METHOD FOR PREDICTING TRAVEL TIME AND OTHER OPERATIONAL MEASURES IN REAL-TIME DURING FREEWAY INCIDENT CONDITIONS

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This paper presents the development of a method for predicting the travel time required by a motorist to travel from any selected freeway location to the end of the freeway system during incident conditions. It is predictive in that it computes an estimate of a motorist’s travel time if he were to enter the freeway several minutes in the future. Speeds, volumes, and other operational measures can be predicted also. These calculations are made immediately after the incident is detected and the necessary operational measures have been evaluated. Speeds of the various shock waves and travel-time results are also presented. The model was developed following the kinematic wave theory of Lighthill and Whitham for possible use in an operational control strategy of variable-message signs whereby motorists would be diverted to alternate routes if conditions on the freeway relative to selected alternate routes justified the diversion. The model could also be used to predict queue backups and delays due to lane closures caused by scheduled maintenance operations.

*FREeways ramp control systems have proved their effectiveness in relieving freeway congestion when operations are free of incidents. Incident conditions, however, are a frequently occurring phenomenon on urban freeways. Goolsby found that, within a 6-mile section on the Gulf Freeway in Houston (1), more than 13 lane-blocking incidents occur on the average during the time period of 6 a.m. to 7 p.m. from Monday through Friday. Stalled vehicles and accidents were the contributing causes of 97 percent of the incidents observed. Approximately 80 percent of the incidents reduced the capacity of the freeway by about one-half or more.

Freeway operational improvements have been implemented or proposed to improve the level of service provided during incidents. Several of these systems have consisted of some form of variable-message signs (2-6). One of the chief operational objectives of these signs is to increase the effective capacity of the freeway corridor during incidents on the freeway by achieving a higher utilization of the adjacent frontage road and surface street system. Driver preference questionnaire studies indicate that drivers will divert around congestion if accurate, reliable, and timely traffic information is provided to them. This diversion could occur from the freeway, at the frontage roads, or at major intersections located within the freeway corridor (7). One measure of the likelihood and desirability of diversion is the travel-time saving that may occur to motorists if they are diverted (7, 8). This evaluation requires an estimate of the travel times along the alternate route and along the freeway during the incident conditions.

This paper presents the development of a method for predicting the time a motorist will need to travel from selected freeway locations to the end of the freeway system during incident conditions on the freeway. It is predictive in that it computes an esti-

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mate of what a motorist's travel time would be if he were to enter the freeway at a selected location at a given time. Speeds, volumes, and other operational measures together with the speed and location of shock waves can also be predicted. Previous methods for calculating travel times have been based on measured or average speeds in fixed subsections \(^{(9, 10)}\) rather than on predictions of changing traffic flow conditions.

**DEVELOPMENT OF METHOD**

**Traffic Flow Theory**

The deterministic theory of traffic flow has been shown to be very useful in describing freeway traffic conditions and in providing a basis for a rational explanation of certain observed traffic phenomena \((11, 12, 13)\). The results of several approaches to the deterministic theory of traffic flow have been summarized by Drew \((14)\) in his textbook on traffic flow theory and control. In general, the traffic flow theory has presented several mathematical models that interrelate the traffic flow variables of volume, speed, and density.

One of the more used deterministic theories of traffic flow is Greenshields' well-known linear speed-density model \((11)\):

\[
u = u_r - \frac{u_r}{k_i} k
\]

or

\[
k = k_i - \frac{k_i}{u_r} u
\]

where

\[u = \text{speed of the traffic stream,}\]
\[u_r = \text{free speed,}\]
\[k = \text{density of the traffic stream, and}\]
\[k_i = \text{jam density.}\]

Using the general equation of the traffic stream, \(q = ku\), where \(q\) is the mean rate of traffic flow, we can formulate the parabolic relations between traffic speed \(u\) and volume. Substituting from Eq. 2 for density \(k\) into \(q = ku\) yields

\[
q = k_i u - \frac{k_i}{u_r} u^2
\]

A similar relation exists between volume \(q\) and density \(k\). Substituting from Eq. 1 for speed \(u\) in \(q = ku\) yields

\[
q = u_r k - \frac{u_r}{k_i} k^2
\]

Equations 1, 3, and 4 are shown in generalized form in Figures 1a, 1b, and 1c respectively. Also shown is the point on each of the respective curves that represents an assumed traffic flow condition existing on a section of freeway during normal operating conditions. Normal operating conditions are assumed to be free of traffic congestion or incidents that might cause congestion to develop.

**Initial Effects of Incident**

When an accident occurs on a high-volume freeway, it has been widely observed that a queue forms at the location of the accident. The queue and its resulting congestion then begin backing upstream from the scene of the bottleneck, often for several miles during peak-hour operations. Whitson \((15)\) has presented volume-density plots of freeway operations in Houston during an incident, which clearly illustrate this upstream...
progression of the queuing area and its corresponding congestion. The frontal boundary of this queue, as it moves upstream, is commonly called the shock wave. Freeway surveillance of traffic operations during incidents has indicated that the shock wave commonly travels from 10 to 20 mph during moderate to heavy traffic conditions.

Whitson (15) also noted that a wave moves downstream from the incident location. This wave denotes the change that occurs downstream of the incident, from normal traffic flow to a much lighter flow. The reduction in the capacity of the freeway caused by an accident, or other lane-blocking incident, thus meters the freeway flow downstream from the site of the incident but causes a queue and congestion to form upstream of it.

Figure 2 shows a graphic summary of freeway traffic conditions upstream and downstream of the incident location while the incident blocks the freeway. The congested queue is bounded by the shock wave and the incident location with the queue having a nearly saturated density \( k_q \) that is much higher than the normal density \( k_n \) (Fig. 3). Downstream of the incident in the metered flow region, the density is reduced from the normal density \( k_n \) existing before the incident to a much lighter metered density \( k_m \), reflecting a higher mean traffic speed. The location of the clearing wave defines the boundary between the metered flow and the as yet undisturbed normal flow region.

Wave Theory

Lighthill and Whitham have presented a theoretical model for computing the speed of a shock wave based on changes in volume and density. The speed of the shock wave is given (16) by

\[
W_u = \frac{q_\delta - q_\alpha}{k_q - k_n}
\]

where

- \( W_u \) = the speed of the shock wave,
- \( k_q \) = traffic density in the congested queue,
- \( k_m \) = traffic density during normal operations,
- \( q_\delta \) = stream flow rate in the congested queue, and
- \( q_\alpha \) = stream flow rate during normal operations.

The wave subscript notation refers to the direction of travel of the wave and the position number. That is, \( W_{u1} \), the shock wave, is the speed of the first wave that travels upstream during incident conditions. \( W_{d1} \) would be the first wave traveling downstream. As shown in Figure 2, the density \( k_q \) in the congested queue is greater than the normal density \( k_n \). The incident is assumed to reduce the capacity of the freeway to less than the normal flow \( q_\alpha \), which is a requirement if congestion is to form. Thus, the speed of the shock wave \( W_{u1} \) will be negative, indicating the wave is moving upstream.

As shown in the volume-density curve in Figure 3, the speed of \( W_{u1} \), the shock wave moving upstream from the location of the incident, is the slope of the chord that connects the point characterizing the traffic condition within the congested queue with the point characterizing normal traffic conditions. The negative speed of \( W_{u1} \) is also shown in Figure 3 because the slope of the chord that defines \( W_{u1} \) from Eq. 5 is negative.

As shown in Figure 3, the traffic flow rate \( q_\delta \) in the clearing metered section downstream of the bottleneck incident is the same as the bottleneck flow rate \( q_\delta \), but the density \( k_m \) within the metered area is much lower than the density \( k_q \) in the congested queueing section. The speed of the metered wave, which is the boundary between the metered and normal traffic operation, is

\[
W_{d1} = \frac{q_\delta - q_\alpha}{k_m - k_n} = \frac{q_\delta - q_\alpha}{k_m - k_n}
\]

where

- \( W_{d1} \) = speed of the clearing metered wave being the first wave moving downstream from the incident,
\( q_m = \) flow rate in the metered section, 
\( q_q = \) flow rate in the queue and equals \( q_m \), 
\( q_n = \) normal flow rate, 
\( k_m = \) density in metered section, and 
\( k_n = \) normal density.

Because both the numerator and the denominator of Eq. 6 are negative, \( W_d \) is positive, indicating that the clearing metered wave is traveling downstream from the site of the incident bottleneck.

After a time \( T \) has elapsed since the incident occurred, the incident is assumed to be completely removed from the freeway (Fig. 4). When the incident is removed, the capacity of the freeway is increased, and the vehicles stored upstream of the site of the incident then begin to travel downstream. The flow of these vehicles out of the downstream end of the congested queue also begins to shorten or clear up the queue upstream of the site of the incident. Figure 4 shows a summary of the traffic operating conditions along the affected sections of freeway from the time the incident begins until the freeway traffic operations return to normal sometime after the incident is removed. The shock wave \( W_{s1} \) and the clearing metered wave \( W_{d1} \) are depicted as the boundary vectors emanating upstream and downstream respectively from point A in Figure 4, which defines the beginning of the incident. The equations shown in Figure 4 for the wave speeds will be developed in a later section.

The freeway traffic flow in the high-density, high-flow region, denoted as region \( c \) (capacity) in Figure 4, may be described as generally being unstable flow at or slightly under the maximum flow at normal capacity. For the purposes of this analysis, the average flow and density within this high-density, high-volume section are assumed to be at capacity, noted as the capacity point in Figure 3. As soon as the incident bottleneck is removed from the freeway, this unstable, near-capacity region of flow begins to travel upstream from the incident location (point B, Fig. 4), reducing the queue length, and downstream from the incident, increasing the flow and density downstream.

Associated with the movement upstream of the capacity flow region is the wave \( w_{s2} \) shown in Figure 4. Likewise, the wave \( W_{d2} \) moves downstream from the site of the incident (when it is removed) that defines the boundary between the capacity flow and the metered regions. Using Figure 3, it follows that

\[
W_{s2} = \frac{q_m - q_q}{k_m - k_q} \tag{7}
\]

where \( W_{s2} \) is the speed of the capacity boundary wave moving upstream, and \((q_m, k_m)\) and \((q_q, k_q)\) define the volume-density operating conditions in the capacity flow region \( c \) and congested queue region \( q \) respectively shown in Figure 3. Note in Figure 4 that \( W_{s2} \) is the second wave that travels upstream.

The boundary of the high-density, capacity-flow region travels downstream at a speed of

\[
W_{d2} = \frac{q_m - q_q}{k_m - k_q} = \frac{q_n - q_m}{k_m - k_n} \tag{8}
\]

where \( W_{d2} \) is the speed of the boundary wave, and \((q_n, k_n)\) and \((q_m, k_m)\) define the volume-density operating conditions in the capacity flow region \( c \) and the clear metered region \( m \) respectively noted in Figures 3 and 4. Note again that \( q_m = q_n \).

As shown in Figure 4, one remaining wave occurs before the freeway traffic conditions return to normal. Sometime after the incident is removed, the capacity flow wave \( W_{s2} \) will catch the shock wave \( W_{s1} \), and the congested queue will have been dissipated. At this point, the final clearing wave \( W_{d3} \) forms and begins to move downstream. This wave defines the boundary between the high-density capacity flow region and normal traffic flow. The speed of the wave is

\[
W_{d3} = \frac{q_m - q_n}{k_n - k_m} \tag{9}
\]
where \( W_{d3} \) is the speed of the last clearing wave, and \((q_0, k_0)\) and \((q_1, k_1)\) define the volume and density in the capacity flow and normal regions respectively shown in Figures 3 and 4.

**Computing Shock Waves From Speed**

Freeway surveillance of incidents in Houston has indicated that a very useful and reliable method for readily detecting the occurrence of an incident on the freeway is to measure the change that occurs in the stream speed (or occupancy) in the queuing area immediately upstream of the scene of the incident. This suggests that it would be desirable if the entire freeway traffic flow existing during incident conditions (in essence a mathematical description of Fig. 4) could be related to the normal speed \( u_0 \) existing before the incident occurred and the average speed within the congested queue \( u_q \). The average speed in the queue could be determined from the incident bottleneck capacity \( q_b \) using Eq. 3.

Figure 4 shows that a description of freeway traffic conditions during an incident depends heavily on knowing the speeds and locations of the various waves in time and space and on knowing the duration of the incident. The following development is directed toward relating the previously discussed wave speeds to the normal traffic speed \( u_0 \) and the queue speed \( u_q \).

The two wave speeds \( W_{s1} \) and \( W_{d1} \) are of primary interest while the incident forms a bottleneck on the freeway. Note that the shock wave \( W_{s1} \) in Eq. 5 can be written as a function of only the normal traffic speed \( u_0 \) and the speed \( u_q \) in the congested queue because \( q = f(u) \) from Eq. 3 and \( k = f(u) \) from Eq. 2. Because the speed of the shock wave is

\[
W_{s1} = \frac{q_b - q_s}{k_q - k_s}
\]

based on Eq. 5, substituting for \( k = f(u) \) and \( q = f(u) \) from Eqs. 2 and 3 yields

\[
W_{s1} = \frac{k_1 u_0 - k_1 u_q^2 - k_1 u_q + k_1 u_0^2}{k_1 u_q - k_2 + k_2 u_0}
\]

and subtracting the \( k_i 's \) and rearranging yield

\[
W_{s1} = \frac{k_1 (u_q - u_0) - k_1 (u_q^2 - u_0^2)}{-k_1 (u_q - u_0)}
\]

Dividing by \(-k_1/u_q\) and by \((u_q - u_0)\) leaves

\[
W_{s1} = -u_q + u_0 + u_q
\]

where \( W_{s1} \) is the speed of the shock wave, \( u_s \) is the free speed, and \( u_q \) and \( u_q \) are the normal and queue speeds respectively. For the Greenshields linear speed-density model being used, the speed-volume curve of Figure 1b is symmetrical about the speed at capacity. Thus, the sum of \( u_q + u_q \) will be less than \( u_s \) so long as the bottleneck capacity flow \( q_b \) is less than the normal flow \( q_s \) that existed before the incident occurred.

The speed of the clearing metered wave \( W_{d1} \), progressing downstream from the scene of the incident, can be developed in a similar manner because

\[
W_{d1} = \frac{q_b - q_s}{k_q - k_s}
\]
from Eq. 6. However, $k_\alpha$ must first be related to traffic conditions existing within the queuing section. By referring to Figure 3 and using Eq. 4, which relates $q = f(k)$, we can show that $k_\alpha = f(q)$ is

$$k_\alpha = \frac{k_\alpha}{2} - \sqrt{\frac{k_\alpha^2}{4} - \frac{k_\alpha}{u_r} q_\alpha}$$  \hspace{1cm} (13)

Substituting $q_\alpha = f(u_\alpha)$ from Eq. 3 into Eq. 13 yields

$$k_\alpha = \frac{k_\alpha}{2} - \sqrt{\frac{k_\alpha}{4} - \frac{k_\alpha}{u_r} (k_\alpha u_\alpha - k_\alpha u_r^2)}$$  \hspace{1cm} (14)

which reduces to

$$k_\alpha = \frac{k_\alpha}{u_r} u_\alpha$$  \hspace{1cm} (15)

Returning to the equation for the metered wave speed of Eq. 6,

$$W_{d1} = \frac{q_\alpha - q_n}{k_\alpha - k_n}$$

the results of Eq. 15 are then substituted for $k_\alpha$, which yields

$$W_{d1} = \frac{q_\alpha - q_n}{\frac{k_\alpha}{u_r} u_\alpha - k_n}$$  \hspace{1cm} (16)

Next, the volume and density relations as a function of speed, Eqs. 2 and 3, are then substituted into Eq. 16, yielding

$$W_{d1} = \frac{k_\alpha u_\alpha - k_\alpha u_r^2}{\frac{k_\alpha}{u_r} u_\alpha - k_\alpha}$$  \hspace{1cm} (17)

$$W_{d1} = \frac{k_\alpha (u_\alpha - u_r) - k_\alpha (u_\alpha^2 - u_r^2)}{\frac{k_\alpha}{u_r} (u_\alpha + u_r) - k_\alpha}$$  \hspace{1cm} (18)

Dividing by $u_r/k_\alpha$ yields

$$W_{d1} = \frac{u_r (u_\alpha - u_r) - (u_\alpha - u_r) (u_\alpha + u_r)}{u_r + u_\alpha - u_r}$$  \hspace{1cm} (19)

Factoring $-(u_\alpha - u_r)$ results in

$$W_{d1} = \frac{- (u_\alpha - u_r) (u_\alpha + u_r - u_r)}{u_r + u_\alpha - u_r}$$  \hspace{1cm} (20)
and dividing out \((u_i + u_n - u_q)\) yields

\[
W_{d1} = u_n - u_i
\]  

(21)

where \(W_{d1}\) is the clearing metered wave speed, \(u_n\) is the normal speed on the freeway before the incident, and \(u_i\) is the speed in the congested queue.

As has been shown in Figure 4, when the bottleneck incident is removed, three additional waves are generated. The equations for computing these waves have also been presented. The procedures used to relate the wave speeds to the normal speed \(u_n\) and the speed in the congested queue \(u_q\) follow the two previous examples. Hence, only the results of these three analyses will be presented:

\[
W_{d2} = \frac{u_n}{2} + u_q
\]  

(22)

\[
W_{d2} = \frac{u_n}{2} - u_i
\]  

(23)

\[
W_{d3} = \frac{u_n}{2} + u_i
\]  

(24)

Discussion of Model

The results of the previous equations are summarized in Figure 4. The bottleneck incident occurs at point A in time and space, and it lasts until the time of point B is reached. The maximum queue backup along the freeway from the location of the incident is shown as point C in Figure 4.

Comparisons of different wave speeds are made in the interest of providing additional insight and information on the model’s description of freeway operation during the incident. Because it is proposed that \(W_{d2}\) must catch the initial shock wave \(W_{d1}\), the difference between them yields the rate of queue dissipation, or

\[
W_{d2} - W_{d1} = \frac{u_n}{2} - u_i
\]  

(25)

This difference will be negative as expected because the normal speed \(u_n\) is greater than the speed at normal capacity flow \(u_c/2\) using Greenshields’ linear model of traffic flow. The expected negative difference also follows from the initial assumption that the normal flow was stable before the incident occurred with operating speeds above the speed at capacity (Fig. 1). Eq. 25 confirms the expectation that, the lighter the normal traffic flow is before the incident (a larger \(u_n\)), the quicker the queue is dissipated.

For the three waves traveling downstream, the differences indicate that each subsequent wave is slower than the previous one. This suggests that these waves never intersect downstream of the incident—as if all three waves were rays emanating from a common point source. These results are based on the differences between

\[
W_{d1} - W_{d2} = u_n - \frac{u_n}{2}
\]  

(26)

and

\[
W_{d2} - W_{d3} = u_r - u_n - u_i
\]  

(27)

Both of these differences are positive, indicating that \(W_{d1}\) is faster than \(W_{d2}\), which in turn is faster than \(W_{d3}\), the third and final wave traveling downstream. These results are reflected in the respective slopes of the waves shown in Figure 3.
PREDICTION OF FREEWAY TRAVEL TIMES

The procedure for computing the travel times of vehicles on the freeway during incident conditions requires a knowledge of freeway traffic speeds as a function of time and distance. Figure 4 has been shown to define the time and space locations of the four different freeway traffic flow conditions that exist during incident conditions. The average volumes and densities existing within each of these flow regions have been shown in Figure 3. Thus, the average traffic speed within each of the flow regions can be determined using Eq. 1. The traffic speed within each region and two examples of vehicles traveling through a congested freeway section during an incident are shown in Figure 5. Again all speeds are being computed from only two traffic variables, the normal speed \( u_n \) and the speed within the congested queue \( u_q \). Recall that \( u_f \) is the free-speed parameter in Greenshields' linear speed-density model.

The procedure for computing the travel times of two vehicles will be illustrated. Shown as point A in Figure 5, one vehicle is assumed to be at an entrance ramp at \( t_0 \), the time the incident occurs. This vehicle would travel at a speed \( u_n \) until it intercepts the shock wave backing up the freeway at B. The speed of the vehicle would then drop considerably to \( u_q \) while the vehicle travels through the congested queue. When it passes the incident location at C, the vehicle then enters the high-speed metered region at a speed \( u_h = u_f - u_q \). The vehicle is assumed to leave the freeway system at D. The travel time for this vehicle would be \( t_D - t_0 \).

One feature of the travel-time model is that it permits an immediate prediction, as soon as the incident is detected, of the travel times of vehicles that may enter the freeway some time after the incident occurs. Assume that a vehicle enters the freeway at the on-ramp, point I in Figure 5, 10 min after an incident occurred. Entering the freeway, the vehicle then intercepts the shock wave at J and remains in the queue until the capacity flow wave at K is reached. The vehicle then remains in the capacity flow region, leaving the system at L. The travel time on the freeway from point I to L would be \( t_L - t_0 \).

The time-distance path that a vehicle would trace, e.g., path IJKL in Figure 5, is not known initially for an incident and must be computed in a trial-and-error manner. A computer program, which requires only a few seconds to execute, was written to compute these travel times.

A travel-time solution will be presented for a typical lane blockage incident that occurred on the inbound Gulf Freeway in Houston. A vehicle stalled in the median lane at 8:16 a.m., reducing the capacity by about one-half, and was removed 6 min later at 8:22, as shown in Figure 6. This figure also shows the predicted operating speeds, wave speeds, and average traffic conditions during incident conditions. The incident generated a shock wave having a speed of 11 mph. It moved upstream for 13 min, until 8:29, resulting in a queue backup of about 2 1/2 miles. The shock wave was predicted to arrive at the Griggs ramp at 8:24 a.m. and was observed to arrive at 8:25.

Figure 7 shows the predicted travel time from any freeway location shown to the end of the system if the vehicle were to begin its trip at the time shown. The predicted travel times at 8:16, the time the incident occurred, are higher than the travel times expected just before the incident occurred. Note that the predicted travel times at the Griggs and Lombardy ramps located upstream of the incident increase for about 10 min, 4 min after the blockage was removed.

FEASIBILITY STUDY

The method presented for predicting travel times requires estimates for several variables and parameters. The location, duration, and severity of the incident must be established in addition to the normal average operating speed, speed in queue, and free speed. During real-time operations, all of these would have to be estimated within a short period of time based on real-time traffic data. The accuracy of these estimates would directly affect the accuracy of the travel-time prediction model. Based on the literature available and freeway operations experience, it would appear that an accurate prediction of incident duration would be the most difficult variable to determine (1). Research is currently being conducted in this area to develop the necessary detection and estimation techniques.
Figure 1. Speed, volume, and density relations using Greenshields' model.

Figure 2. Existing freeway traffic conditions until incident is removed from freeway.

Figure 3. Deterministic relations of five waves caused by incident.

Figure 4. Time-space model of freeway traffic flow conditions and wave speeds.

Figure 5. Method of predicting travel times of vehicles traveling through incident conditions.

Figure 6. Time-space diagram of traffic conditions for incident on Gulf Freeway.

Figure 7. Travel times predicted by model for incident on Gulf Freeway.
An initial feasibility study was conducted, however, to determine the accuracy of the method in predicting travel times if all the necessary variables and parameters were accurately determined. One off-peak- and three peak-period incidents that occurred on the Gulf Freeway in Houston were evaluated. Freeway traffic flow was normal and not congested before the lane blockages occurred. The incident data were accurately recorded from television surveillance available in the freeway surveillance center, and resulting freeway traffic flow data were available from computer printout. Ten automobile travel times were manually recorded from the television surveillance for each incident. All travel-time computations were made at a later date. Because each incident occurred at a different location on the freeway, the free speed $u$, used in the method was adjusted slightly to provide the best possible fit of the recorded data.

Figure 8 shows the cumulative percentage of the relative percentage of error among the 40 samples of the automobile travel times taken during the four incidents and the computer travel times. Two-thirds of the observed travel times were within 10 percent of the computer travel times, a level felt satisfactory for consideration as reliable information. Most of the larger errors arose when travel times were being predicted for times 10 to 20 min after the incidents occurred. Again, based on the available data, this is the highest accuracy that could be expected to be obtained with accurate estimates of the incident variables under ideal conditions. It remains to be determined how accurately the incident variables can be estimated in real time.

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REFERENCES

DISCUSSION

Robert L. Gordon, Sperry Systems Management Division, Sperry Rand Corp.

The authors have developed an application of the theory of kinematic waves for freeway control. The techniques described in the paper permit the calculation of wave propagation rates under incident conditions and allow travel times to be computed under these conditions. The authors, however, use the linear Greenshields' speed-density model that results in parabolic relations for volume-speed and volume-density plots. A considerable amount of empirical data exists (e.g., 15) that indicate that speed-volume and speed-density curves that are skewed are generally more representative of freeway traffic flow, particularly where speed limits influence flow. This discussion is intended to describe the general magnitudes of the possible errors that can result in the wave propagation rates computed by the algorithms suggested in the paper as a result of differences among the parabolic model for the flow-density curve and certain empirical data.

Figure 9 shows a volume-speed curve obtained as a result of measurements over three lanes of roadway made on the Van Wyck Expressway in New York City. The data are based on 5-min volume and speed samples, most of which were obtained between speeds of 15 and 50 mph. The figure also shows a hypothetical plot based on the parabolic relation, i.e., Eq. 3 of the paper.

It was necessary to make certain assumptions concerning the parameters of the parabola to develop this plot. Because the maximum flow is a measurable quantity, this quantity was selected to be one of the specified values. Either jam density or free speed may be selected as another value. Jam density was obtained by extrapolating measured data, and the resultant free speed is seen (Fig. 9) to be close to the measured value for this quantity.

Figure 10 shows the plots of Figure 9 converted to volume-density plots by the use of the relation

\[ k = \frac{q}{u} \]

In this conversion, differences between mean space speeds and mean time speeds (17) have been ignored, and this may lead to some error in the results of the computation in this discussion.

A normal (upstream) flow of 5,200 vehicles per hour and an incident providing a capacity flow of 2,800 vehicles per hour were assumed in order to compare wave velocities calculated from both the measured data and the parabolic representation. The shock wave speeds, \( W_{11}, W_{41}, W_{22}, W_{42}, \) and \( W_{43} \), were calculated for the following situations:
1. Case 1—Wave velocities were calculated using the measured volume-density data shown in Figure 10 and Eqs. 5, 6, 7, 8, and 9 of the paper. These calculations are assumed to provide the correct wave velocities.

2. Case 2—Wave velocities were calculated using the parabolic representation of the volume-density data shown in Figure 10 in conjunction with Eqs. 5, 6, 7, 8, and 9. These calculations describe the basic fundamental wave speeds if the parabolic representation for the flow conditions described was in fact valid.

3. Case 3—Wave velocities were calculated using the speed measurements for the actual data curve in Figure 9 applied to the computational algorithms (based on the parabolic assumption) described in Eqs. 12, 26, 27, 28, and 29 of the paper. These computations describe the wave velocities that would be calculated based on vehicular-speed measurements that would actually be made on the roadway under the flow conditions described and then processed by the algorithms described in the paper.

Comparison of the results for the three sets of calculations is as follows:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Case 1, Correct Wave Speed</th>
<th>Case 2, Computed Wave Speed</th>
<th>Case 3, Computed Wave Speed</th>
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<tr>
<td>$W_{k2}$</td>
<td>27.2</td>
<td>19.0</td>
<td>19.2</td>
</tr>
<tr>
<td>$W_{k3}$</td>
<td>11.2</td>
<td>7.5</td>
<td>17.5</td>
</tr>
</tbody>
</table>

The results show that, when upstream flow conditions are in regions where actual and parabolic curves do not coincide, significant errors can be made in calculating the wave velocities based on the parabolic assumption. It is suggested that the accuracy of the wave velocity computation could be improved by the use of an empirical volume-speed curve based on measured relations, conversion to density by using $k = q/u$, and use of Eqs. 5, 6, 7, 8, and 9 to calculate wave velocity.

Another alternative is to use Eqs. 5, 6, 7, 8 and 9 in conjunction with a more representative analytic model for the moderate- and high-volume cases. Such models that might be considered are the Greenberg model (13) or alternatively a higher order polynomial than that provided by Eq. 4.

REFERENCE

Joseph A. Wattleworth, University of Florida

The authors are to be commended for the development of an extremely interesting mathematical model of conditions near a freeway incident and the application of this model to the very practical purpose of real-time estimation of the measures of freeway operations. This model has the potential of increasing the accuracy of measures intended for the estimation of travel time for the individual vehicles on freeway sections.

Past systems for estimating travel time were based on the measurement of speed at several points along a route and the assumption that this speed was constant over a length of roadway. Normally, the speed at a detector is assumed constant from a point halfway to an upstream detector station to a point halfway to a downstream detector station. This assumption may be reasonably valid in cases in which no shock wave formation takes place. Most practical applications, however, are those in which shock waves do form a congested freeway and an arterial sheet, to name but two.

The model presented by the authors uses simple measurements to infer the location of the shock wave. A vehicle is assumed to travel at one speed until it reaches the shock wave and to travel at another speed thereafter. This model is more refined than the earlier models in that it calculates the (changing) position of the speed change point and does not simply assume that it is halfway between detector stations.

One can examine the maximum speed error between a pair of the detector stations if the present logic is employed, namely, assuming a constant speed from a detector to a point halfway to the next detector. Figure 11 shows a schematic of this situation. The upstream detector station measures a speed \( V_1 \), while the downstream station measures a speed \( V_2 \). Under the conventional systems, the speed would be assumed to be \( V_1 \) from the upstream station to the midpoint and would be assumed to be \( V_2 \) from the midpoint to the downstream station.

Under these assumptions the speed of a vehicle in traversing the distance between the detectors can be calculated to be

\[
V = \frac{2V_1V_2}{V_1 + V_2}
\]

If the actual speed over the section is actually \( V_1 \) from the upstream detector to a point very close to the downstream detector station, the actual average speed over the distance is \( V_1 \). Thus the error in speed is

\[
V_1 - V = V_1 - \frac{2V_1V_2}{V_1 + V_2} = \frac{V_1^2 - V_1V_2}{V_1 + V_2} = \frac{V_1(V_1 - V_2)}{V_1 + V_2}
\]

In the case of a shock wave due to congestion, \( V_1 \) might be about 40 mph and \( V_2 \) about 20 mph. The error in this case would be 13 mph. Thus, the method suggested by the authors will potentially improve the accuracy of speed and travel-time estimates over a length of a facility.

Figure 11. Schematic of detector stations.
One must ask several questions regarding the measurement technique and model presented by the authors. First, the shock wave speeds are easily calculated from detector speeds or speed parameters if the linear speed-density relation is assumed (Greenshields' model). Have investigations been made into the relations for calculating shock wave speeds if other traffic flow models are assumed? Are the relations workable?

Second, has an investigation been undertaken into the detector station frequency and travel-time accuracy? The accuracy should be greater than for existing techniques for a given detector spacing, and this analysis would be interesting.

Third, the authors have assumed a straight-pipe section for their analysis. What changes must be made in order to analyze a realistic system such as an urban freeway with exit and entrance ramps? What detector configuration would be required in order to implement the model? In this regard, has consideration been given to the implementation of this model on an arterial street where the incident is a traffic signal and there is an incident at each signalized intersection once each cycle? The model would appear to have the greatest potential for improvement in travel-time estimations on streets (because of the number of incidents) if the model can be adapted to such an application.

In summary, the authors have presented a very interesting and potentially very useful model. The discussion has raised some questions regarding the implementation of the model.

Eugene F. Reilly, New Jersey Department of Transportation

The authors have done a good job in applying the theoretical flow-concentration curve as expressed by Greenshields to the determination of travel time on the freeway. Freeway incident detection and the associated handling of traffic present the engineer with many problems when he attempts to give accurate delay information to the motorist.

Without the aid of a television camera, we must use remote-sensing devices and historical traffic data. The sensors will detect the reduced speeds and flow, or the increased occupancy and density of traffic, which will then be used to estimate the time of occurrence and location of a possible incident. With the passage of sufficient time to verify an incident, action is immediately taken to meter upstream entry ramp traffic or to inform entry traffic of the advisability of taking alternate routes or both.

The task of diverting traffic is the final objective of the authors in the paper under discussion. For the engineer to achieve that same objective, he must have defined his usable network within the corridor; he must have chosen the optimum locations for his changeable-message signs that will be used to give the delay information to the motorist; he must have detailed the messages that will be displayed; and he may also have to develop a method of estimating or measuring the diverted or alternate route traffic flow so that he can determine the travel time of motorists using the alternate route. The difference in travel time between the alternate route and the freeway is one inducement to the motorist to use the alternate.

If the freeway corridor conditions are known, we can concentrate on the items that directly affect the freeway corridor travel time during the time of an incident and consider those motorists who will be given the information about the freeway delay.

The data received at a central control station from the remote sensor will be received up to several minutes after the incident has occurred. If the flow conditions are severe enough, the programmed logic will assume the existence of an incident. Up to this point in time, all traffic that has passed decision points will become part of the backed-up queue. But perhaps the most severe problem now exists for the engineer: How long will the incident last? If the engineer assumes a short time interval for the incident, there may be no reason to divert traffic because the savings in travel time may be but a few minutes for the diverted traffic. Historically, the distribution of the length of time of previous incidents can give the engineer a few alternatives in this
regard. It may be reasoned that the engineer should assume short time interval inci-
dents as a matter of policy and then relate the resulting short delays to the motorists
if they chose the freeway as their route. What might be expected to result is a growing
acceptance of the displayed delay information by the motorists who use the freeway.
These motorists will always experience some delay, but rarely less than they are told to
expect; and, because the time length of incident will be longer than the engineer as-
sumes, the motorist choosing the freeway will learn to expect that the displayed delay
information is usually a minimum amount.

The other alternatives left to the engineer would be the choice of either an average
length incident or an incident of long duration. In either of these latter cases, the
motorist can never be sure when he would save time by diverting. In at least half the
cases when he chooses the freeway, he will experience less delay than he was told. The
results of a system that would function on such a policy would be one of total unreli-
ability. The motorist could not accept the displayed delay or time savings information
based on his previous experiences, and the engineer would have no reliable method of
estimating traffic flows on any of the routes in the corridor.

The second most important aspect of this entire process is determining the location
of the incident and verifying the extent of capacity restriction. The information that is
displayed to the motorist is again vitally dependent on these factors. Regardless of
whether speed, volume, or occupancy, as measured by the remote sensors, is the im-
portant parameter, a certain amount of inaccuracy will exist in determining the reduced
capacity of the roadway. With the arrival of police on the scene and subsequent traffic
handling, the roadway capacity could be further affected. The spacing of the detectors
allows additional inaccuracy because the location of the incident has to be estimated down-
stream of the sensor. Assuming that the incident is half the distance to the next down-
stream detector, the engineer can estimate the time of its occurrence using the mea-
sured flow data to give him the speed of the initial shock wave. By using estimates of
location and by making allowances for time to verify the existence of an incident, the
engineer maintains as high a degree of accuracy as methods will currently allow.

The first-hand information that television surveillance gives does not overcome the
problem of estimating the length of time of the incident. This aspect of the problem
will be the major barrier to reliable and timely display data until ongoing research
programs can satisfactorily estimate clearance intervals.

With the continued success of the authors and other researchers in this field, the
solution of the other problems related to incident detection will make a significant con-
tribution to the engineer in his handling of freeway operations.

AUTHORS' CLOSURE

The authors would like to express their appreciation to Gordon, Wattleworth, and
Reilly for their stimulating reviews of the paper. These discussions, in themselves,
will provide considerable guidance for future research and development.

The question was raised as to the need for using a traffic flow model other than the
Greenshields model used in the paper. Traffic flow data were presented that do not
totally follow Greenshields' model. If linear regression had been used to fit the model,
it is suggested that a better fit would have resulted. In order to provide the flexibility
of describing various traffic streams, the authors are considering the use of the gen-
eralized traffic flow model rather than the Greenshields model at the cost of increasing
the complexity of the travel-time model.

It was also noted that the input-output flows to the freeway should also be considered.
Perhaps the generalized traffic flow model, if used, should be calibrated to closed sys-
tem data of total travel and total time rather than point location data.

In the proposed application of the travel-time model to driver information systems,
the consequences of overestimating or underestimating the duration of incidents were
noted. Because it may be difficult to accurately estimate the duration of an incident,
the estimate should tend to underestimate the average duration so that a higher respect of the driver information system can be maintained. Research is currently being conducted to develop improved incident duration estimation techniques based on measured operational and environmental data.