A METHOD FOR DETERMINING THE STRENGTH PARAMETERS OF SOILS

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A simple method for determining the cohesion c and internal friction angle ϕ of soils and stabilized materials, which requires knowledge of only the unconfined compressive strength and tensile strength, is presented. The tensile strength may be conveniently determined by the newly developed double-punch test. A procedure is outlined for establishing c and ϕ from the Mohr-Coulomb failure envelope constructed by using the proposed method. Comparisons showing good agreement between strength parameters calculated from the proposed method and those measured by more conventional direct shear and split-tensile strength tests for various types of soils are given.

•CONVENTIONAL ANALYSES of the stability of soil-pavement systems require knowledge of one or more of the strength parameters: cohesion c, internal friction angle ϕ, unconfined compressive strength q_u, and tensile strength σ_t. Commonly used methods for establishing c and ϕ include direct and triaxial shear tests. These test methods are generally time-consuming, expensive, and particularly poorly suited to testing stabilized pavement material because of the large particle sizes and high strengths involved. This frequently necessitates the use of large test specimens, which results in the need for larger test equipment and higher test loads.

This paper presents a simple method for determining the (undrained) cohesion and internal friction angle of soils and stabilized materials if the tensile and compressive strength of the material are known. The compressive strength can be determined conventionally, and the tensile strength may be simply established by using the newly developed double-punch test (7, 8). A comparison of loading conditions, types of failure planes, and failure envelopes for the direct shear test, triaxial test, and proposed method is shown in Figure 1.

The method assumes that the cohesion may be adequately expressed as a function of soil type and tensile strength. Both graphical and analytical methods of establishing c and ϕ are given. Comparisons between strength parameters calculated from the proposed method and those measured by more conventional direct shear and split-tensile strength tests are presented and discussed.

THEORETICAL CONSIDERATIONS

The modified Mohr-Coulomb failure envelope used in this paper was suggested by Chen and Drucker (3). The failure envelope (Fig. 2) is denoted by AG′H where AG′ is part of the circle and G′H is a straight line. The distance AB is equal to the magnitude of the tensile strength. BE is equal to the radius of the unconfined compressive strength Mohr circle, and distance BG is equal to the cohesion. ϕ is the slope of the line GH.

To establish the failure envelope requires that at least three points on the envelope be given. AB can be determined from a simple indirect tensile test such as the double-punch test. Distance BF is equal to the compressive strength and may be determined by a conventional unconfined compression test.

This information provides two of the three points necessary to define the envelope. Experimental data indicate that c is related to the tensile strength of the material (see Fig. 5). From Figure 2, the unconfined compressive strength can be computed by

$$q_u = 2c \tan\left(45^\circ + \frac{\phi}{2}\right) \tag{1}$$

where q_u = unconfined compressive strength. Rearranging Eq. 1 gives

$$\phi = 2 \tan^{-1} \frac{q_u}{c} - \frac{\pi}{4} \tag{2}$$

where $\phi < \frac{\pi}{2}$. If

$$\xi = \frac{\sigma_t}{c} \tag{3a}$$

then

$$c = \frac{\sigma_t}{\xi} \tag{3b}$$

where ξ is the ratio of tensile strength to cohesion. It will be shown later that ξ can be determined experimentally and is a function of plasticity index (10, 11, 13). Therefore, ϕ may be calculated by Eq. 2 or graphically by connecting points G and H as shown in Figure 2.

To establish the failure envelope, we should know the curve distance AG′, inasmuch as AG′ is part of the circle whose center is D and whose radius is R. The radius may be determined from the following formula (3):

$$R = \frac{q_u}{2} - \frac{\sigma_t \sin \phi}{1 - \sin \phi} \tag{4}$$

The circle shown in Figure 2 must pass through point A and be tangent to line GH at point G′. AG′H, therefore, represents the failure envelope of the material.

EXPERIMENTAL ANALYSIS

From the preceding discussion, it has been suggested that to determine the cohesion and internal friction angle for soils requires that two tests, double-punch and unconfined compression, be performed. In addition, the plasticity index of the material is required.

The double-punch test may be briefly described as follows: Two steel disks (punch) centered on both top and bottom surfaces of a cylindrical soil specimen are used, and the vertical load is applied on the disks until the specimen reaches failure. The tensile strength of the specimen can be calculated from the maximum load by the formula

$$\sigma_t = \frac{P}{\pi (KbH - a^2)} \tag{5}$$

where

σ_t = tensile strength,
P = load at failure,
b = radius of specimen,
H = height of specimen,
a = radius of disk, and
K = constant.

Recommended values of K are as follows:

Figure 1. Comparison of direct shear test, triaxial test, and proposed method.

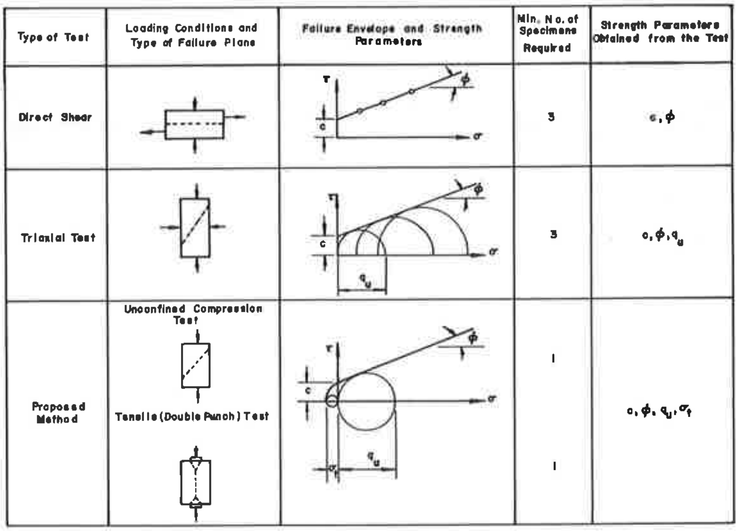


Figure 2. Modified Mohr-Coulomb failure envelope.

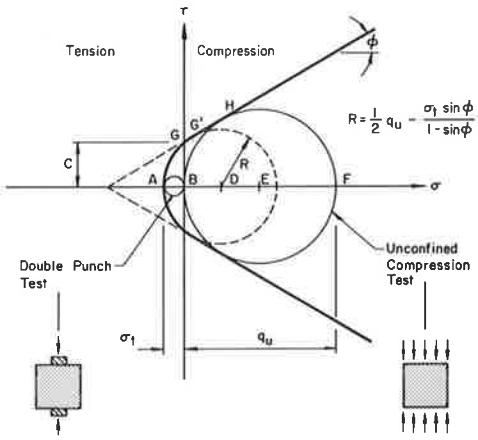


Figure 3. Comparison of tensile strengths determined by double-punch and split tensile tests.

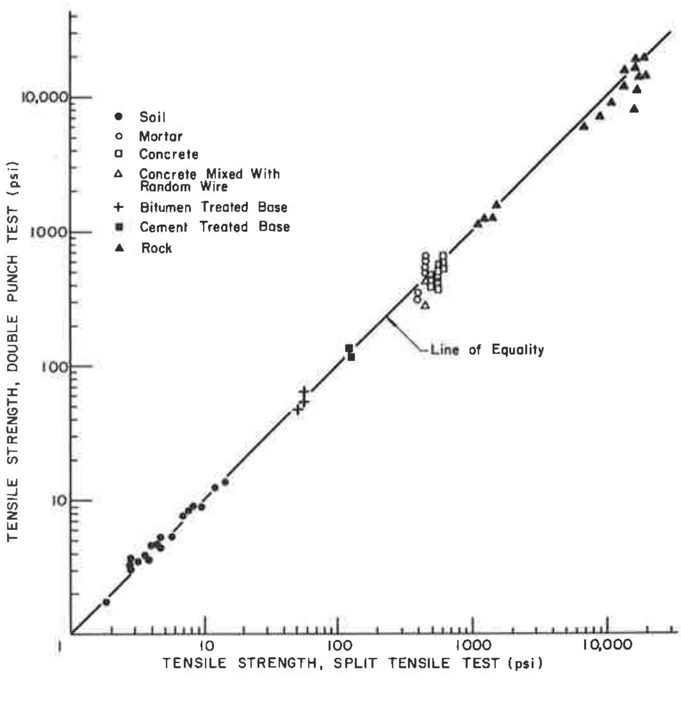


Figure 4. Relationship between tensile strength and plasticity index.

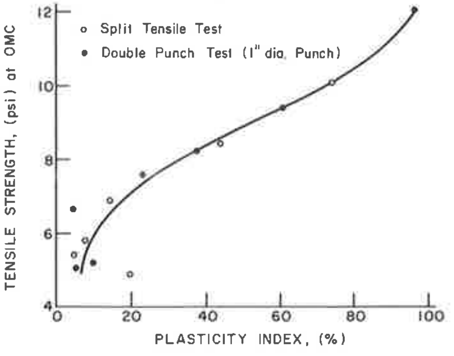


Figure 5. Tensile strength-cohesion ratio versus plasticity index for laboratory-prepared silty clay soils.

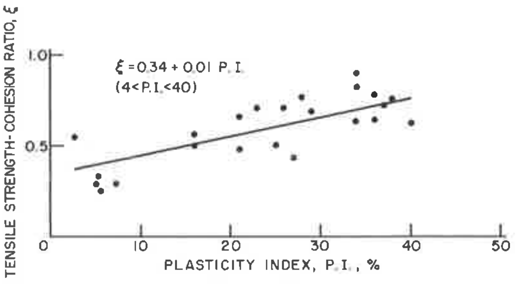


Figure 6. Comparison of computed cohesion and that measured by direct shear test.

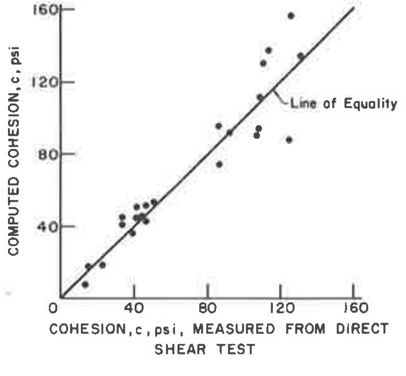
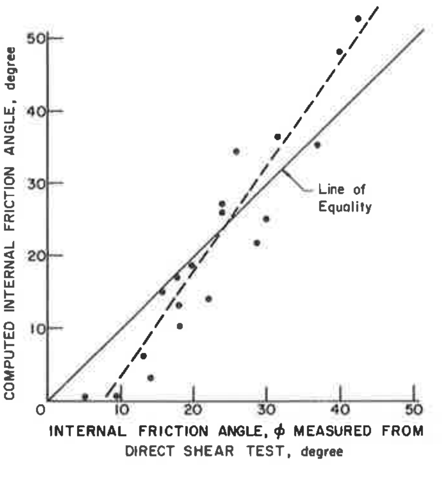


Figure 7. Comparison of computer internal friction angle and that measured by direct shear test.



Mold	Size (in.)	K-Value Soil	K-Value Stabilized Material
Proctor	4 × 4.6	1.0	1.2
CBR	6 × 7	0.8	1.0

The effect of sample-punch size and rate of strain on the results of tensile strength tests has been studied by Fang and Chen (8). They concluded that a height-to-diameter ratio of the specimen varying from 0.8 to 1.2 and a ratio of the diameter of the specimen to the diameter of the disk varying from 0.2 to 0.3 are suitable for the test. The rate of strain used for the double-punch test is the ASTM loading rate for unconfined compression tests (1).

For the unconfined compression test, the same size of specimen is used as for the tensile strength test. A 4- × 4.6-in. Proctor mold was used in the tests reported herein. The test procedure follows ASTM D 2116 (1).

TEST RESULTS AND DISCUSSIONS

The validity of tensile strength determined by the double-punch test has been confirmed by the split-tensile test. It has proved to be a simple and reliable test (6, 7, 8, 9). Figure 3 shows a comparison of the tensile strength of soils and other materials determined by double-punch and split-tensile tests. These materials include concrete (4, 5), mortar, bitumen- and cement-treated base (8), and rock (6). Good agreement between both tensile strength results is observed. Figure 4 shows the tensile strength versus plasticity index (PI). It can be seen that the tensile strength increases as plasticity index increases. Similar conclusions were drawn by Narain and Rawat (12).

It has been found experimentally that c is related to the tensile strength. For rocks (10) cohesion is equal to two times the tensile strength ($\xi = 0.5$). For soils it is shown that the relationship between cohesion and tensile strength varies with soil type (11, 13) Figure 5 shows the tensile strength-cohesion ratio versus PI for laboratory-prepared silty clay soils. The following equation expresses the linear relationship shown in Figure 5 for PIs between 4 and 40:

$$\xi = 0.34 + 0.01\ \text{PI} \qquad (6)$$

If the plasticity index is known, ξ can be determined from Eq. 6 and c can be determined from Eq. 3.

Comparisons between c and ϕ measured in direct shear tests (2) and computed from Eqs. 2 and 3 are shown in Figures 6 and 7. Good agreement is observed for both values.

SUMMARY AND CONCLUSIONS

1. A simple method for determining the undrained strength parameters, c and ϕ, of soils and stabilized materials from the tensile strength and unconfined compressive strength has been presented. The ϕ and c values can be determined from Eqs. 2 and 3 or graphically from Figure 2. For silty clay soils, the ξ value can be found from Eq. 6 if the PI of the material is known.
2. The unconfined compression and double-punch tests are both simple and easy to perform. No additional equipment is needed, and the tests can be conveniently performed in conjunction with routine CBR and compaction tests.
3. The proposed method for determining c and ϕ can save up to two-thirds the time necessary for conventional direct shear and triaxial shear tests.

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