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## FOREWORD

The papers published in this RECORD relate to construction management. Three of them draw on techniques from other disciplines in solving construction problems while the fourth is a pragmatic discussion of current practice and options. Cabrera and Maher utilize queuing theory to develop an optimal mix of equipment for earthmoving operations. Ghare and Bidwell use simulation to optimize haul fleet size and distance between plant moves on highway paving projects. These are well-known techniques in operations research but are not too often used by construction managers. Fishback and Dickson use thermoscience techniques to optimize size and spacing of hot-mix asphalt windrows. Lee's paper dwells on the complex choice of options available to the construction manager in equipping a project. All of these papers are recommended to the attention of alert constructors for use as analytical tools.
-James Douglas

# EQUIPPING THE PROJECT 

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#### Abstract

The pre-bid study and careful planning of construction methods together with the selection of the most efficient equipment conforming to the planned method is an essential ingredient for successful participation in the heavy construction industry. The use of new, higher horsepower, larger capacity equipment will produce significant savings in the cost or projects. Computer studies using vehicle simulation programs have greatly simplified the selection of equipment that will produce the best costs. Another essential ingredient in a successful and prositable operation is the establishment of a well-planned cost and budget system that provides current production and cost information. The problems presented by high equipment inventories, the obsolescence of older models, and their effect on bonding capacity are restrictive to otherwise technically highly qualified contractors. A more liberal use of mobilization advances would encourage the utilization of newer, more productive equipment. Consequently, the contractor, and ultimately the taxpayer, would benefit from the resulting lower bids on public-works projects.


-EQUIPPING the project is a subject that has been almost "beat to death" over the years by contractors, equipment manufacturers, and engineers. The subject will always survive, however, because the contractor who wants to stay in business must use the newest and latest proven equipment available if he is to participate effectively in what has become one of the most highly competitive industries of our economy.

As one generation of equipment is retired, a new, more productive, and generally more sophisticated breed takes its place. Each successive generation has its own special uses, its own capabilities and, quite frequently, its individual limitations. One very common trait is that, although a piece of equipment may be newer, more productive, bigger, faster, and generally more efficient than its predecessor, it is usually more expensive-more expensive to buy and more expensive to operate. So the contractor faces an ever-growing challenge to get the most from his equipment in order to attain all of the manufacturer's promises. And, to maintain a consistently high production rate, he must find ways to reduce downtime.

In this report I attempt to review the processes that usually influence equipment selection for a project and what effect this has on the persons for whose benefit the work is being done and who ultimately foot the bill-us, the taxpayers.

## SELECTING EQUIPMENT FOR THE PROJECT ESTIMATE

It has often been said that a good estimate is half the battle and that the other half of the battle is making it work. In our experience of making estimates for the larger civil projects, we find we must go one step further. We believe that a thorough study and preliminary cost comparison of various construction methods must precede the development of a project estimate. A good estimate and a poor work method will not get the job done. What is required is a good estimate using the best possible method.

The study of methods must not be limited to single transportation schemes. Many times, a combination of haulage vehicles with material-handling systems such as conveyors-or even railroads-will produce the lowest unit costs, but this requires a

[^0]careful study of borrow sources, haul routes, construction sequence, production requirements, and equipment capabilities.

To illustrate the cost-saving opportunities of new or higher productivity units versus older models, I would like to point out the benefits that can be obtained by using newer, higher horsepower, larger capacity units. I will purposely avoid comparing the relative merits of methods, such as bottom dump haulage versus scrapers, but rather will compare the same type of equipment as a class, older models versus newer models. The details supporting my observations are given in Tables 1 and 2.

The haul road on which the vehicles were studied is an actual project involving six different dump locations along the same route, ranging from $7,400 \mathrm{ft}$ to $29,100 \mathrm{ft}$ or about 1.4 to 5.5 miles.

If the 70 -ton bottom dump is compared with the 110 -ton bottom dump, the larger unit, with a 57 percent greater load capacity, hauls for 17 percent less cost on the short haul and 19 percent less on the long haul. Fleet cost for the larger unit is also less, by 7 and 9 percent for the short and long hauls respectively.

Similarly, comparing the 35 -ton rear dump with the 50 -ton rear dump shows that the larger unit with a 43 percent greater load capacity hauls for 10 percent less cost on the short haul and 12 percent less on the long haul. Fleet cost is reduced from 27 to 28 percent for the short and long hauls respectively by using the larger truck.

Now, let us compare the $24-\mathrm{cu}$ yd scraper with the $40-\mathrm{cu}$ yd unit. With a 67 percent greater capacity, the larger unit will haul for 16 percent less cost on the short haul and 18 percent less on the longer haul. Here again, fleet cost favors the larger unit, which reduced cost 6 and 9 percent for short and long hauls respectively.

There can be no question that a significant reduction in estimated project costs can result from the application of newer, larger, and more productive equipment. This reduction appears not only in unit costs but also in overall capital cost.

## EQUIPMENT AVAILABILITY

The use of more productive equipment reduces the number of vehicles required to obtain a given rate of production. The examples cited show an average of 30 percent fewer bottom dumps and 27 percent fewer rear dumps or scrapers required if large units are chosen. Obviously, fewer units in operation make the operation more sensitive to the mechanical availability of the equipment.

Manufacturers are responding to the need for higher mechanical availability. Many vehicles of recent design include features such as unitized components that are easily removed and replaced, on-board lubrication systems, rapid refueling devices, and more wear-resistant liner material, all of which reduce downtime. The selection of machinery used in the project estimate must consider the availability record of new machinery.

Table 1. Haul road profiles.

| Road Section | Haul No. 1 |  | Haul No. 2 |  | Haul No. 3 |  | Haul No. 4 |  | Haul No. 5 |  | Haul No. 6 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Feet | $\begin{aligned} & \text { Per- } \\ & \text { cent } \end{aligned}$ | Feet | Percent | Feet | Per- cent | Feet | Percent | Feet | Percent | Feet | Per- cent |
| 1 | 2,500 | +2.6 | 2,500 | $+2.6$ | 2,500 | +2.6 | 2,500 | +2.6 | 2,500 | +2.6 | 2,500 | +2.6 |
| 2 | 3,700 | $+1.7$ | 3,700 | +1.7 | 3,700 | +1.7 | 3,700 | +1.7 | 3,700 | $+1.7$ | 3,700 | +1.7 |
| 3 | 0 | $+2.0$ | 2,300 | +2.0 | 6,100 | $+2.0$ | 9,500 | $+2.0$ | 15,300 | $+2.0$ | 19,700 | $+2.0$ |
| 4 | 900 | +4.0 | 1,900 | +4.0 | 1,700 | +4.0 | 2,900 | +4.0 | 2,200 | $+4.0$ | 2,900 | +4.0 |
| 5 | 300 | +5.0 | 300 | +5.0 | 300 | $+5.0$ | 300 | $+5.0$ | 300 | $+5.0$ | 300 | +5.0 |
| Haul distance | 7,400 |  | 10,700 |  | 14,300 |  | 18,900 |  | 24,000 |  | 29,100 |  |
| 5 | 300 | $+5.0$ | 300 | +5.0 | 300 | +5.0 | 300 | +5.0 | 300 | +5.0 | 300 | +5.0 |
| 4 | 900 | +4.0 | 1,900 | +4.0 | 1,700 | $+4.0$ | 2,900 | $+4.0$ | 2,200 | $+4.0$ | 2,900 | +4.0 |
| 3 | 0 | +2.0 | 2,300 | +2.0 | 6,100 | +2.0 | 9,500 | +2.0 | 15,300 | +2.0 | 19,700 | $+2.0$ |
| 2 | 3,700 | +2.3 | 3,700 | +2.3 | 3,700 | +2.3 | 3,700 | +2.3 | 3,700 | +2.3 | 3,700 | $+2.3$ |
| 1 | 1,900 | +3.4 | 1,900 | +3.4 | 1,900 | +3.4 | 1,900 | +3.4 | 1,900 | +3.4 | 1,900 | +3.4 |
| Return distance | 6,800 |  | 10,100 |  | 13,700 |  | 18,300 |  | 23,400 |  | 28,500 |  |
| Cycle distance | 14,200 |  | 20,800 |  | 28,000 |  | 37,200 |  | 47,400 |  | 57,600 |  |

[^1]Table 2. Vehicle comparisons (performance data from published specifications).

| Item | Euclid Bottom Dumps |  |  | Euclid Rear Dumps |  | Euclid Scrapers |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | B30 | B70 | B110 | R35 | R50 | SS24 | SS40 |
| Travel time (minutes) |  |  |  |  |  |  |  |
| Haul No. 1 | 5.82 | 7.24 | 6.91 | 5.64 | 5.69 | 6.01 | 6.91 |
| Haul No. 2 | 8,14 | 10.30 | 9.84 | 7.85 | 8.00 | 8.36 | 10.04 |
| Haul No. 3 | 10.50 | 12.92 | 12.45 | 10.11 | 10.39 | 10.63 | 12.68 |
| Haul No. 4 | 13.80 | 17.09 | 16.50 | 13.23 | 13.66 | 13.99 | 16.90 |
| Haul No. 5 | 17.02 | 20.58 | 19.97 | 16.36 | 16.97 | 17.02 | 20.40 |
| Haul No. 6 | 20.56 | 24.85 | 24.15 | 19.73 | 20.51 | 20.56 | 24.70 |
| Haul unit capacity (cubic yards) | 21 | 47 | 67.5 | 23.3 | 33.3 | 24 | 40 |
| Number of units required to deliver $3,000 \mathrm{cu}$ yd per $50-\mathrm{min}$ hour |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |
| Haul No. 1 | 17 | 10 | 7 | 15 | 11 | 15 | 11 |
| Haul No. 2 | 24 | 14 | 9 | 21 | 15 | 21 | 15 |
| Haul No. 3 | 30 | 17 | 11 | 26 | 19 | 27 | 19 |
| Haul No. 4 | 40 | 22 | 15 | 34 | 25 | 35 | 26 |
| Haul No. 5 | 49 | 27 | 18 | 43 | 31 | 43 | 31 |
| Haul No. 6 | 59 | 32 | 22 | 51 | 37 | 52 | 37 |
| Haul unit hourly cost (dollars) | 22.97 | 30.75 | 36.27 | 25.23 | 30.71 | 32.91 | 37.68 |
| Cost per cubic yard to deliver |  |  |  |  |  |  |  |
| 3,000 cu yd per $50-\mathrm{min}$ hour (dollars) |  |  |  |  |  |  |  |
| Haul No. 1 | 0.130 | 0.103 | 0.085 | 0.126 | 0.113 | 0.165 | 0.138 |
| Haul No. 2 | 0.184 | 0.144 | 0.109 | 0.177 | 0.154 | 0.230 | 0.188 |
| Haul No. 3 | 0.230 | 0.174 | 0.133 | 0.219 | 0.194 | 0.296 | 0.239 |
| Haul No. 4 | 0.306 | 0.226 | 0.181 | 0.286 | 0.256 | 0.384 | 0.327 |
| Haul No. 5 | 0.375 | 0.277 | 0.218 | 0.362 | 0.317 | 0.472 | 0.389 |
| Haul No. 6 | 0.452 | 0.328 | 0.266 | 0.429 | 0.378 | 0.570 | 0.465 |
| Approximate purchase price (thousand dollars) | 47.3 | 85.6 | 113.9 | 86.3 | 85.5 | 80.3 | 102.8 |
| Approximate fleet cost without allowance for spares (thousand dollars) |  |  |  |  |  |  |  |
| Haul No. 1 | 804 | 856 | 797 | 1,295 | 941 | 1,205 | 1,131 |
| Haul No. 2 | 1,135 | 1,198 | 1,025 | 1,812 | 1,283 | 1,686 | 1,542 |
| Haul No. 3 | 1,419 | 1,455 | 1,253 | 2,244 | 1,625 | 2,168 | 1,953 |
| Haul No. 4 | 1,892 | 1,883 | 1,709 | 2,934 | 2,138 | 2,811 | 2,673 |
| Haul No. 5 | 2,318 | 2,311 | 2,050 | 3,711 | 2,651 | 3,453 | 3,187 |
| Haul No. 6 | 2,791 | 2,739 | 2,506 | 4,401 | 3,164 | 4,176 | 3,804 |

## USE OF COMPUTER STUDIES

The use of vehicle simulation by computer makes the selection of methods, vehicle characteristics, and optimum fleet size a matter of routine input of job data, once realistic and truly representative facts have been determined.

Computer studies must carefully tie down all of the variables as a part of input if the results are to have any validity. This is most easily done with an in-house computer. The work, of course, can be farmed out to computer centers. Some major manufacturers of earthmoving construction machinery make their computer services available to prospective purchasers for equipment studies. The customer's own estimator is then usually invited to supply all the special job or application requirements he feels should be considered in applying the equipment to the job.

There is doubt in some quarters regarding the value of computer studies. The magic of electronics has, in a few instances, fallen prey to the "numbers game" that some equipment people play to promote their products. The end result of this type of computer use is often what could be expected only under optimum conditions of grades, road maintenance, performance, availability, tire life, and other factors.

Guarantees for vehicle performance have even been given to prospective buyers based on computerized vehicle applications studies. In a few instances, these computergenerated performance or availability expectations did not materialize, and the guarantees based on them have been disputed. These are isolated cases, however, and
should not be construed as an indictment against the use of computers for estimating or other purposes. We must remember that information obtained from a computer is only as good as the information programmed into it.

Properly used, computers are invaluable for studying methods through simulation programs, projecting information, analyzing complex scheduling problems, and accumulating and recording data from a great many sources into the various bid items of a project estimate. Our company uses its computer extensively for these and other purposes.

## COMPLETION OF THE PROJECT ESTIMATE

Once construction methods have been determined and the equipment has been selected, a good project estimate must go all the way, in a detailed plan and schedule, from the date of notice to proceed with the work to the final release of contractual liability. The estimate must account for every man- and equipment-hour required to build the job, together with the cost of supplies, permanent material, subcontracts, and overhead necessary to support, equip, and de-equip the project. Add profit, interest on investment, contingencies for escalation in the cost of labor, supplies, and equipment, then add the cost of the bond, and you have the bid estimate. All this may sound simple, but for a complex project it is a long, arduous, and expensive job that usually must be completed in a very short period of time. Some contractors think it is just too muchtrouble and too costly. Consequently, they rely on a unit cost approach for estimating and a "seat-of-the-pants" approach for planning. This may be one reason, among others, why profits are marginal or nonexistent for some contractors in highway and heavy construction work.

## ESTABLISHING A WORK PROGRAM

The game plan of a project estimate is essential. It means a melding of the equipment, the method, and the performance of the work according to some ordered discipline. Many firms use CPM or PERT as added tools, while still others use various unnamed systems that may work best for them.

Most contracts require some sort of schedule to accompany the bid, and many specify periodic updating to assure tinuely completion of the work. Today's well-managed construction firms recognize that more emphasis must be placed on the study of equipment and methods, the careful preparation of a job estimate, and the development of a detailed construction schedule that is updated as the work progresses. They have learned that any compromise or substitution in equipment, methods, or scheduling without a careful study of the results with the owner can be disastrous. This is the stuff of which unresolved claims and extensive post-job litigation cases are made.

## TRYING NEW EQUIPMENT

In view of my foregoing statements, it is appropriate to discuss instances in which a new machine or operating concept pops up elsewhere within the industry during the course of a job. The urge to try something new is almost irresistible. However, the prudent contractor, if he is to try a new machine or new method, must make provision for proving it with the least disruption to his established plan.

Complete reliance on a new piece of equipment can be dangerous even if there is a definite understanding between the manufacturer or dealer and the contractor concerning guaranteed availability and performance before the equipment is put to use. When problems do occur, productive time is invariably lost before corrective measures can be taken. These corrective measures themselves, such as adding more units, cause congestion on roads and in work areas. Congestion disrupts the work sequence, leading to a reduction in productive time and, as a consequence, an increase in the cost of the work being performed. This latter phenomenon can occur even while scheduled production is being maintained. Loading, spreading, processing, or handling machinery geared to a particular mode of delivery may be very sensitive to disruptions. Our company's studies indicate, for instance, that a fill spread whose routine operation is
disrupted 25 percent of the time would show a 10 percent increase in its cost; the same spread disrupted 35 percent of the time would show a 17 to 20 percent increase in its cost.

Obviously, no newly designed equipment should be considered as a primary producer until it has been thoroughly proved under actual work conditions. This same principle, to a lesser degree, applies also to new components employed in standard production units. Battles can still be lost for want of the proverbial horseshoe nail in the form of the "we don't stock that part yet" response to field problems.

## KEEPING COST RECORDS

The importance of keeping detailed cost records, prepared on a current basis, cannot be emphasized enough. It is only by this means that management knows where the project stands from week to week or month to month, where the trouble spots are, and where corrective measures should be taken to keep the job on schedule and maintain anticipated earnings.

In dealing primarily with contractors involved in larger civil works, our company has frequently noted that even some big firms do not have an accounting system geared to develop detailed equipment costs. A good equipment cost system is invaluable in determining when one should trade in or sell a piece of equipment. Otherwise, a project may be burdened with equipment that has passed the point of diminishing returns.

Accurate and detailed project cost records are essential in making meaningful costrevenue projections during the course of the work. They also are invaluable for future use as a check on the reasonableness of costs generated in future project estimates.

## EQUIPMENT OBSOLESCENCE

[^2]
## EQUIPMENT AND BONDING CAPACITY

A contractor's bonding capacity is the after-effect of his overall financial position. It is the result of his ability to complete work using the most productive equipment at the least cost to create earnings equal to, or exceeding, his project estimate projec-
tions. Only then can he create a favorable financial condition that maintains or increases his bonding capacity. If the contractor, on completion of a project, carries equipment into his inventory with contingent conditional sales contracts or lease or rental commitments, his bonding capacity may be adversely affected. This is particularly true if the equipment involved is not suitable for work being bid. Thus, a completely experienced and capable contractor may have difficulty in getting a bond.

## THE PUBLIC INTEREST

I have discussed some of the problems that face contractors bidding on public-works projects, dealing primarily with these problems as they relate to equipment. Now, let us assume a situation where

1. The best planned method was selected by the contractor;
2. The bid estimate was realistic and supportable;
3. The contractor was awarded the job;
4. Adequate equipment, selected to conform to the method planned by the contractor, was acquired;
5. The game plan was meticulously followed;
6. The job was well managed and good cost and equipment records were maintained; and
7. The job made a reasonable profit.

If all the foregoing took place, the obvious answer is that the contractor bid the job for the best possible price. To that extent, the public interest was served: The taxpayer got the job done for the least apparent cost. The question is, could the taxpayer have benefited more?

It seems to me that, in looking at contracting for public projects, what we as taxpayers are really buying is the expertise to manage the construction of our projects at the least possible cost. However, much of project management's time is now spent on matters involving financing a project-finding means to stretch available capital to cover equipment purchases and other cost matters-rather than on the job itself.

Financing a project, which is primarily an investment in equipment, is one of the major problems facing the contracting business. Some of the burden of financing should be borne by the owner of the project. Mobilization advances, to the extent of major production equipment requirements, would have the effect of lowering the net cost of projects. The procedures involved in such advances have been used successfully for many years by some government agencies. I see no good reason why the same principle cannot be used more frequently, right down to the level of some of the larger municipal projects.

The funds to finance public works usually have been appropriated by the time jobs are awarded. Therefore, there is little added cost to the public for mobilization advances. The bid would not include the cost of commercial interest. The low bidder would have the opportunity to acquire the most productive equipment. Thus, the contractor, properly bonded, would then be giving an implied assurance to the public that it is getting the best performance for the least cost. And that, after all, is what we as taxpayers are striving for.

# OPTIMIZING EARTHMOVING PLANT: SOLUTION FOR THE EXCAVATOR-TRUCKS SYSTEM 

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#### Abstract

This paper presents a convenient graphical solution for the optimization of an excavator-truck earthmoving system by considering it as a cyclic queuing system. Four different situations are analyzed with reference to the variability of the service time of the excavator and the transit time of the trucks: constant service time and constant transit time; random (negative exponentially distributed) service time and random transit time; constant service time and random transit time; and random service time and constant transit time. Mathematical solutions are presented for the first three situations, and the solution of the fourth situation is obtained via simulation. The optimum number of trucks is determined as a function of two ratios-cost per hour of excavator/cost per hour of truck and transit time/service time. The unit costs of earthmoving are obtained as a function of transit time/service time and the optimum number of trucks, N . There is a point at which optimal values of N are independent of the variability of service and transit times.


-EARTHWORKS are undoubtedly a major activity in modern highway construction. In terms of unit cost per unit area of roadway constructed, plant costs in earthworks amount to at least 50 percent of such costs (1). A considerable part of these costs arises from earthmoving operations, typical of which are excavating-hauling activities carried out using plant systems composed of excavators and hauling units. The efficiency of these systems and consequently the reduction in costs per unit of earth moved is dependent primarily on the appropriate selection of the number and size of units that are served by an excavator. Various investigators have developed methods of optimization for this particular type of problem ( $2, \underline{3}, \underline{4}, \underline{5}$ ). Nevertheless, their use as a tool in the management of the highway construction industry is, to say the least, very limited.

The purpose of this paper is to present an analysis of the excavator-truck combination as an earthmoving system in which the object is to calculate the optimum number of trucks for a particular size of excavator. The optimum number of trucks is expressed as a function of the ratio of costs of excavator to costs of trucks and the ratio of transit time to loading time for different assumptions about the variability of transit and loading times.

## STATEMENT OF THE PROBLEM

An earthmoving system composed of one excavator and N trucks is shown diagrammatically in Figure 1. This may be considered as a queuing system that is described as follows: A truck is loaded, travels to the tip, and returns to the back of the queue or, if there is no queue, begins loading immediately. If there were a continuous queue of trucks, the excavator would move an average of $X$ cubic yards per hour. If the excavator is idle for a proportion $P_{0}$ of the time, the cost per cubic yard of earth moved will then be

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$$
\begin{equation*}
\mathrm{C}_{\mathrm{N}}=\frac{\mathrm{K}_{1}+\mathrm{NK}_{2}}{\mathbf{X}\left(1-\mathrm{P}_{0}\right)}=\frac{\mathrm{K}_{2}}{\mathbf{X}} \mathrm{~F}_{\mathrm{N}}\left(\frac{\mathrm{~K}_{1}}{\mathrm{~K}_{2}}+\mathrm{N}\right) \tag{1}
\end{equation*}
$$

where $\mathrm{K}_{1}$ is the cost per hour of the excavator, $\mathrm{K}_{2}$ is the cost per hour of a truck, and $F_{\mathrm{N}}$ is defined as $1 /\left(1-P_{o}\right)$.

The problem is, then, to determine $P_{0}$ (and hence $F_{N}$ ) for any particular $N$ and any particular set of assumptions about the service time and the transit time.

The service time is defined as the time that elapses from the start of loading one truck until the excavator is available to start loading the next truck. The transit time is the time taken by a truck from leaving the excavator to arriving at the back of the queue. Both these times will, in general, be subject to random fluctuations.

If the mean service time is $T_{s}$ and the mean transit time is $T_{t}, R$ is defined as the ratio $T_{t} / T_{8}$. The standard deviations are $c_{s} T_{g}$ and $c_{t} T_{t} ; c_{8}$ and $c_{t}$ are then "coefficients of variation".

In the simplest theory, a completely deterministic one, $c_{8}=c_{t}=0$, whereas in the queuing theory approach (5), the probability distributions are negative exponential and thus $c_{s}=c_{t}=1$. These two situations may be regarded as extremes between which any practical situation will lie.

This paper carries out the analysis of the optimization problem in four sections, each corresponding to a different set of assumptions with regard to the variations of loading time and transit time. The variations are as follows:

1. Constant service time and constant transit time ( $\mathrm{c}_{\mathrm{g}}=\mathrm{c}_{\mathrm{t}}=0$ );
2. Random service time and random transit time ( $c_{\mathrm{g}}=\mathrm{c}_{\mathrm{t}}=1$ );
3. Constant service time and random transit time ( $c_{\mathrm{g}}=0, \mathrm{c}_{\mathrm{t}}=1$ ); and
4. Random service time and constant transit time ( $c_{s}=1, c_{t}=0$ ).

## CONSTANT SERVICE TIME AND CONSTANT TRANSIT TIME

When $c_{s}=c_{t}=0$, clearly, the optimum value of $N$ in this completely deterministic analysis is either the integer immediately below $R+1$ or the integer immediately above. If $R_{o}$ is the highest integer that is less than $R$, the choice lies between $N\left(=R_{o}+1\right)$ and $\mathrm{N}+1$.

With $N=R_{o}+1$, the shovel is idle a fraction $\left(R-R_{o}\right) /(R+1)$ of the time, so that the $\mathrm{F}_{\mathrm{N}}$ value is $(\mathrm{R}+1) /\left(\mathrm{R}_{0}+1\right)$. With $\mathrm{N}+1$ trucks, the shovel is never idle and therefore $\mathrm{F}_{\mathrm{N}+1}=1$.

From Eq. 1 it can be seen that the two systems are equally good if $C_{N}=C_{N+1}$, or

$$
\frac{\mathrm{R}+1}{\mathrm{R}_{\mathrm{o}}+1}\left(\frac{\mathrm{~K}_{1}}{\mathrm{~K}_{2}}+\mathrm{N}\right)=\frac{\mathrm{K}_{1}}{\mathrm{~K}_{2}}+\mathrm{N}+1
$$

or

$$
\begin{equation*}
\frac{\mathrm{K}_{1}}{\mathrm{~K}_{2}}=\frac{1-\mathrm{E}}{\mathrm{E}}\left(1+\mathrm{R}_{0}\right) \tag{2}
\end{equation*}
$$

where $E=R-R_{0}$.
The regions of optimal $N$ are shown in Figure 2 in the parameter space which has axes $R$ and $K_{1} / K_{2}$ at right angles.

## RANDOM SERVICE TIME AND RANDOM TRANSIT TIME

When service time and transit time are random, the distributions of both times are negative exponential, and the system is then the simple cyclic queuing system analyzed by Griffis ( 5 ):

$$
\begin{equation*}
P_{o}=\frac{1}{\sum_{i=0}^{N} \frac{N:}{(N-i)!} \frac{1}{R^{1}}} \tag{3}
\end{equation*}
$$

Figure 1. Basic layout of an excavator-truck earthmoving system.


Figure 2. Regions of optimal $\mathbf{N}$ in the parameter space $\log _{\mathrm{e}}$ $\left(K_{1} / K_{2}\right)-R$ for the condition $c_{s}=c_{t}=0$.


Figure 3. Regions of optimal $\mathbf{N}$ in the parameter space $\log _{\text {s }}$ $\left(K_{1} / K_{2}\right) \cdot R$ for the condition $c_{s}=c_{t}=1$.


To avoid numerical calculations using tables of the cumulative Poisson distribution, the values of $F_{N}$ have been calculated for various values of $R$. The critical values of $\mathrm{K}_{1} / \mathrm{K}_{2}$ have been calculated following the procedure of the previous section. Below a critical value of $\mathrm{K}_{1} / \mathrm{K}_{2}, N$ is the optimal number of trucks, whereas immediately above it $\mathrm{N}+1$ is better. The results of this are shown in Figure 3.

This representation has the advantage of convenience for the engineer on site, as he does not need to perform any calculations other than those to find $K_{1} / K_{2}$ and $R\left(=T_{t} / T_{\mathrm{a}}\right)$. If, for example, the ratio of mean transit time to mean service time was 7 while the ratio of hourly costs was $1.5\left(\log _{e} \mathrm{~K}_{1} / \mathrm{K}_{2}=0.405\right)$, the optimum N value is read off from Figure 3 as being 6. From Figure 2, in the deterministic analysis, the choice would have been 8 trucks.

Figure 4 shows the values of $\mathrm{F}_{\mathrm{N}}$ plotted against R , from which it can be seen that, for $R=7, F_{6}=1.50$, whereas, for $R=8, F_{8}=1.22$. Since, from Eq. 1, $C_{v}$ is proportional to $F_{N}\left(K_{1} / K_{2}+N\right)$, the difference between $\mathrm{C}_{6}$ and $\mathrm{C}_{8}$ (according to the queuing theory calculations) is about 3 percent.

## CONSTANT SERVICE TIME AND RANDOM TRANSIT TIME

In this section the service time is assumed constant $\left(=T_{s}\right)$ while the distribution of transit time is negative exponential (with mean $T_{t}$ ). Again, $R=T_{t} / T_{s}$. The state of the system is defined oy the number of trucks left behind in the queue at the moment when a truck has just completed its loading. The equilibrium probability of being in state $i$ is $Q_{1}$. The transition probability between successive states $i$ and $j$ is written as $q(i, j)$, which means that (if $i>0$ ), during a service time $T_{s},(j-i+1)$ trucks have arrived (out of a possible maximum of $\mathrm{N}-\mathrm{i}$ ). The probability of any particular truck arriving in a time $\mathrm{T}_{\mathrm{B}}$ is

$$
\begin{equation*}
\int_{0}^{T_{s}} \frac{1}{T_{t}} \exp \left(-t / T_{t}\right) d t=1-\exp \left(-T_{s} / T_{t}\right)=1-\exp (-1 / R)=1-r \tag{4}
\end{equation*}
$$

where $r=\exp (-1 / R)$ and $t=a$ random transit time.
The distribution of the number of trucks arriving during the loading time is binomial, with parameters ( $1-\mathrm{r}$ ) and $\mathrm{N}-\mathrm{i}$. Therefore,

$$
\begin{equation*}
q(i, j)={ }^{N-1} C_{j-1+1}(1-r)^{j-1+1} r^{N-1-j}(j=i-1, \ldots, N-1)(i>0) \tag{5}
\end{equation*}
$$

When $\mathbf{i}=0$, the excavator is idle until the first truck arrives. Since there are N trucks out and their arrivals are Poisson events, the expected time to the first arrival is $\mathrm{T}_{\mathrm{t}} / \mathrm{N}$. After the first one has arrived, the number that arrive during the first loading time, $\mathrm{T}_{\mathrm{s}}$, is again binomial, with parameters $(1-\mathrm{r})$ and $\mathrm{N}-1$. Therefore,

$$
\begin{equation*}
q(0, j)=^{N-1} C_{j}(1-r)^{J} r^{N-1-j} \tag{6}
\end{equation*}
$$

The transition probabilities, $q(i, j)$, are therefore known for all $i(=0,1, \ldots, N-1)$ and $\mathrm{j}(\mathrm{i}-1, \mathrm{i}, \ldots, \mathrm{N}-1$ ).

In equilibrium,

$$
Q_{1}=\sum_{i=0}^{j+1} Q_{i} q(i, j) \quad \text { for } j=0,1, \ldots, N-2
$$

and

$$
\begin{equation*}
Q_{N-1}=\sum_{i=0}^{N-1} Q_{1} q(i, N-1) \tag{7}
\end{equation*}
$$

The form of these equations allows them to be solved by successive substitution, so that

$$
\begin{aligned}
& \frac{Q_{1}}{Q_{o}}=\frac{1-q(0,0)}{q(1,0)} \\
& \frac{Q_{2}}{Q_{0}}=\left\{\frac{Q_{1}}{Q_{0}}-q(0,1)-\frac{Q_{1}}{Q_{0}} q(1,1)\right\} / q(2,1)
\end{aligned}
$$

..., finding successively,

$$
\frac{\mathrm{Q}_{1}}{\mathrm{Q}_{0}}, \frac{\mathrm{Q}_{2}}{\mathrm{Q}_{0}}, \ldots, \frac{\mathrm{Q}_{N-1}}{\mathrm{Q}_{0}}
$$

Finally, the condition $\sum_{j=0}^{N-1} Q_{j} \equiv 1$ may be applied, so that

$$
\begin{equation*}
Q_{0} \equiv\left\{1+\frac{Q_{1}}{Q_{0}}+\frac{Q_{2}}{Q_{0}}+\ldots+\frac{Q_{4-1}}{Q_{0}}\right\}^{-1} \tag{8}
\end{equation*}
$$

All transitions from $i$ to $j$ (where $i>0$ ) take a time $T_{8}$. The average time for any transition from 0 to j is $\mathrm{T}_{s}+\mathrm{T}_{\mathrm{t}} / \mathrm{N}$, so that, after a state 0 , there is, on average, a time $\mathrm{T}_{\mathrm{t}} / \mathrm{N}$ during which the excavator is idle. Over a long time, the proportion of time during which the excavator is idle is

$$
P_{o}=\frac{\frac{T_{t}}{N} Q_{0}}{T_{s}+\frac{T_{t}}{N} Q_{0}}
$$

Therefore,

$$
\begin{align*}
& \mathrm{F}_{n}=\frac{1}{1-\frac{T_{t}}{\mathrm{~N}} \mathrm{Q}_{0}} \mathrm{~T}_{\mathrm{a}}+\frac{\mathrm{T}_{\mathrm{t}}}{\mathrm{~N}} \mathrm{Q}_{\mathrm{o}}
\end{align*}=\frac{\mathrm{T}_{\mathrm{s}}+\frac{\mathrm{T}_{\mathrm{t}}}{\mathrm{~N}} \mathrm{Q}_{0}}{\mathrm{~T}_{\mathrm{s}}} .
$$

where $Q_{0}$ is given by Eq. 8.
The results are again shown in graphical form in Figure 5, showing regions of optimal N in the ( $\mathrm{R}, \mathrm{K}_{1} / \mathrm{K}_{2}$ ) parameter space. The curves from Figure 2, the completely deterministic analysis, are superimposed on these present results. The shaded regions are those in which the optimal values of N given by the deterministic and this present analysis are identical. In these regions, naturally enough, the queuing theory approach also gives the same results.

The shaded areas lie approximately on the straight line

$$
\begin{equation*}
\log _{e} \frac{\mathrm{~K}_{1}}{\mathrm{~K}_{2}}=0.5+0.135 \mathrm{R} \tag{10}
\end{equation*}
$$

For example, if $R=4$ and $\log _{4}\left(K_{1} / K_{2}\right)=1$, the analyses under the assumptions already considered give the same solution of $\mathrm{N}_{\text {opt }}=5$. As R becomes larger, however, the practical values of $\log _{0}\left(\mathrm{~K}_{1} / \mathrm{K}_{2}\right)$ tend to lie below the shaded areas, indicating that the
estimates of $\mathrm{N}_{\text {opt }}$ by the three analyses differ. For example, if $\mathrm{R}=10$ and $\log _{\mathrm{e}}\left(\mathrm{K}_{1} / \mathrm{K}_{2}\right)=$ 0 , the values of $\mathrm{N}_{\text {opt }}$ estimated by the three analyses are 11 by the deterministic theory, 7 by the queuing theory analysis, and 8 by the analysis using constant loading time and negative exponential transit time.

## RANDOM SERVICE TIME AND CONSTANT TRANSIT TIME

It was not found possible to analyze the situation of random service time and constant transit time theoretically, and therefore simulation was used. The service time is negative exponentially distributed with mean $\mathrm{T}_{s}$ while the transit time is constant and equal to $T_{t}$. If the "headway" before the $j$ th truck is $h_{\text {, [that is, the time from the mo- }}$ ment the $(j-1)$ th truck leaves the excavator until the moment the $j$ th truck leaves the excavator ], then the $j$ th truck will join the back of a queue if

$$
\sum_{i=1}^{N-1} h_{y_{-1}}>T_{t}
$$

and the headway $h_{s}$ will be a random service time, from the distribution $\frac{\exp \left(-x / T_{s}\right)}{T_{s}}$;
whereas, if

$$
\sum_{i=1}^{N-1} h_{j-i}<T_{t}
$$

the headway $h_{j}$ will be the sum of a random service time and the different $T_{t}-\sum_{h_{j-1}}^{N-1}$ N-1
and there will have been an idle time $T_{t}-\sum_{i=1} h_{j-1}$. Therefore, over a large number of headways $H$, the time for which the system has run will be $\sum h_{j}$. All this is equivalent to saying

$$
z_{j}=y_{j} \text { if } \sum_{i=1}^{N-1} z_{j-1}>1
$$

where $z_{j}=\frac{h_{j}}{T_{t}}$ and $y_{j}=$ a ratio of a random service time and $R$, and

$$
z_{j}=y_{j}+1-\sum_{i=1}^{N-1} z_{j-1} \text { if } \sum_{i=1}^{N-1} z_{j-i}<1
$$

where the $y_{j}$ are independent negative exponential variables with mean $1 / R$.
Under the first set of conditions, the idle time, expressed in terms of $T_{z}$, is $I_{j}=0$; under the second set of conditions,

$$
I_{j}=1-\sum_{i=1}^{N-1} z_{j-1}
$$

Figure 4. Relationship between $\mathrm{F}_{\mathrm{N}}$ and R for different values of $\mathrm{N}_{\text {opt }}$.


Figure 5. Regions of optimal N in the parameter space $\log _{\mathrm{e}}\left(\mathrm{K}_{1} / \mathrm{K}_{2}\right)-\mathrm{R}$ for the condition $\mathrm{c}_{\mathrm{s}}=0, \mathrm{c}_{\mathrm{t}}=\mathbf{1}$; curves from Figure 2 have been superimposed.


Figure 6. Regions of optimal $\mathbf{N}$ in the parameter space $\log _{\text {e }}$ $\left(K_{1} / K_{2}\right)-R$ for the condition $c_{s}=1, c_{t}=0$.


Figure 7. Regions of optimal $\mathbf{N}$ in the parameter space ( $c_{s}, c_{t}$ ) for $R=10, \log _{e}$ ( $\mathrm{K}_{1} / \mathrm{K}_{2}$ ) $=0$; the dotted line is $\mathrm{N}_{\text {opt }}=8.75$ from the interpolation formula.


The proportion of time, $P_{0}$, that the excavator is idle is

$$
P_{0}=\frac{E\left(I_{j}\right)}{E\left(z_{j}\right)} \text { as } j \rightarrow \infty
$$

Therefore,

$$
P_{o}=1-\frac{E\left(y_{j}\right)}{E\left(z_{j}\right)}=1-\frac{1}{R E\left(z_{j}\right)} \text { as } j \rightarrow \infty
$$

Since $\mathbf{F}_{\mathrm{N}}=1 /\left(1-\mathbf{P}_{\mathrm{o}}\right), \mathbf{F}_{\mathrm{N}}=$ R.E $\left(\mathbf{z}_{\mathrm{j}}\right)$ as $\mathrm{j} \rightarrow \infty$.
The problem has not been solved analytically but has been simulated on the University of Leeds KDF 9 computer. The graphs resulting from this analysis are shown in Figure 6.

An extensive empirical analysis of the simulated values of $N_{\text {opt }}$ with $c_{s}=1, c_{t}=0$ together with the values of $N_{D D}, N_{R R}$, and $N_{D R}$ and other simulations with "real" values of $c_{b}$ and $c_{t}$ have shown that the best interpolation formula is of the form

$$
N_{o p t}=\left(1-c_{s}\right)\left(1-c_{t}\right) N_{D D}+c_{s}\left(1-c_{t}\right) N_{R D}+c_{t}\left(1-c_{s}\right) N_{D R}+c_{g} c_{t} N_{R R}
$$

This amounts to a linear interpolation when either $c_{s}$ or $c_{t}$ is fixed. For example, if $c_{t}=1$, the expression is reduced to

$$
N_{o p t}=\left(1-c_{s}\right) N_{D R}+c_{s} N_{R R}
$$

If $R=10$ and $\log _{e}\left(K_{1} / K_{2}\right)=0, N_{00}=10.6$ (from Fig. 2), $N_{R R}=6.9$ (from Fig. 3), $N_{0 R}=$ 7.6 (from Fig. 5), and now (from Fig. 6) it can be seen that $\mathrm{N}_{\mathrm{RD}}=7.1$. The interpolation formula above may be used to give $N_{o p t}$ for any "real" values of $c_{s}$ and $c_{t}$. For example, if $\mathrm{c}_{\mathrm{n}}=0.33$ and $\mathrm{c}_{\mathrm{t}}=0.33, \mathrm{~N}_{\mathrm{opt}}$ is estimated to be 8.75, or in integer form, 9 trucks. Figure 7 shows, for the example already chosen, the regions of optimal $N$ in the ( $c_{s}, c_{t}$ ) space. Now $N_{o_{p t}}=8.75$ is the average of $N_{D O}$ and $N_{R R}$, the two estimates found by existing standard techniques, and so it may be said that the $N_{\text {opt }}=8.75$ curve in Figure 7 divides the space into two regions. Above and to the right of it, the Griffis model is better than the deterministic model, whereas below and to the left of it, the deterministic model is better. The fact that the first of these regions is larger than the second shows that, in the absence of any information about the values of $c_{8}$ and $c_{t}$, the Griffis queuing theory model is likely to give better results than the deterministic model.

## CONCLUSIONS

The graphical solution presented in this paper has the advantage of simplicity and expedience for the man at the site.

By careful selection of excavator-truck combinations, i.e., by selecting an appropriate ratio ( $\mathrm{K}_{1} / \mathrm{K}_{2}$ ) in function of $R$, optimization of $N$ can be made independent of the variability of service and cycle times, thereby making the optimizing exercise far simpler.

If for practical considerations such as availability of plant the determination of optimal N becomes dependent on the variability of service and cycle times, then the 4point interpolation proposed may be used by initially assuming values for $c_{s}$ and $c_{t}$ that can be adjusted by obtaining data during the field operations.

The simplicity of the procedure may be of great advantage, especially in road construction where the length of haul roads and face of excavation change rapidly.

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# OPTIMIZATION OF HAUL FLEET SIZE AND DISTANCE BETWEEN PLANT MOVES ON HIGHWAY PAVING PROJECTS 

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#### Abstract

The study investigates different equipment configurations and procedures on highway paving projects and discusses a means of optimizing haul fleet size and distance between plant moves. The simulation program used to predict production rates is described, and a method is given for obtaining production in truckloads per hour from the plots of the simulation results without the need to run through several approximations. Six paving configuration and procedure models are analyzed with regard to costper cubic yard of concrete in place for different truck fleet sizes or for rates of paving advance in conjunction with different distances between plant moves for each model, least-cost combination, and least-cost model. Conclusions drawn from the study are as follows: Simulation is one means of obtaining the production rates needed for analysis of paving spread configurations and procedures; plots on $\log -\log$ graph paper provide an economical means to extrapolate the simulated data; simulation shows that steady state is always reached in a paving operation and usually between the second and third hours; mathematical modeling allows a means for analyzing different paving spread configurations and procedures; there is considerable difference in cost of concrete in place, depending on the paving spread configuration and procedure used; and picking the least-cost combination within a model is difficult because models are quite sensitive in the areas of production and number of trucks.


-DURING the past 18 years, since the introduction of the slip-form paver and mobile paving plant, highway paving contractors have increased their paving rate more than fourfold. This increase in rate has come about mainly because both contractors and equipment manufacturers have spent considerable time and money developing better equipment. However, much less time and effort have been spent developing least-cost configurations and procedures for using this equipment. [Cost is defined here in dollars per cubic yard of concrete in place; configurations refers to number and size of equipment a paving contractor employs in the paving process; and procedures refers to the way the paving contractor employs the equipment configurations he decides on.] Because of this void in configuration and procedure criteria, an analytical study was conducted.

The account of this study is divided into two major sections. The first section describes how simulation was used to obtain the necessary production rates for determining the least-cost configurations and procedures analytically. The simulation is described in detail elsewhere (1,2). The second section describes the analytical study and results; these are also covered extensively elsewhere (2).

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## THE SIMULATION PROGRAM

To derive the least-cost combination analytically, it was necessary to obtain labor, equipment, and material costs and production rates. It was assumed that present estimating methods gave satisfactory labor and equipment costs per hour and material costs per unit. However, there was some doubt about the accuracy of the production rate estimates because of the stochastic nature of the process. It was therefore decided to use computer simulation to determine these production rates.

The simulation program was written in GPSS and contained as decision variables the average truck speed, average rate of paving advance, and truck fleet size. The program also contained as input empirical probability distributions describing queues at plant and paving train; time between failures for plant, paving train, and individual trucks; repair times for any equipment down; load and unload times; and travel times for plant to paving train and return. These distributions were derived empirically from data gathered by the Bureau of Public Roads during summers from 1963 through 1966. Because the objectives of the Bureau's studies were different from those of this study, not all data necessary for the probability distributions used in the simulation program could be obtained from a single project or even from a single equipment spread. Rather, truck travel times and corresponding distribution came from one study, plant loading times from another, and so forth. A summary of all probability distribution information used in the simulation program is given in Table 1.

The program also provided for such details as starting the trucks in small groups at designated time intervals in the morning and shutting them down in the same order at night and for the repairs on plant, paving train, or individual trucks taking longer than 24 hours.

Reduction of Data
After the preliminary computer runs were completed it was apparent that the cost of computer time was going to be a limiting factor in contractor acceptance and use of the study. That is, if a contractor were interested in finding his least-cost configuration and procedure, data from his own equipment spread would have to be used in the simulation. If, for example, he simulated 19 different haul fleet sizes ( 1 to 18 and 21 trucks), 4 different rates of paving advance ( $1 / 2,1,1 \frac{1}{2}$, and 2 miles per day), and 3 average truck speeds ( 15,30 , and 45 mph ), he would need more than 120 hours of simulation time. It was therefore necessary to analyze the data obtained in the simulation and then attempt to find either a direct mathematical solution or some relationships that would greatly shorten the simulation time. The latter approach turned out to be the more feasible and is described here.

The data were first reduced to hourly production rates and plotted on rectangular coordinate paper. Figure 1 shows such a plot. The plot suggested that steady state might be reached within 2 or 3 hours after starting from an idle state each morning. Other plots verified this fact and showed that steady state was usually reached between the second and third hours. Figure 1 also shows that the production rate might be constant over some part of the region. It was determined that this occurred when the number of trucks working in the system was equal to or greater than the average roundtrip time divided by the average loading time.

It was noticed that the steady state portion of the trucks-per-hour curve formed one continuous curve for every fleet size when extended over several days. To incorporate this into a single continuous curve and to avoid the distortions caused by the start-ups each morning, the daily average rates were plotted on rectangular coordinate paper. Figure 2 shows such a plot. Again each curve was seen to consist of two identifiable regions, the horizontal region and the decreasing region. The formula for the horizontal region has already been stated. It was conjectured that the decreasing region could be represented by a general formula. The daily totals were therefore tried on $\log -\log$ paper. Figure 3 shows such a plot. Because the different fleet sizes plotted as straight lines, the curves represent a general hyperbolic form. The slope unfortunately depends on the average truck speed and the average rate of pavement advance.

Figure 3 also shows that not only were the lines parallel but also there was a constant separation between lines where the fleet size was doubled. Hence the distance between lines was a logarithmic function of the fleet size, and there was no reason to run a simulation for each fleet size. Instead a simulation could be run for, say, fleet sizes of 3,6 , and 12 , and the other fleet size lines could be constructed from these. Figure 4 shows such a plot.

Next it seemed reasonable to investigate the other decision variables, average rate of pavement advance and average truck speed, with the same objective in mind. It was discovered that, for all values of average truck speed divided by average rate of pavement advance (designated a " B " value) and the same fleet size, the plot lines coincided. That is, for a fleet size of, say, 3, the line for a speed of 15 mph and a rate of advance of $1 / 2$ mile per day, the line for a speed of 30 mph and a rate of advance of 1 mile per day, and the line for a speed of 45 mph and a rate of advance of $11 / 2$ miles per day all coincided. Figure 5 shows such a plot for "B" values of 7.5, 15, 30, and 60.

A closer examination of Figure 5 reveals that any line drawn perpendicular to the " B " line of slope 1 shows an equal distance between all lines where the " B " value is doubled. That is, the distance along this line perpendicular to the " B " line of slope 1 between " $B$ " values of 15 and 30 was the same as between 30 and 60 . Therefore the relation along this perpendicular line for all " B " values was logarithmic, and all " B " values for one fleet size could be drawn on one graph after simulating only two or three different ' $B$ " value combinations. Figure 6 shows such a plot.

Now with the relationships found in Figures 3 and 5, a whole group of graphs like Figure 6 (one for each fleet size) could be constructed from only 5 simulations (" $B$ " value of 30 and fleet sizes of 3,6 , and 12 and fleet size of 3 and " $B$ " values of 15 and 60 ). The simulation time could then be reduced from about 120 hours to less than 2 , and a contractor could simulate his equipment spread for less than $\$ 1,000$. Therefore, simulation costs were no longer a limiting factor to contractor acceptance.

One problem still remained before this group of graphs (one for each truck fleet size) became useful. As can be seen from Figure 6, if both the average truck speed and number of trucks remain constant, the amount of concrete delivered to the paving train per hour decreases as the distance from plant to paving train increases. Because it takes a fixed amount of concrete per lineal foot of pavement, the rate of paving advance also decreases. Hence a series of approximations was necessary to find the correct production rate at different distances from the plant because the " $B$ " value did not remain constant. By preparing a series of tables like Table 2 (one for each average truck speed), the necessity for this series of approximations was eliminated. Column 1 of Table 2 is a listing of rates of paving advance. Column 2 is the corresponding " B " value for each rate of paving advance in column 1 and the average truck speed of 30 mph . Column 3 is the number of $8-\mathrm{cu}$ yd truckloads per day needed for a $24-\mathrm{ft}$ by $9-\mathrm{in}$. pavement if the corresponding rates of pavement advance in column 1 are to be obtained. Other truck capacities and other pavement dimensions could also be used as appropriate. Column 4 is column 3 divided by 8 because the simulation was run for an 8 -hour day. If the " B " values (column 2) are plotted against truckloads per hour (column 4) these points line up as a straight line on graphs of the type in Figure 6. Then, to find the hourly output for each day's distance from the plant, a straightedge is held along these points and the output is found at the point where this straightedge intersects the desired day of production away from the plant. At this point then it was thought that acceptable average production rates were obtainable and the development of the different models could begin.

## ANALYTICAL STUDY AND RESULTS

This section deals with quantitative modeling of different equipment configurations and construction procedures. Any quantitative model supposedly representing a realworld situation is by necessity only an approximation. This is partly because some factors cannot be expressed quantitatively and partly because it is often difficult or impossible to find a mathematical solution unless some factors are ignored.

Table 1. Probabilistic distribution information used in the simulation program.

| Use | Distribution Type | Parameters |  |
| :---: | :---: | :---: | :---: |
| Truck loading times | Empirical | Mean | = 73 seconds |
| Truck travel times (plant to paving train) | Normal 0,1 with corrections | Mean Variance | = variable <br> = variable |
| Truck travel times (paving train to plant) | Normal 0,1 with corrections | Mean Variance | $\begin{aligned} & =\text { variable } \\ & =\text { variable } \end{aligned}$ |
| Truck unloading times | Empirical | Mean | = 62 seconds |
| Trucks' interfailure rate | Empirical | Mean | $=15.1$ hours |
| Trucks' downtime | Empirical (closely resembles exponential except for a few lengthy delays) | Mean | $=1,786$ seconds |
| Plant interfailure rate | Empirical | Mean | $=6.9$ hours |
| Plant downtime | Empirical (closely resembles exponential except for a few lengthy delays) | Mean | $=1,920$ seconds |
| Paving train interfailure rate | Empirical | Mean | $=5.9$ hours |
| Paving train downtime | Empirical (closely resembles exponential except for a few lengthy delays) | Mean | = 2,142 seconds |

Figure 1. Hourly production curves for selected fleet sizes (average truck speed $=\mathbf{3 0} \mathbf{~ m p h}$; average rate of truck advance $=1$ mile per day).


Figure 3. Daily production lines for selected truck fleet sizes.


DAYS OF PRODUCTION AWAY FROM PLANT

Figure 2. Daily production curves for selected truck fleet sizes.


Figure 4. Daily production lines for truck fleet sizes of 1 to 18 and 21.


DAYS OF PRODUCTION AWAY FROM PLANT

Six models were investigated for optimum truck fleet size and distance between plant moves: Model 1, constant truck fleet size with box spreader and special sidedump trucks; Model 2, constant truck fleet size with belt spreader and standard reardump trucks; Model 3, constant rate of pavement advance; Model 4, pavement length divided by distance between plant moves may not be an integer; Model 5, drive-through method; and Model 6, leapfrog method. The cost per cubic yard in place was found for different configurations and procedures in each model, and the least-cost combination within the model was identified. Then these least-cost combinations for the different models were compared to try to identify the best least-cost configuration and procedure. All models assume some longitudinal steel and thus require a means of discharging the load at the side of the pavement.

## Model 1-Constant Truck Fleet Size With Box Spreader and Special Side-Dump Trucks

Model 1 assumed that the contractor owned his own side-dump trucks, and, because they had special truck bodies, was unable to rent additional units. Therefore, the truck fleet size was fixed. If the truck fleet size was constant and the distance from plant to paving train first decreased and then increased, by necessity the plant output must vary. So there was a trade-off between cost to move the plant and cost from loss in production because of lack of trucks. When the cost from loss of production became greater than the cost to move the plant, it was time to move the plant.

The total cost per cubic yard of concrete in place included the cost of all equipment, labor, and material pertinent to the decision. The Appendix contains a tabulation of how these costs were calculated for a fleet of 3 trucks and an average truck speed of 30 mph if the plant was located 2 days' production from the beginning of the project and the distance between plant moves was twice 2 days' production. Labor and equipment costs were estimated at $\$ 258.72$ per hour by following the usual cost-estimating procedures. The average truck speed was arbitrarily picked as 30 mph . It could be estimated for any given project, however, either through time studies or, if that proved costly or impossible because the job had not started, by using the information presented in any one of at least 6 construction texts (3). In any case there would always be an average truck speed for a given paving project under given weather and haul-road conditions.

The production rate was found by using the procedure described earlier in this paper. This gave an average rate of pavement advance of 0.75 mile per day and a production rate of 41.25 truckloads per hour during the first day's production away from the plant and an average rate of pavement advance of $0.52+$ mile per day and a production rate of 28 truckloads per hour during the second day's production away from the plant. At 8 cubic yards per truckload and 8 working hours per day, the estimated 2 -day output (one each way from the plant) would be
(41.25) ( $8 \mathrm{cu} y d$ ) ( 8 hours) ( 2 days) $=5,280 \mathrm{cu} \mathrm{yd}$

This considers only one lane; for both lanes the estimated output would be

$$
(2 \text { lanes })(5,280 \mathrm{cu} y d / \text { lane })=10,560 \mathrm{cu} \mathrm{yd}
$$

If the plant were located 2 production days from the beginning of the project, total production between moves would be

First day out each way both lanes
Second day out each way both lanes = (28 loads/hour) (8 hours) (8 cu yd/load) (2 days) $(2$ lanes $)=\underline{7,168}$
Total concrete placed $\quad=17,728 \mathrm{cu} \mathrm{yd}$
The distance between plant moves when the plant was located 2 days' production from the beginning of the project would then be

Figure 5. Selected "B" value lines for truck fleet size of 3.


Table 2. Required production for a $24-\mathrm{ft}$ by 9 -in. pavement, given selected average rates of paving advance and an average truck speed of 30 mph .

Figure 6. "B" value lines for a truck fleet size of 3.

$\left.\left.\begin{array}{llll}\hline \begin{array}{l}\text { Average Rate of } \\ \text { Paving Advance } \\ \text { (miles per day) }\end{array} & & & \\ \text { (1) } & & & \\ \hline \text { (2) Value }\end{array} \quad \begin{array}{l}\text { Truckloads } \\ \text { per Day } \\ (3)\end{array}\right) \begin{array}{l}\text { Truckloads } \\ \text { per Hour } \\ \text { (4) }\end{array}\right]$

Table 3. Comparison of concrete costs in dollars per cubic yard of concrete placed for Model 1.

| Distance Between <br> Plant Moves in <br> Production Days | Number of Trucks in Haul Fleet |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :---: |
|  | 3 | 4 | 5 | 6 | 7 | 8 | 9 |  |
| 2 | 12.171 | 12.025 | 12.070 | 12.116 | 12.161 | 12.207 | 12.253 |  |
| 4 | 11.976 | 11.835 | 11.782 | 11.781 | 11.744 | 11.784 | 11.821 |  |
| 6 | 11.986 | 11.835 | 11.761 | 11.722 | 11.693 | 11.703 | 11.726 |  |
| 8 | 12.040 | 11.872 | 11.780 | 11.746 | 11.704 | 11.699 | 11.714 |  |
| 10 | 12.097 | 11.922 | 11.821 | 11.779 | 11.734 | 11.720 | 11.732 |  |
| 12 | 12.164 | 11.980 | 11.866 | 11.818 | 11.774 | 11.752 | 11.757 |  |

[^3]Table 3 shows the cost per cubic yard for the plant located 1, 2, 3, 4, 5, and 6 days' production from the beginning of the project and truck fleet sizes of 3 through 9 trucks.

Model 2-Constant Truck Fleet Size With Belt Spreader and Standard Rear-Dump Trucks
Model 2 makes all the same assumptions as Model 1 and was introduced to study whether a box spreader requiring special side-dump trucks or a belt spreader and standard rear-dump trucks gave the lower cost per cubic yard in place. The results of the study for Model 2 are given in Table 4.

## Model 3-Constant Rate of Pavement Advance

Model 3 assumed a constant rate of pavement advance and therefore a constant production rate. A belt spreader allowed the use of standard rear-dump trucks and, because these standard rear-dump trucks could be rented, the number of trucks in the haul fleet could realistically be allowed to vary. So Model 3 introduced the other procedure now in general use with paving contractors-constant rate of paving progress and variable truck fleet size. It was reasoned that the contractor should own 3 trucks and rent the rest because these 3 trucks would be needed to move the plant and because at least 3 would be needed in most cases for even the first day's production away from the plant. The model assumed that any extra trucks above 3 needed to maintain constant production could be rented at $\$ 15.00$ per hour.

In Model 3, because the production rate was constant and the haul fleet size varied, more than one graph of the Figure 6 type was needed in finding the number of trucks required to maintain a given production rate. Here the method was the same as described for Models 1 and 2 except that it was necessary to use the straightedge technique on a number of graphs until the fleet size that would just sustain the required production rate for the given number of days' production away from the plant was found.

In Model 3, costs per cubic yard were calculated for 5 different rates of pavement advance ( $0.25,0.40,0.55,0.70$, and 0.845 miles per day) and the plant initially placed 1 through 6 days of production from the beginning of the project. These costs are given in Table 5.

## Model 4-Pavement Length Divided by Distance Between Plant Moves May Not Be an Integer

Model 4 recognized the fact that projects are of a definite length and may not give an integer value when divided by the distance between plant moves. That is, Models 1, 2 , and 3 handled the project length as infinite, whereas Model 4 handled it as if it were finite. All other assumptions were the same as those in Model 3.

The solution procedure for Model 4 was developed in 2 steps. Step 1 figured the distance between plant moves if the plant were moved $0,1,2,3$, etc., times on the project and then copied the costs per cubic yard from Model 3 for the integer days of production between moves on both sides of the one just calculated. That is, for a 12 -mile project, a rate of advance of 0.845 mile per day and 2 moves on the project, the distance between moves is 12 miles divided by 3 (a move-in plus 2 additional moves), then that quantity divided by 0.845 mile per day which equals 4.73 days. Model 3 gave a cost of $\$ 11.684$ for 4 days of production between moves and a cost of $\$ 11.638$ for 5 days of production between moves.

In step 2 some 4 to 6 combinations of rates of pavement advance and moves on the project that showed the lowest costs per cubic yard in step 1 were analyzed to obtain an exact cost for each of these.

Model 4 thus added nothing to the study except to recognize that projects are of finite length and that a reasonably short procedure could be developed to handle this. It will not be mentioned further in the analysis.

Table 4. Comparison of concrete costs in dollars per cubic yard of concrete placed for Model 2.

| Distance Between <br> Plant Moves in <br> Production Days | Number of Trucks in Haul Fleet |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :---: |

Model 2-Constant fleet size, variable production, standard rear-dump trucks, and belt spreader.

Table 5. Comparison of concrete costs in dollars per cubic yard of concrete placed for Model 3.

| Distance Between <br> Plant Moves in <br> Production Days | Rates of Paving Progress in Miles per Day |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  | 0.25 | 0.40 | 0.55 | 0.70 | 0.845 |
| 2 | 15.532 | 13.639 | 12.777 | 12.284 | 12.017 |
| 4 | 14.225 | 12.825 | 12.202 | 11.870 | 11.684 |
| 6 | 13.796 | 12.551 | 12.052 | 11.775 | 11.684 |
| 8 | 13.578 | 12.439 | 11.994 | 11.758 | 11.634 |
| 10 | 13.444 | 12.389 | 11.985 | 11.775 |  |
| 12 | 13.360 | 12.370 | 11.995 | Not calculated |  |

Model 3-Variable haul fleet size, constant production, standard rear-dump trucks, and belt spreader.

Figure 7. Moving and paving schedule for Model 5, given that first plant location is $\mathbf{6}$ days' production from start of project (numbers indicate the location of each paving train in days of production away from plant; circles around numbers indicate it is paving train No. 2).


Table 6. Comparison of concrete costs in dollars per cubic yard of concrete placed for Model 5.

| Distance Between <br> Plant Moves in <br> Production Days | Number of Trucks in Haul Fleet |  |  |
| :--- | :--- | :--- | :--- |
|  | 7 | 8 | 9 |
| 12 (both rigs, one lane) | 11.931 | 11.941 | 11.973 |
| 14 (both rigs, one lane) | 11.898 | 11.903 | 11.927 |
| 16 (both rigs, one lane) | 11.908 | 11.883 | 11.895 |
| 18 (both rigs, one lane) | 11.916 | 11.897 | 11.885 |
| 20 (both rigs, one lane) | 11.939 | 11.915 | 11.898 |

Model 5-Constent haul fleet size, variable production, two complete paving trains with belt spreader, and standard rear-dump trucks.

Model 5 assumed a constant fleet size but tried to make better use of the haul trucks by including two complete paving trains. This would be possible because trucks could be shifted from one paving train to the other as demand dictated. Unfortunately, the simulation program was not written to cover this explicit case but rather provided for one paving train and one plant with single channels at each. Therefore the costs per cubic yard in Model 5 must be considered as less exact than those of the other models.

The construction procedure would be as follows (Fig. 7): One paving train starts at half the distance between plant moves and proceeds toward the plant. The other paving train starts at the plant and moves toward the beginning of the project. When the paving train moving away from the plant reaches the beginning of the project, the other paving train should hopefully be at the plant. Next, both paving trains move to the other lane and pave back to their starting points. When this is completed the plant is moved to the second location and the procedure starts all over again. Table 6 gives costs for this method.

## Model 6-Leapfrog Method

The previous models all had shortcomings. Model 1 required special haul trucks and the refore had a plant output that varied. Model 2 eliminated the use of special haul trucks but still gave variable output. Model 3 allowed a constant production rate but required varying the haul fleet size. Theoretically, varying the number of trucks daily is feasible, but in actual practice superintendents have found this hard to do in many locations. Most owner-drivers would prefer the promise of more than 1 day's work at a time, especially during the busy months when demand is high. Model 4 was Model 3 adapted to a given length of pavement and thus has the same problems. Model 5 returns to the idea of constant fleet size but, as in Models 1 and 2, does it at the expense of not always utilizing full capacity.

Model 6 was developed to try to overcome all the shortcomings discovered in the other models. It allows a constant rate of production and a constant number of haul trucks. To do this it was necessary to go to 2 complete plants and 2 complete paving trains. Model 6 is therefore really just a revision of the method suggested by Maxon and Miller (4).

In Model 6, the first plant (called Plant A) is placed a predetermined distance (number of days' production) from the beginning of the project. The second plant (called Plant B) is placed a distance equal to 3 times Plant A's distance from the beginning of the project plus 1 day's production. That makes the distance between Plants A and B equal to twice Plant A's distance from the beginning of the project plus 1 day's production (production is constant).

Plant A's paving train starts paving at the plant and moves down one set of lanes to the beginning of the project and then back up the other set of lanes past Plant A and on an equal number of production days toward Plant B. Then Plant A's paving train moves back to the set of lanes it started on and paves back to Plant A (Fig. 8).

Plant B's paving train starts a half day's production closer to Plant B than Plant A. It starts at the same time as Plant A's paving train and paves toward Plant B. After reaching Plant B , it continues on an equal number of days' production the other side of Plant B. It then paves back up the other set of lanes to the place it started.

Next, while Plant A and its paving train are moved to their new location past Plant $B$ and an equal distance the other side, Plant $B$ and its paving train pave the remaining 1 day's production in each set of lanes between the original locations of Plants A and B. This allows the 2 days necessary to move Plant A. Then while Plant B is moving to its new location an equal distance the other side of Plant A, Plant A and its paving train can start by paving the center day's paving in each set of lanes between Plant B's original location and Plant A's present location. Both paving trains are now ready to repeat the complete process.

Table 7 gives cost per cubic yard for pavement advances of 0.70 and 0.845 mile per day and days of production between plant moves of $5,7,9,11,13$, and 15.

Figure 8. Moving and paving schedule for Model 6, given that Plant A's first location is $\mathbf{3}$ days' production from start of project (numbers indicate the location of the paving train in days of production away from plant; circles around numbers indicate it is paving train $B$ ).


Table 7. Comparison of concrete costs in dollars per cubic yard of concrete placed for Model 6.

| Parameter | Production Days Between Plant Moves |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 5 | 7 | 9 | 11 | 13 | 15 |
| At 0.70 Mile per Day Paving Progress for Each Paving Train |  |  |  |  |  |  |
| Distance between plant moves in miles | 7 | 9.8 | 12.6 | 15.4 | 18.2 | 21 |
| Number of trucks in haul fleet | 9 | 11 | 14 | 16 | 19 | 21 |
| Cost per cubic yard of concrete placed | 11.751 | 11.675 | 11.667 | 11.661 | 11.685 | 11.699 |
| At 0.845 Mile per Day Paving Progress for Each Paving Train |  |  |  |  |  |  |
| Distance between plant moves in miles | 8.45 | 11.83 | 15.21 | 18.59 | 21.97 | 25.35 |
| Number of trucks in haul fleet | 11 | 15 | 18 | 22 | 25 |  |
| Cost per cubic yard of concrete placed | 11.566 | 11.533 | 11.524 | 11.538 | 11.567 | 11.606 |

Model 6-Constant haul fleet size, constant production, two complete plants, two complete paving trains with belt spreader, and standard dump trucks.

Figure 9. Minimum cost curves for each truck
fleet size in Models 1, 2, and 5.


Analysis
Models 1 and 2 compared equipment configurations using different spreaders and different truck bodies. For the values used in the example simulation and cost analysis, the belt spreader and standard dump trucks gave a smaller cost for each fleet size and distance in days of production between plant moves (Tables 3 and 4 and Fig. 9). However, there is not enough difference to claim Model 2 is always better. Both models find the least-cost combination at 7 trucks and 4.7 miles between plant moves. Again, other data may give other equipment configurations and construction procedures.

Models 2 and 3 compared constant versus variable haul fleet size. Model 3 gave the smaller least cost (Tables 4 and 5). Here again different cost data, especially the $\$ 15.00$ per hour haul truck rental charge, could give different results. The least-cost combination for the example data used indicated that the plant should be run at full capacity (not surprising) and the optimum distance between plant moves should be about 5 miles.

Model 5 was an attempt to hold both production and haul fleet size constant. While it held fleet size constant, it did not hold production constant, and ended up giving the largest least cost per cubic yard combination (Table 6 and Fig. 9). Model 6 represented a second attempt to hold both fleet size and production constant. This time it gave the smallest least-cost combination (Table 7 and Fig. 10). In fact Model 6 gave costs throughout its range that were less than the least-cost combination of any of the other models (Fig. 11). Although it would be unwise to claim it was always the best procedure from among those considered, it would be safe to say that it showed enough promise to be considered seriously by paving contractors.

## Sensitivity

Sensitivity appeared to be a problem within all models as far as equipment configuration was concerned but did not seem to be a problem as far as least cost was concerned, either within the model or between models. In Models 1, 2, and 5, estimated truckloads per hour appeared to be the most critical. Consider, for example, the combination of 7 trucks and 6 days' production between plant moves, which gave the least cost for Model 2. A decrease in output of $1 / a$ truckload per hour gave a percentage decrease of only about 1.5 percent, which in turn changed the least-cost combination by $\$ 0.013$, or less than 1 percent. However, this was enough to change the ieasi-cost combination to 8 trucks and 8 days of production between plant moves. Fortunately, the curves for Models 1 and 2 were flat in the least-cost range (Fig. 9), because it would be most difficult to estimate the output within $1 / 2$ truckload per hour from graphs of the Figure 6 type.

For Models 3 and 6 the number of trucks in the fleet on any given day is most critical because the plant should always be run as close to full production as possible ( 0.845 mile per day was full production for the example). If the number of trucks in the haul fleet was increased by 1 for one-fourth the time between plant moves, the least-cost combination of Model 6 would be increased by less than 1 percent. This, however, would be enough to change the distance between plant moves from 15.2 miles to 18.6 miles. Fortunately, the cost curve is again very flat in the least-cost region (Fig. 11).

## CONCLUSIONS

The study demonstrated that

1. Simulation is one means of obtaining the production rates needed for an analytical examination of paving spread configurations and procedures.
2. Plots on $\log -\log$ graph paper provide an economically feasible procedure to extrapolate the simulation data.
3. Simulation shows that steady state is always reached in a paving operation and usually between the second and third hour after production starts each day.
4. Mathematical modeling does allow a means for analyzing different paving spread configurations and procedures.

Figure 10. Minimum cost curves for each rate of paving progress in Models 3 and 6.

Figure 11. Curve of costs for Model 6 with a rate of paving progress of 0.845 mile per day and least-cost lines for Models 1, 2, 3, and 5.

5. There is considerable difference in cost of concrete per cubic yard in place for different paving spread configurations and procedures.
6. Sensitivity analysis shows that the models are quite sensitive in the areas of production and number of trucks, and picking the least-cost combination within a particular model is difficult. However, a near-least-cost combination can be picked.

It may be impossible to place the plant exactly where the mathematical model dictates because of variables not considered in the mathematical model. However, the least-cost distance does provide a basis on which a decision can be made in light of experience and judgment.

Model 6 appears to be one of the better choices for a paving project over about 8.5 miles in one set of lanes. This is not to say that a contractor should go out and buy all the equipment needed for such a configuration from the information and analysis presented here. However, if he already owns enough equipment to form such a paving spread, he may want to try this method in actual practice.

## REFERENCES

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## APPENDIX

## CALCULATION OF CONCRETE PAVEMENT COSTS

Given: Plant is located 2 days' production from the beginning of the job and moved twice 2 days' production.

Labor and equipment costs
Production

| (\$258.72 per hour) (8 hours) (4 days) (2 lanes) | $\$ 16,558.08$ <br> Moving |
| :--- | ---: |
| $4,299.52$ <br> Total labor and equipment costs | $\$ 20,857.60$ |

Labor and equipment costs per cubic yard $\$ 20,857.60$
$17,728 \mathrm{cu} \mathrm{yd}$
\$ $\quad 1.177$ per cu yd 9.027

Materials
Well $\frac{\$ 4,500}{17,728 \mathrm{cu} \mathrm{yd}}$
Other items 1.000

Subtotal
General overhead 4 percent of $\$ 11.458$

Bonds 0.5 percent of $\$ 11.916$
Subtotal
0.458

Total cost in place
$\$ 20,857.60$

## DISCUSSION

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We congratulate Ghare and Bidwell on a comprehensive simulation of an important set of problems. As they rightly point out, too little fundamental analytical work has been done on such problems. The methods presented should enable the contractor to reduce his costs considerably, but not every contractor has access to a computer or is willing to spend a thousand dollars to obtain the solution. Simple equations or graphs are more likely to be used by the site engineer than are computer simulation packages. It is toward this end, therefore, that the following comments are made.

From the $\log -\log$ curves of production rate versus days of production from the plant, it would seem that

$$
\frac{\mathrm{r}}{\mathrm{r}_{\mathrm{o}}}=\frac{\mathrm{knV}}{\mathrm{~d}} \quad(\text { for } \mathrm{d}>\mathrm{knV})
$$

where
$r$ = rate of production (in truckloads per hour),
$r_{\mathrm{o}}=$ maximum rate of production ( $=46.5$ truckloads per hour),
$\mathrm{n}=$ number of trucks,
$\mathrm{V}=$ average truck speed (in mph),
$\mathrm{d}=$ distance of paver from plant (in miles), and
$\mathrm{k}=\mathrm{a}$ constant, approximately 0.0082 .
We feel that this relationship between $r$ and $d$ is rather more fundamental than that between $r$ and time, since the assumption of a constant rate of pavement advance is unattainable in practice. The relationship demonstrates the points noted by Ghare and Bidwell:

1. If the rate of pavement advance, p , is constant, the graph of $\log \mathrm{r}$ versus $\log$ time is straight;
2. There is a cutoff point such that, for $n>d / k V, r=r_{0}$;
3. When n and B are held constant, the production rate versus time curve is constant;
4. The doubling of $n$ leads to a doubling of $r$ (that is, a constant spacing in the loglog curves); and
5. The doubling of $B$ leads to a doubling of $r$.

As d increases, $r$ decreases. If the connection between the rate of pavement advance $p$ (in miles per day) and the rate of production $r$ (in truckloads per hour) is $p=$ gr, then an equation that links time, $t$, and $d$ can be built up:
When $\mathrm{d}<\mathrm{knV}$,

$$
\mathrm{t}=\mathrm{gr} \mathrm{r}_{\mathrm{a}}
$$

When $d>k n V$,

$$
\mathrm{t}=\frac{\mathrm{knV}}{\mathrm{gr} \mathrm{r}_{\mathrm{o}}}+\int_{\mathrm{knV}}^{\mathrm{u}} \frac{\mathrm{u}}{\mathrm{gr}} \mathrm{du}=\frac{\mathrm{knV}}{2 \mathrm{gr} \mathrm{r}_{\mathrm{o}}}\left\{1+\left(\frac{\mathrm{d}}{\mathrm{knV}}\right)^{2}\right\}
$$

The use of this relationship obviates the need for the "straightedge" method and gives instead a simple means of calculating the time taken to pave a length d starting from the plant.

Before making use of this relationship to investigate the various models proposed, we will comment further on the pattern of the $r$ versus $d$ curves. By setting up a cyclic queuing theory model, assuming negative exponentially distributed loading, unloading,
and transit times, the rate of production can be found as a function of $d$. The pattern of these curves is rather different from those obtained by Ghare and Bidwell; in particular, there is no cutoff point, and the lines in the $\log -\log$ graphs are not straight. A simpler, deterministic analysis, however, does display a cutoff point. For n > $\frac{(L+U+2 d / V)}{U}$ (where $L=$ mean loading time and $U=$ mean unloading time), $r=r_{o}$. For $n<\frac{(L+U+2 d / V)}{U}$, the rate of production is $\frac{r_{0} n U}{(L+U+2 d / V)}$. Now if the sum of loading and unloading times is small compared with the sum of the transit times, this reduces to $\frac{r}{r_{0}}=\frac{U}{2} \frac{n V}{d}$, which is of the same form as the relationship observed by Ghare and Bidwell. All of this indicates that the simulation model gives similar results to those obtained by a deterministic model. It would be interesting to know whether the full complexity of the simulation model is necessary. If the probability distributions of the interfailure times and down times of plant, paver, and trucks were taken out of the model, would the results be significantly different? And what effect do the variances of the loading, unloading, and transit times have on the results ?

With regard to the second part of the paper (that concerned with the comparison of the different models of paving processes), broadly speaking the models fall into two categories. In the first, a length 2 D is paved before the plant is moved, the plant being placed in the middle of each section. The number of trucks is kept constant, and the problem is then to optimize $n$ and $D$. In the second category, the number of trucks in a plant-paver combination is variable and chosen at each point of time so as to have the optimum n . This n will be equal to $\mathrm{d} / \mathrm{kV}$ so as to maintain full production without any waste. It remains to optimize $D$ as in the first type of model. The ability to vary $n$ depends on circumstances such as using two paving trains arranged so that the total number of trucks is constant and the desired rates of increase and decrease of $n$ in the two paving trains match exactly.

There are three basic costs to be introduced: $c_{1}$ is the fixed hourly costs (of plant, paver, etc.); $c_{2}$ is the hourly cost of a truck; $c_{3}$ is the cost of moving the plant.

In the first type of model, the cost of paving a length 2 D is $\left(\mathrm{c}_{1}+\mathrm{nc}_{2}\right) \mathrm{T}+\mathrm{c}_{3}$, where T , the time taken to pave that length, is given by $\frac{\mathrm{knV}}{\mathrm{gr}}\left\{1+\left(\frac{\mathrm{D}}{\mathrm{knV}}\right)^{2}\right\}$. Therefore, the cost per unit length of road may be written as

$$
\mathrm{M}_{1}=\frac{\mathrm{c}_{2}}{\mathrm{gr}_{\mathrm{o}}}\left\{\frac{\mathrm{a}}{2 \mathrm{D}_{0}}+\left(\frac{\mathrm{b}+\mathrm{n}}{\mathrm{D}_{\mathrm{o}}}\right)\left(\mathrm{n}+\frac{\mathrm{D}_{\mathrm{o}}^{2}}{\mathrm{n}}\right)\right\}
$$

where $\mathrm{a}=\frac{\mathrm{gr}_{6} \mathrm{c}_{3}}{\mathrm{KV} \mathrm{c}_{2}}, \quad \mathrm{~b}=\frac{\mathrm{c}_{1}}{\mathrm{c}_{2}}$, and $\mathrm{D}_{0}=\frac{\mathrm{D}}{\mathrm{kV}}$; a and b are the (nondimensional) input parameters of the problem and $n$ and $D_{0}$ are to be optimized. It can be shown that these optimum values are given by

$$
4 n^{2}(n+b)=a b
$$

and

$$
\mathrm{D}_{\mathrm{o}}^{2}=\mathrm{n}^{2}\left(1+\frac{2 \mathrm{n}}{\mathrm{~b}}\right)
$$

In the second type of model, $n$ should be kept equal to $d / k V=D_{0}$ for all values of $d$. The cost per unit length of road is given by

$$
\mathrm{M}_{2}=\frac{\mathrm{c}_{2}}{\mathrm{gr}_{\mathrm{e}}}\left\{\frac{\mathrm{a}}{2 \mathrm{D}_{\mathrm{e}}}+\mathrm{b}+\frac{\mathrm{D}_{0}}{2}\right\}
$$

and this is minimized by putting $\mathrm{D}_{o}^{2}=a$.

It can be shown that the minimum cost of model 2 is between 29 and 50 percent lower than the minimum cost of model 1 , depending on the values of a and $b$. Both these analyses, of course, have treated $n$ as if it were a continuous variable, whereas in fact it may take only integer values. Because of this, the minimum value of $\mathrm{M}_{2}$ given by the analysis above is not strictly attainable. It does, however, show the scale of the savings that are possible by arranging the paving system so that it allows the variation of n so as to maintain optimality.

## AUTHORS' CLOSURE

We thank Cabrera and Maher for an excellent discussion of our paper. We agree wholeheartedly with their suggestion of developing simple equations or graphs that can be used by field engineers as a day-to-day decision tool. Both authors remember too well from their time as field engineers of being frustrated by the lack of good decision tools of this type.

We wish to point out, however, that the comprehensive simulation is not intended to be used as a day-to-day decision tool for field work. It is intended to study the interrelationships of the paving process during the planning stage. During the planning stage a constant rate of pavement advance is taken as a simplifying assumption to develop the curves of production rate versus days of production from the plant. The simulation can be run only once, but the results (the log-log paper curves) can then be incorporated in the form of simple decision tools for day-to-day field work. These simple decision tools can take the form of equations, as pointed out by Cabrera and Maher, or can be in the form of a set of guidelines or graphs.

Cabrera and Maher also raise the possibility of obtaining r-d curves analytically by setting up a cyclic queuing model. Unfortunately, closed-form solution to such queuing models can be obtained only if the probability distributions of times can be assumed to be either deterministic or negative exponential. Neither of the two assumptions is realistic. Analysis of actual times would indicate a large deterministic time component and a smaller component following a beta distribution. The ratio ( $\sqrt{\text { variance }} / \mathrm{mean}$ ) is neither zero as required by deterministic assumption or 1 as required by the negative exponential assumption, but is a small fraction between zero and 1. Hence during the simulation the probability distributions for loading times, unloading times, travel times, and interfailure rates were taken to be either empirical or an approximating normal.

We would like to point out that during the research we did conduct a sensitivity analysis and found that models 1,2 , and 5 are quite sensitive to changes in the ratio ( $\sqrt{\text { variance }} /$ mean ).

Again, in regard to the authors' reason for making the simulation program so allinclusive, it was hoped that a study of the interrelationships of the process would lead to the discovery of a general equation covering all combinations and there would be no further need for simulation runs. At present we still have not been able to derive such a general equation, but we will continue to search and feel if we and others such as Cabrera and Maher continue the research and discussion of results someone will find an answer.

# TWO-DIMENSIONAL FINITE DIFFERENCE TECHNIQUES APPLIED TO TRANSIENT TEMPERATURE CALCULATIONS IN HOT-MIX ASPHALT CONCRETE WINDROWS 

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#### Abstract

Temperature is an important variable in the compaction of hot-mix asphalt concrete. This study calculates the change with time in average bulk temperature of hot-mix windrows of different sizes and initial temperatures considering the following environmental conditions: the base material temperature, the ambient temperature, the net absorbed solar radiation, and the wind velocity. The analytical solution of the mathematical model describing the windrow and its immediate surroundings cannot be obtained. Therefore, the approach used was a 2-dimensional transient heat balance model formulated by explicit finite-difference techniques in FORTRAN-IV. The results of the finite-difference solution were those readily predicted by the laws of heat transfer. The most significant variable affecting the cooling rates was the size of the windrow. Large windrows, having a lower surface-to-volume ratio than smaller windrows, were less affected by all environmental conditions and thus cooled more slowly. For the same initial temperature ( 300 F ) under the same environmental conditions (the most severe case used was 10 F base temperature, 10 F ambient temperature, overcast day, and still wind), the temperature drop after 30 minutes was 67 F for a 2- by 1 -ft windrow but only 24 F for a 6 - by 3 - ft windrow. The computer programs developed in the study may be used to calculate bulk temperature versus time curves for an almost limitless number of combinations of windrow sizes, initial temperatures, environmental conditions, and cooling times.


- THE PURPOSE of this study was to determine the average bulk temperature of windrows of hot-mix asphalt concrete as a function of time. The bulk temperature of the asphalt windrow is important because this is the effective input temperature of the hotmix asphalt at the laydown machine. The average bulk temperature will vary considerably, depending on the environmental conditions to which the asphalt is exposed. The environmental conditions that most affect the bulk temperature of the asphalt and that were considered in this study are the base material temperature, the ambient temperature, the net absorbed solar radiation, the wind velocity, the size of the windrow, and the initial temperature of the windrow.

The success of a paving operation using hot-mix asphalt concrete depends significantly on the temperature of the asphalt when it is compacted into the road surface mat. Before the asphalt enters the paving machine to be laid down in a mat, it is sometimes dumped from trucks into long triangular windrows ahead of the paver, where it begins to cool before it can be used. If the temperature of the asphalt becomes too low, the asphalt viscosity will be so high that specified compaction densities cannot be obtained and a poor road surface will result.

[^4]The results of this study enable an asphalt contractor to predict the bulk temperature of an asphalt windrow under given environmental conditions at any time after dumping from the truck. If the actual temperature of the hot mix being fed to the laydown machines is known, he can decide in advance of starting the operation whether or not enough time exists to complete the paving operation.

Previous studies of temperature effects in hot-mix asphalt concrete have been concerned with the effect of temperature on compactibility in the final mat (1). Experimental work has also been conducted to measure temperature changes as a function of time at given positions in the mat (1).

A logical next goal was a mathematical analysis of heat transfer in asphalt windrows to allow prediction of bulk asphalt temperatures in advance of paving jobs. Previous studies have assumed that the laydown temperature is approximately that of the hot-mix plant. When hot mix is dumped in windrows rather than directly into the laydown machine, this assumption will not in all cases be true. The results of this study provide the means to predict effective laydown temperature based on windrow cooling.

## STATEMENT OF THE PROBLEM

As noted earlier, the purpose of this study was to determine the average bulk temperature of windrows of hot-mix asphalt concrete as a function of time under different environmental conditions. The problem was to mathematically model the windrow system and its immediate surroundings to achieve the stated purpose. This mathematical model must include realistic boundary conditions that can be calculated accurately from easily measurable physical quantities. The analytical mathematical solution of the simultaneous, 2 -dimensional, nonlinear, unsteady-state partial differential equations that result from the energy balances in this problem cannot be obtained. Therefore, finite-difference mathematical solutions must be calculated. The problem, in finite-difference form, must be solved with a digital computer and with a node size small enough to closely approximate the real situation yet within the storage capacity of the computer and within reasonable expenditures of computer time.

## APPROACH TO THE PROBLEM

The approach used to calculate a feasible and reasonably accurate solution to the problem of heat losses from an asphalt windrow was a 2 -dimensional transient mathematical heat-balance model of the windrow formulated by explicit finite-difference techniques in FORTRAN-IV for computer solution on a PDP- 10 computer.

The necessary theoretical considerations were first incorporated into the solution; these included applications of Fourier's law for conduction effects, Newton's law of cooling (or heating) for convection effects, Nusselt-type heat transfer correlations to predict the convection heat transfer coefficient, the Stefan-Boltzmann law for radiative effects, and empirical expressions for incident solar flux.

The asphalt windrow model was then divided into a grid system of specific nodes, as shown in Figure 1. Overall transient energy balances were developed for each of 8 different types of nodes. The type of node depends on the combinations of boundary conditions to which the windrow is subjected at various locations throughout the windrow.

The overall transient energy balances were then converted from the differential form, for which an analytical solution is mathematically unobtainable, to the finitedifference form, for which iterative approximate solutions can be obtained with digital computers. The change in temperature of each node with time was used to calculate the change in average bulk temperature of the windrow as a function of time.

Finally, the computer program model was executed with various combinations of values of important variables such as base temperature, ambient temperature, net absorbed solar radiation, wind velocity, size of windrow, and initial temperature of windrow, as shown in Table 1. The average bulk temperatures of the windrow at 10 , 20 , and 30 minutes after it is dumped onto the base material were calculated and plotted in Figures 2 through 5. These figures illustrate the effect of each significant variable independent of the others and indicate permissible paving conditions that can be used readily by asphalt contractors.

Figure 1. Schematic cross section of asphalt windrow and base material with typical nodal system overlay for finite-difference analysis indicating various modes of thermal energy transfer at a given node.


Table 1. Combinations of variables for each execution of computer program.

| Run | Base <br> Temperature <br> (F) | Ambient Temperature (F) | Solar Flux (Btu/ft ${ }^{2} /$ hour ) | Convective Heat Transfer Coefficient (Btu/ $\mathrm{ft}^{2} /$ hour $/$ deg F ) | Width of Base of Windrow (ft) | Initial Mix <br> Temperature <br> (F) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 10 | 10 | 80 | 1.5 | 6.0 | 300 |
| 2 | 10 | 10 | 80 | 1.5 | 6.0 | 275 |
| 3 | 10 | 10 | 80 | 1.5 | 6.0 | 250 |
| 4 | 50 | 50 | 175 | 1.5 | 6.0 | 300 |
| 5 | 50 | 50 | 175 | 1.5 | 6.0 | 275 |
| $\overline{6}$ | 50 | 50 | 175 | 1.5 | 6.0 | 250 |
| 7 | 100 | 90 | 265 | 1.5 | 6.0 | 300 |
| 8 | 100 | 90 | 265 | 1.5 | 6.0 | 275 |
| 9 | 100 | 90 | 265 | 1.5 | 6.0 | 250 |
| 10 | 10 | 10 | 80 | 1.5 | 4.8 | 300 |
| 11 | 10 | 10 | 80 | 1.5 | 4.8 | 275 |
| 12 | 10 | 10 | 80 | 1.5 | 4.8 | 250 |
| 13 | 50 | 50 | 175 | 1.5 | 4.8 | 300 |
| 14 | 50 | 50 | 175 | 1.5 | 4.8 | 275 |
| 15 | 50 | 50 | 175 | 1.5 | 4.8 | 250 |
| 16 | 100 | 90 | 265 | 1.5 | 4.8 | 300 |
| 17 | 100 | 90 | 265 | 1.5 | 4.8 | 275 |
| 18 | 100 | 90 | 265 | 1.5 | 4.8 | 250 |
| 19 | 10 | 10 | 80 | 1.5 | 2.8 | 300 |
| 20 | 10 | 10 | 80 | 1.5 | 2.8 | 275 |
| 21 | 10 | 10 | 80 | 1.5 | 2.8 | 250 |
| 22 | 50 | 50 | 175 | 1.5 | 2.8 | 300 |
| 23 | 50 | 50 | 175 | 1.5 | 2.8 | 275 |
| 24 | 50 | 50 | 175 | 1.5 | 2.8 | 250 |
| 25 | 100 | 90 | 265 | 1.5 | 2.8 | 300 |
| 26 | 100 | 90 | 265 | 1.5 | 2.8 | 275 |
| 27 | 100 | 90 | 265 | 1.5 | 2.8 | 250 |
| 28 | 10 | 10 | 80 | 1.5 | 2.0 | 300 |
| 29 | 10 | 10 | 80 | 1.5 | 2.0 | 275 |
| 30 | 10 | 10 | 80 | 1.5 | 2.0 | 250 |
| 31 | 50 | 50 | 175 | 1.5 | 2.0 | 300 |
| 32 | 50 | 50 | 175 | 1.5 | 2.0 | 275 |
| 33 | 50 | 50 | 175 | 1.5 | 2.0 | 250 |
| 34 | 100 | 90 | 265 | 1.5 | 2.0 | 300 |
| 35 | 100 | 90 | 265 | 1.5 | 2.0 | 275 |
| 36 | 100 | 90 | 265 | 1.5 | 2.0 | 250 |
| 37 | 10 | 10 | 80 | 4.9 | 6.0 | 300 |
| 38 | 10 | 10 | 80 | 2.8 | 6.0 | 300 |
| 39 | 10 | 10 | 80 | 5.1 | 4.8 | 300 |
| 40 | 10 | 10 | 80 | 5.7 | 2.8 | 300 |
| 41 | 10 | 10 | 80 | 2.3 | 2.0 | 300 |

Figure 2. Comparison of calculated temperatures for a 6- by 3 -ft windrow under different environmental conditions at different initial temperatures.

Figure 3. Comparison of calculated temperatures for a 4.8 - by $2.4-\mathrm{ft}$ windrow under different environmental conditions at different initial temperatures.



Figure 4. Comparison of calculated temperatures for a 2.8 - by 1.4 -ft windrow under different environmental conditions at different initial temperatures.

Figure 5. Comparison of calculated temperatures for a 2.0 - by $1.0-\mathrm{ft}$ windrow under different environmental conditions at different initial temperatures.



## DISCUSSION OF RESULTS

The results of this study consist of the effect that each of the variable changes listed in Table 1 has on the cooling rate of the hot-mix asphalt concrete windrow.

## Effect of Environmental Conditions

Because of the large number of possible combinations of the variables given in Table 1, executing the computer program for 3 values of each of 6 variables would require $6^{3}$ or 216 computer runs. The average total elapsed time for each computer run was approximately 10 minutes. Therefore, the execution of all runs would have required 2,160 minutes or 36 hours of computer time. This prohibitive time requirement made it necessary to combine the effects of base temperature, ambient temperature, and solar flux into 3 groups representing most severe, moderate, and least severe environmental conditions. The most severe conditions were those that caused the fastest cooling of the asphalt windrow. The most severe conditions were used in runs $1,2,3$, 10, 11, 12, 19, 20, 21, 28, 29, 30, and 37 through 41 in Table 1. The moderate conditions were used in runs $4,5,6,13,14,15,22,23,24,31,32$, and 33 . The least severe conditions were used in runs $7,8,9,16,17,18,25,26,27,34,35$, and 36 . The effects of these sets of environmental conditions can be seen in Figures 2, 3, 4, and 5. Figure 2 shows that for the largest windrow ( 6 by 3 ft ) at the highest initial temperature ( 300 F ) the difference in average bulk temperature for the least severe minus the most severe case after 30 minutes of cooling time is $8 \mathrm{~F}(284 \mathrm{~F}-276 \mathrm{~F})$. Figure 6 shows that for the smallest windrow ( 2 by 1 ft ) at the same initial temperature ( 300 F ) the difference in average bulk temperature after 30 minutes is much greater ( 22 F ). This reflects the fact that a larger windrow has less surface-to-volume ratio than a smaller windrow, so that the surface effects are reduced on larger windrows. Each of the environmental effects is a surface effect; therefore, the same change in severity of environmental conditions does not affect the larger windrow as much as the smaller one.

## Effect of Initial Asphalt Temperature

Three different initial temperatures of $300 \mathrm{~F}, 275 \mathrm{~F}$, and 250 F were used in the study, as given in Table 1 and shown in Figures 2 through 5. In Figure 2 we see that for the largest windrow the total temperature drop under the most severe conditions at an initial temperature of 300 F was 24 F ( 300 F to 276 F ) in 30 minutes. For the same windrow under the same severity of conditions after the same cooling time, the temperature drop was $20 \mathrm{~F}(250 \mathrm{~F}$ to 230 F ) for an initial temperature of 250 F . These differences are even more pronounced for the smaller windrows, as can be seen from Figures 3 through 5. This is because conductive heat transfer increases as the temperature difference between the windrow and the base increases, because convective heat transfer increases as the temperature difference between the windrow and air increases, and because radiative heat transfer increases as the temperature of the windrow increases. Therefore, a hotter windrow will always cool faster than a cooler windrow under the same conditions. This fact was conclusively supported by Figures 2 through 5.

## Effect of Size of Windrow

The most significant variable affecting cooling rates analyzed in this study was the size of the windrow. Windrow sizes were varied over a larger percentage range than the other variables, which accounted for some of the effect. For example, the initial temperature was varied from 300 F to 250 F or 83.3 percent of the highest value. The convective heat transfer coefficient was varied from $5.7 \mathrm{Btu} / \mathrm{ft}^{2} / \mathrm{hour} / \mathrm{deg} \mathrm{F}$ to $1.5 \mathrm{Btu} /$ $\mathrm{ft}^{2}$ /hour/deg F or 26.3 percent of the highest value. But the windrow size was varied from 6 by 3 ft or $9 \mathrm{ft}^{3}$ per linear foot to 2 by 1 ft or $1 \mathrm{ft}^{3}$ per linear foot, a reduction to 11.1 percent of the highest value. These relative ranges of the variables investigated do not minimize the pronounced effect of windrow size on cooling rate.

Comparison of Figure 2 and Figure 5 shows that for the same initial temperatures ( 300 F ) under the same environmental conditions (the most severe case), the tempera-
ture drop after 30 minutes is 67 F for the 2 - by 1 - ft windrow whereas it is only 24 F for the 6 - by 3 - ft windrow.

All of these significant temperature drops due to windrow size are based on the surface-to-volume ratio effect in the windrow. The larger the windrow, the less surface it has per unit volume; thus the interior of the windrow is more effectively insulated from all boundary conditions that affect the cooling rates, and therefore the entire mass cools more slowly.

## SUMMARY AND CONCLUSIONS

From the application of finite-difference techniques to a real-world problem impossible to solve by analytical methods, a significant amount of quantitative information has been presented about cooling rates of hot-mix asphalt concrete windrows under varied environmental conditions. This quantitative information can be put to valuable use by asphalt contractors. Just by knowing the approximate environmental conditions at a given time, they will be able, with the help of this study, to judge more accurately the allowable time to complete paving jobs.

The true value of the study, however, lies not only in the information presented here. The computer programs developed in this study may be used to calculate bulk temperatures for any of an almost limitless number of combinations of windrow sizes, initial temperatures, environmental conditions, and cooling times that might be of interest.

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## REFERENCE

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[^0]:    Publication of this paper sponsored by Committee on Highway Equipment.

[^1]:    Note: Percent shown is total of grade and rolling resistance.

[^2]:    "Engineering News-Record" in its April 6, 1972, issue reported that contractors engaged in heavy and highway construction in 1971 had an average current replacement cost investment in equipment ranging from $\$ 221,000$ to $\$ 384,000$ per $\$ 1,000,000$ of contracts. Their average annual equipment purchase was pegged at $\$ 27,000$ to $\$ 43,000$ per $\$ 1,000,000$ in contracts. Thus, the average contractor in this category is apparently carrying on his books, at replacement cost, 5 to 14 years' accumulation of equipment purchases. Although some of this machinery is undoubtedly in the form of highinvestment items such as large shovels or similar units, much of it must be machinery that is outdated and long ago superseded by technologically improved items. It has been said that some contractors become emotionally involved with their equipment and are therefore reluctant to dispose of a once-profitable spread, perhaps thinking that the same equipment will perform just as profitably on the next job.

    A review of estimates recently made by us that involved 13 domestic earth-filled dam projects having a total value of approximately a half billion dollars and ranging between 3 and 110 million dollars each showed that contractors involved solely with this type of work would require an investment in machinery of approximately \$190,000 per $\$ 1,000,000$ of contract value. The higher equipment investment ratio reported by "Engineering News-Record" suggests a tendency by the larger highway and heavy construction firms to carry higher equipment inventories than is necessary.

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[^3]:    Model 1-Constant fleet size, variable production, special side-dump trucks, and box spreader.

[^4]:    Publication of this paper sponsored by Committee on Flexible Pavement Construction.

