

# A VARIABLE-SEQUENCE MULTIPHASE PROGRESSION OPTIMIZATION PROGRAM

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A traffic signal progression program has been developed that maximizes progression along a facility having multiphase signals. The main street green phase sequences of left turns first, through movements first, leading green, and lagging green can be evaluated at each intersection. The progression program can determine which of the four phase sequences provides maximum progression. Conventional two-phase signal operation is a special case of the through-movements-first sequence. The computer program, written in FORTRAN IV, can also compute movement durations and phase splits if desired. The progression program was adapted for use in the real-time control of an arterial pilot control system in Dallas. The controllers were modified to permit variable-phase sequence operation. Good progression was obtained, and no apparent problems have occurred due to the variable-phase sequencing.

• WITH ever-increasing demands being placed on urban traffic facilities, traffic engineers need efficient traffic control systems and strategies to improve the level of service being provided. New solid-state traffic controllers, digital process control computers, and minicomputers now provide increased computational and control capabilities for use in improving traffic operations. Modern telecommunications equipment enables efficient gathering, transmission, and receiving of large quantities of traffic data. Integrated circuit design now also permits flexible signal phase implementation. These new computational and control capabilities have removed several of the hardware constraints that have restrained the implementation of more responsive and efficient traffic control strategies.

Traffic control strategies have been developed for optimizing large-network signal control such as SIGOP (1, 2) and TRANSYT (3). The analysis is done off-line on large digital computers. However, the intersection control operation is usually conventional two-phase operation. In the area of arterial optimization, researchers have concentrated on developing optimization techniques that minimize delay, such as the delay-offset technique (4), or that maximize the progression bands, such as the algorithms developed by Little (5) or Brooks (6) as investigated by Bleyl (7). A recent research study (4) recommends that Webster's method (8) for computing cycle length and splits be used in conjunction with the arterial optimization techniques.

The previously noted arterial progression programs determine the offsets that yield the maximum progression only for conventional two-phase signal operation. These programs do not analyze multiphase (greater than two-phase) signal operation or a control process having variable multiphase sequencing. With modern electronics, variations in phase sequencing are possible. For example, a lagging green phase sequence can easily follow a leading green phasing arrangement using the new hardware.

## ARTERIAL PILOT STUDY CONTROL SYSTEM

For purposes of illustration, the program is discussed as applied to a pilot arterial control system operated in Dallas as a research project conducted by the Texas Trans-

portation Institute for the Federal Highway Administration in cooperation with the Texas Highway Department and the city of Dallas. The general-use progression program was used with a real-time data acquisition and signal driver program to produce the real-time control program.

The arterial pilot study site is located on Mockingbird Lane, a six-lane divided major urban arterial that serves as a crosstown facility and a feeder street to the North Central Expressway. Within the 1½-mile study section (Fig. 1), there are three high types of intersections, having separate protected left-turning movements, and a diamond interchange at the expressway. These intersections have traffic-actuated controllers; however, with the installation of additional electronics, the actuated controllers were completely bypassed during computer control, thus permitting a variable selection of nonconflicting phase sequences. The diamond interchange is operated with four-phase overlap control (9, 10). Progression is provided in both directions along the arterial from the interchange through the three intersections.

## DEVELOPMENT OF PHASING SEQUENCES AND PROGRESSIVE GREENS

### Actuated Control Phase Sequence

Each high type of intersection has eight separate and protected movements as shown in Figure 2. When the actuated controllers are operating, traffic movements are separated into two basic phases: the A-phase for the arterial and the B-phase for the cross street. The relationships between traffic movements and resulting phase sequence are shown in Figure 3. Within the A-phase, the normal quad-left operation with all movements calling would be left turns first (movements 1 + 3), followed by a left-turn drop-out (movements 2 + 3 or 1 + 4) depending on the durations of movements 1 and 3, followed by the through movements (2 + 4). Transfer of control to the B-phase would then be made, which would result in a similar sequence.

### Analysis of Four Phase Sequences

The program determines the signal phase sequence and offset at each intersection that will maximize the progression. Within the basic two-phase framework consisting of an A-phase followed by a B-phase, the following four A-phase sequences (Fig. 4) can be analyzed: (a) left turns first (e.g., dual or quad left), (b) through movements first, (c) leading main street (arterial) green, or (d) lagging main street green. The latter two sequences are with respect to the outbound direction from the diamond interchange. A single protected left-turn movement would be either a leading sequence (sequence 3) or a lagging sequence (sequence 4). Conventional two-phase signal operation would be represented by through movements first (sequence 2) with no left turns present.

The three intersections in the pilot study are permitted to have any one of the four possible A-phase sequences with a different sequence permitted with each new real-time evaluation. The diamond interchange in the progression analysis is considered as having only one possible phase sequence, a leading green on the side of the interchange connecting to the remainder of the study section.

### Traffic Movement Durations

The movement green times, consisting of the green plus amber, for a given cycle length are based on the demand-capacity ratio concept as presented by Webster (8). The smallest movement green  $g_{1n}$  (on movement  $m$  of intersection  $i$ ) that will satisfy the present average movement demand  $D_{1n}$  is computed from

$$g_{1n} = \frac{D_{1n}}{S_{1n}} C + L_{1n} \quad (1)$$

where  $g_{1n}$  must be greater than or equal to a minimum permitted movement length and where  $S_{1n}$  is the movement saturation or capacity flow,  $L_{1n}$  is the lost time per movement (11), and  $C$  is the cycle length. The cycle length used by the progression program is the one that results in the most efficient progression as described later.



The minimum A-phase (arterial) and B-phase (cross street) lengths are then computed from

$$A_{min} = \max \left| \begin{array}{l} (g_{11} + g_{12}) \\ (g_{13} + g_{14}) \\ Ped_b \end{array} \right| \quad (2)$$

$$B_{min} = \max \left| \begin{array}{l} (g_{15} + g_{16}) \\ (g_{17} + g_{18}) \\ Ped_a \end{array} \right|$$

where  $Ped_a$  and  $Ped_b$  are the minimum pedestrian crossing times when activated. The respective movements are as shown in Figure 2. Any slack, or difference between the sum of the minimum A and B phase lengths and the cycle length being analyzed, is first prorated to the two basic A- and B-phases. The corresponding phase slack is then proportioned to the related movements within the phase.

### Queue Clearance Option

The objective of an arterial progressive signal system is to allow platoons of vehicles to travel through the signal system without having to stop. These vehicles are impeded when they arrive during a red signal or when they arrive during a green but are blocked by a queue of vehicles still stopped at the signal. Even for an arterial having good two-way progression, queues can form at intersections due to traffic turning onto the arterial from adjacent intersections or to parking facilities.

If the average stopped queue for each of the through movements ( $g_{12}$ ,  $g_{14}$ ) is known, then the queue clearance time per movement can be calculated from

$$Q_{1m} = \frac{N_{1m}}{S_{1m}} + u_{1m} \quad (3)$$

where

- $Q_{1m}$  = the queue clearance time in seconds required at movement  $m$  of intersection  $i$ ,
- $N_{1m}$  = average number in queue at start of green on movement  $m$  of intersection  $i$ ,
- $S_{1m}$  = capacity flow of movement  $m$  of intersection  $i$ , and
- $u_{1m}$  = queue start-up time of vehicles on movement  $m$  of intersection  $i$ .

The queue clearance option is a logical addition to the progression analysis. To allow progressive movement on the arterial when stopped queues exist, we subtracted the queue clearance times  $Q_{1m}$  from the two through movement green times  $g_{12}$  or  $g_{14}$  to determine the resulting progressive through green times  $G_{12}$  or  $G_{14}$ :

$$G_{1m} = g_{1m} - Q_{1m} \quad m = 2, 4 \quad (4)$$

where  $m$  refers to the movements shown in Figure 2. The progression program can skip this option if desired by letting  $Q_{1m} = 0$  or  $G_{1m} = g_{1m}$ .

## MULTIPHASE PROGRESSION OPTIMIZATION

### Theory

The procedure used to determine the maximum progression bands that can be found along an arterial having multiphase signal sequences is an extension of Brooks's interference algorithm (6) illustrated by Bleyl (7). Reference is also made to Little's maximum bandwidth algorithm (5). Both algorithms analyze only two-phase progression and use the half-integer synchronization technique, which does not apply for multiphase signal operation because the inbound and outbound progression greens at an intersection having multiphase signal operation are generally of unequal lengths and are offset in time relative to one another.

Figure 1. Pilot control system site.

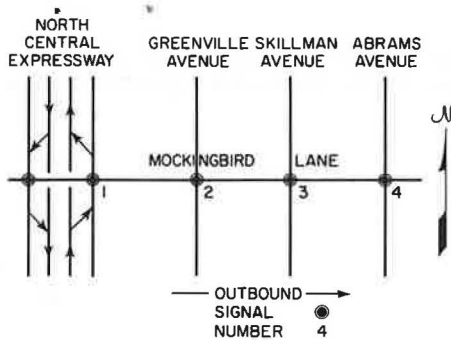


Figure 2. Traffic movements on high type of intersection.

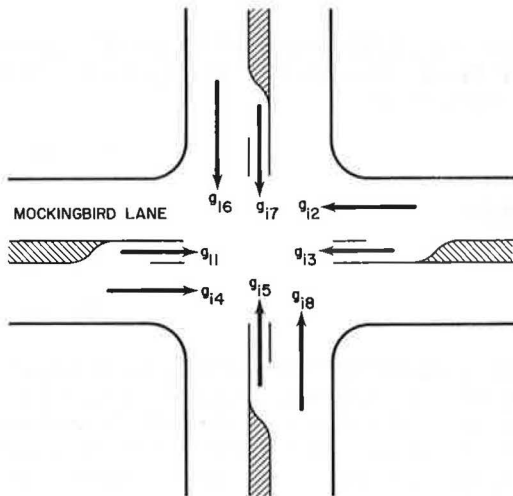


Figure 3. Relationships between traffic movement durations and resulting typical quad-left sequence.

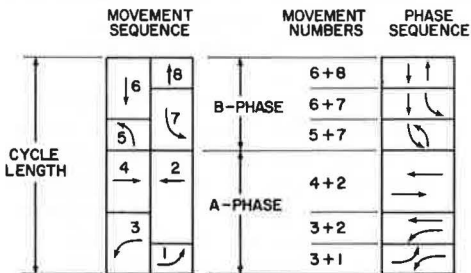


Figure 4. Four A-phase sequences.

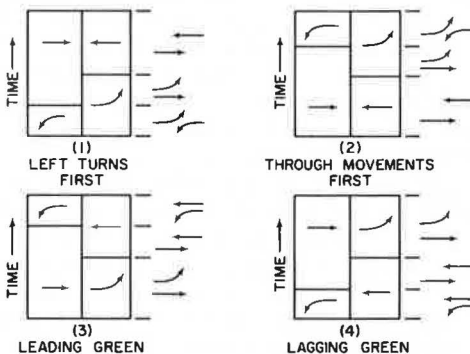
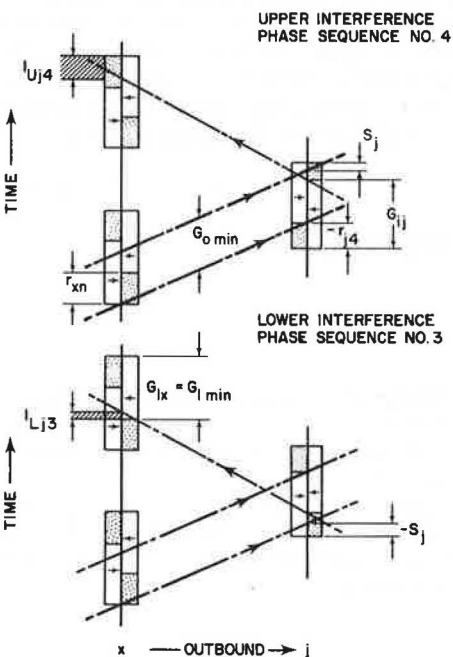


Figure 5. Example of upper and lower interferences to inbound progression band caused by an intersection.



However, Little's unequal bandwidth equation at an optimized condition does hold within given constraints. This equation is that the sum of the bandwidths at optimization is a constant, or

$$B_o + B_i = B_{max} \quad (5)$$

subject to

$$B_o \leq G_o_{min} \quad (6)$$

$$B_i \leq G_i_{min}$$

where  $B_o$  equals the width of the progression band in seconds in the outbound (an arbitrary selection) direction along the arterial and  $G_o_{min}$  is the minimum outbound progressive through green. Likewise,  $B_i$  is the bandwidth and  $G_i_{min}$  the minimum progressive through green in the inbound direction.

Extending Brooks's interference theory, it can be shown that

$$B_{max} = G_o_{min} + G_i_{min} - I_{i_{min}} \quad (7)$$

where  $G_o_{min}$  and  $G_i_{min}$  are the minimum outbound and inbound progressive greens respectively and  $I_{i_{min}}$  is the minimum possible inbound band interference as described subsequently. The minimization of inbound interference must be achieved without causing any outbound interference to occur. Thus, to maximize the sum of the progressive bands, the total inbound interference should be minimized.

The minimum inbound interference  $I_{i_{min}}$  is computed by setting the initial intersection signal phasings for outbound one-way progression as shown in Figure 5. The width of the outbound green progressive band is  $B_o = G_o_{min}$ . The intersection, denoted as  $x$ , that has the minimum progressive green in the inbound direction  $G_i_{min}$  is then located. The inbound green bands of all other intersections are then projected onto this smallest inbound green to determine their interference to the inbound progression. Because these interference projections are with respect to the smallest green in the inbound direction, it is not possible for another intersection to have both upper and lower interferences simultaneously. However, it is possible for the projection of an inbound green onto the minimum green to completely cover or straddle the minimum green causing neither upper nor lower interference.

To evaluate the upper interference values  $I_{UjP}$  for phase sequence  $p$  of intersection  $j$ , all signal phases are offset for perfect one-way progression in the outbound direction as shown in the upper section of Figure 5. The upper interference is computed by first accumulating the elapsed time from the inbound minimum green  $G_{ix}$  located at intersection  $x$  to intersection  $j$  and returning to project the upper edge of the inbound band onto  $G_{ix}$ . This total time is then scaled to modulus  $C$  and then subtracted from  $G_{ix}$ . That is,

$$I_{UjP} = G_{ix} - (-r_{xn} + t_{xj} + r_{jp} + G_{ij} + t_{jx}) \text{ mod } C \quad (8)$$

and after regrouping terms

$$I_{UjP} = G_{ix} - (t_{xj} + t_{jx} - r_{xn} + r_{jp} + G_{ij}) \text{ mod } C \quad (9)$$

where

$I_{UjP}$  = upper interference caused by phase sequence  $p$  of intersection  $j$  ( $0 \leq I_{UjP} < C$ )  
mod  $C$ ,

$G_{ix}$  = minimum inbound progressive green located at intersection  $x$  (i.e.,  $G_{ix} = G_{i_{min}}$ ),

$t_{xj}$  = cumulative travel time from intersection  $x$  to intersection  $j$ ,

$t_{jx}$  = cumulative travel time from intersection  $j$  to intersection  $x$ ,

$r_{xn}$  = the relative offset of  $G_{ix}$  with respect to  $G_{ox}$  with up (lag) positive for phase sequence  $n$ ,

- $r_{jp}$  = the relative offset of  $G_{1j}$  with respect to  $G_{0j}$  with up (lag) positive for phase sequence  $p$ ,  
 $G_{ij}$  = the inbound progressive green time at intersection  $j$ , and  
 $C$  = cycle length.

Travel times are considered positive if intersection  $j$  is outbound of intersection  $x$ , the intersection having the minimum green in the inbound direction. Conversely, travel times are negative if intersection  $j$  is inbound of intersection  $x$ . Travel times are computed from the directional link distances between intersections and the corresponding running speeds. Because only cumulative travel times are used, directional link speeds or distances or both between intersections may be different.

According to the lower section of Figure 5, the lower interferences are computed similarly from the following equation:

$$I_{Ljp} = (-r_{xn} + t_{xj} - S_j + r_{jp} + t_{jx}) \bmod C \quad (10)$$

or

$$I_{Ljp} = (t_{xj} + t_{jx} - r_{xn} + r_{jp} - S_j) \bmod C \quad (11)$$

where

$$I_{Ljp} = \text{lower interference } (0 \leq I_{Ljp} < C) \bmod C \text{ and}$$

$$S_j = G_{0j} - G_{0x}.$$

Note that, in the lower interference computation, the phase sequence at intersection  $j$  is slipped down an amount  $S_j$  to reduce the lower interference as much as possible without causing any interference to the outbound band. Lower interferences are also checked for the possibility of an inbound minimum green straddle condition occurring where neither upper nor lower interference occurs. This can occur if  $I_{Ljp} \geq C - (G_{ij} - G_{ix})$ .

After upper and lower interferences have been computed for all of the four-phase sequences permitted at each intersection, the minimum upper  $I_{Uj}$  and lower  $I_{Lj}$  interference values are determined at each intersection  $j$  for all intersections within the progressive system.

### Optimization

As noted in Eq. 7, the optimization criterion is to maximize the sum of the progression bands by minimizing the total inbound interference without causing any outbound interference to occur. The total inbound interference is the sum of the maximum upper and lower interferences that are in the solution at any time. That is,

$$I = I_{U \max} + I_{L \max} \quad (12)$$

where  $I$  is the total interference for a progression solution having a maximum upper interference of  $I_{U \max}$  and a maximum lower interference of  $I_{L \max}$ . Either an upper or lower interference can be selected at an intersection. It is possible to select all upper or lower interferences or any combination of the two. Brooks's minimization of interference concept (6) can now be used to determine the appropriate combination of upper and lower interferences that will yield the minimum interference.

### Interference Minimization

An example of the minimization of interference will be presented for the four-intersection Mockingbird Lane pilot control system. The existing intersections with their allowable phase sequences and corresponding computed upper and lower interferences are given in Table 1. These values were determined using a 70-sec cycle and a uniform speed of 40 fps in each direction for clarity of presentation.

The minimum total interference can be evaluated from Eq. 12 in the following manner. The minimum upper and lower interferences at each intersection from Table 1 are

ranked in descending order according to upper interferences as given in Table 2. If all the  $I_{U_j}$  in Table 2 were used as a solution, the total interference would be the largest upper interference, 24 sec. However, if, for the first intersection that has the largest upper interference, the lower interference of 1 sec were selected, the total interference for this second trial would be  $I_2 = I_{U(2)} + I_{L \text{ max}}$  or  $I_2 = 12 + 1 = 13$  sec. Continuing, if the second ranked intersection also used a lower interference, then the total interference of the third alternative would be  $I_3 = I_{U(3)} + I_{L \text{ max}}$  or  $I_3 = 5 + \max(1, 5) = 10$  sec. Lastly, if all three of the possible interference intersections used lower interferences, then the total interference of the fourth trial would be  $I_4 = I_{U(4)} + I_{L \text{ max}}$  or  $I_4 = 0 + \max(1, 5, 11) = 11$  sec.

Thus, the minimum possible interference for this cycle  $I_{1 \text{ min}}$  is 10 sec from trial three. This minimum is obtained by selecting lower interferences for the first two listed intersections (intersections 4 and 3) and upper interferences for the remainder (intersection 2 in this case). If the upper and lower interferences that yield the minimum interference are known, the corresponding phase sequence to be used at each intersection is determined. As given in Table 2, intersections 4, 3, 2, and 1 would use the phase sequences shown in Figure 4 of 1, 4, 4, and 3 respectively.

The maximum sum of the inbound and outbound progression bands  $B_{\text{max}}$  is determined from Eq. 7.

$$B_{\text{max}} = G_{o \text{ min}} + G_{i \text{ min}} - I_{1 \text{ min}}$$

For this example,  $G_{o \text{ min}} = 20$  sec,  $G_{i \text{ min}} = 15$  sec, and  $I_{1 \text{ min}} = 10$  sec. Thus,  $B_{\text{max}} = 20 + 15 - 10 = 25$  sec.

In the example presented, the intersection having the minimum green in the inbound direction, intersection x, was the diamond interchange. It has only one possible phase sequence. Thus, only one analysis of interferences projected onto it had to be made. At the present time, the progression program has to evaluate as many total interference calculations, similar to the previous example, as are the number of phase sequences existing at the intersection having the minimum inbound green.

The diamond interchange, which had the minimum inbound green, also had the minimum outbound green. Thus, the phase sequence timing band could not be "slipped down" to reduce the initial upper interference value of 24 sec at the first or all upper interference trials. Normally, some reduction can be achieved. Because of the way the program is structured, no similar all lower interference evaluation is required.

### Selection of Cycle Length

The system cycle length that the progression program will finally select is the one that will maximize progression band efficiency. The procedure is similar to the two-phase signal operation described by Bleyl (7). The percentage of efficiency  $E_c$  of an optimal progression solution for a given cycle length  $C$  is defined as

$$E_c = \frac{100 \times B_{c \text{ max}}}{2C} \quad (13)$$

where  $B_{c \text{ max}}$  is the maximum sum of the progression bands at cycle length  $C$ . In the example being presented,  $B_{70 \text{ max}} = 25$  sec and  $C = 70$  sec; therefore, the percentage of progression efficiency is

$$E_{70} = \frac{100 \times 25}{2 \times 70} = 14.9$$

The upper curve in Figure 6 shows the variation in efficiency of the optimal progression solutions for the Mockingbird Lane arterial system as a function of system cycle lengths ranging from 50 to 80 sec in 1-sec increments. The most efficient cycle length is 53 sec, which results in an efficiency of 28.4 percent. The 70-sec cycle is one of the least efficient cycle lengths that could have been selected. Selecting the maximum efficiency rather than the maximum sum of the progression bands results in



the selection of a shorter cycle length; this is desirable because it tends to further minimize delays at the intersections (8).

The lower curve in Figure 6 is an efficiency plot of the optimal progression solutions for the Mockingbird Lane arterial system but where the three high types of intersections were restricted to operate with only the left-turns-first sequence (phase sequence number 1) permitted. The differences between the two curves in Figure 6 clearly demonstrate the improvement in progression efficiency that may occur in variable-sequence multiphase progression analysis. These plots should not be interpreted, however, as a direct comparison of the progression efficiencies of multiphase progression analysis and conventional two-phase signal operation.

In two-phase operation, approximately 60 percent of the cycle is devoted to the arterial phase, whereas, with multiphase operation having protected left turns, perhaps only 40 percent of the cycle is devoted to arterial through movements. If the arterial has large unprotected turning movements, rather sizable stopped queues may develop that may block the through movements and, as a consequence, reduce the actual progressive green time of a two-phase system from 60 to 40 percent of the cycle or less. In this case, it may be possible to provide a multiphase protected turning movement signal operation that has as much if not more actual progression than a conventional two-phase signal system. In addition, traffic flow using multiphase control would result in smoother, more orderly, and safer traffic operations.

### Attainability

Attainability is a measure of the ability of the progression strategy to utilize the available progressive greens of the intersections within the system. Attainability shows how good the progression solution is compared to the maximum possible solution for given traffic conditions and green splits. The percentage of attainability  $A_c$  for a given cycle length is defined as

$$A_c = 100 - \frac{I_{l \min}}{G_{o \min} + G_{l \min}} \times 100 \quad (14)$$

Thus, if  $I_{l \min}$  is reduced to zero, the attainability would be 100 percent. An attainability plot for the 50- to 80-sec cycle lengths previously evaluated for the Mockingbird Lane control system is shown in Figure 7. This plot shows that several solutions with different cycle lengths have the largest progression bands that could have been determined or 100 percent attainability. The inbound and outbound progression bands must be equal to the minimum greens in each direction to reach 100 percent attainability.

### Time-Space Diagram

It has been previously shown that the most efficient progression occurs at a 53-sec cycle length within the Mockingbird Lane pilot system for the given traffic conditions. As illustrated in the corresponding time-space diagram (Fig. 8), the solution uses phase sequences 3, 4, 4, and 1 for the four intersections 1, 2, 3, and 4 respectively. That is, the diamond interchange uses the leading phase sequence, intersections 2 and 3 the lagging phase sequence, and intersection 4 the left-turns-first sequence. Uniform speeds were used for clarity. This solution has an efficiency of 28.4 percent and an attainability of 100 percent. The relatively low efficiency is due to the low minimum bandwidth limits placed on the system in both directions by the diamond interchange. The attainability of 100 percent shows that no interference to the progression bands occurs. Thus, the phase sequences of the three high types of intersections were selected such that they did not interfere with the progression bands generated from the diamond interchange. Efficiencies on the order of 35 to 40 percent would likely have been obtained had the diamond interchange been a high type of intersection.

### Testing

Although this example has only four progressive signals, the general-use progression program can analyze any practical number of signals. It has been tested against the 10-



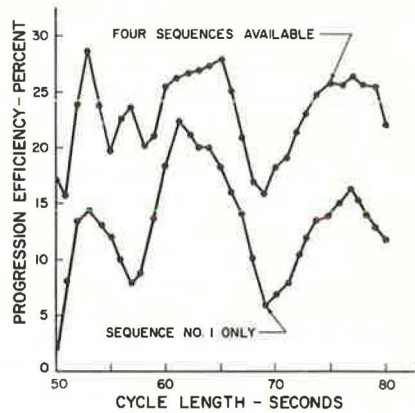
**Table 1. Upper and lower interferences by phase sequence for Mockingbird Lane arterial system.**

Intersection	Intersection Number	Phase Sequence <sup>a</sup>	$I_{U,JP}$	$I_{L,JP}$	
North Central Expressway	1	3	0	0	
		2	17	20	
			30	16	
Greenville	2	3	9	28	
		4	5 <sup>b</sup>	11 <sup>b</sup>	
		3	1	26	8
			2	16	18
3	12 <sup>b</sup>		24		
Skillman	3	4	18	5 <sup>b</sup>	
		4	1	30	1 <sup>b</sup>
			2	24 <sup>b</sup>	5
			3	26	13
4	35		4		

<sup>a</sup>Phase sequences are shown in Figure 4.

<sup>b</sup>Minimum upper or lower interference at intersection.

**Figure 6. Optimal progression efficiency as a function of cycle length for Mockingbird Lane system.**

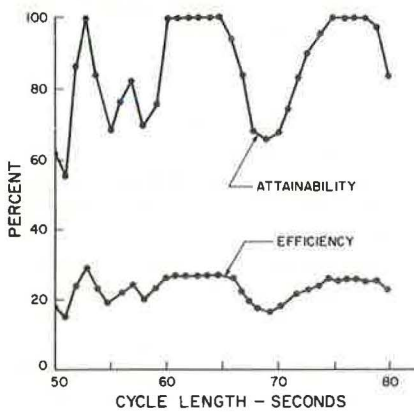


**Table 2. Ranking of upper interference values for determining minimum interference for Mockingbird Lane arterial system.**

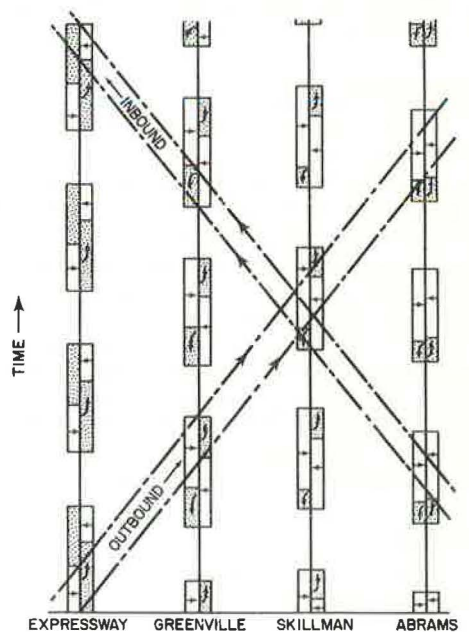
Rank k	Intersection Number	Minimum $I_{Uj}$	Phase Sequence	Minimum $I_{Lj}$	Phase Sequence
1	4	24	2	1 <sup>a</sup>	1
2	3	12	3	5 <sup>a</sup>	4
3	2	5 <sup>a</sup>	4	11	4
4	1	0 <sup>a</sup>	3	0	3

<sup>a</sup>Interferences used in minimum interference solution.

**Figure 7. Relationships between attainability and efficiency for Mockingbird Lane system.**



**Figure 8. Optimal time-space diagram and selected phase sequences for Mockingbird Lane system.**



intersection conventional two-phase progression solution presented by Little (5) with identical optimal results. The real-time control version has been implemented on the Mockingbird Lane computer control system. Preliminary travel time studies reveal that the expected high-quality two-way progression is obtained. Variable-phase sequencing within the arterial A-phase has not caused the motorists traveling the arterial any noticeable difficulty.

#### SUMMARY

A highly flexible general-use computer program has been developed that can be used to determine the most efficient optimal progression along an arterial where the signal phasing can range from the conventional two-phase operation to the flexible selection of multiphase sequences. The program can also compute the initial phase splits if desired. Any practical number of intersections can be analyzed on most digital computers. Intersection types can include normal, jogged, high type, and diamond having three-phase or four-phase with overlap operation. Speeds or distances or both between intersections can also be different in each direction.

The program was developed with the objective of providing a new progression analysis technique that could be used to provide more efficient traffic operations and a higher level of service on signalized traffic facilities. The traffic operations characteristics of the program appear to be well suited for computer control. However, the program can also serve to analyze more typical progression problems such as the desirability of adding leading or lagging left turns at signalized intersections currently having two-phase operation.

#### ACKNOWLEDGMENT

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