OPTIMIZING EARTHMOVING PLANT: SOLUTION FOR THE EXCAVATOR-TRUCKS SYSTEM

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This paper presents a convenient graphical solution for the optimization of an excavator-truck earthmoving system by considering it as a cyclic queuing system. Four different situations are analyzed with reference to the variability of the service time of the excavator and the transit time of the trucks: constant service time and constant transit time; random (negative exponentially distributed) service time and random transit time; constant service time and random transit time; and random service time and constant transit time. Mathematical solutions are presented for the first three situations, and the solution of the fourth situation is obtained via simulation. The optimum number of trucks is determined as a function of two ratios—cost per hour of excavator/cost per hour of truck and transit time/service time. The unit costs of earthmoving are obtained as a function of transit time/service time and the optimum number of trucks, N. There is a point at which optimal values of N are independent of the variability of service and transit times.

•EARTHWORKS are undoubtedly a major activity in modern highway construction. In terms of unit cost per unit area of roadway constructed, plant costs in earthworks amount to at least 50 percent of such costs (1). A considerable part of these costs arises from earthmoving operations, typical of which are excavating-hauling activities carried out using plant systems composed of excavators and hauling units. The efficiency of these systems and consequently the reduction in costs per unit of earth moved is dependent primarily on the appropriate selection of the number and size of units that are served by an excavator. Various investigators have developed methods of optimization for this particular type of problem (2, 3, 4, 5). Nevertheless, their use as a tool in the management of the highway construction industry is, to say the least, very limited.

The purpose of this paper is to present an analysis of the excavator-truck combination as an earthmoving system in which the object is to calculate the optimum number of trucks for a particular size of excavator. The optimum number of trucks is expressed as a function of the ratio of costs of excavator to costs of trucks and the ratio of transit time to loading time for different assumptions about the variability of transit and loading times.

STATEMENT OF THE PROBLEM

An earthmoving system composed of one excavator and N trucks is shown diagrammatically in Figure 1. This may be considered as a queuing system that is described as follows: A truck is loaded, travels to the tip, and returns to the back of the queue or, if there is no queue, begins loading immediately. If there were a continuous queue of trucks, the excavator would move an average of **X** cubic yards per hour. If the excavator is idle for a proportion P_0 of the time, the cost per cubic yard of earth moved will then be

Publication of this paper sponsored by Committee on Construction Management.

$$C_N = \frac{K_1 + NK_2}{X(1 - P_o)} = \frac{K_2}{X} F_N \left(\frac{K_1}{K_2} + N\right) \tag{1}$$

where K_1 is the cost per hour of the excavator, K_2 is the cost per hour of a truck, and F_N is defined as $1/(1 - P_o)$.

The problem is, then, to determine P_o (and hence F_N) for any particular N and any particular set of assumptions about the service time and the transit time.

The service time is defined as the time that elapses from the start of loading one truck until the excavator is available to start loading the next truck. The transit time is the time taken by a truck from leaving the excavator to arriving at the back of the queue. Both these times will, in general, be subject to random fluctuations.

If the mean service time is T_s and the mean transit time is T_t, R is defined as the ratio T_t/T_s. The standard deviations are c_sT_s and c_tT_t; c_s and c_t are then "coefficients of variation".

In the simplest theory, a completely deterministic one, $c_s = c_t = 0$, whereas in the queuing theory approach (5), the probability distributions are negative exponential and thus $c_s = c_t = 1$. These two situations may be regarded as extremes between which any practical situation will lie.

This paper carries out the analysis of the optimization problem in four sections, each corresponding to a different set of assumptions with regard to the variations of loading time and transit time. The variations are as follows:

1. Constant service time and constant transit time ($c_s = c_t = 0$);
2. Random service time and random transit time ($c_s = c_t = 1$);
3. Constant service time and random transit time ($c_s = 0$, $c_t = 1$); and
4. Random service time and constant transit time ($c_s = 1$, $c_t = 0$).

CONSTANT SERVICE TIME AND CONSTANT TRANSIT TIME

When $c_s = c_t = 0$, clearly, the optimum value of N in this completely deterministic analysis is either the integer immediately below R + 1 or the integer immediately above. If R_o is the highest integer that is less than R, the choice lies between N ($= R_o + 1$) and N + 1.

With $N = R_o + 1$, the shovel is idle a fraction $(R - R_o)/(R + 1)$ of the time, so that the F_N value is $(R + 1)/(R_o + 1)$. With N + 1 trucks, the shovel is never idle and therefore $F_{N+1} = 1$.

From Eq. 1 it can be seen that the two systems are equally good if $C_N = C_{N+1}$, or

$$\frac{R + 1}{R_o + 1}\left(\frac{K_1}{K_2} + N\right) = \frac{K_1}{K_2} + N + 1$$

or

$$\frac{K_1}{K_2} = \frac{1 - E}{E}(1 + R_o) \tag{2}$$

where $E = R - R_o$.

The regions of optimal N are shown in Figure 2 in the parameter space which has axes R and K_1/K_2 at right angles.

RANDOM SERVICE TIME AND RANDOM TRANSIT TIME

When service time and transit time are random, the distributions of both times are negative exponential, and the system is then the simple cyclic queuing system analyzed by Griffis (5):

$$P_o = \frac{1}{\sum_{i=0}^{N} \frac{N!}{(N-i)!} \frac{1}{R^i}} \tag{3}$$

Figure 1. Basic layout of an excavator-truck earthmoving system.

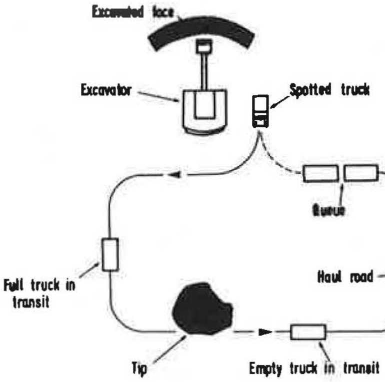


Figure 2. Regions of optimal N in the parameter space $\log_e (K_1/K_2)$ - R for the condition $c_s = c_t = 0$.

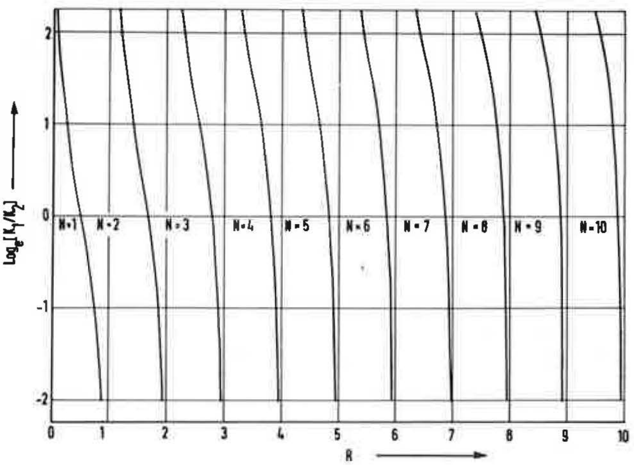
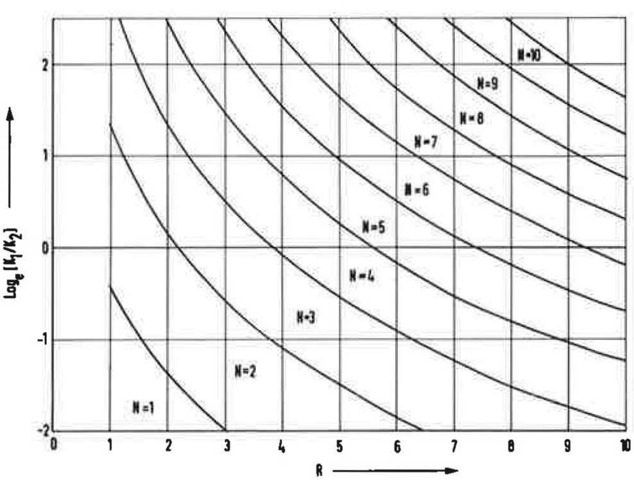


Figure 3. Regions of optimal N in the parameter space $\log_e (K_1/K_2)$ - R for the condition $c_s = c_t = 1$.



To avoid numerical calculations using tables of the cumulative Poisson distribution, the values of F_N have been calculated for various values of R. The critical values of K_1/K_2 have been calculated following the procedure of the previous section. Below a critical value of K_1/K_2, N is the optimal number of trucks, whereas immediately above it N + 1 is better. The results of this are shown in Figure 3.

This representation has the advantage of convenience for the engineer on site, as he does not need to perform any calculations other than those to find K_1/K_2 and R $(= T_t/T_s)$. If, for example, the ratio of mean transit time to mean service time was 7 while the ratio of hourly costs was 1.5 ($\log_e K_1/K_2 = 0.405$), the optimum N value is read off from Figure 3 as being 6. From Figure 2, in the deterministic analysis, the choice would have been 8 trucks.

Figure 4 shows the values of F_N plotted against R, from which it can be seen that, for R = 7, $F_6 = 1.50$, whereas, for R = 8, $F_8 = 1.22$. Since, from Eq. 1, C_N is proportional to $F_N (K_1/K_2 + N)$, the difference between C_6 and C_8 (according to the queuing theory calculations) is about 3 percent.

CONSTANT SERVICE TIME AND RANDOM TRANSIT TIME

In this section the service time is assumed constant $(= T_s)$ while the distribution of transit time is negative exponential (with mean T_t). Again, $R = T_t/T_s$. The state of the system is defined by the number of trucks left behind in the queue at the moment when a truck has just completed its loading. The equilibrium probability of being in state i is Q_i. The transition probability between successive states i and j is written as q(i, j), which means that (if i > 0), during a service time T_s, (j - i + 1) trucks have arrived (out of a possible maximum of N - i). The probability of any particular truck arriving in a time T_s is

$$\int_0^{T_s} \frac{1}{T_t} \exp(-t/T_t)dt = 1 - \exp(-T_s/T_t) = 1 - \exp(-1/R) = 1 - r \tag{4}$$

where $r = \exp(-1/R)$ and t = a random transit time.

The distribution of the number of trucks arriving during the loading time is binomial, with parameters (1 - r) and N - i. Therefore,

$$q(i,j) = {}^{N-i}C_{j-i+1} (1-r)^{j-i+1} r^{N-1-j} \quad (j = i-1, \ldots, N-1) \; (i > 0) \tag{5}$$

When i = 0, the excavator is idle until the first truck arrives. Since there are N trucks out and their arrivals are Poisson events, the expected time to the first arrival is T_t/N. After the first one has arrived, the number that arrive during the first loading time, T_s, is again binomial, with parameters (1 - r) and N - 1. Therefore,

$$q(0,j) = {}^{N-1}C_j (1-r)^j r^{N-1-j} \tag{6}$$

The transition probabilities, q(i, j), are therefore known for all i (= 0, 1, . . . , N - 1) and j (i - 1, i, . . . , N - 1).

In equilibrium,

$$Q_j = \sum_{i=0}^{j+1} Q_i \, q(i,j) \quad \text{for } j = 0, 1, \ldots, N-2$$

and

$$Q_{N-1} = \sum_{i=0}^{N-1} Q_i \, q(i, N-1) \tag{7}$$

The form of these equations allows them to be solved by successive substitution, so that

$$\frac{Q_1}{Q_o} = \frac{1 - q(0,0)}{q(1,0)}$$

$$\frac{Q_2}{Q_o} = \left\{\frac{Q_1}{Q_o} - q(0,1) - \frac{Q_1}{Q_o} q(1,1)\right\} \Big/ q(2,1)$$

. . . , finding successively,

$$\frac{Q_1}{Q_o}, \frac{Q_2}{Q_o}, \ldots, \frac{Q_{N-1}}{Q_o}$$

Finally, the condition $\sum_{j=0}^{N-1} Q_j \equiv 1$ may be applied, so that

$$Q_o \equiv \left\{1 + \frac{Q_1}{Q_o} + \frac{Q_2}{Q_o} + \ldots + \frac{Q_{N-1}}{Q_o}\right\}^{-1} \tag{8}$$

All transitions from i to j (where i > 0) take a time T_s. The average time for any transition from 0 to j is $T_s + T_t/N$, so that, after a state 0, there is, on average, a time T_t/N during which the excavator is idle. Over a long time, the proportion of time during which the excavator is idle is

$$P_o = \frac{\frac{T_t}{N} Q_o}{T_s + \frac{T_t}{N} Q_o}$$

Therefore,

$$F_N = \frac{1}{1 - \dfrac{\frac{T_t}{N} Q_o}{T_s + \frac{T_t}{N} Q_o}} = \frac{T_s + \frac{T_t}{N} Q_o}{T_s}$$

$$F_N = 1 + \frac{R}{N} Q_o \tag{9}$$

where Q_o is given by Eq. 8.

The results are again shown in graphical form in Figure 5, showing regions of optimal N in the (R, K_1/K_2) parameter space. The curves from Figure 2, the completely deterministic analysis, are superimposed on these present results. The shaded regions are those in which the optimal values of N given by the deterministic and this present analysis are identical. In these regions, naturally enough, the queuing theory approach also gives the same results.

The shaded areas lie approximately on the straight line

$$\log_e \frac{K_1}{K_2} = 0.5 + 0.135\,R \tag{10}$$

For example, if R = 4 and $\log_e (K_1/K_2) = 1$, the analyses under the assumptions already considered give the same solution of $N_{opt} = 5$. As R becomes larger, however, the practical values of $\log_e (K_1/K_2)$ tend to lie below the shaded areas, indicating that the estimates of N_{opt} by the three analyses differ. For example, if R = 10 and $\log_e (K_1/K_2) = 0$, the values of N_{opt} estimated by the three analyses are 11 by the deterministic theory, 7 by the queuing theory analysis, and 8 by the analysis using constant loading time and negative exponential transit time.

RANDOM SERVICE TIME AND CONSTANT TRANSIT TIME

It was not found possible to analyze the situation of random service time and constant transit time theoretically, and therefore simulation was used. The service time is negative exponentially distributed with mean T_s while the transit time is constant and equal to T_t. If the "headway" before the j th truck is h_j [that is, the time from the moment the (j - 1) th truck leaves the excavator until the moment the j th truck leaves the excavator], then the j th truck will join the back of a queue if

$$\sum_{i=1}^{N-1} h_{j-i} > T_t$$

and the headway h_j will be a random service time, from the distribution $\frac{\exp(-x/T_s)}{T_s}$; whereas, if

$$\sum_{i=1}^{N-1} h_{j-i} < T_t$$

the headway h_j will be the sum of a random service time and the different $T_t - \sum_{i=1}^{N-1} h_{j-i}$ and there will have been an idle time $T_t - \sum_{i=1}^{N-1} h_{j-i}$. Therefore, over a large number of headways H, the time for which the system has run will be $\sum_{j=1}^{H} h_j$. All this is equivalent to saying

$$z_j = y_j \text{ if } \sum_{i=1}^{N-1} z_{j-i} > 1$$

where $z_j = \frac{h_j}{T_t}$ and y_j = a ratio of a random service time and R, and

$$z_j = y_j + 1 - \sum_{i=1}^{N-1} z_{j-i} \text{ if } \sum_{i=1}^{N-1} z_{j-i} < 1$$

where the y_j are independent negative exponential variables with mean 1/R.

Under the first set of conditions, the idle time, expressed in terms of T_z, is $I_j = 0$; under the second set of conditions,

$$I_j = 1 - \sum_{i=1}^{N-1} z_{j-i}$$

Figure 4. Relationship between F_N and R for different values of N_{opt}.

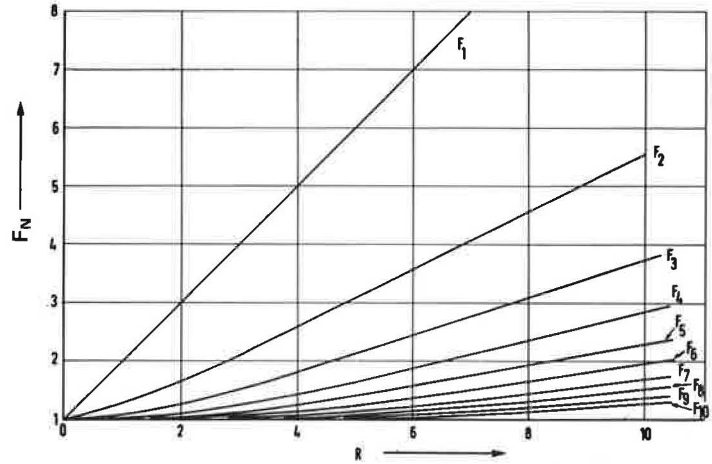


Figure 5. Regions of optimal N in the parameter space $\log_e (K_1/K_2)$ - R for the condition $c_s = 0$, $c_t = 1$; curves from Figure 2 have been superimposed.

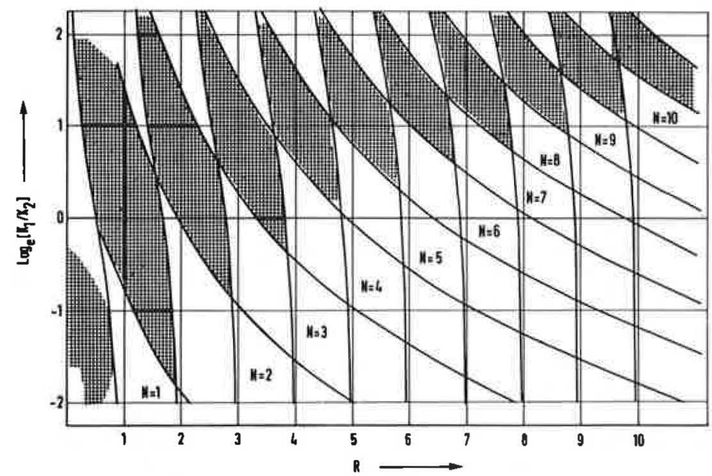


Figure 6. Regions of optimal N in the parameter space $\log_e (K_1/K_2)$ - R for the condition $c_s = 1$, $c_t = 0$.

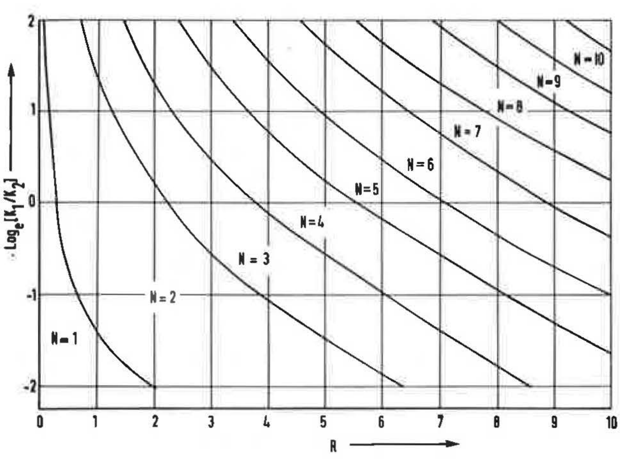
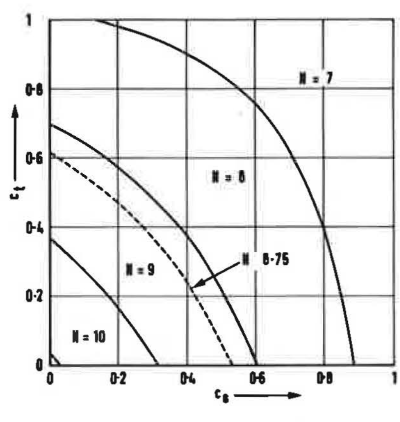


Figure 7. Regions of optimal N in the parameter space (c_s, c_t) for R = 10, $\log_e (K_1/K_2) = 0$; the dotted line is $N_{opt} = 8.75$ from the interpolation formula.



The proportion of time, P_o, that the excavator is idle is

$$P_o = \frac{E\ (I_j)}{E\ (z_j)} \text{ as } j \to \infty$$

Therefore,

$$P_o = 1 - \frac{E\ (y_j)}{E\ (z_j)} = 1 - \frac{1}{R\ E(z_j)} \text{ as } j \to \infty$$

Since $F_N = 1/(1 - P_o)$, $F_N = R \cdot E\ (z_j)$ as $j \to \infty$.

The problem has not been solved analytically but has been simulated on the University of Leeds KDF 9 computer. The graphs resulting from this analysis are shown in Figure 6.

An extensive empirical analysis of the simulated values of N_{opt} with $c_s = 1$, $c_t = 0$ together with the values of N_{DD}, N_{RR}, and N_{DR} and other simulations with "real" values of c_s and c_t have shown that the best interpolation formula is of the form

$$N_{opt} = (1 - c_s)\ (1 - c_t)N_{DD} + c_s\ (1 - c_t)N_{RD} + c_t\ (1 - c_s)N_{DR} + c_s\ c_t\ N_{RR}$$

This amounts to a linear interpolation when either c_s or c_t is fixed. For example, if $c_t = 1$, the expression is reduced to

$$N_{opt} = (1 - c_s)N_{DR} + c_s\ N_{RR}$$

If $R = 10$ and $\log_e\ (K_1/K_2) = 0$, $N_{DD} = 10.6$ (from Fig. 2), $N_{RR} = 6.9$ (from Fig. 3), $N_{DR} = 7.6$ (from Fig. 5), and now (from Fig. 6) it can be seen that $N_{RD} = 7.1$. The interpolation formula above may be used to give N_{opt} for any "real" values of c_s and c_t. For example, if $c_s = 0.33$ and $c_t = 0.33$, N_{opt} is estimated to be 8.75, or in integer form, 9 trucks. Figure 7 shows, for the example already chosen, the regions of optimal N in the (c_s, c_t) space. Now $N_{opt} = 8.75$ is the average of N_{DD} and N_{RR}, the two estimates found by existing standard techniques, and so it may be said that the $N_{opt} = 8.75$ curve in Figure 7 divides the space into two regions. Above and to the right of it, the Griffis model is better than the deterministic model, whereas below and to the left of it, the deterministic model is better. The fact that the first of these regions is larger than the second shows that, in the absence of any information about the values of c_s and c_t, the Griffis queuing theory model is likely to give better results than the deterministic model.

CONCLUSIONS

The graphical solution presented in this paper has the advantage of simplicity and expedience for the man at the site.

By careful selection of excavator-truck combinations, i.e., by selecting an appropriate ratio (K_1/K_2) in function of R, optimization of N can be made independent of the variability of service and cycle times, thereby making the optimizing exercise far simpler.

If for practical considerations such as availability of plant the determination of optimal N becomes dependent on the variability of service and cycle times, then the 4-point interpolation proposed may be used by initially assuming values for c_s and c_t that can be adjusted by obtaining data during the field operations.

The simplicity of the procedure may be of great advantage, especially in road construction where the length of haul roads and face of excavation change rapidly.

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