MULTIPLE RAMP CONTROL FOR A FREEWAY BOTTLENECK

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Many current approaches to freeway control use deterministic models of traffic flow based on the continuous flow-density curve. This paper proposes a control strategy based on a two-state traffic flow pattern with the primary control parameter being the probability of transition from uncongested flow to congested flow. The objective of the control is to maximize the reward associated with free flow. Trial solutions indicate that feasible numerical values for optimum control can be easily obtained, and these will be dependent on the length of the peak period. The approach should have direct applicability to existing surveillance and control hardware.

BOTTLENECK, which is the primary cause of congestion on a limited-access highway, is a term defining some operational constriction. It is usually identified with a local area rather than a precise point of the highway. Physical bottlenecks are related to the design features of the highway and are fixed in space, and dynamic bottlenecks are related to traffic incidents and can occur at any location. Regardless of the type, all bottlenecks have a disruptive effect on traffic, which will be some combination of increased accident potential, reduced traffic volume, and detrimental environmental effects. Bottleneck control, simplistically overstated, contends that more traffic can be served at a higher level of service if congestion is eliminated. The control concept is to sustain the best operational level and, by preventing congestion, to yield benefits in increased safety and reduced delay.

Many control methodologies, theoretical and applied, have been based on the traditional flow-density relationship (1), which suggests a point of maximum flow (capacity). Initial controls in the Lincoln Tunnel in New York (2) and the Eisenhower Expressway experiments in Chicago (3) were based on the assumption that traffic could be controlled to this maximum flow condition. This maximum flow point, however, turned out to be very sensitive to breakdown, and it could not be maintained in practice without the rapid onset of congestion. Accordingly, most strategies have backed off from the theoretical ideal of maximum flow, and the emphasis is now on delaying or preventing congestion.

New York used density as the control parameter, whereas Chicago used the directly measurable equivalent, occupancy (4). Experimental work on the Gulf Freeway in Houston (5) combined parameters in various functions of volume, speed, and density. The deterministic approaches to these systems required ongoing empirical refinement of their control functions to balance the risk of congestion against higher allowable flows.

PEAK-PERIOD BOTTLENECK CONTROL

Early literature on freeway characteristics alludes to traffic operation as a two-state process. Mika, Kreer, and Yuan (5) identified two modes of operation corresponding to congested and uncongested flow on a freeway. Refined measurements of flow and density indicate that the q-k curve masks the underlying traffic process. The curve is a regression fitted to historical data and, as such, does not necessarily provide a suitable model for real-time control. In particular, the curve does not model operating differences from day to day caused by weather or short-term variations caused by individual driver characteristics.

A pilot control scheme that has been tested in Chicago (6) modifies some of the previous approaches. The control is aimed at peak-period flow, and its primary objective is to delay the onset of congestion by limiting bottleneck flow. During any time period, which in this case is 1 min, the probability of congestion setting in, i.e., the probability of breakdown, is assumed to be a function of flow and density in the bottleneck. Given a suitable probability and the bottleneck density, the controller sets the desired bottleneck flow for each time period. The ramps upstream of the bottleneck are then metered to achieve the appropriate bottleneck flow. In the Chicago experiment the probability of breakdown and its functional relationships were heuristically determined and then empirically refined from freeway data.

In the following sections, a more analytical approach is suggested. This approach involves techniques that should be within the capability of current controllers and uses functions that, although not yet empirically validated, will require only currently available freeway data for their estimation.

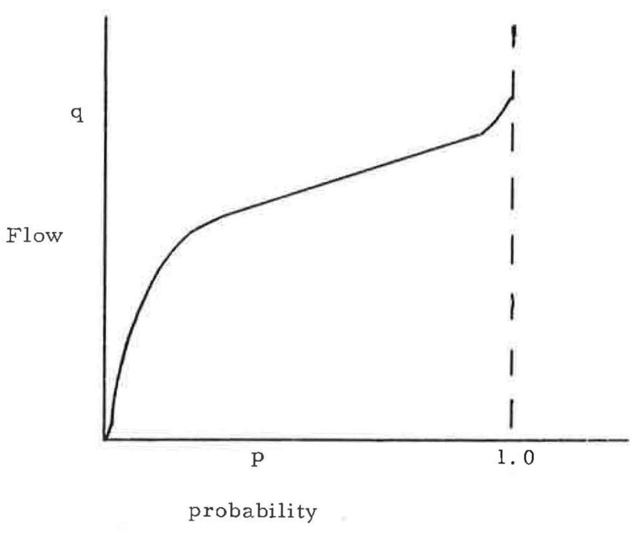
PROBLEM FORMULATION

Bottleneck operation is formulated as a process in which finite probabilities of breakdown are associated with each level of operation. The control stragegy considers the peak period as a series of successive time intervals where the probability of breakdown is set to optimize overall performance. It is assumed that, once congestion has set in, recovery during the peak period cannot be effected, a characteristic common in many practical situations.

The peak-period operation of the bottleneck consists of an uncongested period of some length followed by a congested period. If some reward is associated with the uncongested period, then the control objective would be to maximize the expected reward of the system. The reward, which can be some combination of increased flow, reduced accident risk, reduced emissions, and the like, will be some function of the probability of breakdown and the length of the uncongested period.

The reward function considered here will be in terms of traffic flow, and the objective considered for this problem is that of maximizing the expected value of the uncongested peak-period flow. The exact form of the reward function will require field testing and estimation. Its general characteristics, however, can be deduced from operational bottleneck experience and should follow the approximate form shown in Figure 1. Initially the function will rise rapidly to a substantial traffic flow before the probability of breakdown becomes significant, but then the rate of flow increase will decline. Because only uncongested traffic flow is being considered, the function will be continuous and for this particular problem only the left region ($p < 0.25$) is significant. This will simplify empirical validation.

Figure 1. Flow at a bottleneck as a function of the probability of breakdown.



For a suitable function to fit this general shape we use here the incomplete beta function:

$$R(p) = \frac{\gamma(a+b)}{\gamma(a)\gamma(b)} \int_0^p t^{a-1}(1-t)^{b-1}dt \tag{1}$$

where

$0 \le p < 1$,
a and $b > 0$, and
$\gamma(x)$ = the gamma function.

AN INFINITE PEAK PERIOD

Consider a controlled bottleneck where the probability of breakdown is set equal to p for each time period during uncongested flow. Then the probability $P(k)$ of an uncongested period of k time periods will be given by

$$P(k) = (1-p)^k \times p \qquad k = 0, 1, 2 \ldots \tag{2}$$

Because we are dealing with traffic at the macro level (1-min averages) and not the micro level of individual vehicles, the assumption of independence of trials is valid.

The expected return $E(R)$ will be

$$E(R) = \sum_{k=0}^{\infty} (1-p)^k p \times R(k, p) \tag{3}$$

The optimal control strategy will be to choose p to maximize the return.

$$\frac{dE(R)}{dp} = \frac{d}{dp} \sum_{k=0}^{\infty} (1-p)^k p \times R(k, p) = 0 \tag{4}$$

for optimum and

$$\frac{d^2E(R)}{dp^2} = \frac{d^2}{dp^2} \sum_{k=0}^{\infty} (1-p)^k p \times R(k, p) < 0 \tag{5}$$

for maximum.

To check the feasibility of the reward function developed in the previous section we assume a form

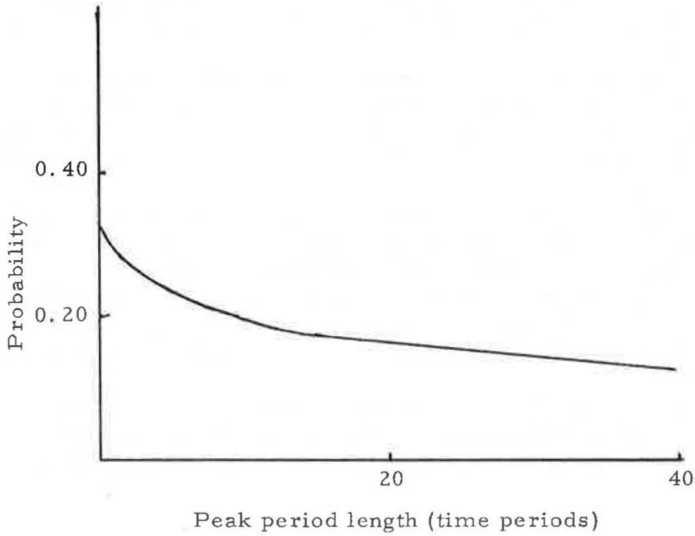
$$R(k, p) = k \frac{\gamma(a+b)}{\gamma(a)\gamma(b)} \int_0^p t^{a-1}(1-t)^{b-1}dt \tag{6}$$

and the optimal strategy would be given by

$$\frac{d}{dp} \left\{ \sum_{k=0}^{\infty} \left[(1-p)^k \times p \times k \frac{\gamma(a+b)}{\gamma(a)\gamma(b)} \int_0^p t^{a-1}(1-t)^{b-1}df \right] \right\} = 0 \tag{7}$$

For values of $a = 2$ and $b = 9$, this gave a numerical solution of $p = 0.12$. Although this is a hypothetical example, the solution is in a practical range although perhaps rather high inasmuch as it gives an expected length of the uncongested period of only 7.5 time periods. Values of p below 0.1 would give somewhat better results.

Figure 2. Optimum probability of breakdown for a finite peak period.



A FINITE PEAK PERIOD

Suppose now that the peak period has a finite length of n time periods. Then the probability of an uncongested period of k time periods is given by

$$P(k) = (1 - p)^k p \qquad k = 0, 1, \ldots (n - 1)$$

$$P(n) = (1 - p)^n$$

The expected reward during the uncongested period will be

$$E(R) = \sum_{k=0}^{n-1} \left[(1 - p)^k pR(k, p) \right] + (1 - p)^n R(n, p)$$

And, again, setting $[dE(R)]/dp = 0$ will give the optimum control value for the probability of breakdown. In this case, however, this optimum value will be a function of n.

This affects the bottleneck control strategy in at least two ways. First, if uncongested flow has been continuously maintained during the peak period, then the control stragegy at any time period is dependent only on the length of the peak period remaining. As the peak period progresses, therefore, the value of n steadily declines, and, accordingly, the parameter values for the control may change.

The second circumstance is when uncongested flow is recovered from the congested state during the peak period, which can occur, for example, when demand temporarily declines because of an incident upstream. The control strategy for uncongested flow will then depend on the length of the peak period remaining.

To indicate the possible magnitude of this dependence, we used the same reward function used as an example in the previous section as a numerical example of the finite length case. The optimum values for p are shown in Figure 2, which clearly indicates the effect of the peak period length n.

CONCLUSION

This paper offers a different analytical approach to freeway control. Inasmuch as it provides a limited theoretical supplement to the empirical control algorithms already in operation, it should be suitable for practical implementation. The essential traffic functions required, such as the probability of system breakdown, are not yet generally available; their estimation, however, will require only data that are a normal output of most current freeway surveillance systems.

The methodology can be used on existing freeway control systems by changing the computer programming but without modification to the hardware. New operational parameters can be developed from the system itself based on a new datum of controlled bottleneck operations. Data from an uncontrolled bottleneck serve as the first approximation in developing the control strategy.

To sustain congestion-free bottleneck operations for longer control periods requires that the optimal value of the probability of breakdown be maintained considerably below 0.1 (assuming 1-min time periods). This finding is contrary to the control strategy of operating at maximal flow developed by most theoretical studies but agrees with operational experience where "overcontrol" is necessary to prevent breakdown. The explanation lies with the q-k relationship, its probable discontinuous character, and its nonregular short-term behavior.

For a normal peak period, the length of the peak period has little effect on the selection of the optimum probability. Where the control is operating near the end of the peak period, however, the length of the peak period remaining should be taken into consideration.

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