reducing the maximum particle size. Exploratory work indicated that a rapid freeze-thaw procedure similar to ASTM Designation C666-71 would be suitable. A failure criterion of 0.032 to 0.033 percent expansion in 350 or fewer cycles was established on the basis of the service records of 15 sources from which the test materials were obtained. Studies of the effect of maximum particle size on durability indicated that decreasing the size from 1½ in. to 1 in. and ½ in. reduced expansions to varying degrees. These findings are in line with the critical size concept for aggregate that was developed in previous work. It is recommended that, where D-cracking is a problem, similar testing programs be set up to evaluate coarse aggregate sources on an individual basis and to determine the benefits to be derived by reducing maximum particle sizes to improve durability.

•IN certain areas of eastern and west-central United States, and in Canada, D-cracking is a serious and costly durability problem affecting highways and airfields. Although it was initially observed in pavements more than 30 years ago, only within the past 10 years have efforts been made to develop an understanding of the mechanism of distress. The Portland Cement Association has been investigating this problem in the United States and Canada and has found that deterioration is initiated through freezing and thawing of coarse aggregate particles located in the lower and middle portions of pavement slabs. Observations have also indicated that subbase drainage affects the rate of development of deterioration but that factors such as joint spacing, air entrainment, and cement composition are of little or no significance in this problem.

Recent work on D-cracking has been concerned primarily with characterizing durable and nondurable coarse aggregates by laboratory test methods and determining ways by which the durability of materials from existing coarse aggregate sources can be improved. This paper will describe field observations and laboratory tests that indicate that, where durable coarse aggregates in existing gradations are not available, reducing the maximum particle size is an effective method of eliminating or reducing the rate of development of D-cracking.

## DEFINITION

The term D-cracking dates back to the 1930s and was used in reference to deterioration due to weathering as evidenced by the appearance of fine, parallel cracks along transverse and longitudinal joints and the free edges of pavement slabs (Fig. 1). Often these cracks contain deposits of secondary reaction products, primarily $CaCO_3$. Because our studies have revealed that, with few exceptions, this cracking is initiated through freezing and thawing of coarse aggregate particles, the term in this report will denote a cause and source of distress as well as the nature of the crack pattern observed at the pavement wearing surface.

## OBSERVATIONS OF PAVEMENTS

Observations of pavement concrete have indicated that the development of D-cracking depends primarily on the source of coarse aggregate. This is shown in Figure 2, where coarse aggregate from a different source was used on each side of the transerse joint. After 8 years of exposure, D-cracking is well developed on one side of the joint but not evident at the wearing surface on the other side. Examination of cores confirmed the absence of distress in the lower levels of the one slab, as shown by the comparison of core sections in Figure 3. Laboratory tests and other field observations indicate that D-cracking is unlikely to develop in the currently unaffected pavement.

A second example of this relation is shown in Figure 4. Here, it is seen that, after 15 years of exposure, one traffic lane is free of distress, whereas, with a change in source of coarse aggregate, D-cracking has developed in the abutting lane. Unlike the previous example, however, the examination of cores revealed that distress had developed in the lower levels of the pavement slab in the traffic lane that is free of distress at the wearing surface. In this case, both sources are vulnerable to D-cracking, but to varying degrees.

From the preceding figures it is apparent that coarse aggregates from different sources in a given area may show varying susceptibilities to D-cracking in concrete pavements. In some areas, however, materials from available sources all show marked susceptibilities to D-cracking, in which case there is no advantage in selecting one source over another. Under these circumstances, limited field observations have indicated that reducing the maximum particle size can greatly reduce the rate of development of D-cracking or possibly eliminate it. The following examples illustrate the improved durability to be gained.

In a pavement built in 1965, crushed-limestone coarse aggregates taken from the same source but containing 1½-in. and 1-in. maximum sizes were used in different sections. In this pavement, D-cracking is apparent at the joint intersections where the large aggregate was used (Fig. 5), whereas, in the section with the small size, D-cracking is not visible at the wearing surface. However, petrographic examination of cores taken from selected joint areas revealed that distress has developed in the pavement concrete with the 1-in. maximum particle size. In addition, cracking was observed in coarse aggregate particles of less than ½-in. size. Thus, although improved performance was obtained at the age of 5 years by reducing the maximum particle size of the coarse aggregate, the change only reduced the rate of development of D-cracking.

A second example of improved performance with a reduction in maximum particle size was seen in the Topeka test road in Kansas, which is a project of the "long-time study" (1). Here, a crushed-limestone coarse aggregate was used in all sections of pavement, and the maximum particle sizes were ¾ in. and 1½ in. After 13 years, D-cracking was well advanced in concrete containing the 1½-in. maximum particle size (Fig. 6), but it was not then apparent at the wearing surface in concrete containing the ¾-in. maximum size. It should be noted here that mix proportions and placing and curing procedures were also variables in this road, but observations elsewhere indicate that variables of these types are of little or no significance in this type of deterioration. Therefore, the conclusion on the effect of maximum particle size on the development of D-cracking in this pavement appears to be fully justified.

A secondary factor that has been found to affect the rate of development of D-cracking is subbase drainage. Where artificial drains have not been installed, conditions are most conducive to the development of D-cracking where a potentially non-durable coarse aggregate has been used. Where longitudinal tile drains have been installed, the rate of development of D-cracking is reduced if the fines in the subbase do not seal off subbase moisture migration to the drain inlets. However, to the best of our knowledge, a pavement drainage system has not yet been found that will obviate

Figure 1. Severe D-cracking along longitudinal and transverse joints of 9-year old pavement.

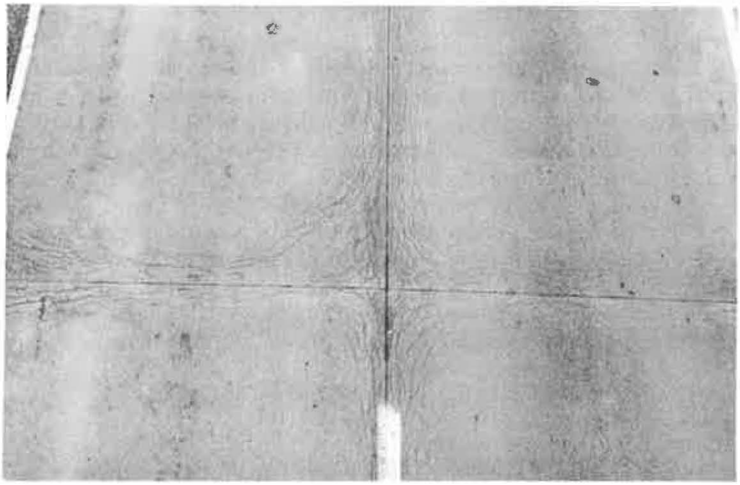

Figure 2. D-cracking along one side of transverse joint of 8-year old pavement.

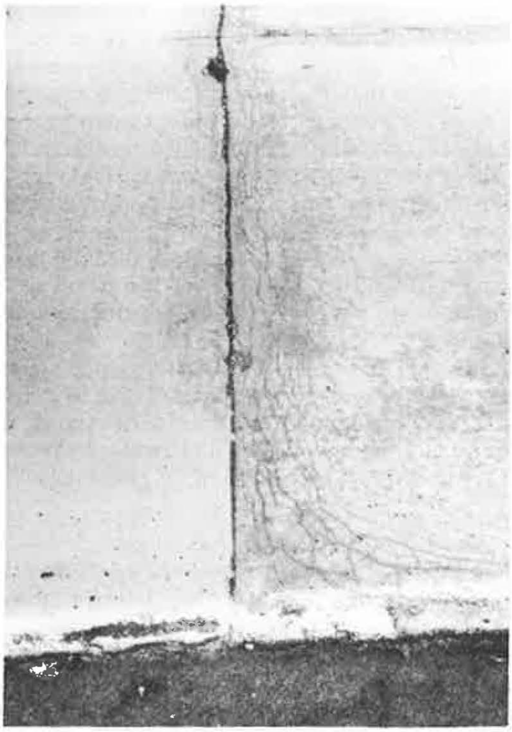

Figure 3. Vertical sections of cores taken on each side of transverse joint shown in Figure 2.

durability deficiencies in the coarse aggregate and eliminate the development of D-cracking.

## LABORATORY STUDIES

Laboratory studies were undertaken to determine if freezing and thawing tests could substantiate the service records of coarse aggregate sources with respect to D-cracking. It was anticipated that a failure criterion could be established that would distinguish between durable and nondurable materials and also provide a reference to judge the merits of decreasing maximum particle sizes to reduce or eliminate the susceptibility of various aggregates to D-cracking. Exploratory work indicated that a rapid freeze-thaw procedure in water, similar to ASTM Designation C 666-71, would be most suitable. In the procedure used, 3- by 3- by $11\frac{1}{4}$-in. concrete prisms were frozen and thawed in water at the rate of two cycles per day, and length-change measurements were made after every 25 cycles up to 300 cycles and at 350 cycles. Details of the materials, mix designs, procedures used, and test results are given in the following sections.

### Materials and Service Records

Coarse aggregates from 15 sources were used in this series of tests. These sources are given in Table 1 and are categorized according to service record for gradations with a $1\frac{1}{2}$-in. maximum particle size. Three categories of service record are shown: one for which D-cracking appears at the wearing surface in less than 8 years, one for which D-cracking appears between 8 and 15 years, and one for which D-cracking is not apparent in more than 15 years. For two sources in the latter group, 3C and 3E, D-cracking has not appeared at the wearing surface in more than 22 years. Examination of cores from pavement containing coarse aggregates from all sources in this group revealed no evidence of incipient distress in the lower portions of the pavement slabs. At these ages, this absence of distress would indicate that D-cracking will not appear at the wearing surface for at least 25 years. Service records of this length can be considered as satisfactory and will serve as a basis for evaluating laboratory test results.

It may be noted in Table 1 that all of the aggregate types with satisfactory service records are crushed dolomites. This does not mean that, where D-cracking is a problem, dolomite is the type of material that should be used. In much more extensive field observations, crushed dolomites as well as siliceous and other types of carbonate rock have been found to perform both poorly and satisfactorily. Most of the present work was based on observations in an area of dolomitic bedrock where the most extensive and complete service records were available.

Single sources of fine aggregate (mixed carbonate) and cement (ASTM Type I) were used for these tests. Both have been used with a variety of coarse aggregates in concrete pavements and, in comparing the performance of these pavements with others using coarse aggregates from the same sources but fine aggregates and cements from different sources, have shown no singular effect on the development of D-cracking.

### Mix Design and Procedure

Coarse aggregate gradations and mix design data for all tests are given in Tables 2 and 3 respectively. The fine aggregate gradations were as follows: No. 4 to No. 8 sieve size, 10 percent; No. 8 to No. 16, 20 percent; No. 16 to No. 30, 25 percent; No. 30 to No. 50, 25 percent; No. 50 to No. 100, 16 percent; and -No. 100, 4 percent. The compound composition of the cement was as follows: $C_3S$, 59 percent; $C_2S$, 16 percent; $C_3A$, 9.9 percent; and $C_4AF$, 7.9 percent. Chemical analysis of the cement showed the following: CaO, 64.85 percent; $SiO_2$, 20.90 percent; $Al_2O_3$, 5.42 percent; $Fe_2O_3$, 2.61 percent; MgO, 1.85 percent; $Na_2O$, 0.15 percent; $K_2O$, 0.74 percent; $SO_3$, 2.14 percent; loss value, 1.17 percent; and Blaine value, 3,640 $cm^2/g^2$. All concrete was mixed in a Lancaster mixer, $\frac{2}{3}$-$ft^3$ capacity, for $2\frac{1}{2}$ min. Neutralized Vinsol resin was added as an air-entraining admixture at the mixer. Air contents were measured immediately after mixing and were maintained at 5.5 to 6.5 percent. Slumps

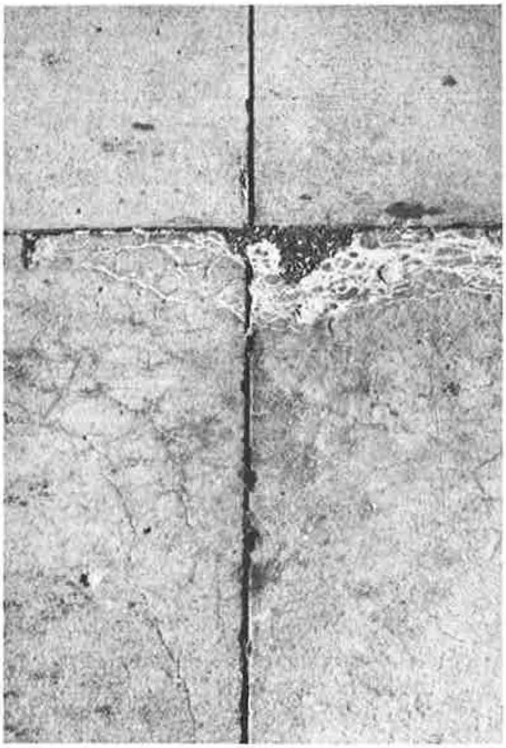

Figure 4. D-cracking in one (lower) traffic lane; abutting lane free of visible distress.

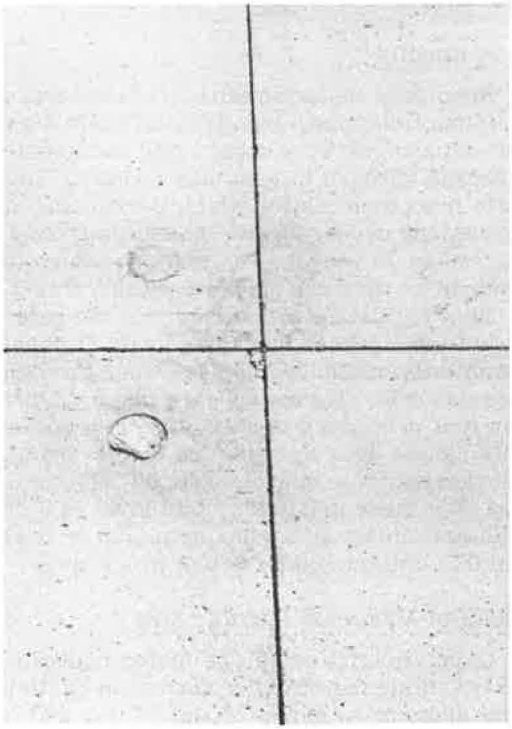

Figure 5. Early D-cracking at longitudinal-transverse joint intersection where 1½-in. maximum-sized coarse aggregate from source 1A was used.

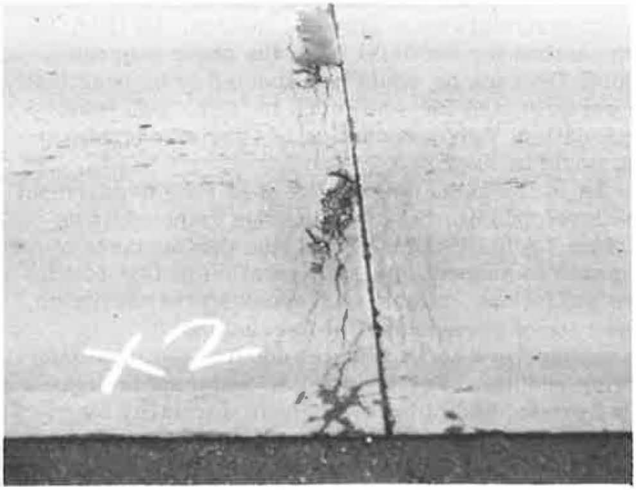


Figure 6. Typical D-cracking along a transverse joint in the Topeka test road, where 1½-in. maximum-sized aggregate particles were used.

Table 1. Service record of coarse aggregates used in laboratory tests.

| Designation | Type |
|---|---|
| D-Cracking Appears in Less Than 8 Years | |
| 1A | Crushed limestone |
| 1B | Mixed carbonate gravel |
| 1C | Mixed siliceous gravel |
| 1D | Crushed dolomitic limestone |
| 1E | Crushed limestone |
| D-Cracking Appears in 8 to 15 Years | |
| 2A | Mixed carbonate gravel |
| 2B | Mixed carbonate gravel |
| 2C | Crushed dolomite |
| 2D | Crushed dolomite |
| 2E | Mixed carbonate gravel |
| D-Cracking Has Not Appeared in More Than 15 Years | |
| 3A | Crushed dolomite |
| 3B | Crushed dolomite |
| 3C | Crushed dolomite |
| 3D | Crushed dolomite |
| 3E | Crushed dolomite |

were held to between ½ in. and 2 in. Four 3- by 3- by 11¼-in. concrete prisms were made from each batch. One of the prisms was continuously moist-cured at 73 F and 100 percent relative humidity, whereas the remaining three were subjected to freezing and thawing following a 14-day moist cure.

## Test Results

Results of tests for all aggregate sources, using a gradation with a 1½-in. maximum particle size, are given in Table 4 and shown in Figure 7. These data indicate that expansions were most rapid and greatest for materials from sources for which D-cracking appears in less than 8 years. Intermediate expansions developed with materials from sources for which D-cracking appears in 8 to 15 years, whereas the smallest expansions occurred with materials from sources with satisfactory service records of more than 15 years. The test procedure thus appears to substantiate the field service records as they are categorized in Table 1.

The test data also indicate that the rate of expansion continues to increase for materials from sources associated with D-cracking, whereas the rate of expansion remains essentially unchanged or decreases for materials from sources with satisfactory service records. It thus appears that a failure criterion could be established for which this test procedure would distinguish between durable and nondurable coarse aggregates. This can be done by using the length-change level separating sources associated with D-cracking from sources with satisfactory service records. The test data indicate that, for these materials, this level is 0.032 to 0.033 percent expansion at 350 cycles. This percentage of expansion in 350 or fewer cycles is therefore considered to be a suitable criterion for the test procedure.

## Effect of Maximum Particle Size

Using this criterion, we tested materials from nine of the coarse aggregate sources to determine the effect of maximum particle size on durability. Three maximum particle sizes were used: 1½ in., 1 in., and ½ in. Mixing, curing, and testing were carried out as previously described.

The results of these tests are given in Table 5, whereas those for selected sources are shown in Figures 8 through 11. In the group for which D-cracking appears in less than 8 years, decreasing the maximum particle size from 1½ in. to 1 in. and ½ in. progressively reduced expansions as much as two to four times. However, decreasing the maximum particle size only to 1 in. failed to reduce expansions sufficiently for any of the sources to meet the selected criterion. When the maximum particle size was further reduced to ½ in., expansions for materials from only one source, 1B (Fig. 8), almost met the criterion, whereas expansions for material from the other sources still greatly exceeded this range. Thus, D-cracking would be expected to be practically eliminated by reducing the maximum particle size for source 1B to ½ in., whereas, for the other sources with the same gradation, only a reduction of varying degrees in the rate of development of D-cracking could be expected.

It should be noted here that source 1A is the same one as that used for the pavement described earlier in which the rate of development of D-cracking was reduced by decreasing the maximum particle size from 1½ in. to 1 in. Thus, the performance of the coarse aggregate in this pavement appears to support the interpretation of test results that, where expansions are still above the failure criterion, decreasing the maximum particle size serves only to reduce the rate of development of D-cracking.

In the group for which D-cracking appears in 8 to 15 years, reducing the maximum particle size produced somewhat varying results. For source 2A, reducing the maximum particle size from 1½ in. to 1 in. produced no improvement in durability as expansions remained essentially unchanged and well in excess of the failure criterion. When the maximum particle size was further reduced to ½ in., expansions were reduced by one-half but still exceeded the failure criterion. For this source, the results indicate that the maximum particle size would have to be reduced to less than 1 in. to show any improved durability, whereas a reduction to ½ in. would have the effect of only delaying and not eliminating the development of D-cracking.

**Table 2. Coarse aggregate gradation.**

| Maximum Particle Size (in.) | Sieve Size (percent retained) 1½ to 1 in. | 1 to ¾ in. | ¾ to ½ in. | ½ to ⅜ in. | ⅜ in. to No. 4 | No. 4 to No. 8 |
|---|---|---|---|---|---|---|
| 1½ | 35 | 18 | 28 | 16 | 3 | – |
| 1 | – | 10 | 55 | 30 | 5 | – |
| ½ | – | – | – | 15 | 70 | 15 |

**Table 3. Mix design for concrete prisms.**

| Maximum Particle Size (in.) | Aggregate Content (percent) Coarse | Fine | Cement Content ($lb/yd^3$) | Water Content ($lb/yd^3$) | Air Content (percent) |
|---|---|---|---|---|---|
| Gravel | | | | | |
| 1½ | 64 | 36 | 611 | 258 to 267 | 5.5 to 6.5 |
| 1 | 60 | 40 | 611 | 258 to 267 | 5.5 to 6.5 |
| ½ | 55 | 45 | 611 | 267 to 283 | 5.5 to 6.5 |
| Crushed stone | | | | | |
| 1½ | 60 | 40 | 611 | 267 to 275 | 5.5 to 6.5 |
| 1 | 56 | 44 | 611 | 267 to 275 | 5.5 to 6.5 |
| ½ | 50 | 50 | 611 | 275 to 283 | 5.5 to 6.5 |

**Table 4. Test results using gradation with 1½-in. maximum-sized particles.**

| Aggregate Source | Percentage of Expansion at Cycle Indicated 50 | 100 | 150 | 200 | 250 | 300 | 350 |
|---|---|---|---|---|---|---|---|
| D-Cracking Appears in Less Than 8 Years | | | | | | | |
| 1A | 0.009 | 0.019 | 0.043 | 0.071 | 0.100 | – | – |
| 1B | 0.004 | 0.011 | 0.020 | 0.036 | 0.060 | 0.112 | – |
| 1C | 0.003 | 0.012 | 0.027 | 0.041 | 0.062 | 0.093 | 0.127 |
| 1D | 0.021 | 0.048 | 0.111 | – | – | – | – |
| 1E | 0.011 | 0.023 | 0.054 | 0.080 | 0.110 | – | – |
| D-Cracking Appears in 8 to 15 Years | | | | | | | |
| 2A | 0.005 | 0.007 | 0.012 | 0.018 | 0.028 | 0.045 | 0.078 |
| 2B | 0.008 | 0.011 | 0.018 | 0.026 | 0.034 | 0.046 | 0.068 |
| 2C | 0.005 | 0.010 | 0.012 | 0.017 | 0.022 | 0.027 | 0.034 |
| 2D | -0.001 | 0.001 | 0.006 | 0.008 | 0.017 | 0.027 | 0.046 |
| 2E | 0.005 | 0.005 | 0.011 | 0.015 | 0.021 | 0.031 | 0.039 |
| D-Cracking Has Not Appeared in More Than 15 Years | | | | | | | |
| 3A | 0.002 | 0.002 | 0.005 | 0.005 | 0.008 | 0.011 | 0.011 |
| 3B | 0.008 | 0.011 | 0.014 | 0.019 | 0.022 | 0.029 | 0.031 |
| 3C | 0.006 | 0.008 | 0.009 | 0.013 | 0.015 | 0.017 | 0.019 |
| 3D | 0.004 | 0.008 | 0.010 | 0.013 | 0.018 | 0.020 | 0.022 |
| 3E | 0.002 | 0.006 | 0.006 | 0.008 | 0.013 | 0.014 | 0.015 |

**Figure 7. Comparison of length changes during freezing and thawing.**

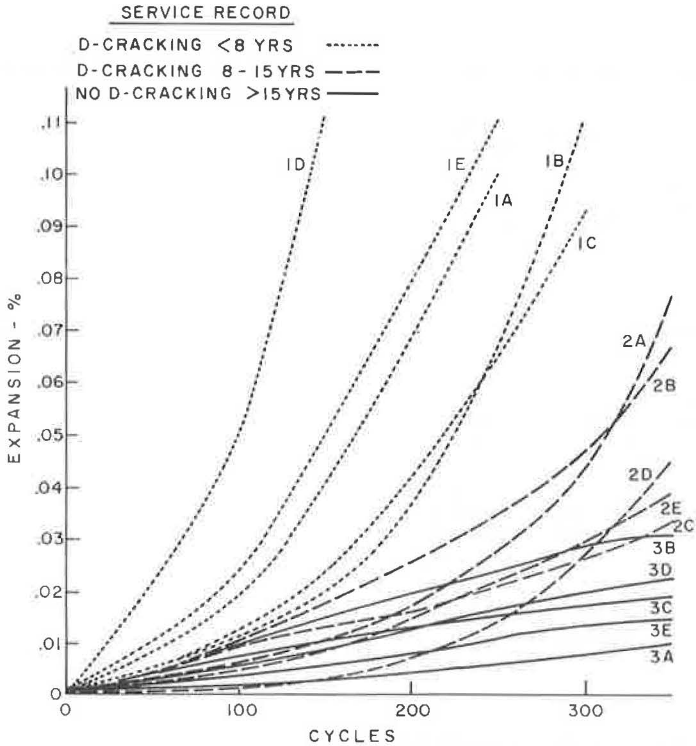


**Table 5. Effect of maximum particle size on durability.**

| Aggregate Source | Maximum Size (in.) | Percentage of Expansion at Cycle Indicated | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | 50 | 100 | 150 | 200 | 250 | 300 | 350 |
| **D-Cracking Appears in Less Than 8 Years** | | | | | | | | |
| 1A | 1½ | 0.009 | 0.019 | 0.043 | 0.071 | 0.100 | – | – |
| | 1 | 0.010 | 0.014 | 0.032 | 0.050 | 0.074 | – | – |
| | ½ | 0.009 | 0.013 | 0.023 | 0.044 | 0.076 | – | – |
| 1B | 1½ | 0.004 | 0.011 | 0.020 | 0.036 | 0.060 | 0.112 | – |
| | 1 | 0.004 | 0.007 | 0.015 | 0.021 | 0.031 | 0.048 | 0.061 |
| | ½ | 0.003 | 0.004 | 0.009 | 0.014 | 0.019 | 0.030 | 0.035 |
| 1C | 1½ | 0.003 | 0.012 | 0.027 | 0.041 | 0.062 | 0.093 | 0.127 |
| | 1 | 0.001 | 0.007 | 0.015 | 0.025 | 0.040 | 0.062 | 0.091 |
| | ½ | 0.001 | 0.005 | 0.012 | 0.018 | 0.026 | 0.039 | 0.049 |
| 1D | 1½ | 0.021 | 0.048 | 0.111 | – | – | – | – |
| | 1 | 0.010 | 0.022 | 0.045 | 0.069 | 0.097 | 0.144 | 0.183 |
| | ½ | 0.006 | 0.009 | 0.018 | 0.027 | 0.040 | 0.061 | 0.079 |
| 1E | 1½ | 0.011 | 0.023 | 0.054 | 0.080 | 0.110 | – | – |
| | 1 | 0.009 | 0.021 | 0.044 | 0.081 | 0.122 | – | – |
| | ½ | 0.007 | 0.016 | 0.024 | 0.039 | 0.052 | 0.063 | 0.069 |
| **D-Cracking Appears in 8 to 15 Years** | | | | | | | | |
| 2A | 1½ | 0.005 | 0.007 | 0.012 | 0.018 | 0.028 | 0.045 | 0.078 |
| | 1 | 0.004 | 0.006 | 0.010 | 0.016 | 0.028 | 0.050 | 0.083 |
| | ½ | 0.003 | 0.006 | 0.008 | 0.014 | 0.019 | 0.027 | 0.041 |
| 2D | 1½ | -0.001 | 0.001 | 0.006 | 0.008 | 0.017 | 0.027 | 0.046 |
| | 1 | -0.001 | 0.003 | 0.009 | 0.013 | 0.023 | 0.035 | 0.057 |
| | ½ | -0.002 | 0.000 | 0.005 | 0.007 | 0.014 | 0.021 | 0.033 |
| 2E | 1½ | 0.005 | 0.005 | 0.011 | 0.015 | 0.021 | 0.031 | 0.039 |
| | 1 | 0.004 | 0.005 | 0.010 | 0.013 | 0.019 | 0.027 | 0.032 |
| | ½ | 0.003 | 0.004 | 0.008 | 0.010 | 0.014 | 0.021 | 0.024 |
| **No D-Cracking in More Than 15 Years** | | | | | | | | |
| 3A | 1½ | 0.002 | 0.002 | 0.005 | 0.005 | 0.008 | 0.011 | 0.011 |
| | 1 | 0.001 | 0.001 | 0.003 | 0.003 | 0.006 | 0.009 | 0.009 |

The data for source 2D (Fig. 9) indicate, like source 2A, no improvement in durability when the maximum particle size is reduced from 1½ in. to 1 in. In fact, somewhat greater expansions were recorded with the 1-in. maximum particle size. However, when the maximum size was reduced to ½ in., expansions met the failure criterion, which indicates that, for this source, D-cracking would be essentially eliminated by using the ½-in. maximum size.

The results for source 2E (Fig. 10) indicate progressively lesser expansions as the maximum particle size is reduced from 1½ in. to 1 in. and ½ in. For this source, however, the data indicate that a reduction in maximum particle size only to 1 in. would be sufficient to essentially eliminate the development of D-cracking because the expansion at 350 cycles is 0.032 percent. A further reduction would appear to be unnecessary to eliminate D-cracking.

The effect of maximum particle size on durability was studied for only one source, 3A (Fig. 11), in the group in which D-cracking has not appeared in more than 15 years. Here, expansions of 0.011 and 0.009 percent developed for gradations with maximum particle sizes of 1½ in. and 1 in. respectively. These expansions are well below the failure criterion range and substantiate the field observations that a reduction in maximum particle size is not needed for this material.

## DISCUSSION OF RESULTS

In the work here reported it is apparent that the nature of the coarse aggregate is a primary factor affecting freeze-thaw durability and the development of D-cracking in concrete pavements. The evidence that decreasing the maximum particle size improves durability and reduces the rate of development of D-cracking is in line with the work of Powers (2) and Verbeck and Landgren (3), who developed the critical-size concept to explain the behavior, during freezing and thawing, of aggregates with varying pore characteristics. Briefly, in a given concrete and environment, this concept involves the distances certain quantities of water can be moved through an aggregate particle or surrounding mortar to prevent the generation of excessive hydraulic pressures during freezing. With a given size, certain types of critically saturated particles may fail if their permeabilities are sufficiently low to prevent expulsion of adequate water into the surrounding cement paste and the relief of internal hydraulic pressures. For other types of particles, porosities and permeabilities may be sufficiently high to allow excessive quantities of water to be expelled into the surrounding cement paste where critical saturation can then be reached and excessive hydraulic pressures be generated. In either case, and in intermediate cases, a reduction in particle size would reduce the magnitude of hydraulic pressures generated during freezing.

In line with this concept, the failure criterion established in these laboratory tests can be used to provide an indication of the critical particle size above which the aggregate is not immune to the effects of freezing and thawing and may cause D-cracking. In this work, if expansions exceed 0.032 to 0.033 percent, the critical size is less than the maximum particle size of the aggregate gradation being tested. If expansions are less, the critical size is greater than the maximum particle size. The critical sizes for the coarse aggregate sources in this test series are given in Table 6. According to these data, D-cracking would be essentially eliminated with two of the nine sources if ½-in. maximum particle sizes were specified. With one source, a 1-in. size would be permissible, whereas, with another source, a 1½-in. or possibly larger size could be tolerated. For the remaining five sources, the maximum particle size would have to be less than ½ in. to eliminate the development of D-cracking.

## CONCLUSIONS

The rapid freeze-thaw test procedure described in this paper appears to have successfully distinguished between coarse aggregates from sources associated with the development of D-cracking and those from sources with known satisfactory service records. The test procedure also appears to have substantiated the limited but meaningful field observations on the benefits of reducing the maximum particle size of coarse aggregates from sources associated with D-cracking. It is thus recommended

**Figure 8. Effect of maximum particle size on durability for source 1B.**

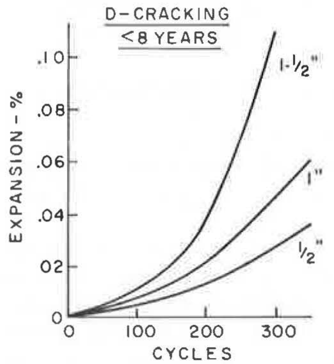


**Figure 9. Effect of maximum particle size on durability for source 2D.**

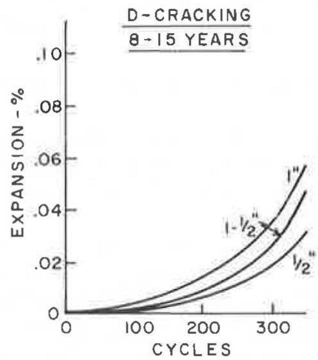


**Figure 10. Effect of maximum particle size on durability for source 2E.**

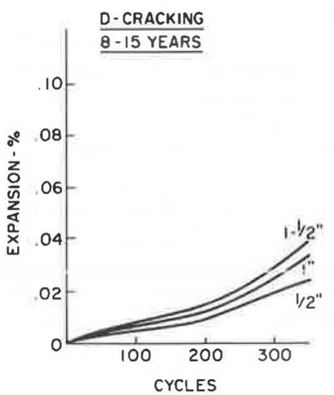


**Figure 11. Effect of maximum particle size on durability for source 3A.**

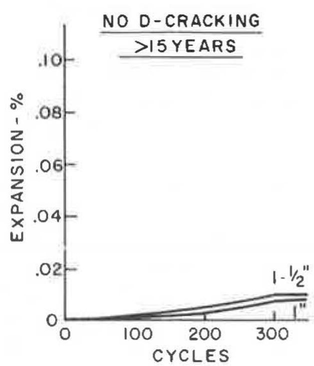


**Table 6. Critical sizes for aggregate sources.**

| Service Record | Source | Critical Size (in.) |
|---|---|---|
| D-cracking appears in less than 8 years | 1A | < 1/2 |
| | 1B | 1/2 |
| | 1C | < 1/2 |
| | 1D | < 1/2 |
| | 1E | < 1/2 |
| D-cracking appears in 8 to 15 years | 2A | < 1/2 |
| | 2D | 1/2 |
| | 2E | 1 |
| No D-cracking in more than 15 years | 3A | > 1 1/2 |

that, where D-cracking is a problem, similar testing programs be set up to evaluate coarse aggregate sources on an individual basis and to determine the benefits to be derived by reducing maximum particle sizes to improve durability.

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# INFLUENCE OF THE GRADING OF AGGREGATES ON CONCRETE MIX PROPORTIONS

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It is well known that aggregate grading directly influences concrete mix proportions and that satisfactory concrete can be produced with aggregates whose gradings do not fall entirely within normal specifications or conform with typical grading curves. In developing countries and particularly on small islands, such aggregates are often the only ones locally available or within economic distance, and the concrete manufacturer has no choice but to select the most suitable mix proportions. The paper discusses the influence of grading on mix proportions and describes the determination of economic proportions using aggregates available on three islands of the eastern Caribbean. On one island, the coarse aggregate consists of coral limestone containing a significant percentage of fines; the fine aggregate is a very fine, uniformly sized sand. On another island, the excessive use of beach sand has necessitated the investigation of concrete-making properties of a local "pumice," a significant fraction of which passes the No. 100 sieve. Results show that, with suitable selection of mix proportions, these aggregates make satisfactory concrete.

•CONCRETE is a mixture that consists principally of water, cement, and aggregates, and one of the most important features of its production is the selection of the proportions of these constituents. The selection must be done in a way that ensures economy and achieves the properties required in the freshly mixed and hardened states.

Among the several important factors that influence the proportions of the constituents of a concrete mixture is the grading of the aggregates. Experience has shown that satisfactory concrete can be made with a wide range of aggregates so long as the mix proportions are suitably selected in accordance with the grading. The selection of these proportions becomes a matter of major importance in developing countries (and particularly on small islands), where it often happens that only one type of aggregate is locally available or within economic distance.

In this paper, the influence of grading on mix proportions is discussed, and a description is given of the determination of economic proportions for a wide range of concrete strengths using aggregates available on three islands of the eastern Caribbean.

## GRADING

The term grading refers to the distribution of particle sizes in the aggregates. This distribution is readily obtained by sieve analyses whose results can be easily understood and compared by graphical representation. For this reason, grading charts are used on an extensive scale throughout the world and form the basis of mix design methods recommended by many concrete institutes.

An alternative method of representing grading is the fineness modulus, which is a single factor computed from sieve analyses by adding the total percentages of material retained on each of the standard sieve sizes starting with British Standard or ASTM size No. 100 up to the largest size available. The fineness modulus method is subject to the criticism that it describes the average particle size only, the same modulus representing a large variety of gradings. Nevertheless, it has been shown that, in combination with other factors (1-4), the fineness modulus is very useful in determining mix proportions, and its use has found favor in the Americas, South Africa, Australia, and other places.

It has been established for some time that grading is a major factor in workability of concrete, and it is appropriate therefore to consider the nature, definition, and measurement of this property of concrete.

## WORKABILITY

In its fresh state, concrete must have the property of being handled, consolidated, and finished without segregation or bleeding. This property has a direct influence on its qualities in the hardened state, the most important of which are strength and durability. This influence derives from the well-established relation that exists between the degree of compaction of concrete and its strength (5). The greater the degree of compaction, the smaller the number and size of voids there are, and it is the presence of voids that affects the strength of concrete. Compaction to maximum density can only be achieved if the fresh concrete has adequate workability.

Although there has long been agreement on the conception of what constitutes a workable concrete, there is still little agreement as to its definition. Terms such as workability, plasticity, mobility, and consistence are often used synonymously to describe the same characteristic, whereas they really refer to different attributes of the concrete. Terms such as workable plasticity, which appear in the famous formulation of the Duff Abrams water-cement ratio law (6), make the situation no clearer.

Probably the first serious attempt at a precise and quantitative definition was made in 1947 (5). Workability was defined as "that property of the concrete which determines the amount of useful work necessary to produce full compaction." This property was related to a compacting factor that could be used as a measure of workability. Cusens (7) has shown, however, that the compacting factor is not an accurate measure of the work required to compact dry, harsh mixes with factors of less than 0.80.

Newman (8) has stated that the compacting factor is really a measure of "compactibility" only and has suggested that workability is a composite property of concrete that should be defined in terms of three separate properties: compactibility, mobility, and stability. Hughes (9) has added a fourth property, "finishability," which has been rejected by Uzomaka (10) as not being peculiar to concrete. Uzomaka has further confused the issue, however, by replacing "mobility" with "spreadability" and naming the three properties "consistence."

In view of the confusion surrounding its definition and the complexity of factors involved in its conception, it seems improbable that workability can ever be precisely defined or quantified. The American Concrete Institute, in its excellent recommendations on mix proportioning and consolidation (4, 11), has probably arrived at the best answer to the problem by defining workability as the property of fresh concrete that determines the ease with which it can be placed, consolidated, and finished without harmful segregation. It goes on to say that workability embodies concepts such as flowability, moldability, cohesiveness, and compactibility and relates all three to consistency, which is defined as "the ability of freshly mixed concrete to flow." It further states that, "once the materials and mix proportions are selected, the primary control over workability is through changes in the consistency brought about by changing the water content."

Nearly 60 methods using slump, flow, penetration, drop, mixer, deforming, compacting, and other techniques have been developed over the past 50 years to measure workability. As can be inferred from the foregoing discussion, these have really only succeeded in correlating some aspect of workability or consistency with an easily determinable physical measurement. The vast majority of these methods have found only limited application, and the ones most commonly used today are the slump, consistometer, and compacting methods.

The slump and consistometer methods are tests of consistency, and a direct relation seems to exist between them for ranges of slump from 25 to 100 mm (11, 12). The slump test is relatively insensitive for the drier, stiffer mixes, the consistometer test being similarly so for the wetter, more plastic mixes.

Although there is no obvious connection between slump results and workability, and although the test is so liable to random variations that one cannot readily distinguish among the slumps of mixes of different workabilities, such a relation is of real value as a field control of mixes and is widely and successfully used to indicate the consistency of mixes used in normal construction. In particular, the test will quickly detect a change in water content or grading of a mix of given materials and proportions. The test is of wide application in the Americas and is used as a basis for mix proportioning by the American Concrete Institute (4).

The compacting factor test, developed in Britain, is used there and in many parts of the British Commonwealth mainly as a laboratory tool. Although it is really a test of compactibility only, it provides a reasonably good measure of workability and, in combination with a rough range of slump values (13, 14), is the basis for most mix design methods used in the British Commonwealth.

## EFFECT OF GRADING ON WORKABILITY

Apart from hydrating the cement, water lubricates the cement and aggregates in a concrete mix. It is this lubrication that makes the mix workable. The lubricating water evaporates and causes voids when the concrete dries; these should be kept to a minimum. There is therefore an optimum water content that ensures both a workable mix and a minimum number and volume of voids.

Two main characteristics of the aggregates affect the water content of a mix: total surface area and particle interference. Both are functions of grading.

The greater the surface area of the mix constituents, the larger will be the amount of water required to lubricate the surfaces of the particles. A large surface area can be the result of a fine grading or the presence of a large proportion of sharp, angular particles or a combination of both. Obviously, the shape of the particles must influence the grading if a particular surface area is to be maintained, and one cannot escape the conclusion that aggregates having a wide variety of shapes and gradings can have the same total surface area and thus lead to the same water content and workability. It should be noted, however, that the surface area concept breaks down for very fine particles, which appear to have their own lubricating qualities and require less water to wet them. This has led Murdock (15) to devise the surface index method. The apparently inseparable relation between grading and particle shape has also led him to combine them in a formula for the compacting factor.

It is to be especially noted that the packing of the particles must be such as to make it possible for the cement paste to fill the voids in the fine aggregate and for the mortar to fill the voids in the coarse aggregate. This brings us to the second characteristic of the aggregates, particle interference. This occurs when the distance between the larger particles is not sufficient to allow free passage of the smaller ones. The lubricating effect is thus hindered, thereby reducing workability. Moreover, the voids ratio is increased, with consequent effects on strength. The addition of cement paste and/or fine sand forces the larger particles apart, thus increasing the lubricating effect, but this addition creates a need for more water to wet the larger surface area so created and causes a strength reduction if the cement content is not correspondingly increased. Particle interference usually occurs as a result of the fine aggregate containing an excess of larger sizes or the presence of a large proportion of sharp, angular particles in the coarse aggregate, or a combination of both. The situation is probably worst when a finely graded, angular coarse aggregate is combined with a coarsely graded fine aggregate.

It should be clear from the foregoing that the grading of the fine aggregate is far more critical in influencing workability than that of the coarse aggregate because of the larger surface area of the former. If the fine aggregate has an excess of finer particles, with correspondingly large surface area, low workability results. If, on the other hand, it has an excess of coarser particles, the tendency to higher workability caused by the smaller surface area may be offset by the occurrence of particle interference. The ratio of fine to coarse aggregate is therefore of major importance, and it seems that, for any given set of aggregates, there is an optimum combination that effects a balance between the opposing tendencies of surface area and particle interference (9, 16-20). It should also be pointed out that, for obvious reasons, the effects of surface area and particle interference and hence grading are much less in rich mixes than in lean ones. In the case of rich mixes, it is therefore possible to produce mixes with wide limits of grading and the same workability. Murdock's compacting factor formula (15) takes account of this fact.

As indicated previously, aggregate particles of a given size and shape pack in such a way that free passage between them of smaller ones can only take place if they are sufficiently small. This has led some people to advocate gap grading as the optimum solution to particle interference. It has also been stated (21) that, by using the largest size of coarse aggregate consistent with clearance in structural sections, the reduction of surface area so obtained will lead to higher workability. It has been shown, however, that gap-graded concretes in the more workable ranges are more prone to segregation than continuously graded ones (22), and gap grading is therefore recommended for concretes of low workability that are to be compacted by vibration. Gap-graded concretes also necessitate much closer control than continuously graded ones because they are much more sensitive to changes in water content. With proper control, however, gap-graded concretes have the advantage of requiring less water for a particular workability, thus having a lower water-cement ratio and hence higher strength for a particular cement content.

The use of the largest possible size of coarse aggregate has been mentioned as a way of reducing surface area and thus minimizing the water requirement and cement content. For instance, increasing the maximum aggregate size from 10 to 63 mm can, under certain conditions, reduce the water requirement for a constant consistency by as much as 50 $kg/m^3$ of concrete and the water-cement ratio by as much as 0.15 (23). Caution must, however, be exercized in increasing the size beyond a certain point because it has been shown that, above a maximum size of 40 mm, the strength gain due to the lower water-cement ratio is offset by the increased heterogeneity of the concrete and the discontinuities created by the presence of very large particles in the mortar matrix (24, 25). For many aggregates, the critical maximum size seems to be as small as 20 mm (26).

## WATER REQUIREMENT AND AGGREGATE PROPORTIONS

Tests and experience have shown that the amount of water per unit volume of concrete made with any given set of aggregates and required to have a particular workability is substantially constant regardless of the cement content or water-cement ratio. The different water requirements of various mixes can only be due therefore to the differences in those aggregate properties that influence workability. As indicated previously, the most important of these are grading and particle shape.

If the coarse aggregate is kept the same and if different fine aggregates are used for a series of mixes, it will be found that each mix has a different water requirement that may be called the "water requirement of the fine aggregate." Conversely, we can find the "water requirement of the coarse aggregate." It has been established that there is a much greater variation in the water requirement of the fine aggregate than in that of the coarse aggregate, a condition that can be inferred from the earlier discussion on the surface area of aggregates. As can also be inferred from the foregoing discussion, there is an optimum percentage of fine aggregate (17, 20) that, for any given fine and coarse aggregates used in combination and for a given degree of workability, will require the least amount of water and hence the least amount of cement for a given strength. Therefore, the selection of an overall grading of fine and coarse aggregate is basically the choice of an appropriate percentage of fine aggregate.

## MIX PROPORTIONING METHODS

As has been stated, the whole purpose of mix proportioning is to ensure that the properties of freshly mixed and hardened concrete are economically achieved. This reduces primarily to a choice of a suitable combination of materials that are readily available or within economic distance. It is clearly absurd, for instance, to attempt to adhere to particular aggregate gradings when it is neither possible nor economic to do so. Even when it is possible to do so, one may well find that the grading chosen does not give expected results because the aggregates may differ significantly in shape, texture, and specific gravity from those for which the particular gradings have been developed. The use of particular gradings and other proportions related to them can only serve as "a means of making an intelligent guess at a starting point for the first tests to be made" (27).

During the past 70 years, many methods of mix proportioning have been proposed, but most of these have not been adopted for general use. Among those that are commonly used or are receiving serious attention are those based on arbitrary selection, optimum aggregate content, and specified grading curves. A very useful method combining the concepts of optimum aggregate content and specified grading curves has been developed by Frost (19).

## Arbitrary Proportions

The method of arbitrary proportions, in which fixed quantities of fine and coarse aggregate are mixed regardless of size and grading, is highly unsatisfactory and should have been abandoned long ago. It is a matter for great astonishment that specification of concrete in such proportions is still widespread in civil engineering works today.

## Optimum Aggregate Content Methods

From the discussion on the existence of an optimum percentage of fine aggregate, it follows that there must also be an optimum percentage of coarse aggregate. This is the basis of the excellent method of proportioning recommended by the American Concrete Institute (4) and, with some slight variations, by the Portland Cement Association (26) and the South African Portland Cement Institute (20). It is also recommended as an alternative in Australia (28).

This method, which is very simple, consists of choosing the consistency of the concrete by selecting an appropriate slump range and the maximum size of coarse aggregate that is economically available and consistent with the dimensions of the structure and the limitations imposed by the heterogeneity of the concrete mentioned earlier. The water requirement is then estimated from published tables or charts or from experience with particular types of aggregates, and the cement content is calculated from the water-cement ratio appropriate to the strength and durability requirements. The optimum volume of coarse aggregate, on a dry-rodded basis and appropriate to the maximum size of aggregate and the fineness modulus of the fine aggregate, is then estimated from published tables. Finally, the content of fine aggregate is calculated by using either the estimated unit weight of concrete or the more accurate method of absolute volumes. Trial batches are made and the proportions suitably adjusted to achieve the required workability.

It is to be noted that the method makes use of the fineness modulus, which is open to certain criticisms. The fineness modulus is only an index of average particle size and hence of the fineness or coarseness of a particular fine aggregate. It gives no indication of particle size distribution, but Fulton (1) has overcome this by using the statistical approach of standard deviation from the fineness modulus as a measure of this distribution. His charts for estimating water requirement use both the fineness modulus and the standard deviation, and they are the basis for the method used for mix proportioning in South Africa. Popovics, in a spirited defense of the fineness modulus (2), proposes the use of the D-m-s method where D is the maximum particle size, m is the fineness modulus, and s is the specific surface. A dispersion index together with the fineness modulus is proposed by Lecompte (3), and Frost (19) has proposed a method combining the concepts of fineness modulus, particle shape, and specified grading curves. One certainty about the fineness modulus is that it makes possible the use of supposedly unorthodox fine aggregate gradings that have been shown to make strong and workable concrete.

In North America, the range of approved fineness moduli is 2.3 to 3.1, but it has been found necessary in other countries to increase the range so that as wide a variety as possible of fine aggregates can be used. The author has successfully used fine aggregates with moduli as low as 1.9.

The fineness modulus seems to be anathema in Britain where methods based on optimum aggregate content have been developed by Hughes (9, 16, 18, 29) and, in a somewhat different form, by Murdock (15). Both have preferred to use the characteristics of surface area, Hughes developing a grading modulus that he combines with the equivalent mean diameter of the fine aggregate and Murdock combining surface and angularity indexes. Their methods include formulas and charts based on the compacting factor concept of workability as related to the surface area characteristic. Their methods, although excellent contributions to the literature, lack the simplicity and practical nature of the American method based on the fineness modulus of the fine aggregate. It is to be noted that the use of the dry-rodded bulk density of the coarse aggregate in the American method automatically takes account of particle shape.

## Specified Grading Curve Methods

Methods using specified grading curves seem to be very popular in the British Commonwealth and some European countries. Perhaps the best known of these is the one given in Road Note No. 4 (13). It is based primarily on combining aggregates in such a way that the grading curve will conform with one of a group of curves that tests at the Road Research Laboratory (5) have shown will give good results.

The method consists of initially choosing an appropriate water-cement ratio and then a related aggregate-cement ratio that will give a particular workability based on the compacting factor. For the particular workability, there are four aggregate-cement ratios corresponding to four specified grading curves. Account is taken to some extent of particle shape by having separate tables of these relations for rounded, irregular, and angular aggregates of 20- and 40-mm maximum size. Similar relations have been published by the Cement and Concrete Association (14) for aggregates having a maximum size of 10 mm.

Although the use of this method results in the production of strong, workable concrete for gradings that are sufficiently close to those specified, it possesses a number of limitations. Some of these are described as follows:

1. The tables and grading curves apply to combinations of similar fine and coarse aggregate commonly available in some parts of Britain. Such aggregates are not necessarily available elsewhere, and it may be impossible to combine them in such a way as to conform with any of the specified grading curves. The combination of crushed stone and natural sand, which occurs in many countries, is not covered at all, and the curves take no account of the fact that many fine aggregates contain a significant fraction of particles passing the No. 100 sieve and that these have a significant influence on the water requirement. Moreover, gap gradings that can make excellent concrete seem to be completely overlooked.
2. It seems that use of the method leads to larger proportions of fine aggregate as this aggregate gets finer. The exact opposite should apply, as has been indicated previously. It also seems that the proportion of fine aggregate to be used is independent of the water-cement ratio, i.e., the wetness of the paste. Experience has shown, however, that the wetter the paste, the larger is the proportion of fine aggregate that can be used in the mix.
3. The water requirement is obtained only indirectly by this method, and it has often been found that significant adjustments to the water content are required to achieve satisfactory workability. The water content is also affected by the percentage of fine aggregate, which is sometimes markedly different from the optimum figure as determined by other methods.

## ECONOMIC MIX PROPORTIONS IN THE EASTERN CARIBBEAN

A description is now given of the determination of economic mix proportions of concrete undertaken by the writer, using aggregates available on the three islands of Barbados, St. Lucia, and Trinidad in the eastern Caribbean. These proportions are based on small-scale laboratory tests and full-scale field trials on concretes with a slump range of 35 to 70 mm and a wide range of water-cement ratios. The slump range chosen represents concrete of medium consistency because this is the consistency required for most of the concrete used on the islands. It should be noted here that, because of the accelerated stiffening of fresh concrete caused by the warm climate and drying winds of the Caribbean (30), the slump range for similar concrete would be about 50 to 90 mm in more temperate climates.

Barbados is an island consisting almost entirely of coral limestone. This rock is the only available source of coarse aggregate on Barbados. The only available fine aggregate except for beach sand, whose use is prohibited, is an inland marine deposit of very fine sand that is largely retained between the No. 25 and No. 52 British Standard sieves. This is clearly shown in Table 1 and Figure 1 for the type A aggregate grading. The sieve sizes shown are based on the British Ready Mixed Concrete Association metric description (31).

Such fine aggregate is not normally considered suitable for strong, workable concrete and, combined with the type A 20-mm coarse aggregate, produces a gap-graded concrete requiring a high degree of control. Fortunately, the type A 10-mm aggregate contains a significant fraction of smaller sizes (Table 1), and a combination of 1:1:1 of the three aggregates produces the grading shown as type A in Table 2 and Figure 2. This grading, based on a judicious assessment of curve No. 4 in Road Note No. 4, has produced a good workable concrete of suitable strength with economic cement content (Table 3 and Fig. 3). The density of the concrete, which is somewhat lower than that of normal-weight concrete and is the result of the specific gravity of the aggregates, appears to have no appreciable effect on strength. The specific gravities are as follows:

| Aggregate | Type A | B | C | D |
|---|---|---|---|---|
| Coarse | 2.40 | 2.28 | 2.62 | 2.60 |
| Fine | 2.45 | 2.42 | 2.56 | 2.57 |

It is of particular interest to note that concrete in the higher strength range was successfully used for the post-tensioned ring of the satellite station tower in Barbados and for parking aprons and runways for heavy jet aircraft.

St. Lucia is an island of volcanic origin with long stretches of beautiful beach, which has been the traditional source of fine aggregate for concrete. The boom in construction caused by the rapid development of the tourist industry has led to excessive use of beach sand. This is now threatening the tourist industry itself, and efforts have been made to find alternative sources of fine aggregate. The island has large deposits of pyroclastic material that is called pumice by local geologists. Its specific gravity is, however, much higher than that of normal pumice, and it has satisfactory grading for concrete production (type B aggregate, Table 1 and Fig. 1). The one possible limitation to its use is the rather high fraction passing the No. 100 sieve.

Tests have been made recently on concrete manufactured with this pumice in combination with a sharp, angular, volcanic coarse aggregate of rather low specific gravity (2.28), the only coarse aggregate readily available. A balance had to be struck between the high water requirement necessitated by the grading of the fine aggregate and the gap grading that would be caused by using a relatively small percentage of this aggregate with the available coarse aggregate, the gradation of which is given under type B in Table 1. Gap gradings are not recommended in St. Lucia because the degree of control is suspect. Moreover, the writer's tests have shown a tendency to segregation for such gradings. The combined grading is given as type B in Table 2 and Figure 2 and the mix proportions in Table 3. It is to be noted that the fineness modulus of the St. Lucian fine aggregate is not significantly different from that of the Barbadian one but that the greater standard deviation indicates a need for a somewhat higher water content. The strengths are shown in Figure 3 and are very satisfactory despite the somewhat low density of the concrete resulting from the low specific gravity of the aggregates.

**Table 1. Gradings of coarse and fine aggregates.**

| British Sieve Size | Percent Passing | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 20-mm Aggregate | | | | 10-mm Aggregate | | | Fine Aggregate | | | |
| | A | B | C | D | A | B | C | A | B | C | D |
| 20 mm | 97 | 100 | 98 | 87 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |
| 10 mm | 12 | 8 | 26 | 16 | 87 | 99 | 100 | 100 | 100 | 100 | 100 |
| 5 mm | 2 | 0 | 2 | 0 | 40 | 15 | 55 | 100 | 100 | 98 | 94 |
| No. 7 | 1 | 0 | 1 | 0 | 23 | 0 | 7 | 100 | 99 | 85 | 71 |
| No. 14 | 0 | 0 | 0 | 0 | 14 | 0 | 3 | 100 | 92 | 63 | 52 |
| No. 25 | 0 | 0 | 0 | 0 | 8 | 2 | 2 | 88 | 68 | 42 | 34 |
| No. 52 | 0 | 0 | 0 | 0 | 5 | 0 | 1 | 14 | 35 | 25 | 17 |
| No. 100 | 0 | 0 | 0 | 0 | 5 | 0 | 0 | 0 | 15 | 11 | 6 |
| Fineness modulus | | | | | | | | 1.98 | 1.91 | 2.76 | 3.26 |
| Standard deviation | | | | | | | | 0.51 | 1.18 | 2.61 | 1.65 |

**Figure 1. Grading curves for fine aggregate.**

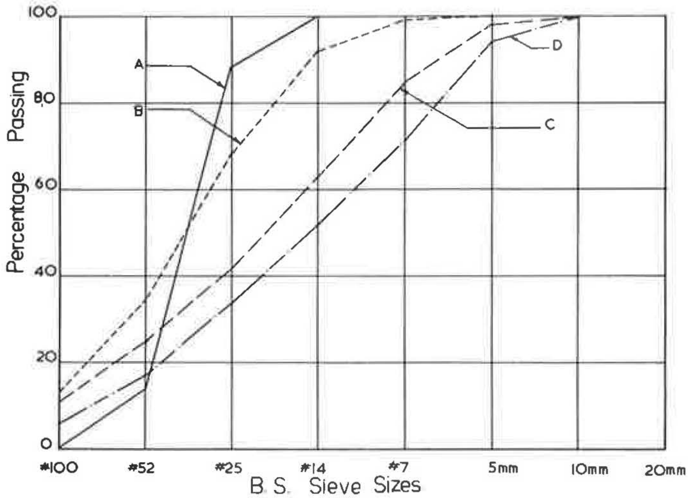

**Figure 2. Grading curves for combined aggregate.**

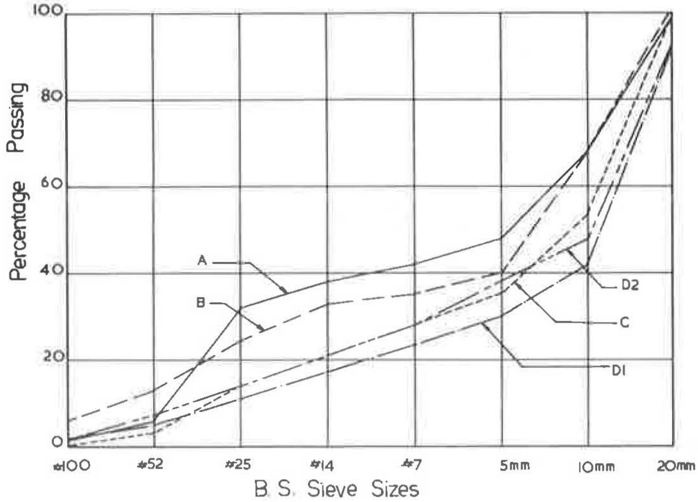


**Figure 3. Relation between strength and water-cement ratio.**

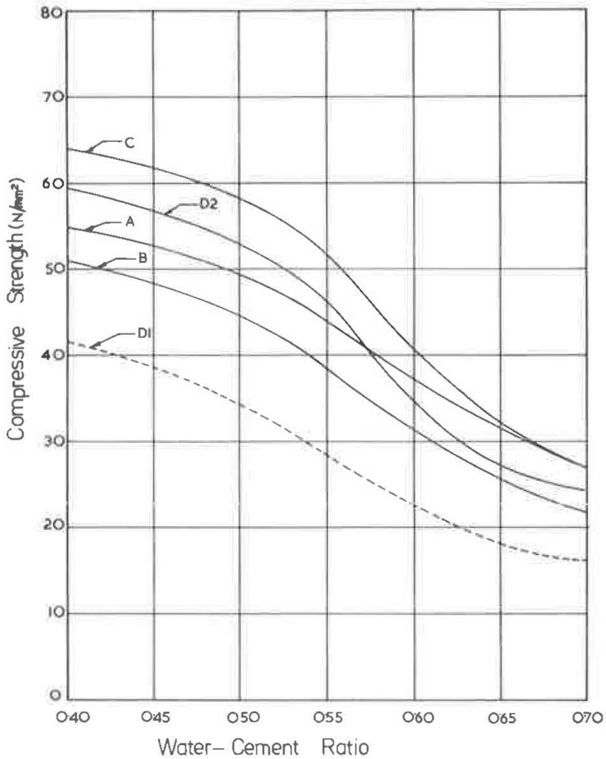

**Table 2. Combined aggregate gradings.**

| British Sieve Size | Percent Passing: A (33 percent fine) | B (35 percent fine) | C (32 percent fine) | D1 (32 percent fine) | D2 (40 percent fine) |
|---|---|---|---|---|---|
| 20 mm | 99 | 100 | 98 | 91 | 92 |
| 10 mm | 68 | 68 | 53 | 42 | 48 |
| 5 mm | 48 | 40 | 35 | 30 | 38 |
| No. 7 | 42 | 35 | 28 | 23 | 28 |
| No. 14 | 38 | 33 | 21 | 17 | 21 |
| No. 25 | 32 | 24 | 14 | 11 | 14 |
| No. 52 | 6 | 13 | 3 | 5 | 7 |
| No. 100 | 1 | 6 | 0 | 2 | 2 |

**Table 3. Mix proportions.**

| Water Cement Ratio | Aggregate-Cement Ratio | Material Weights (kg/m³): Water | Cement | Fine | 10-mm Aggregate | 20-mm Aggregate | Density (kg/m³) | Field Slump (mm) |
|---|---|---|---|---|---|---|---|---|
| Barbados Aggregate | | | | | | | | |
| 0.40 | 3.0 | 203 | 507 | 525 | 525 | 525 | 2,285 | 50 |
| 0.45 | 3.5 | 203 | 454 | 530 | 530 | 530 | 2,247 | 50 |
| 0.50 | 4.0 | 203 | 407 | 540 | 540 | 540 | 2,230 | 60 |
| 0.55 | 4.5 | 203 | 372 | 550 | 550 | 550 | 2,225 | 60 |
| 0.60 | 5.0 | 203 | 337 | 560 | 560 | 560 | 2,220 | 50 |
| 0.65 | 5.4 | 203 | 312 | 567 | 567 | 567 | 2,216 | 60 |
| 0.70 | 5.9 | 203 | 293 | 572 | 572 | 572 | 2,212 | 60 |
| St. Lucia Aggregate | | | | | | | | |
| 0.40 | 2.9 | 207 | 509 | 517 | 436 | 524 | 2,189 | 30 |
| 0.45 | 3.3 | 207 | 459 | 530 | 448 | 536 | 2,180 | 35 |
| 0.50 | 3.6 | 207 | 413 | 540 | 456 | 546 | 2,162 | 35 |
| 0.55 | 4.2 | 207 | 374 | 549 | 463 | 553 | 2,146 | 45 |
| 0.60 | 4.6 | 207 | 345 | 554 | 469 | 560 | 2,135 | 50 |
| 0.65 | 5.1 | 207 | 319 | 560 | 437 | 566 | 2,125 | 50 |
| 0.70 | 5.5 | 207 | 295 | 566 | 477 | 572 | 2,117 | 55 |
| Trinidad Melajo Aggregate | | | | | | | | |
| 0.40 | 3.7 | 189 | 474 | 553 | 88 | 1,090 | 2,394 | 40 |
| 0.45 | 4.3 | 189 | 419 | 568 | 88 | 1,121 | 2,385 | 50 |
| 0.50 | 4.8 | 189 | 377 | 579 | 91 | 1,141 | 2,377 | 40 |
| 0.55 | 5.4 | 189 | 342 | 590 | 91 | 1,158 | 2,370 | 50 |
| 0.60 | 5.9 | 189 | 314 | 596 | 94 | 1,172 | 2,365 | 50 |
| 0.65 | 6.5 | 189 | 289 | 602 | 95 | 1,186 | 2,361 | 60 |
| 0.70 | 7.1 | 189 | 268 | 607 | 95 | 1,197 | 2,356 | 65 |
| Trinidad Guanapo Aggregate | | | | | | | | |
| 0.40 | 4.0 | 177 | 443 | 709 | – | 1,063 | 2,392 | 40 |
| 0.45 | 4.6 | 177 | 393 | 723 | – | 1,086 | 2,379 | 45 |
| 0.50 | 5.1 | 177 | 354 | 735 | – | 1,100 | 2,366 | 40 |
| 0.55 | 5.8 | 177 | 321 | 744 | – | 1,112 | 2,354 | 50 |
| 0.60 | 6.3 | 177 | 295 | 749 | – | 1,121 | 2,342 | 50 |
| 0.65 | 6.9 | 177 | 272 | 751 | – | 1,130 | 2,330 | 50 |
| 0.70 | 7.4 | 177 | 254 | 755 | – | 1,132 | 2,318 | 55 |

In Trinidad, there are two main sources of aggregate. One is Melajo limestone, designated as type C in the tables and figures, the fine aggregate of which is a naturally occurring sand and the coarse aggregate an irregularly shaped crushed stone. The aggregates can be easily combined in a grading that approximates curve No. 2 in Road Note No. 4 and have specific gravities of the values that produce normal-weight concrete. The fine aggregate falls within the British Zone 2 range (Table 1 and Fig. 1) and thus requires less water than the Barbadian one, despite the percentages being almost exactly the same. The mix proportions are given in Table 3 and the strengths shown in Figure 3 where it can be seen that an economical cement content produces quite high strengths. It is of interest to note that the information shown in the tables and figures was used to design a mix of somewhat different proportions, with a maximum aggregate size of 10 mm and a slump of 100 mm, for the post-tensioned ring of the satellite station tower in Trinidad. The concrete had a strength of 70 $N/mm^2$.

The other main source of aggregate in Trinidad is Guanapo limestone, a river deposit having a specific gravity almost identical to that of the Melajo. The fine aggregate is significantly coarser than that of the Melajo, indicating a lower water requirement for the same percentage. When the specified-grading-curve method was used, however, and a grading that approximates curve No. 1 of Road Note No. 4 obtained (aggregate type D1, Table 2), it was discovered that the water content was much higher than expected (216 $kg/m^3$). The strengths were also much lower than expected for the various water-cement ratios (D1 curve, Fig. 3). This was a clear example of the specified-grading-curve method being unsuitable for particular mix designs. Another approach was then used, employing the method of estimating the water requirement of the fine aggregate. Not only were the water requirement and cement content significantly less, but the strengths were considerably higher as can be seen from the D2 curve shown in Figure 3. It should be noted that the fine aggregate content was 40 percent, a figure to be expected from its coarse grading. It is also significant that, with this percentage, the combined grading was similar in many respects to that of the Melajo aggregate.

## CONCLUSION

Grading and its effect on workability, water requirement of mixes, and mix proportions have been discussed. Emphasis has been placed on the nature, definition, and measurement of workability, the single most important property of concrete in its fresh state. Mix proportioning methods have been discussed and a description given of the determination of economic mix proportions in the eastern Caribbean. It has been shown that satisfactory concrete can be made with a wide variety of aggregates so long as suitable mix proportions are selected. The writer hopes that this paper will widen some horizons and indicate that there is still considerable research to be done on the grading of aggregates and the properties that it influences.

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# AGGREGATE GRADING AND THE INTERNAL STRUCTURE OF CONCRETE

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It is shown that there are many gradings for concrete aggregate that can be considered as optimum for one purpose or another. In addition to the behavior of the concrete in the fresh as well as in the hardened state, visual inspection of the internal structure of concrete reveals whether the grading used is close enough to the desired optimum. A method, adopted from a procedure used in petrography, is presented for such evaluation of the structure of concrete. This is based on the measurement of the intercepts of mortar layers among coarse aggregate particles in the finished concrete. Various kinds of concretes are analyzed by this method. The results indicate that there exists a needed minimum value for the average mortar layer intercept below which the workability of the concrete is inadequate; this needed minimum mortar intercept is dependent on several factors such as the method of compaction and the grading of the sand; and the minimum value seems to be independent of the particle shape of the coarse aggregate. The effect of the particle shape of coarse aggregate on the grading optimum is also discussed. Based on this, two methods for measuring the angularity of the particles are recommended. Finally, internal structures of concretes made with continuous gradings and comparable gap gradings are contrasted.

•THE term optimum grading is used in this paper to represent a grading that can provide the optimum of a given concrete property under a set of circumstances, including specified, reasonable amounts of cement and water. Economical considerations will be excluded because they vary drastically from location to location. Neither is there space in this paper to discuss the various methods for checking whether a grading is optimum without trial mixtures (ideal sieve curve, optimum fineness modulus, maximum denseness, etc.); these subjects have been treated thoroughly in a recent book by Powers (1). Rather, the internal structure of the hardened concrete will be discussed here in connection with the grading of the aggregate used. Nevertheless, two comments seem appropriate to resolve certain misconceptions about optimum gradings:

1. It is unrealistic to expect that there is any single grading that can optimize all or most concrete properties simultaneously. Although it is a common feature of all optimum gradings that they provide good workability under the given circumstances, there are numerous optimum gradings. For one thing, gradings that are optimum for one concrete property, say, the strength of concrete, may not be, and usually are not, optimum for another concrete property, such as impermeability. Second, the optimum of a grading for, say, the concrete strength is also influenced by the method of consolidation, the type of aggregate, and the amount of cement to be used. Third, different particle-size distributions, under otherwise identical conditions, can have practically identical concrete-making properties; i.e., they can produce the same optimum of the concrete property in question.

2. There is no single grading for fine aggregate that can provide an optimum combined grading, in any sense of the term optimum, with every coarse aggregate. Fine aggregates complying with accepted grading specifications, such as ASTM Designation C 33, can make good concretes with conventional coarse aggregates. However, combined with unusual coarse aggregates, in gap gradings for instance, the same fine aggre-

gates can be less than optimum. The effects of using a finer sand on the properties of concrete can usually be compensated for by using a smaller proportion of it (1, 2). In other words, the optimum grading, as well as the optimum quantity of a sand in concrete aggregate, depends on the grading, the type of coarse aggregate used, and other factors. The same is true in a reverse sense for the optimum grading of the coarse aggregate.

The amount of a given fine aggregate and the amount of a given coarse aggregate are well balanced in an optimum grading, but the nature of this balance is dependent on numerous factors. In the remainder of the paper, an attempt will be made to give a sense of this balance to the reader using photographs of well-graded concrete aggregates and internal structures of concretes made with such aggregates. The discussion is restricted to the most common case when the concrete strength is to be optimized.

## RELATION BETWEEN AGGREGATE GRADING AND INTERNAL STRUCTURE OF THE CONCRETE

A simple petrographic method is also recommended in the paper for the numerical evaluation of the macrostructure of hardened concrete. Although there is a considerable amount of literature on the petrographic examination of concrete, especially on the microscopic determination of its air void content, no published systematic treatment has been found concerning the relation between the macrostructure of hardened concrete and the grading of the aggregate used. Thus, it is hoped that this paper, as a first step, will induce further research in this neglected area.

Figure 1 shows the internal structure of an air-entrained gravel concrete through a cut surface. The continuous grading used is shown as curve 1 in Figure 2. The fineness modulus of this grading is 5.7. The cement content of the concrete is 570 lb/cu yd (340 kg/m$^3$) of portland cement and 170 lb/cu yd (100 kg/m$^3$) of fly ash. The water-cement ratio is 0.50 by weight, and the air content is about 3 percent. The unit weight of the fresh concrete was 146 lb/cu ft (2,340 kg/m$^3$), and the slump was about 1 in. (2.5 cm). This concrete had a compressive strength of 5,960 psi (420 kg/cm$^2$) and a flexural strength of 525 psi (37 kg/cm$^2$) at the age of 28 days.

The high strength values indicate that this concrete, and thus the grading, too, is good. It is a good grading but not quite optimum. The behavior of this fresh concrete in the laboratory, especially during the compaction by rodding performed according to ASTM Designation C 192-68, indicated that there was a slight excess in the amount of mortar. This observation is supplemented by the fact that the optimum fineness modulus of the aggregate recommended for the maximum strength of the particular mixture is approximately 6.2 (3, 4).

Figure 1 shows that the concrete aggregate used is well graded: The matrix is a dense mortar in which a number of coarse particles are embedded in a random manner, although perhaps a few more gravel pieces could have been placed in it without overcrowding the internal structure. This evaluation is of course highly subjective and only qualitative. An attempt to make this approach numerical is presented as follows.

## MORTAR-INTERCEPT METHOD

For a simple numerical analysis of the internal macrostructure of hardened concrete, the intercepts of mortar layers between coarse aggregate particles in a cut, plain concrete surface can be utilized. The coarser the overall grading, the smaller these intercepts become along with the actual thicknesses of the coatings of mortar surrounding the coarse aggregate particles. The mortar intercepts can be measured with the linear traverse method, which is similar to the procedure used in petrography or as described in ASTM Designation C 457 for the air content determination in the hardened concrete, except that there is no need for a microscope in this case. That is, a random cut is taken through the concrete, a regular grid is randomly placed thereon, and linear intercepts are measured along the lines of the grid with the lengths as intercepts in mortar among coarse aggregate particles. These measured intercepts can be summed and averaged, the result of which is a number called "average mortar intercept."

Some difficulty has been encountered during the preliminary intercept measurements in distinguishing between fine and coarse aggregate particles. As a result, it was decided that any aggregate particle having a cut surface larger than $\frac{5}{32}$ in. (4.0 mm) in diameter be accepted as coarse aggregate because the probability is extremely low that all the particles in any given cut will be intersected in the equatorial plane and that all these will have the grid line crossing the particle at the diameter. It also follows that the average mortar intercept is always greater than the average mortar layer between the closest points of two neighboring coarse aggregate particles in the finished concrete.

The linear traverse method can provide the proportion of the cement paste or that of the mortar in the concrete with a fair accuracy (5, 6). It is also conceivable to obtain the equivalent of an aggregate sieve analysis or the specific surface of the aggregate from linear traverse measurements. But the mathematical difficulties are formidable even for a spherical aggregate. If the aggregate is assumed cuboidal, the situation becomes more complex because lines passing through a cube can be longer or shorter than the cube side. Irregular shapes or mixed shapes present even more difficult problems; thus, the reliability of such grading analysis is highly questionable (7). An advantage of the mortar-intercept method is that it is free from these difficulties.

## APPLICATION OF THE MORTAR-INTERCEPT METHOD

The method of intercept, as described previously, was applied to the concrete section shown in Figure 1. The total length of the measured grid lines was about 25 in. The frequency distribution of these intercepts is given in Table 1. As can be seen, the average mortar intercept in this concrete is 0.148 in. (3.75 mm). Because there was a slight excess of mortar in this concrete, one could estimate that, under the given circumstances (that is, traditional gradings of the fine and coarse aggregate for a smooth and continuous combined particle-size distribution, medium cement content, etc.), an average mortar intercept of 0.14 in. (3.5 mm) still would have provided a workability that is suitable for compaction by standard hand-rodding. This would have been the grading that is usually considered as an optimum for the strength of this concrete. A series of similar analyses (8-11) of internal structures performed on photographs of various but comparable concretes resulted in the same needed minimum average for mortar intercepts. It is important also that this 0.14-in. value seems to be the needed minimum in these concretes, not only for gravel particles but for crushed coarse particles as well. That is, this criterion of workability seems independent of the particle shape. Thus, one can define the coarsest permissible gradings for the preceding concretes as those that provide a dense mortar and an average mortar layer intercept of 0.14 in. (3.5 mm) in the finished concrete. Any amount of mortar less than this minimum in a continuously graded concrete would not provide enough lubrication for the coarse aggregate particles for a workability that is adequate for hand compaction. This does not mean, however, that smaller mortar layer intercepts result always in an inadequate workability. For one thing, a reduction in workability caused by a moderate reduction in the mortar quantity can be overcome by intensive mechanical compaction, such as vibration. This is in accordance with the experience that coarser gradings can be used in the concrete when it is compacted by vibration (1). Second, less mortar can also do the job when its lubricating ability is improved. This can be done to a certain extent either by increasing the cement content or by using a finer sand.

An illustration of this latter statement is shown in Figure 3. This shows a cut surface of an air-entrained concrete made with beach sand and crushed reef shell as a coarse aggregate (11). The aggregate grading used is shown in Figure 2 as curve 2. The maximum particle size in the sand is $\frac{3}{128}$ in. (0.6 mm), and its fineness modulus is approximately 1.3. The fineness modulus of the complete grading is 4.95. The cement content of the concrete is 570 lb/cu yd (340 kg/m$^3$) of portland cement and 170 lb/cu yd (100 kg/m$^3$) of fly ash. The water-cement ratio is 0.69 by weight, and the air content is about 3 percent. The unit weight of the fresh concrete was 140 lb/cu ft (2,240 kg/m$^3$), and the slump was about 3.5 in. (9 cm). This concrete had a compressive strength of 2,830 psi (200 kg/cm$^2$), a flexural strength of 590 psi (42 kg/cm$^2$), and a splitting strength of 325 psi (23 kg/cm$^2$) at the age of 28 days.

A series of trial mixtures indicated that the use of 62 percent crushed reef shell as coarse aggregate is the maximum that still provides a reasonable workability with beach sand. Figure 3 indicates that attempts to insert more shell pieces would overcrowd the internal structure, causing interference among the particles.

The results of mortar-intercept measurements for this concrete are given in Table 1. The average intercept thickness among the shell particles is 0.120 in. (3.04 mm). When the beach sand in this concrete was substituted by a traditional, coarser sand in the same quantity, the workability of the concrete became poor. Thus, the reducibility of the permissible minimum average layer intercept of mortar from 0.14 in. (3.5 mm) in the previous concrete to 0.12 in. (3.0 mm) in this concrete is attributed to the fine grading of the beach sand used.

## EFFECT OF PARTICLE SHAPE ON THE STRUCTURE OF CONCRETE

For the illustration of the effect of particle shape of coarse aggregate on the internal structure of concrete, crushed stone and gravel concretes were made with compositions similar to the composition of the reef shell concrete shown in Figure 3. The macrostructures of these concretes are shown in Figure 4. It can be easily seen that, despite the application of an identical grading (curve 2, Fig. 2), the distances between the coarse aggregate particles, particularly in the gravel concrete, are, by and large, greater than in the reef shell concrete. Mortar-intercept measurements given in Table 1 confirm this judgment numerically for these concretes. The average intercept of mortar layers in the crushed stone concrete shown in Figure 4 is 0.148 in. (3.75 mm), and that in the gravel concrete is 0.159 in. (403 mm), as compared to the 0.120-in. (3.04 mm) value in the reef shell concrete shown in Figure 3.

The behavior of these two concretes in the fresh state indicated also that there was an excess amount of mortar present, particularly in the gravel concrete.

Thus, the well-known rule-of-thumb, the more unfavorable is the particle shape of a concrete aggregate the finer the grading required for an adequate workability, can be explained using these photographs. Therefore, fewer coarse aggregate particles can be packed without interference into concrete when the particle shape is unfavorable, elongated for instance, than when the shape is spherical.

Incidentally, the reverse of this sequence of thoughts helps us select two promising approaches from the many available methods (12) for the numerical evaluation of particle shape. First, one can utilize the ratio of the actual specific surface of the aggregate particles in question to a hypothetical (minimum) specific surface, which is calculated using the premise that each particle in the sample keeps its volume but changes its shape into a sphere. The closer this surface ratio is to unity, the better the particle shape is for the workability of concrete. The second approach is the measurement of the denseness of packing of a one-sized fraction of the given aggregate under well-defined conditions. The denser the packing is, the more favorable the particle shape is. This latter principle was utilized in a test method developed by Shergold (13) for the measurement of the so-called angularity number of coarse aggregate particles. This angularity number (AN) is defined as

$$AN = \text{the percentage of voids among coarse aggregate particles} - 33$$

when the aggregate sample of a narrow size range is compacted in a prescribed manner in a specific container. The approximate angularity numbers for the reef shell, crushed stone, and gravel shown in Figures 3 and 4 are 30, 10, and 5 respectively. A test method on the same principle has been incorporated in the British standards (14).

It does not seem too difficult to establish the needed minimum amount of mortar as a function of the shape of the coarse aggregate particle when this shape is characterized by a properly selected number, such as the ones mentioned previously.

## SUPPORTING EVIDENCE

The findings concerning the effects of the sand grading and the particle shape of the coarse aggregate on the grading optimum were obtained by the analysis of the internal

Figure 1. Concrete made with traditional sand and gravel according to curve 1 shown in Figure 2 (6).

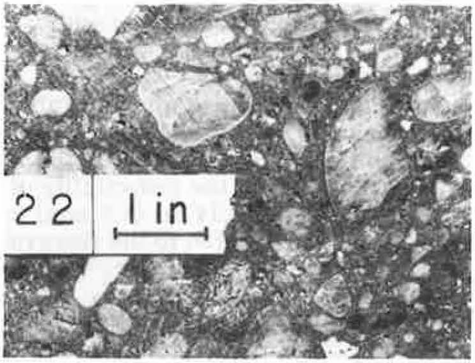


Figure 3. Concrete made with beach sand and crushed reef shell according to curve 2 shown in Figure 2 (7).

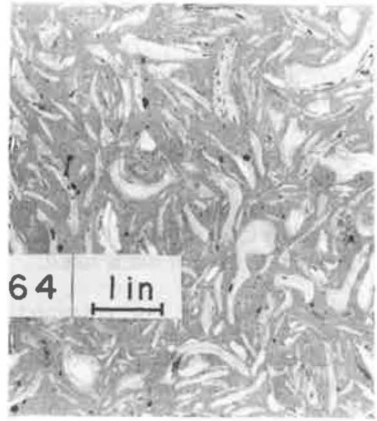


Figure 2. Gradings used in test concrete.

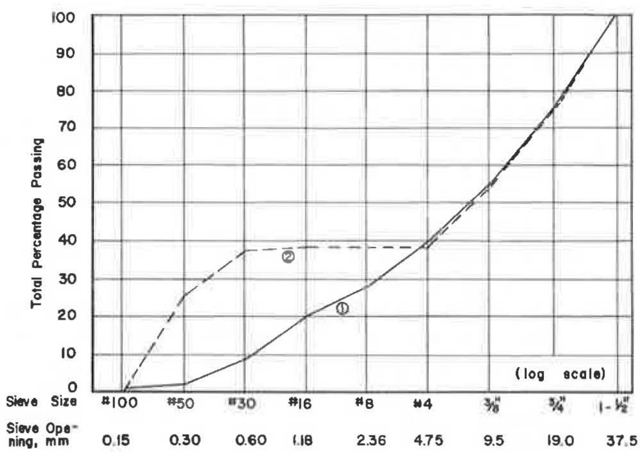


Figure 4. Crushed stone and gravel concretes.

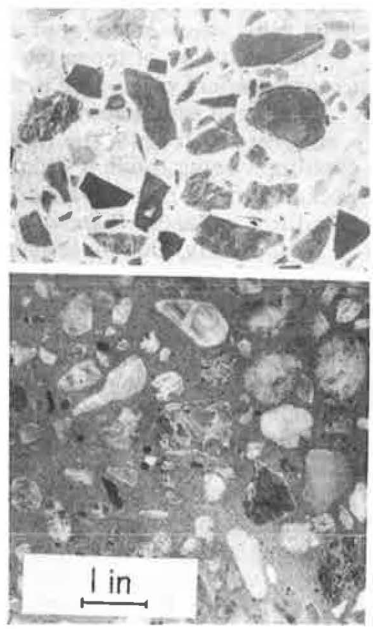


Table 1. Frequency distributions of intercepts of mortar layers.

| Concrete Presented In | Aggregate | | Intercept of the Mortar Layers Among Coarse Particles (in.) | | | | | | Total Number of Intercepts | Average Mortar Intercept (in.) |
|---|---|---|---|---|---|---|---|---|---|---|
| | Grading | Type | 0 to 0.1 | 0.1 to 0.2 | 0.2 to 0.3 | 0.3 to 0.4 | 0.4 to 0.5 | 0.5 to 0.6 | | |
| Figure 1 | Curve 1, Figure 2 | Sand and gravel | 20 | 20 | 9 | 3 | - | 1 | 53 | 0.148 |
| Figure 3 | Curve 2, Figure 2 | Beach sand and crushed shell | 37 | 37 | 8 | 2 | - | - | 84 | 0.120 |
| Figure 4 (upper) | Curve 2, Figure 2 | Beach sand and crushed stone | 20 | 30 | 8 | 4 | 1 | - | 63 | 0.148 |
| Figure 4 (lower) | Curve 2, Figure 2 | Beach sand and gravel | 17 | 22 | 11 | 4 | 1 | - | 55 | 0.159 |

structure of the hardened concrete. Because this writer has been unable to find other numerical analyses published concerning the relation between the grading and the internal structure of concrete, a direct comparison with results by other authorities was impossible. It is still encouraging, however, that published data on optimum gradings based on differing approaches are in line with the findings presented in this paper. Reference is made here to (a) the suggested modification for various particle shapes of the optimum values of the fineness modulus that are valid for rounded gravel aggregate (15); (b) the recommended $b/b_0$ values, as a function of the sand grading, for the needed amount of coarse aggregate in proportioning concrete (16); and (c) the determination of the optimum amount of coarse aggregate in concrete as recommended by Hughes (17).

## CONTINUOUS GRADING VERSUS GAP GRADING

The internal structure of concrete depends not only on the coarseness of the grading and on the particle shape of the aggregate but also on the details of the employed particle-size distribution. This is shown in Figure 5, which is a portion of a larger investigation (18). Here, three non-air-entrained concretes of differing but identically coarse gradings and of otherwise identical compositions are presented. The cement contents are 520 lb/cu yd (310 kg/m$^3$), and the water-cement ratios are 0.62 by weight. Concrete 10 was made with a continuous grading ($C_1$), concrete 15 with a one-gap grading ($O_1$), and concrete 18 with a two-gap grading ($T_1$). Details of the particle-size distributions are shown in Figure 6. These three gradings were set up so that the three important characteristics of coarseness, namely, the maximum particle size, the fineness modulus, and the calculated specific surface of the aggregate, were kept practically constant despite the obvious differences in the particle-size distributions.

The differences in the internal structures shown in Figure 5 are quite obvious. Especially the concrete made with the grading of one large gap ($O_1$) appears different; the grading seems coarser (which is actually not true), and excessive mortar seems to be present (which is true). This latter fact again supports the statement that a thinner average mortar layer is acceptable when a finer sand is used. Mortar intercepts were not measured on these three concretes because the available cut surfaces were too small to make such meaningful measurements. For the sake of comparison, the fractured surfaces of these concretes are shown in Figure 7, as obtained by the compression test of 3- by 6-in. (7.5- by 15-cm) cylinders. The three graded aggregates are shown in Figure 8.

A general remark seems appropriate for closing. The photographs of gap-graded and continuously graded concretes presented in this paper, and elsewhere in the literature, prove that the internal structure of concrete does not resemble at all the picture that is proposed by advocates of the maximum denseness principle, i.e., a structure that consists of symmetrically arranged, contacting circles of identical d diameter, representing the coarse aggregate particles, where the remaining holes are filled first with circles of identical 0.15-d diameter and then with subsequently smaller circles. In actuality, the coarse aggregate particles are distributed in a random manner in the mortar. This is true even in the case of concretes where the aggregate was carefully blended to obtain gradings of maximum denseness (19). Thus, it is little wonder that none of the "ideal gradings" derived mathematically from this unrealistic picture of maximum denseness in the aggregates has been proved optimum from the standpoint of concrete technology.

## CONCLUSIONS

The various optimum gradings have the common feature that they provide a dense mortar in which coarse aggregate particles are embedded in appropriate quantity. The coarser the grading, the closer the coarse aggregate particles are packed in the concrete. This evaluation of the aggregate grading from the structure of concrete can be made numerical by measuring the intercepts of mortar layers among coarse aggregate particles in a cut surface of the finished concrete. The results of such analyses indicate that, for the strength of traditional concretes (that is, usual gradings of the fine and

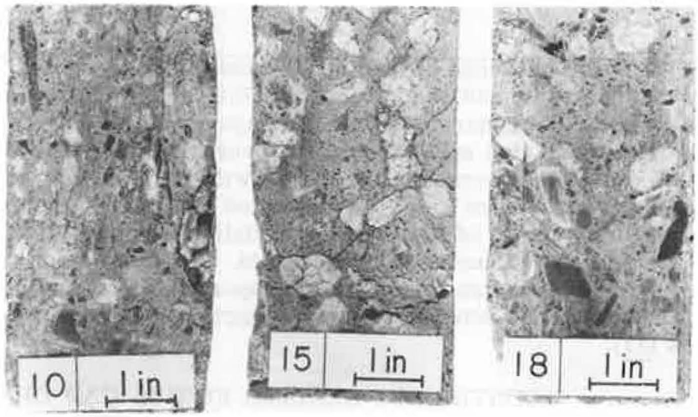


Figure 5. Concretes made with gradings of identical coarseness according to curves shown in Figure 6 (11).

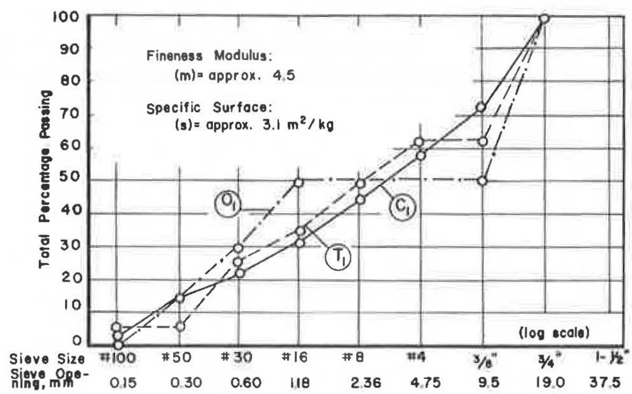


Figure 6. Gradings having the same maximum size, fineness modulus, and specific surface values (11).

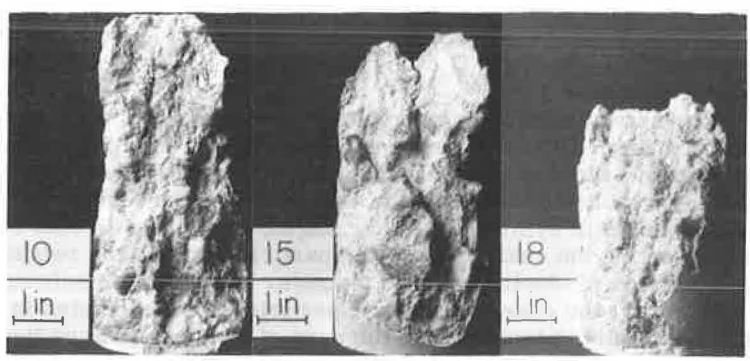


Figure 7. Fractured surfaces of the concretes shown in Figure 5 (11).

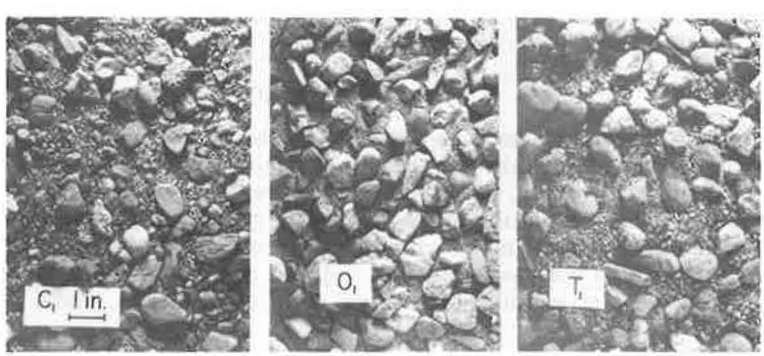


Figure 8. Appearance of continuous gradings shown in Figures 5 through 7.

coarse aggregates for a smooth and continuous combined particle-size distribution, medium cement content, etc.), the coarsest permissible gradings for compaction by hand-rodding are those that provide an average mortar layer intercept of 0.14 in. (3.5 mm). Under other circumstances, however, this mortar quantity can be reduced. For instance, when a very fine sand was used in the concrete, 0.12-in. (3.0-mm) average mortar layer intercept still provided a reasonable workability.

The needed minimum average intercept of mortar layers does not seem to be affected by the shape of the coarse aggregate particles. The primary effect of the particle shape on the grading optimum is that fewer coarse aggregate particles can be packed without interference in concrete when the particle shape is unfavorable than when it is spherical. This observation supports the test method developed by Shergold for the measurement of the angularity number of coarse aggregates.

These considerations seem valid both for continuous and for gap gradings even though the usual lack of the middle-sized particles in a gap grading may change the appearance of the internal structure of concrete; the gap grading may seem coarser than it actually is, and excessive mortar may be present in the concrete.

## ACKNOWLEDGMENT

The contributions by Bryant Mather, especially for his definition of the "average mortar intercept," are gratefully acknowledged. The photographs presented are by courtesy of Gilbert Pogany.

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## DISCUSSION

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The interesting photograph (Fig. 3) of a slice through a sample of concrete using reef shell as coarse aggregate reveals a structure that is unfamiliar to many students of concrete. A study of this photograph suggests quite strongly that the structure represents not only one in which the coarse aggregate particles are flat and elongated but also one in which there is a quite pronounced preferred orientation of these particles, specifically an orientation in which they have their minimum dimension generally in the plane of the slice. For many purposes, when dealing with the structure of a substance consisting of non-equidimensional particles in a matrix, when the question of preferred orientation exists, it is useful to examine slices cut in three mutually perpendicular planes or, at least, in planes both parallel and perpendicular to the direction of the influence that may have been at work to induce a preferred orientation. This discussion invites the author to comment as to whether he has prepared or can prepare slices of this concrete in a plane or planes perpendicular to the one illustrated and to comment on what they do or might reveal as it relates to the matters discussed in the paper.

## AUTHOR'S CLOSURE

The experiments presented in the paper were conducted years ago, so there is no possibility of producing the cuts recommended by Mather, no matter how instructional they would be. Still, it is useful that he points out the importance of the preferred orientation of aggregate particles because this is one of the primary sources of the anistropy of concrete. Although it is a practically important property, many of us "students of concrete" are more than willing to neglect it.

# METHODS FOR THE DETERMINATION OF REQUIRED BLENDING PROPORTIONS FOR AGGREGATES

Sandor Popovics, Northern Arizona University

The general numerical method and several of its special forms, along with three graphical forms, are presented for finding the required blending proportions for mineral aggregates. These methods are analyzed, their limits of applicability are given, and the special features, including the advantages and disadvantages of each method, are discussed. Eleven examples illustrate the application of the presented methods. These are also used to make the comparisons among the various methods more meaningful.

•IT often occurs in practice that two or more mineral aggregates of differing grading should be blended to comply with a given grading specification.

A set of blending proportions with given aggregates for approximation of a required grading can usually be obtained by trial and error, provided that such a set does exist. This method, however, is quite time-consuming and cumbersome. There are numerous other methods for estimating the needed proportions more directly.

The linear theory of blending, as well as several of its particular forms, is discussed in this paper. Although the methods discussed here pertain primarily to grading, most of them are suitable for other blending estimations as well.

## GENERAL THEORY OF BLENDING

The common mathematical basis of the various graphical, semigraphical, and numerical methods for finding the needed blending proportions can be stated in the following way.

The fulfillment of (n - 1) grading conditions requires, in general, the combination of at least n aggregates of differing grading. These conditions can be written in the form of a system of (n - 1) simultaneous linear equations and inequalities containing the needed blending proportions as unknowns. To these an n th equation should be added to take care of the obvious condition, namely, that the sum of the blended aggregates is equal to the total of the combined aggregate. Thus, the general mathematical form of this system is something like

$$\left.\begin{array}{l}
c_{11} x_1 + c_{12} x_2 + \dots + c_{1n} x_n = b_1 \\
c_{21} x_1 + c_{22} x_2 + \dots + c_{2n} x_n = b_2 \pm \Delta b_2 \\
\vdots \\
c_{i1} x_1 + c_{i2} x_2 + \dots + c_{in} x_n \geq b_i \\
\vdots \\
c_{(p-1)1} x_1 + c_{(p-1)2} x_2 + \dots + c_{(p-1)n} x_n < b_{(p-1)} \\
x_1 + x_2 + \dots + x_n = 1
\end{array}\right\} \qquad (1)$$

where

$c_{ij}$ = the i th grading characteristic of the j th aggregate to be blended,
$x_j$ = the sought blending proportion for the j th aggregate,
$b_i$ = the given i th condition for the combined grading,
p = number of grading conditions plus one, and
n = number of the aggregates to be blended.

n can be smaller, equal to, or larger than p. So long as $n \geq p$, there are, in general, an infinite number of sets of $x_i$-values (but at least one set) that satisfy this system. Out of these, however, only those sets represent blending proportions where none of the included $x_i$-values is less than zero because a negative blending proportion is physically meaningless. The lack of such a nonnegative set of solutions indicates that it is impossible to fulfill all the given (p - 1) grading conditions with the given n aggregates.

In other words, system 1 is applicable for every kind of blending problem of aggregates. In most cases it provides the exact solution (if there is any) to a grading problem. The disadvantage of the numerical method is that it becomes increasingly lengthy and cumbersome when n and/or p is larger than three, unless a computer is used.

It follows from the space-filling role of the aggregate particles that the correct way to express their blending proportions is by absolute volume. In this case, the grading of the combined aggregate should also be expressed in terms of absolute volume. Only when the specific gravities of all the given aggregates to be blended can be considered identical may the proportions be expressed by weight.

## SPECIFIC FORM OF SYSTEM 1

The mathematically simplest form of system 1 is when p = n, and each condition is represented by an equation with a single number on the right-hand side. Here the number of the aggregates available for blending is one greater than the number of the given grading conditions. Mathematically this problem is represented by a system of n linear equations with n unknowns. Because the determinant of the coefficients cannot be zero in blending problems, there is one and only one set of solutions of this system. If none of the solutions is less than zero in this set, the solutions represent the only combination of the n given aggregates that can fulfill all the (n - 1) grading conditions.

When there are more equations than unknowns in the system, it is mathematically impossible, in general, to find an exact set of solutions for the system of equations. In other words, when the number of aggregates to be blended is less than the number of grading conditions plus one, all the grading conditions cannot be fulfilled in an exact fashion. It still may be possible, however, to find a set of blending proportions by the least-squares method that provides the best approximation for the combined grading to all the given conditions (1).

A typical case for the application of the system of n linear equations with p = n unknowns is when the conditions specify (n - 1) points within the 0 and 100 percent points through which the cumulative sieve curve of the combined grading must pass. A numerical illustration, for three aggregates, is given in example 1.

## EXAMPLE 1

Three mineral aggregates, A, B, and C, are given. Their gradings are shown in Figure 1. Determine their $x_A$, $x_B$, and $x_C$ blending proportions required to yield a combined grading that passes through the 20 percent point at the No. 30 particle size and the 40 percent point at the No. 4 particle size.

The needed three blending proportions can be calculated from the following system of linear equations where the coefficients were taken from Figure 1:

$$\left.\begin{aligned} 0.52x_A + 0.10x_B + 0.05x_C &= 0.20 \\ 0.92x_A + 0.60x_B + 0.10x_C &= 0.40 \\ x_A + x_B + x_C &= 1.00 \end{aligned}\right\} \qquad (2)$$

From this, $x_A = 0.303$, $x_B = 0.102$, and $x_C = 0.595$; that is, approximately 30 percent should be blended from aggregate A, 10 percent from aggregate B, and 60 percent from aggregate C.

When only two aggregates are to be blended, so that the combined sieve curve must pass through the $y_o$-point, the blending proportions can be calculated directly from the following two formulas, provided that $y_o$ is between $y_1$ and $y_2$:

$$x_1 = \frac{y_2 - y_o}{y_2 - y_1} \tag{3}$$

$$x_2 = \frac{y_1 - y_o}{y_1 - y_2} \tag{4}$$

with the availability of the check

$$x_1 + x_2 = 1 \tag{5}$$

where $y_1$ and $y_2$ are the pertinent ordinates of the sieve curves of aggregates 1 and 2 respectively.

Equations 3 and 4 are obviously the solutions of the corresponding system of two linear equations but otherwise similar to Eq. 2. Thus, they take into account only one point, through which the combined grading curve is required to pass.

## EXAMPLE 2

Use Eqs. 3 and 4 for the determination of the blending proportions for aggregates B and C shown in Figure 1 such that the amount of particles in the combined grading passing through a No. 4 sieve is 40 percent.

By taking the appropriate y-values from Figure 1,

$$x_B = \frac{10 - 40}{10 - 60} = 0.60 \text{ and } x_C = \frac{60 - 40}{60 - 10} = 0.40$$

The graphical interpretation of Eqs. 3 and 4 reveals that (a) the sieve curve of any combination of two aggregates must fall in its total length within the sieve curves of the two aggregates and (b) the distance between these two sieve curves at any particle size is divided by the combined sieve curve in the ratio of the $x_1$ and $x_2$ blending proportions employed.

This consideration may help one select a suitable value of $y_o$ in a blending problem. It can also be seen in Figure 1 that there is no combination of aggregate A and B that could fulfill the 40 percent condition given in example 2.

Note also that there is a range of solutions for the linear system, instead of a unique solution, when (a) the number of aggregates to be blended is greater than the number of grading conditions plus one, and/or (b) some or all of the grading conditions are given in the form of inequalities or in the form of ranges rather than as single numbers.

The range of solutions provides a certain flexibility for the engineer in selecting the most suitable set of blending proportions based on economical or other considerations.

## EXAMPLE 3

Calculate again the blending proportions for the case discussed in example 2, but with the condition that $y_o = 40 \pm 5$.

The upper and lower limits of the sought $x_B$ blending proportion and the corresponding $x_C$-value can be calculated again by substituting successively $y_o = 45$ and $y_o = 35$ into Eqs. 3 and 4. The results are $(x_B)_{up} = 0.70$ and $(x_C)_{up} = 0.30$ and $(x_B)_{lo} = 0.50$ and $(x_C)_{lo} = 0.50$.

That is, any quantity of aggregate B between 50 and 70 percent combined with aggregate C in the corresponding quantity of 100 $(1 - x_B)$ percent yields a combined grading in which the amount of particles passing the No. 4 sieve is the required 40 $\pm$ 5 percent. From this range, one pair of blending proportions can be selected on the basis of the local conditions, such as the price and availability of the two aggregates.

Example 2 is a special case of the more general blending problem previously discussed.

A system similar to Eq. 2, and consequently formulas similar to Eqs. 3 and 4, can be used for the calculation of blending proportions when the fineness modulus, or the specific surface, is used for the grading characterization rather than the sieve curves. This is shown for two aggregates in the following example.

EXAMPLE 4

In what proportions should a fine aggregate with a fineness modulus of $m_f = 2.2$ be blended with a coarse aggregate of $m_c = 8.0$ to obtain a combined grading with a fineness modulus of $m_o = 5.4$?

By using formulas similar to Eqs. 3 and 4,

$$x_f = \frac{8.0 - 5.4}{8.0 - 2.2} = 0.45 \text{ and } x_c = \frac{2.2 - 5.4}{2.2 - 8.0} = 0.55$$

## Graphical Methods in General

Certain forms of the linear system presented as system 1 may lend themselves conveniently to graphical or semigraphical solutions. Such methods can be advantageous beyond their visuality because they may be faster than numerical methods, they may provide the totality of the solutions, and they may be applicable for computer graphics. Their common disadvantage is that they are valid only for more or less special cases.

The limited accuracy or, in certain cases, the approximate nature of graphical methods is not particularly harmful because fluctuations in the gradings of the aggregates to be blended make a blending proportion precision better than 1 percent meaningless anyway.

## First Graphical Method

An approximate graphical method for the determination of blending proportions, very much the same as the one to be described, was probably first offered by Rothfuchs (2). There are two restrictions concerning the applicability of the method: It is suitable only for grading problems when all the gradings are characterized by sieve curves, and no more than two of the n given individual sieve curves may overlap significantly at any point, as viewed from the horizontal axis. Even with these restrictions, however, the method cannot provide exact solutions for two reasons: The actual sieve curves of the aggregates to be combined are not used, but straight lines are substituted for their curves; and the larger the overlapping of two such grading straight lines, the poorer is the approximation of the solution obtained.

The main advantage of this method is its simplicity, and this simplicity is hardly affected by the number of aggregates to be blended. Thus, it is particularly useful for large n-values.

The mathematical justification of this method is that, when there are no double or triple overlappings in the n sieve curves, the system of the corresponding n linear equations can be arranged in the form of a Gaussian elimination algorithm. This permits the successive determinations of the unknowns (blending proportions) one by one. The graphical method discussed here is an approximate form of this procedure, as can be seen from a comparison with the presented graphical interpretation of Eqs. 3 and 4.

The use of the method is illustrated in example 5 for four aggregates.

EXAMPLE 5

Determine the blending proportions for the four aggregates shown by the continuous lines in Figure 2 such that the combined gradings approximate the specified sieve curve shown by dotted line. The procedure for solution is as follows.

The sieve curves of the four aggregates to be blended are approximated by fitting straight lines. These are represented by the dashed lines shown in Figure 2. The opposite ends of these straight lines are joined together as shown by the dot-and-dash lines. The blending proportions sought then can be determined as the differences of the ordinates of the successive points, marked by circles, where the joining dot-and-dash lines intersect the dotted line representing the required grading. In our example, as shown on the right-hand side of Figure 2, 4 percent of aggregate 1, 15 percent of aggregate 2, 24 percent of aggregate 3, and 57 percent of aggregate 4 should be blended for the approximation of the specified grading.

The difference between the specified sieve curve and the actual sieve curve of the determined combination of the four aggregates is also shown in Figure 2 by the shaded areas.

If the combined grading is specified by a pair of limit curves rather than a single sieve curve, there are, as a rule, an infinite number of solutions to the blending problem. Here the acceptable ranges of the blending proportions can be obtained by applying the foregoing method for the upper limit and then repeating the procedure for the lower limit. On the other hand, if any of the dot-and-dash lines do not intersect the sieve curve specified for the combined grading, the grading problem has no solution.

Another simple graphical method is recommended by the British Road Research Laboratory (3) for the determination of n blending proportions for passing through (n - 1) specified points of a combined sieve curve. The mathematical form of this method is again the Gaussian elimination algorithm; therefore, no double or triple overlapping of the sieve curves is permitted here, either. In other words, the grading problem in example 5 can be solved with this method, but the one in example 1 cannot. Under these conditions, this method provides the exact solution to the given system of linear equations within the accuracy of a graphical procedure. On the other hand, the method becomes more and more complicated with the increase of the value of n.

Various applications of the method are demonstrated in examples 6 through 9.

## EXAMPLE 6

Determine the blending proportions for aggregates 1 and 2 (Fig. 2) by using the British graphical method such that the amount of particles in the combined grading passing through a No. 50 sieve is 14 ($\approx 100 \times 4/27.5$) percent. The steps of the solution are as follows (Fig. 3).

A square diagram is prepared with percentage scales along three sides as shown in Figure 3. The amount of material in aggregate 1 that passes through a No. 50 sieve (50 percent) is marked off along the left-hand vertical axis. The amount of material in aggregate 2 that passes through a No. 50 sieve (0 percent) is marked off along the right-hand axis and is joined by a straight line to the point on the left-hand axis (thick sloping line called sieve-size line). This sieve-size line, representing the No. 50 sieve size, intersects the horizontal line representing the required percentage of material passing the No. 50 sieve in the combined grading (14 percent). This intersection is marked again by a circle. The sought percentage of aggregate 1 is indicated on the top scale by the vertical line (line of combination) drawn through this point of intersection. In this example, it is approximately 28 percent.

When aggregates 3 and 4 are to be combined such that the amount of particles in the combined grading passing through a ½-in. sieve is 22 ($\approx 100 \times 15.5/72.5$) percent, the same graphical method provides approximately 20 and 80 percent blending proportions for aggregates 3 and 4 respectively (Fig. 4). Note that this combination does not contain particles passing through the ⅜-in. sieve.

This graphical method also has the advantage that it can provide the complete combined grading with little additional work. This is shown in example 7.

## EXAMPLE 7

Determine several points of the sieve curve representing the combination of aggregates 1 and 2 that was obtained in example 6. The steps of this determination are as follows (Fig. 3).

The sieve sizes for the complete grading of aggregate 1 are marked along the left-hand axis of Figure 3 according to the corresponding percentage passing values. Aggregate 2 is represented similarly on the right-hand axis. Each point on the left-hand axis is joined by a sloping straight line (sieve-size line) to the point with the same sieve size on the right-hand axis. Any point of the combined sieve curve (that is, the percentage of the combined aggregate passing through any sieve) is the ordinate of the point of intersection that the vertical line of combination (as defined in example 6) makes with the corresponding sloping sieve-size line. The process is shown in Figure 3 by the thin continuous lines. It can be seen that, in our example, the points of the combined sieve curve related to the Nos. 100, 50, 30, 16, 8, and 4 sieve sizes are approximately 1, 14, 42, 58, 72, and 100 percent respectively.

The British graphical method is also applicable to the case when the aggregates should be blended such that the sieve curve of the combined grading falls within a pair of specified limit curves.

EXAMPLE 8

Determine the blending proportions for aggregates 3 and 4 (Fig. 2) by using the British method such that the combined gradings fall within the following limits:

| Sieve Size | Specified Limits, Total Percentage Passing |
|---|---|
| $1\frac{1}{2}$ in. | 100 |
| 1 in. | 50 to 73 |
| $\frac{3}{4}$ in. | 30 to 60 |
| $\frac{1}{2}$ in. | 18 to 45 |
| $\frac{3}{8}$ in. | 15 to 40 |
| No. 4 | 7 to 20 |
| No. 8 | 0 |

The steps of the procedure are as follows (Fig. 4).

The sieve-size lines are constructed for aggregates 3 and 4 in the same way as was discussed in example 7. The specified upper and lower limits are marked off on each sieve-size line as shown in Figure 4. The points representing the lower limit are connected, and so are the points representing the upper limit. Any vertical line of combination that can be placed between these limits without intersecting either of the limits represents a blending proportion for aggregates 3 and 4, which produces a combined grading falling between the specified limits. Figure 4 shows that any proportion for aggregate 3 between 18 and 35 percent combined with the corresponding amount (between 82 and 65 percent) of aggregate 4 will yield gradings that comply with the specification.

If the upper limit and lower limit curves intersect each other, the blending problem, as stated, has no solution.

When more than two aggregates are to be combined, the blending proportions can be obtained by the repeated application of the British method: Two appropriately selected aggregates should be blended first, and then this combination should be treated as one material to be combined with another, and so forth.

An example for four aggregates is presented in the following section.

EXAMPLE 9

Determine again the blending proportions for the case discussed in example 5 but by using the British graphical method. The procedure is as follows.

An appropriate combination of aggregates 1 and 2 is found. Then an appropriate combination of aggregates 3 and 4 is found. These steps have been taken in example 6. Thus, these two combinations should be blended again such that the amount of particles passing through the No. 8 sieve in the new combination is 19 percent. Because it has

Figure 1. Aggregate gradings for examples 1 and 10.

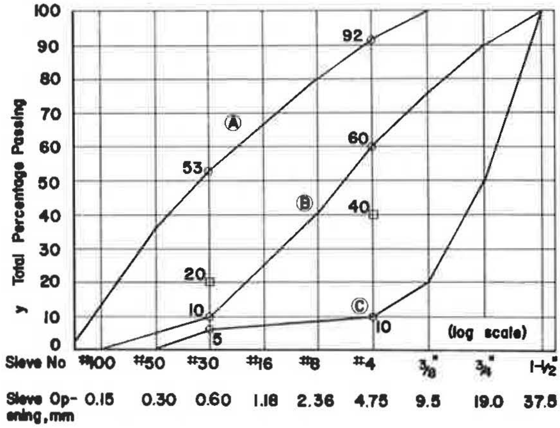



Figure 2. Graphical determination of the blending proportions needed to approximate the specified grading.

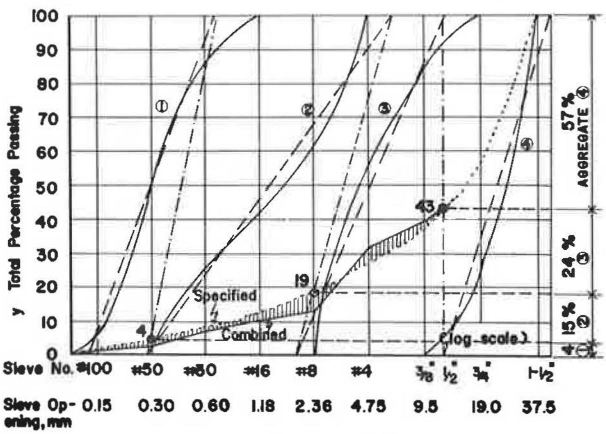



Figure 3. Application of British method.

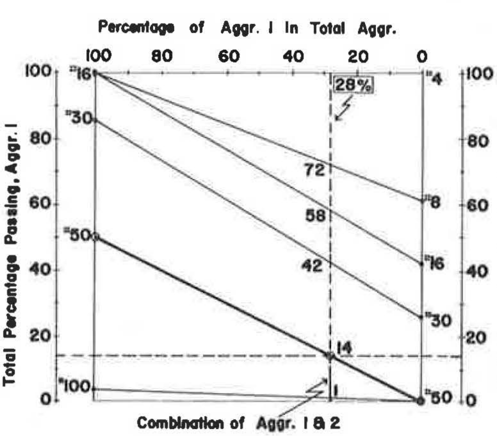
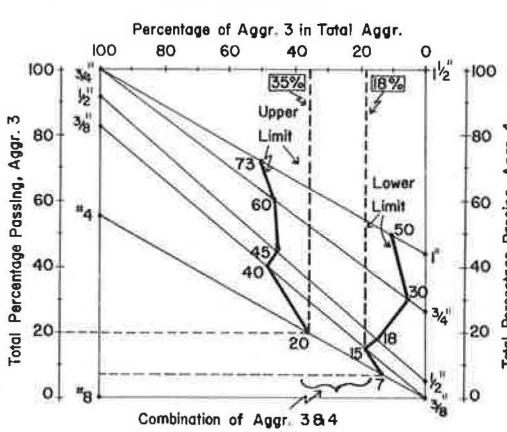



Figure 4. Determining range of blending proportions using the British graphical method.



also been established in example 7 that the amount of particles passing through a No. 8 sieve in the combination of aggregates 1 and 2 is 72 percent, whereas it is 0 percent in the combination of aggregates 3 and 4, the needed blending proportions for the combination of combinations can be determined as shown in Figure 5 by a thick line. Accordingly, approximately 26 percent should be taken from the combination of aggregates 1 and 2 and 74 percent from the combination of aggregates 3 and 4. Therefore, the needed percentages of the four individual aggregates in the total aggregate can be calculated from these values and from the values obtained in example 6 as follows: aggregate 1 (100 x 0.26 x 0.28 = 7.3 percent), aggregate 2 (100 x 0.26 x 0.72 = 18.7 percent), aggregate 3 (100 x 0.74 x 0.20 = 14.8 percent), and aggregate 4 (100 x 0.74 x 0.80 = 59.2 percent).

There are noticeable differences between these blending proportions and those determined in example 5, but these new proportions are practically identical to the exact values that could have been calculated by the numerical method from the same grading conditions. As a result of this identity, the combined grading of example 9 passes through the No. 50, No. 8, and ½-in. points of the specified sieve curve (Fig. 2), whereas the combined sieve curve of example 5 does not. Nevertheless, one can ascertain from a comparison of the line of combination in Figure 5 with the specified and combined sieve curves in Figure 2 that the quality of the overall fit provided by the two graphical methods is more or less the same.

Note also that, if the specified sieve-size line does not intersect the horizontal line representing the required percentage of passing material in the combined grading, the grading problem, as stated, has no solution.

## The Triangular Method

This semigraphical method utilizing triangular charts is much more general than the previous two in that it can handle sieve curve, fineness modulus, specific surface, or other grading specifications, or combinations of these, in the form of both equalities and inequalities. It is also suitable for blending on the basis of the probability of grading, that is, with a consideration of the expected grading fluctuations in the aggregates to be blended (4). The triangular method provides the exact solutions within the accuracy limit of graphical methods. Without the application of computer graphics, however, it is restricted essentially to cases where all the gradings included in the blending problem are considered as consisting of not more than three appropriately defined aggregate fractions.

The mathematical basis of the method (5) is that, in the case of transformation of a triangular system of coordinates into another one, the $x_i$ new coordinates, on the one hand, are solutions of a system of three linear equations similar to Eq. 2 but, on the other hand, they can also be determined by measuring certain p distances in the new system (Fig. 6) and substituting them into the following formulas:

$$\left.\begin{aligned} x_A &= \frac{p_2}{p_1 + p_2}\,\frac{p_3}{p_3 + p_4} \\ x_B &= \frac{p_1}{p_1 + p_2}\,\frac{p_3}{p_3 + p_4} \\ x_C &= \frac{p_4}{p_3 + p_4} \end{aligned}\right\} \qquad (6)$$

where it can be verified that $x_A + x_B + x_C = 1$.

Shaefer was probably the first to recommend these formulas, without an exact mathematical justification, for the determination of blending proportions (6).

Before the application of this method is demonstrated, it should be pointed out that the grading in a triangular system is represented by a point rather than a curve. For instance, if an aggregate consists of 53 percent of pan to No. 30 particles (fine sand), 39 percent of No. 30 to No. 4 particles (coarse sand), and 8 percent of No. 4 to 1½-in.

particles (gravel), this grading is represented by point A in the triangular system shown in Figure 7. Incidentally, this is the same grading that is represented by sieve curve A in Figure 1.

The use of triangular diagrams for the determination of blending proportions is illustrated in the following two examples. More details and pertinent information are given elsewhere (5).

## EXAMPLE 10

Determine again the blending proportions for the case discussed in example 1 but by using the triangular-diagram method. The procedure is as follows (Fig. 7).

All gradings should be considered as consisting of three fractions, the limits of which are determined by the specified points of the combined grading. In our example the specified points are at the No. 30 and No. 4 sieve sizes. Therefore, the three fractions to deal with are fine sand, coarse sand, and gravel.

An equilateral triangular system of coordinates is prepared with percentage scales along the three sides as shown in Figure 7. Each side is an axis for one of the three aggregate fractions previously mentioned. The grading point of aggregate A can be obtained by marking off 53 percent on the fine sand scale and 39 percent on the coarse sand scale and checking whether this point cuts out 8 percent on the gravel scale. This is shown in Figure 7 for point A with dashed lines. The grading points for aggregates B and C, as well as for the specified combined grading (P), are similarly plotted. Points A, B, and C are connected with straight lines to form another triangle. Then another straight line is drawn to pass through point P and one of the A, B, or C vertexes. In our example, point B is selected arbitrarily, and point X is marked off where the BP line intersects the AC side. The p distances are measured from Figure 7 as follows:

$p_1 = AX = 2.65$ in. $\qquad p_3 = BP = 1.50$ in.
$p_2 = CX = 1.35$ in. $\qquad p_4 = PX = 0.15$ in.

By substituting these values into Eq. 6, the sought blending proportions can be calculated as follows:

$$x_A = \frac{1.35}{4.00}\,\frac{1.50}{1.65} = 0.307$$

$$x_B = \frac{0.15}{1.65} = 0.091$$

$$x_C = \frac{2.65}{4.00}\cdot\frac{1.50}{1.65} = 0.602$$

$$\text{Total} \qquad \overline{1.000}$$

A comparison of these results with the solutions of Eq. 2 in example 1 shows good agreement. Note that only such blending problems have solutions where the point P is within the ABC triangle.

## EXAMPLE 11

In this example the triangular method is applied for a more complicated blending problem (5).

Assume that materials a, b, and c are given. Their compositions are given in Table 1. Determine the blending proportions required to yield the specified grading given in Table 1.

If the gradings of the available materials are plotted as points a, b, and c in Figure 8, then the points of the shaded area represent the full range of all possible mix proportions that will comply with the grading requirements. It can be seen that in this

Figure 5. Determination of blending proportions for four aggregates using the British graphical method.

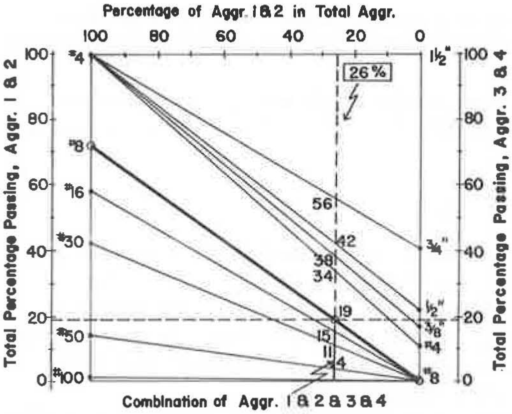



Figure 6. Determination of the trilinear coordinates.

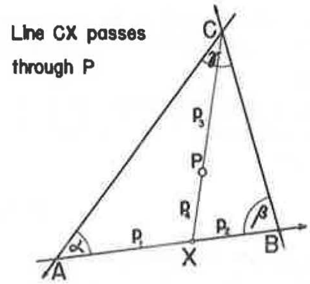



Figure 7. Triangular method for the determination of the blending proportions.

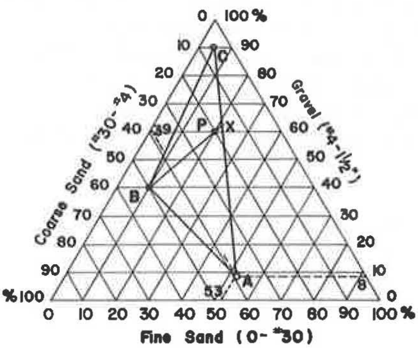



Figure 8. Proportioning mineral aggregates by the triangular method.

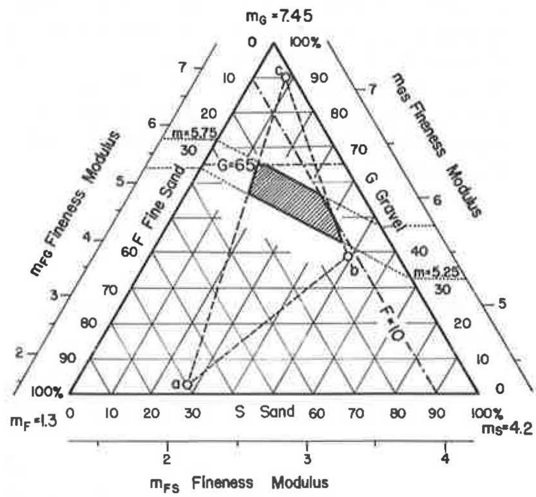



Table 1. Grading data for materials a, b, and c.

| Material | Gravel[a] (percent) | Sand[b] (percent) | Fine Sand[c] (percent) | Fineness Modulus (ASTM Designation C 125-68) |
|---|---|---|---|---|
| a | 3 | 27 | 70 | 2.3 |
| b | 38 | 49 | 13 | 5.05 |
| c | 90 | 7 | 3 | 7.05 |
| Specified grading | < 65 | None | > 10 | 5.5 ± 0.25 |

[a]Coarser than No. 4 sieve. [b]No. 4 to No. 16 sieve. [c]Finer than No. 16 sieve.

Table 2. Limits for permissible blending proportions of materials a, b, and c.

| Material | Limit | Extreme Proportions Within Specifications (percent) | | | |
|---|---|---|---|---|---|
| | | Blend 1 | Blend 2 | Blend 3 | Blend 4 |
| b | Lower | 0 | 4 | 61 | 80 |
| | Upper | 4 | 61 | 80 | 92 |
| c | Lower | ← -0.6b + 63 → | | | |
| | Upper | -0.43b + 72 | -0.6b + 73 | -0.84b + 87 | 100-b |
| a | Lower and upper | ← 100 - b - c → | | | 0 |

case there are many such mix proportions. The construction of the limits of the shaded area is as follows: The positions of the two parallel, sloping dotted lines are determined by the specified values ($5.5 \pm 0.25$) of the fineness modulus of the combined grading; the two dot-and-dash lines are determined by the specified limits for the respective gravel and fine sand contents; and the three dashed lines are determined by the actual gradings of aggregates a, b, and c to be blended.

One of the mix proportions can be determined by the semigraphical method from any point of the shaded area and the triangle abc. This has been illustrated in the preceding example. By the successive application of the same method to each corner point of the shaded area, the lower and upper limits of the desired mix proportions can be obtained for all three mineral aggregates. Thus, the full range of the blending proportions that will comply with the specified grading requirements is determined. These lower and upper limits are given in Table 2. It can be seen that the amount of material b may vary between 0 and 92 percent, and the amount of material c may vary between 7.8 and 72.0 percent. If it has been decided to take, for instance, 50 percent of material b, then, according to Table 2, the amount of material c may vary between 33 and 43 percent, and the amount of material a may vary between 7 and 17 percent to comply with the specified grading requirements.

## CONCLUSIONS

The general mathematical form of the blending problem is system 1, which contains the sought blending proportions as unknowns. Depending on the number and nature of the grading conditions, there can be an infinite number of sets of blending proportions, a single set, or no set at all that satisfies all the given conditions. The numerical method is applicable for every kind of blending problem of aggregates. In most cases, it provides the exact solutions; but it becomes increasingly cumbersome when the number of aggregates to be blended is more than three, unless a computer is used.

The graphical method discussed first is simple even for a large number of aggregates to be blended, but it is applicable only in special, although practically important, cases. Also, the solutions it provides are usually not exact.

The restrictions concerning the applicability of the British graphical method are essentially the same as those for the first graphical method. The British method becomes more complicated with the increase of the number of aggregates, but the solutions obtained are the exact solutions. It also provides the grading of the combined aggregate with little additional work.

The triangular method is simple and still more general than the other two graphical methods. It provides also the exact solutions. It is an ideal method for blending three aggregates. In cases more complex than this, however, it may become overly complicated, unless computer graphics is used.

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# COMPATIBLE GRADATION OF AGGREGATES AND OPTIMUM VOID-FILLING CONCRETE PROPORTIONING FOR FULL CONSOLIDATION

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Through the authors' extensive research and development in the key issues of concrete technology and through their wide practical applications from prior to 1949 to the present time, they have recognized the importance of compatible gradation of aggregates and have evolved an optimum void-filling method of concrete proportioning to ensure full consolidation. The method is based on four mutually dependent principles as follows: Compatibility between the grading of the coarse and fine aggregates is an essential factor in accurately controlling the total void content per unit volume of the combined aggregates; by applying the principle of gradation compatibility, engineering quality concretes can be readily designed from sound aggregates although full information may be lacking on some other characteristics of these materials; the compatible gradation of the aggregates and their optimum void-filling proportioning will lead to the formation of a plastic concrete that may be fully compacted even at large aggregate-cement ratios; and it is this concrete of largest aggregate-cement ratio with consistently minimum specific surface that will ensure the greatest economy and still retain compactible workability.

•FOR all given sound coarse and fine aggregates, concrete technology should start with their grading to allow the voids of the compacted coarse aggregate particles in a given volume of concrete to be filled with compacted fine aggregate particles and, in turn, to allow the combined remaining voids of a system of compacted coarse and fine aggregates to be filled with a cement paste at a stipulated water-cement (w-c) ratio, adjusted for admixtures if used, to meet the strength requirement.

This desideratum can be best achieved through gap grading with "permissible" maximum size of coarse aggregate and "admissible" maximum size of fine aggregate. The fixed ratio of permissible and admissible sizes constitutes the criterion for compatible gradation. The fundamental basis of compatible gradation forms the backbone of gap grading. It permits an optimum void-filled concrete, proportioned with mathematical accuracy, to provide the full consolidation that cannot be otherwise obtained.

Full consolidation of fresh concrete is of prime importance in achieving high-quality concrete in the hardened state. First, it enhances density, impermeability, durability, and resistances to wear, freeze-thaw, scaling, and spalling. Second, it greatly reduces shrinkage and creep. Third, it increases compressive strength, modulus of rupture, and moduli of elasticity and rigidity.

Instead of relying on post-placing vibration alone, the best results of full consolidation should start with compatible gradation and optimum void-filling proportioning. These subjects will constitute the central theme of the present paper, as prerequisites to full consolidation before mixing.

# FUNDAMENTAL BASIS OF COMPATIBLE GRADATION

## Basic Principle

The basic principle of gap gradation or gap grading is to omit the undesirable intermediate sizes lying between a narrow range of the maximum allowable size of the coarse aggregate and the largest admissible size of the fine aggregate and to delete the extreme fines from the conventional continuous gradings. To explain clearly gap grading requires that the terms introduced herein be explained further.

## Maximum Sizes of Coarse and Fine Aggregates

In plain concrete, the maximum allowable size of the coarse aggregate will be limited by the available equipment for efficient handling, mixing, and conveyance and by the available vibrating devices for effective consolidation. Where the concrete sections are reinforced or prestressed, the maximum allowable size of the coarse aggregate will be further limited by (a) its free passage through spacings of the reinforcing or prestressing steel or ducts and (b) the clear cover required.

The largest admissible size of the fine aggregate should be slightly less than the size of the side interstices when the coarse aggregate particles are closely compacted. As will be shown in the following rhombohedral form of packing, the largest size of the fine aggregate is not determined by the largest size of voids among the coarse aggregate particles but limited by the admissibility under vibration through its side interstices, often described as "throat openings."

## Rhombohedral Form of Packing

As the depletion of suitable natural aggregates becomes more pronounced and the substitution of kiln-fired spherical pellets of uniform size comes into more general use, especially in lightweight aggregates, the condition of packing coarse aggregate nearly to a rhombohedral form will be reached in actual practice. Not only is this form of packing attainable with uniform-sized spherical artificial aggregates, but also it is well known from long experience that rounded aggregates have always produced more workable concretes, whereas elongated or flat particles do not and hence are usually required to be picked out by the more rigid specifications. Thus, in the idealized geometrical model for crushed stone or gravel, it is reasonable to take spheres to represent the aggregates. In general, the average size of coarse aggregate passing one size of sieve and retained on the next smaller size, elongated and flat pieces being removed, does statistically represent an equivalent spherical size.

When such average spheres are packed together, the most stable and efficient packing is in the rhombohedral form. In other words, if the maximum-sized spheres having a diameter D are hand packed in the most efficient way in a container whose three dimensions are large as compared with the diameter D, a rhombohedral form of packing will result.

If these particles with a diameter D are termed "major spheres," the largest voids among their packing will be able to accommodate smaller spheres of diameter 0.414 D, which may be called "major occupational spheres." The then remaining largest voids would geometrically be able to accommodate still smaller spheres of diameter 0.225 D, which may be termed "minor occupational spheres." The residual voids now remaining can be geometrically filled with further smaller spheres of diameter 0.155 D, which may be called "admittance spheres." This geometrical model can go on down to the extreme fines.

In an ordinary sense, for a coarse aggregate of maximum-sized D, and if D is equal to 1 in., those particles with a diameter larger than 0.225 D would be all termed coarse aggregate, and those equal to or smaller than 0.225 D would be all termed fine aggregate. However, the rhombohedral packing is feasible only in very careful hand packing according to a predetermined geometrical model. In practice, neither the major occupational spheres of 0.414 D nor the minor occupational spheres of 0.225 D can enter the side interstices after the major spheres have been closely packed in the rhombohedral form. Only the admittance spheres can enter such side interstices.

The fact just stated will at once become convincing by performing a visual experiment with a three-dimensional container of plate glass sides whose inside dimensions are large as compared with the diameter D of the maximum-sized spherical particles. After laying them closely in rhombohedral form, one may try to overlay them with any size of aggregate smaller than the maximum size D but greater than 0.155 D; it will be seen that it is impossible for any overlying intermediate sizes to settle through the interstices into the voids even with the aid of slight vibration. By changing the overlay to particles equal to or smaller than 0.155 D, it will be seen that they can pass through the interstices, fill the voids, and become compacted even without the aid of vibratory motion, provided both materials are dry. It can thus be proved visually, in confirmation with the geometrical theory, that

1. Any intermediate size of coarse aggregate smaller than a narrow range of the maximum size but greater than 0.155 D cannot be consolidated to achieve the function for filling the voids of the maximum-sized coarse aggregate; and
2. The largest size of fine aggregate particles, in order to serve their function properly as a filler material, must have a diameter not greater than 0.155 D.

This leads to the undesirability of having intermediate sizes of the coarse aggregate in any combination of the conventional continuous gradings.

## Undesirability of the Intermediate Sizes of Coarse Aggregate

A close and thorough examination of the undesirable effects of the intermediate sizes of coarse aggregate will reveal the following:

1. Any efficient grading must achieve strength from efficient packing such that compression loads will be mainly transmitted by direct contact among particles of the coarse aggregate rather than through the generally weaker mortar, though tensile and shearing stresses must be transmitted through the latter. The condition of such efficient packing can only be obtained if the coarse aggregate particles are within a narrow maximum-sized range and general spherical shape in the absence of intermediate sizes. Adjacent to point contacts among coarse particles, the mortar shares relatively less compression load by virtue of its generally lower elastic modulus than that of the coarse aggregate.
2. Although the two sizes consisting of major occupational spheres and minor occupational spheres could be laid by hand in a predetermined sequence in the voids of the major spheres, it is, nevertheless, impossible to produce this result through any available process of mixing, depositing, and vibrating. This is because the major spheres or major aggregate particles have greater momentum and arrange themselves first into a compact form, after which the intermediate-sized coarse aggregate particles have no way to fill up the voids by virtue of their inadmissible sizes through the side interstices of the earlier compacted maximum-sized coarse aggregate particles.
3. In continuous gradings, the situation is even worse because the coarse aggregate will contain all sizes, from the maximum size to the size of admissible fine aggregate.
4. All the intermediate sizes of coarse aggregate lying between its maximum size and the admissible size of fine aggregate, in all compositions of the conventional continuous grading, will upon consolidation by vibration have an adverse effect by interfering with the close compaction of the maximum-sized coarse aggregate particles, wedging them out from contacting each other and thereby requiring more mortar and hence more cement and water for any stipulated w-c ratio. The separation of the larger sized coarse aggregate particles prevents them from transmitting compression by direct contact and compels such transmission through the generally weaker mortar.
5. The increased mortar content required by a continuous grading represents the need for additional cement and water per unit volume of concrete, and hence more shrinkage and creep can be expected in continuous grading because of the wedging action of intermediate sizes. There is also a further adverse effect of increased "specific surface" in significantly augmenting the cement-paste requirement. This leads to the necessary consideration of the physical influence of specific surface of aggregates in the next section.

## Specific Surface of Aggregates

Apart from the conventional definition of specific surface as the surface area per unit weight, which is erroneous except in one instance where the particles under consideration, crushed stone or gravel or sand, have the same specific gravity, the correct definition of this term should be changed to "surface area per unit volume" in concrete technology.

What one is really interested in with regard to this term is the necessary cement paste for binding the coarse and fine aggregates together after the water needed for lubricating and saturating the aggregates in a surface-dry condition has been otherwise provided. It is, therefore, the cement paste needed at a stipulated w-c ratio for strength, with a quantity of water just sufficient to hydrate the cement without excess that will cause bleeding. In this context, the less the mortar, the less the cement paste, the less the cement, and the consequent reduction in water will give the advantages of producing higher unit weight, higher strength, higher moduli of elasticity and rigidity, less shrinkage, less creep, and higher creep recovery. For all these advantages, the optimum condition can be achieved only when the specific surface is at a minimum.

The specific surface may be defined as k/L, where k is a constant depending on the particle under study. In practice, the calculation of specific surface, except for particles of regular geometric shape, is laborious, particularly so in the case of the fine aggregates applicable to concretes. The determination of the specific surface of such a material may be made by assessment of an equivalent mean dimension, L, representative of the average particle in each size group considered. Except in the cases of flaky and elongated particles, a single mean dimension will allow a simple equivalent geometric solid to be calculated, conveniently a sphere, from which a reasonably accurate determination of the true specific surface may be made.

Alternatively, an average particle volume may be established by weighing a representative group of grains, counting the number, n, in the group, which should be not less than 100, and then dividing the group weight, W, by n and the appropriate specific gravity, p. Then

$$D = \sqrt[3]{\frac{6W}{\pi np}}$$

where D = average particle diameter. Comparing the specific surface of spheres per cubic inch of material where the diameters are 3 in. and 0.003 in., it will be found that the former provides 2 sq in., whereas the latter possesses 2,000 sq in. (The ASTM and British Standard No. 200 sieves coincide as to a square opening of 0.003-in. size.)

## Minimization of Void Surface

Concrete is known to lose strength as its void content increases. The minimization of voids can be effectively achieved by appropriately arranging the grading of the selected coarse and fine aggregates to the extent that the total volume of voids left will accept just sufficient cement paste at the required w-c ratio to give the fresh mixture the desired workability.

In a three-dimensional container having sides that are large as compared with the size of particles, it can be visualized that the number of voids is at least equal to the number of particles packed in the system. If there is no arching effect, the size of the individual voids is directly proportional to that of the particles forming the system. Although mathematically the percentage of voids remains constant for any system of equal-sized ideal aggregate, the surface area will increase greatly with the decrease in size of particles.

Consider two identical assemblies of single-sized spheres arranged in their most compact and stable rhombohedral packing in two identical containers large enough to minimize the side effect. In this form of packing every sphere touches 12 others. Each assembly will leave the same total void volume of 26 percent. If the spheres in one of this pair of assemblies are of twice the diameter as those in the other, the number of spheres as well as the number of voids formed by the smaller spheres will be the cube of two or eight times the number in the other system. As the surface of each sphere is equal to $\pi$(diameter)$^2$ and the volume of each sphere is equal to one-sixth of $\pi$(diameter)$^3$, both the total surface and the specific surface of the smaller sphere assembly will be twice that of the larger assembly.

In assessing the probable workability of a concrete mixture by the fineness-modulus approach, it has thus been recognized that the total surface area to be coated with cement paste has more significance than either the number or the size of voids.

Consider next any conventional continuous grading. It should be obvious that the voids formed by the packing together of the particles retained on one sieve cannot be filled by those retained on the next smaller sieve because they cannot enter the side interstices of the immediately larger size. The effect of combining two successive aggregate sizes is to induce particle interference, cause wedging action upon compaction, increase the number of voids, reduce the average size, and increase the specific surface as well as the total surface while achieving little or no reduction in total voids. The ultimate effect of this so-called continuous grading is to increase the demand for mortar and, hence, sand, cement, and water.

Therefore, for identical aggregate-cement (a-c) ratios, gap grading will virtually manifest a richer mortar as the result of reduced specific surface than its continuously graded counterpart for the same maximum size of coarse aggregate.

## Compatibility of Coarse and Fine Aggregate Sizes

The advantage of using as much permissible maximum-sized coarse aggregate as possible to the point of filling the concrete spatial configurations with its compacted bulk volume is obvious from the foregoing consideration of specific surface. Mathematically, the single maximum size of coarse aggregate is the best. In practice, a narrow range of maximum size may be preferable. If the maximum permissible size of coarse aggregate is 1¼ in., the best range may be passing the 1¼-in. sieve and retained on a 1-in. sieve, calling this lesser average diameter as D.

The compacted bulk volume of this coarse aggregate can only admit through its side interstices fine-aggregate sizes not greater than 0.155 D. A maximum No. 8 sand can then be used, allowing a mortar film on the opposed surfaces.

If intermediate-sized coarse aggregates ranging from those passing the 1-in. sieve to those retained on the No. 8 sieve were also used, they would have a wedging action, increase the specific surface, require more mortar, cause more shrinkage and creep, and produce a concrete lower in density, weaker in strength, and lower in moduli of elasticity and rigidity. The removal of intermediate sizes virtually lowers the specific surface, reduces the cement requirement, and increases the workability.

Additionally, to get optimum results in saving cement, all very fine sand particles should be deleted. Because the specific surface for a No. 200 sand doubles that of the No. 100 sand, there is no advantage in using sand particles finer than No. 100, and in many cases the cutoff point can be set at No. 50 or even No. 30, except when it becomes necessary to vary the range for adjusting workability.

On this basis, as an example, Table 1 gives one way for providing gap-grading ranges of coarse aggregate and compatible fine aggregate for gap-graded concrete.

**Table 1. Examples of gap-grading ranges of coarse and compatible fine aggregate sizes.**

| Gap-Grading Range of Coarse Aggregate | | Gap-Grading Range of Fine Aggregate | |
|---|---|---|---|
| Passing Through Sieve (in.) | Retained on Sieve (D) (in.) | Maximum Size $\not>$ 0.155 D | Deleting Fines Finer Than (No.) |
| 6 | 4 | ½ in. | 30 |
| 4 | 3 | ⅜ in. | 50 |
| 3 | 2 | ¼ in. | 50 |
| 2 | 1½ | No. 8 | 100 |
| 1½ | 1 | No. 8 | 100 |
| 1 | ½ | No. 16 | 100 |
| ¾ | ⅜ | No. 30 | 100 |

The data given in Table 1 further indicate that all available sizes from 6 in. down to No. 100 can be effectively utilized as categorically outlined in the following section.

Full Utilization of All Sizes

The compatible gradation just stated amounts to what is technically termed "optimum gap grading." Despite its successful use in England, Israel, India, and West Germany, there remains in the United States an unfounded argument against gap grading from all those who cannot see how to utilize all the available coarse and fine aggregates except the dust. The following four different versatile applications of gap grading can dispel their argument completely:

1. In highway bridge projects, the larger sizes can be best used in abutments, pier footings, and pier shafts; the upper medium sizes in pier bents and girders; the lower medium sizes in stringers and floor beams; and the smaller sizes in slabs, railings, and light posts. The construction sequence is such that they can never be poured at the same time.
2. In precasting plants, the larger sizes of the coarse aggregate can be used in heavy girders and columns, the medium sizes in lighter beams and columns, and the smaller sizes in floor slabs, roof slabs, and wall panels. As a matter of fact, these precast elements will be prefabricated in groups.
3. In building work cast in situ, the larger sizes of the coarse aggregate can appropriately go to foundation mats and retaining walls, the upper medium sizes to lower tier heavy columns and girders, the lower medium sizes to lighter columns and beams, and the smaller sizes to floors, roofs, partitions, and parapet walls. By necessity, they are to be cast in stages.
4. In mass concrete work, the necessity to pour in sections and lifts offers the special advantage of being able to vary aggregate sizes in different sections, in lifts, and even in the hearting and facings within a lift.

If we know the available quantities of each size range of the aggregates, it is an easy matter to program the concreting work with different gap gradings for different stages or for different sections and lifts.

The uniform strength of all stages or of all sections and lifts may be easily controlled by the appropriate w-c ratio for each gap grading.

Thus, there could be no waste in any size of aggregate in any project or plant of a fair size. The concrete technologists should be able to adapt their aggregates in a versatile way, to best utilize their available sizes, rather than be controlled by the conventional continuous grading that requires excessive quantities of cement but does not produce the best concrete possible.

## FURTHER CONSIDERATIONS IN AGGREGATE GRADING

Regardless of the view one takes with regard to the grading of coarse and fine aggregates for any concrete, it is obviously most desirable to grade them such that they (a) require the least effort for aggregate synthesis, (b) need the minimum control to achieve the mean strength at the narrowest deviation from the minimum required strength, (c) minimize the cement and water requirements for a given w-c ratio, and (d) maximize the a-c ratio. The latter two are the key to economy in concrete production.

If the conventional continuous grading of all sizes is used to achieve maximum density, as is commonly believed, disappointment will result because the interference and wedging action of the intermediate sizes work to produce the opposite effect. There can never be a guarantee to attain the goal. It is, therefore, futile to work to a grading curve for aggregate synthesis, which is labor- and time-consuming and hence an expensive chore. This process will become much simpler as the number of coarse aggregate sizes is reduced.

The simplest way, as has been confirmed numerous times, is to use gap grading, consisting of a single-sized coarse aggregate (deleting the intermediate sizes), and a range of fine aggregate whose maximum size is not greater than that admissible into the side interstices of the coarse aggregate when compacted, at the same time deleting the relatively extreme fines. Such coarse and fine aggregates will both minimize the cement and water requirements as the result of much reduced specific surface. Not only has the grading variation been virtually eliminated but so also has any inaccuracy in moisture content correction been minimized.

It is in the fines that the moisture content may vary between 1 and 12 percent, and a check on this must be made if any continuous grading is to be used. Although the sand content of the combined aggregate in continuous grading may be 35 percent or even more, the sand content in gap grading is likely to be between 30 and 28 percent of the combined weight of aggregates, depending on the a-c ratio. Hence, the possible error in moisture content for gap-graded aggregates should be at a minimum.

Thus, with gap-graded aggregates, there is an inherent (a) minimization of variation in grading, (b) minimization of error in moisture correction, (c) minimization of segregation as the result of only one narrow range in size of coarse aggregate, (d) minimization of cement-paste content due to reduced specific surface and hence water and cement requirements, and (e) minimization of heat of hydration, drying, shrinkage, and creep.

The compressive strength of concrete at a constant w-c ratio increases with reduced workability; i.e., an increase in the a-c ratio will, on compaction, yield a higher strength. Experience not only has shown the importance of grading, particle shapes, and other physical and chemical properties of the aggregates but has also indicated that the grading of the coarse aggregate has a major influence on strength, workability, and cement economy. All these have led to the emergence of statistical quality control.

For maintaining any desired constant workability and w-c ratio to meet a target strength, an increase in the a-c ratio toward economy in construction can be accomplished by an increase in the maximum and mean sizes of the coarse fraction of the aggregates so as to reduce the specific surface area. In heavy concrete work, unrestricted by the spacing of reinforcing or prestressing steel or ducts, it is the larger sizes that will save more of the cement requirement; the crushing of these larger sizes into smaller sizes should be avoided, thereby saving production costs.

Further, all sizes below the input size to the crusher, because of their reduced size and tendency to have an angular and flaky nature, will reduce workability and even strength and hence demand a higher cement content. It is not merely that the intermediate sizes cause particle interference, produce wedging action, and hence require more mortar, but, additionally, the $^3/_{16}$-in. or finer crushings of very bad shape, when fed back into the natural sand, will degrade the fine aggregate.

## THE VOID-FILLING METHOD OF OPTIMUM PROPORTIONING

The foregoing fundamental basis of gap grading and further considerations of aggregate grading will naturally suggest a rational approach to an optimum proportioning method for all available sound aggregates with a mathematical accuracy and an utmost simplicity that can never be achieved with the conventional continuous gradings.

With isolated construction sites and a demand for large quantities of coarse and fine aggregates to be either locally quarried or obtained from the river run, the use of any continuous grading may suffer because of a lack of the required amount of certain critical sizes to meet a supposedly ideal grading curve. In the case of gap grading, however, all available sizes of sound aggregates may be successfully utilized to give an optimum proportioning.

In essence, for each cubic yard of concrete, we need to fill the space with compacted coarse aggregate within a narrow range of the maximum size, fill the voids therein with compacted admissible sizes of fine aggregate to save cement paste, and then fill the remaining voids with a cement paste at the required w-c ratio. Thus, a cubic yard of absolute volume of constituent materials will, in practice, yield a cubic yard of consolidated gap-graded concrete if allowance is made for any mortar that may stick to the inside of a mixer.

This easy-to-use optimum proportioning method is extremely simple. It is adaptable to any given sound coarse and fine aggregate. The unit weight of clean water and the specific gravity of standard portland cement are known. For any sound coarse and fine aggregate to be used, it is only necessary to determine (a) the weight of a unit compacted bulk volume of saturated surface-dry coarse aggregate, (b) its specific gravity, (c) the weight of a unit compacted bulk volume of saturated surface-dry fine aggregate, and (d) its specific gravity. The rest can be mathematically determined. To be more specific, the method may be stated as follows:

1. To achieve the highest possible a-c ratio, and therefore the highest economy in making concrete, fill each unit volume of concrete with compacted bulk volume of saturated surface-dry coarse aggregate of the maximum permissible size within a narrow range.
2. Determine the bulk weight per unit volume of the compacted coarse aggregate and its specific gravity, compute its absolute solid volume, and get its total void volume.
3. Fill this total void with compacted bulk volume of saturated surface-dry fine aggregate from the maximum admissible size down through a moderate range, deleting all the extreme fines.
4. Determine the bulk weight per unit volume of the compacted fine aggregate and its specific gravity, compute its absolute solid volume, and find the remaining void volume of the combined coarse and fine aggregates.
5. The remainder of the void is to be filled with cement paste at a stipulated w-c ratio corresponding to the required strength, adjusted with the absolute volume of any desired admixture(s) that may be used.
6. Compute the absolute volumes of the cement and water required to fill the remaining voids and check that the water content is sufficient to provide a suitable degree of workability for the expected placing conditions. This may be checked in the following manner: Calculate the weights of the individual solids in unit volume of concrete and multiply each by the relevant percentage of water requirement. Thus cement is multiplied by 28 percent, fine aggregate by 10 percent, and coarse aggregate by 0.5 to 2 percent, depending on the maximum size in accordance with the following data:

| Coarse Aggregate (in.) | Water Demand (percent) | Remark |
|---|---|---|
| $1\frac{1}{2}$ | 0.5 to 0.75 | The lower value is for rounded and the higher for crushed material |
| $\frac{3}{4}$ | 1.0 to 1.5 | |
| $\frac{3}{8}$ | 1.7 to 2.0 | |

By summing the weights of water calculated and dividing by that of cement per unit volume, the probable water content and w-c ratio for a reasonable degree of workability will be given, which can be compared with that specified. Although this method of checking the water content of the mix is empirical, it is based on theoretical considerations of specific surface and surface tension and has been checked experimentally by Stewart.

7. The a-c ratio by weight will directly follow from the foregoing results.

The value of gap-graded concrete technology lies in the ease with which the seven simple procedures can be applied to the proportioning of concrete mixtures involving any unknown coarse and fine aggregates, with a degree of mathematical accuracy considerably greater than would be possible if a continuous grading were employed.

## ADJUSTMENTS FOR VARIANTS IN CONCRETE CONSTITUENTS

All concrete is affected by seasonal diurnal weather conditions both on site and frequently at the points where the aggregates are won. Probably the greatest cause of variations in the strength, density, and yield of the concrete is the lack of control of the water carried into the mix by the aggregates. Should this water fail to be taken into account, then the true a-c ratio would be reduced, and at the same time the w-c ratio would be increased. This may reduce the true yield because part of the water will be expelled or at least brought to the surface as the concrete is compacted and later more volume will be lost as a result of sedimentation (plastic shrinkage).

It is quite essential that all aggregates are monitored at least once a day to check their gradings and moisture contents and in certain circumstances their temperatures.

As the magnitude of a concrete construction work increases in size, because of the large quantities of constituent materials needed, there may exist nonuniformity in quarry products or river-run gravels, in sand pits, in water quality, and even in the composition, melt, and grinding of cements because all of them may not be supplied from a single source of constant quality. For these reasons, rigid quality control must be exercised on all the ingredients of concrete. To reduce the possible variants to a minimum, appropriate control and adjustments should be made from time to time on the following:

1. Quality, fineness, and performance of cement;
2. Quality, maximum and minimum size, and grading control of coarse aggregate;
3. Quality, maximum and minimum size, and grading control of fine aggregate;
4. Moisture content measurements and adjustments in water content;
5. Quality of mixing water;
6. Batching control of all ingredients by weight; and
7. Appropriate adjustments of weights of cement and water if admixtures are introduced.

These controls and adjustments apply whether the concrete is site-mixed or supplied ready-mixed.

It must be emphasized that, with gap-graded aggregates, not only can the maximum, minimum, and average sizes be easily controlled but also the grading control will, in the absence of intermediate-sized coarse aggregate, become greatly simplified, thereby giving minimum liability to segregation, increased uniformity in the characteristics of the aggregates, and least variation in the quality and strength of the concrete they produce.

## FULL CONSOLIDATION OF DRY MIXTURES

There is no problem in consolidating readily flowing fresh concrete of very wet mixtures. But under no circumstances will such a mixture make a durable concrete in the hardened state. Consolidation must by nature associate itself with a process of compacting the rather dry mixtures with very low slumps.

In fact, because of the many types of vibrators available for consolidating fresh concrete, there is not the slightest justification today to use any mixture with a slump higher than 2 in., and it is preferable to keep it within 1 in.

Slump is by no means as good a measure for the workability of dry mixtures (with slumps from 0 to less than 2 in.) as is Vebe time. It is being referred to here only for its wide use.

Nearly all reasonably proportioned mixes in the range of a-c ratios from 2:1 to 10:1 with corresponding w-c ratios from 0.25 to 0.65 can be consolidated by vibration. In the hearting of mass concrete, large-sized coarse aggregates ($4\frac{1}{2}$ in. to 3 in.) can be used. Even an a-c ratio of as high as 14:1 was used and consolidated in a Scottish dam.

Any mix can have its own particular maximum consolidation by driving out all air entrapped during batching, mixing, transporting, and depositing and by compacting fresh concrete around all reinforcements and inserts and into all reentrant corners. Such particular maximum consolidation, however, may not be the obtainable optimum, and it does not guarantee the best quality at the minimum cost.

Full consolidation has to be achieved first through optimum proportioning to remove all the adverse conditions. This requires that the optimum functions of the constituents of concrete be utilized and adhered to when proportioning aggregate sizes, quantities, water and cement contents, and ratios for the desired strengths.

With $\frac{3}{4}$-in. maximum-sized coarse aggregate, a "zero-slump" concrete may have a low compacting factor of around 0.67 to 0.69 and can be consolidated without difficulty by vibro-pressure compaction. A concrete of 0- to 1-in. slump may have a medium compacting factor of around 0.77 to 0.79 and can be consolidated with fairly intense vibration. A concrete of 1/4- to 2-in. slump may have a rather high compacting factor of around 0.84 to 0.86 and can be easily consolidated either manually or mechanically. With gap grading, though the fresh concrete may appear harsh and even have the lower compacting factor, it is really more workable, requiring no more vibration time, by virtue of its inherent smaller specific surface, than continuous grading.

The generally drier, but not less compactible, gap-graded concrete exerts much less pressure on the formwork and permits earlier stripping of forms to allow sooner reuse and more timely finishing than does the conventional concrete. These advantages, coupled with the much-improved physical and mechanical properties and the great savings in cement, have made gap-graded concrete a more economical concrete that is more readily consolidated.

## CONCLUSIONS

The foregoing concepts have been evolved through the authors' extensive research and development in the key issues of concrete technology and through their wide practical applications from prior to 1949 to the present.

To recapitulate in the simplest terms, they are based on four mutually dependent principles:

1. Compatible gradation of coarse and fine aggregates will ensure the feasibility of an optimum void-filling method for concrete proportioning and make it a unique method of mathematical accuracy;
2. Optimum void-filling concrete proportioning will always constitute the easiest and simplest method for rationally designing concrete mixtures out of sound aggregates, even of otherwise unknown characteristics;
3. Compatible gradation of aggregates and optimum void-filling proportioning will realize full consolidation of plastic concrete of the largest a-c ratio; and
4. It is this concrete of largest a-c ratio, having consistently minimum specific surface, that will ensure the greatest economy yet still retain compactible workability.

# THE EFFECT OF GRADING ON LEAN-MIX CONCRETE

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Severe nontraffic-load-associated cracking of lean-mix concrete (cement-treated) bases in the highveld region of the Republic of South Africa led to a laboratory investigation into some engineering properties of this material. This paper covers the first part of the laboratory study in which the effect of grading on flexural strength, tensile strength, strain at break, and drying shrinkage strain were investigated. Three aggregate gradings using the same aggregate with varying cement and water contents were tested at approximately 96 percent modified AASHO density in beam flexure, direct and indirect tension, and drying shrinkage. A mix having a relatively low water-cement ratio and the coarsest grading yielded the highest flexural strength, static elastic modulus, and tensile strength and lowest short-term drying shrinkage strain. These findings point to a possible revision of current specifications for lean-mix concrete bases that could lead to an improved performance of this base material in practice.

•THE active implementation of a long-term rural freeway plan in the Republic of South Africa has not come without its attendant problems to the materials engineer. Predicted traffic volumes and loadings that will use these pavements during their design life have demanded materials of the highest quality. Because of the element of uncertainty that existed during the early 1960s with respect to the long-term performance of untreated crushed-rock bases under heavy freeway traffic, certain freeway pavements were constructed with lean-mix concrete bases under relatively thin bituminous surfacings. Lean-mix concrete bases in this paper are equivalent to cement-treated bases where crushed-rock aggregate is used. Under the climatic conditions of the highveld in the Transvaal, these pavements showed severe block cracking even before the freeways were opened to public traffic (Fig. 1).

Initial investigations into these nontraffic-load-associated cracks pointed to drying shrinkage and possibly temperature effects as being the principal causative factors. Material properties that were considered to be important in the formation of cracks were tensile strength, strain at break, and amount of short-term drying shrinkage.

A research program was therefore initiated to investigate the effect of aggregate grading on these material parameters. The research program is still in progress, and this paper constitutes a progress report of some of the initial findings.

## LABORATORY STUDY

The laboratory study was divided into two phases: one involving materials recovered from actual in-service pavements and the other laboratory-prepared materials. The first phase of the study was necessary to establish the elastic properties of lean-mix concrete to analyze the traffic-load-associated stresses and strains that occur in practice. This aspect, although important in the overall performance of lean-mix concrete bases, will not be dealt with in this paper.

A number of laboratory testing techniques were used to measure the properties of prepared specimens of lean-mix concrete. These techniques are briefly described.

### Beam Flexure Test

Prismatic beam specimens measuring 75 by 75 by 450 mm were subjected to a third-point loading test. The load was applied gradually and was measured throughout the test with a load cell. The resultant deflection of the beam was measured at the midpoint by two linear variable differential transformers (LVDTs). Compression at the points of load transfer was eliminated from the deflection measurements by measuring the deflection of the beam relative to a datum that was at a fixed distance from the initial top surface of the beam. A continuous record of the load and deflections was made, and the average deflection was used in conjunction with the applied load to calculate the flexural bending strength, static elastic modulus, and strain at break. The flexural bending strength was defined as the maximum bending moment divided by the section modulus (Z) of the beam. Using the load and deflection data and applying normal elastic beam theory, we obtained a stress-strain relation. The slope of the straight-line portion of this relation was taken as the static elastic modulus in bending. Figure 2 shows a view of the apparatus during a test.

### Direct Tensile Tests

Direct tensile tests were performed on 75- by 75- by 215-mm prismatic beam specimens by applying a gradually increasing load to the ends of the specimen. Load transfer was achieved by gluing metal loading heads to the ends of the specimen with quick-drying synthetic polyester resin. So far as possible, eccentric loading was eliminated by using ball-and-socket joints between the platens of the testing machine. The tensile load was measured by means of a load cell, and the deformation, in the direction of the applied load, was measured by means of an LVDT fixed between the ends of the specimen as shown in Figure 3. The load and deformation were recorded simultaneously on the vertical and horizontal axes of an X-Y recorder.

From the trace obtained, it was possible to calculate the tensile strength, strain at break, and tangent elastic modulus in tension of the lean-mix concrete tested.

### Indirect Tensile Test

Cylindrical specimens measuring approximately 100 mm in diameter and 65 mm high were tested in the Brazilian test by applying a gradually increasing load at diametrically opposite points on the cylindrical face of the specimen. Case-hardened loading strips 25 mm wide were used to spread the applied load as recommended by Hudson and Kennedy (1). The maximum load resisted by the test specimen was recorded on the dial of the testing machine. The indirect tensile strength was calculated from $S = (2P/\pi td)$ where S = indirect tensile strength (Pa), P = maximum load (N), t = average height of specimen (m), and d = nominal diameter of specimen (m).

### Drying Shrinkage Test

Prismatic beam specimens, similar to those used for direct tensile tests, were used for continuous short-term drying shrinkage measurements. Both ends of each prism were capped with a 25- by 25-mm brass plate that was glued in position by applying the glue over a small area on the axis of the beam. A solid metal frame designed to hold two rows of six specimens in a vertical position provided a datum from which the shrinkage measurements could be made. The frame containing the specimens was placed in an environmental cabinet set at 25 C and 90 percent relative humidity. The humidity level was chosen to simulate, as accurately as possible, the drying rate pertaining to a lean-mix concrete base under the local environment. Expansion or contraction of the metal frame could be disregarded because all measurements were made at a constant temperature. Each specimen was supported on three pins and located directly below an LVDT fixed to the top of the frame. Shrinkage of each specimen was measured by the LVDT and automatically recorded on a data logger. Measurements were recorded at hourly intervals for the first 2 days and thereafter at 3-hour intervals throughout the testing period.

Shrinkage tests were continued until the shrinkage was considered to be negligible over a period of 2 days. It generally took at least 3 weeks before this stable condition developed.

## TESTING OF MATERIALS RECOVERED FROM THE ROAD

The first phase of the laboratory testing program consisted of beam flexure tests on lean-mix concrete base materials recovered from various pavements. Major samples, consisting of slabs measuring approximately 600 by 700 mm, were recovered and thereafter cut by means of a diamond saw into six test specimens of the required dimensions. The span-to-depth ratio of the cut specimens was chosen to exceed the minimum value of five recommended by Pretorius (2). The ratio of the minimum specimen size to the maximum aggregate size used varied between two and four. Results of seven pavements sampled are given in Table 1. Values reported are the average of six results.

Current specifications for lean-mix concrete base material in South Africa generally impose an upper limit of 5 percent on the cement content and give an envelope within which the aggregate grading should fall (3). The water content used is that which provides maximum density under modified AASHO compaction. The specified envelope of aggregate grading is shown in Figure 4 as gradings B and C.

## TESTING OF LABORATORY-PREPARED MATERIAL

One of the objectives of the laboratory study was to isolate the effect of aggregate grading on the engineering properties of lean-mix concrete. Laboratory-prepared mixtures were thus tested in gradings A, B, and C as shown in Figure 4. The inclusion of a finer grading than that allowed by the current specification envelope was based on the work of Johnson (4) who showed that, for concrete, both tensile and compressive strengths increase with a decrease in mean aggregate size.

### Description of Material Used

The types of both aggregate and ordinary portland cement used during the laboratory investigation were kept constant. Domestic water was used in all the mixtures. The aggregate was a reef quartzite with typical physical properties as given in Table 2.

The cement, while still in a hot condition, was obtained directly from a local cement factory. Results of physical and chemical tests on the cement used are given in Table 3.

For the three gradings investigated, the cement content was varied between 3 and 7 percent; generally the water content used was that which gave a maximum density at modified AASHO compactive effort for 5 percent cement content. Results of these tests are given in Table 4, which include dry densities obtained at 5 percent content and optimum moisture content between 3 and 7 percent.

Test prisms were compacted in collapsible steel molds, on a vibrating table operating at 50 cycles per second, to approximately 96 percent modified AASHO density. It required approximately 2 min of compaction to attain the required density.

## INFLUENCE OF GRADING ON TEST PARAMETERS

### Beam Flexure Tests

Test specimens for beam flexure tests were compacted and then cured in a moist room at 25 C ± 3 C for a period of 28 days before testing.

Figures 5, 6, and 7 show respectively the results of flexural bending strength, static elastic modulus, and strain at break for gradings A, B, and C at cement content levels of 3 and 7 percent. The aggregate gradings were defined in terms of mean particle diameter of the aggregate or more simply mean aggregate size. According to Johnson (4), mean aggregate size is defined as $\Sigma(\sqrt{d_1 d_2}/f)$ (percentage of particles between sieve sizes $d_1$ and $d_2$) where d is the sieve size in mm and f is an angularity factor defined by Loudon (5) taken as 1.5 for the quartzite used. Grading C, having the largest mean aggregate size, gave the highest values of flexural bending strength, static elastic modulus, and strain at break, and grading A gave the lowest values.

### Direct Tensile Tests

Direct tensile tests were performed on specimens that were cured at 25 C ± 1 C for 7 days in sealed plastic bags and thereafter dried at 90 percent relative humidity at

Figure 1. Pavement showing block cracking of lean-mix concrete base.

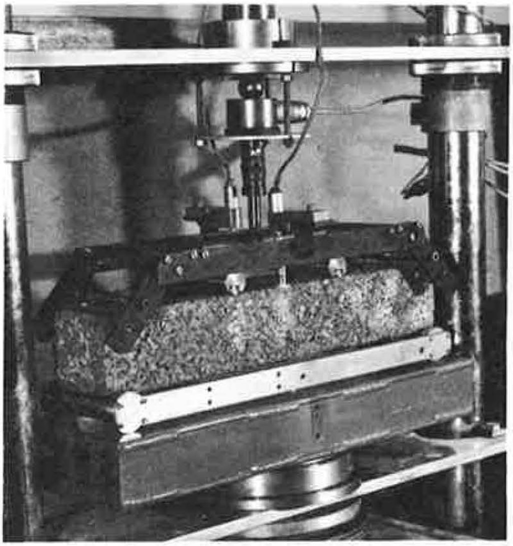


Figure 2. Beam flexure test apparatus.

Table 1. Results of flexure tests on specimens of lean-mix concrete removed from existing pavments.

| Contract | Flexural Bending Strength (MPa) | Coefficient of Variation (percent) | Strain at Break ($\mu\epsilon$) | Coefficient of Variation (percent) | Static Elastic Modulus (GPa) | Coefficient of Variation (percent) |
|---|---|---|---|---|---|---|
| A | 1.39 | 6 | 113 | 4 | 18.5 | 10 |
| B | 2.68 | 9 | 181 | 11 | 23.8 | 11 |
| C | 3.07 | 15 | 145 | 12 | 27.9 | 12 |
| D | 2.05 | 19 | 134 | 19 | 20.6 | 14 |
| E | 0.78 | 7 | 114 | 23 | 16.4 | 17 |
| F | 4.38 | 9 | 148 | 11 | 38.9 | 18 |
| G | 0.45 | 20 | 182 | 10 | 5.9 | 23 |

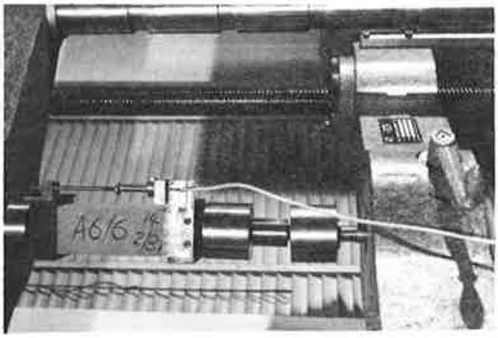



Figure 3. Direct tensile test apparatus.

Table 2. Typical results of physical tests on aggregate used in study.

| Property | Value | Unit | Coefficient of Variation (percent) |
|---|---|---|---|
| Elastic modulus | 79 | GPa | 4 |
| Uniaxial compressive strength | 240 | MPa | 4 |
| Uniaxial tensile strength | 10.8 | MPa | 11 |
| Poisson's ratio | 0.13 | – | – |
| Density | 2,710 | $kg/m^3$ | – |
| Water absorption (ASTM) | 1 | Percent | – |

Figure 4. Three aggregate gradings used in laboratory investigation.

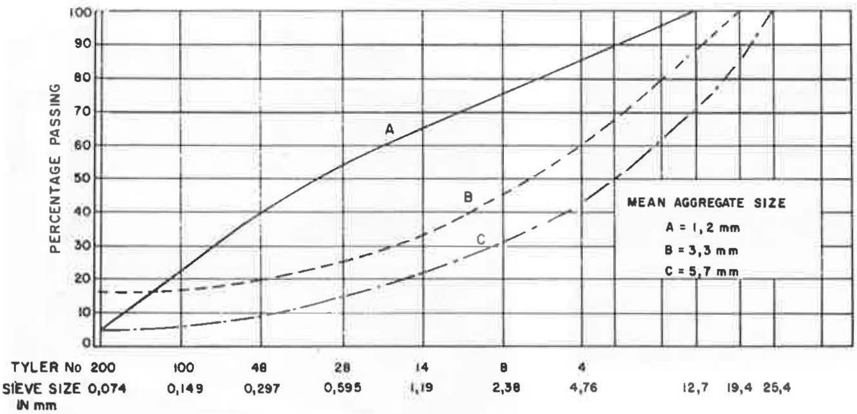



Table 3. Results of physical and chemical tests on concrete used in study.

| Physical Test | Value | Chemical Analysis | Percent |
|---|---|---|---|
| Standard consistency (percent) | 26.3 | $SiO_2$ | 21.59 |
| Initial set (min) | 150 | Insoluble residue | 0.38 |
| Final set (min) | 195 | $Fe_2O_3$ | 2.83 |
| False set (mm) | 1 | $Mn_2O_3$ | 0.07 |
| Expansion (mm) | 1 | $Al_2O_3$ | 4.68 |
| Autoclave expansion (mm) | 0.11 | CaO | 63.0 |
| 170 mesh residue (percent) | 6.6 | MgO | 3.54 |
| 72 mesh residue (percent) | 0.2 | $SO_3$ | 2.47 |
| Surface area ($cm^2/g$) | 2,670 | Loss on ignition (900 C) | 0.80 |
| Compressive strength | — | — | — |
| Vibration machine, 3 days (MPa) | 28.9 | Free lime | 1.66 |
| Vibration machine, 7 days (MPa) | 38.8 | — | — |

Table 4. Results of water content and dry density tests.

| Grading | Cement Content (percent by mass of aggregate) | Water Content (percent by mass of aggregate and cement) | Dry Density ($kg/m^3$) |
|---|---|---|---|
| A | 3 | 7 | 2,115 |
| | 5 | 5 | 2,091 |
| | 5 | 7[a] | 2,158[a] |
| | 5 | 9 | 2,110 |
| | 7 | 7 | 2,126 |
| B | 3 | 6 | 2,270 |
| | 5 | 4 | 2,215 |
| | 5 | 6[a] | 2,225[a] |
| | 5 | 8 | 2,200 |
| | 7 | 6 | 2,259 |
| C | 3 | 5 | 2,345 |
| | 5 | 3 | 2,247 |
| | 5 | 5[a] | 2,323[a] |
| | 5 | 7 | 2,302 |
| | 7 | 5 | 2,339 |

[a]Optimum water content and maximum dry density at 5 percent cement content.

**Figure 5. Relation between bending strength and mean aggregate size for lean-mix concrete.**

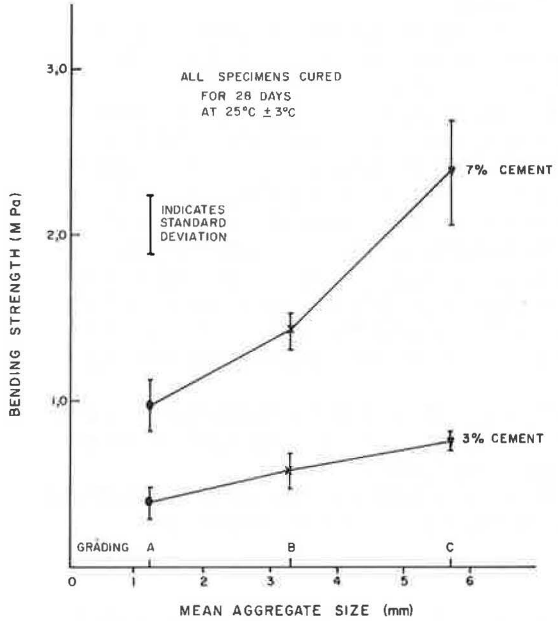



**Figure 6. Relation between static elastic modulus in flexure and mean aggregate size of lean-mix concrete.**

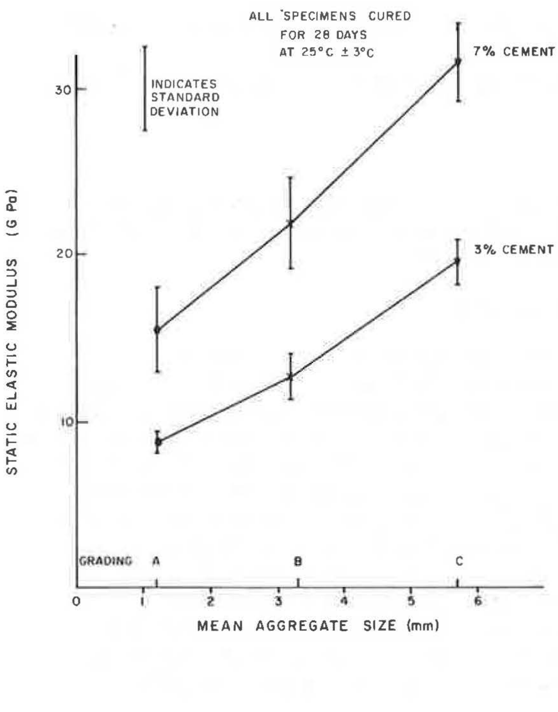
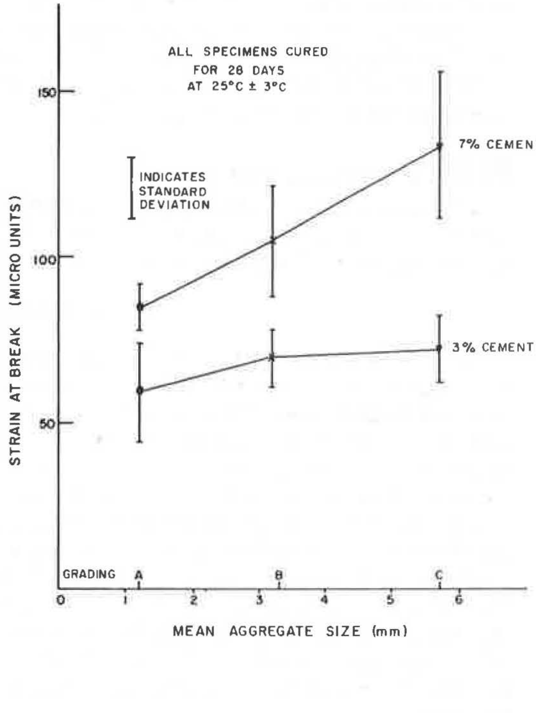



**Figure 7. Relation between strain at break and mean aggregate size of lean-mix concrete.**



25 C in an environmental cabinet for periods of 0, 7, 14, and 28 days. Gradings A, B, and C were investigated at cement contents of 3, 5, and 7 percent and at optimum water contents of 7, 6, and 5 percent respectively (Table 4).

To vary the water-cement (w-c) ratio, we made a second series of tests at a constant cement content of 5 percent where the water content was varied by 2 percent above and below the optimum value for gradings A, B, and C.

Certain difficulties were experienced in obtaining acceptable values of the strain at break. This was caused by the relatively weak material sometimes bending during the tensile test, thus giving completely meaningless strain values. So far as could be ascertained, this bending was not due to eccentric loading being applied. It resulted because, under tension, failure did not occur uniformly in a plane at right angles to the axially applied load, but rather microcracks formed on one or two faces of the prism and propagated slowly through the material. During this phase internal eccentric loading conditions prevailed, causing slight bending of the prism. This condition was immediately obvious from the load-deformation curve recorded on the X-Y recorder, and unacceptable results could thus be eliminated. An entirely satisfactory solution to this problem has not yet been found.

Values of strain at break varied considerably, and there was no meaningful statistical relation with respect to grading, w-c ratio, or drying period. However, average values and their coefficients of variation are given in Table 5.

The relation between direct tensile strength and w-c ratio for the various periods of drying for gradings A, B, and C are shown in Figures 8, 9, and 10 respectively.

### Indirect Tensile Tests

Indirect tensile tests were performed on cylindrical specimens of lean-mix concrete compacted to approximately 96 percent modified AASHO density in a Hveem kneading compactor. Before testing, the specimens were cured for 7 days in sealed plastic bags at 25 C ± 1 C. For this series of tests the water content was kept constant at 7, 6, and 5 percent for gradings A, B, and C respectively, and the cement content varied over a wide spectrum of 3 to 12 percent.

Figure 11 shows the relation between indirect tensile strength and mean aggregate size for cement contents of 3, 5, 7, and 10 percent.

The relation between indirect tensile strength and w-c ratio for the three gradings investigated is shown in Figure 12.

### Drying Shrinkage Tests

Shrinkage strains obtained at cement contents of 3, 5, and 7 percent are shown against mean aggregate size in Figure 13. The effect of w-c ratio on shrinkage strain for the three gradings investigated is shown in Figure 14.

## DISCUSSION OF RESULTS

The first phase of this laboratory study, where specimens of lean-mix concrete were recovered from in-service pavements and tested in beam flexure, proved to be of great value in the planning of the subsequent study of laboratory-prepared specimens. Results of flexural bending strength and static elastic modulus obtained on the recovered specimens (Table 1) show variations by factors of 10 and 7 respectively. These large variations occurred in spite of the aggregate gradings used on the seven contracts falling within the envelope given by gradings B and C (Fig. 4) and the cement content being constant at the 4 percent level. An explanation of this large variation in properties could be that different curing, compaction, and construction techniques were used by the various contractors. It is important to note that the ratio of flexural bending strength to static elastic modulus is nearly constant, varying only by a factor of 1.6. This ratio is a function of the strain at break and is obviously much less affected by construction and curing techniques than the flexural bending strength and static elastic modulus.

**Table 5. Analysis of strain at break under direct tension.**

| Material Composition | Average Strain at Break ($\mu\epsilon$) | Coefficient of Variation (percent) |
|---|---|---|
| All available results (all gradings, w-c ratios, and drying periods) | 120.6 | 50 |
| All results on grading A (all w-c ratios and drying periods) | 129.7 | 43 |
| All results on grading B (all w-c ratios and drying periods) | 129.8 | 49 |
| All results on grading C (all w-c ratios and drying periods) | 99.3 | 56 |

**Figure 8. Relation between direct tensile strength and w-c ratio of lean-mix concrete grading A.**

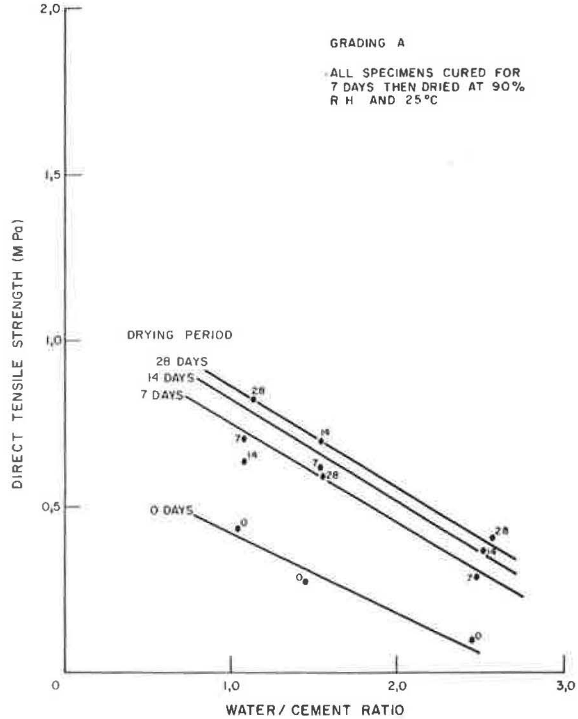



**Figure 9. Relation between direct tensile strength and w-c ratio of lean-mix concrete grading B.**

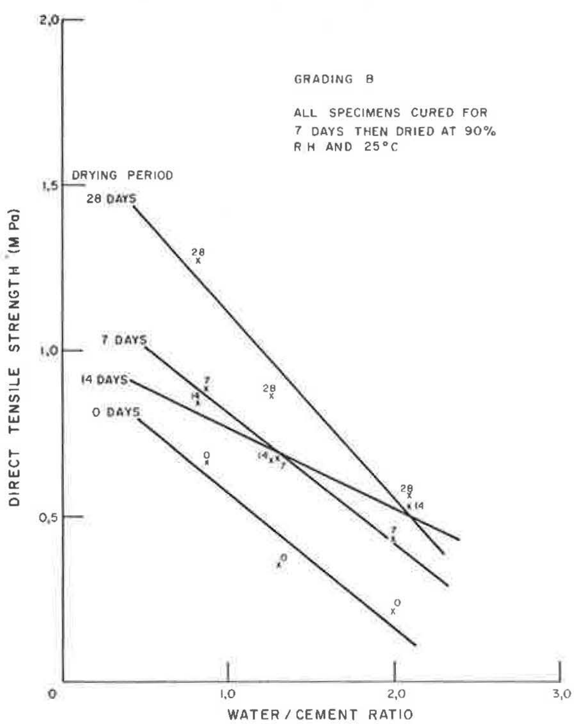



**Figure 10. Relation between direct tensile strength and w-c ratio of lean-mix concrete grading C.**

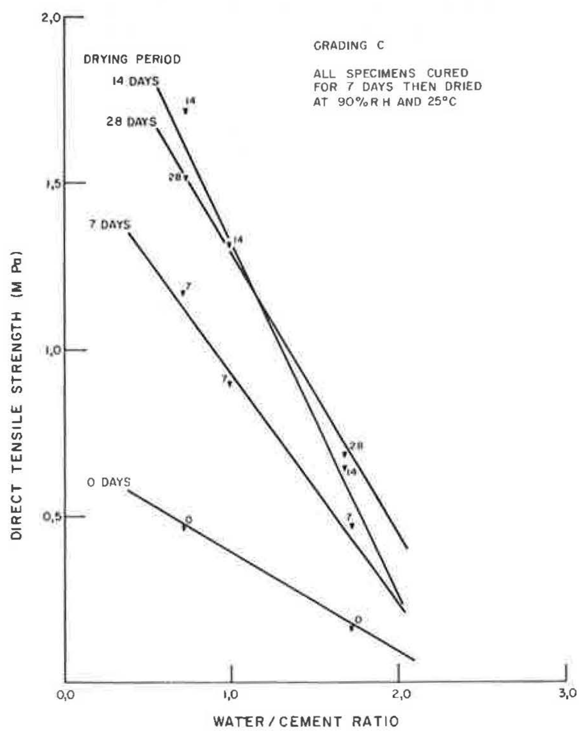


Figure 11. Relation between indirect tensile strength and mean aggregate size of lean-mix concrete.

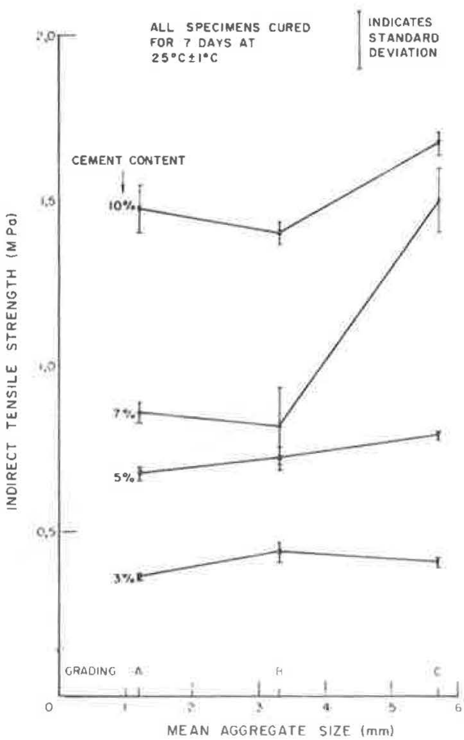



Figure 12. Relation between indirect tensile strength and w-c ratio of lean-mix concrete gradings A, B, and C.

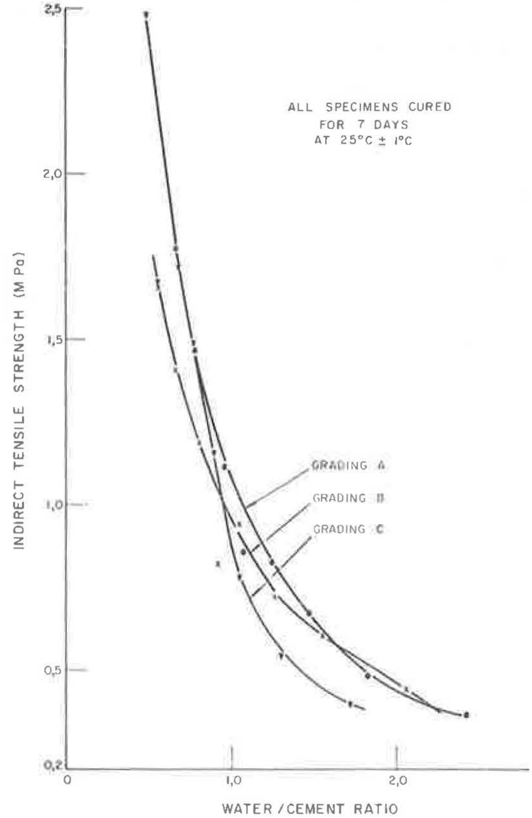



Figure 13. Relation between shrinkage strain and mean aggregate size of lean-mix concrete.

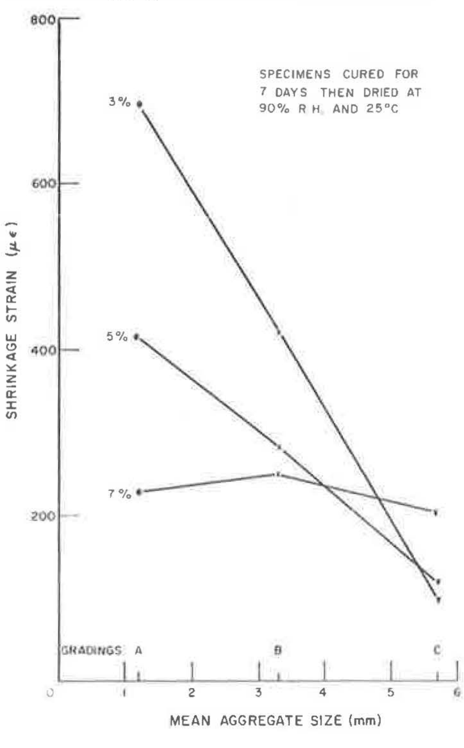
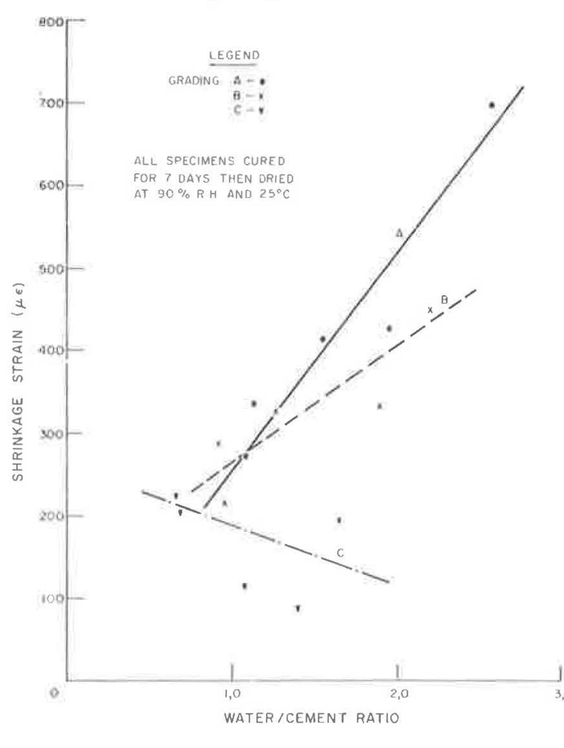



Figure 14. Relation between shrinkage strain and w-c ratio of lean-mix concrete gradings A, B, and C.



Because of the importance of the strain at break in the structural design of lean-mix concrete bases, part of the second phase of the laboratory study was aimed at investigating the effect of grading on this and other engineering properties of lean-mix concrete.

Results from the beam flexure test on laboratory-prepared specimens show that the strain at break for aggregate gradings A, B, and C at 3 percent cement content does not vary significantly; however, at the 7 percent cement content level, there is a significant difference among the strains for all gradings. Grading C gives the largest strain at break, approximately 130 $\mu\epsilon$ (Fig. 7). This value is not very different for the strain at break obtained from samples with an average age of about 1.5 years recovered from the in-service pavements that gave an average value of 145 $\mu\epsilon$ with a standard deviation of $\pm 28$ $\mu\epsilon$. These results are important in terms of the structural design of lean-mix concrete bases under traffic loading and give a clear indication that improved performance will result from gradings that approximate grading C rather than gradings A or B. Grading C is the finest of the three gradings investigated when a cement content of at least 7 percent is used. Beam flexure tests also show a significant improvement in bending flexural strength and static elastic modulus for grading C, which has a mean aggregate size of 5.7 mm. This trend is more marked with the mixtures containing 7 percent cement than those of the 3 percent cement content level (Figs. 5 and 6).

Hughes and Ash (6), summarizing the findings of various researchers (7, 8), concluded that, when the workability of a concrete mix is kept constant by altering the w-c ratio, increasing the aggregate size results in a gain in strength for low cement contents; for high cement contents, however, a drop in strength results.

In these tests the cement content levels of 3 and 7 percent used may be regarded as low, and the workability of the mixes was kept constant by using the optimum water content, which gave a maximum density under modified AASHO compactive effort for mix gradings A, B, and C. From Table 4 it can be seen that the coarser gradings have lower optimum water contents, resulting in a decreasing w-c ratio at a constant cement content. The results obtained in this study therefore confirm the findings of the other researchers.

Direct tensile test results show a relation between tensile strength and w-c ratio; low w-c ratios give higher tensile strengths for all three gradings investigated (Figs. 8, 9, and 10). An improvement in tensile strength is also obtained on drying at 90 percent relative humidity; however, this is not very marked after 14 days. Here again, grading C has given the highest tensile strength, particularly after drying periods of 7 days and longer where the w-c ratio is below about 1.5.

Indirect tensile test results show a marked increase in tensile strength for grading C at cement content levels of 7 percent and above (Fig. 11). Below this level of cement content there is not a significant effect on indirect tensile strength with the three gradings investigated. The results of indirect tensile strength for various w-c ratios after a period of 7 days' curing, which is equivalent to specimens tested in direct tension without any drying, show the same trend; i.e., under these conditions a rapid increase in tensile strength is obtained for a decrease in w-c ratio. However, the variation of grading tested does not have a significant effect on the early tensile strength of the lean-mix concrete (Fig. 12). It should be noted that the indirect tensile strength values obtained are roughly two to three times the direct tensile strength values.

In terms of drying shrinkage cracking, the tensile strength and strain at break are of paramount importance. The strain at break values obtained in direct tension are very variable, there being no indication that either grading, cement content, or period of drying from 0 to 28 days has a significant effect on the level of strain at break.

Unrestrained shrinkage strain values vary considerably depending on grading and cement content; however, the effect of cement content is reduced as the mean aggregate size increases. For grading C, the effect of cement content is small as is shown in Figure 13. The w-c ratio has a significant effect on shrinkage strain, especially for gradings A and B; grading C is rather insensitive. It is of interest to note that, at a w-c ratio of between 0.8 and 1.0, the shrinkage strain is practically independent of grading and reaches a level of about 240 $\mu\epsilon$ (Fig. 14). From these results, it is obvious that grading C gives the lowest shrinkage strains for all cement contents and w-c ratios tested. The lowest strains obtained are in the order of 200 $\mu\epsilon$. From the results of both direct tensile and flexure tests, it is clear that, unless creep has a significant effect on the relaxation of tensile stresses during the drying shrinkage of lean-mix concrete, cracking will not be prevented but could be reduced by the correct selection of aggregate grading, cement, and water content.

## CONCLUSIONS

Initial results from a laboratory study have been very interesting, and tentative conclusions may be made as follows:

1. The grading of the aggregate plays an important role in the strength and shrinkage characteristics of lean-mix concrete;
2. Of the three gradings investigated, grading C, having a mean aggregate size of 5.7 mm, has the highest strength and strain at break and lowest drying shrinkage strain values within the range of cement and water contents tested;
3. The levels of drying shrinkage strain are in excess of the strain levels that can be tolerated by lean-mix concrete, and it is therefore unlikely that cracking of lean-mix concrete bases can be prevented using ordinary portland cement and normal aggregate gradings suitable to meet the strength requirements demanded by heavy dynamic traffic loads;
4. From considerations of both flexural strength and the reduction of drying shrinkage cracking of lean-mix concrete, a coarse grading, such as grading C with a mean aggregate size of 5.7 mm, should be used at a cement content of about 7 percent with a w-c ratio of between 0.8 and 1.0; and
5. A revision of current South African specifications for lean-mix concrete should be considered with respect to grading and cement content requirements to increase the strength, elastic modulus, and bending strain at break and to reduce the potential drying shrinkage.

## ACKNOWLEDGMENT

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# EFFECT OF VARIATIONS IN COARSE-AGGREGATE GRADATION ON PROPERTIES OF PORTLAND CEMENT CONCRETE

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The effect of variations in coarse-aggregate gradation on properties of a highway concrete mixture was studied by testing numerous batches of laboratory-prepared concrete. The water-cement and aggregate-cement ratios were the same for all batches; the only variable was the grading of the natural gravel used as coarse aggregate throughout the investigation. Each batch contained a known coarse-aggregate grading, and the entire group of gradings examined represented the variation determined to occur in typical pavement concrete production. The data were obtained and analyzed in a manner that allowed evaluation of the importance of variations in gradation of natural gravel as a source of variation in slump, compressive strength, and unit weight of field-produced concrete. On the basis of the results obtained in the investigation, typical field variations in coarse-aggregate gradation appear to have a relatively small effect on slump variability compared to the combined effect of other factors, a surprisingly large effect on compressive strength, and little effect on the "compactibility" of a low-slump paving mixture.

•VARIATIONS in coarse-aggregate gradation can be expected to occur on nearly any construction job because of variations in the source material, production, or segregation or degradation or both in the stockpiling operation. The extent of these variations is, of course, partly an economic matter, for close control can be achieved but generally only at greater cost to both the producer and the specifying inspecting agency. Unique gradations are therefore questionable economically and may or may not be better than other gradations.

This paper reports an investigation directed toward determining the quantitative effect of variation in coarse-aggregate gradation on the workability and strength of a highway concrete mixture.

The variation in coarse-aggregate gradation within a given production situation is nothing more than the occurrence of different gradings, within and among batches of concrete, caused by segregation, degradation, or inherent randomness. Though little work has been directed toward the effect of variation specifically, extensive research has been done on the influence of coarse-aggregate gradings on the properties of concrete. These studies provide information from which the qualitative effects of variability can be inferred (1-5).

In 1960 Bloem and Walker (6) dispelled traditional beliefs about strength being unaffected by gradation. They showed that strength is higher for smaller sizes of well-graded aggregates than for larger ones when the cement factor and slump are the same.

Gap gradings have been found to require a lower cement factor for the same workability. Li (5), in a study comparing the workability of mixes with 1½-in. and smaller maximum-sized aggregates, obtained more workable concrete from gap gradings than from continuous gradings, other factors (water-cement ratio, cement content, and maximum size of aggregate) being the same.

The following pertinent deductions can be made from these previous investigations by considering variation as the occurrence of fine, coarse, or gap gradings:

1. Variation in coarse-aggregate gradation causes changes in the workability of concrete, with relatively lower workability resulting from finer gradings (2, 3).
2. Variation causes changes in the compressive strength of concrete, with gradings having smaller maximum sizes increasing in strength (6).
3. Variations in the form of gap gradings may not cause changes in workability if the mix proportions are suitable (5).
4. The cement mortar-coarse aggregate volume ratio affects the sensitivity of the mix to changes in grading (4).

An experiment that parallels this investigation was done by Tynes (7), in which he concluded that mass concrete containing crushed limestone or natural gravel meeting the gradation specification did not show significant differences in compressive strength or workability.

The conclusions made by Tynes were applicable to concrete produced with gradings not exceeding the specification boundaries. The assumption that the extent of gradation variability in highway construction is within typical specification limits has been shown to be invalid by statistical evaluations of concrete aggregates. In a study by Mills and Fletcher (8), 20 percent of the individual sieve analysis results failed to conform to the specification limits.

The variability in some properties of highway concrete, also produced under conditions representative of the highway construction industry, has been measured and is discussed in connection with the results of this investigation.

## INVESTIGATION PLAN

The scope of gradation variability investigated was that associated with typical highway concrete production. An estimate of this variability (standard deviation) was obtained from the statistical studies of other investigators and from the authors' statistical analyses of sieve analysis data from the Indiana State Highway Commission (ISHC) aggregate inspectors' test reports. Batches of concrete that contained gradings having percentage passing curves within an envelope representing the estimated gradation variability were mixed in the laboratory and tested for slump, unit weight, compacting factor (3), and compressive strength.

The investigation was limited to a single set of proportions for a six-bag, low-slump mix and to natural gravel from one plant and one pit. Other factors affecting the variability of the concrete were reduced to a practical minimum. Cement from within a continuous run of the source plant and sand from a single stockpile were used throughout the investigation. The aggregates, both fine and coarse, were dried to eliminate the effect of varying moisture content. Though all variation from the many sources could not be totally eliminated, every effort was made to establish coarse-aggregate gradation as the sole variable within a concrete mixture designed to meet the ISHC requirements for slip-form pavement. The particular gradings examined were representative of all gradings within a percentage passing envelope covering the variation occurring in typical highway concrete aggregate.

The laboratory tests performed on each batch were intended to provide data that could be used to evaluate the effect of gradation on properties used to control and measure the quality and uniformity of field-produced concrete and to detect changes in workability. Standard slump, compressive strength, and unit-weight tests were made to evaluate the effect of grading on the commonly measured properties of field-produced concrete. A compacting factor apparatus was used to observe effects of grading on workability that might not be detected by the slump test. Unit-weight or density measurements were also made after vibrating the concrete for 10 and 20 sec. These data, combined with those from the standard rodded unit weight and compacting factor unit-weight tests, permitted evaluation of the effect of grading on compactibility. Those specimens observed to bleed most extremely during vibration were sawed after hardening and visually examined for segregation. The air content of five representative hardened specimens was also determined by microscopic methods in accordance with ASTM Designation C 457.

## SELECTION OF THE GRADINGS INVESTIGATED

Statistical evaluations carried out by state highway organizations have identified the magnitude of variation (standard deviation) in coarse-aggregate gradation occurring in paving projects under their jurisdiction (8, 9). The results of such studies were found to be comparable to the standard deviations for percentage passing obtained from statistical analysis of ISHC sieve reports done by the authors. Estimates were established on the basis of the various studies and were used in defining a range of gradings that would be representative of typical gradation variability for natural gravels used in paving concrete. These estimates are as follows:

| Sieve Size | Standard Deviation of Percentage Passing |
|---|---|
| 1 in. | 3.0 |
| ³⁄₄ in. | 8.0 |
| ¹⁄₂ in. | 9.0 |
| ³⁄₈ in. | 8.0 |
| No. 4 | 2.0 |
| No. 8 | 1.0 |

The analysis of ISHC sieve reports also showed the average percentage passing for each sieve to closely approximate the norm of the No. 5 gradation limits specified by the ISHC Standard Specification (10). The term "norm" is used to identify the grading defined by the average percentage passing for each sieve. The estimates were applied to the norm to obtain the $3\hat{\sigma}$ limits shown in Figure 1. The envelope outside the $3\hat{\sigma}$ limits shown in Figure 1 was established as the boundary for the range of gradings to be investigated. Figure 2 shows the continuous gradings of major interest. Figure 3 shows four gradings that typify the variations examined for each sieve. Table 1 gives all 33 gradings examined. As a group, the gradings are representative of variations that occur in natural gravel gradation during production of paving concrete.

The inclusion of continuous gradings having a variation of $\pm 3\sigma$ occurring simultaneously on each sieve is considered conservative. Statistically, the assumption of a normal distribution with regard to the concurrent variation for the series may be extreme, for a binomial distribution might better describe the combinations of individual variation that occur. More practically interpreted, a positive variation on one sieve might more typically be accompanied by negative variation on one or more of the other sieves than by other positive variations. In defense of the possible overestimation, it should be pointed out that a lens of fine or coarse particles within a stockpile could result in the extreme gradings represented by the $\pm 3\sigma$ continuous gradings.

It is noted that the ISHC No. 5 coarse-aggregate gradation envelope is located at approximately the $1.5\sigma$ level of the estimated gradation variation. This indicates that approximately 15 percent of sieve analysis test results would be expected to exceed the No. 5 limits, which is comparable to the 20 percent found by Mills and Fletcher (8).

## LABORATORY TESTS

The presentation and discussion of results make frequent reference to the fineness index of each coarse-aggregate grading. The index is comparable to the fineness modulus (ASTM Designation C 125) commonly used to describe the relative fineness of fine-aggregate gradings. The fineness index was calculated as $\Sigma r_i/100$, where $r_i$ equals the cumulative percentage retained on the individual sieves in the ISHC No. 5 gradation series, except for the No. 200 sieve. Table 2 gives the calculation of the fineness index for the norm grading. The terms "fine" and "coarse" with regard to grading are relative classifications on the basis of fineness index, finer gradings having lower indexes than coarser gradings. It should also be noted that the surface area of the coarse aggregate is inversely proportional to the fineness index; thus, the finer gradings have more surface area.

The grading numbers used to identify gradings refer to those given in Table 1.

**Figure 1. Range of gradings investigated.**

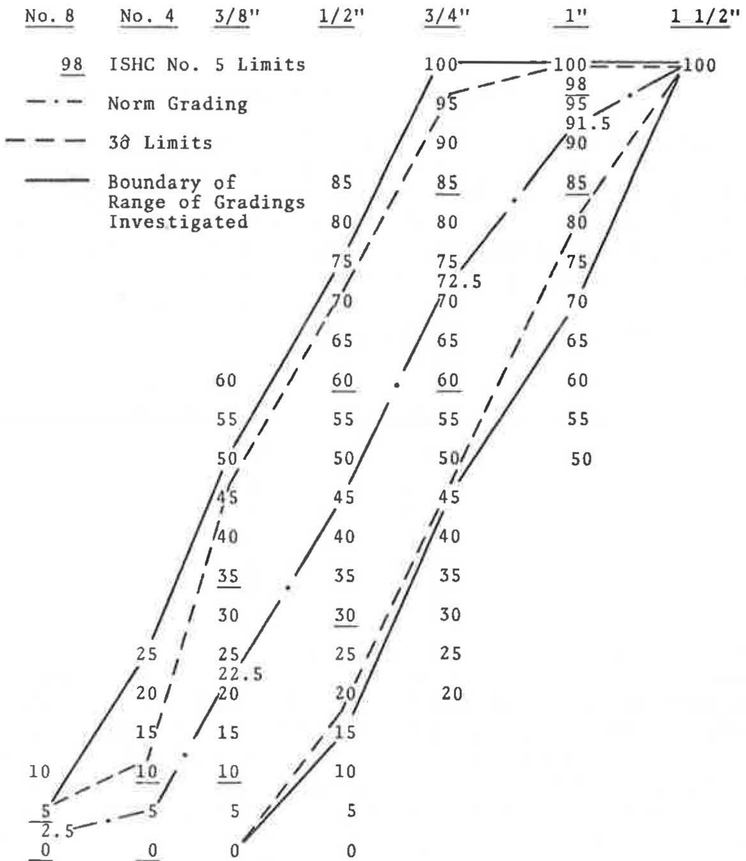


**Figure 2. Continuous gradings of major interest.**

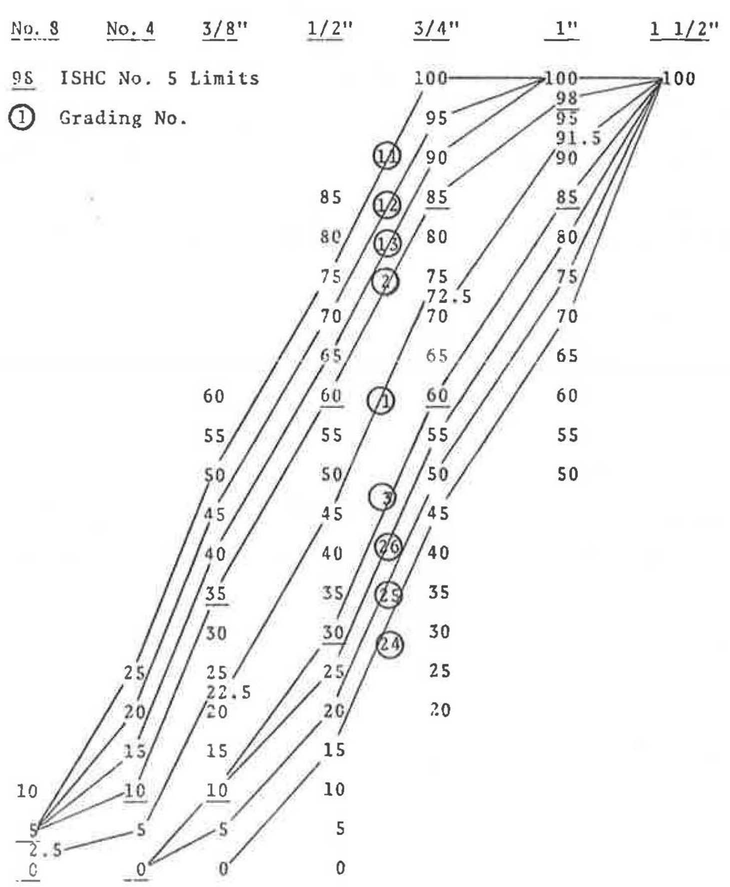

**Figure 3. Gradings typifying variations.**

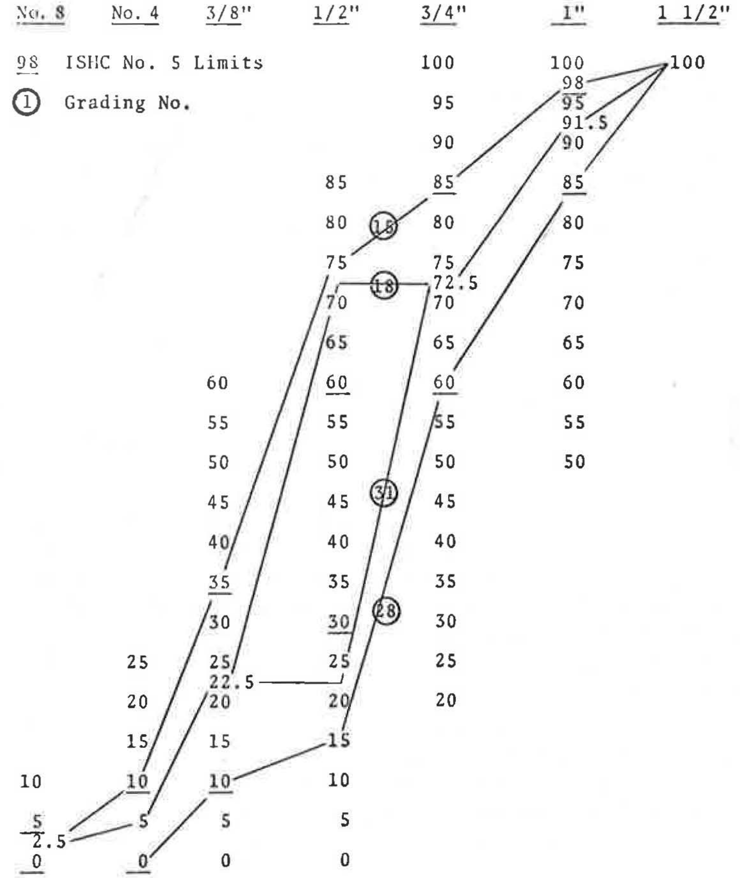


**Table 1. Cumulative percentage passing for gradings investigated.**

| Grading Number | Sieve Size (percent passing) | | | | | | |
|---|---|---|---|---|---|---|---|
| | 1½ In. | 1 In. | ¾ In. | ½ In. | ⅜ In. | No. 4 | No. 8 |
| 1 | 100.0 | 91.5 | 72.5 | 45.0 | 22.5 | 5.0 | 2.5 |
| 2 | 100.0 | 98.0 | 85.0 | 60.0 | 35.0 | 10.0 | 2.5 |
| 3 | 100.0 | 85.0 | 60.0 | 30.0 | 10.0 | 0.0 | 0.0 |
| 4 | 100.0 | 85.0 | 60.0 | 60.0 | 35.0 | 10.0 | 2.5 |
| 5 | 100.0 | 98.0 | 70.0 | 30.0 | 30.0 | 0.0 | 0.0 |
| 6 | 100.0 | 91.5 | 70.0 | 30.0 | 10.0 | 0.0 | 0.0 |
| 7 | 100.0 | 91.5 | 72.5 | 45.0 | 22.5 | 10.0 | 2.5 |
| 8 | 100.0 | 91.5 | 72.5 | 45.0 | 22.5 | 0.0 | 0.0 |
| 9 | 100.0 | 85.0 | 85.0 | 30.0 | 30.0 | 0.0 | 0.0 |
| 10 | 100.0 | 98.0 | 85.0 | 60.0 | 35.0 | 5.0 | 2.5 |
| 11 | 100.0 | 100.0 | 100.0 | 75.0 | 50.0 | 25.0 | 5.0 |
| 12 | 100.0 | 100.0 | 95.0 | 70.0 | 45.0 | 20.0 | 5.0 |
| 13 | 100.0 | 100.0 | 90.0 | 65.0 | 40.0 | 15.0 | 5.0 |
| 14 | 100.0 | 98.0 | 98.0 | 60.0 | 35.0 | 10.0 | 2.5 |
| 15 | 100.0 | 98.0 | 85.0 | 75.0 | 35.0 | 10.0 | 2.5 |
| 16 | 100.0 | 98.0 | 85.0 | 60.0 | 50.0 | 10.0 | 2.5 |
| 17 | 100.0 | 91.5 | 91.5 | 45.0 | 22.5 | 5.0 | 2.5 |
| 18 | 100.0 | 91.5 | 72.5 | 72.5 | 22.5 | 5.0 | 2.5 |
| 19 | 100.0 | 91.5 | 72.5 | 45.0 | 45.0 | 5.0 | 2.5 |
| 20 | 100.0 | 98.0 | 90.0 | 70.0 | 40.0 | 5.0 | 2.5 |
| 21 | 100.0 | 98.0 | 95.0 | 60.0 | 30.0 | 0.0 | 0.0 |
| 22 | 100.0 | 85.0 | 70.0 | 70.0 | 45.0 | 5.0 | 2.5 |
| 23 | 100.0 | 98.0 | 95.0 | 85.0 | 60.0 | 5.0 | 2.5 |
| 24 | 100.0 | 70.0 | 45.0 | 15.0 | 0.0 | 0.0 | 0.0 |
| 25 | 100.0 | 75.0 | 50.0 | 20.0 | 5.0 | 0.0 | 0.0 |
| 26 | 100.0 | 80.0 | 55.0 | 25.0 | 10.0 | 0.0 | 0.0 |
| 27 | 100.0 | 85.0 | 30.0 | 30.0 | 10.0 | 0.0 | 0.0 |
| 28 | 100.0 | 85.0 | 60.0 | 15.0 | 10.0 | 0.0 | 0.0 |
| 29 | 100.0 | 85.0 | 60.0 | 30.0 | 0.0 | 0.0 | 0.0 |
| 30 | 100.0 | 91.5 | 45.0 | 45.0 | 22.5 | 5.0 | 2.5 |
| 31 | 100.0 | 91.5 | 72.5 | 22.5 | 22.5 | 5.0 | 0.0 |
| 32 | 100.0 | 91.5 | 72.5 | 45.0 | 0.0 | 0.0 | 0.0 |
| 33 | 100.0 | 100.0 | 45.0 | 45.0 | 45.0 | 5.0 | 2.5 |

The various components of the variance associated with concrete production are discussed throughout the following sections. The term "overall" as used herein refers to the combination of testing, sampling, and actual variation of the material property being discussed. The term "testing" refers to the combined sampling and testing variation.

The coarse-aggregate gradings taken as a group represent the gradation variability associated with typical paving variations, and the results are often discussed in terms of the effect of gradation variability.

## RESULTS OF SLUMP TESTS

The average result for each grading is given in Table 3.

Homogeneity of variance among the test results was accepted after evaluating the data by the Foster-Burr test (11). Analysis of variance No. 1 (ANOV 1) showed the variation in slump, which resulted from the entire set of gradings examined, to be significant (Table 4).

Figure 4 shows the average slump plotted versus the fineness index for each grading examined. The plot suggests a general trend of increased slump over the entire range of gradings. A simple linear regression of slump on fineness index was found to be significant. This is not to say that the relation is best defined as simple linear but rather that finer gradings generally resulted in lower slump.

The following observations were made after examination of the average results for the various gradings:

1. Variation of the smaller sizes of aggregate seemed to have a greater effect on slump than similar variation of larger sizes,
2. The average result for each grading within the ISHC No. 5 limits did not exceed the ½- to 2-in. specification requirement, and
3. Gap gradings did not seem to affect slump differently from continuous gradings having approximately the same fineness index.

The variance $\sigma_g^2$ from ANOV 1 (Table 4) provides a quantitative measure of the effect of gradation, a measure that is more meaningful than the individual slump result for each grading. $\sigma_g^2$ is the variance in slump caused solely by all gradings examined during the investigation. The entire set of gradings represents the variation in gradation that might occur on typical highway projects, and so $\sigma_g^2$ can be considered representative of the variance in slump that results from the variation in gradation in typical highway concrete production. To use $\sigma_g^2$ in this manner it is necessary to assume that the batch-to-batch variations were minimized by the laboratory procedures used. ANOV's for other test data that permitted identification of among-batch variation showed batches to be significant, but this should not detract from the understanding that the degree of control maintained on the materials and mixing could not be matched in the field or reduced, to any great extent, by methods other than those used. Thus, $\sigma_g^2$ was taken as a maximum variance that could be observed above a practically minimized batch-to-batch variance.

As mentioned previously, the variation in slump for typical highway concrete has been measured. Table 5 gives the standard deviations, $s_a$, of the slump test results from a study by Hanna, McLaughlin, and Lott (12). The study involved field tests of highway concrete being produced under contract for the ISHC. The testing plan was conducted so as to identify and separate the testing and actual material variation. The portion of the actual material variation that can be attributed to variation in gradation can be calculated (13) as follows: $(\sigma_g^2/s_a^2) \times 100$. In making this calculation the gradation variability represented by the gradings examined in this laboratory investigation is assumed to characterize that same factor in the statistical model for the field studies.

The values of $s_a^2$ and $\sigma_g^2$ are of course estimates, and any comparison of them provides only a means of evaluating the relative importance of coarse-aggregate gradation variability. It should also be reemphasized that $\sigma_g^2$ is the result of conservative estimates of the $\pm 3\sigma$ variation in gradation, which would include 99.7 percent of the gradings occurring in typical production. Using the average standard deviation for slump of Indiana projects (12), we derive $(\sigma_g^2/s_a^2) \times 100 = (0.16/0.92) \times 100 = 17$ percent.

**Table 2. Calculation of fineness index for norm grading.**

| Sieve Size | Percentage Passing | Percentage Retained | Cumulative Percentage Retained |
|---|---|---|---|
| 1½ in. | 100.00 | 0.00 | 0.00 |
| 1 in. | 91.50 | 8.50 | 8.50 |
| ¾ in. | 72.50 | 19.00 | 27.50 |
| ½ in. | 45.00 | 27.50 | 55.00 |
| ⅜ in. | 22.50 | 22.50 | 77.50 |
| No. 4 | 5.00 | 17.50 | 95.00 |
| No. 8 | 2.50 | 2.50 | 97.50 |

Note: Fineness index = 361/100 = 3.61.

**Table 3. Average results of slump tests.**

| Grading Number | Fineness Index | Number of Batches | Average Slump (in.) |
|---|---|---|---|
| 1 | 3.61 | 7 | 1.3 |
| 2 | 3.09 | 2 | 1.0 |
| 3 | 4.15 | 2 | 1.9 |
| 4 | 3.47 | 1 | 1.3 |
| 5 | 3.72 | 1 | 1.8 |
| 6 | 3.98 | 1 | 1.8 |
| 7 | 3.56 | 1 | 0.5 |
| 8 | 3.69 | 3 | 1.6 |
| 9 | 3.70 | 1 | 1.8 |
| 10 | 3.14 | 2 | 1.3 |
| 11 | 2.45 | 3 | 0.8 |
| 12 | 2.65 | 1 | 0.5 |
| 13 | 2.85 | 3 | 1.0 |
| 14 | 2.96 | 3 | 1.3 |
| 15 | 2.94 | 3 | 1.0 |
| 16 | 2.94 | 3 | 0.8 |
| 17 | 3.42 | 3 | 1.2 |
| 18 | 3.33 | 3 | 1.6 |
| 19 | 3.38 | 1 | 1.0 |
| 20 | 2.94 | 2 | 0.8 |
| 21 | 3.17 | 1 | 1.0 |
| 22 | 3.22 | 1 | 1.0 |
| 23 | 2.55 | 1 | 0.3 |
| 24 | 4.70 | 3 | 1.8 |
| 25 | 4.50 | 1 | 1.5 |
| 26 | 4.30 | 3 | 2.3 |
| 27 | 4.30 | 3 | 1.9 |
| 28 | 4.30 | 3 | 1.7 |
| 29 | 4.25 | 3 | 1.6 |
| 30 | 3.88 | 2 | 1.4 |
| 31 | 3.86 | 2 | 1.4 |
| 32 | 3.91 | 3 | 1.8 |
| 33 | 3.58 | 1 | 1.5 |

**Figure 4. Average slump and fineness index for coarse-aggregate grading.**

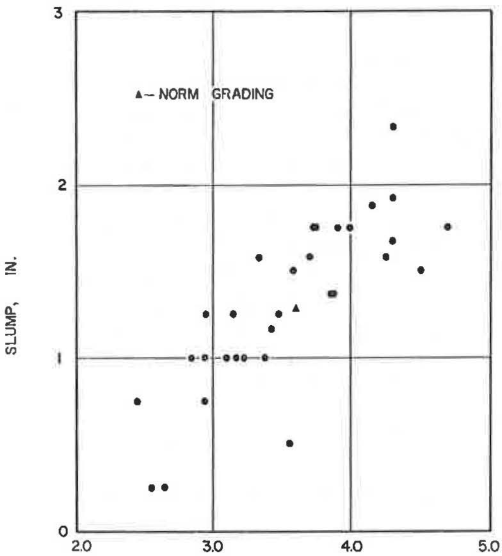


**Table 4. ANOV 1 for slump.**

| Source of Variation | Degrees of Freedom | Sum of Squares | Mean Square | Expected Mean Square | F-Test | F-Test (0.05) |
|---|---|---|---|---|---|---|
| Gradation | 32 | 13.7648 | 0.4301 | $\sigma_t^2 + 2.2\ \sigma_g^2$ | 5.62[a] | 1.73 |
| Error | 40 | 3.0640 | 0.0766 | $\sigma_t^2$ | — | — |

Note: $\sigma_g^2 = 0.16$.

[a]Significant.

Table 6 gives the results of studies by other state agencies, which were reported by Newlon (14) and the Federal Highway Administration (9). The results are for different projects having different types of paving equipment. Though a range in variation would be expected from job to job and state to state, the differences in the reported results are extreme and thus leave in doubt the actual variation one could expect in slump. In a summary statement, Newlon suggested 0.70 in. as an achievable overall standard deviation and 0.50 as desirable. Using these values to evaluate the importance of gradation variation, we derive the following: achievable control, $\sigma_g^2/s_o^2 \times 100 = 33$ percent; and desirable control, $\sigma_g^2/s_o^2 \times 100 = 64$ percent.

The results show that gradation significantly affected slump. This is consistent with accepted principles regarding the relation of gradation and workability. The results further suggest that the combined effect of sources other than coarse-aggregate gradation is the predominate factor in slump variability of paving concrete. For a magnitude of slump variability such as reported by Hanna, McLaughlin, and Lott (12), gradation variability would be of minor importance.

## RESULTS OF UNIT-WEIGHT TESTS ON SPECIMENS MADE BY RODDING

The average result for each grading is given in Table 7 along with the fineness index and number of batches.

Homogeneity of variance among the weights for each batch was tested and accepted using the Foster-Burr test. ANOV 2 (Table 8) showed batches to be significant and gradation to be not significant. This indicated either no effect of gradation on unit weight or no identifiable effect because of the significantly high batch-to-batch variance ($\sigma_b^2 = 2.0336$). Relating ANOV 2 to a t-test evaluation, we can use $\sigma_b^2$ as an estimate of variance for establishing the observable difference in the mean unit weight of any two gradings. It can be shown that, with three observations (batches) per grading and $\sigma_b^2 = 2.0336$, a difference of approximately 4 lb/ft$^3$ can be distinguished at a reasonable level of confidence ($\alpha = 0.05$ and $\beta = 0.1$). Thus it can be concluded that the unit weights resulting from all the gradings examined did not differ more than approximately 4 lb/ft$^3$ or 3 percent. A plot of the average results for each grading is shown in Figure 5.

Hanna, McLaughlin, and Lott (12), in a study cited by Newlon (14), obtained a testing variance of 1.15 lb/ft$^3$. Comparing this with the testing variance ($\sigma_t^2 = 0.3904$) from ANOV 2, the sampling and testing procedures of this investigation were seen to be relatively good.

The unit weights were determined from weights of fresh concrete compacted in 6- by 12-in. paperboard cylinder molds (ASTM Designation C 470). The size and construction of the container were expected to cause a relatively high testing variance, but the results did not indicate this.

The results are of interest from two aspects: the relation of unit weight or density to air content and strength and the use of unit weight for evaluating uniformity in the yield. The results showed that coarse-aggregate gradation variability did not substantially affect the unit weight of the concrete as measured by the standard testing procedure. The results (Fig. 5) suggest that, if density is affected at all by gradation, it is less so for fine gradings. Thus any effect of density on the strength of the hardened specimens made by rodding should have resulted in lower compressive strength for finer gradings. This result is important to the interpretation of the compressive strengths obtained for these same specimens. The calculated yield of the concrete mix would vary less than 3 percent as a result of the maximum variation (4 lb/ft$^3$) in unit weight.

The results are discussed further with regard to compactibility in the next section.

## RESULTS OF UNIT-WEIGHT TESTS AS A MEASURE OF COMPACTIBILITY

Four different compactive efforts were applied to the concrete for each batch, and unit-weight measurements were made after each effort. The four levels of compactive

**Table 5. Variability in slump of field-produced concrete (12).**

| Site | Material Standard Deviation (in.) | Testing Standard Deviation (in.) |
|---|---|---|
| 1 | 1.12 | 0.48 |
| 2 | 0.75 | 0.34 |
| 3 | 1.01 | 0.27 |
| Average | 0.96 | 0.36 |

**Table 6. Variability in slump of pavement concrete.**

| Cited By | Testing Variance ($in.^2$) | Sampling Variance ($in.^2$) | Material Variance ($in.^2$) | Overall Variance ($in.^2$) |
|---|---|---|---|---|
| Newlon (14) | | | | 0.25 |
| | | | | 0.64 |
| | | | | 0.36 |
| | | | | 0.16 |
| | | | | 0.14 |
| | | | | 0.27 |
| | | | | 0.90 |
| | | | | 0.64 |
| | 0.10 | | | |
| FHWA (9) | 0.16 | 0.04 | 0.26 | 0.46 |
| | 0.13 | 0.02 | 0.45 | 0.64 |
| | 0.25 | 0.09 | 0.46 | 0.79 |
| | 0.07 | 0.00 | 0.15 | 0.22 |
| | 0.08 | 0.02 | 0.42 | 0.53 |
| | 0.03 | 0.03 | 0.14 | 0.21 |
| | 0.08 | 0.09 | 0.20 | 0.38 |
| | 0.16 | 0.05 | 0.50 | 0.71 |

**Figure 5. Average unit weight and fineness index for coarse-aggregate grading.**

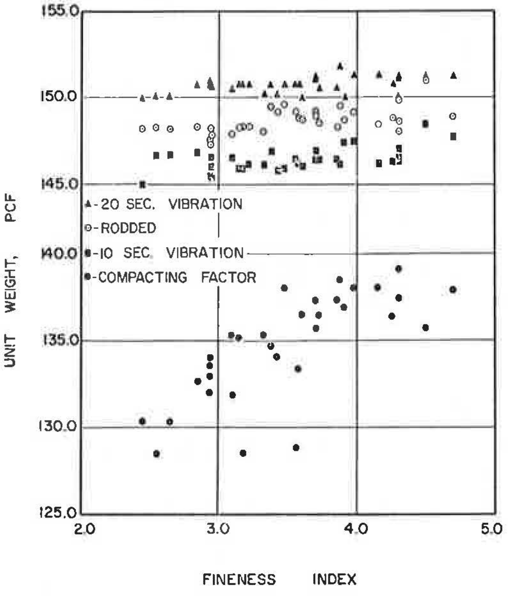


**Table 7. Average results of unit-weight tests on rodded specimens.**

| Grading Number | Fineness Index | Number of Tests | Number of Batches | Average Unit Weight ($lb/ft^3$) |
|---|---|---|---|---|
| 1 | 3.61 | 14 | 7 | 148.72 |
| 2 | 3.09 | 4 | 2 | 147.89 |
| 3 | 4.15 | 4 | 2 | 148.47 |
| 4 | 3.47 | 2 | 1 | 149.62 |
| 5 | 3.72 | 2 | 1 | 148.60 |
| 6 | 3.98 | 2 | 1 | 149.23 |
| 7 | 3.56 | 2 | 1 | 148.23 |
| 8 | 3.69 | 6 | 3 | 148.90 |
| 9 | 3.70 | 2 | 1 | 149.24 |
| 10 | 3.14 | 4 | 2 | 148.28 |
| 11 | 2.45 | 6 | 3 | 148.22 |
| 12 | 2.65 | 2 | 1 | 148.21 |
| 13 | 2.86 | 6 | 3 | 148.34 |
| 14 | 2.96 | 6 | 3 | 147.87 |
| 15 | 2.94 | 6 | 3 | 147.36 |
| 16 | 2.94 | 6 | 3 | 147.62 |
| 17 | 3.42 | 6 | 3 | 149.15 |
| 18 | 3.33 | 6 | 3 | 148.05 |
| 19 | 3.38 | 2 | 1 | 149.49 |
| 20 | 2.94 | 4 | 2 | 148.21 |
| 21 | 3.17 | 2 | 1 | 148.34 |
| 22 | 3.22 | 2 | 1 | 148.34 |
| 23 | 2.55 | 2 | 1 | 148.34 |
| 24 | 4.70 | 6 | 3 | 148.04 |
| 25 | 4.50 | 2 | 1 | 150.02 |
| 26 | 4.30 | 6 | 3 | 148.00 |
| 27 | 4.30 | 6 | 3 | 149.87 |
| 28 | 4.30 | 6 | 3 | 148.64 |
| 29 | 4.25 | 6 | 3 | 148.81 |
| 30 | 3.88 | 4 | 2 | 149.55 |
| 31 | 3.86 | 4 | 2 | 148.21 |
| 32 | 3.91 | 6 | 3 | 148.72 |
| 33 | 3.58 | 2 | 1 | 148.85 |

effort were (a) dropping through a compacting factor apparatus in accordance with British Standard 1881, (b) 10 sec of vibration with a poker vibrator, (c) rodding in accordance with ASTM Designation C 192, and (d) 20 sec of vibration with a poker vibrator.

The average unit weight resulting from each compactive effort and grading is given in Table 9 and shown in Figure 5. The ANOV's for the results of each compactive effort are given in Table 8 and Tables 10 through 12. The results are discussed in terms of the curves or lines that are defined by the sets of data points shown in Figure 5. (The discussion refers to the slopes of the curves with the knowledge that they may well be curvilinear.)

ANOV's 2, 3, and 4 for the last three compacting efforts previously given show grading to be not significant. This indicates that the slopes of the upper three lines shown in Figure 5 are not significantly different from zero (if a linear relation is assumed) or, as explained for the rodding effort, the differences in unit weight are so minimal as to be indistinguishable. The slope of the lower line is seen by ANOV 5 to be significantly greater than zero.

The lower curve shows grading to have affected the workability or compactibility of the mix. The lower unit weights obtained for the finer gradings indicate more difficulty in densifying or compacting the specimens having finer gradings. This may be attributed to the increased water demand imposed by finer gradings, resulting in a harsher mix.

The effect of grading on compactibility is seen to be inconsequential for compacting efforts greater than 10 sec of vibration as indicated by the zero slopes or insignificant differences in unit weights for the upper three curves.

The slopes of the curves for compaction efforts between the compacting factor and 10 sec of vibration would be expected to move from positive to zero. The minimum compactive effort above which the effect of grading is inconsequential is left undefined.

A conclusion as to the importance of gradation variability with regard to compactibility of field-produced concrete rests with one's opinion as to the similarity of field vibration and that used in the laboratory. In the opinion of the authors, the 10 sec of vibration applied to the specimen 6 in. in diameter and 12 in. in length is a reasonable representation of field vibration. It is observably less than rodding and also substantially less than 20 sec of vibration. It must be recognized that the test was conducted with a particular electric vibrator ($^3/_4$-in. diameter spud) representative of what might be used on small sections in the field. The results would no doubt have different values if another vibrator, differing in size and performance, had been used.

## RESULTS OF AIR-CONTENT TESTS

The results of air-content tests on five hardened specimens are given in Table 13. The average for the two finer gradings appears to be higher than for the two coarser gradings, and a trend of increased air content from coarser to finer gradings is seen. Though these observations cannot be statistically substantiated, the results agree in concept with the rodded unit-weight results. The finer gradings appear to have resulted in slightly lower unit weights, which would be indicative of higher air content.

Thus, the results of the air-content tests are taken as further evidence that any reduction in strength from lower density would be expected in specimens having fine gradings rather than those having coarse ones.

Several hardened concrete cylinders, which were observed to have bled during vibration, were sawed longitudinally and examined for segregation, none of which had any extreme settling of coarse particles.

## RESULTS OF 7-DAY COMPRESSIVE STRENGTH TESTS ON CYLINDERS COMPACTED BY RODDING

The average result for each grading is given in Table 14 along with the fineness index.

Homogeneity of variance among the test results was accepted after evaluating the data by the Foster-Burr test. ANOV 6 (Table 15) showed the variation in compressive strength, which resulted from the entire set of gradings examined, to be significant.

**Table 8. ANOV 2 for unit weight of rodded specimens.**

| Source of Variation | Degrees of Freedom | Sum of Squares | Mean Square | F-Test | F-Test (0.05) |
|---|---|---|---|---|---|
| Gradation | 32 | 70.9835 | 2.2182 | 1.09 | 1.73 |
| Batches | 40 | 81.3452 | 2.0336 | 5.21[a] | 1.58 |
| Error | 73 | 28.5000 | 0.3904 | — | — |

[a]Significant.

**Table 9. Average results of unit-weight tests.**

| Grading Number | Fineness Index | 20-Second Vibration (lb/ft³) | Rodded Unit Weight (lb/ft³) | 10-Second Vibration (lb/ft³) | Compacting Factor (lb/ft³) |
|---|---|---|---|---|---|
| 1 | 3.61 | 149.96 | 148.72 | 146.03 | 136.55 |
| 2 | 3.09 | 150.51 | 147.89 | 146.56 | 135.33 |
| 3 | 4.15 | 151.28 | 148.47 | 146.17 | 138.01 |
| 4 | 3.47 | 150.76 | 149.62 | 145.92 | 138.01 |
| 5 | 3.72 | 150.51 | 148.60 | 146.43 | 136.48 |
| 6 | 3.98 | 151.28 | 149.23 | 147.45 | 138.01 |
| 7 | 3.56 | 150.76 | 149.23 | 146.43 | 128.83 |
| 8 | 3.69 | 151.19 | 148.89 | 146.43 | 137.32 |
| 9 | 3.70 | 151.02 | 149.23 | 146.94 | 135.71 |
| 10 | 3.14 | 150.76 | 148.28 | 145.92 | 135.20 |
| 11 | 2.45 | 149.91 | 148.21 | 144.98 | 130.36 |
| 12 | 2.65 | 150.00 | 148.21 | 146.68 | 130.36 |
| 13 | 2.85 | 150.76 | 148.34 | 146.85 | 132.65 |
| 14 | 2.96 | 150.59 | 147.87 | 145.41 | 134.01 |
| 15 | 2.94 | 150.68 | 147.36 | 146.00 | 133.67 |
| 16 | 2.94 | 150.93 | 147.62 | 145.58 | 133.00 |
| 17 | 3.42 | 150.08 | 149.15 | 145.83 | 134.01 |
| 18 | 3.33 | 150.17 | 148.05 | 145.15 | 135.37 |
| 19 | 3.38 | 150.76 | 149.49 | 146.94 | 134.69 |
| 20 | 2.94 | 150.76 | 148.21 | 146.68 | 132.01 |
| 21 | 3.17 | 150.76 | 148.34 | 145.92 | 128.57 |
| 22 | 3.22 | 150.76 | 148.34 | 146.17 | 131.89 |
| 23 | 2.55 | 150.00 | 148.34 | 146.68 | 128.57 |
| 24 | 4.70 | 151.19 | 148.94 | 147.70 | 137.92 |
| 25 | 4.50 | 151.27 | 151.02 | 148.47 | 135.71 |
| 26 | 4.30 | 150.00 | 148.00 | 146.34 | 139.12 |
| 27 | 4.30 | 151.19 | 149.87 | 147.02 | 139.11 |
| 28 | 4.30 | 151.10 | 148.64 | 146.51 | 137.41 |
| 29 | 4.25 | 150.76 | 148.81 | 146.34 | 136.39 |
| 30 | 3.88 | 151.78 | 149.55 | 146.17 | 138.52 |
| 31 | 3.86 | 150.51 | 148.21 | 146.43 | 137.37 |
| 32 | 3.91 | 149.91 | 148.72 | 147.36 | 136.90 |
| 33 | 3.58 | 150.76 | 148.85 | 146.19 | 133.42 |

**Table 10. ANOV 3 for unit weight from 20 sec of vibration.**

| Source of Variation | Degrees of Freedom | Sum of Squares | Mean Square | F-Test | F-Test (0.05) |
|---|---|---|---|---|---|
| Gradation | 32 | 18.3298 | 0.5728 | 0.7553 | 1.73 |
| Error | 40 | 30.3358 | 0.7584 | — | — |

**Table 11. ANOV 4 for unit weight from 10 sec of vibration.**

| Source of Variation | Degrees of Freedom | Sum of Squares | Mean Square | F-Test | F-Test (0.05) |
|---|---|---|---|---|---|
| Gradation | 32 | 35.5722 | 1.1126 | 1.0215 | 1.73 |
| Error | 40 | 43.5303 | 1.0883 | — | — |

Figure 6 shows the average strength plotted versus the fineness index for each grading examined. A simple linear regression of strength on fineness index was found to be significant. As explained previously in the discussion of slump results, this analysis provides statistical substantiation of the trend of decreased strength with increased fineness index. Thus, finer gradings resulted in higher strength for the range of gradings investigated. It is noted that the higher strength was not attributable, even partially, to higher density, for both unit-weight and air-content results suggest that cylinders having finer gradings have been less dense.

The following observations were also made after examination of the average results for the various gradings:

1. Variation of the smaller sizes of aggregate seemed to have a greater effect on compressive strength than similar variation of larger sizes,
2. The average compressive strengths of gap gradings were higher than those of continuous gradings with approximately the same fineness index, and
3. With regard to the range in strengths obtained from the various gradings, the decrease in compressive strength, from that associated with the norm grading, was notably less than the increase (Fig. 6).

As shown in Figure 6, even though the range in fineness index (i.e., the variation in grading) is centered on the norm grading, the strength associated with the norm grading is notably lower than the center of the range of strengths.

As discussed previously, $\sigma_g^2$ from the ANOV 6 can be used to evaluate the portion of actual variation in the compressive strength of field-produced paving concrete attributable to variation in coarse-aggregate gradation. Also, the significance of batch-to-batch variation does not detract from the analysis if it is realized that the laboratory control was maintained at the lowest level practically possible. Statistical studies of paving concrete, such as those reported by Newlon (14), have analyzed 28-day compressive strength. Because the data obtained in this investigation were for 7-day strength, it is necessary to assume that 7-day and 28-day variations are the same. This assumption has been substantiated by at least one study (15).

Table 16 gives the results of research of 28-day compressive strength variations for paving concrete (9). The results agree with Newlon's suggestion of 500 to 550 psi as an overall standard deviation achievable in normal highway construction. Using the average of the $s_o$'s and $\sigma_g^2$ from this investigation (9), we derive $(\sigma_g^2/s_o^2) \times 100 = (10.85 \times 10^4/30.03 \times 10^4) = 36$ percent.

The degree of importance of gradation depends on the level or amount of overall variation occurring. In this regard, it is convenient to evaluate the relation of $\sigma_g^2/s_o^2$ for a range of $s_o^2$. If production is to be tightly controlled, a desirable $s_o$ might be 400 psi; for less critical work, an $s_o$ of 700 psi might be acceptable. The relative importance of variation in gradation for these two cases can be seen by comparing $\sigma_g^2/s_o^2$ as follows: good control $(\sigma_g^2/s_o^2) \times 100 = 69$ percent, and poor control $(\sigma_g^2/s_o^2) \times 100 = 22$ percent.

The compressive strength was related to the fineness of the coarse-aggregate grading over the entire range of gradings examined. All gradings, except the finest continuous gradings, are classifiable within a single maximum aggregate size. Thus, it appears that strength is directly influenced not only by water-cement ratio, density, and maximum aggregate size but also by the fineness of coarse-aggregate grading.

## SUMMARY

The variations in gradation of natural gravel, typically occurring in paving concrete, were shown to significantly affect workability and compressive strength of a low-slump mixture. Relatively finer gradings resulted in significantly lower slump and compacting factor. However, for a reasonable compaction effort, typical variations in the gradation of natural gravel did not result in any substantial variations in density.

Compressive strength was found to be related to the fineness of the coarse-aggregate grading. Relatively finer gradings resulted in higher strength. This relation was seen to be independent of water-cement ratio, density, and maximum aggregate size.

**Table 12. ANOV 5 for unit weight from compacting factor test.**

| Source of Variation | Degrees of Freedom | Sum of Squares | Mean Square | F-Test | F-Test (0.05) |
|---|---|---|---|---|---|
| Gradation | 32 | 538.5341 | 16.8292 | 4.7447[a] | 1.73 |
| Error | 40 | 141.8792 | 3.5470 | — | — |

[a]Significant.

**Table 13. Results of microscopic air-content determinations.**

| Grading Number | Fineness Index | Air Content (percent) |
|---|---|---|
| 11 | 2.45 | 6.1 |
| 12 | 2.65 | 5.9 |
| 1 | 3.61 | 5.7 |
| 32 | 3.91 | 4.9 |
| 26 | 4.30 | 5.1 |

**Table 14. Average results of 7-day compressive strength tests on rodded specimens.**

| Grading Number | Fineness Index | Number of Tests | Number of Batches | Average Strength (psi) |
|---|---|---|---|---|
| 1 | 3.61 | 5 | 3 | 3,780 |
| 2 | 3.09 | 4 | 2 | 4,270 |
| 3 | 4.15 | 4 | 2 | 3,780 |
| 4 | 3.47 | 2 | 1 | 3,950 |
| 5 | 3.72 | 2 | 1 | 3,840 |
| 6 | 3.98 | 2 | 1 | 3,760 |
| 7 | 3.56 | 2 | 1 | 4,140 |
| 8 | 3.69 | 6 | 3 | 4,040 |
| 9 | 3.70 | 2 | 1 | 3,770 |
| 10 | 3.14 | 4 | 2 | 4,200 |
| 11 | 2.45 | 4 | 2 | 4,720 |
| 12 | 2.65 | 2 | 1 | 4,520 |
| 13 | 2.85 | 5 | 3 | 4,730 |
| 14 | 2.96 | 6 | 3 | 4,270 |
| 15 | 2.94 | 6 | 3 | 4,240 |
| 16 | 2.94 | 6 | 3 | 4,390 |
| 17 | 3.42 | 6 | 3 | 4,140 |
| 18 | 3.33 | 4 | 2 | 3,930 |
| 19 | 3.38 | 2 | 1 | 4,170 |
| 20 | 2.94 | 4 | 2 | 4,630 |
| 21 | 3.17 | 2 | 1 | 4,240 |
| 22 | 3.22 | 2 | 1 | 4,280 |
| 23 | 2.55 | 2 | 1 | 4,520 |
| 24 | 4.70 | 4 | 2 | 3,660 |
| 25 | 4.50 | 2 | 1 | 3,700 |
| 26 | 4.30 | 6 | 3 | 3,430 |
| 27 | 4.30 | 5 | 3 | 3,590 |
| 28 | 4.30 | 4 | 2 | 3,720 |
| 29 | 4.25 | 6 | 3 | 3,640 |
| 30 | 3.88 | 4 | 2 | 3,940 |
| 31 | 3.86 | 2 | 1 | 4,080 |
| 32 | 3.91 | 4 | 2 | 3,580 |
| 33 | 3.58 | 2 | 1 | 3,830 |

**Figure 6. Average compressive strength and fineness index for coarse-aggregate grading.**

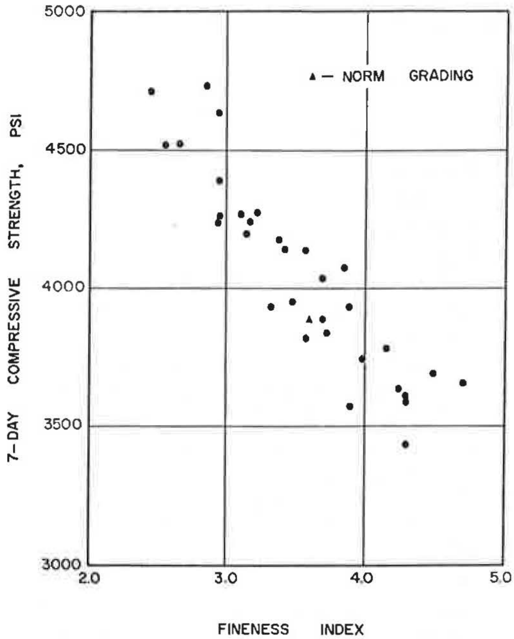


**Table 15. ANOV 6 for 7-day compressive strength tests.**

| Source of Variation | Degrees of Freedom | Sum of Squares | Mean Square | Expected Mean Square | F-Test | F-Test (0.05) |
|---|---|---|---|---|---|---|
| Gradation | 32 | 16,197,795 | 506,181 | $\sigma_t^2 + a\sigma_b^2 + 3.7\ \sigma_g^2$ | 4.806[a] | 1.83 |
| Batches | 30 | 3,159,877 | 105,329 | $\sigma_t^2 + a\sigma_b^2$ | 7.337[a] | 1.65 |
| Error | 60 | 861,377 | 14,356 | $\sigma_t^2$ | — | — |

Note: $\sigma_g^2 = 10.85 \times 10^4$.

[a]Significant.

**Table 16. Variability of 28-day compressive strength for paving concrete (9).**

| Project | Testing Standard Deviation (psi) | Material Standard Deviation (psi) | Overall Standard Deviation (psi) |
|---|---|---|---|
| 1 | 377 | 386 | 545 |
| 2 | 322 | 270 | 420 |
| 3 | 318 | 495 | 575 |
| 4 | 200 | 420 | 467 |
| 5 | 585 | 545 | 733 |

On the basis of the results of this investigation, the effect of large variations in the gradation of natural gravel as a source of variability in the slump of paving concrete appears to be smaller than the combined effect of other factors. The effect of variations in gradation on compressive strength appears to be surprisingly large and may be a major source of strength variability.

## ACKNOWLEDGMENT

The information contained in this paper was extracted from a report entitled An Investigation of the Effects of Variations in Coarse Aggregate Gradation on Properties of Portland Cement Concrete, Joint Highway Research Project Report No. 8, 1973. The report is available through the School of Civil Engineering at Purdue University.

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# REVIEW OF AGGREGATE BLENDING TECHNIQUES

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A number of aggregate blending methods have been proposed and used since the work of Fuller and Thompson on proportioning concrete. In this paper, a review of these methods is made, from the simplest trial-and-error method for two aggregates to the most sophisticated mathematical method based on theories of least squares and use of electronic computers. Six major methods are examined: trial and error, triangular chart, rectangular-chart or straight-line method and variations, Sargent's (triangular-prism projection) method, Rothfuchs' (balanced area) method and modification by the Japanese Highway Institution, and mathematical method and its variations. A general discussion of grading requirements and the general principles and procedures of aggregate blending problems are presented. For each method reviewed, the specific theories and steps are described, the applications and limitations are suggested, and one or more examples are given to illustrate the method.

•FOR various reasons mostly associated with achieving maximum density, certain desirable gradation limits are usually required of aggregates and soils for all portland cement concretes, asphalt concretes, granular bases and subbases, and soils as embankments, stabilized bases, subbases, or subgrades. Because it is unlikely that natural materials will meet these specifications, modification of in-place materials and blending of two or more aggregates or soils of different gradations to meet specification limits, or more importantly for economic considerations, have presented problems and challenges to highway engineers, contractors, plant operators, and aggregate producers.

Gradation or particle size distribution of an aggregate can be expressed in terms of total percentages, by weight, passing each sieve of a series; total percentages retained on each sieve; or percentages passing one sieve and retained on the next (size fractions). The nature of particle size distribution can be examined by graphically representing the gradation by (a) a cumulative distribution curve on a semilog scale (Fig. 1), (b) the cumulative percentage passing versus the exponential function of the sieve size (1), or (c) a histogram or bar diagram of "percent fractions" among sieves and sieve sizes (Fig. 1).

The gradation specifications are usually in terms of upper and lower limits of total or cumulative percentages passing each sieve or percentages of fractions between successive sieves (percentage passing one sieve and retained on the next).

If the specifications are expressed in terms of the total percentage passing each sieve, they can be plotted as bands or envelopes (Fig. 2). A method of transforming a "passing-retained" specification to an approximate equivalent "total percentage passing" specification was described by Dalhouse (2). The transformation enables the plotting of the passing-retained specification on the total percentage passing chart and makes visual examination and comparison of different specifications possible. However, in this paper, all gradations and specification are expressed on a total percentage passing basis, unless otherwise stated.

## GENERAL PROCEDURES

A large number of blending methods (techniques of determining relative proportions of various aggregates to obtain a desired gradation) have been developed since the

Figure 1. Typical aggregate grading chart on a semilog scale.

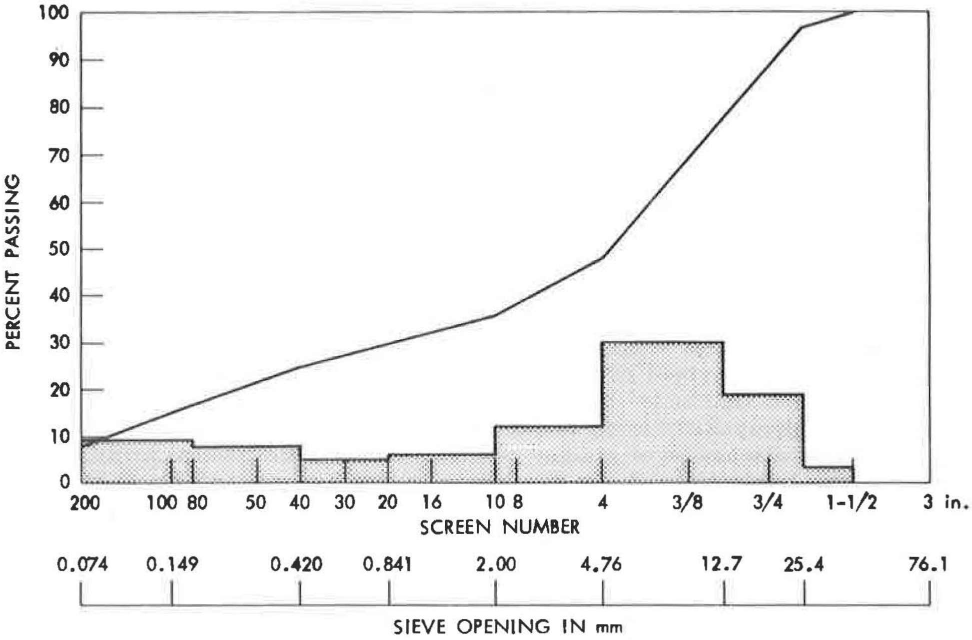
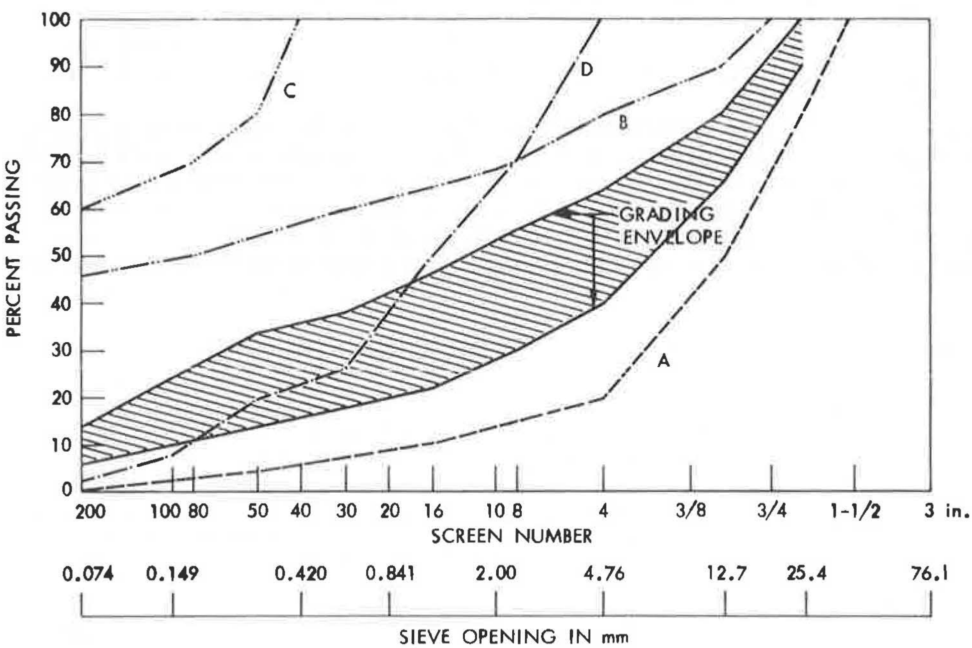



Figure 2. Aggregate blending charts.



suggestion of "ideal gradation" by Fuller and Thompson (3). The suitability of these methods depends on the types of specifications and number of aggregates involved, the experiences of the individual, and the major emphasis of the blending (closeness to the desired gradation or economics).

Regardless of the number of aggregates involved or of the method to be used, the basic formula expressing the combination is

$$p = Aa + Bb + Cc + \dots \tag{1}$$

where

p = the percentage of material passing (or retained on) a given sieve for the combined aggregates A, B, C, ...;
A, B, C, ... = percentages of material passing (or retained on) the given sieve for aggregates A, B, C, ...; and
a, b, c, ... = proportions (decimal fractions) of aggregates A, B, C, ... used in the combination and where a + b + c + ... = 1.00.

It is desirable, no matter which method is used, to first plot the gradations of the aggregates to be blended and the specification limits on a gradation chart (Fig. 2) before actual blending is attempted. From these plots, decisions can be made prior to any calculation on (a) whether a blend(s) can be found using the available aggregates to meet the specification limits, (b) where the critical sizes are, and (c) if trial-and-error method is used, the approximate trial proportions to be selected. These decisions can be made based on the following simple facts:

1. The gradation curves for all possible combinations of aggregates A and B fall between curves A and B. It is impossible to blend aggregates C and B to meet the specification regardless of the method used (Fig. 2).
2. If two curves cross at any point (B and C, Fig. 2), the grading curves for all possible combinations pass through that point.
3. The curve for a blend containing more of aggregate A than B is closer to curve A than B and vice versa.

## REVIEW OF MAJOR BLENDING TECHNIQUES

In the following sections, six major blending methods and their variations are reviewed using examples. For clarity and ease of comparison among methods, only three blending problems involving blending of two, three, and four aggregates to meet respective gradation specifications (Tables 1, 2, and 3) are used.

### Trial-and-Error Method

As the name implies, in this method, trial blends are selected (aided by experience and plots of individual gradation curves and specification) and calculated for each sieve for the combined grading (using Eq. 1). The grading that results is compared with the specification. Adjustments can be made for the second or the third trial blends and the calculations repeated for the critical sieves until the satisfactory or optimum blend is obtained. This method, guided by a certain amount of reasoning, mathematics, and graphics, is the easiest procedure to determine a satisfactory blend for two or even three aggregates.

For example, blend aggregates using the trial-and-error method (problem 1, Table 1).

Examination of grading curves indicates that it is possible to find a blend that falls within the specification limits, possibly a 50-50 blend because of the relative distance of the curves to the center of the band. The first trial blend can be determined more intelligently if certain critical sizes or control sizes are selected. By inspecting the gradations, it can be noted that all fractions retained on the $^3/_8$-in. sieve (100 - 80 = 20 percent) have to come from aggregate A and all that are less than the No. 30 aggregate must be furnished by aggregate B. With regard to aggregate A, because 100 - 59 = 41 percent retained on the $^3/_8$-in. sieve from A, the percentage needed from A to retain

20 percent on this sieve (specification median) is a = 20/41 = 0.49; the percentage b will be 1 - 0.49 = 0.51. With regard to aggregate B, because there is 51 percent passing No. 30, the percentage of aggregate B required to arrive at the desired 24 percent passing this sieve is b = 24/51 = 0.47; the percent of aggregate A in the blend will be 1 - 0.47 = 0.53. The general equation for combining two aggregates for any sieve can be expressed by substituting a = 1 - b, into Eq. 1, giving

$$b = \frac{p - A}{B - A} \tag{2}$$

Using Eq. 2 and selecting the No. 8 sieve as the control size, b = (43 - 3)/(82 - 3) = 0.51, and a = 1 - 0.51 = 0.49.

Calculation of actual blended gradation can be done conveniently by a form such as shown in Figure 3. This form and trial blends of 49 - 51 and 53 - 47 are used to show the combined gradations (Fig. 3).

## Triangular-Chart Method

Development of the triangular-chart method is credited to the Indiana State Highway Commission (4). It is based on the fact that the sum of the distances of any point within an equilateral triangle from the three sides is constant and equals the length of one side. Theoretically this can be used to blend any number of aggregates.

It is necessary to first divide the aggregates, as well as the specification, into three size separates by two more or less arbitrarily selected sieves, so that the total of the three size fractions is 100 percent. By using a triangular chart, each aggregate can be represented by a point on the chart, and specification can be represented by an area. Proportions of the various aggregates needed in the blend can be determined by measuring the distances from each aggregate to a point in the specification area and applying the principle of moments.

For example, blend three aggregates using the triangular-chart method.

The gradations of A, B, and C and the specifications given in Table 2 are retabulated in terms of three separates defined by the No. 4 and No. 200 sieves (Table 4.)

The next step is to plot points A, B, and C on the triangular chart (Fig. 4). Each of the three separates is measured from one of the sides as a base, being 0 percent at the base and increasing to 100 percent at the opposite vertex. The specifications are then plotted on the same chart as an area. Draw a line AB. Then draw a line from C through the center of the specification area E until it intersects AB at D. The proportions to be used in the blend can be determined by measuring the lengths of various lines using any convenient scale and applying the following facts:

1. Any possible blend of two materials A and B can be represented by a point on a line joining A and B. The proportions of A and B in a blend represented by a point on the line AB are inversely proportional to the distances from the point to A and B.
2. All possible blends of three materials A, B, and C lie in the triangular area ABC.
3. Any number of materials can be blended by successive blends of two materials. This blend can be considered as one material with proportions of the two materials determined by the distances.

For three materials, in this case, point D can be considered as a blend between A and B with A = DB/AB = 10.9/15.8 = 69.0 percent and B = AD/AB = 4.9/15.8 = 31.0 percent. Then blend D and C to meet gradation E by proportions of D = CE/CD = 13.6/14.6 = 93.2 percent and C = DE/CD = 1.0/14.6 = 6.8 percent. The proportions of aggregates A, B, and C in the final blend are A = 93.2 percent × 69.0 percent = 64.3 percent, B = 93.2 percent × 31.0 percent = 28.9 percent, and C = 6.8 percent, totaling 100.0 percent.

It is to be noted that this blend meets the specification in terms of the three fractions defined by the No. 4 and No. 200 sieves, but may or may not meet other sieve limits. A check calculation for all sieves with these proportions should always be

**Table 1. Blending of two aggregates, problem 1 (based on percentage passing).**

| Sieve Size | Aggregate A | Aggregate B | Specification | Median of Specifications |
|---|---|---|---|---|
| ¾ in. | 100 | 100 | 100 | 100 |
| ½ in. | 90 | 100 | 80 to 100 | 90 |
| ⅜ in. | 59 | 100 | 70 to 90 | 80 |
| No. 4 | 16 | 96 | 50 to 70 | 60 |
| No. 8 | 3 | 82 | 35 to 50 | 43 |
| No. 30 | 0 | 51 | 18 to 29 | 24 |
| No. 50 | 0 | 36 | 13 to 23 | 18 |
| No. 100 | 0 | 21 | 8 to 16 | 12 |
| No. 200 | 0 | 9 | 4 to 10 | 7 |

**Table 2. Blending of three aggregates, problem 2 (based on percentage passing).**

| Sieve Size | Aggregate A | Aggregate B | Aggregate C | Specification | Median of Specifications |
|---|---|---|---|---|---|
| 1 in. | 100 | 100 | 100 | 94 to 100 | 97 |
| ½ in. | 63 | 100 | 100 | 70 to 85 | 78 |
| No. 4 | 19 | 100 | 100 | 40 to 55 | 48 |
| No. 10 | 8 | 93 | 100 | 30 to 42 | 36 |
| No. 40 | 5 | 55 | 100 | 20 to 30 | 36 |
| No. 80 | 3 | 36 | 97 | 12 to 22 | 17 |
| No. 200 | 2 | 3 | 88 | 5 to 11 | 8 |

**Figure 3. Trial-and-error calculations for problem 1.**

Iowa State University
Civil Engineering Materials Laboratory
Aggregate Blending

Design to meet________ Specification No.________

Date______________

Operator______________

| Sieve No. | A: % Passing A. | A: % Total Agg. a | A: % Passing Total Agg. This Agg. Aa | B: % Passing B | B: % Total Agg. b | B: % Passing Total Agg. This Agg. Bb | % Passing | % Total Agg. | % Passing Total Agg. This Agg. | % Passing | % Total Agg. | % Passing Total Agg. This Agg. | % Passing | % Total Agg. | % Passing Total Agg. This Agg. | % Passing Total Agg. | Specification |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 3/4 | 100 | 0.49<br>0.53 | 49.0<br>53.0 | 100 | 0.51<br>0.47 | 51.0<br>47.0 | | | | | | | | | | 100<br>100 | 100 |
| 1/2 | 90 | | 44.1<br>47.7 | 100 | | 51.0<br>47.0 | | | | | | | | | | 95.1<br>94.7 | 80-100 |
| 3/8 | 59 | | 28.9<br>31.3 | 100 | | 51.0<br>47.0 | | | | | | | | | | 79.9<br>78.3 | 70-90 |
| 4 | 16 | | 7.8<br>8.5 | 96 | | 49.0<br>45.1 | | | | | | | | | | 56.8<br>53.6 | 50-70 |
| 8 | 3 | | 1.5<br>1.6 | 82 | | 41.8<br>38.5 | | | | | | | | | | 43.3<br>40.1 | 35-50 |
| 30 | 0 | | 0<br>0 | 51 | | 26.0<br>24.0 | | | | | | | | | | 26.0<br>24.0 | 18-29 |
| 50 | 9 | | 0<br>0 | 36 | | 18.4<br>16.9 | | | | | | | | | | 18.4<br>16.9 | 13-23 |
| 100 | 0 | | 0<br>0 | 21 | | 10.7<br>9.9 | | | | | | | | | | 10.7<br>9.8 | 8-16 |
| 200 | 0 | | 0<br>0 | 9 | | 4.6<br>4.2 | | | | | | | | | | 4.6<br>4.2 | 4-10 |
| | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | |

done. A calculation of the gradation of the blend by these proportions indicates that the blend meets specification limits for all sieve sizes (Table 5).

Ranges of percentages for aggregates A, B, and C can be determined by plotting lower and upper limits of the specification on Figure 4 as L and M and determining percentages of A, B, and C for L and for M separately (5). To improve the likelihood of the proportions determined from the triangle chart defined by two sieves or three fractions to meet requirements of other sieves, two procedures can be used as follows:

1. Select the two control sieves by comparing the steepness of the grading curves, so that the sum of the coarse fraction of coarse aggregate (i.e., A in this example), medium fraction of medium aggregate (aggregate B), and fine fraction of the fine aggregate (C) is a maximum (6). For the materials in this example, sieves No. 8 and No. 200 may be chosen because the sum is 90 + 92 + 88 = 270 as compared to the sum of 81 + 97 + 88 = 266 for sieves No. 4 and No. 200.
2. The procedures can be repeated using a second or third triangle chart, each time using different control sieves to divide the aggregates and specification into three fractions and each time determining the ranges of needed materials. By comparing the lower and upper limits determined from different triangle charts, ranges of materials can be determined that meet the limits of all sieves selected.

## Modified Triangular-Chart Methods

Modified triangular-chart methods were introduced by Driscoll (7) and Aron (8) by which one solution, showing all possible blends of three aggregates, can be obtained graphically. Though no calculation or guesswork is needed and steps are simple to follow, the procedures are quite cumbersome, especially if more sieves are involved in the specifications.

To find the most economical blend, "iso-cost" lines ("isopleths") can be drawn either on the blending chart or on a separate transparent triangular chart.

## Rectangular-Chart or Straight-Line Method

The rectangular-chart method is possibly the best method for blending two aggregates because it is simple, it requires no computation, it considers all sieve requirements at once, and it provides solutions for all possible blends that will aid in selection of the most economical blend. Even though the procedure can be used to blend three or more aggregates, repeated trials may be necessary, and there is no assurance of obtaining the optimum or the most economical blend.

Example 1—Blend two aggregates (Table 1) using the rectangular-chart method.

This method is illustrated in Figure 5, which shows a diagram with vertical percentage scales for the two aggregates and horizontal scales for proportions of aggregates in the blend. Gradation of aggregate A is represented by points on a 100 percent A vertical scale; gradation of aggregate B is plotted on a 100 percent B (0 percent A) scale. Points on the vertical scales common to the same sieve sizes are connected and labeled. This line will contain all possible percentages of that size material for any blend of A and B. A vertical line intersecting any sloping line indicates that the blend of A and B, as measured from horizontal scales, will yield a mixture with a percentage passing that sieve as indicated by the vertical scales.

For a particular sieve size, specification limits are indicated on the sloping line. That portion of the line between the two points represents the proportions of aggregates A and B, measured on the horizontal scale, that will meet specification limits for that sieve. When all specification limits on all sieves are plotted and points of lower and upper limits of consecutive sieves are connected, a specification envelope will be formed by which all possible blends can be defined. In this example, 43 to 55 percent A and 45 to 57 percent B will meet the specifications when blended. If the midpoint of all possible blends is selected, the best blend will be 49 percent A and 51 percent B. The gradation of the blend can be read directly off the vertical scale, as indicated by the points of intersections of the vertical line and the respective sieve sloping lines.

**Table 3. Blending of four aggregates, problem 3 (based on percentage passing).**

| Sieve Size | Aggregate A | B | C | D | Specification | Median of Specifications |
|---|---|---|---|---|---|---|
| ½ in. | 100 | 100 | 100 | 100 | 95 to 100 | 98 |
| ⅜ in. | 79 | 100 | 100 | 100 | 80 to 95 | 88 |
| No. 4 | 18 | 68 | 100 | 100 | 58 to 75 | 67 |
| No. 8 | 6 | 11 | 95 | 100 | 43 to 60 | 52 |
| No. 30 | 1 | 2 | 41 | 95 | 20 to 35 | 28 |
| No. 100 | 1 | 1 | 20 | 30 | 6 to 12 | 9 |

**Table 4. Blending three aggregates using the triangular-chart method.**

| Size Fraction | Aggregate (percent) A | B | C | Specification (percent) |
|---|---|---|---|---|
| > No. 4 | 81 | 0 | 0 | 45 to 60 |
| No. 4 to No. 200 | 17 | 97 | 12 | 50 to 29 |
| < No. 200 | 2 | 3 | 88 | 5 to 11 |
| Total | 100 | 100 | 100 | 100 to 100 |

**Figure 4. Blending of aggregates in triangular-chart method example.**

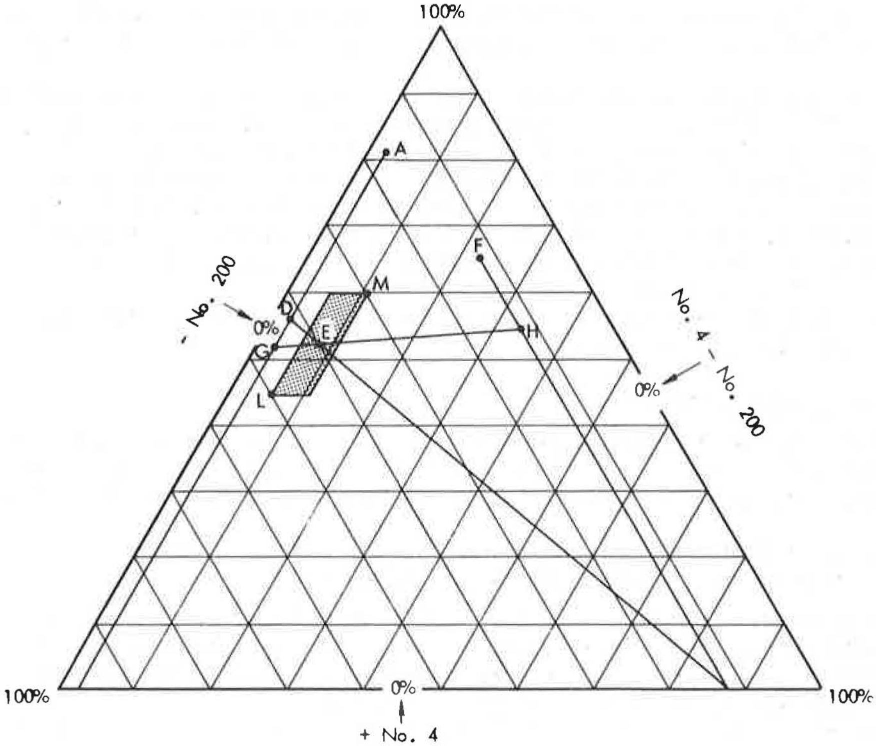



**Table 5. Comparison of results for blending three aggregates by different methods (based on percentage passing).**

| Sieve Size | Specification | Midpoint Specification | Method: Triangular-Chart | Rectangular-Chart | Rothfuch | Japanese | Mathematical Method: General | Least-Squares |
|---|---|---|---|---|---|---|---|---|
| 1 in. | 94 to 100 | 97 | 100.0 | 100 | 100.0 | 100.0 | 100.0 | 100.0 |
| ½ in. | 70 to 85 | 78 | 76.3 | 77 | 74.1 | 76.0 | 76.0 | 76.0 |
| No. 4 | 40 to 55 | 48 | 48.2 | 49 | 43.3 | 47.4 | 48.0 | 47.1 |
| No. 10 | 30 to 42 | 36 | 39.1 | 41 | 33.9 | 38.5 | 38.9 | 38.0 |
| No. 40 | 20 to 30 | 25 | 26.2 | 27 | 22.7 | 27.1 | 25.9 | 25.1 |
| No. 80 | 12 to 22 | 17 | 19.1 | 18 | 16.5 | 20.7 | 18.8 | 18.1 |
| No. 200 | 5 to 11 | 8 | 8.4 | 7 | 7.4 | 10.9 | 8.0 | 7.5 |
| Blend Proportions | | | | | | | | |
| Aggregate A | | | 64 | 62 | 70 | 65 | 64.2 | 65.5 |
| Aggregate B | | | 29 | 33 | 24 | 25 | 29.2 | 28.7 |
| Aggregate C | | | 7 | 5 | 6 | 10 | 6.6 | 5.8 |
| Total | | | 100 | 100 | 100 | 100 | 100.0 | 100.0 |

Two ingenious procedures for using this method, for repeated blending of two materials to meet different specifications or repeated blending of two different materials to meet one given specification, have been suggested (9, 10).

Example 2—Blend three aggregates (Table 2) using the rectangular-chart method. When more than two aggregates are to be combined and the rectangular-chart method is used, the best combinations of two of the materials must first be selected on one chart as previously described. This combination is then considered as a single aggregate, and its combination with a third material is determined in the same way on a second chart. The procedure is illustrated by solving this problem as follows (Fig. 6):

1. Plot gradations of aggregates A, B, and C on scales A, B, and C.
2. Connect the percentage passing for each respective sieve size by straight lines between scales A and B. Mark on each sieve line the specification limits for that particular sieve.
3. Choose a vertical line that will strike the best average between the specification limits. In this example, the vertical line selected represents 65 percent A and 35 percent B.
4. Project horizontally the intersections of each sieve line with the selected vertical line to scale A. The values projected on scale A represent the gradation for an aggregate blend composed of 65 percent aggregate A and 35 percent aggregate B.
5. Repeat steps 2 and 3 to determine the final proportions for blending aggregate C with the combination of aggregates A and B. In this example, the vertical line chosen represents 5 percent aggregate C and 95 percent aggregates A and B, or aggregate C = 5 percent, aggregate B = 0.95 × 35 percent = 33 percent, and aggregate A = 0.95 percent × 65 percent = 62 percent, totaling 100 percent.
6. The gradation of the final blend can be obtained by horizontally projecting the intersections of each line with the selected vertical line (Table 5).

## Rothfuchs' Balanced Area Method

The method developed by Rothfuchs (11-13) is widely used outside the United States and has been considered in many countries as one of the most useful graphical procedures. It is reasonably quick and simple and can be applied to mixtures of any number of aggregates.

For example, blend three aggregates (Table 2) using the Rothfuchs method.

The solution and procedure, shown in Figure 7, are as follows:

1. Plot the median or midpoint of the specifications using linear ordinates for the percentage passing, but choose a scale of sieve size such that the grading plots as a straight line. This can be done readily by drawing an inclined line, S, and marking on it the sizes corresponding to the various percentages passing.
2. The gradings of aggregates A, B, and C are plotted on this scale (curves A, B, and C).
3. Straight lines that most nearly approximate the grading curves of the individual aggregates are drawn (lines A', B', and C'). This is done by selecting a straight line for each curve such that the areas enclosed between it and the curve are a minimum and are balanced about the straight line.
4. The opposite ends of these straight lines are joined together; the proportions for the blend can be read off from the points where these joining lines intersect the straight line representing the specification grading ($P_1$ and $P_2$). Figure 7 shows that this method yields the following proportions: aggregate A = 70 percent, aggregate B = 24 percent, and aggregate C = 6 percent, totaling 100 percent. The grading for this combination is given in Table 5.

It is to be noted that calculations using the proportions determined graphically are necessary and that the blend may or may not meet the specification. The only case in which the blend determined from the chart yields the exact specification grading (S) is when (a) all aggregate grading curves are linear as plotted on the chart and (b) there is neither gap nor overlapping among the aggregates; i.e., the opposite ends of the straight gradation lines of the aggregates can be joined by vertical lines as shown in Figure 8.

Figure 5. Blending two aggregates by straight-line chart method.

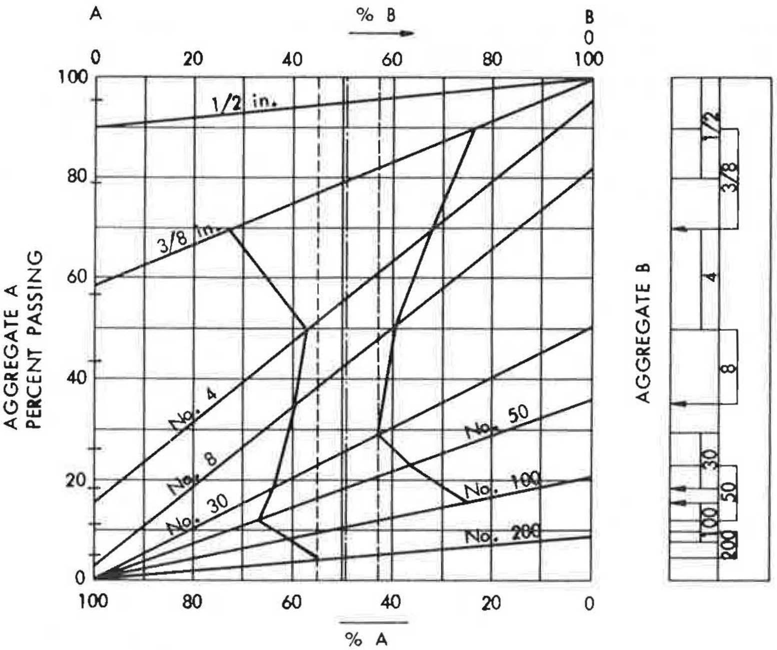
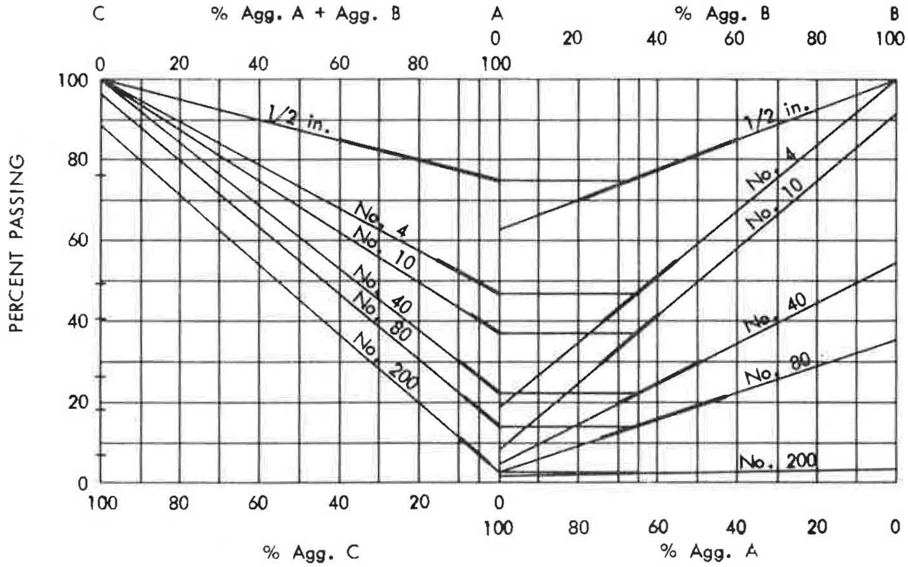



Figure 6. Blending three aggregates by rectangular-chart method.



## Japanese Method

A method, recommended by the Japanese Highway Institution (14), is based on a concept similar to that of Rothfuchs' method. In the Japanese method, a straight-line specification median and aggregate gradings are plotted as described in Rothfuchs' method. But instead of approximating the grading curves to straight lines, vertical lines are drawn between the opposite ends of adjacent curves so that, if the two gradations overlap, the percentage retained on the upper curve equals the percentage passing on the lower curve (curves A and B, Fig. 9), and, if there is a gap between the two gradations curves, the horizontal distances from the opposite ends of the curves to the vertical line are equal (curves C and D, Fig. 9). Again, the proportions for the blend can be read off from the point where these vertical lines cross the specification median. This method is shown in Figure 9.

Example 1—Blend three aggregates (Table 2) using the Japanese method.

The solution for this problem is shown in Figure 10. This gives the following proportions: aggregate A = 65 percent, aggregate B = 25 percent, and aggregate C = 10 percent, totaling 100 percent. The resulting gradation of the blend was calculated and is given in Table 5.

It has been demonstrated (13) that the plotting of gradations on a straight-line specification scale is not necessary in the Japanese method. Identical results can be obtained if vertical lines between adjacent aggregate grading curves are drawn on a regular semilog gradation chart.

Example 2—Blend four aggregates (Table 3) using the modified Japanese method.

The blending is shown in Figure 11. The grading of the blend is given in Table 6. Though the Japanese method is very quick, simple, and exact, it is less reliable than Rothfuchs' method. However, the latter method is not exact because of the judgment factor involved when straight lines are drawn to approximate the grading curves of the individual aggregates. In either case, adjustments of the proportions obtained graphically may have to be made when sieve-by-sieve comparison between the gradation of the blend and the specification is examined.

## Projection-Triangular Chart or Triangle-Rectangular Chart Method

A method using a combination of rectangular-chart and triangular-chart techniques for an economical blend of three aggregates was developed independently by Sargent (15) and Sheeler (16). In both approaches, the possibilities of blending two aggregates at a time to meet certain sieve limits are evaluated on rectangular charts. The ranges for each sieve are plotted on a triangular chart as an area. The common area enclosed by ranges for all sieves represents all possible blends of the three aggregates that will meet the specification.

## Sargent's Method

The procedure for this method is illustrated first by blending three aggregates to meet a hypothetical specification in terms of limitations by only one sieve size (Fig. 12) and then by solving problem 2 (Table 2 and Fig. 13). In Figure 12, the blending problem is as follows:

| Sieve Size | Aggregate Gradations (percent) | | | Specification |
|---|---|---|---|---|
| | A | B | C | (percent) |
| No. 4 | 60 | 30 | 10 | 40 to 50 |

The ranges of proportions that will meet the specifications with respect to the No. 4 sieve (as indicated by the shaded area) are aggregate A = 33 to 82 percent, aggregate B = 0 to 67 percent, and aggregate C = 0 to 40 percent.

The ranges of proportions that will meet the specifications of problem 2 are as follows (shaded areas in Fig. 13): aggregate A = 62 to 74 percent, aggregate B = 16 to 36 percent, and aggregate C = 4 to 10 percent.

Figure 7. Blending three aggregates by Rothfuchs' method.

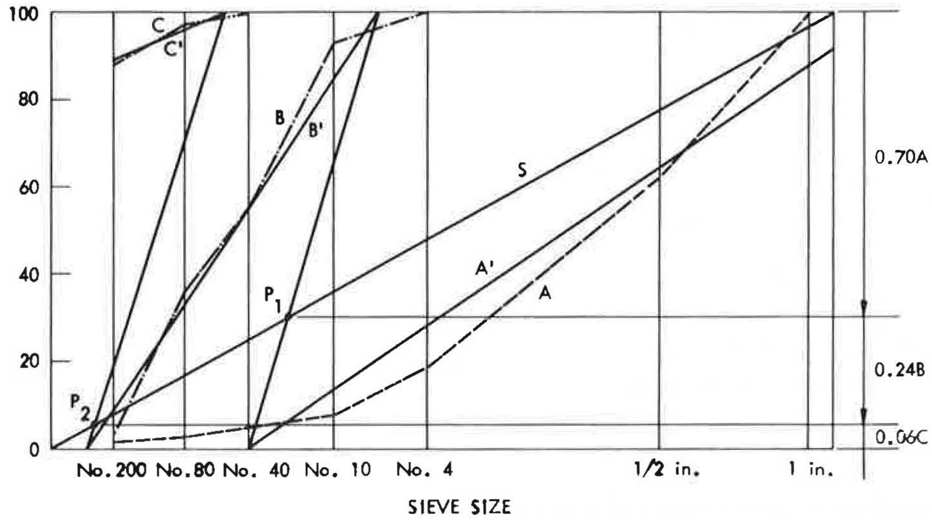



Figure 8. Ideal conditions for Rothfuchs' method.

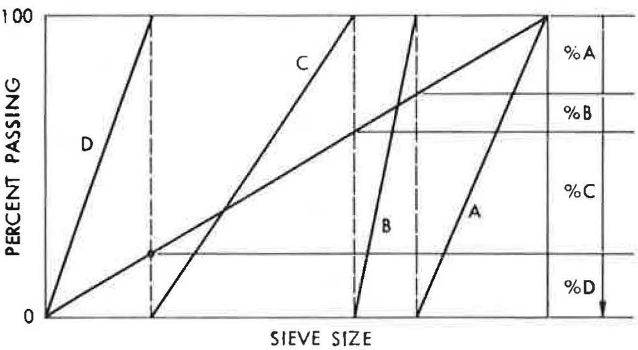



Figure 9. Blending by Japanese Highway Institution method.

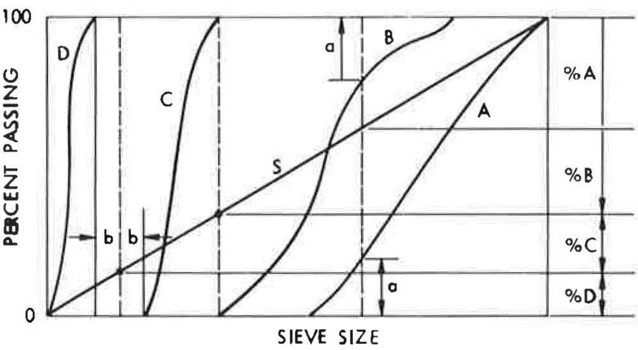


**Figure 10. Blending three aggregates by Japanese Highway Institution method.**

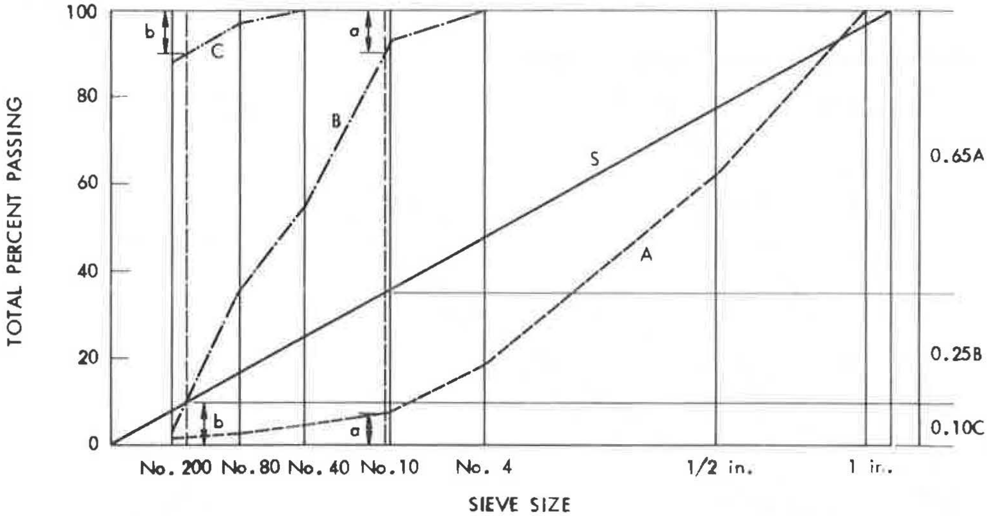



**Figure 11. Blending four aggregates by modified Japanese method.**

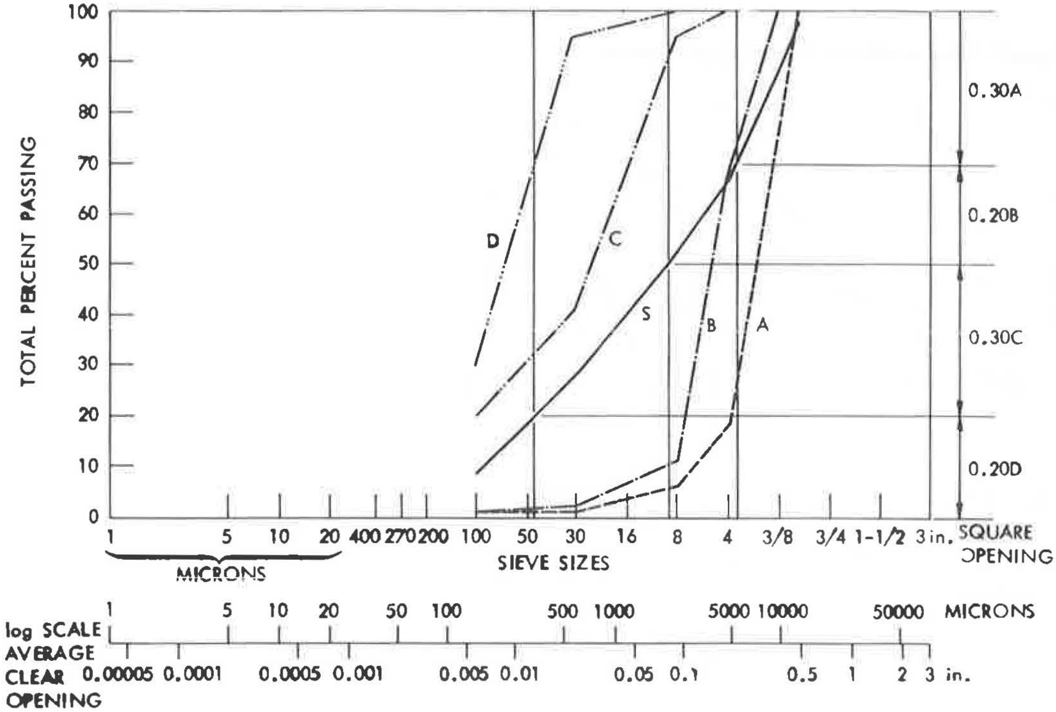



**Table 6. Comparison of results for blending four aggregates by different methods (based on percentage passing).**

| Sieve Size | Specification | Median Specification | Method: Japanese | Method: Rothfuch | Method: Mathematical |
|---|---|---|---|---|---|
| 1/2 in. | 95 to 100 | 98 | 100.0 | 100.0 | 100 |
| 3/8 in. | 80 to 95 | 88 | 93.7 | 95.2 | 92 |
| No. 4 | 58 to 75 | 67 | 69.0 | 72.5 | 65 |
| No. 8 | 43 to 60 | 52 | 52.5 | 51.2 | 53 |
| No. 30 | 20 to 35 | 28 | 32.0 | 24.0 | 27 |
| No. 100 | 6 to 12 | 9 | 12.6 | 11.0 | 11 |
| **Blend Proportions** | | | | | |
| Aggregate A | | | 30 | 23 | 38.6 |
| Aggregate B | | | 20 | 27 | 9.5 |
| Aggregate C | | | 30 | 45 | 42.9 |
| Aggregate D | | | 20 | 5 | 9.0 |

Figure 12. Blending three aggregates to meet one-size specification by Sargent's method.

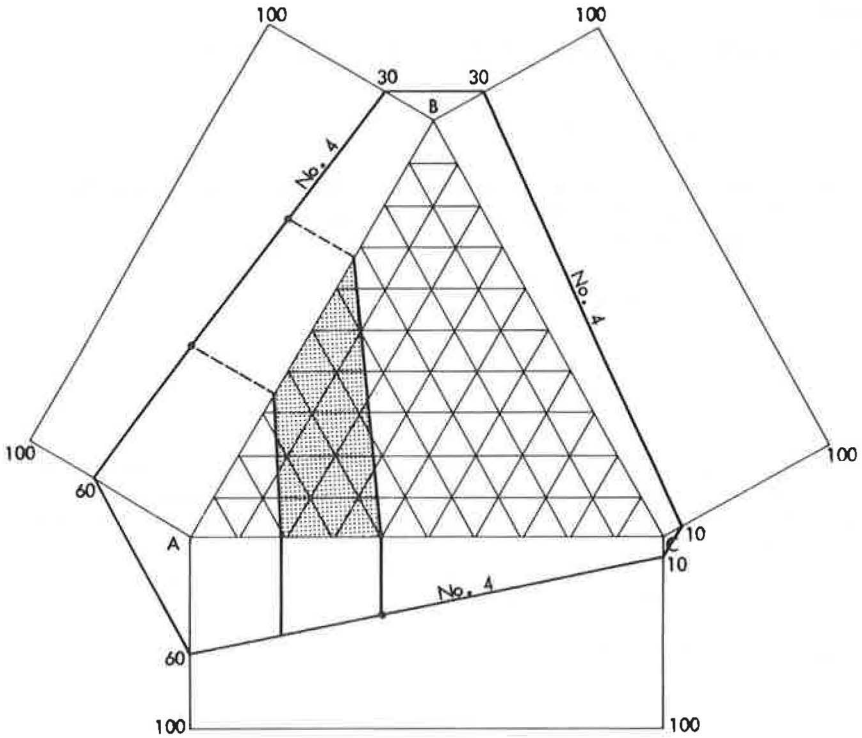
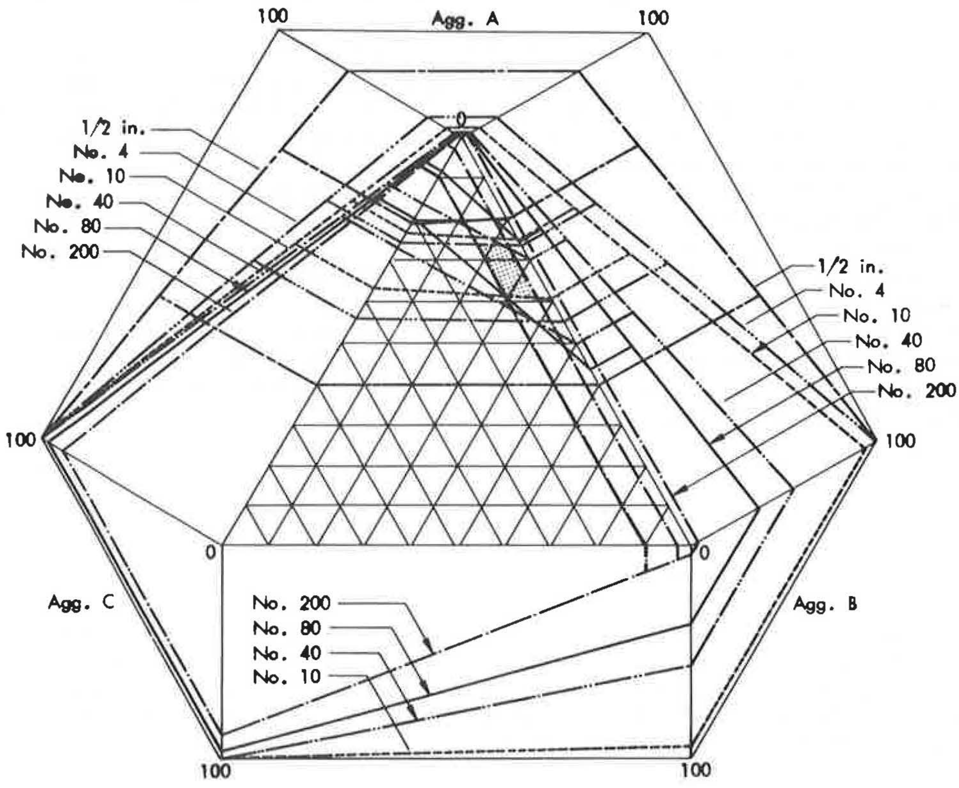



Figure 13. Blending three aggregates by Sargent's method.



Mathematical Method

General Equations—As stated previously, the basic equation for aggregate blending is

$$aA + bB + cC + \ldots = S \qquad (3)$$

which can be obtained for each sieve. In this equation, S is the percentage either passing or retained on the particular sieve for the midrange of specification limits. Thus, for two aggregates, the equations will be (for sieve i)

$$aAi + bBi = Si \qquad (4)$$

and

$$a + b = 1 \qquad (5)$$

For blending three aggregates, the following simultaneous equations can be obtained for control sieves 1 and 2: $aA_1 + bB_1 + cC_1 = S_1$, $aA_2 + bB_2 + cC_2 = S_2$, and $a + b + c = 1$.

Example—Blend the three aggregates given in Table 2 using the mathematical method. For percentages retained on the No. 4 sieve,

$$81a + 0b + 0c = 52 \qquad (6)$$

For percentages passing the No. 200 sieve,

$$2a + 3b + 88c = 8 \qquad (7)$$

Also,

$$a + b + c = 1 \qquad (8)$$

Solving Eqs. 6, 7, and 8 for a, b, and c yields a = 0.642, b = 0.292, and c = 0.066.

It is obvious that solutions for the simultaneous equations can meet the requirements of only n - 1 sieves, with n being the number of aggregates in the blend. Therefore, check calculations for all other sieves in the specifications are necessary. Gradations of the blend with the preceding proportions are computed and given in Table 5.

Least-Squares Method—Least-squares method, developed by Mackintosh (6) and Neumann (17), provides the best possible blend from the available aggregates by constructing simultaneous normal equations with unique solutions. The equations are constructed based on fractions retained on each sieve and minimization of the sum of the squared residual terms for all sieves.

Assume that M aggregates are to be blended to meet midpoint specification X and that the proportion (in decimal fractions) of each aggregate to produce the desired blend will be A, B, C, ..., M for aggregates A, B, C, ..., M respectively. M represents the last aggregate; therefore, we have

$$A + B + C + \ldots + M = 1.00 \qquad (9)$$

and

$$A, B, C, \ldots, M \geq 0. \qquad (10)$$

Let the percentage retained on each of the n sieves in the gradations for the first aggregate be a1, a2, a3, ..., ai, ..., an; for the second aggregate b1, b2, b3, ..., bi, ..., bn; and for the last aggregate m1, m2, m3, ..., mi, ..., mn, etc., where the subscript indicates an arbitrarily assigned sieve number. The percentage retained on each sieve for the specification median is x1, x2, x3, ..., xi, ..., xn.

By the use of the least-squares criterion, simultaneous equations, termed "normal equations," can be obtained (17):

$$
\begin{aligned}
A\Sigma\, Ai^2 + B\Sigma\, AiBi + C\Sigma\, AiCi + \ldots &= \Sigma\, AiXi \\
A\Sigma\, AiBi + B\Sigma\, Bi^2 + C\Sigma\, BiCi + \ldots &= \Sigma\, BiXi \qquad (11) \\
A\Sigma\, AiCi + B\Sigma\, BiCi + C\Sigma\, Ci^2 + \ldots &= \Sigma\, CiXi
\end{aligned}
$$

in which, Ai = (ai - mi), Bi = (bi - mi), Ci = (ci - mi), . . . , Xi = (xi - mi), etc.

This set of equations may be solved for the proportions A, B, C, etc. Under the assumption of the least-squares criterion, the solution for A, B, C, etc., resulting from Eq. 11, provides the closest possible solution to the desired gradation; no better solution is possible. Should this solution fail to satisfy the specification limits with respect to all sieves, the aggregates available will not satisfy the design specification, and other aggregate sources or modifications of the aggregates or specification must be considered. Negative-value solutions indicate that more aggregates are being considered in the problem than are required to obtain the best approximation to the desired specification.

Example—Blend the three aggregates given in Table 2 by using the least-squares method.

The calculations for constructing the normal equations are given in Table 7. The procedures are as follows:

1. Convert the gradations for percentage passing to percentage retained on each sieve;
2. Compute ci - ai = -Ai, ci - bi = -Bi, and ci - xi = -Xi;
3. Compute the sums of squares and cross products;
4. The normal equations thus obtained are 10,896 A + 7,293 B = 9,063 and 7,293 A + 9,550 B = 7,382;
5. Solving the simultaneous equations, we have A = 0.6432 and B = 0.2818;
6. Because A + B + C = 1.00, we have C = 0.0750; and
7. All variable solutions are positive. The solution is the optimum for the available aggregates for the specification.

Even though the mathematical derivation of the normal equations is based on the percentage retained on each sieve ai, bi, . . . , similar simultaneous equations can also be constructed by assuming ai, bi, ci, and xi as percentage passing. Solutions of this example using the percentage passing approach are given in Table 8. Equations derived from this approach are 4.1651 A + 1.7963 B = 3.2439 and 1.7963 A + 1.3020 B = 1.5503, from which the following results are obtained: A = 0.6551, B = 0.2809, and C = 0.0580. This set of results also meets the specification. Solution of problem 3 (Table 3) was obtained by this method and is given in Table 9.

A linear-programming method of the mathematical solution for blending a large number of aggregates from various sources, not only considering gradation and other physical requirements but also minimizing costs, was presented by Ritter and Shaffer (18). Methods for blending by volume and grading adjustment by wasting are described elsewhere (13, 19).

## SUMMARY

Six major aggregate blending methods and their variations are reviewed. For two aggregates, although highly experienced engineers can work out satisfactory solutions rapidly by the trial-and-error method, graphical solution by the rectangular-chart method is the best for economical blending. For three aggregates, the triangular-prism projection (combination of triangular-chart and rectangular-chart methods) methods give all possible blends and thus are best for economical blending. For four or more aggregates, the Rothfuchs method or the Japanese method provides the quickest and simplest solution. The mathematical method based on least-squares

**Table 7. Blending three aggregates by least-squares method (based on percentage retained).**

| Sieve Size | Gradation: Aggregate A (ai) | B (bi) | C (ci) | Midpoint Specification [X(xi)] | Value: -Ai (ci - ai) | -Bi (ci - bi) | -Xi (ci - xi) | Ai² | AiBi | AiXi | Bi² | BiXi |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 in. | 0 | 0 | 0 | 3 | 0 | 0 | -3 | 0 | 0 | 0 | 0 | 0 |
| ½ in. | 37 | 0 | 0 | 19 | -37 | 0 | -19 | 1,369 | 0 | 703 | 0 | 0 |
| No. 4 | 44 | 0 | 0 | 30 | -44 | 0 | -30 | 1,936 | 0 | 1,320 | 0 | 0 |
| No. 10 | 11 | 7 | 0 | 12 | -11 | -7 | -12 | 121 | 77 | 132 | 49 | 84 |
| No. 40 | 3 | 38 | 0 | 11 | -3 | -38 | -11 | 9 | 114 | 33 | 1,444 | 418 |
| No. 80 | 2 | 19 | 3 | 8 | 1 | -16 | -5 | 1 | -16 | -5 | 256 | 80 |
| No. 200 | 1 | 33 | 9 | 9 | 8 | -24 | 0 | 64 | -192 | 0 | 576 | 0 |
| Pan | 2 | 3 | 88 | 8 | 86 | 85 | 80 | 7,396 | 7,310 | 6,880 | 7,225 | 6,800 |
| Total | 100 | 100 | 100 | 100 | | | | 10,896 | 7,293 | 9,063 | 9,550 | 7,382 |

Note: 10,896 A + 7,293 B = 9,063; 7,293 A + 9,550 B = 7,382; and A + B + C = 1. A = 0.6432, B = 0.2818, and C = 0.0750.

**Table 8. Blending three aggregates by least-squares method (based on percentage passing).**

| Sieve Size | Gradation: Aggregate A | B | C | Specification (X) | Value: -Ai (C - A) | -Bi (C - B) | -Xi (C - X) | Ai² | AiBi | AiXi | Bi² | BiXi |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 in. | 100 | 100 | 100 | 97 | 0 | 0 | 0.03 | 0 | 0 | 0 | 0 | 0 |
| ½ in. | 63 | 100 | 100 | 78 | 0.37 | 0 | 0.22 | 0.1369 | 0 | 0.0814 | 0 | 0 |
| No. 4 | 19 | 100 | 100 | 48 | 0.81 | 0 | 0.52 | 0.6561 | 0 | 0.4212 | 0 | 0 |
| No. 10 | 8 | 93 | 100 | 36 | 0.92 | 0.07 | 0.64 | 0.8464 | 0.0644 | 0.5888 | 0.0049 | 0.0448 |
| No. 40 | 5 | 55 | 100 | 25 | 0.95 | 0.45 | 0.75 | 0.9025 | 0.4275 | 0.7125 | 0.2025 | 0.3375 |
| No. 80 | 3 | 36 | 97 | 17 | 0.94 | 0.61 | 0.80 | 0.8836 | 0.5734 | 0.7520 | 0.3721 | 0.4880 |
| No. 200 | 2 | 3 | 88 | 8 | 0.86 | 0.85 | 0.80 | 0.7396 | 0.7310 | 0.6880 | 0.7225 | 0.6800 |
| Total | | | | | | | | 4.1651 | 1.7963 | 3.2439 | 1.3020 | 1.5503 |

Note: $A\Sigma Ai^2 + B\Sigma AiBi = \Sigma AiXi$, $A\Sigma AiBi + B\Sigma Bi^2 = \Sigma BiXi$, and A + B + C = 1.00. A = 0.6551, B = 0.2869, and C = 0.0580.

**Table 9. Blending four aggregates by least-squares method.**

| Sieve Size | Aggregate A | B | C | D | Desired Specification (X) | Value: -Ai (D - A) | -Bi (D - B) | -Ci (B - C) | -Xi (D - X) | Ai² | AiBi | AiCi | AiXi | Bi² | BiCi | BiXi | Ci² | CiXi |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ½ in. | 100 | 100 | 100 | 100 | 98 | 0 | 0 | 0 | 0.02 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| ⅜ in. | 79 | 100 | 100 | 100 | 88 | 0.21 | 0 | 0 | 0.12 | 0.0441 | 0 | 0 | 0.0252 | 0 | 0 | 0 | 0 | 0 |
| No. 4 | 18 | 68 | 100 | 100 | 67 | 0.82 | 0.32 | 0 | 0.33 | 0.6724 | 0.2624 | 0 | 0.2706 | 0.1024 | 0 | 0.1056 | 0 | 0 |
| No. 8 | 6 | 11 | 95 | 100 | 52 | 0.94 | 0.89 | 0.05 | 0.48 | 0.8836 | 0.8366 | 0.0470 | 0.4512 | 0.7921 | 0.0445 | 0.4272 | 0.0025 | 0.024 |
| No. 16 | 2 | 3 | 60 | 100 | | | | | | | | | | | | | | |
| No. 30 | 1 | 2 | 41 | 95 | 28 | 0.94 | 0.93 | 0.54 | 0.67 | 0.8836 | 0.8742 | 0.5076 | 0.6298 | 0.8649 | 0.5022 | 0.6231 | 0.2916 | 0.361 |
| No. 50 | 1 | 2 | 31 | 66 | | | | | | | | | | | | | | |
| No. 100 | 1 | 1 | 22 | 39 | | | | | | | | | | | | | | |
| No. 200 | 1 | 1 | 20 | 30 | 9 | 0.29 | 0.29 | 0.10 | 0.21 | 0.0841 | 0.0841 | 0.0290 | 0.0609 | 0.0841 | 0.0290 | 0.0609 | 0.0010 | 0.021 |
| Total | | | | | | | | | | 2.5678 | 2.0573 | 0.5836 | 1.4377 | 1.8435 | 0.5757 | 1.2168 | 0.2951 | 0.406 |

Note: 2.5678 A + 2.0573 B + 0.5836 C = 1.4377; 2.0573 A + 1.8435 B + 0.5757 C = 1.2168; 0.5836 A + 0.5757 B + 0.2951 C = 0.4068; and A + B + C + D = 1.000. A = 0.386, B = 0.095, C = 0.429, and D = 0.090.

criterion gives the best possible blend from all available materials. The linear-programming method of solution can be used if minimizing costs is the major consideration, a large number of aggregates of different characteristics are considered, other linear restrictions are placed in addition to gradation limits, and an electronic computer is readily available.

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