STRESSES AND STRAINS IN VISCOELASTIC MULTILAYER SYSTEMS SUBJECTED TO MOVING LOADS

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A method programmed for a high-speed computer is developed for determining the stresses and strains in viscoelastic multilayer systems subjected to moving loads. The method, which can be applied to systems consisting of any number of layers and any type of linear viscoelastic materials, is based on the principle of elastic-viscoelastic correspondence and an approximate procedure of collocation. Numerical solutions, which do not consider the inertia effect, are presented for the stresses and strains in a four-layer system. Of particular interest are the compressive stresses and strains on the surface of the subgrade, layer 4, and the tensile strains at the bottom of the asphalt-bound layer, layer 1, because these stresses and strains have been suggested as criteria for pavement design and evaluation. A study of these critical stresses and strains shows that they all decrease with the increase in speed of the moving load. If the compressive stress on the subgrade is considered as a criterion, either the vertical or the principal stress can be used with little difference. However, if the compressive strain on the subgrade or the tensile strain in the asphalt-bound layer is considered, a criterion based on the vertical or the radial strain will be certainly quite different from that based on the principal strain because of the significant effect of the shear strain.

BECAUSE of the time-dependent behavior of paving materials, there has been a growing belief that pavement design and evaluation should be based on viscoelastic theory rather than the conventional elastic theory. Although elastic theory has been used successfully for determining the stresses and strains in flexible pavements under moving loads, its successful application requires the judicious selection of a Young's modulus and a Poisson's ratio for each of the component layers. An alternative, which is more direct and not so arbitrary, is first to determine the viscoelastic properties of the materials forming the component parts of the pavement and then apply the viscoelastic theory for computing the stresses and strains in the pavement under actual moving loads. This approach was employed by Perloff and Moavenzadeh (1) for determining the surface deflection of a viscoelastic half-space, by Chou and Larew (2) for the stresses and displacements in a viscoelastic two-layer system, and by Elliot and Moavenzadeh (3) for those in a three-layer system. For viscoelastic systems of more than three layers, very little work has been done in the case of moving loads, although a method based on the Duhamel superposition integral was presented by Barksdale and Leonards (4) for analyzing four-layer viscoelastic systems under repeated loads, which was later employed by Elliot and Moavenzadeh (3) for analyzing both the repeated and the moving loads. Because actual flexible pavements are generally composed of multiple layers, an analysis of viscoelastic multilayer systems subjected to moving loads is of practical significance.

The purpose herein is to present a new and more effective method, programmed for a high-speed computer, for determining the stresses and strains in a viscoelastic

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multilayer system due to a circular load moving at a constant speed on the surface. Although the example given is limited to a four-layer system with each layer characterized by a constant Poisson's ratio and a simple mechanical model, the method can be applied to systems consisting of any number of layers and any type of linear viscoelastic materials. In view of the general belief that the fatigue cracking of asphalt pavements is caused by the repeated application of excessive tensile strains at the bottom of the asphalt-bound layer and that the rutting of pavement surfaces is caused by excessive compressive stresses or strains at the surface of the subgrade (5, 6), numerical results on the strains at the bottom of the first layer and the stresses and strains at the top of the fourth layer are presented to illustrate the effect of time and speed on these critical stresses and strains. In line with all previous studies, inertia forces are not considered in the analysis.

STATEMENT OF PROBLEM

Figure 1 shows an n-layer system, the Young's modulus and the Poisson's ratio of the j th layer being $(E)_j$ and $(\nu)_j$ respectively. For a linear elastic system, both $(E)_j$ and $(\nu)_j$ are constants independent of time; for a linear viscoelastic system, they are linear time operators. A uniform load of intensity q is applied on the surface over a circular area of radius a and moves with a constant velocity, v, from a distant point toward a point 0 on the surface along a straight path. It is convenient to consider the time as zero, i.e., $t = 0$, when the center of the load just arrives at point 0, as negative before the load reaches point 0, and as positive after the load passes point 0. The distance, r, between the load and point 0 at any given time, t, is $r = v|t|$. The problem now on hand is to determine the stresses and strains at any point directly beneath point 0 as a function of time. The restriction that the load moves toward point 0 along a straight path is a practical way to simplify the problem because experience indicates that most pavement distress is along the wheelpath; so point 0 may be considered as a given point in the wheelpath.

It is further assumed that the layers are in continuous contact as indicated by the continuity in vertical stress, shear stress, vertical displacement, and radial displacement and that the surface is free of shear stress.

ELASTIC SOLUTION

Before viscoelastic solutions can be developed, elastic solutions must be obtained. The stresses in an elastic multilayer system under a circular loaded area can be expressed as (7)

$$\sigma_z = q\alpha \int_0^\infty J_1(m\alpha) J_0(m\rho)\phi(m)\,dm \tag{1a}$$

$$\sigma_r = q\alpha \int_0^\infty J_1(m\alpha)\left\{\left[J_0(m\rho) - \frac{J_1(m\rho)}{m\rho}\right]\phi_1(m) + J_0(m\rho)\phi_2(m)\right\} dm \tag{1b}$$

$$\sigma_\theta = q\alpha \int_0^\infty J_1(m\alpha)\left[\frac{J_1(m\rho)}{m\rho}\,\phi_1(m) + J_0(m\rho)\phi_2(m)\right] dm \tag{1c}$$

$$\tau_{rz} = q\alpha \int_0^\infty J_1(m\alpha) J_1(m\rho)\phi_3(m)\,dm \tag{1d}$$

in which σ_z, σ_r, σ_θ, and τ_{rz} = vertical, radial, tangential, and shear stresses respectively; q = intensity of the uniformly applied load; $\alpha = a/H$; a = radius of loaded area;

H = depth from surface to the upper boundary of the lowest layer; J_0 and J_1 = Bessel functions of the first kind and order, 0 and 1 respectively; $\rho = r/H$; r = radial distance from the center of loaded area to the point at which stresses are to be determined; m = a parameter of integration; and ϕ, ϕ_1, ϕ_2, and ϕ_3 = functions of elastic constants, layer thicknesses, the vertical coordinate of the point in question, as well as the parameter of integration, m. Once the stresses are known, the strains can be determined by

$$(\epsilon_z)_j = \frac{1}{(E)_j}\left[\sigma_z - (\nu)_j(\sigma_r + \sigma_\theta)\right] \tag{2a}$$

$$(\epsilon_r)_j = \frac{1}{(E)_j}\left[\sigma_r - (\nu)_j(\sigma_\theta + \sigma_z)\right] \tag{2b}$$

$$(\epsilon_\theta)_j = \frac{1}{(E)_j}\left[\sigma_\theta - (\nu)_j(\sigma_z + \sigma_r)\right] \tag{2c}$$

$$(\Upsilon_{rz})_j = \frac{1}{(G)_j}\tau_{rz} \tag{2d}$$

in which ϵ_z, ϵ_r, ϵ_θ, and Υ_{rz} = vertical, radial, tangential, and shear strains respectively; E = Young's modulus; ν = Poisson's ratio; and the subscript j outside the parentheses indicates the j th layer.

VISCOELASTIC SOLUTION

In presenting numerical results, it is desirable to use a dimensionless velocity, V, and a dimensionless time, T, defined as

$$V = \frac{v\tau^*}{H} \tag{3}$$

$$T = \frac{t}{\tau^*} \tag{4}$$

in which τ^* is one of the retardation times used for describing material properties. Note that $\rho = V|T|$.

For brevity, only the vertical stress, σ_z, will be used for illustration. It must be borne in mind that the same procedure can be employed for determining other stresses and strains as well.

The viscoelastic solution can be obtained by applying the elastic-viscoelastic correspondence principle originally developed by Lee (8). Instead of considering directly the stress, strain, and load, the Laplace transform of stress, the Laplace transform of strain, and the Laplace transform of load are considered. Application of the correspondence principle involves the following steps:

1. Taking the Laplace transform of the time-dependent boundary conditions;
2. Changing the elastic field equations by replacing E, which is a ratio between stress and strain, with $\bar{E}(p)$, which is a ratio between the Laplace transform of stress and the Laplace transform of strain, and also replacing ν with $\bar{\nu}(p)$;
3. Solving the resulting problem in terms of the transformed variable p; and
4. Inverting the solution involving the transformed variables into time variable.

Based on the preceding principle, Eq. 1a, which is based on elasticity, can still be applied for viscoelastic media, if the stress, σ_z, is replaced by the Laplace transform of stress, $\bar{\sigma}_z$, the time-dependent boundary or the moving load $qJ_0(mV|T|)$ by $q\bar{J}_0(mV|T|)$, as indicated in step 1, and $\phi(m)$, which involves E and ν, by $\bar{\phi}(m,p)$, as indicated in step 2. Note that $\bar{\phi}(m,p)$ is obtained by replacing E and ν in the expression for $\phi(m)$ by $\bar{E}(p)$ and $\bar{\nu}(p)$. Consequently, the resulting problem in terms of p, as indicated in step 3, can be written as

$$\bar{\sigma}_z = q\alpha \int_0^\infty J_1(m\alpha)\bar{J}_0(mV\,|T\,|)\bar{\phi}(p, m)dm \tag{5}$$

The bar on top of σ_z, J_0, and ϕ denotes the Laplace transform and implies that they are functions of the transformed variable p. Note that $\bar{J}_o$ is a function of load, and $\bar{\phi}$ is a function of material properties. The determination of $\bar{\phi}$ from material properties was described elsewhere (9). To complete step 4, Eq. 5 can be inverted by the convolution theory as

$$\sigma_z = q\alpha \int_0^\infty \int_{-\infty}^{T} J_1(m\alpha) \left[J_0(mV\,|\tau\,|)\ \phi(T - \tau, m)d\tau\right] dm \tag{6}$$

The same procedure can be used for determining other stresses. The radial stress has three terms involving Bessel functions $J_o(mV\,|\tau\,|)$ and $J_1(mV\,|\tau\,|)$; each must be evaluated independently and then combined. The same is true for the tangential stress, which involves two terms to be evaluated.

By the correspondence principle, Eq. 2a can be written as

$$(\bar{\epsilon}_z)_J = \frac{\bar{\sigma}_z}{(\bar{E})_J} - \frac{(\bar{\nu})_J}{(\bar{E})_J}\,\bar{\sigma}_r - \frac{(\bar{\nu})_J}{(\bar{E})_J}\,\bar{\sigma}_\theta \tag{7}$$

$\bar{\sigma}_z/(\bar{E})_J$ can be inverted in the same way as $\bar{\sigma}_z$, except that $\bar{\phi}$ is replaced by $\bar{\phi}/(\bar{E})_J$. The same procedure can be applied to $-(\bar{\nu})_J\bar{\sigma}_r/(\bar{E})_J$ and $-(\bar{\nu})_J\bar{\sigma}_\theta/(\bar{E})_J$; their summation gives the vertical strain $(\epsilon_z)_J$.

LAPLACE INVERSION

The major difficulty of the preceding procedure lies in the Laplace inversion of $\bar{\phi}(p, m)$ to $\phi(T, m)$. Because exact inversion is difficult, if not impossible, for a multilayer system, an approximate method of collocation is employed.

The function of $\bar{\phi}(p, m)$ is a ratio of two polynomials in p. Depending on the stress or strain to be determined and the models used to characterize the materials, the degree of the polynomial in the denominator may be equal to or greater than that in the numerator. If both have the same degree, $\bar{\phi}(p, m)$ must be separated into a constant, s_o, plus a ratio, $\bar{\Psi}(p, m)$, the denominator of which has degrees higher than the numerator.

$$\bar{\phi}(p, m) = s_o + \bar{\Psi}(p, m) \tag{8}$$

The value of s_o can be determined from $\bar{\phi}(p, m)$ by assigning a very large value, e.g., 10^{10}, to p.

$$s_o = [\bar{\phi}(p, m)]_{p=10^{10}} \tag{9}$$

If the denominator of $\bar{\phi}(p, m)$ has a higher degree, s_o automatically approaches zero when a large p is assigned. Substituting Eq. 8 into Eq. 5 gives

$$\bar{\sigma}_z = q\alpha \int_0^\infty J_1(m\alpha)\ \bar{J}_o(mV\,|T\,|)\ [s_o + \bar{\Psi}(p, m)]dm \tag{10}$$

The inversion of Eq. 10 is

$$\sigma_z = q\alpha \int_0^\infty J_1(m\alpha) \left[s_o J_o(mV\,|T\,|) + \int_{-\infty}^{T} J_o(mV\,|\tau\,|)\ \Psi(T - \tau, m)d\tau\right] dm \tag{11}$$

In the collocation method, it is assumed that $\Psi(T, m)$ can be expressed approximately by a Dirichlet series of decaying exponentials

$$\Psi(T, m) = \sum_{i=1}^{n} s_i \exp(-p_i T) \tag{12}$$

in which s_i and p_i = constants and n = number of terms. This approximation is possible because $\Psi(T, m)$ is a monotonic increasing or decreasing function of T. The collocation method cannot be applied directly to the stresses and strains, as in the case of stationary loads, because under moving loads they are not monotonic functions of T. Take the Laplace transforms of Eq. 12 and then multiply by p:

$$p\bar{\Psi}(p, m) = \sum_{i=1}^{n} \frac{s_i}{1 + \frac{p_i}{p}} \tag{13}$$

In this study, eight exponential terms are used to approximate $\Psi(T, m)$. The assumed values of p_i are 0.02, 0.05, 0.1, 0.2, 0.5, 1, 10, and 100. Theoretically, the values of p selected depend on the time range within which the stresses and strains are to be analyzed. Schapery (10) suggested the relation between p and T as $p = \frac{1}{2}T$, so a range of p from 0.02 to 100 is equivalent to a range of T from 25 to 0.005. By successively assigning p in Eq. 13 to each of the preceding values, eight simultaneous equations are obtained that will give a solution to the unknowns, s_1 through s_8.

$$\begin{Bmatrix} \frac{1}{1+\frac{0.02}{0.02}} & \frac{1}{1+\frac{0.05}{0.02}} & \cdots & \frac{1}{1+\frac{100}{0.02}} \\ \frac{1}{1+\frac{0.02}{0.05}} & \frac{1}{1+\frac{0.05}{0.05}} & \cdots & \frac{1}{1+\frac{100}{0.05}} \\ \vdots & \vdots & & \vdots \\ \frac{1}{1+\frac{0.02}{100}} & \frac{1}{1+\frac{0.05}{100}} & \cdots & \frac{1}{1+\frac{100}{100}} \end{Bmatrix} \begin{Bmatrix} s_1 \\ s_2 \\ \vdots \\ s_8 \end{Bmatrix} = \begin{Bmatrix} p[\bar{\phi}(p, m) - s_0]_{p=0.02} \\ p[\bar{\phi}(p, m) - s_0]_{p=0.05} \\ \vdots \\ p[\bar{\phi}(p, m) - s_0]_{p=100} \end{Bmatrix} \tag{14}$$

Once s_i is known, $\Psi(T, m)$ can be determined from Eq. 12 and σ_z from Eq. 11.

NUMERICAL INTEGRATION

Most of the computer time used in the viscoelastic analysis is for the evaluation of two infinite integrals, one with respect to time and the other to m. Without involving excessive computer times, accuracy can be ensured by selecting the proper increments in performing the numerical integration.

The inner integral in Eq. 11 is evaluated by a five-point Gaussian quadrature formula. The zeros of Bessel functions, J_0 and J_1, and the values of the functions at the five points between 2 zeros are stored in the computer and used repeatedly to save computer time.

When $T \leq 0$, let $x = -mV\tau$; the inner integral in Eq. 11 becomes

$$I = \frac{1}{mV} \int_{mV|T|}^{\infty} J_0(|x|)\Psi(T + \frac{x}{mV} m)\, dx \tag{15}$$

Equation 15 is integrated numerically from $mV|T|$ to a large value until the integral converges. It is found that integration up to 40 cycles of the Bessel functions is sufficient, so a limit of 40 cycles is imposed to save computer times. A five-point Gaussian quadrature formula is used to evaluate the integral between 2 zeros of the Bessel function and between the starting point and the next nearest zero.

When $T > 0$, the inner integral in Eq. 11 can be divided into two parts:

$$I = \int_{-\infty}^{0} J_0(mV|\tau|)\Psi(T - \tau, m)d\tau + \int_{0}^{T} J_0(mV|\tau|)\Psi(T - \tau, m)\, d\tau \tag{16}$$

Because the shear stress and strain change signs when passing point 0, a negative sign should be used for the integral from 0 to T.

Let $x = -mV\tau$ for the first integral and $x = mV\tau$ for the second; Eq. 16 becomes

$$I = \frac{1}{mV}\int_{0}^{\infty} J_0(|x|)\Psi(T + \frac{x}{mV}, m)dx + \frac{1}{mV}\int_{0}^{mVT} J_0(|x|)\Psi(T - \frac{x}{mV}, m)\, dx \tag{17}$$

The outer integral in Eq. 11 is evaluated by Simpson's one-third rule. The values of m used are 0, 0.01, 0.02, 0.21, 0.4, 0.7, 1.0, and then every 0.5 until the integral converges. Because the increments of integration used in Simpson's rule are 0.005 for $m < 0.02$, 0.085 for $0.02 < m < 0.4$, and 0.1 for $m > 0.4$, a three-point parabolic interpolation formula is used to determine the values of the integrand at the intermediate points.

The analysis of viscoelastic multilayer systems can be summarized as follows:

1. Assign successive values of m starting from zero to a rather large positive value until σ_z converges;
2. For each m, determine s_0 from Eq. 9, s_1 through s_8 from Eq. 14, and the values of the integrand at various times, T, from Eq. 11; and
3. Integrate Eq. 11 by Simpson's rule and parabolic interpolation.

COMPARISON WITH EXISTING METHODS

The method developed in this study is very effective for analyzing multilayer systems and differs significantly from the existing methods by Chou and Larew for two-layer systems and Elliot and Moavenzadeh for three-layer systems (2, 3).

Chou and Larew (2) considered a concentrated moving load instead of a circular load. Because they assumed that the load started from a fixed point at a given distance, R_o, from point 0, they had difficulty in integrating Eq. 6. To obtain correct solutions, they had to exchange the order of integration, i.e., integrating first with respect to m and then with respect to τ. This is a very cumbersome process. First, they divided the time into small increments. At the end of each time increment, a series of m values, e.g., 25, was assumed, and integration was performed with respect to m. If there are 200 time increments, as is usually the case when the load starts from $-\infty$, the collocation method will be applied 200 x 25 or 5,000 times. Therefore, because the computer time for the double integration was just too excessive, they could not obtain numerical results for systems with more than two layers.

The result of this study shows that, by changing the starting point of the load from $-\infty$, instead of from a fixed distance R_o, Eq. 6 could be integrated first with respect to τ and then with respect to m. This means that, for each m, only one collocation is necessary, or a total of only 25 collocations instead of 5,000. The reason that Eq. 6

does not yield correct solutions when the load starts from a fixed distance R_o instead of from $-\infty$ is that the load $J_0(mV|\tau|)$ at the starting time is not negligible when m is small, and the system cannot be considered initially undisturbed as assumed in the Laplace transform of the field equations. However, if the order of integration is exchanged, the problem becomes the determination of stress at various times. Unless R_o is very small, the stress at point 0 when the load starts will be very small, and the system can be considered initially undisturbed.

Elliot and Moavenzadeh (3) employed the Duhamel integral for determining the stresses and displacements under a circular moving load. Their method is different from the author's in that they applied the Duhamel integral, instead of the correspondence principle, in obtaining the viscoelastic solutions. Using the Duhamel integral, the vertical stress in a viscoelastic-layered system, corresponding to Eq. 6, can be written as

$$\sigma_z = q\alpha \int_0^{\infty} \int_{-\infty}^{T} J_1(m\alpha) \left[\frac{\partial}{\partial \tau} J_o(mV|\tau|)\right] \phi_s(T - \tau, m)\, d\tau\, dm \tag{18}$$

in which ϕ_s is the response of the system to a static load and can be obtained by inverting $\bar{\phi}(p, m)/p$. After differentiation, the equation becomes

$$\sigma_z = -q\alpha \int_0^{\infty} \int_{-\infty}^{T} \frac{J_1(m\alpha)}{mV} J_1(mV|\tau|) \phi_s(T - \tau, m)\, d\tau\, dm \tag{19}$$

The reason that the Duhamel integral was not employed in this study is that the differentiation of one term of Bessel function such as J_1 for determining radial, tangential, and shear stresses may generate two terms of Bessel functions, and the evaluation of these additional Bessel functions requires additional computer times. This is particularly significant when determining the strains due to the large number of terms involved.

COMPUTER PROGRAM

The method described in this paper was programmed in FORTRAN IV for an IBM 360 computer available at the University of Kentucky. It is not the intention herein to describe in any detail the computer program. Nevertheless, the capability of the program will be pointed out, so that readers interested in using the program can obtain a complete listing of the program from the author.

Theoretically, the method developed in this study is quite general and can be used to determine the stresses and strains at any point in a multilayer system consisting of any number of layers and any types of linear viscoelastic materials. However, a general program of this type requires considerable computer times and may not be desirable from a practical viewpoint. Consequently, the program was written in a more restrictive way to obtain useful information at a reasonable cost. It is hoped that these restrictions will promote the application of the program instead of limit its usefulness.

The program can only provide solutions for the vertical displacement on the surface and the vertical, radial, tangential, and shear stresses or strains at both the bottom of layer 1 and the top of the lowest layer because these stresses and strains have been considered as important criteria for pavement design and evaluation. The user must specify whether the stresses or strains are to be computed.

In computing the stresses, the transformed shear and bulk moduli of each layer are represented by the quotients of two polynomials, the degrees and coefficients of which must be specified by the user. In computing the strains, the transformed shear moduli and the Poisson's ratios of each layer must be specified. The latter are assumed to be elastic and independent of time. Because the Poisson's ratios have relatively small effect on the strains, this simplification will save a great deal of computer time without affecting the results significantly.

Computer storage is reduced by dimensioning to take care of up to six layers using a maximum of 12 values of the transformed variables for the collocation. The displacement, stresses, or strains at 20 different times are computed simultaneously. These restrictions can easily be removed by merely increasing the dimensions of the parameters involved.

NUMERICAL RESULTS

The applicability of the method is demonstrated by using a four-layer system (Fig. 2) to simulate a highway pavement. It is assumed that the surface course of the pavement is asphaltic concrete, the behavior of which under pure shear is characterized by a Burgers model; the base and subbase courses are granular materials, which are considered as elastic; and the subgrade is a soft clay, which is represented by a Maxwell model. The Poisson's ratios of the layers from top down are assumed to be 0.4, 0.3, 0.3, and 0.5 respectively. The total computer time for obtaining all the data presented in this paper by an IBM 360/65 was about 15 min.

Let $(G_1)_1$ be the shear spring constant and $(\tau_1)_1$ be the shear retardation time of the Kelvin element in layer 1. Because $(G_1)_1$ and $(\tau_1)_1$ are assumed unity in Figure 2, the spring constants and relaxation times shown in the various models are not their actual values but are their ratios to $(G_1)_1$ and $(\tau_1)_1$ respectively. If $T = t/(\tau_1)_1$ and the transformed variable of T is p, the transformed shear moduli of the materials are $(\bar{G})_1 = [20p(p + 1)(G_1)_1]/[(p + 1)(2p + 1) + 20p]$, $(\bar{G})_2 = 2(G_1)_1$, $(\bar{G})_3 = (G_1)_1$, and $(\bar{G})_4 = 5p(G_1)_1/(10p + 1)$. Because the Poisson's ratios are assumed time-independent, the transformed Poisson's ratio and the original Poisson's ratio are the same, so

$$(\bar{E})_j = 2[1 + (\upsilon)_j](\bar{G})_j \qquad (20)$$

Once the transformed moduli of elasticity and Poisson's ratios are known, $\bar{\phi}(p, m)$ can be evaluated and the stresses and strains determined by the method presented.

Figures 3, 4, and 5 show the stresses and strains in the four-layer system at nine different dimensionless times, T, i.e., -1, -0.75, -0.5, -0.25, 0, 0.25, 0.5, 0.75, and 1. The narrow range of times from -1 to 1 is used because the most critical stresses and strains generally occur within this range, especially when the velocity is high. Two different dimensionless velocities are used: V = 0.25, as indicated by the small circles, and V = 1 by the small triangles. Integration is carried out to m = 10.

In the figures, compressive stresses and strains are considered positive and tensile stresses and strains negative. In addition to the tangential stress or strain, which is one of the principal stresses or strains, two other principal stresses or strains are also shown, the major and minor principal stress or strain. These are the major and minor principal stresses or strains in the rz plane only and may not always be the largest or the smallest of the three principal stresses or strains.

Figure 3 shows the vertical, radial, tangential, shear, and principal stresses at the top of layer 4. The vertical, tangential, and major principal stresses decrease with the increase in speed and become maximum sometime after the load passes point 0. The fact that all three principal stresses are positive implies the nonexistence of tensile stresses. The nearly equal magnitude of vertical and major principal stresses in this critical range indicates that, if rutting is caused by excessive stress in the subgrade, either vertical or major principal stress can be used as a criterion with no significant difference.

Figure 4 shows the vertical, radial, tangential, shear, and principal strains at the top of layer 4. Although both the vertical and the major principal strains decrease with increasing speed, the maximum vertical strain occurs after the load passes point 0, whereas the maximum major principal strain, because of the large component of the shear strain, occurs before the load reaches point 0. In this particular case, where shear strains are large and contribute to a significant portion of the principal strains, a criterion based on the vertical compressive strain will certainly be different from that based on the major principal strain. Note that part of the radial strains and all of the tangential and minor principal strains are negative, even though all stresses are positive at the top of layer 4.

Figure 1. A multilayer system subjected to a moving load.

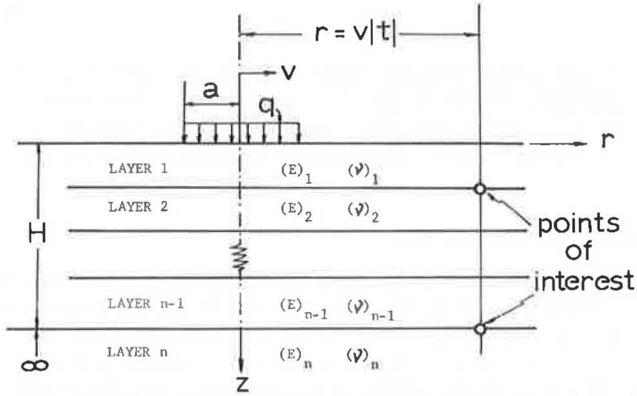

Figure 2. Models characterizing a four-layer system.

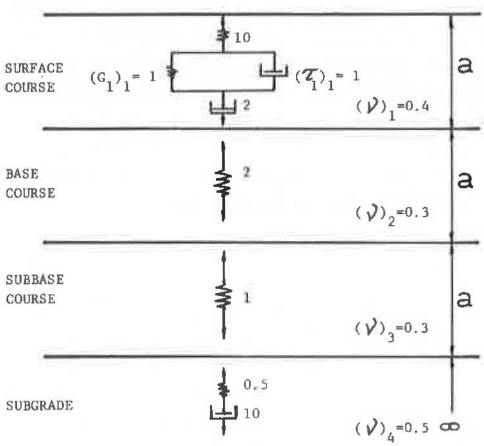

Figure 3. Stresses at top of layer 4.

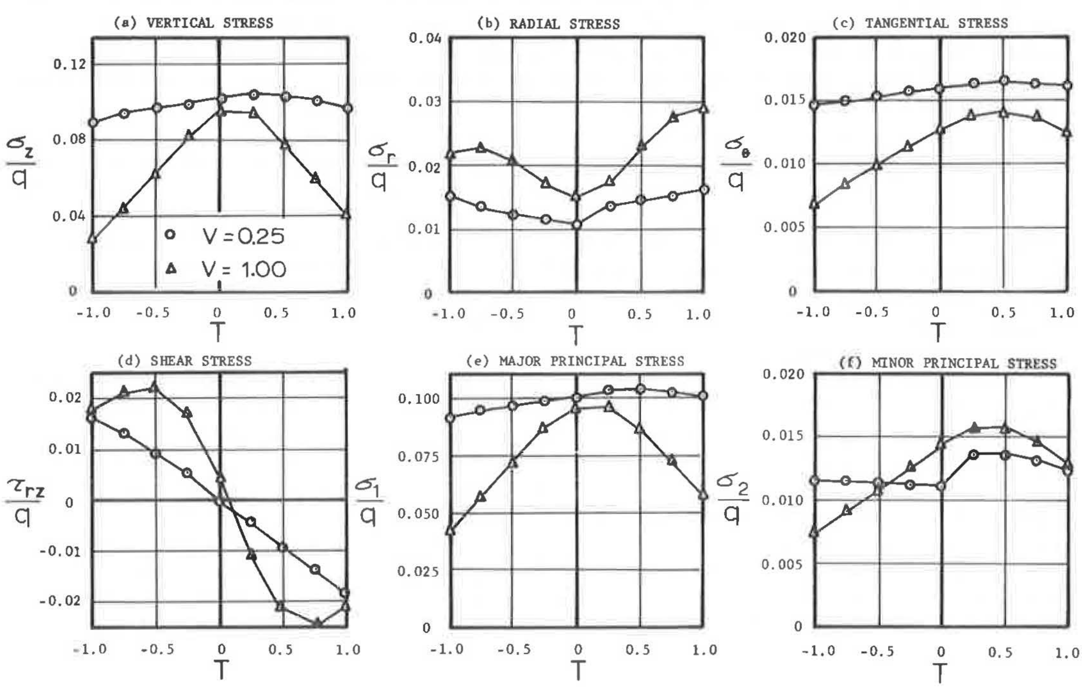

Figure 4. Strains at top of layer 4.

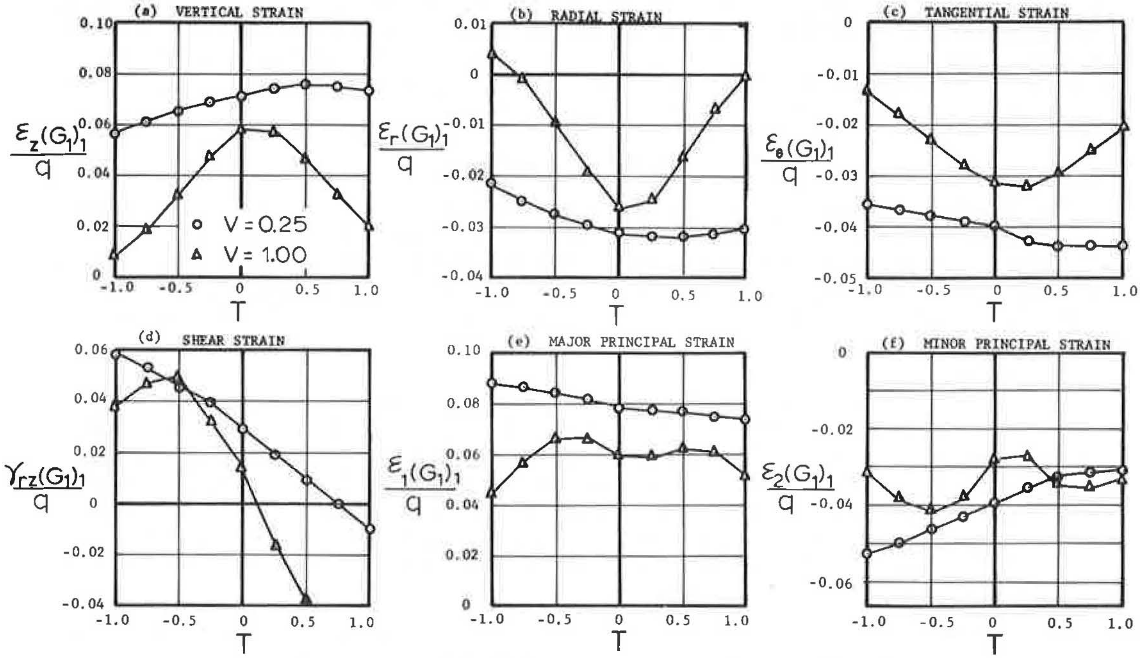

Figure 5. Strains at bottom of layer 1.

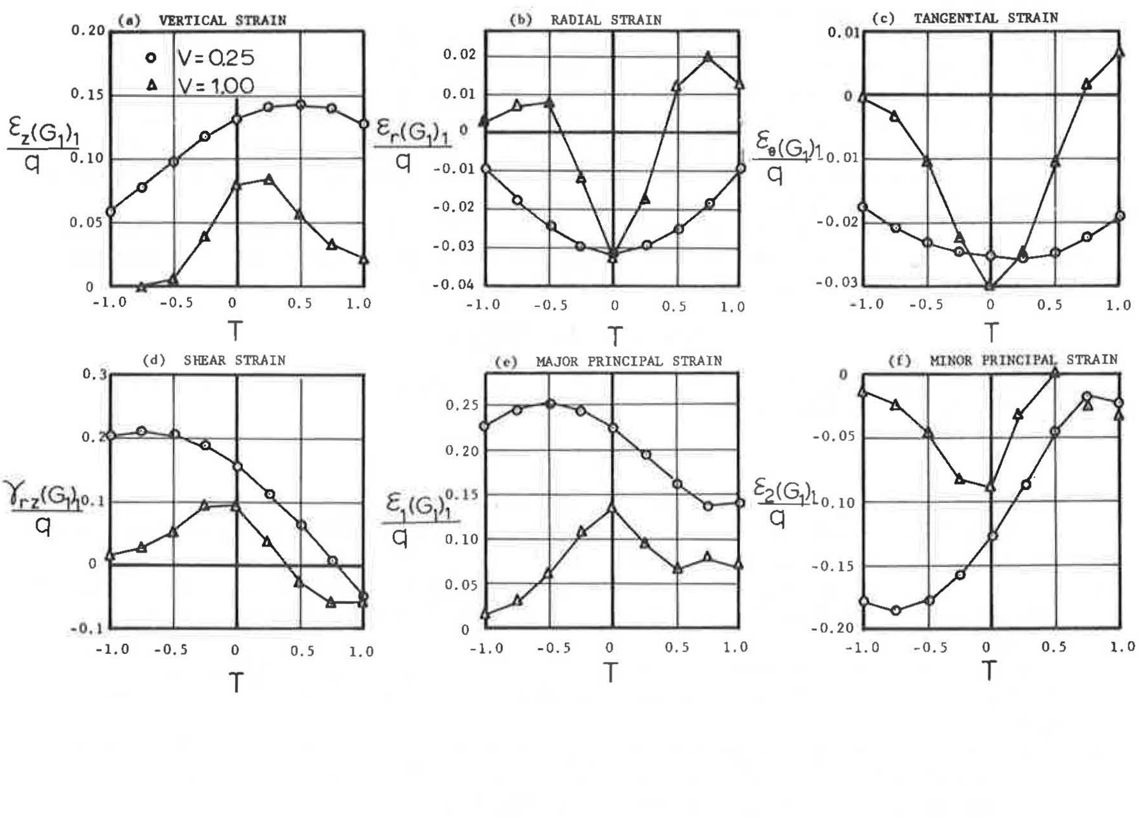

Figure 5 shows the vertical, radial, tangential, shear, and principal strains at the bottom of layer 1. The general trend for the change of strain with time is quite similar to that at the top of layer 4. Because fatigue is caused by excessive tensile strains at the bottom of the asphalt-bound layer, the minor principal strains are of particular interest. The principal tensile strains also decrease appreciably with the increase in speed and arrive at a maximum value before point 0 is reached.

CONCLUSIONS

A method programmed for a high-speed computer is presented for determining the stresses and strains in viscoelastic multilayer systems subjected to moving loads. The elastic solution is briefly described, and the application of the elastic-viscoelastic correspondence principle to change the viscoelastic problem to an associated elastic problem is illustrated. The inversion of the associated elastic problem to the viscoelastic problem is facilitated by using the convolution theory and an approximate method of collocation.

Numerical results are presented for the stresses and strains in a four-layer system consisting of an asphalt-bound surface course, granular base course, granular subbase course, and soil subgrade. Of particular interest are the vertical and principal compressive stresses and strains on the surface of the subgrade, layer 4, and the principal tensile strains at the bottom of the asphalt-bound layer, layer 1, because these stresses and strains have been suggested as criteria for pavement design and evaluation. A study of these critical stresses and strains in the four-layer system subjected to moving loads reveals the following facts:

1. Both the vertical compressive stresses and the major principal stresses at the top of layer 4 decrease with the increase in speed; their maximum values occur at or immediately after the load passes point 0. The predominant contribution of the vertical stress to the principal stress indicates that, if rutting is caused by excessive stresses in the subgrade, there is very little difference whether the design or evaluation is based on the vertical or the principal stress.
2. The vertical compressive strains and the major principal strains at the top of layer 4 also decrease with the increase in speed. However, the maximum vertical strain occurs after the load passes point 0, whereas the maximum principal strain, because of the large component of shear strain, occurs before the load reaches point 0. When shear strains are large and contribute to a significant portion of the principal strains, a criterion based on the vertical compressive strain will certainly be different from that based on the major principal strain.
3. The principal tensile strains at the bottom of layer 1 also decrease with the increase in speed, and, because of the existence of the large shear strain, the maximum principal tensile strain occurs before point 0 is reached.

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