EXPERIMENTAL VALIDATION OF LANE-CHANGING HYPOTHESES FROM AERIAL DATA

P. K. Munjal and Y. S. Hsu, System Development Corporation, Santa Monica

Lane changing is a very important component in highway traffic flow. Many researchers have recently presented mathematical models to describe lane-changing behavior. This paper focuses on the linear model by Gazis, Herman, and Weiss, the nonlinear model by Oliver and Lam, and the stochastic model by Worrall, Bullen, and Gur. Our objective is to evaluate the validity of these models by using aerial photographic data. Unknown parameters of the linear and nonlinear models, as well as the probability transition matrix of the stochastic model, are estimated by using the experimental data. Some statistical analyses are carried out to measure their validity.

•LANE CHANGING is a very common and complex phenomenon in highway travel. There may be a variety of reasons why a driver changes lanes: driver's lane preference, local traffic concentration, and average speed, to name just a few. It is impossible to model lane changing in mathematical forms that would take into account all causes for a lane change. Even if we could do that, the model would be too complex to have any practical value. This is one of the reasons why we want to study the lane-changing phenomenon in a macroscopic fashion. Another reason is that, even though traffic is a nondeterministic process, we cannot identify each individual driver's behavior. Thus, the best we can do is to study their average behavior.

The objective of this study is to validate and compare the available lane-changing models. There is a definite need to understand the relation between lane-change maneuvers and traffic flow conditions. The results we found may be directly applicable to the development of freeway traffic control strategies.

Several lane-changing studies have been made before. Oliver (7) proposed a theoretical model for lane changing on a two-lane, unidirectional roadway. In his paper, traffic was assumed to behave as a compressible fluid, obeying the equation of continuity.

$$\frac{\partial k_1}{\partial t} + \frac{\partial q_1}{\partial x} = P_{21}(x, t) - P_{12}(x, t)$$

$$\frac{\partial k_2}{\partial t} + \frac{\partial q_2}{\partial x} = P_{12}(x, t) - P_{21}(x, t) \qquad (1)$$

where

k_i = concentration of lane i, i = 1, 2;
q_i = flow of lane i, i = 1, 2;
$P_{12}(x, t)$ = lane-change function that describes transfer of vehicles from lane 1 to lane 2; and
$P_{21}(x, t)$ = lane-change function that describes transfer of vehicles from lane 2 to lane 1.

Furthermore, the lane-changing functions were assumed to satisfy

$$P_{12}(x,t) = \alpha k_1^2(x,t)[k_{2j} - k_2(x,t)]$$

$$P_{21}(x,t) = \beta k_2^2(x,t)[k_{1j} - k_1(x,t)] \tag{2}$$

where α and β are unknown constants to be estimated from experimental data, and k_{1j} and k_{2j} are jam concentrations of lanes 1 and 2 respectively. Experimental results were given in a later paper by Oliver and Lam (8).

A different approach was given by Worrall, Bullen, and Gur (12) where an elementary stochastic model was hypothesized. They made the following assumptions:

1. Lane changes were independent with an equal probability of occurrence for all vehicles; and
2. $X_{ij}^{mt} \geq 0$, if $|i-j| = 1$; $X_{ij}^{mt} = 0$, if $|i-j| \neq 1$ where X_{ij}^{mt} = number of lane changes observed between lanes i and j within subsection m during time t, and

$$Pr(X_{ij}^{mt} = N) = \frac{\exp(-\lambda_{ij}^m \times t)(\lambda_{ij}^m \times t)^N}{N!} \tag{3}$$

(N = 0, 1, 2, ...) as a Poisson process where λ_{ij}^m equals average number of lane changes between lanes i and j within subsection m during unit time and may depend on the flow or density.

It is assumed that the probability of a vehicle changing lanes in section m is a function only of its position in section m-1 and of the lane into which the change is made. The position of the vehicle is proposed as an outcome of a finite Markov process that defines a probability transition matrix T within section m. Specifically,

$$T(m) = \begin{bmatrix} t_{11}(m), & t_{12}(m)\ldots, & t_{1r}(m) \\ t_{21}(m), & t_{22}(m)\ldots, & t_{2r}(m) \\ \cdot & & \\ \cdot & & \\ \cdot & & \\ t_{r1}(m), & t_{r2}(m)\ldots, & t_{rr}(m) \end{bmatrix}$$

for an r-lane highway, where $t_{ij}(m)$ is the probability that a vehicle in lane i in section m-1 will make a lane change to lane j in section m. For simplicity, T(m) is further assumed to be independent of m. The probability transition matrix is to be estimated from experimental data.

The compressible fluid approach was also applied by Gazis, Herman, and Weiss (1) and later extended by Munjal and Pipes (5) to multilane freeway on-ramp perturbation studies. In these studies, the rate of lane changes was hypothesized as

$$\frac{\partial q_1}{\partial x} + \frac{\partial K_1}{\partial t} = a(K_2 - K_1)$$

$$\frac{\partial q_2}{\partial x} + \frac{\partial K_2}{\partial t} = a(K_1 - K_2) \tag{4}$$

for a two-lane uniform unidirectional freeway and as

$$\frac{\partial q_1}{\partial x} + \frac{\partial K_1}{\partial t} = aK_2 - bK_1$$

$$\frac{\partial q_2}{\partial x} + \frac{\partial K_2}{\partial t} = bK_1 - aK_2 \tag{5}$$

for a two-lane non-uniform unidirectional freeway, where K_i is the deviation from the equilibrium concentration in lane i, i = 1, 2. No experimental studies were made for these on- and off-ramp models. However, a similar study was carried out for a freeway lane drop by Munjal and Pipes (6) in which aerial photographic data were used for experimental validation and gave encouraging results.

The work by Levin (3) is concerned with the mathematical modeling of the delay and distance experienced by a vehicle making a lane change by using gap-acceptance concepts.

Although we have mentioned four lane-changing hypotheses, Levin's work (3) is not considered further here because of the complexity of his delay and distance models and the excessive data required for validation. Therefore, we have reduced our study to three models, the linear lane-changing model (Eq. 5), the nonlinear lane-changing model (Eqs. 1 and 2), and the stochastic model (Eq. 3).

The aerial photographic data available from the Federal Highway Administration are of the three-lane Long Island Expressway. Our first task is to extend the two-lane linear and nonlinear models to three-lane models. The unknown parameters of the linear and nonlinear models, as well as the probability transition matrix of the stochastic model, are estimated by using the aerial data. Some statistical analysis is also carried out to provide a quantitative measure of the validity of each model.

DATA ACQUISITION AND REDUCTION

Data were supplied from two sites that were selected to study the traffic flow on grade- and curvature-free multilane freeway sections with no nearby on- and off-ramps. These were the Long Island Expressway (three lanes wide) in New York and the Palisades Interstate Parkway (two lanes wide) in New Jersey. Traffic count studies showed both sites to carry a medium-to-high flow of traffic. The present analysis is carried out for the Long Island Expressway only because it provides more accurately reduced data. Daily 5-min traffic counts were taken for a week to determine a reasonable estimate of different time periods for various constant traffic-flow levels. The Long Island Expressway site is free of access for a distance of 3.2 miles; the westbound direction was chosen for data collection. This section is between the interchanges at Guinea Woods Road and Jericho Turnpike (Fig. 1).

The data were collected by aerial photography. A sequence of 70-mm color photographs was taken at 2-sec intervals with a Maurer 220 pulsed-sequence camera, a lightweight camera designed for aerial reconaissance. A 38-mm Zeiss Biogen wide-angle lens with a relative aperture of f/4.5 was used, allowing filming of about 1 mile of freeway from the helicopter, a Bell 47G3B1, hovering at a constant altitude of 4,000 ft. Magazines of 225 ft were used, which allowed continuous filming of about 30 min of traffic by using the 2-sec frame. Photographs obtained in this manner were projected on a film reader on-line with a computer, and this system permitted an accurate measure of the coordinates of vehicles in the photographic image.

The Benson-Lehner 29E film-reading system containing two crosswires and 10x magnification optics was used to read the x-y coordinates of the vehicles. This information was stored in electronic accumulators in the 282E Telecordex, which was connected to an IBM 1800 computer through a special interface. The data-reduction processing immediately followed the reading of data. There was a real-time feedback to the operator if any rereading of data was required.

Details of the film-reading technique and the associated computer software to develop trajectories of vehicles relative to an actual ground-based coordinate are given by Tashjian and Knobel (9).

Some of the important features are

1. All photographic image points of interest corrected for the optical distortion produced by the combined effect of the aerial camera lens and film magazine,
2. Position and orientation of the aerial camera in ground coordinates determined from camera and ground reference points,
3. Reference points transformed from film coordinates to ground coordinates,
4. Automobile coordinates transformed from film coordinates to ground coordinates,

5. Automobile coordinate points translated into distance and lane position relative to a prespecified ground point,
6. Position and speed of a car in its previous frame predicted from the position and speed of that car in any subsequent frame,
7. Automobile position matched in previous frame to the trajectory (i.e., the predicted position computed during processing of every frame),
8. Car trajectories smoothed for improved estimate of position and speeds, and
9. Data tape generated that contains a car number, associated distances along the road, and corresponding times, speeds, and lane numbers of the car.

The concentration k and the number of lane changes P_{ij} from lane i to lane j are needed for model validations. For reasons to be explained in the next section, we subdivide the film into 3-min time periods. Within each time period, we calculate the flow q, space-mean speed $\bar{v}$, and concentration k. Because the aerial data were taken at an interval of 2 sec, these parameters can be obtained from the following procedures.

Let R_1, R_2, ... R_k denote a set of points along the roadway. For each point R_j that a car passes during the filmed period, we will have the following situation. We observe that the car at time t_o is at a distance x_o and at time $t_o + \Delta t$ is at a distance x_1; R_j is between x_o and x_1. Δt is the time interval of the photograph taken, say 2 sec. The relation is shown in Figure 2. Then the time that the vehicle passes R_j is, by linear interpolation, given by

$$t = \frac{R_j - x_o}{x_1 - x_o} \Delta t + t_o \text{ sec} \qquad (6)$$

The velocity at R_j is estimated by

$$v = \left(\frac{R_j - x_o}{x_1 - x_o}\right) v_1 + \left(\frac{x_1 - R_j}{x_1 - x_o}\right) v_o \qquad (7)$$

where v_1 and v_o are the velocities at x_1 and x_o respectively. Thus, when R_j tends to x_o, v would tend to v_o.

If the number of cars passing R_j in the photographed time period (3 min) is n, then the average velocity (space-mean speed) is

$$\bar{v} = \frac{n}{\sum_{i=1}^{n} \frac{1}{v_i}} \qquad (8)$$

and the density is

$$k = q/\bar{v} \qquad (9)$$

where q = n × 20 is the hourly flow.

We shall make use of these statistics in the next section.

MODEL VALIDATIONS

Because the aerial data were taken from a three-lane site, we need first to extend the two-lane model to three lanes for a non-uniform roadway. The non-uniformity means that the three lanes have different q-k relations. The model as extended by Munjal and Pipes (5) is

$$(D_t + c_1 D_x)K_1 = aK_2 - bK_1$$

$$(D_t + c_2 D_x)K_2 = bK_1 - aK_2 + cK_3 - aK_2$$

$$(D_t + c_3 D_x)K_3 = aK_2 - cK_3 \qquad (10)$$

The time interval of 3 min is chosen for calculating concentrations because

1. It is long enough to average out fluctuations over a constant flow period,
2. It contains enough cars to be statistically meaningful, and
3. It is short enough that a sufficient number of intervals are available from the constant flow period.

The first algorithm is used to estimate a, b, and c in Eq. 10 and is as follows:

1. Find $k_1(i)$, $k_2(i)$, $k_3(i)$ for each 3-min interval i, $i = 1, 2, \ldots, n$ by using Eqs. 8 and 9, where the subscripts stand for the lane number. Next, compute $K_1(i) = k_1(i) - \bar{k}_1$, where $\bar{k}_1$ is the mean of $k_1(i)$. $K_2(i)$ and $K_3(i)$ are similarly computed.
2. Find the number of net lane changes for each lane and for each time period i, over a 3,200-ft stretch of road section and denote them as $\ell_1(i)$, $\ell_2(i)$, and $\ell_3(i)$.
3. Use a least squares procedure to obtain estimates of a, b, and c. That is, we find a, b, and c that minimize

$$f = \sum_{i=1}^{n} \{[aK_2(i) - bK_1(i) - \ell_1(i)]^2 + [bK_1(i) - aK_2(i) + cK_3(i)$$

$$- aK_2(i) - \ell_2(i)]^2 + [aK_2(i) - cK_3(i) - \ell_3(i)]^2\}$$

The minimizing values of a, b, and c are denoted by a_o, b_o, and c_o respectively.

The aerial data used are from two films, the first of 849 frames, starting at 9:50 a.m. on August 21, 1969, and the second of 821 frames starting at 5:55 p.m. on August 22, 1969. The numbers of car trajectories are 271 for lane 1, 538 for lane 2, and 584 for lane 3 on the first film and 249 for lane 1, 496 for lane 2, and 562 for lane 3 on the second film. The flow and space-mean speed are 2,953 cars/hour and 84.8 ft/sec for the first period and 2,852 cars/hour and 86.7 ft/sec for the second period. We feel that these two films can be considered to have the same constant flow rate. They both belong to service level B [according to the Highway Capacity Manual (2)]. Net lane changes for lanes 1, 2, and 3 are obtained and are given in Table 1 under the "experimental" columns. Estimates of a, b, and c by using the above data are $a_o = 15.04$, $b_o = -15.84$, and $c_o = -0.39$. These parameters are used in the linear model, and the theoretical net lane changes are computed by

$$L_1 = a_o K_2(i) - b_o K_1(i)$$

$$L_2 = b_o K_1(i) - a_o K_2(i) + c_o K_3(i) - a_o K_2(i)$$

$$L_3 = a_o K_2(i) - c_o K_3(i), \quad i = 1, 2, \ldots, n \tag{11}$$

The theoretical values of $L_j (j = 1, 2, 3)$ are those under the "model" columns in Table 1.

The two-lane model by Oliver and Lam (8) is now extended to a three-lane model resulting in the following form

$$D_t k_1 + D_x q_1 = P_{21} - P_{12}$$

$$D_t k_2 + D_x q_2 = P_{12} - P_{21} + P_{32} - P_{23}$$

$$D_t k_3 + D_x q_3 = P_{23} - P_{32} \tag{12}$$

with

$$P_{12}(i) = \alpha k_1^2(i)[k_{2_J} - k_2(i)]$$

$$P_{21}(i) = \beta k_2^2(i)[k_{1_J} - k_1(i)]$$

Figure 1. Long Island Expressway test site.

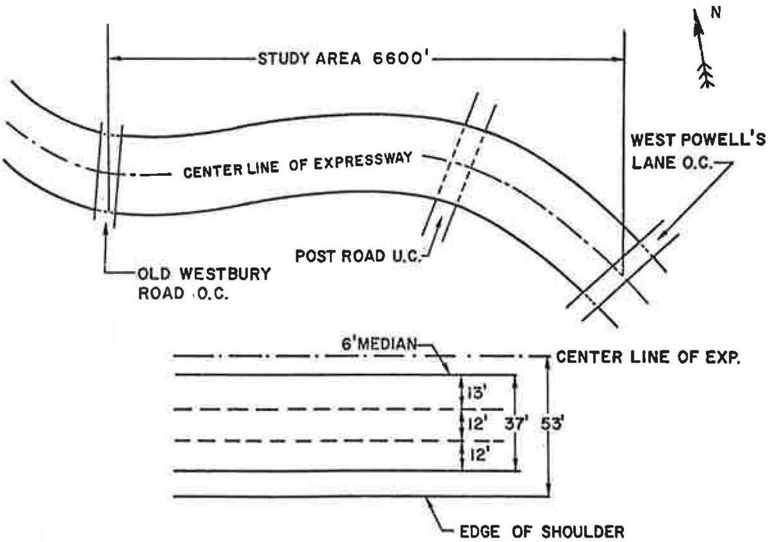


Figure 2. Relation between vehicle position and time.

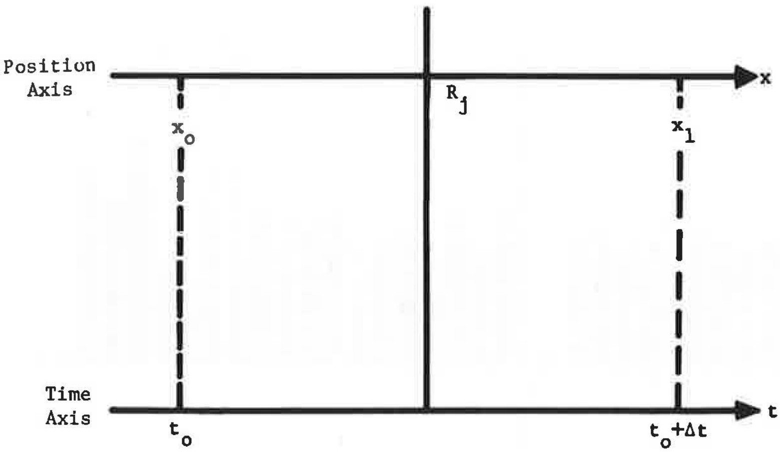


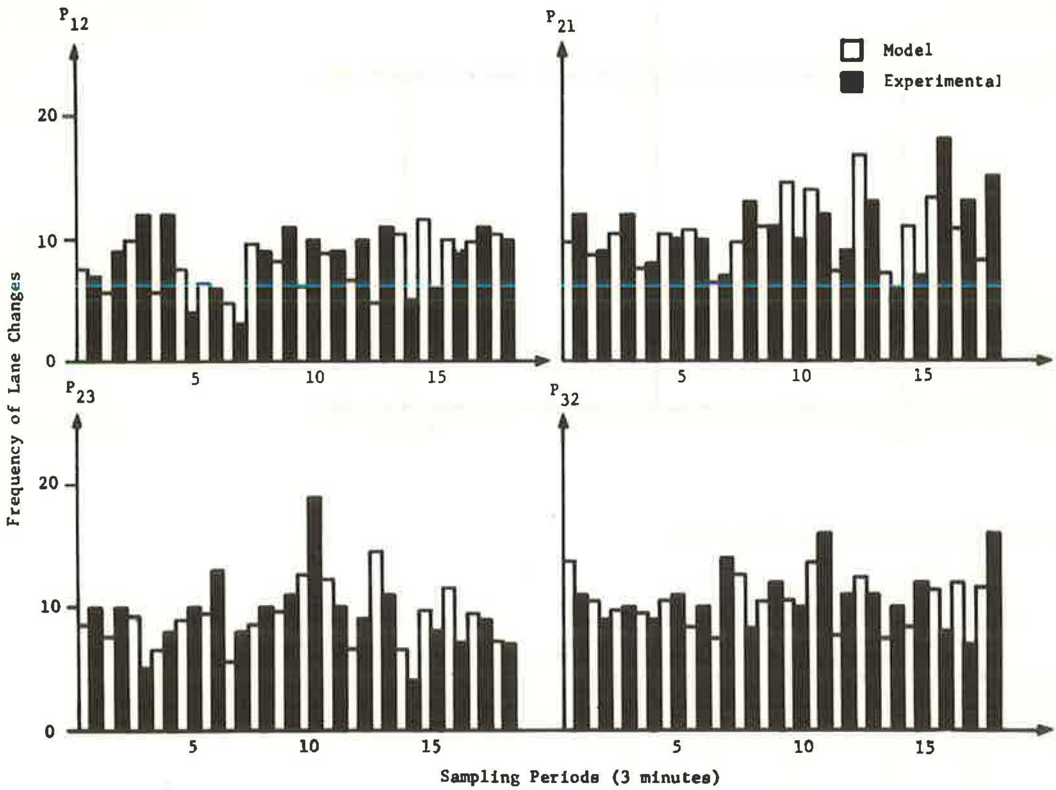
Table 1. Net lane changes from linear model.

	Number of Net Lane Changes					
	Lane 1		Lane 2		Lane 3	
Interval	Model	Experimental	Model	Experimental	Model	Experimental
1	-0.2	5	0.3	-4	-0.1	-1
2	1.5	0	-0.8	-1	-0.7	1
3	0.7	-1	-0.8	6	0.1	-5
4	-2.2	-4	3.5	5	-1.3	-1
5	-0.1	6	0.1	-5	0	-1
6	-0.4	4	0.2	-7	0.2	3
7	-3.1	4	1.1	2	2.0	-6
8	0.4	4	-0.3	-6	-0.1	2
9	0.4	0	-0.7	1	0.3	-1
10	1.1	0	-2.8	-9	1.7	9
11	1.9	3	-3.5	3	1.6	-6
12	-1.8	-1	3.2	3	-1.4	-2
13	1.4	2	-3.9	-2	2.5	0
14	-0.7	1	2.3	5	-1.6	-6
15	1.5	1	-1.8	3	0.3	-4
16	2.0	9	-3.3	-8	1.3	-1
17	0.9	2	-1.2	-4	0.3	2
18	-0.2	5	1.1	4	-0.9	-9
Mean	0.173	2.28	-0.406	-0.83	0.233	-1.45

Table 2. Results of nonlinear model.

	Number of Lane Changes From Lane i to j								Net Gain (Cars) of Lane i					
	P_{12}		P_{21}		P_{23}		P_{32}		L_1		L_2		L_3	
Interval	Model	Experimental	Model	Experimental	Model	Experimental	Model	Experimental	Model	Experimental	Model	Experimental	Model	Experimental
1	7.6	7	9.9	12	8.6	10	13.7	11	2.3	5	2.8	-4	-5.1	-1
2	5.8	9	8.7	9	7.6	10	10.4	9	2.9	0	-0.1	-1	-2.8	1
3	9.9	12	10.4	12	9.2	5	9.9	10	0.5	0	0.2	5	-0.7	-5
4	5.3	12	7.7	8	6.7	8	9.6	9	2.4	-4	0.5	5	-2.9	-1
5	7.5	4	10.3	10	9.0	10	10.6	11	2.8	6	-1.2	-5	-1.6	-1
6	6.3	6	10.8	10	9.5	13	8.3	10	4.5	4	-5.7	-7	1.2	3
7	4.9	3	6.4	7	5.7	8	7.5	14	1.5	4	0.3	2	-1.8	-6
8	9.6	9	9.8	13	8.6	10	12.7	8	0.2	4	3.9	-6	-4.1	2
9	8.2	11	11.0	11	9.7	11	10.3	12	2.8	0	-2.2	1	-0.6	-1
10	6.1	10	14.6	10	12.7	19	10.6	10	8.5	0	-10.6	-9	2.1	9
11	8.9	9	14.0	12	12.2	10	13.8	16	5.1	3	-3.5	3	-1.6	-6
12	6.7	10	7.4	9	6.6	9	7.7	11	0.7	-1	0.4	3	-1.1	-2
13	4.8	11	16.8	13	14.5	11	12.4	11	12.0	2	-14.1	-2	2.1	0
14	10.5	5	7.1	6	6.4	4	7.4	10	-3.4	1	4.4	5	-1.0	-6
15	11.7	6	10.8	7	9.8	8	8.4	12	-0.9	1	-0.5	3	1.4	-4
16	9.9	9	13.3	18	11.7	7	11.4	8	3.4	9	-3.7	-8	0.3	-1
17	9.8	11	10.8	13	9.5	9	12.0	7	1.0	2	1.5	-4	-2.5	2
18	10.3	10	8.1	15	7.2	7	11.6	16	-2.2	5	6.6	4	-4.4	-9
Mean	7.88	8.55	10.44	10.83	9.18	9.38	10.46	10.83	2.56	2.28	-1.28	-0.83	-1.28	-1.45

Figure 3. Lane-change frequencies (nonlinear model).



$$P_{23}(i) = \gamma k_2^2(i)[k_{3j} - k_3(i)]$$

$$P_{32}(i) = \delta k_3^2(i)[k_{2j} - k_2(i)] \qquad (13)$$

(i = 1, 2, . . . , n) where

P_{ij} = number of lane changes from lane i to lane j,
k_{ij} = jam concentration, and
α, β, γ, and δ = parameters to be estimated.

The second algorithm that is used in this paper estimates α, β, γ, and δ of Eq. 13 and compares the theoretical and experimental statistics in the following manner:

1. Use the values $k_1(i)$, $k_2(i)$, and $k_3(i)$ calculated from the first algorithm;
2. Find α, β, γ, and δ such that all of the following

$$S_1 = \sum_{i=1}^{n} \{P_{12}(i) - \alpha k_1^2(i)[k_{2j} - k_2(i)]\}^2$$

$$S_2 = \sum_{i=1}^{n} \{P_{21}(i) - \beta k_2^2(i)[k_{1j} - k_1(i)]\}^2$$

$$S_3 = \sum_{i=1}^{n} \{P_{23}(i) - \gamma k_2^2(i)[k_{3j} - k_3(i)]\}^2$$

$$S_4 = \sum_{i=1}^{n} \{P_{32}(i) - \delta k_3^2(i)[k_{2j} - k_2(i)]\}^2 \qquad (14)$$

are minimized, and let α_o, β_o, γ_o, and δ_o be the minimizing values of α, β, γ, and δ respectively; and

3. Find theoretical values of P_{ij} by using α_o, β_o, γ_o, and δ_o in Eq. 13, and compare the differences

$$L_1(i) = P_{21}(i) - P_{12}(i)$$

$$L_2(i) = P_{32}(i) - P_{23}(i) + P_{12}(i) - P_{21}(i)$$

$$L_3(i) = P_{23}(i) - P_{32}(i) \qquad (15)$$

for i = 1, 2, . . . , n, which are net gains or losses for each lane due to lane changes.

Using the same data as used for the linear model, we summarized the experimental and computed results by using the nonlinear model as given in Table 2 and shown in Figure 3. More information is provided by Table 2 than by Table 1 in that not only is the net gain for each lane due to lane changes recorded (L_1, L_2, L_3) but also each individual lane-changing flow is recorded (P_{12}, P_{23}, P_{32}). If we consider the average behavior of lane changers, i. e., if we look at the mean values of the samples we collected, the nonlinear lane-changing hypothesis seems to be superior to the linear lane-changing hypothesis. This is not surprising because the nonlinear model has more mechanisms than the linear model.

To validate the stochastic model, we divide the road stretch into 16 sections of 200 ft each and calculate the probability distribution of cars in each lane of each road section for the entire filmed period (821 + 849 = 1,670 frames). The probability transition matrix T for sections 1 and 16 is also calculated:

$$T = \begin{bmatrix} T_{11} & T_{12} & T_{13} \\ T_{21} & T_{22} & T_{23} \\ T_{31} & T_{32} & T_{33} \end{bmatrix}$$

where T_{ij} is the ratio of the number of lane changes from lane i to lane j to the total flow in lane i. The 200-ft road section is chosen mainly to ensure that $T_{ij} = 0$ for $|i-j| > 1$.

The algorithm proposed by Worrall and Bullen (10) was used to calculate the approximate T, say $\hat{T}$, for the entire road stretch. This $\hat{T}$ is employed for each 3-min interval to obtain the theoretical value of lane changes. That is, we want to find $\hat{T}$ such that

$$\hat{a}_{16} = a_1 \times \hat{T}^{15}$$

is as close to the experimental value of a_{16} as possible, where a_1 and a_{16} are the distributions of vehicles by lane in section 1 and section 16 respectively, and $\hat{a}_{16}$ is the distribution, by lane, of vehicles in section 16 estimated by using the transition matrix $\hat{T}$. Therefore, the probability transition matrix of the entire road stretch (16 sections) is just $R = \hat{T}^{15}$. The estimated R from the algorithm is

$$R = \begin{bmatrix} 0.7235 & 0.2478 & 0.0287 \\ 0.1513 & 0.7038 & 0.1449 \\ 0.0211 & 0.1398 & 0.8391 \end{bmatrix}$$

This transition matrix, R, is used for each 3-min time period for estimating the number of lane changes. Results are given in Table 3 and shown in Figure 4.

STATISTICAL COMPARISONS

A better comparison can be made if we employ some quantitative measure of the validity of each model. The approximate normal statistic u can provide such a measure. We outline the procedure in the following.

1. Calculate

$$e_j(i) = L_j(i) - \hat{L}_j(i), \quad j = 1, 2, 3, \quad i = 1, 2, \ldots, n \tag{16}$$

where j is the lane number, n is the total sample size, and $L_j(i)$ and $\hat{L}_j(i)$ indicate the number of net lane changes for lane j in sample i from the experimental data and computed data, in turn.

2. Obtain

$$\bar{e}_j = \frac{1}{n} \sum_{i=1}^{n} e_j(i) \tag{17}$$

3. Compute

$$s_j = \left\{ \sum_{i=1}^{n} [e_j(i) - \bar{e}_j]^2 \Big/ (n-1) \right\}^{1/2} \tag{18}$$

4. The approximate u-statistic is

Table 3. Results of stochastic model.

	Number of Lane Changes From Lane i to j								Net Gain (Cars) of Lane i					
	P_{12}		P_{21}		P_{23}		P_{32}		L_1		L_2		L_3	
Interval	Model	Experimental	Model	Experimental	Model	Experimental	Model	Experimental	Model	Experimental	Model	Experimental	Model	Experimental
1	7.8	7	9.8	12	9.2	10	11.2	11	2.0	5	0	-4	-2.0	-1
2	6.7	9	9.3	9	8.8	10	10.1	9	2.6	0	-1.3	-1	1.3	1
3	9.0	12	9.9	12	9.7	5	9.6	10	0.9	0	-1.0	5	0.1	-5
4	6.4	12	8.5	8	7.8	8	9.4	9	2.1	-4	-0.5	5	-1.6	-1
5	8.1	4	8.8	10	9.4	10	10.2	11	0.7	6	0.1	-5	-0.8	-1
6	8.0	6	9.7	10	8.8	13	9.1	10	1.7	4	-1.4	-7	-0.3	3
7	6.4	3	8.0	7	7.6	8	8.6	14	1.6	4	-0.6	2	-1.0	-6
8	9.0	9	9.8	13	9.4	10	10.9	8	0.8	4	0.7	-6	-1.5	2
9	8.1	11	9.9	11	9.7	11	9.9	12	1.8	0	-1.6	1	-0.2	-1
10	6.4	10	11.0	10	10.4	19	9.8	10	4.6	0	-5.2	-9	0.6	9
11	8.1	9	11.0	12	10.5	10	11.4	16	2.9	3	-2.0	3	-0.9	-6
12	7.0	10	8.1	9	7.8	9	8.5	11	1.1	-1	-0.4	3	-0.7	-2
13	6.2	11	11.7	13	11.0	11	10.9	11	5.5	2	-5.6	-2	0.1	0
14	9.0	5	8.1	6	8.0	4	8.5	10	-0.9	1	1.4	5	-0.5	-6
15	9.5	6	9.5	7	9.4	8	8.6	12	0	1	-0.8	3	0.8	-4
16	9.0	9	10.5	18	10.3	7	10.3	8	1.5	9	-1.5	-8	0	-1
17	8.7	11	9.9	13	9.5	9	10.8	7	1.2	2	0.1	-4	-1.3	2
18	8.7	10	8.8	15	8.4	7	10.3	16	0.1	5	1.8	4	-1.9	-9
Mean	7.92	8.55	9.6	10.83	9.25	9.38	10.07	10.83	1.68	2.28	-0.99	-0.83	-0.69	-1.45

Figure 4. Lane-change frequencies (stochastic model).

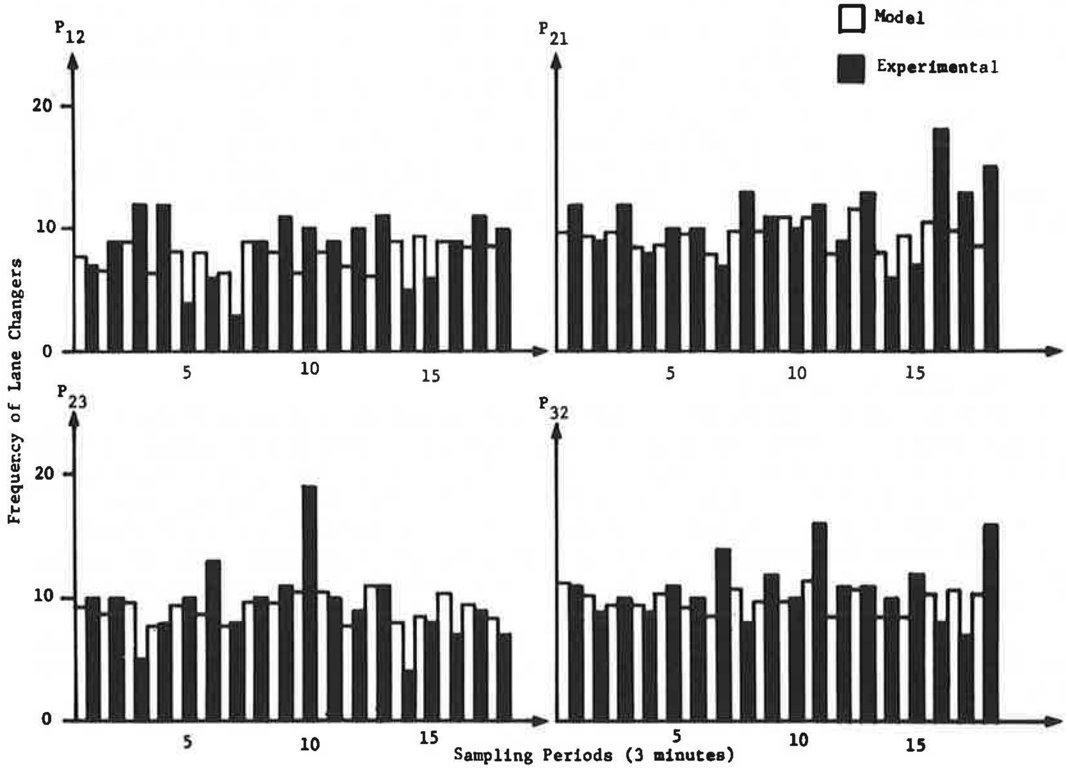


Table 4. u-statistic of lane-changing models.

	Lane 1		Lane 2		Lane 3		Composite
Model	$\|u\|$	α[a]	$\|u\|$	α[a]	$\|u\|$	α[a]	Tail Statistic[b]
Linear	3.09	0.002	0.36	0.719	1.79	0.073	14.325
Nonlinear	0.168	0.867	0.295	0.768	0.182	0.856	1.124
Stochastic	0.677	0.498	0.147	0.883	0.807	0.420	3.378

[a]Represents the exceedance (tail) probability given by the formula $\alpha = \text{prob}\ \{|u| > u_\alpha\}$.
[b]Using Fisher's Combination-of-Tests Statistic (11)

$$\Lambda = -2 \sum_{1}^{k} \ln \alpha_j \sim \text{chi-square (2k)}$$

where k is the degree of freedom.

$$u_j = \frac{\sqrt{n-1}\,\bar{e}_j}{s_j} \tag{19}$$

The summary of the u-statistics for both models is given in Table 4.

The entries in the last column of Table 4 obey the chi-square distribution with 6 degrees of freedom in accordance with Fisher's Combination-of-Tests Statistic where we have combined the results over all three lanes for each model. Referring to the tabulation of chi-square values, it is seen that an observed value of Λ or greater for the linear model has occurrence probability of less than 0.05, whereas the corresponding values for the nonlinear and stochastic models are approximately 0.98 and 0.75. These results are only approximate, but they do indicate relative ranking. On this basis we rank the three models as the nonlinear model, the stochastic model, and the linear model in this order according to the u-statistics.

The superiority of a model over others can also be viewed by the complexity of the model or the amount of information used in parameter estimation. The nonlinear model clearly has more mechanisms than the linear model. Moreover, the value of jam concentration is used in the nonlinear model, whereas it is not used in the linear model. The stochastic model is a completely different approach and does not hypothesize a density oscillation between lanes. It estimates the frequency of lane changes by fixing the boundary conditions at both ends of the road stretch. Because of this complexity, it is very difficult to evaluate this model against the other two models. However, the extensiveness of data preparation is about the same for all three models.

One shortcoming of the stochastic model is that the algorithm suggested by Worrall and Bullen only gives an approximate solution of the probability transition matrix. This probably explains why, with more parameters to estimate (six for a three-lane highway), it is still not better than the nonlinear model according to the validation results. (It is noted that, although T has nine unknowns, the constraints

$$\sum_{j=1}^{3} t_{ij}, \quad i = 1, 2, 3$$

reduce the unknowns to six.)

We should be more careful here to interpret the u-statistics given in Table 4. Any deviation from the assumptions in which the u-statistic is derived can result in a large value of u, e.g., the non-zero mean of $\bar{e}_j$ in Eq. 17, the mutual dependence of the samples. If corrections can be made, the u-statistics will, in general, be improved.

Both models assume the unknown parameters to be independent of concentration. These parameters have been estimated by first fitting the experimental data by using a constant flow of traffic. How valid this assumption is can be tested by using data from a different constant flow level of traffic and computing the lane changes by using the same parametric values. Available aerial data do not provide enough samples for this kind of study. However, some preliminary inspection suggests that the unknown parameters are concentration-dependent.

CONCLUSIONS

Three lane-changing models, the linear lane-changing model by Gazis, Herman, and Weiss, the nonlinear model by Oliver and Lam as extended here, and the stochastic model by Worrall and Bullen, were selected for experimental validation with aerial photographic data supplied by the Federal Highway Administration. The unknown parameters of the linear and nonlinear models, as well as the probability transition matrix of the stochastic model, were estimated from the data. The number of lane changes was then calculated by using models with estimated parameters. It was found, statistically, that the nonlinear model gave excellent validation results for every lane of the three-lane Long Island Expressway, and the linear and stochastic models gave excellent results only for lane 2. Generally speaking, the three models are ranked as the nonlinear model, the stochastic model, and the linear model.

REFERENCES

1. Gazis, D. C., Herman, R., and Weiss, G. H. Density Oscillations Between Lanes of Multilane Highways. Operations Research, Vol. 10, 1962, pp. 658-667.
2. Highway Capacity Manual—1965. HRB Spec. Rept. 87, 1965.
3. Levin, M. Some Investigations of the Freeway Lane Changing Processes. Civil Engineering Dept., Texas A&M Univ., PhD dissertation, 1970.
4. Munjal, P. K., Hsu, Y. S., and Lawrence, R. L. Analysis and Validation of Lane-Drop Effects on Multi-Lane Freeways. Transportation Research, Vol. 5, 1971, pp. 257-266.
5. Munjal, P. K., and Pipes, L. A. Propagation of On-Ramp Density Waves on Non-Uniform Unidirectional Two-Lane Freeways. System Development Corp., Tech. Memo. TM-3858/017/00, 1969.
6. Munjal, P. K., and Pipes, L. A. Propagation of On-Ramp Density Waves on Uniform Unidirectional Multi-Lane Freeways. Transportation Science, Vol. 5, 1971, pp. 390-402.
7. Oliver, R. M. A Two-Lane Traffic Model. Operations Research Center, Univ. of California, Berkeley, Rept. 64-34, 1965.
8. Oliver, R. M., and Lam, T. Statistical Experiments With a Two-Lane Flow Model. Proc. Third International Symposium on Theory of Traffic Flow, New York, 1965, pp. 170-180.
9. Tashjian, Z. S., and Knobel, H. C. On-Line Processing of Aerial Photographic Data. Univ. of California, Los Angeles, Eng. 7144, 1971.
10. Worrall, R. D., and Bullen, A. G. R. Lane Changing on Multilane Highways. Northwestern Univ., Evanston, Ill., final rept., 1969.
11. Kendall, M. G. The Advanced Theory of Statistics, Vol. II. Hafner Publ. Co., 1951.
12. Worrall, R. D., Bullen, A. G. R., and Gur, Y. An Elementary Stochastic Model of Lane Changing on a Multilane Highway. Highway Research Record 308, 1970, pp. 1-12.