COMPUTERIZED SLOPE STABILITY ANALYSIS: THE SLIDING BLOCK

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This paper presents a computer program for a common type of analysis of the slope stability problem: the possibility of slope failure by translation of a massive block along a weak layer of soil. The problem, which can occur in either natural or man-made slopes, is most generally referred to as the sliding block problem. Variation in the water surface position requires three subroutines or cases. The program automatically sequences selected potential sliding surfaces one by one, then selects the desired water surface case, and finally computes the factor of safety against sliding along the base of the central block. The analysis is based on total unit weights and boundary forces. It is possible to consider 10 different soil types having very different soil parameters, such as unit weight, Mohr-Coulomb cohesion intercept, and Mohr-Coulomb angle of friction. A maximum of 12 continuous soil layers at any inclination can be considered in the present program. A total of 10 vertical strip loads of different intensities can be placed on the ground surface anywhere below the toe and above the crest. Finally, with all this information, 10 sliding surfaces can be concurrently analyzed for the factor of safety. This factor is applied to the strength of the soil at the base of the central block, assuming that there is limiting equilibrium for the active and passive earth pressure forces at the ends of the central block.

•THE stability of man-made and natural slopes has always been an important topic of discussion in the field of civil engineering. Yet, failure of man-made fills and cuts probably occurs more frequently than all other failures of civil engineering structures combined. Although an understanding of the major factors that contribute to failure of slopes has improved considerably, our predictive ability remains less than satisfactory.

This paper addresses the problem of the sliding block, i.e., an essentially rigid mass sliding in a weak layer. At first glance, this seems to be a rather simple problem; however, when practical variations in soil profile are considered, as well as water levels, boundary geometries and loadings, and uncertainties of position and shape of the most critical sliding surface, the solutions require reasonably large computer systems.

When a slope is underlain by one or more strata of very soft or loose materials, the most critical sliding surface may not be even approximately circular, as shown in Figure 1. Rather there is a three-plane surface of potential sliding in which a maximum amount of the surface lies within the weak material.

An initial programmed solution (11) was quite general with respect to the shape of the three-plane surface, but to accommodate this feature the profile was simplified to two soil layers, i.e., a strong soil over a weak one. A second program, reported in this paper, makes simplifying assumptions with respect to the shape of the sliding surface but is quite versatile with respect to the profile and boundaries. This second

Publication of this paper sponsored by Committee on Embankments and Earth Slopes.

program seems to better meet the analytical requirements of the Indiana State Highway Commission.

SLOPE FAILURE BY SLIDING

The type of failure usually assumed in slope stability analysis is the one-piece slide (10). The failure is one in which the moving body is essentially rigid and the failing mass is separated from the unmoved one by a surface of assumed shape. Where the soil is grossly homogeneous, it seems logical that the failure surface would be roughly circular, and, in the interest of simplicity, it is usually made exactly so. A recent overview of the circular analysis, involving the well-known methods of slices, is given elsewhere (4).

Where there is evidence of definite differences in shearing resistance in the soil profile, it is well to consider potential failure surfaces that follow the surfaces of weakness. Several methods of handling irregular surfaces are reported elsewhere (4, 12, 14).

The irregular sliding surface is shown in Figure 1, where the potential failure planes have a maximum length in the weaker materials. The potential failing block is actually a combination of active and passive wedges, with a central trapezoidal block based in a weak layer. Examples of simplified solutions to this problem are given elsewhere (5, 6, 11).

GENERAL SOLUTION TO THE SLIDING BLOCK PROBLEM

Figure 2 shows the free body diagram with a full quota of complexities in boundary geometries and forces; i.e., these could be simpler in a given instance. Incorporation of a water surface and associated water forces into the problem makes it convenient to consider three cases, each with its appropriate subroutine in the computer solution. The upper boundary slopes reading left to right in Figure 2 are referred to as the down slope and the middle slope or simply the slope and the upper slope. The cases are as follows:

1. Case 1—when the water surface is below the trial sliding surface,
2. Case 2—when the water surface is partly above and below the ground surface but above the trial sliding surface, and
3. Case 3—when the water surface is anywhere below the ground surface but above the trial sliding surface.

It is assumed that the right-hand wedges are in a state of limiting active earth pressure, and the left-hand wedges are in a state of limiting passive earth pressures. Simplifying assumptions are employed with respect to the inclinations of the wedge surfaces and the directions of the earth pressure forces. Although the right-hand and left-hand wedges are assumed to be on the verge of sliding, there is in general an incomplete mobilization of the shearing resistance along the base of the block; i.e., the factor of safety is defined with respect to the shearing resistance-shearing force ratio along this surface.

The wedge inclination and earth pressure force direction assumptions are those that apply for a simple Rankine case. They are employed by others (5) and have been shown to be good approximations of the most critical values for a number of cases tested by Mendez (11).

To be certain that all assumptions inherent in the solution are understood, we are listing them as follows:

1. Problem is two-dimensional;
2. The ground surface is defined by three slopes and a well-defined toe and crest;
3. Soil strata are laterally continuous;
4. Soil properties in layers are defined by γ, c, and ϕ (where c or ϕ can be equal to zero);
5. Sliding surface is at the base of the block, and between the slide wedges is a plane;

6. All lateral forces on vertical wedge boundaries are normal to these boundaries (i.e., there are no shear forces on these boundaries);

7. The factor of safety is figured for the base of the sliding block only, and the movement required to mobilize limiting active and passive pressures is smaller than the movement required to mobilize the shearing strength of the weak soil strata;

8. The wedge slip surfaces are at $45 + \phi/2$ and $45 - \phi/2$ with the horizontal for active and passive wedges respectively;

9. The active and passive forces are computed by satisfying static equilibrium (after verifying assumptions 6 and 8); and

10. Seepage, if any, is in a steady state; however, water pressures are calculated at any point as if they were hydrostatic.

The analysis of forces is shown in Figures 2, 3, and 4. The analysis is divided into three parts: calculation of forces on central block due to active wedge, calculation of forces on central block due to passive wedge, and calculation of base forces on central block and of the factor of safety against sliding along this base.

The analysis of forces is illustrated for water surface case 2, but the other cases follow directly.

Figure 2 shows a rather complex problem space section, with multiple soil layers at variable inclinations and with very different soil properties.

Analysis of Active Forces on Central Block

Figure 3 shows the active wedge (Fig. 2) divided into small wedges governed by the intersection of the assumed slip surface and soil boundaries.

Let us consider a typical polygon of forces for any (n th) wedge shown in Figure 3. Summing all the forces in the x- and y-directions and equating to zero yield the following equations:

For $\Sigma F_x = 0$,

$$\begin{aligned} PAn = {} & UARn - UALn - UAn \cos(45 - \phi n/2) + NA'n \cos(45 + \phi n/2) \\ & - CAn \cos(45 + \phi n/2) \end{aligned} \tag{1}$$

and for $\Sigma F_y = 0$,

$$WAn = CAn \sin(45 + \phi n/2) + UAn \sin(45 - \phi n/2) + NA'n \sin(45 + \phi n/2) \tag{2}$$

Elimination of NA′n from Eqs. 1 and 2 yields an expression for the incremental active force for the n th wedge,

$$\begin{aligned} PAn = {} & WAn \tan(45 - \phi n/2) - 2\,CAn \cos(45 + \phi n/2) \\ & + (UARn - UALn) + UAn\,[\cos(45 - \phi n/2) \\ & - \tan(45 - \phi n/2) \sin(45 - \phi n/2)] \end{aligned} \tag{3}$$

Analysis of Passive Forces on Central Block

Figure 4 shows the forces acting on the passive wedge shown in Figure 2. Consider a typical polygon of forces acting for an n th passive wedge in Figure 4. Forces in the x- and y-directions are summed, and equilibrium is equated to zero.

For $\Sigma F_x = 0$,

$$\begin{aligned} PPn = {} & U\beta_n \sin\beta_1 + CP_n \cos(45 - \phi n/2) + ULn + UPn \cos(45 + \phi n/2) \\ & - URn + NP'n \cos(45 - \phi n/2) \end{aligned} \tag{4}$$

and for $\Sigma F_y = 0$

$$\begin{aligned} WPn = {} & NP'n \sin(45 - \phi n/2) - U\beta_n \cos\beta_1 - CPn \sin(45 - \phi n/2) \\ & + UPn \sin(45 + \phi n/2) \end{aligned} \tag{5}$$

Figure 1. Slope in stratified soil profile.

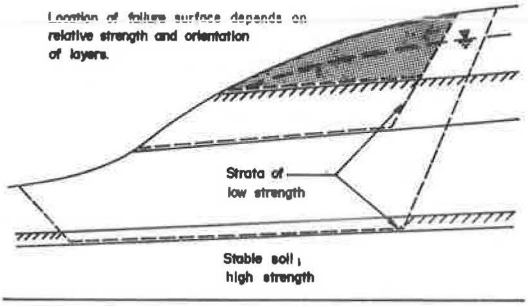


Figure 2. General problem of sliding block with submerged water (case 2).

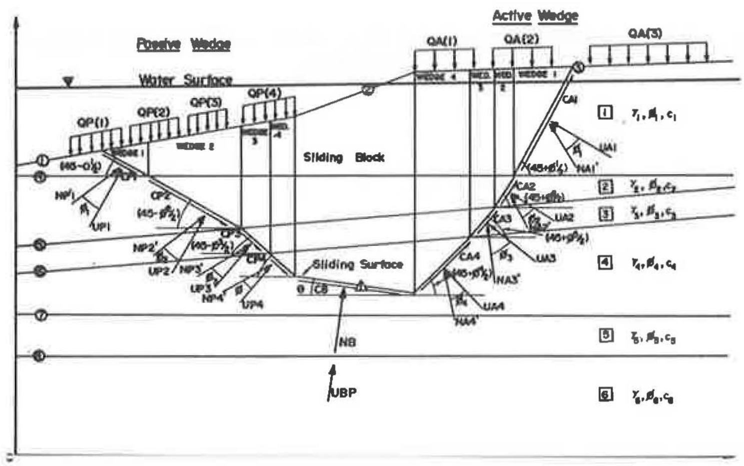
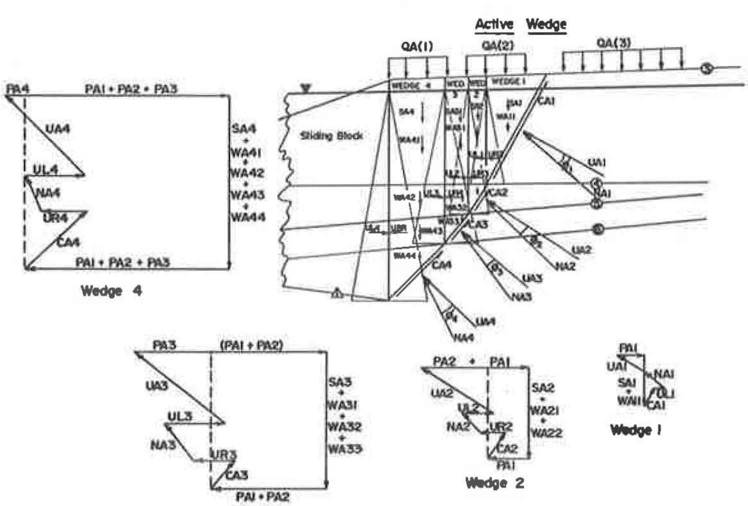


Figure 3. Analysis of forces on active wedge (case 2).



$V = 196.25$ metric tons $= 196{,}250$ kg. The influence values S for $m = 3.00$ are taken from Figures 1 and 2 and are given in column 6 of Table 1. The settlements as successive displacement differences,

$$\Delta s_n = \left(\frac{V}{E \times R_o}\right)(S_n - S_{n-1}) \tag{15}$$

for each layer are given in column 9 of Table 1. By stepwise calculations, the total settlement s of this soil-foundation-load system is found as the sum of the settlements of each individual layer:

$$s = \sum_{1}^{4}(\Delta s_n) = 2.41 \text{ cm}$$

Method of Equivalent Layers

Because in the multilayered soil system each layer may be of different homogeneity with large differences in elastic characteristics, use of these charts (Figs. 1, 2, and 3), which are prepared for a homogeneous monolayer, to make $\bar{\sigma}_z$ stress and settlement s calculations necessitates that the stratified, multilayered soil system be converted or homogenized into an equivalent (fictitious), homogeneous hemispace. The homogenization is accomplished by the so-called method of equivalent layers h_e. This method works with fictitious substitute heights h_e for each thickness $h_1, h_2, h_3, \ldots, h_n$ of the real strata. The principle involved in this method is to determine an ideal, equivalent thickness h_e of a uniform, homogeneous soil column (or beam) that, upon loading, will bring about a deflection equal to the sum of the deflections (viz., settlements) of each of the strata in the multilayered system. Thus, the charts here developed are directly applicable to such an equivalent, homogeneous hemispace, or, in other words, the charts are also indirectly applicable as an approximation of multilayered soil systems.

The homogenization of multiple layers into a single, ideal, equivalent homogeneous monolayer is based on the principle that two layers of various thickness with differing moduli of elasticity are equivalent when these two layers are of the same stiffness, i.e., when

$$E_1 \times I_1 = E_2 \times I_2 \tag{16}$$

or

$$\frac{E_1 \times h_1^3}{(12)\left(\frac{m_1^2 - 1}{m_1^2}\right)} = \frac{E_2 \times h_2^3}{(12)\left(\frac{m_2^2 - 1}{m_2^2}\right)} \tag{17}$$

where

h_1 and h_2 = heights of a column of soil, namely, thickness of soil layers, and
m_1 and m_2 = Poisson's numbers for layer one and layer two respectively.

If it can be assumed that $m_1 \approx m_2$ for all courses, then the equivalent height h_e for a two-layered system calculates as

$$h_2 = h_e = h_1 \sqrt[3]{\frac{E_1}{E_2}} \tag{18}$$

Thus, it is here assumed that the rigid circular foundation and the individual layers form a compound unit. Upon deformation, a two-layered soil system, it is assumed, would deflect uniformly and retain its unity. When $\sqrt[3]{E_1/E_2} = 1$, we have a uniform mass—a monolayer.

Figure 4. Self-settlement influence values (SO) for rigid circular foundation.

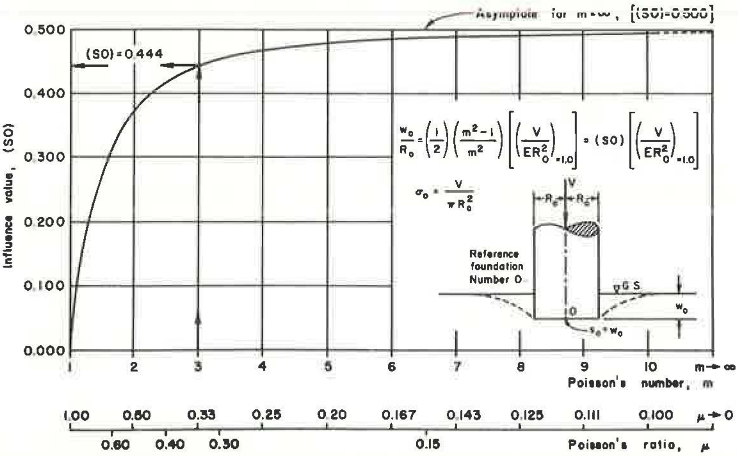


Table 1. Calculation of settlement.

Number of Layers (1)	Soil Material (2)	Thickness of Each Layer (m) (3)	Depth to Bottom of Each Layer (m) (4)	Relative Depth (5)	Influence Value (6)	Modulus of Elasticity of Each Layer (kg/cm²) (7)	Load Factor (cm) (8)	Successive Settlement Difference for Each Layer, $\Delta s_n = (8)(S_n - S_{n-1})$ (9)
1	Sand	3.0	3.0	$z_1/R_o = 3/2.50 = 1.20$	$s_1 = 0.14351$	$E_1 = 150$	$\frac{785}{150} = 5.233$	$\Delta s_1 = (5.233)(0.14351) = 0.75$
2	Silt	2.0	5.0	$z_2/R_o = 5/2.50 = 2.00$	$s_2 = 0.22838 - \frac{0.14351}{0.08487}$	$E_2 = 120$	$\frac{785}{120} = 6.542$	$\Delta s_2 = (6.541)(0.22838 - 0.14351) = (6.541)(0.08487) = 0.56$
3	Silty clay	4.0	9.0	$z_3/R_o = 9/2.50 = 3.60$	$s_3 = 0.31306 - \frac{0.22838}{0.08468}$	$E_3 = 100$	$\frac{785}{100} = 7.8500$	$\Delta s_3 = (7.850)(0.31306 - 0.22838) = (7.850)(0.08468) = 0.66$
4	Clay	5.0	14.0	$z_4/R_o = 14/2.50 = 5.60$	$s_4 = 0.35772 - \frac{0.31306}{0.04466}$	$E_4 = 80$	$\frac{785}{80} = 9.8125$	$\Delta s_4 = (9.8125)(0.35772 - 0.31306) = (9.813)(0.04466) = 0.44$
5	Gravel	—	—	—	—	$E_5 = \infty$	—	—

Figure 2. Settlement influence-value chart for rigid circular foundations.

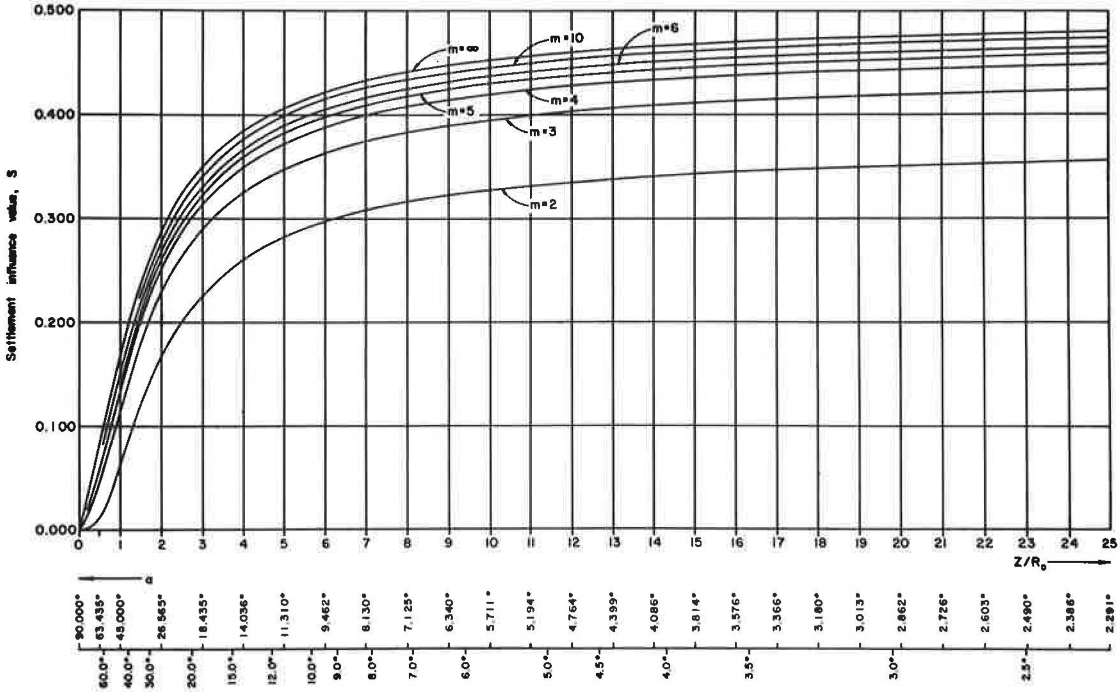
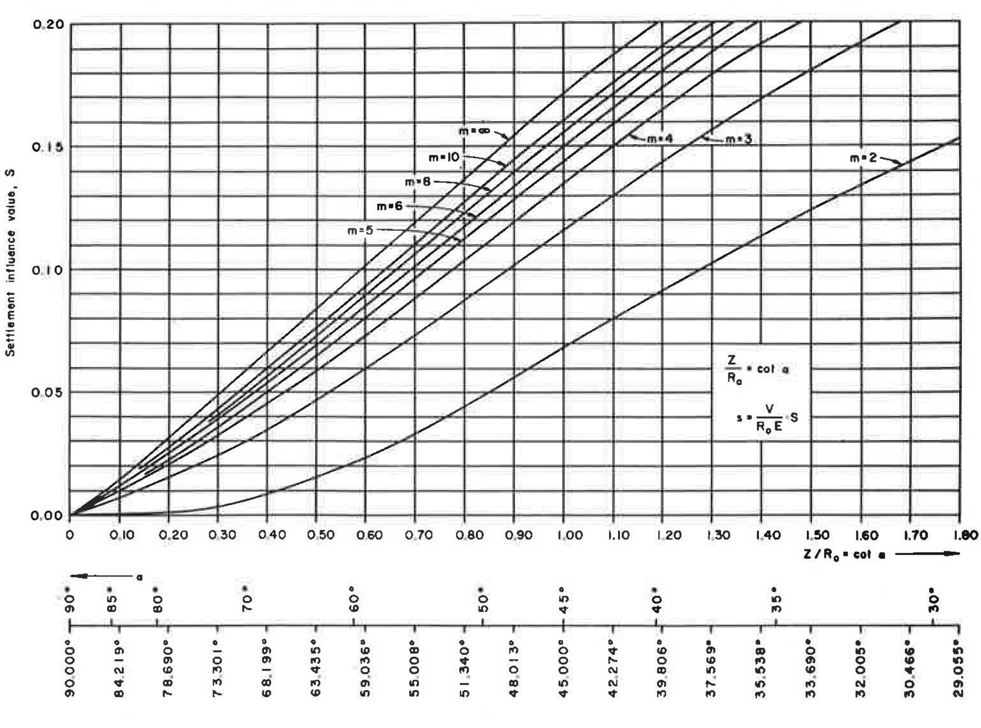


Figure 3. Enlargement of Figure 2 for z/R_o ratios.



Settlement s may be obtained by multiplying the corresponding settlement influence value S by the factor $V/(E \times R_o)$:

$$s = (S)\left(\frac{V}{E \times R_o^2}\right)(R_o) = (S)\left(\frac{V}{E \times R_o}\right) \tag{12}$$

Settlement Influence-Value Charts

The dimensionless influence values were programmed for computer calculations and compiled in tabular form for various Poisson's numbers (13). By means of such tables, the settlement influence-value charts (Figs. 1, 2, and 3) were prepared for quick, effective, and practical use. Figure 3 is an enlargement of Figure 2 for z/R_o ratios from 0 to 1.80 (for settlement influence values from S = 0.00 to S = 0.20). For the purpose of comparison, in Figure 1 there is also shown for m = 2 the vertical σ_z stress influence-value curve $i_z = \sigma_z/\sigma_o$ for limply arranged single uniform stresses (σ_o) over a circular area on a homogeneous, hemispatial medium.

For a hemispace of infinite extent, $z = \infty$ and $\alpha = 0$; thus the total settlement [elastic settlement according to Boussinesq's theory of elasticity (9, 12-16)] is

$$s_o = w_o = \left(\frac{1}{2}\right)\left(\frac{m^2 - 1}{m^2}\right)\left(\frac{V}{E \times R_o}\right) \tag{13}$$

or, in terms of influence value,

$$\frac{w_o}{R_o} = \left(\frac{1}{2}\right)\left(\frac{m^2 - 1}{m^2}\right)\left(\frac{V}{E \times R_o^2}\right) = (SO)\left(\frac{V}{E \times R_o^2}\right) \tag{14}$$

Here (SO) is the influence value of the so-called self-settlement $s_o = w_o$ of the circular rigid foundation (Fig. 4). If, for example, m = 3.0, E = 120 kg/cm^2, V = 196,250 kg, and R_o = 2.50 m, then the self-settlement $s_o = w_o$ of the rigid circular foundation (which is the total vertical displacement in the z = 0 plane) calculates (Fig. 4) as

$$s_o = w_o = (SO)\left(\frac{V}{E \times R_o}\right) = (0.444)\left[\frac{196{,}250}{(120)\ (250)}\right] = (0.444)\ (6.541) = 2.90 \text{ cm}$$

MULTILAYERED SOIL SYSTEM

Stress distribution and settlement in a multilayered soil system differ considerably from those in a uniform, massive (infinitely thick), homogeneous monolayer only in cases where there exists a sharp difference in elasticity characteristics of the various individual deformable component layers in the multilayered soil. If the differences are small, then the influence-value chart renders a reasonably satisfactory approximation for determining settlement of such multilayered soil systems.

Example for Use of the Settlement-Influence Chart

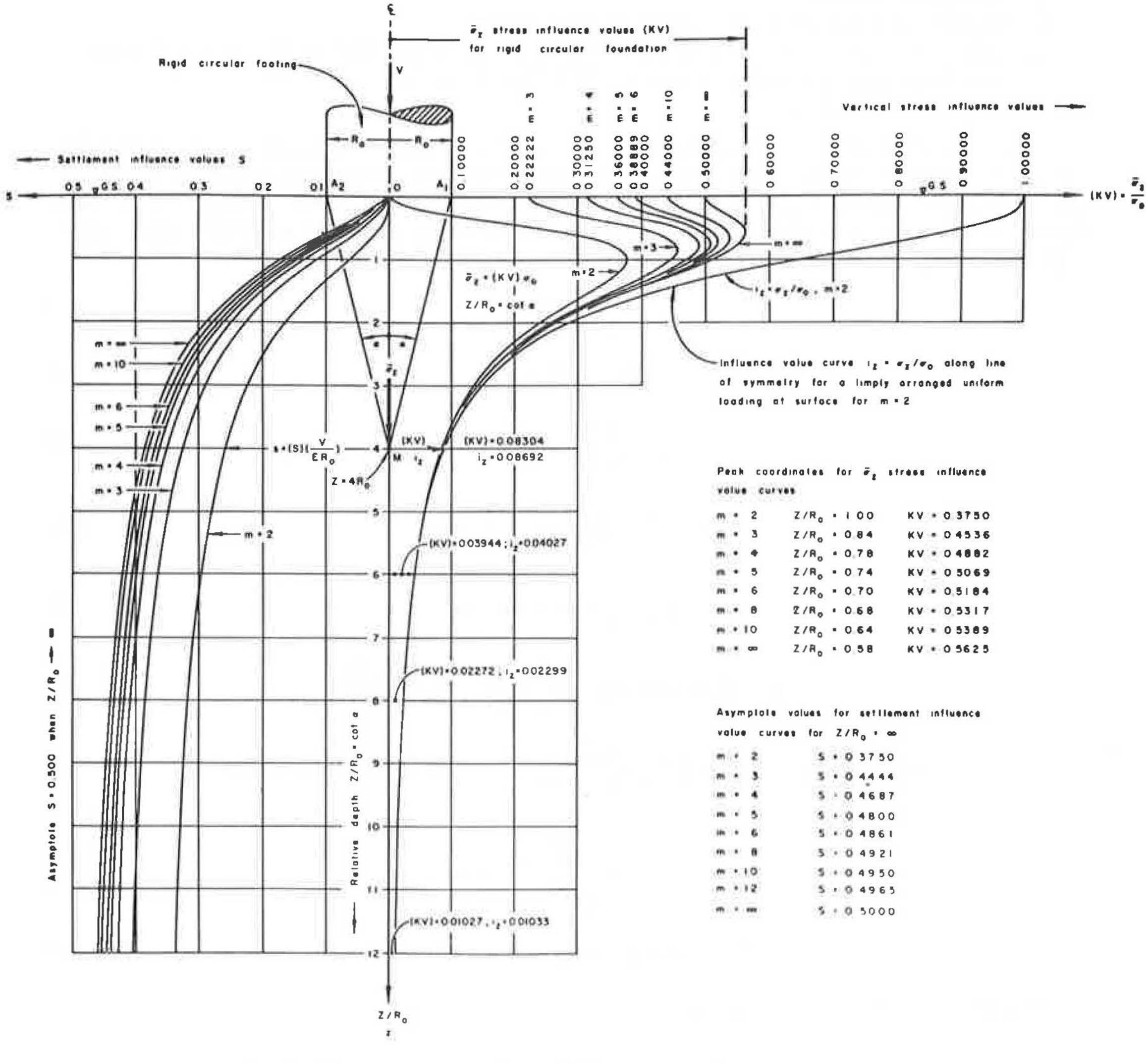
The influence chart may also be used for calculating approximative total elastic settlement of a multilayered soil system. There exist two principal methods for doing this, namely,

1. The method of successive displacement-difference steps where the total settlement is obtained by adding the so-called partial settlements (i.e., settlements of each layer in the multilayered soil system), and
2. The method of equivalent layers.

Successive Settlement-Difference Step Method

A multilayered soil system consisting of four layers of compressible soil over a gravel (Table 1) is given here. The radius of the rigid circular foundation is R_o = 2.50 and m = 2.50 cm. The magnitude of the central-symmetrical load is given here as

Figure 1. Vertical stress and settlement influence-value charts.



Peak coordinates for $\bar{\sigma}_z$ stress influence value curves

m = 2	Z/R_0 = 1.00	KV = 0.3750
m = 3	Z/R_0 = 0.84	KV = 0.4536
m = 4	Z/R_0 = 0.78	KV = 0.4882
m = 5	Z/R_0 = 0.74	KV = 0.5069
m = 6	Z/R_0 = 0.70	KV = 0.5184
m = 8	Z/R_0 = 0.68	KV = 0.5317
m = 10	Z/R_0 = 0.64	KV = 0.5389
m = ∞	Z/R_0 = 0.58	KV = 0.5625

Asymptote values for settlement influence value curves for $Z/R_0 = \infty$

m = 2	S = 0.3750
m = 3	S = 0.4444
m = 4	S = 0.4687
m = 5	S = 0.4800
m = 6	S = 0.4861
m = 8	S = 0.4921
m = 10	S = 0.4950
m = 12	S = 0.4965
m = ∞	S = 0.5000

m = $1/\mu$ = Poisson's number;
μ = Poisson's ratio;
$\bar{\sigma}_z$ = spatial (triaxial) vertical stress (Eqs. 8-14);
α = arc cot (z/R_o) = one-half of the central angle at point M on the vertical centerline beneath the center of the circle (Fig. 1); and
z = depth coordinate.

The derivation of the settlement influence values S can be developed using the cylindrical coordinate system: Boussinesq's contact pressure distribution under a central symmetrically loaded rigid circular foundation with a smooth base, laying of the foundation on the ground surface, and use of the following system of equations (especially Eqs. 8 through 14):

$$\sigma_{zo} = \frac{V}{2\pi R_o (R_o^2 - x^2)^{1/2}} \tag{1}$$

$$V = \pi R_o^2 \times \sigma_o \tag{2}$$

$$\epsilon_z = \frac{1}{E}\left(\sigma_z - \frac{2}{m} \times \sigma_x\right) \tag{3}$$

$$\epsilon_z \times E = \bar{\sigma}_z = \sigma_z - \frac{2}{m} \times \sigma_x \tag{3a}$$

$$\sigma_z = \frac{1}{2} \times \sigma_o \times \sin^2 \alpha \tag{4}$$

$$\sigma_x = \frac{1}{4} \times \sigma_o \times \sin^2 \alpha \left(\frac{m+2}{m} - 2\cos^2 \alpha\right) \tag{5}$$

$$\bar{\sigma}_z = \frac{1}{2} \times \sigma_o \times \sin^2 \alpha \left(\frac{m+1}{m}\right)\left(\frac{m-2}{m} + 2\cos^2 \alpha\right) \quad \text{(Eqs. 8-14)} \tag{6}$$

$$s = \frac{1}{E}\int_0^z \bar{\sigma}_z \times d_z \tag{7}$$

$$z/R_o = \cot \alpha, \text{ or } \alpha = \text{arc cot}\,(z/R_o) \tag{8}$$

The settlement s calculated as

$$s = \left(\frac{1}{2\pi}\right)\left(\frac{m+1}{m}\right)\left[2\left(\frac{m-1}{m}\right)\left(\frac{\pi}{2} - \alpha\right) - \sin\alpha\cos\alpha\right]\left(\frac{V}{E \times R_o}\right) \tag{9}$$

Settlement Influence-Value Equation

The settlement equation (Eq. 9) can be rewritten in a dimensionless form as

$$\frac{s}{R_o} = \underbrace{\left(\frac{1}{2\pi}\right)\left(\frac{m+1}{m}\right)\left[2\left(\frac{m-1}{m}\right)\left(\frac{\pi}{2} - \alpha\right) - \sin\alpha \times \cos\alpha\right]}_{S}\left(\frac{V}{E \times R_o^2}\right) \tag{10}$$

or

$$\frac{s}{R_o} = (S)\left(\frac{V}{E \times R_o^2}\right) \tag{11}$$

where S is the elastic settlement influence value for a homogeneous, elastic hemispace of infinite depth (a monolayer).

1. Boussinesq's theory of elasticity is applicable.
2. Particularly, the foundation-supporting monolayer of soil is an elastic, homogeneous, isotropic, weightless, linearly deformable solid of semi-infinite extent. It obeys Hooke's law of proportionality between stress and strain. Hence, stresses are compatible with strains.
3. The modulus of elasticity E of the hemispatial material is constant throughout.
4. Originally, before loading, the soil is free of stress caused by force fields or thermal effects.
5. The circular footing laid on the ground surface is assumed to be a completely rigid, nondeformable body as compared with the rigidity of the soil—a situation that is frequently encountered in practice.
6. Cohesive and frictional forces between soil and foundation are disregarded in the development of this chart although slips or horizontal displacements can occur along the contact surface between the base of the footing and the soil. Only vertical displacements are reckoned with. In practice, these assumptions are used in the case of many uniform soils and many stress-strain problems in soil mechanics. Hence, they can also be used for elastic settlement calculations.

Relative to theoretical settlement calculations in multilayered soil systems, the following further assumptions are made:

1. The individual, horizontal soil layers are weightless and of infinite lateral extent.
2. Each individual soil layer in the multilayered system has its own elastic properties and is of perfect homogeneity and isotropy.
3. The stress distribution used is that predicted by Boussinesq's theory for a homogeneous half-space; the varying moduli of each layer are assumed not to influence the stress distribution.
4. Also, in this study, no consideration is given to the drainage (filtration) and rate problems as in a consolidating soil. Only total, elastic settlements are dealt with.
5. These charts do not apply to eccentrically loaded, rigid circular footings.
6. In these elastic settlement calculations of rigid circular foundations loaded only vertically and centrally and shallowly laid on the ground surface, only the vertical reactive soil resistance at the base of the foundation is considered. Hence, soil lateral resistance against the walls of the foundation does not enter into these calculations. Even if the footing were laid below the ground surface, a shallow embedment would mobilize a soil lateral resistance that would be relatively small as compared with the relative resistance vertically induced by a structural load. Also, if in the future there should arise a need for excavating a part of the embedded foundation and hence loosening the soil around it (for the purpose of repairs, utility installations, or laying of new foundations adjacent to a structure already in service, for example), then the soil lateral surcharge would be removed, and the stability of the soil-foundation system may become impaired.

SETTLEMENT EQUATION

The following notation is used in this paper:

σ_{zo} = Boussinesq's general vertical contact pressure at the base of a rigid circular die;
V = externally applied single, vertical, concentrated load on a rigid circular foundation;
R_o = radius of circle;
x = a coordinate;
$\sigma_o = V/(\pi \times R_o^2)$ = average calculated vertical stress from applied load V;
ϵ_z = elastic, vertical relative deformation (stress) of an element at depth z under a triaxial (spatial) stress condition;
E = modulus of elasticity of soil;
σ_x, σ_z = orthogonal stress components;