INTERNAL ENERGY OF TRAFFIC FLOWS

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The objective of this paper is to establish an acceptable parameter for the internal energy of traffic flow so that further exploration of traffic dynamics can be pursued. Through a boundary condition analysis of traffic flows, it has been found that the currently suggested "acceleration noise" is not a good measure of the internal energy. Results of a theoretical analysis of analogous compressible fluid conditions indicate that, if the kinetic energy of a traffic stream is defined as αku^2, the principle of conservation of energy will not apply. This is because, when density is used instead of total number of vehicles, the system is not confined; thus energy will not be conserved. The compressible fluid analogy further suggests that a term $P_t[(k_t/k_o) - 1]$ may be used to represent the internal energy of traffic flows. Because the accuracy of this compressible fluid analogy is questionable, the term $P_t[(k_t/k_o) - 1]$ is not directly applicable to traffic flows. Instead, an empirical approach is used in the search for a suitable internal energy parameter. Aerial photographic traffic data were used in this effort. Four vehicle-interaction-related parameters were analyzed. One of the parameters tested, the coefficient of variation of speed, not only has exhibited a variational pattern that agrees with that of the $P_t[(k_t/k_o) - 1]$ but also satisfies the boundary condition requirements. It is, therefore, proposed as a suitable measure of the internal energy of traffic flow.

•AN UNDERSTANDING of the dynamics involved in traffic movement is no doubt a basis for design of an efficient and safe highway system. However, fundamentals of traffic dynamics have not been so fully developed as have other physical phenomena such as movement of discrete or continuous masses. One difficulty has been defining energy parameters in the involved macroscopic traffic dynamic system. In an attempt to provide a solution to this problem, a traffic parameter is discussed here that can be used to measure the internal energy of a traffic flow.

Drew (1) introduced the energy concept into traffic flow analysis by considering the traffic stream to be analogous to the flow of a compressible fluid in a constant-area duct. He suggested that a kinetic energy term on the order αku^2 might be used to describe certain properties of a traffic stream inasmuch as a similar term, $\frac{1}{2}\rho V^2$, is defined in fluid mechanics as the kinetic energy of a compressible fluid. In the traffic case, α is a dimensionless constant, k is the density of the traffic stream, and u is the average speed of the stream. Then, by applying the well-known principle of conservation of energy, Drew further suggested that an internal energy term be added to the system to yield an expression for total traffic energy. The proposed relationship may be written as

$$T = E + I \tag{1}$$

where

T = total energy of a traffic stream (constant),
E = kinetic energy of the traffic stream (αku^2), and
I = internal energy of the traffic stream.

In most cases, the kinetic energy of a traffic stream can be easily obtained by measuring the density and average velocity of the stream. The internal energy, however, is thought to be related to the interactions among vehicles in the stream, and it is very difficult to define. Drew has proposed that the parameter "acceleration noise" (3) be used as a measure of internal energy. His proposal was based on two observations. First, the acceleration noise obtained by finding the standard deviation of the acceleration distribution of one vehicle traveling along a stretch of roadway has the same dimensions as kinetic energy. Second, a plot of acceleration noise and αku^2 versus density revealed that the acceleration noise values are generally low when the kinetic energy values are high, thus yielding a near constant value for total energy.

Using acceleration noise as a measure of internal energy, we can rewrite Eq. 1

$$T = \alpha ku^2 + \sigma_t = \text{constant} \tag{2}$$

where σ_t is the derived acceleration noise parameter.

Although this expression represents a significant concept for studying traffic characteristics, it appears to have certain shortcomings.

If the expression $\alpha ku^2 + \sigma_t = \text{constant}$ is applied at the boundary conditions of a traffic stream, certain discrepancies become apparent. Consider first the internal energy term σ_t. According to Drew, σ_t is derived from $\sigma - \sigma_n$ where σ is the measured acceleration noise of a vehicle and σ_n is the natural acceleration noise displayed by the same vehicle subjected to no traffic interference.

For the boundary condition where the density is zero ($k = 0$), the acceleration noise value σ has to equal σ_n by definition. Therefore, $\sigma_t = \sigma - \sigma_n$ would reduce to $\sigma_n - \sigma_n$ or zero. Because there are no vehicles on the road at zero density, the kinetic energy at this point would also be equal to zero. Consequently, the total energy of the traffic stream when $k = 0$ would be $T = E + I = 0$. At the other end of the density domain, jam density ($k = k_j$), all vehicles on the roadway are stopped. Because there is no movement, σ would necessarily be zero. Also, because the idea of a natural acceleration noise makes no sense for such extremely high-density conditions, σ_n is undefined at k_j. Thus, no meaningful value for total energy can be found for the jammed condition by using the proposed definition of internal energy. If the principle of conservation of energy holds true for a traffic stream using the parameters suggested by Drew, σ_t does not seem to represent a good measure of internal energy.

Intuitively, the internal energy of a traffic stream should express the degree to which vehicle interactions exist in the stream. From this point of view, the internal energy should be equal to zero when there are no vehicles on the road and should reach its maximum value when the density is maximum inasmuch as the greatest amount of vehicle interaction can be expected to occur at this point. A parameter that fulfills these boundary conditions is required. If it is assumed that such a parameter, call it I, exists, then the condition for the conservation of energy would be written as

$$T = \alpha ku^2 + I = \text{constant} \tag{3}$$

where $I = 0$ at $k = 0$ and $I = I_{max}$ at $k = k_j$.

To this point in the analysis, it has been assumed that the principle of conservation of energy can be applied to a traffic stream with the suggested parameters. If Eq. 3 is evaluated at the appropriate boundary conditions, however, the following results are obtained:

$$k = 0 \rightarrow E = \alpha ku^2 = 0 \text{ and } I = 0$$

$$\therefore T = E + I = 0$$

$$k = k_j \rightarrow E = \alpha ku^2 = 0 \text{ since } u = 0 \text{ and } I = I_{max}$$

$$\therefore T = E + I = I_{max}$$

Because I_{max} must be greater than 0, the conservation of energy (Eq. 3) does not hold.

From the analysis documented above, two general conclusions can be drawn. First, acceleration noise is not an adequate parameter for representing the internal energy of a traffic stream if the internal energy is defined in terms of vehicular interaction; and, second, if the kinetic energy of the traffic stream is defined as $\alpha k u^2$ and the internal energy is defined in terms of vehicular interaction, which is zero at zero density and a maximum at jam density, the principle of conservation of energy does not apply.

From these conclusions, it is apparent that some modifications must be made in the energy concept if it is to be used in traffic flow conditions.

THEORETICAL INVESTIGATIONS

Energy System of a Traffic Stream

Consider a platoon of n vehicles. At time t_o, assume that these vehicles are spread along a section of roadway at a low density k_o and are moving at an average speed u_o. Due to a disturbance of some sort, the first vehicle slows down and the vehicles start backing up. At time t_i the average speed has dropped to u_i and the density has increased to k_i. If the cause of the disturbance continues to prevail, a complete stoppage of the platoon will eventually occur. At this time a bumper-to-bumper situation will exist, and the density will have reached its maximum value of k_j. This sequence of occurrences is shown in Figure 1.

These conditions can be considered analogous to the system shown in Figure 2. In this system, a bulk of compressible fluid with mass m is moving through a frictionless pipe with unit cross section. The initial conditions are that at time t_o this bulk of fluid is moving at a velocity v_o with density ρ_o and has length l_o. A varying resistant force is introduced into the system. Because of the resistance, the movement of the fluid mass is retarded and the fluid slows, eventually coming to a stop. At the same time, because of the compressive action of the variable force and the inertia of the fluid, the density of the fluid increases and reaches a maximum density ρ_j when the stoppage occurs.

Now suppose that the fluid mass was completely stopped at time t_j and that the average resistant force from time t_o to t_j was measured as P_j. Also assume that the length of the mass at t_j was l_j. In the intermediate condition at time t_i (Fig. 2b), the mass is moving at a velocity v_i, the density is ρ_i, the length of the mass has been reduced from l_o by an amount Δl_i to l_i, and the average resistant force from time t_o to t_i is represented as P_i.

Consider the condition shown in Figure 2a. There is no external force in the system, and the total energy involved is simply equal to the kinetic energy of the moving mass, $\frac{1}{2}mv_o^2$. After the resistance is applied to the system, the speed of the mass is reduced, and part of the kinetic energy is lost and is transferred to another form of energy. In this confined system the only other form of energy possible is that stored in the fluid itself due to the work done by the compressive action of the resistant force and the inertia of the fluid itself. At time t_i the kinetic energy of the fluid has been reduced to $\frac{1}{2}mv_i^2$. The work done to this time by the resistance is equal to the average compressive force P_i times the distance by which the fluid was compressed Δl_i. The total energy at time t_i is then

$$\frac{1}{2}mv_i^2 + P_i \Delta l_i = T \qquad (4)$$

When the fluid mass is stopped at time t_j, there is no kinetic energy in the system. The stored energy at this time is equal to $P_j \Delta l_j$ where $\Delta l_j = l_o - l_j$. Hence, the total energy would be

$$P_j \Delta l_j = T \qquad (5)$$

With the system confined and no other forces or energy involved, the principle of conservation of energy states that the total energy of the fluid for all three points in time must be equal.

$$\frac{1}{2}mv_o^2 = \frac{1}{2}mv_1^2 + P_1\Delta l_1 = P_J\Delta l_J \tag{6}$$

Now, if the intermediate condition is taken as a reference, the following general expression can be written:

$$\frac{1}{2}mv_1^2 + P_1\Delta l_1 = C \text{ (constant)} \tag{7}$$

Dividing both sides of Eq. 7 by l_1 gives Eq. 8.

$$\frac{1}{2}\frac{m}{l_1}v_1^2 + \frac{P_1\Delta l_1}{l_1} = C/l_1 \tag{8}$$

Now, m/l_1 is the density of the fluid mass at time $t_1(\rho_1)$ and the term $(P_1\Delta l_1)/l_1$ is simply the energy stored in a unit section (I_1). Thus another form of Eq. 8 is

$$\frac{1}{2}\rho_1 v_1^2 + I_1 = C/l_1 \tag{9}$$

Inasmuch as C/l_1 is not a constant but is a function of l_1, the conclusion extracted from this analysis is that, if the kinetic energy of a compressible fluid is expressed as $\frac{1}{2}\rho_1 v_1^2$ and the internal energy is expressed as the energy stored in a unit section of the fluid, then the principle of conservation of energy does not hold because the system is no longer confined; we are not dealing with a certain amount of mass but, instead, the variable mass in a unit volume. From the analogous point of view, if the kinetic energy of a traffic stream is expressed as αku^2, then the energy of the stream will not be conserved, no matter how the internal energy is defined, because the system is no longer confined. This conclusion agrees with the observation made in the previous section from examination of the traffic stream boundary conditions.

Internal Energy of a Traffic Stream

Although it has been demonstrated that the principle of conservation of energy does not hold for a traffic stream when kinetic energy is defined as αku^2, it is thought nevertheless that the concepts of kinetic and internal traffic stream energy are valuable contributions to the understanding of the dynamics of traffic flow. To apply these concepts, however, we must find a parameter that accurately reflects internal energy. This parameter must satisfy the boundary conditions for internal energy, which were discussed previously, and should in general exhibit a compensatory pattern with corresponding kinetic energy.

Consider the compressible fluid discussed previously. The internal energy in general can be expressed as $(P_1\Delta l_1)/l_1$. Because $\Delta l_1 = l_o - l_1$ and $m = \rho_o l_o = \rho_1 l_1$, then Δl_1 can be written as $m/\rho_o - m/\rho_1 = m(1/\rho_o - 1/\rho_1)$. Thus, the internal energy term becomes $P_1\rho_1(1/\rho_o - 1\rho_1)$. The traffic stream analogy of this term would be $P_1k_1(1/k_o - 1/k_1)$ where P_1 is the average of an imaginary resistant force acting on the traffic stream from time t_o to time t_1.

If this resistant force were constant (call it P_c), then the term $P_1k_1(1/k_o - 1/k_1)$ could be written as a linear function of k_1, that is, as $P_c(k_1/k_o - 1)$. A graphical presentation of this force is shown in Figure 3. It can be seen that the greater the density k_1 becomes, the greater the internal energy becomes, and when $k_1 \rightarrow k_o \rightarrow 0$ the internal energy also approaches zero. This behavior satisfies the boundary conditions previously postulated for the internal energy of a traffic stream.

The relationship shown in Figure 3 was based on the assumption that the resistant force was constant. When a traffic stream is considered, however, this force is invisible and might be imagined to be a function of the internal friction inherent in traffic flow. From our general knowledge of traffic behavior, it seems more logical to assume a variable force in these circumstances than a constant force. In mechanics a force F is related to the mass of an object m and its acceleration a by Newton's second law of motion, $F = ma$. Because m is a constant, the force can be written simply as a function of acceleration: $F = f(a)$. This argument suggests that the imaginary force that acts

Figure 1. Traffic queue-forming condition.

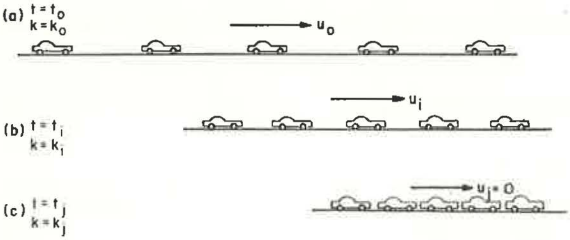


Figure 2. Compressible fluid condition analogous to traffic flow.

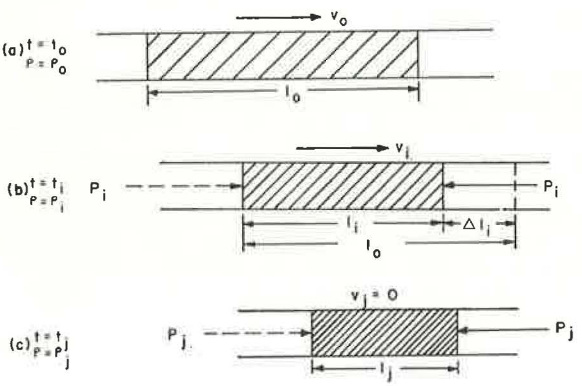
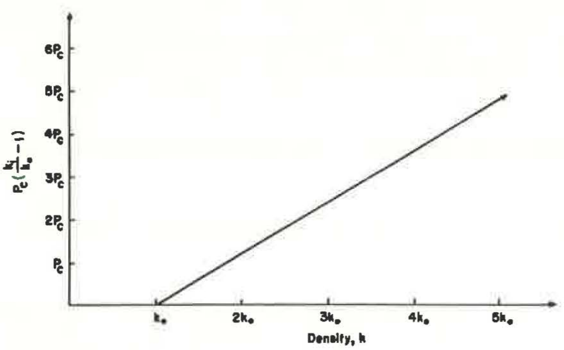


Figure 3. Density versus $P_c\,(k_i/k_o - 1)$.



on a vehicular platoon is a function of the acceleration distribution of the stream with mean value $\dot{u}$. It is generally accepted that the velocity of traffic flow is a function of traffic density: $u = f(k)$. If we differentiate this expression with respect to time, the following relationship is obtained:

$$\dot{u} = f'(k)\frac{dk}{dt} \tag{10}$$

where $f'(k) = df/dk$. This implies that the imaginary resistant force P is a function of $f'(k)(dk/dt)$. In this expression, $f'(k)$ would be a known function if the relationship between speed and density were defined. The term dk/dt, which is the time rate of change of density, however, does not present a functional pattern according to existing knowledge. For this reason, no exact expression for the variation of internal energy as a result of the analysis of this section provides a valuable guide in the search for a suitable internal energy parameter.

EXPERIMENTAL INVESTIGATION

Methodology

From the previous analysis, it seems that, if a traffic flow could closely resemble the properties of a compressible fluid, the term $P_1(k_1/k_0 - 1)$ would be used to indicate its internal energy. Because the fluid analogy is not strictly applicable throughout the density domain and because it has been shown that the imaginary resistance for a traffic flow does not take a specific functional pattern, the term $P_1(k_1/k_0 - 1)$ cannot be used as a direct measure of internal energy in itself. It does seem to provide, however, a good approximation of the true internal energy pattern.

A good internal energy parameter should satisfy the following requirements:

1. It should be a measure of vehicular interaction,
2. It should satisfy the boundary requirements of traffic conditions, and
3. It should have a variational pattern that approximates the variational pattern of the fluid analogous term $P_1(k_1/k_0 - 1)$.

With these criteria in mind, an empirical approach is used in the search for an internal energy parameter. This approach is dictated by our inability to establish a theoretical expression for the internal energy of a traffic system. The following activities direct the empirical analysis:

1. Establish the variational pattern of the term $P_1(k_1/k_0 - 1)$ versus density from appropriate data;
2. Choose vehicular interaction related parameters, and plot their variational pattern against density;
3. Compare the plots obtained from procedures 1 and 2 to see whether they agree; and
4. If they do, check the boundary requirements.

The data used for this investigation were collected by an aerial photogrammetry technique (2). The selected platoon is displayed on the vehicle trajectories shown in Figure 4.

Variations of the Imaginary Resistant Force for the Platoon Studied

To calculate the imaginary resistant force, we treat the platoons as confined masses of compressible fluid. The arithmetic mean speed of the platoon is taken as the speed of the fluid mass, and the force is considered to be a function of the time rate of change of the average speed (acceleration). Figure 5 shows the relationships between average acceleration and density for the platoon. Disregarding scale differences, the average imaginary force would have the same variational pattern with density as does average acceleration.

Figure 4. Identification of platoon studied.

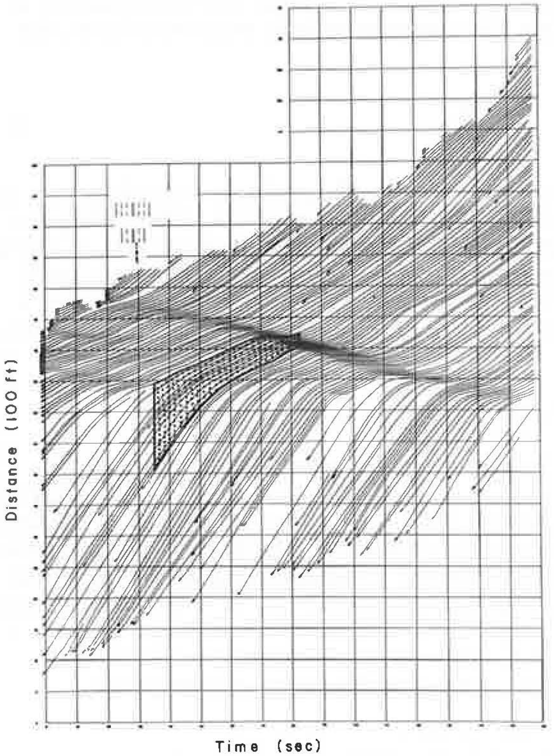
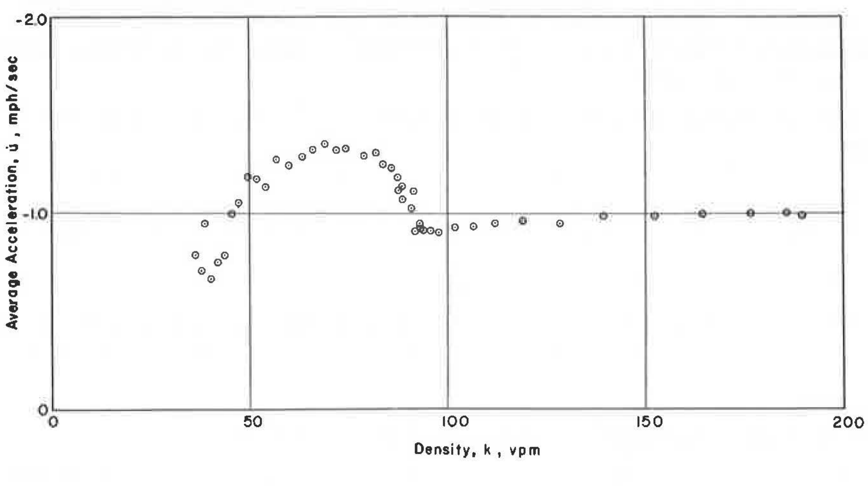


Figure 5. Average acceleration versus density for platoon studied.



If we recall that the expression for internal energy of the traffic stream is $P_i(k_i/k_o - 1)$ and that $(k_i/k_o - 1)$ is an increasing linear function of density, then the variation of internal energy with density can be specified. The internal energy will be a generally increasing function of density with a hump at that value of density where the average resistance is at a maximum (Fig. 5).

Alternative Internal Energy Parameters

With the theoretical pattern for the variation of internal energy with density determined for the selected platoon, it is now possible to investigate the applicability of several possible internal energy parameters. Four different parameters that are considered to be vehicular interaction related have been analyzed. These are

1. Standard deviation of the acceleration distribution of a platoon σ_a (this is in contrast to the acceleration noise value, discussed earlier, that considers only one vehicle),
2. Average of the absolute value of acceleration of the vehicles in a platoon $|\bar{a}|$,
3. Standard deviation of the platoon speed distribution σ_s, and
4. Coefficient of variation of the platoon speed distribution defined as the standard deviation of speed divided by the arithmetic mean speed (CV_u).

Standard Deviation of Acceleration—The relationship between the standard deviation of acceleration and density for the selected platoon is shown in Figure 6. No recognizable pattern similar to the one desired for internal energy can be identified. In addition, this parameter does not satisfy the boundary condition that requires that it be a maximum at maximum density.

Average Absolute Acceleration—Figure 7 shows a plot of the average absolute acceleration versus density for the selected platoon. The pattern is similar to that obtained for the standard deviation of acceleration and has no value as a representative of internal energy.

Standard Deviation of Speed—Investigation of the standard deviation of the platoon speed distributions yielded much more encouraging results than the acceleration-oriented studies. Figure 8 shows the variation of the standard deviation of speed with density for the selected platoon. A functional variational pattern is presented: The dispersion of speed decreases as density increases until a region is reached where almost all the vehicles in the platoon are moving at about the same speed. As density continues to increase, the dispersion of speeds begins to increase as well. This phenomenon can be explained by the fact that traffic flow at high densities tends to be unstable, and there can exist a large variance among the speeds of the individual vehicles in such a disturbed flow situation. With still further increases in density, the dispersion of speed once again drops because the space available to each vehicle for maneuvering has become severely limited. Finally, when jam density is reached, σ_s falls to zero, for all movement on the roadway has ceased.

This parameter appears to be a good indicator of internal energy in that it is representative of prevailing vehicle interactions. It presents a consistent and recognizable pattern with density and is simple to calculate. It does not, however, satisfy the boundary condition that internal energy be a maximum at jam density.

Coefficient of Variation of Speed—To correct the boundary condition shortcoming displayed by σ_s requires that a modified parameter be formed by dividing the standard deviation of the speed distribution by the arithmetic mean speed at each density level. This parameter, CV_u, is referred to in statistical terms as the coefficient of variation of speed and provides a measure of the relative dispersion of the speed values as a percentage of the mean speed. A plot of CV_u versus density is shown in Figure 9 for the selected platoon. Comparing the exhibited patterns with Figure 5, which were derived earlier, evidences a superb agreement.

Now we check the boundary conditions. According to the definition of the coefficient of variation of speed, we have

$$CV_u = \frac{\left[\frac{1}{n-1}\sum_{i=1}^{n}(\mu_i - \bar{\mu})^2\right]^{1/2}}{\bar{\mu}}$$

Figure 6. Standard deviation of acceleration versus density for platoon studied.

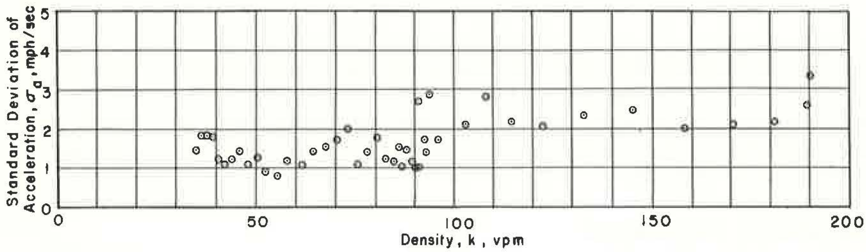


Figure 7. Average absolute acceleration versus density for platoon studied.

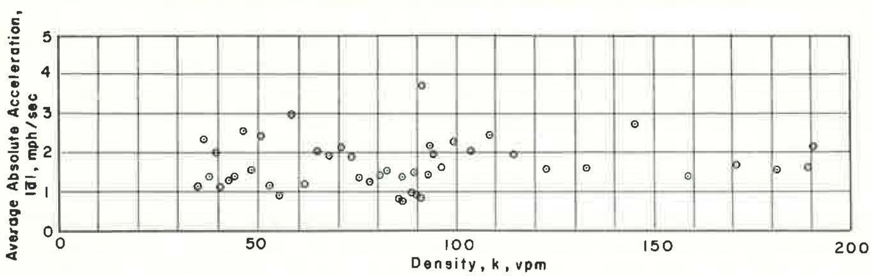


Figure 8. Standard deviation of velocity versus density for platoon studied.

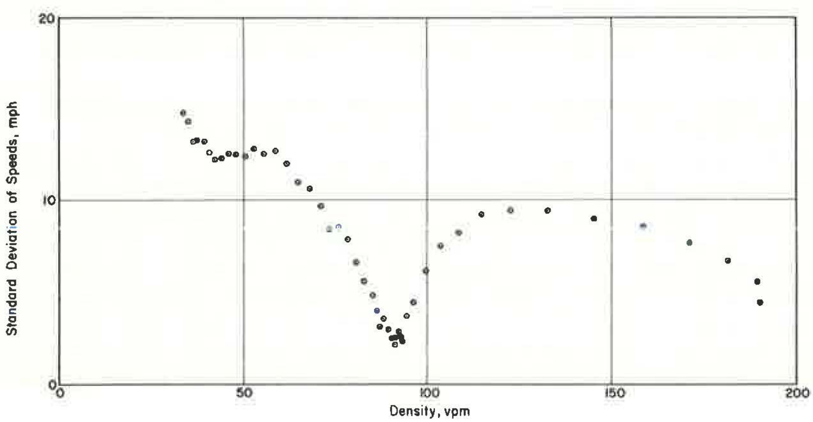
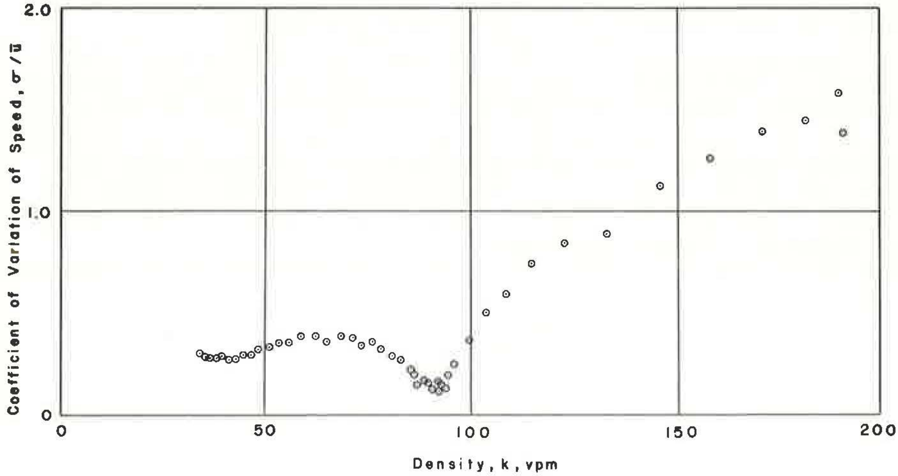


Figure 9. Coefficient of variation of speed versus density for platoon studied.



where

$$\bar{\mu} = \sum_{i=1}^{n} \mu_i/n = \text{average speed},$$

μ_i = speed of ith vehicle in a traffic stream, and
n = number of vehicles.

We can see that, when (a) k = o, $\mu_i \rightarrow \mu_f$ and $\bar{\mu} \rightarrow \mu_f$, and thus CV_u approaches zero; and that when (b) k = k_j, $\mu_i \rightarrow 0$, and $\bar{\mu} \rightarrow 0$, and thus CV_u approaches maximum when $k \rightarrow k_j$. From this analysis, it is evident that the boundary conditions have been satisfied. An interesting point to be noted here is that, when every vehicle is moving at the same speed u_i (in this case $\mu_i = \bar{\mu}$), CV_u approaches zero also.

From the evidence presented, it seems that the coefficient of variation of speed is an excellent choice for measuring the internal energy of traffic flows.

CONCLUSIONS

From the analyses relating to traffic energy presented in this paper the following general conclusions may be drawn:

1. If the kinetic energy of a traffic stream is defined as αku^2 and the internal energy is defined in terms of vehicular interactions, the principle of conservation of energy does not hold. In fact, it will not hold regardless of how internal energy is defined so long as kinetic energy is taken to be αku^2 because we are not dealing with a confined system.
2. Acceleration noise does not represent a good indication of internal energy throughout the entire density domain.
3. If traffic flow is taken to be exactly analogous to compressible fluid flow, internal energy can be expressed as $P_i(k_i/k_o - 1)$ for the ith traffic state. If the analogy is only approximately correct, as seems logical, the term $P_i(k_i/k_o - 1)$ serves as an approximation of the true internal energy.
4. Of the four alternative internal energy parameters studied, only the coefficient of variation of speed fulfilled all the requirements postulated for the desired parameter. It is, therefore, proposed as a suitable measure of the internal energy of a traffic stream.

It is thought that the material contained in this paper represents a further step toward the attainment of an understanding of the dynamics involved in traffic movement. Such an understanding is a necessary prerequisite to the establishment of a safe and efficient highway system and forms a basis for determining control strategies for that system.

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