# SETTLEMENT INFLUENCE-VALUE CHART FOR RIGID CIRCULAR FOUNDATIONS

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The paper presents a settlement influence-value chart for a homogeneous, elastic medium applicable to rigid circular foundations laid on the ground surface. The chart, which is based on Boussinesq's theory of elasticity pertaining to elastic deformation of an ideal, elastic, hemispatial medium from a rigid surface loading, is suitable for quick assessment of uniform, elastic settlement from a circular symmetrically and statically loaded, rigid circular foundation laid on the boundary surface of a homogeneous, hemispatial medium (monolayer). A numerical example of settlement calculation is used to demonstrate the routine of the easy-to-use chart prepared for Poisson's numbers. It may also be used for evaluation of approximative settlement of a multilayered soil system under a rigid circular foundation. Derivations of the settlement equation, settlement value equation, and settlement value charts are explained. The method of successive "displacement-difference steps" and the method of "equivalent layers," which may be used for calculating the approximate total elastic settlement of a multilayered soil system, are detailed.

•FOR uniform, circular elastic loading there are stress and strain tables available for one-, two-, and three-layered elastic hemispace systems. They are prepared based on the theory of elasticity and are used primarily in highway pavement design. Stress and strain tables have been developed by Burmister  $(\underline{1})$ , Acum and Fox  $(\underline{2})$ , and Jones  $(\underline{3})$ . Vertical, spatial stress distribution tables for uniformly loaded, flexible circular plates on the surface of a homogeneous hemispace, for Poisson's number m=2 and for any point in the elastic medium inside and outside the circular contour, have been published by Jumikis (4,5).

Based on Jones's stress tables, Peattie (6) presented stress-strain factors graphically. The Jones tables give stresses at interface points on a vertical centerline under a uniformly distributed load over a circular area for a three-layered soil system for Poisson's number m=2 and with full friction in layer interfaces for various ratios of the modulus of elasticity, the stress-strain relation being linear.

The flexible solutions are not applicable to settlement calculations of foundations whose footings are rigid relative to the foundation-supporting soil. In such cases the displacement, namely, uniform settlement of a rigid foundation, must be computed on the basis of a specified, uniform vertical displacement over the loaded area.

## ASSUMPTIONS

In deriving formulas and charts for calculating theoretical elastic settlement of a single layer of soil of semi-infinite extent under a rigid circular, shallow foundation or die loaded central symmetrically with a single, vertical, concentrated point load V only, the following assumptions are made:

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6. Although the selection of elastic constants, especially of Poisson's ratio, is difficult, finite-element methods can be used to estimate the rough magnitudes of horizontal displacements;

7. Inclinometer data, especially the rates of horizontal displacements, were suc-

cessfully used to monitor the safety against slope failures;

- 8. The inevitable uncertainties in predicting settlement rates and rates of strength increase may be compensated for by including a bid item for delay time in the contract documents, to be invoked if field data indicate rates slower than the fastest anticipated; and
- 9. With this approach, field instrumentation and interpretation become very important parts of the design.

The results and procedures described here are applicable in general to any major project involving sand drains and surcharge, particularly if a strength increase of the clay is required for stability of fill slopes and thus a programmed surcharge is called for. Specifically, the soil data are representative of many clays in coastal and estuarine areas of the U.S. eastern seaboard.

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Figure 6. Cross section A-A showing horizontal displacements.

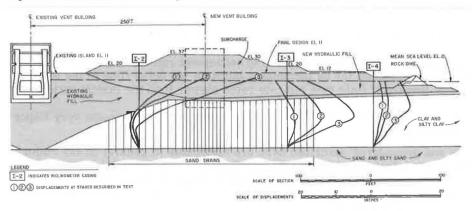
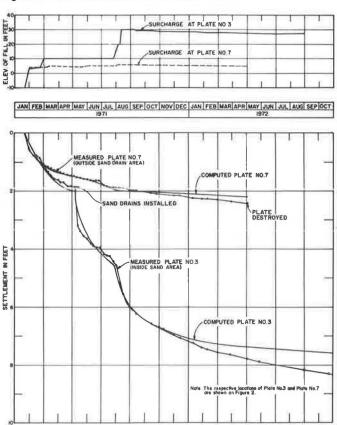


Figure 7. Time settlement curves at station 936+70.



consolidation (5), and the program includes the effect of sand drains according to the theory presented by Richart (6). The program accommodates loading in stages by calculating the average pore pressures in the two layers at the end of each stage and adding the vertical stresses due to the new load to these pore pressures. The computer then calculates the dissipation of the new pore pressures, degree of consolidation, settlement, and compression of the individual layers with time.

The settlement data were analyzed to determine the field values of the consolidation coefficients  $c_{\nu}$  and  $c_{h}$ . A large number of computer analyses were run with different parameters to achieve the best fit with the observed surface and subsurface settlements. Values of  $c_{\nu}$  were determined by fitting settlement data unaffected by sand drains. With these  $c_{\nu}$  values,  $c_{h}$  was determined from settlement data in the sand drain area. The best fit between computed and measured settlement curves was, in general, achieved with values of  $c_{h}$  equal to  $c_{\nu}$ .

In general, it was not possible to fit computed and measured curves unless a reduction of  $c_{\nu}$  and  $c_h$  with time was assumed. Typically, at effective stresses between 1,000 and 2,000 lb/ft² (50 and 100 kN/m²) above the initial stresses, the coefficients were reduced by 50 to 75 percent. This is in reasonable agreement with typical consolidation test data such as those shown in Figure 4.

One year after the beginning of the filling operation, the settlement rate in the sand drain area was about 2 in. (5 cm) per month. At this time, primary consolidation in this area was almost 100 percent complete in the upper clay and about 92 percent complete in the lower clay. Outside the sand drain area, the corresponding values were 96 percent and 34 percent for the upper and lower clays respectively. On the basis of consolidation tests (Table 1), the rate of secondary settlement at this time would be between 0.5 and 1 in. (1.3 to 2.5 cm) per month.

The calculated field values of  $c_{\nu}$  and  $c_{h}$  vary significantly along the island. The initial values for the upper clay vary from more than  $300 \times 10^{-4}$  cm²/sec to about  $50 \times 10^{-4}$  cm²/sec, whereas those for the lower clay vary between about  $50 \times 10^{-4}$  cm²/sec and  $10 \times 10^{-4}$  cm²/sec. The lowest field values are in the high range of those determined in the laboratory, whereas the highest field values correspond to those based on field permeability tests. With progressing consolidation,  $c_{\nu}$  and  $c_{h}$  decreased to within the medium-to-high range of the laboratory values at the appropriate stress levels.

Figure 7 shows typical settlement data from plates inside and outside the sand drain area. The computed curves shown for both plates are based on the back-figured initial values  $c_{\nu}=c_h=100\times10^{-4}~cm^2/sec$  for the upper clay and  $20\times10^{-4}~cm^2/sec$  for the lower clay, decreasing to  $30\times10^{-4}~cm^2/sec$  and  $6\times10^{-4}~cm^2/sec$  at approximately 60 percent consolidation under the full load. In both locations, the measured and computed curves parted after about 300 days because the computations did not include settlements due to secondary consolidation.

#### CONCLUSION

This case history demonstrates that a surcharge and sand drain scheme can be designed reliably on the basis of laboratory data supplemented by field tests. The important specific findings of this study are summarized as follows:

- 1. The values of the  $c/\bar{p}$  ratio determined in the laboratory were verified in the field but were substantially greater than would be expected from correlation with plasticity data;
- 2. The compression index C<sub>c</sub> varied appreciably both in the laboratory and in the field, but it fell within the same ranges;
- 3. The field data showed the horizontal consolidation coefficient  $c_h$  to be essentially equal to the vertical coefficient  $c_v$  for this installation of jetted sand drains;
- 4. The laboratory values of  $c_v$  and  $c_h$  were several times smaller than those indicated by initial settlement data (the values derived from field permeability tests, though on the high side, were in better agreement);
- 5. A substantial decrease of  $c_{\nu}$  and  $c_{h}$  was noted as a result of the compression of the clay with time;

essentially undrained and are governed by a relatively high modulus of deformation and a low volume change, i.e., a Poisson's ratio close to 0.5. Displacements caused by consolidation, on the other hand, are mostly vertical and are associated with considerable volume change. The appropriate Poisson's ratio for these movements would be small, and the modulus would be that determined from consolidation tests.

As a conservative compromise, the movements were analyzed using moduli determined from consolidation tests and a Poisson's ratio of 0.45. In the analyses, the fill load was applied at the clay surface without regard to the rigidity of the fill.

For a typical cross section analyzed in this manner, the maximum predicted settlement would be 7 ft (2.1 m), the maximum predicted horizontal displacement in the clay about 13 in. (0.33 m), and the movement of the existing tunnel about  $\frac{1}{2}$  in. (1.3 cm) with a slight tilt. Tensile stresses and movements toward the new fill were predicted in a wide band between the old and the new islands.

The existing tunnel did, in fact, settle a maximum of  $\frac{1}{2}$  in., but no measurable tilt or horizontal displacement took place. A few tension cracks, about  $\frac{1}{8}$  in. wide and 20 to 30 ft long, were observed on the surface of the existing island above the edge of the tunnel (Fig. 5).

Selected inclinometer data are shown in Figure 6. Curve 1 indicates displacements that occurred during the last 2 months' fill (at elevation of  $\pm 12$ ) but does not include the displacements during filling to elevation  $\pm 12$  and the following 3 months. Filling from  $\pm 12$  to  $\pm 37$  took place in a 2-week period, and curve 2 shows the inclinometer displacements immediately after this filling operation. Curve 3 shows the displacements 6 months later.

Curve 3 for inclinometer I-2 indicates a displacement at elevation zero of 30 in. (76 cm) toward the centerline of the fill or about 6 times the displacement predicted by the computer analysis at that location. This discrepancy was anticipated and is caused by the fact that the computer analyses assumed the same elastic soil properties in tension and in compression.

For inclinometers I-3 and I-4, maximum predicted horizontal displacements were 13 in. (0.33 m) and 9 in. (0.23 m) respectively. The maximum measured displacements were 19.3 in. (0.49 m) and 10.2 in. (0.26 m) respectively. As mentioned previously, these do not include the initial displacement.

At inclinometer I-3, the horizontal displacements that took place in the 6-month consolidation period after placement of the surcharge were approximately equal to the displacements during the 2-week filling period. For comparison, the settlements during the consolidation period were more than three times greater than the settlements during filling.

Inclinometer I-4 was located 130 ft (40 m) away from the surcharge fill. Here, only small horizontal displacements took place during filling, but the consolidation displacements were only slightly smaller than at I-3.

The measured displacements are somewhat larger than those predicted by the finite-element analysis. The analysis considered only displacements of an elastic nature, and it may be presumed that some elastic deformations took place. However, the rate of displacements decreased with time, indicating the relative stability of the slope and its foundation and that delay time was not invoked.

The computer solution used for these problems was rather crude. Better predictions could have been obtained by assigning a low, or zero, tensile modulus to the soil material and by using a compressive modulus decreasing with the shear stress and increasing with the normal stress. Nonetheless, the experience gained from this project indicates the utility of even a simple-minded computer solution.

## SETTLEMENT RATES

A computer program was used to estimate and back-calculate settlements and degree of consolidation in the two clay layers as a function of time. This program uses Boussinesq stress distribution to calculate stresses and simple consolidation theory to calculate total settlements, without accounting for secondary settlements. The degree of consolidation is calculated by using an approximate two-layer solution for vertical

Figure 3. Coefficients of vertical  $(c_{\nu})$  and horizontal  $(c_{h})$  consolidation.

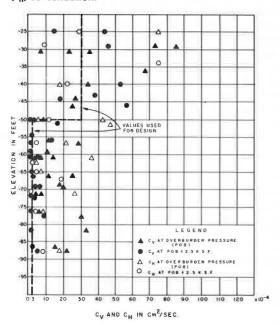


Figure 4. Typical consolidation test data, lower clay.

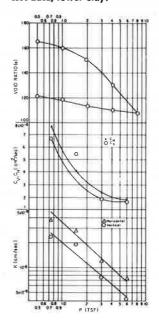
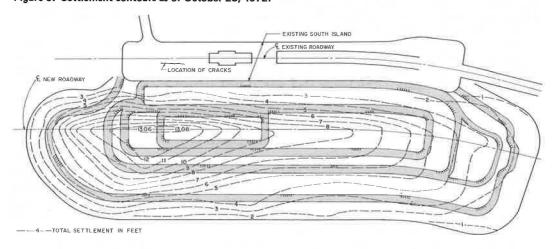


Figure 5. Settlement contours as of October 23, 1972.



alternative. The new tunnel approach should be close enough to the old one to take advantage of the existing island yet far enough away to avoid exposing or endangering the existing structures and generating undue displacements in them. The requirement of minimal residual settlements after construction, stability of fill slopes during construction, and stability of the excavation for the approach and ventilation building in the middle of the new finished island indicated surcharging at a controlled rate, the consolidation being accelerated by sand drains.

Settlement plates were installed after a few feet of hydraulic sand fill were placed, and sand drains were installed by the jetting method in the central areas with three different spacings after the fill reached elevation +12. Piezometers and deep settlement points were installed at the same time. At this time four additional borings were made to verify the strength increase of the clay under the initial loading. Inclinometers were installed to warn against excessive horizontal displacements at any stage of filling. Such excessive horizontal displacements would indicate plastic shear and possible impending slope failure. Figures 1 and 2 show the final configuration of the island with its surcharge and the location of the instruments.

Computerized stability analyses of the Bishop type indicated that slopes to an elevation of +20 (+6 m) would be only marginally stable with the in situ clay strength. At this time, all field data were carefully analyzed to determine if delay time should be invoked. On the basis of the favorable strength increase measured in the additional boreholes, the rate of settlements observed, and the nominal displacements measured by the inclinometers, it was concluded that adequate safety against slope failure had been achieved, and construction was permitted to proceed without invoking the delay time. The surcharge was placed to its full height without incident.

#### SHEAR STRENGTH INCREASE

The original design had been based on conservative ratios of  $c/\overline{p} = 0.15$  for the upper clay and 0.25 for the lower clay, ratios smaller than those obtained by tests but in line with past experience (4). If the field values of  $c/\overline{p}$  were indeed that low, delay time would have been required to achieve adequate strength.

The additional borings, however, verified the laboratory tests. In the upper clay, which was nearly consolidated under the load to elevation +12, the  $c/\overline{p}$  ratio was between 0.30 and 0.35 as determined by field vane shear tests and unconfined compression tests on undisturbed samples. The lower clay was at that time not consolidated, and the strength increase in the lower clay was nominal. The strength of the upper clay, however, was the most critical parameter for slope stability.

#### VERTICAL DISPLACEMENTS

The total vertical settlements occurring about 21 months after filling began, and 15 months after the surcharge reached its final elevation, are shown as contours in Figure 5. The settlements at this time were nearly complete in the sand drain area and more than 70 percent complete outside the sand drain area. The maximum settlement of 13 ft (4.0 m) occurred under the highest fill near the north end of the island, where the clays are thickest and more plastic than average.

Field values of the compression index  $C_{\circ}$ , back-calculated from surface and deep settlement data varied from 0.30 to 0.50 for the upper clay and from 0.65 to 1.10 for the lower clay, increasing from south to north. Thus, the entire range of  $C_{\circ}$  values from laboratory tests was in fact experienced at various points along the island.

## HORIZONTAL DISPLACEMENTS

It was important to estimate the possible movements of the existing tunnel and ventilation structures caused by the weight of the new island and its surcharge. An elastic finite-element computer program was employed for this purpose.

The movements that take place under and around a fill such as this island fill are of two kinds. Movements caused by shear stresses occur relatively quickly after the placement of the fill load and include horizontal displacements. These movements are

Figure 1. Longitudinal profile of South Island.

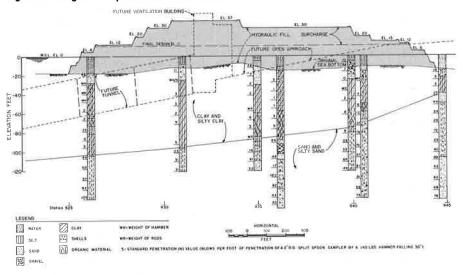


Figure 2. General plan of South Island.

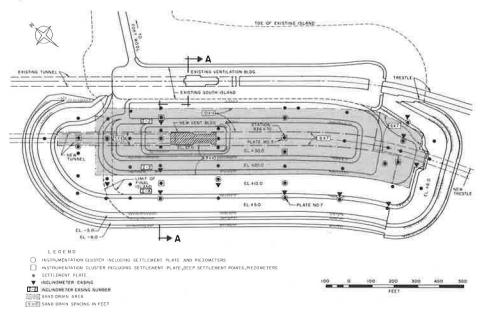


Table 1. Soil properties from laboratory tests.

Test	Upper Clay Layer	Lower Clay Layer
Liquid limit, percent	32 ± 8	75 ± 30
Plasticity index, percent	$15 \pm 10$	$45 \pm 25$
Moisture content, percent	$35 \pm 4$	$65 \pm 15$
Void ratio, e.	$1.1 \pm 0.3$	$1.8 \pm 0.2$
Compression index, C. Coefficient of secondary	$0.45 \pm 0.10$	$0.85 \pm 0.20$
consolidation, Ca	$0.005 \pm 0.002$	$0.011 \pm 0.005$
Shear strength, lb/ft2	$450 \pm 200$	$950 \pm 300$

In its natural state, the soft clay did not have sufficient shear strength to support the full weight of the surcharge. Although the soil would gain strength under the fill load, the time required for adequate strength gain depended on the rate of consolidation, which was only imperfectly predictable through laboratory tests. Furthermore, very large strains would be imposed by anticipated settlements of 10 ft (3 m) or more under the full surcharge load, and the effects of such large strains were not fully predictable.

These concerns were met without excessive cost by basing the design on reasonable assumptions of soil parameters, but the construction contract included an extensive instrumentation program to monitor field performance, with control over the rate of fill placement to be exercised if dangerous conditions were indicated by the field measurements. This was accomplished by providing a separate bid item for delay time, which could be invoked by the engineer after fill had been placed to a specified height, thus compensating the contractor for the cost of idle equipment and crews. The bid price received for delay time was \$4,000 per day, so that a potential delay of 60 days implied a financial risk of \$240,000.

#### SOIL CONDITIONS

Figure 1 shows the soil profile in the longitudinal direction of the island and also the island elevations when the surcharge was at its highest. Figure 2 shows a plan of the island. Through most of the length of the island, the clay extends from sea bottom at elevation -12 to -18 down to elevation -80 to -95, a thickness of 65 to 80 ft (20 to 24 m). In the vertical direction, two strata of clay can be distinguished, with increasing plasticity with depth. A summary of the properties of the two clay layers, separated approximately at elevation -50, as determined by laboratory tests, is given in Table 1.

Although the distinction between the two layers is clear from the test data, there is a considerable random variation, horizontally as well as vertically, of the soil parameters within each layer. There is a trend, however, to greater plasticity and compressibility with increasing clay thickness toward the north.

For the economical design of a sand drain installation, prediction of the time-dependent behavior of the soil is important. For this reason a large number of consolidation tests were run to determine the coefficient of consolidation, on samples cut in both the horizontal and vertical directions. The test results are shown in Figure 3. Not unexpectedly, the scatter of the data is wide, but the coefficients are clearly much greater for the upper layer than for the lower layer. The ratio between the horizontal and vertical coefficients of consolidation,  $c_h/c_v$ , varies but is generally equal to or slightly greater than unity. The results of a typical consolidation test from the lower clay layer are shown in Figure 4. Except for the top parts of the two clay strata, the consolidation tests indicated that the clays were normally consolidated.

Five constant-head permeability tests performed in observation wells, and field pumping tests with observation wells, gave permeabilities between  $2\times10^{-6}$  and  $3\times10^{-6}$  cm/sec for the upper clay and between  $1.8\times10^{-7}$  and  $3.3\times10^{-7}$  cm/sec for the lower clay. Such tests tend to reflect the greater of the horizontal and vertical permeabilities. The consolidation coefficients,  $c_v$  or  $c_h$ , computed from the field permeability data, using average  $a_v$  values from consolidation tests, would be  $300\times10^{-4}$  cm²/sec for the upper clay and  $40\times10^{-4}$  cm²/sec for the lower clay, substantially greater than the values determined in the laboratory.

Using the statistical relation between plasticity index and  $c/\bar{p}$  ratio reported by Bjerrum (4), one would have expected  $c/\bar{p}$  ratios of about 0.15 to 0.20 for the upper clay and 0.25 to 0.30 for the lower clay. Consolidated-undrained and unconsolidated-undrained field vane tests and laboratory triaxial tests, however, indicated a  $c/\bar{p}$  ratio of 0.30 to 0.35 for both layers.

# DESIGN, CONSTRUCTION, AND INSTRUMENTATION

For the South Island of the first crossing, the clay was removed and replaced with hydraulic fill. Environmental restrictions on disposal of dredged materials and the proximity of the first island precluded a similar design for the second island, and a controlled accelerated surcharge compression design was selected as the most economical