# OPTIMIZATION OF HAUL FLEET SIZE AND DISTANCE BETWEEN PLANT MOVES ON HIGHWAY PAVING PROJECTS 

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#### Abstract

The study investigates different equipment configurations and procedures on highway paving projects and discusses a means of optimizing haul fleet size and distance between plant moves. The simulation program used to predict production rates is described, and a method is given for obtaining production in truckloads per hour from the plots of the simulation results without the need to run through several approximations. Six paving configuration and procedure models are analyzed with regard to costper cubic yard of concrete in place for different truck fleet sizes or for rates of paving advance in conjunction with different distances between plant moves for each model, least-cost combination, and least-cost model. Conclusions drawn from the study are as follows: Simulation is one means of obtaining the production rates needed for analysis of paving spread configurations and procedures; plots on $\log -\log$ graph paper provide an economical means to extrapolate the simulated data; simulation shows that steady state is always reached in a paving operation and usually between the second and third hours; mathematical modeling allows a means for analyzing different paving spread configurations and procedures; there is considerable difference in cost of concrete in place, depending on the paving spread configuration and procedure used; and picking the least-cost combination within a model is difficult because models are quite sensitive in the areas of production and number of trucks.


-DURING the past 18 years, since the introduction of the slip-form paver and mobile paving plant, highway paving contractors have increased their paving rate more than fourfold. This increase in rate has come about mainly because both contractors and equipment manufacturers have spent considerable time and money developing better equipment. However, much less time and effort have been spent developing least-cost configurations and procedures for using this equipment. [Cost is defined here in dollars per cubic yard of concrete in place; configurations refers to number and size of equipment a paving contractor employs in the paving process; and procedures refers to the way the paving contractor employs the equipment configurations he decides on.] Because of this void in configuration and procedure criteria, an analytical study was conducted.

The account of this study is divided into two major sections. The first section describes how simulation was used to obtain the necessary production rates for determining the least-cost configurations and procedures analytically. The simulation is described in detail elsewhere (1,2). The second section describes the analytical study and results; these are also covered extensively elsewhere (2).

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## THE SIMULATION PROGRAM

To derive the least-cost combination analytically, it was necessary to obtain labor, equipment, and material costs and production rates. It was assumed that present estimating methods gave satisfactory labor and equipment costs per hour and material costs per unit. However, there was some doubt about the accuracy of the production rate estimates because of the stochastic nature of the process. It was therefore decided to use computer simulation to determine these production rates.

The simulation program was written in GPSS and contained as decision variables the average truck speed, average rate of paving advance, and truck fleet size. The program also contained as input empirical probability distributions describing queues at plant and paving train; time between failures for plant, paving train, and individual trucks; repair times for any equipment down; load and unload times; and travel times for plant to paving train and return. These distributions were derived empirically from data gathered by the Bureau of Public Roads during summers from 1963 through 1966. Because the objectives of the Bureau's studies were different from those of this study, not all data necessary for the probability distributions used in the simulation program could be obtained from a single project or even from a single equipment spread. Rather, truck travel times and corresponding distribution came from one study, plant loading times from another, and so forth. A summary of all probability distribution information used in the simulation program is given in Table 1.

The program also provided for such details as starting the trucks in small groups at designated time intervals in the morning and shutting them down in the same order at night and for the repairs on plant, paving train, or individual trucks taking longer than 24 hours.

Reduction of Data
After the preliminary computer runs were completed it was apparent that the cost of computer time was going to be a limiting factor in contractor acceptance and use of the study. That is, if a contractor were interested in finding his least-cost configuration and procedure, data from his own equipment spread would have to be used in the simulation. If, for example, he simulated 19 different haul fleet sizes ( 1 to 18 and 21 trucks), 4 different rates of paving advance ( $1 / 2,1,1 \frac{1}{2}$, and 2 miles per day), and 3 average truck speeds ( 15,30 , and 45 mph ), he would need more than 120 hours of simulation time. It was therefore necessary to analyze the data obtained in the simulation and then attempt to find either a direct mathematical solution or some relationships that would greatly shorten the simulation time. The latter approach turned out to be the more feasible and is described here.

The data were first reduced to hourly production rates and plotted on rectangular coordinate paper. Figure 1 shows such a plot. The plot suggested that steady state might be reached within 2 or 3 hours after starting from an idle state each morning. Other plots verified this fact and showed that steady state was usually reached between the second and third hours. Figure 1 also shows that the production rate might be constant over some part of the region. It was determined that this occurred when the number of trucks working in the system was equal to or greater than the average roundtrip time divided by the average loading time.

It was noticed that the steady state portion of the trucks-per-hour curve formed one continuous curve for every fleet size when extended over several days. To incorporate this into a single continuous curve and to avoid the distortions caused by the start-ups each morning, the daily average rates were plotted on rectangular coordinate paper. Figure 2 shows such a plot. Again each curve was seen to consist of two identifiable regions, the horizontal region and the decreasing region. The formula for the horizontal region has already been stated. It was conjectured that the decreasing region could be represented by a general formula. The daily totals were therefore tried on $\log -\log$ paper. Figure 3 shows such a plot. Because the different fleet sizes plotted as straight lines, the curves represent a general hyperbolic form. The slope unfortunately depends on the average truck speed and the average rate of pavement advance.

Figure 3 also shows that not only were the lines parallel but also there was a constant separation between lines where the fleet size was doubled. Hence the distance between lines was a logarithmic function of the fleet size, and there was no reason to run a simulation for each fleet size. Instead a simulation could be run for, say, fleet sizes of 3,6 , and 12 , and the other fleet size lines could be constructed from these. Figure 4 shows such a plot.

Next it seemed reasonable to investigate the other decision variables, average rate of pavement advance and average truck speed, with the same objective in mind. It was discovered that, for all values of average truck speed divided by average rate of pavement advance (designated a " B " value) and the same fleet size, the plot lines coincided. That is, for a fleet size of, say, 3, the line for a speed of 15 mph and a rate of advance of $1 / 2$ mile per day, the line for a speed of 30 mph and a rate of advance of 1 mile per day, and the line for a speed of 45 mph and a rate of advance of $11 / 2$ miles per day all coincided. Figure 5 shows such a plot for "B" values of 7.5, 15, 30, and 60.

A closer examination of Figure 5 reveals that any line drawn perpendicular to the " B " line of slope 1 shows an equal distance between all lines where the " B " value is doubled. That is, the distance along this line perpendicular to the " B " line of slope 1 between " $B$ " values of 15 and 30 was the same as between 30 and 60 . Therefore the relation along this perpendicular line for all " B " values was logarithmic, and all " B " values for one fleet size could be drawn on one graph after simulating only two or three different ' $B$ " value combinations. Figure 6 shows such a plot.

Now with the relationships found in Figures 3 and 5, a whole group of graphs like Figure 6 (one for each fleet size) could be constructed from only 5 simulations (" $B$ " value of 30 and fleet sizes of 3,6 , and 12 and fleet size of 3 and " $B$ " values of 15 and 60 ). The simulation time could then be reduced from about 120 hours to less than 2 , and a contractor could simulate his equipment spread for less than $\$ 1,000$. Therefore, simulation costs were no longer a limiting factor to contractor acceptance.

One problem still remained before this group of graphs (one for each truck fleet size) became useful. As can be seen from Figure 6, if both the average truck speed and number of trucks remain constant, the amount of concrete delivered to the paving train per hour decreases as the distance from plant to paving train increases. Because it takes a fixed amount of concrete per lineal foot of pavement, the rate of paving advance also decreases. Hence a series of approximations was necessary to find the correct production rate at different distances from the plant because the " $B$ " value did not remain constant. By preparing a series of tables like Table 2 (one for each average truck speed), the necessity for this series of approximations was eliminated. Column 1 of Table 2 is a listing of rates of paving advance. Column 2 is the corresponding " B " value for each rate of paving advance in column 1 and the average truck speed of 30 mph . Column 3 is the number of $8-\mathrm{cu}$ yd truckloads per day needed for a $24-\mathrm{ft}$ by $9-\mathrm{in}$. pavement if the corresponding rates of pavement advance in column 1 are to be obtained. Other truck capacities and other pavement dimensions could also be used as appropriate. Column 4 is column 3 divided by 8 because the simulation was run for an 8 -hour day. If the " B " values (column 2) are plotted against truckloads per hour (column 4) these points line up as a straight line on graphs of the type in Figure 6. Then, to find the hourly output for each day's distance from the plant, a straightedge is held along these points and the output is found at the point where this straightedge intersects the desired day of production away from the plant. At this point then it was thought that acceptable average production rates were obtainable and the development of the different models could begin.

## ANALYTICAL STUDY AND RESULTS

This section deals with quantitative modeling of different equipment configurations and construction procedures. Any quantitative model supposedly representing a realworld situation is by necessity only an approximation. This is partly because some factors cannot be expressed quantitatively and partly because it is often difficult or impossible to find a mathematical solution unless some factors are ignored.

Table 1. Probabilistic distribution information used in the simulation program.

| Use | Distribution Type | Parameters |  |
| :---: | :---: | :---: | :---: |
| Truck loading times | Empirical | Mean | = 73 seconds |
| Truck travel times (plant to paving train) | Normal 0,1 with corrections | Mean Variance | = variable <br> = variable |
| Truck travel times (paving train to plant) | Normal 0,1 with corrections | Mean Variance | $\begin{aligned} & =\text { variable } \\ & =\text { variable } \end{aligned}$ |
| Truck unloading times | Empirical | Mean | = 62 seconds |
| Trucks' interfailure rate | Empirical | Mean | $=15.1$ hours |
| Trucks' downtime | Empirical (closely resembles exponential except for a few lengthy delays) | Mean | $=1,786$ seconds |
| Plant interfailure rate | Empirical | Mean | $=6.9$ hours |
| Plant downtime | Empirical (closely resembles exponential except for a few lengthy delays) | Mean | $=1,920$ seconds |
| Paving train interfailure rate | Empirical | Mean | $=5.9$ hours |
| Paving train downtime | Empirical (closely resembles exponential except for a few lengthy delays) | Mean | = 2,142 seconds |

Figure 1. Hourly production curves for selected fleet sizes (average truck speed $=\mathbf{3 0} \mathbf{~ m p h}$; average rate of truck advance $=1$ mile per day).


Figure 3. Daily production lines for selected truck fleet sizes.


DAYS OF PRODUCTION AWAY FROM PLANT

Figure 2. Daily production curves for selected truck fleet sizes.


Figure 4. Daily production lines for truck fleet sizes of 1 to 18 and 21.


DAYS OF PRODUCTION AWAY FROM PLANT

Six models were investigated for optimum truck fleet size and distance between plant moves: Model 1, constant truck fleet size with box spreader and special sidedump trucks; Model 2, constant truck fleet size with belt spreader and standard reardump trucks; Model 3, constant rate of pavement advance; Model 4, pavement length divided by distance between plant moves may not be an integer; Model 5, drive-through method; and Model 6, leapfrog method. The cost per cubic yard in place was found for different configurations and procedures in each model, and the least-cost combination within the model was identified. Then these least-cost combinations for the different models were compared to try to identify the best least-cost configuration and procedure. All models assume some longitudinal steel and thus require a means of discharging the load at the side of the pavement.

## Model 1-Constant Truck Fleet Size With Box Spreader and Special Side-Dump Trucks

Model 1 assumed that the contractor owned his own side-dump trucks, and, because they had special truck bodies, was unable to rent additional units. Therefore, the truck fleet size was fixed. If the truck fleet size was constant and the distance from plant to paving train first decreased and then increased, by necessity the plant output must vary. So there was a trade-off between cost to move the plant and cost from loss in production because of lack of trucks. When the cost from loss of production became greater than the cost to move the plant, it was time to move the plant.

The total cost per cubic yard of concrete in place included the cost of all equipment, labor, and material pertinent to the decision. The Appendix contains a tabulation of how these costs were calculated for a fleet of 3 trucks and an average truck speed of 30 mph if the plant was located 2 days' production from the beginning of the project and the distance between plant moves was twice 2 days' production. Labor and equipment costs were estimated at $\$ 258.72$ per hour by following the usual cost-estimating procedures. The average truck speed was arbitrarily picked as 30 mph . It could be estimated for any given project, however, either through time studies or, if that proved costly or impossible because the job had not started, by using the information presented in any one of at least 6 construction texts (3). In any case there would always be an average truck speed for a given paving project under given weather and haul-road conditions.

The production rate was found by using the procedure described earlier in this paper. This gave an average rate of pavement advance of 0.75 mile per day and a production rate of 41.25 truckloads per hour during the first day's production away from the plant and an average rate of pavement advance of $0.52+$ mile per day and a production rate of 28 truckloads per hour during the second day's production away from the plant. At 8 cubic yards per truckload and 8 working hours per day, the estimated 2 -day output (one each way from the plant) would be
(41.25) ( $8 \mathrm{cu} y d$ ) ( 8 hours) ( 2 days) $=5,280 \mathrm{cu} \mathrm{yd}$

This considers only one lane; for both lanes the estimated output would be

$$
(2 \text { lanes })(5,280 \mathrm{cu} y d / \text { lane })=10,560 \mathrm{cu} \mathrm{yd}
$$

If the plant were located 2 production days from the beginning of the project, total production between moves would be

First day out each way both lanes
Second day out each way both lanes = (28 loads/hour) (8 hours) (8 cu yd/load) (2 days) $(2$ lanes $)=\underline{7,168}$
Total concrete placed $\quad=17,728 \mathrm{cu} \mathrm{yd}$
The distance between plant moves when the plant was located 2 days' production from the beginning of the project would then be

Figure 5. Selected "B" value lines for truck fleet size of 3.


Table 2. Required production for a $24-\mathrm{ft}$ by 9 -in. pavement, given selected average rates of paving advance and an average truck speed of 30 mph .

Figure 6. "B" value lines for a truck fleet size of 3.

$\left.\left.\begin{array}{llll}\hline \begin{array}{l}\text { Average Rate of } \\ \text { Paving Advance } \\ \text { (miles per day) }\end{array} & & & \\ \text { (1) } & & & \\ \hline \text { (2) Value }\end{array} \quad \begin{array}{l}\text { Truckloads } \\ \text { per Day } \\ (3)\end{array}\right) \begin{array}{l}\text { Truckloads } \\ \text { per Hour } \\ \text { (4) }\end{array}\right]$

Table 3. Comparison of concrete costs in dollars per cubic yard of concrete placed for Model 1.

| Distance Between <br> Plant Moves in <br> Production Days | Number of Trucks in Haul Fleet |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :---: |
|  | 3 | 4 | 5 | 6 | 7 | 8 | 9 |  |
| 2 | 12.171 | 12.025 | 12.070 | 12.116 | 12.161 | 12.207 | 12.253 |  |
| 4 | 11.976 | 11.835 | 11.782 | 11.781 | 11.744 | 11.784 | 11.821 |  |
| 6 | 11.986 | 11.835 | 11.761 | 11.722 | 11.693 | 11.703 | 11.726 |  |
| 8 | 12.040 | 11.872 | 11.780 | 11.746 | 11.704 | 11.699 | 11.714 |  |
| 10 | 12.097 | 11.922 | 11.821 | 11.779 | 11.734 | 11.720 | 11.732 |  |
| 12 | 12.164 | 11.980 | 11.866 | 11.818 | 11.774 | 11.752 | 11.757 |  |

[^0]Table 3 shows the cost per cubic yard for the plant located 1, 2, 3, 4, 5, and 6 days' production from the beginning of the project and truck fleet sizes of 3 through 9 trucks.

Model 2-Constant Truck Fleet Size With Belt Spreader and Standard Rear-Dump Trucks
Model 2 makes all the same assumptions as Model 1 and was introduced to study whether a box spreader requiring special side-dump trucks or a belt spreader and standard rear-dump trucks gave the lower cost per cubic yard in place. The results of the study for Model 2 are given in Table 4.

## Model 3-Constant Rate of Pavement Advance

Model 3 assumed a constant rate of pavement advance and therefore a constant production rate. A belt spreader allowed the use of standard rear-dump trucks and, because these standard rear-dump trucks could be rented, the number of trucks in the haul fleet could realistically be allowed to vary. So Model 3 introduced the other procedure now in general use with paving contractors-constant rate of paving progress and variable truck fleet size. It was reasoned that the contractor should own 3 trucks and rent the rest because these 3 trucks would be needed to move the plant and because at least 3 would be needed in most cases for even the first day's production away from the plant. The model assumed that any extra trucks above 3 needed to maintain constant production could be rented at $\$ 15.00$ per hour.

In Model 3, because the production rate was constant and the haul fleet size varied, more than one graph of the Figure 6 type was needed in finding the number of trucks required to maintain a given production rate. Here the method was the same as described for Models 1 and 2 except that it was necessary to use the straightedge technique on a number of graphs until the fleet size that would just sustain the required production rate for the given number of days' production away from the plant was found.

In Model 3, costs per cubic yard were calculated for 5 different rates of pavement advance ( $0.25,0.40,0.55,0.70$, and 0.845 miles per day) and the plant initially placed 1 through 6 days of production from the beginning of the project. These costs are given in Table 5.

## Model 4-Pavement Length Divided by Distance Between Plant Moves May Not Be an Integer

Model 4 recognized the fact that projects are of a definite length and may not give an integer value when divided by the distance between plant moves. That is, Models 1, 2 , and 3 handled the project length as infinite, whereas Model 4 handled it as if it were finite. All other assumptions were the same as those in Model 3.

The solution procedure for Model 4 was developed in 2 steps. Step 1 figured the distance between plant moves if the plant were moved $0,1,2,3$, etc., times on the project and then copied the costs per cubic yard from Model 3 for the integer days of production between moves on both sides of the one just calculated. That is, for a 12 -mile project, a rate of advance of 0.845 mile per day and 2 moves on the project, the distance between moves is 12 miles divided by 3 (a move-in plus 2 additional moves), then that quantity divided by 0.845 mile per day which equals 4.73 days. Model 3 gave a cost of $\$ 11.684$ for 4 days of production between moves and a cost of $\$ 11.638$ for 5 days of production between moves.

In step 2 some 4 to 6 combinations of rates of pavement advance and moves on the project that showed the lowest costs per cubic yard in step 1 were analyzed to obtain an exact cost for each of these.

Model 4 thus added nothing to the study except to recognize that projects are of finite length and that a reasonably short procedure could be developed to handle this. It will not be mentioned further in the analysis.

Table 4. Comparison of concrete costs in dollars per cubic yard of concrete placed for Model 2.

| Distance Between <br> Plant Moves in <br> Production Days | Number of Trucks in Haul Fleet |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :---: |

Model 2-Constant fleet size, variable production, standard rear-dump trucks, and belt spreader.

Table 5. Comparison of concrete costs in dollars per cubic yard of concrete placed for Model 3.

| Distance Between <br> Plant Moves in <br> Production Days | Rates of Paving Progress in Miles per Day |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  | 0.25 | 0.40 | 0.55 | 0.70 | 0.845 |
| 2 | 15.532 | 13.639 | 12.777 | 12.284 | 12.017 |
| 4 | 14.225 | 12.825 | 12.202 | 11.870 | 11.684 |
| 6 | 13.796 | 12.551 | 12.052 | 11.775 | 11.684 |
| 8 | 13.578 | 12.439 | 11.994 | 11.758 | 11.634 |
| 10 | 13.444 | 12.389 | 11.985 | 11.775 |  |
| 12 | 13.360 | 12.370 | 11.995 | Not calculated |  |

Model 3-Variable haul fleet size, constant production, standard rear-dump trucks, and belt spreader.

Figure 7. Moving and paving schedule for Model 5, given that first plant location is $\mathbf{6}$ days' production from start of project (numbers indicate the location of each paving train in days of production away from plant; circles around numbers indicate it is paving train No. 2).


Table 6. Comparison of concrete costs in dollars per cubic yard of concrete placed for Model 5.

| Distance Between <br> Plant Moves in <br> Production Days | Number of Trucks in Haul Fleet |  |  |
| :--- | :--- | :--- | :--- |
|  | 7 | 8 | 9 |
| 12 (both rigs, one lane) | 11.931 | 11.941 | 11.973 |
| 14 (both rigs, one lane) | 11.898 | 11.903 | 11.927 |
| 16 (both rigs, one lane) | 11.908 | 11.883 | 11.895 |
| 18 (both rigs, one lane) | 11.916 | 11.897 | 11.885 |
| 20 (both rigs, one lane) | 11.939 | 11.915 | 11.898 |

Model 5-Constent haul fleet size, variable production, two complete paving trains with belt spreader, and standard rear-dump trucks.

Model 5 assumed a constant fleet size but tried to make better use of the haul trucks by including two complete paving trains. This would be possible because trucks could be shifted from one paving train to the other as demand dictated. Unfortunately, the simulation program was not written to cover this explicit case but rather provided for one paving train and one plant with single channels at each. Therefore the costs per cubic yard in Model 5 must be considered as less exact than those of the other models.

The construction procedure would be as follows (Fig. 7): One paving train starts at half the distance between plant moves and proceeds toward the plant. The other paving train starts at the plant and moves toward the beginning of the project. When the paving train moving away from the plant reaches the beginning of the project, the other paving train should hopefully be at the plant. Next, both paving trains move to the other lane and pave back to their starting points. When this is completed the plant is moved to the second location and the procedure starts all over again. Table 6 gives costs for this method.

## Model 6-Leapfrog Method

The previous models all had shortcomings. Model 1 required special haul trucks and the refore had a plant output that varied. Model 2 eliminated the use of special haul trucks but still gave variable output. Model 3 allowed a constant production rate but required varying the haul fleet size. Theoretically, varying the number of trucks daily is feasible, but in actual practice superintendents have found this hard to do in many locations. Most owner-drivers would prefer the promise of more than 1 day's work at a time, especially during the busy months when demand is high. Model 4 was Model 3 adapted to a given length of pavement and thus has the same problems. Model 5 returns to the idea of constant fleet size but, as in Models 1 and 2, does it at the expense of not always utilizing full capacity.

Model 6 was developed to try to overcome all the shortcomings discovered in the other models. It allows a constant rate of production and a constant number of haul trucks. To do this it was necessary to go to 2 complete plants and 2 complete paving trains. Model 6 is therefore really just a revision of the method suggested by Maxon and Miller (4).

In Model 6, the first plant (called Plant A) is placed a predetermined distance (number of days' production) from the beginning of the project. The second plant (called Plant B) is placed a distance equal to 3 times Plant A's distance from the beginning of the project plus 1 day's production. That makes the distance between Plants A and B equal to twice Plant A's distance from the beginning of the project plus 1 day's production (production is constant).

Plant A's paving train starts paving at the plant and moves down one set of lanes to the beginning of the project and then back up the other set of lanes past Plant A and on an equal number of production days toward Plant B. Then Plant A's paving train moves back to the set of lanes it started on and paves back to Plant A (Fig. 8).

Plant B's paving train starts a half day's production closer to Plant B than Plant A. It starts at the same time as Plant A's paving train and paves toward Plant B. After reaching Plant B , it continues on an equal number of days' production the other side of Plant B. It then paves back up the other set of lanes to the place it started.

Next, while Plant A and its paving train are moved to their new location past Plant $B$ and an equal distance the other side, Plant $B$ and its paving train pave the remaining 1 day's production in each set of lanes between the original locations of Plants A and B. This allows the 2 days necessary to move Plant A. Then while Plant B is moving to its new location an equal distance the other side of Plant A, Plant A and its paving train can start by paving the center day's paving in each set of lanes between Plant B's original location and Plant A's present location. Both paving trains are now ready to repeat the complete process.

Table 7 gives cost per cubic yard for pavement advances of 0.70 and 0.845 mile per day and days of production between plant moves of $5,7,9,11,13$, and 15.

Figure 8. Moving and paving schedule for Model 6, given that Plant A's first location is $\mathbf{3}$ days' production from start of project (numbers indicate the location of the paving train in days of production away from plant; circles around numbers indicate it is paving train $B$ ).


Table 7. Comparison of concrete costs in dollars per cubic yard of concrete placed for Model 6.

| Parameter | Production Days Between Plant Moves |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 5 | 7 | 9 | 11 | 13 | 15 |
| At 0.70 Mile per Day Paving Progress for Each Paving Train |  |  |  |  |  |  |
| Distance between plant moves in miles | 7 | 9.8 | 12.6 | 15.4 | 18.2 | 21 |
| Number of trucks in haul fleet | 9 | 11 | 14 | 16 | 19 | 21 |
| Cost per cubic yard of concrete placed | 11.751 | 11.675 | 11.667 | 11.661 | 11.685 | 11.699 |
| At 0.845 Mile per Day Paving Progress for Each Paving Train |  |  |  |  |  |  |
| Distance between plant moves in miles | 8.45 | 11.83 | 15.21 | 18.59 | 21.97 | 25.35 |
| Number of trucks in haul fleet | 11 | 15 | 18 | 22 | 25 |  |
| Cost per cubic yard of concrete placed | 11.566 | 11.533 | 11.524 | 11.538 | 11.567 | 11.606 |

Model 6-Constant haul fleet size, constant production, two complete plants, two complete paving trains with belt spreader, and standard dump trucks.

Figure 9. Minimum cost curves for each truck
fleet size in Models 1, 2, and 5.


Analysis
Models 1 and 2 compared equipment configurations using different spreaders and different truck bodies. For the values used in the example simulation and cost analysis, the belt spreader and standard dump trucks gave a smaller cost for each fleet size and distance in days of production between plant moves (Tables 3 and 4 and Fig. 9). However, there is not enough difference to claim Model 2 is always better. Both models find the least-cost combination at 7 trucks and 4.7 miles between plant moves. Again, other data may give other equipment configurations and construction procedures.

Models 2 and 3 compared constant versus variable haul fleet size. Model 3 gave the smaller least cost (Tables 4 and 5). Here again different cost data, especially the $\$ 15.00$ per hour haul truck rental charge, could give different results. The least-cost combination for the example data used indicated that the plant should be run at full capacity (not surprising) and the optimum distance between plant moves should be about 5 miles.

Model 5 was an attempt to hold both production and haul fleet size constant. While it held fleet size constant, it did not hold production constant, and ended up giving the largest least cost per cubic yard combination (Table 6 and Fig. 9). Model 6 represented a second attempt to hold both fleet size and production constant. This time it gave the smallest least-cost combination (Table 7 and Fig. 10). In fact Model 6 gave costs throughout its range that were less than the least-cost combination of any of the other models (Fig. 11). Although it would be unwise to claim it was always the best procedure from among those considered, it would be safe to say that it showed enough promise to be considered seriously by paving contractors.

## Sensitivity

Sensitivity appeared to be a problem within all models as far as equipment configuration was concerned but did not seem to be a problem as far as least cost was concerned, either within the model or between models. In Models 1, 2, and 5, estimated truckloads per hour appeared to be the most critical. Consider, for example, the combination of 7 trucks and 6 days' production between plant moves, which gave the least cost for Model 2. A decrease in output of $1 / a$ truckload per hour gave a percentage decrease of only about 1.5 percent, which in turn changed the least-cost combination by $\$ 0.013$, or less than 1 percent. However, this was enough to change the ieasi-cost combination to 8 trucks and 8 days of production between plant moves. Fortunately, the curves for Models 1 and 2 were flat in the least-cost range (Fig. 9), because it would be most difficult to estimate the output within $1 / 2$ truckload per hour from graphs of the Figure 6 type.

For Models 3 and 6 the number of trucks in the fleet on any given day is most critical because the plant should always be run as close to full production as possible ( 0.845 mile per day was full production for the example). If the number of trucks in the haul fleet was increased by 1 for one-fourth the time between plant moves, the least-cost combination of Model 6 would be increased by less than 1 percent. This, however, would be enough to change the distance between plant moves from 15.2 miles to 18.6 miles. Fortunately, the cost curve is again very flat in the least-cost region (Fig. 11).

## CONCLUSIONS

The study demonstrated that

1. Simulation is one means of obtaining the production rates needed for an analytical examination of paving spread configurations and procedures.
2. Plots on $\log -\log$ graph paper provide an economically feasible procedure to extrapolate the simulation data.
3. Simulation shows that steady state is always reached in a paving operation and usually between the second and third hour after production starts each day.
4. Mathematical modeling does allow a means for analyzing different paving spread configurations and procedures.

Figure 10. Minimum cost curves for each rate of paving progress in Models 3 and 6.

Figure 11. Curve of costs for Model 6 with a rate of paving progress of 0.845 mile per day and least-cost lines for Models 1, 2, 3, and 5.

5. There is considerable difference in cost of concrete per cubic yard in place for different paving spread configurations and procedures.
6. Sensitivity analysis shows that the models are quite sensitive in the areas of production and number of trucks, and picking the least-cost combination within a particular model is difficult. However, a near-least-cost combination can be picked.

It may be impossible to place the plant exactly where the mathematical model dictates because of variables not considered in the mathematical model. However, the least-cost distance does provide a basis on which a decision can be made in light of experience and judgment.

Model 6 appears to be one of the better choices for a paving project over about 8.5 miles in one set of lanes. This is not to say that a contractor should go out and buy all the equipment needed for such a configuration from the information and analysis presented here. However, if he already owns enough equipment to form such a paving spread, he may want to try this method in actual practice.

## REFERENCES

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## APPENDIX

## CALCULATION OF CONCRETE PAVEMENT COSTS

Given: Plant is located 2 days' production from the beginning of the job and moved twice 2 days' production.

Labor and equipment costs
Production

| (\$258.72 per hour) (8 hours) (4 days) (2 lanes) | $\$ 16,558.08$ <br> Moving |
| :--- | ---: |
| $4,299.52$ <br> Total labor and equipment costs | $\$ 20,857.60$ |

Labor and equipment costs per cubic yard $\$ 20,857.60$
$17,728 \mathrm{cu} \mathrm{yd}$
\$ $\quad 1.177$ per cu yd 9.027

Materials
Well $\frac{\$ 4,500}{17,728 \mathrm{cu} \mathrm{yd}}$
Other items 1.000

Subtotal
General overhead 4 percent of $\$ 11.458$

Bonds 0.5 percent of $\$ 11.916$
Subtotal
0.458

Total cost in place
$\$ 20,857.60$

## DISCUSSION

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We congratulate Ghare and Bidwell on a comprehensive simulation of an important set of problems. As they rightly point out, too little fundamental analytical work has been done on such problems. The methods presented should enable the contractor to reduce his costs considerably, but not every contractor has access to a computer or is willing to spend a thousand dollars to obtain the solution. Simple equations or graphs are more likely to be used by the site engineer than are computer simulation packages. It is toward this end, therefore, that the following comments are made.

From the $\log -\log$ curves of production rate versus days of production from the plant, it would seem that

$$
\frac{\mathrm{r}}{\mathrm{r}_{\mathrm{o}}}=\frac{\mathrm{knV}}{\mathrm{~d}} \quad(\text { for } \mathrm{d}>\mathrm{knV})
$$

where
$r$ = rate of production (in truckloads per hour),
$r_{\mathrm{o}}=$ maximum rate of production ( $=46.5$ truckloads per hour),
$\mathrm{n}=$ number of trucks,
$\mathrm{V}=$ average truck speed (in mph),
$\mathrm{d}=$ distance of paver from plant (in miles), and
$\mathrm{k}=\mathrm{a}$ constant, approximately 0.0082 .
We feel that this relationship between $r$ and $d$ is rather more fundamental than that between $r$ and time, since the assumption of a constant rate of pavement advance is unattainable in practice. The relationship demonstrates the points noted by Ghare and Bidwell:

1. If the rate of pavement advance, p , is constant, the graph of $\log \mathrm{r}$ versus $\log$ time is straight;
2. There is a cutoff point such that, for $n>d / k V, r=r_{0}$;
3. When n and B are held constant, the production rate versus time curve is constant;
4. The doubling of $n$ leads to a doubling of $r$ (that is, a constant spacing in the loglog curves); and
5. The doubling of $B$ leads to a doubling of $r$.

As d increases, $r$ decreases. If the connection between the rate of pavement advance $p$ (in miles per day) and the rate of production $r$ (in truckloads per hour) is $p=$ gr, then an equation that links time, $t$, and $d$ can be built up:
When $\mathrm{d}<\mathrm{knV}$,

$$
\mathrm{t}=\mathrm{gr} \mathrm{r}_{\mathrm{a}}
$$

When $d>k n V$,

$$
\mathrm{t}=\frac{\mathrm{knV}}{\mathrm{gr} \mathrm{r}_{\mathrm{o}}}+\int_{\mathrm{knV}}^{\mathrm{u}} \frac{\mathrm{u}}{\mathrm{gr}} \mathrm{du}=\frac{\mathrm{knV}}{2 \mathrm{gr} \mathrm{r}_{\mathrm{o}}}\left\{1+\left(\frac{\mathrm{d}}{\mathrm{knV}}\right)^{2}\right\}
$$

The use of this relationship obviates the need for the "straightedge" method and gives instead a simple means of calculating the time taken to pave a length d starting from the plant.

Before making use of this relationship to investigate the various models proposed, we will comment further on the pattern of the $r$ versus $d$ curves. By setting up a cyclic queuing theory model, assuming negative exponentially distributed loading, unloading,
and transit times, the rate of production can be found as a function of $d$. The pattern of these curves is rather different from those obtained by Ghare and Bidwell; in particular, there is no cutoff point, and the lines in the $\log -\log$ graphs are not straight. A simpler, deterministic analysis, however, does display a cutoff point. For n > $\frac{(L+U+2 d / V)}{U}$ (where $L=$ mean loading time and $U=$ mean unloading time), $r=r_{o}$. For $n<\frac{(L+U+2 d / V)}{U}$, the rate of production is $\frac{r_{0} n U}{(L+U+2 d / V)}$. Now if the sum of loading and unloading times is small compared with the sum of the transit times, this reduces to $\frac{r}{r_{0}}=\frac{U}{2} \frac{n V}{d}$, which is of the same form as the relationship observed by Ghare and Bidwell. All of this indicates that the simulation model gives similar results to those obtained by a deterministic model. It would be interesting to know whether the full complexity of the simulation model is necessary. If the probability distributions of the interfailure times and down times of plant, paver, and trucks were taken out of the model, would the results be significantly different? And what effect do the variances of the loading, unloading, and transit times have on the results ?

With regard to the second part of the paper (that concerned with the comparison of the different models of paving processes), broadly speaking the models fall into two categories. In the first, a length 2 D is paved before the plant is moved, the plant being placed in the middle of each section. The number of trucks is kept constant, and the problem is then to optimize $n$ and $D$. In the second category, the number of trucks in a plant-paver combination is variable and chosen at each point of time so as to have the optimum n . This n will be equal to $\mathrm{d} / \mathrm{kV}$ so as to maintain full production without any waste. It remains to optimize $D$ as in the first type of model. The ability to vary $n$ depends on circumstances such as using two paving trains arranged so that the total number of trucks is constant and the desired rates of increase and decrease of $n$ in the two paving trains match exactly.

There are three basic costs to be introduced: $c_{1}$ is the fixed hourly costs (of plant, paver, etc.); $c_{2}$ is the hourly cost of a truck; $c_{3}$ is the cost of moving the plant.

In the first type of model, the cost of paving a length 2 D is $\left(\mathrm{c}_{1}+\mathrm{nc}_{2}\right) \mathrm{T}+\mathrm{c}_{3}$, where T , the time taken to pave that length, is given by $\frac{\mathrm{knV}}{\mathrm{gr}}\left\{1+\left(\frac{\mathrm{D}}{\mathrm{knV}}\right)^{2}\right\}$. Therefore, the cost per unit length of road may be written as

$$
\mathrm{M}_{1}=\frac{\mathrm{c}_{2}}{\mathrm{gr}_{\mathrm{o}}}\left\{\frac{\mathrm{a}}{2 \mathrm{D}_{0}}+\left(\frac{\mathrm{b}+\mathrm{n}}{\mathrm{D}_{\mathrm{o}}}\right)\left(\mathrm{n}+\frac{\mathrm{D}_{\mathrm{o}}^{2}}{\mathrm{n}}\right)\right\}
$$

where $\mathrm{a}=\frac{\mathrm{gr}_{6} \mathrm{c}_{3}}{\mathrm{KV} \mathrm{c}_{2}}, \quad \mathrm{~b}=\frac{\mathrm{c}_{1}}{\mathrm{c}_{2}}$, and $\mathrm{D}_{0}=\frac{\mathrm{D}}{\mathrm{kV}}$; a and b are the (nondimensional) input parameters of the problem and $n$ and $D_{0}$ are to be optimized. It can be shown that these optimum values are given by

$$
4 n^{2}(n+b)=a b
$$

and

$$
\mathrm{D}_{\mathrm{o}}^{2}=\mathrm{n}^{2}\left(1+\frac{2 \mathrm{n}}{\mathrm{~b}}\right)
$$

In the second type of model, $n$ should be kept equal to $d / k V=D_{0}$ for all values of $d$. The cost per unit length of road is given by

$$
\mathrm{M}_{2}=\frac{\mathrm{c}_{2}}{\mathrm{gr}_{\mathrm{e}}}\left\{\frac{\mathrm{a}}{2 \mathrm{D}_{\mathrm{e}}}+\mathrm{b}+\frac{\mathrm{D}_{0}}{2}\right\}
$$

and this is minimized by putting $\mathrm{D}_{o}^{2}=a$.

It can be shown that the minimum cost of model 2 is between 29 and 50 percent lower than the minimum cost of model 1 , depending on the values of a and $b$. Both these analyses, of course, have treated $n$ as if it were a continuous variable, whereas in fact it may take only integer values. Because of this, the minimum value of $\mathrm{M}_{2}$ given by the analysis above is not strictly attainable. It does, however, show the scale of the savings that are possible by arranging the paving system so that it allows the variation of n so as to maintain optimality.

## AUTHORS' CLOSURE

We thank Cabrera and Maher for an excellent discussion of our paper. We agree wholeheartedly with their suggestion of developing simple equations or graphs that can be used by field engineers as a day-to-day decision tool. Both authors remember too well from their time as field engineers of being frustrated by the lack of good decision tools of this type.

We wish to point out, however, that the comprehensive simulation is not intended to be used as a day-to-day decision tool for field work. It is intended to study the interrelationships of the paving process during the planning stage. During the planning stage a constant rate of pavement advance is taken as a simplifying assumption to develop the curves of production rate versus days of production from the plant. The simulation can be run only once, but the results (the log-log paper curves) can then be incorporated in the form of simple decision tools for day-to-day field work. These simple decision tools can take the form of equations, as pointed out by Cabrera and Maher, or can be in the form of a set of guidelines or graphs.

Cabrera and Maher also raise the possibility of obtaining r-d curves analytically by setting up a cyclic queuing model. Unfortunately, closed-form solution to such queuing models can be obtained only if the probability distributions of times can be assumed to be either deterministic or negative exponential. Neither of the two assumptions is realistic. Analysis of actual times would indicate a large deterministic time component and a smaller component following a beta distribution. The ratio ( $\sqrt{\text { variance }} / \mathrm{mean}$ ) is neither zero as required by deterministic assumption or 1 as required by the negative exponential assumption, but is a small fraction between zero and 1. Hence during the simulation the probability distributions for loading times, unloading times, travel times, and interfailure rates were taken to be either empirical or an approximating normal.

We would like to point out that during the research we did conduct a sensitivity analysis and found that models 1,2 , and 5 are quite sensitive to changes in the ratio ( $\sqrt{\text { variance }} /$ mean ).

Again, in regard to the authors' reason for making the simulation program so allinclusive, it was hoped that a study of the interrelationships of the process would lead to the discovery of a general equation covering all combinations and there would be no further need for simulation runs. At present we still have not been able to derive such a general equation, but we will continue to search and feel if we and others such as Cabrera and Maher continue the research and discussion of results someone will find an answer.


[^0]:    Model 1-Constant fleet size, variable production, special side-dump trucks, and box spreader.

