PROBABILISTIC CONCEPTS AND THEIR APPLICATIONS TO AASHO INTERIM GUIDE FOR DESIGN OF RIGID PAVEMENTS

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Probabilistic design concepts have been applied to modify the AASHO Interim Guide for the Design of Rigid Pavement Structures; it is now possible to design for any specified level of reliability. The major objective for applying probabilistic concepts to pavement design is to make the design process sensitive to the many variabilities and uncertainties associated with the design, construction, and performance of rigid pavements. This provides a rational means of designing at varying levels of reliability depending on the pavement function. This method makes the design process closer to reality than the present deterministic method and, therefore, upgrades the current procedure. Variance models were developed for the performance equation of the AASHO Guide to predict variation in pavement performance due to statistical variations in traffic estimation, flexural strength and modulus of elasticity of concrete, concrete thickness, joint continuity, foundation modulus, initial serviceability index, and lack of fit of the AASHO performance equation. Estimates of the variations associated with each of the variables listed above are obtained by analysis of data from actual concrete pavement projects. A new revised nomograph is developed to include, among other things, a scale for reliability and a scale for the overall variance determined by the level of quality control exercised, variations associated with design parameters, and errors associated with traffic predictions. The variance models are also applied to determine the relative significance of the design factors associated with rigid pavement design and to quantify the effects of quality control on pavement performance.

•ONE of the most important areas of needed research with regard to the design of asphalt concrete pavement systems, as selected by a Highway Research Board advisory committee in a workshop held at Austin in December 1970, was stated as follows (25):

> So that designers can better evaluate the reliability of a particular design, it is necessary to develop a procedure that will predict variations in the pavement system response due to statistical variations in the input variables, such as load, environment, pavement geometry, and material properties including the effects of construction and testing variables. As part of this research it will be necessary to include a significance study to determine the relative effect on the system response of variations in the different input variables.

Designers of rigid pavements cannot help but rank this research need as one of the most important areas of their interest.

In this paper, probabilistic design concepts have been applied to the AASHO Interim Guide for the Design of Rigid Pavement Structures (1), making it now possible to design

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a pavement for any specified level of reliability. The major purpose for applying probabilistic concepts to pavement design is to make the design much closer to reality than the present deterministic method, therefore upgrading the current procedure. This will provide engineers with means to design a pavement thickness at a desired level of reliability or confidence (i.e., the serviceability of the pavement will not decrease below a specified minimum throughout the design period for which the pavement is being designed).

During the past few years, several investigators (2-9) have suggested that probabilistic concepts be applied to the design and analysis of portland cement concrete and other structures. An excellent summary of these concepts is given in a series of four articles published by the American Concrete Institute (10-13) on the development of a probability-based structural code. The basic reason for the development of such a code is that the loads and resistances in a concrete structure are variable or probabilistic in that they cannot be estimated exactly and they change from point to point in the structure and the foundation soil. For the same reason, random failures in concrete pavement have been observed for many in-service highways and airports as well as the pavements at the AASHO Road Test.

In pavement design, several empirical safety and judgment factors have been applied in the past to "adjust" for the many uncertainties involved without quantitatively considering the magnitude of the uncertainties involved. This generally has resulted in an overdesign or underdesign, depending on the situation and the level of applied safety factor. If the current deterministic pavement design procedures were modified so that the safety factors applied depended on the magnitude of the variation of concrete properties, supporting soil properties, smoothness variations, and uncertainties in traffic estimation, a more realistic design would be achieved. The pavement could be designed for a desired level of reliability, depending on its function and other decision factors.

Whereas the AASHO Guide (1) has been selected to demonstrate the applicability of probabilistic concepts, and at the same time to modify it so that engineers can achieve a pavement design at any desired level of reliability, the concepts have been presented in a general format applicable to any other design model. The method provides a powerful tool to researchers to perform sensitivity analysis of various parameters of a model, to study the effects of quality control and material variability on the output of the model, and to design a structure at any desired level of reliability.

Probabilistic design concepts are discussed first and followed by a short review of the equations involved in the AASHO Guide with emphasis on the uncertainties involved. A derivation of the necessary probability models and a characterization of variabilities associated with concrete pavement design are then presented. A brief sensitivity and quality control analysis illustrating the effects of variability of design factors on pavement performance is then followed by a presentation of the modified nomograph and design procedure. Finally, the results are illustrated by a practical example problem.

PROBABILISTIC CONCEPTS APPLIED TO PAVEMENT DESIGN

The sole underlying reason for formulating a probability-based design procedure is to better account for the variabilities and uncertainties associated with loadings and resistances of a structure. Current design practices are deterministic in that all design factors are assumed to be exact quantities, not subject to variations. However, based on personal judgment and experience, specific safety factors have always been used to account, at least partially, for these uncertainties. These safety factors have generally been the reductions of working strengths of materials, designing for heaviest wheel load, or, in some cases, a gross increase in structure thickness based on personal judgment.

In essence, the three basic types of variations associated with portland cement concrete pavement design that must be considered are as follows:

1. Variability within a design section (or project if the pavement structural design remains the same throughout) such as in flexural strength or subgrade support along a pavement;

2. Variability between assumed average design values and those obtained "as constructed" including, for example, the difference of average measured flexural strength from that specified in design and specifications and/or unforeseen variations to which the pavement may be subjected during its design life, such as traffic and gain or loss of material strengths; and

3. Variability due to the lack of fit of the empirical equation used in the design procedure.

The basic way in which uncertainties in concrete pavement design can be accounted for is through reliability concepts. There are several ways to define the reliability of a pavement. Because the AASHO Guide is based on the serviceability concept, the following definition of reliability is proposed.

Reliability, R, is in general the probability that the pavement will have an adequate serviceability level for the design period. Specifically it may be defined as the expected percentage of length along a project (if the design is constant) that will maintain a serviceability level greater than the specified minimum for a design period. The expected percentage length of failure along a project would correspondingly be defined as 1 - R. Because the failure phenomenon is mostly due to the application of repeated loads, the reliability of a pavement structure will be determined mathematically from the basic concept that a no-failure probability exists when the number of load applications, N, that a given pavement section can withstand to a specified minimum serviceability index is not exceeded by the number of load applications, n, actually applied. Failure as used in this paper refers to a condition of the pavement when the serviceability index drops below its specified minimum level and some sort of repair maintenance or replacement is needed to restore the serviceability.

If the serviceability index is measured along an in-service pavement at any interval, perhaps every 0.2 mile, it is found to vary considerably down the roadway. Each short section will reach failure at different number of load applications because of the variational nature of material strengths, pavement thickness, pavement smoothness, joint conditions, and foundation support. Because of the random nature of fatigue failures, it has been assumed that N is a random variable and the distribution of log N to failure is approximately normally distributed.

The number of load applications that will be applied to a given pavement has been considered as an exact number. However, the actual traffic in most cases has been different from the estimated traffic. It is also assumed that the forecasting error is log normally distributed because of the nature of estimating procedures and the errors associated with various factors that are considered.

Reliability is defined as the probability that N will exceed n as presented in the following expression:

$$R = P[(\log N - \log n) > 0] = P(D > 0) \tag{1}$$

where $D = \log N - \log n$.

Because log N and log n are both normally distributed, D will also be normally distributed. Using bars above the expressions to represent their mean values, we can write the following equation as

$$\overline{D} = \overline{\log N} - \overline{\log n} \tag{2}$$

The standard deviation of D will be computed as s_D by the following equation:

$$s_D = \sqrt{s^2_{\log N} + s^2_{\log n}} \tag{3}$$

where

$s_{\log N}$ = standard deviation of log N, and
$s_{\log n}$ = standard deviation of log n.

Reliability is given by the following expression:

$$R = P[0 < (\log N - \log n) < \infty] = P(0 < D < \infty) \tag{4}$$

The transformation that relates D and the standardized normal variable Z is

$$Z = \frac{D - \overline{D}}{s_D} \tag{5}$$

For D = 0,

$$Z = Z_o = -\frac{\overline{D}}{s_D} = -\frac{\overline{\log N} - \overline{\log n}}{\sqrt{s^2_{\log N} + s^2_{\log n}}} \tag{6}$$

For D = ∞,

$$Z = Z_\infty = \infty \tag{7}$$

The expression for reliability may be rewritten as

$$R = P(Z_o < Z < Z_\infty) \tag{8}$$

The reliability may now be determined very easily by means of the normal distribution table. The area under the normal distribution curve between the limits of $Z = Z_o$ and $Z = \infty$ gives the reliability of a design.

An example for the calculation of reliability will be given here. If we assume that $(\overline{\log N},\ s_{\log N}) = (7.100,\ 0.400)$ and $(\overline{\log n},\ s_{\log n}) = (6.500,\ 0.200)$,

$$Z_o = -\frac{7.100 - 6.500}{\sqrt{(0.4)^2 + (0.2)^2}} = -1.342$$

From normal distribution tables the area from -1.342 to ∞ is 0.91. Therefore, R is 91 percent.

The applicability of these concepts is demonstrated in the following sections.

AASHO RIGID PAVEMENT DESIGN MODEL

The serviceability trends of the rigid pavement sections at the AASHO Road Test led to the following equation (15), which predicts the number of 18-kip single-axle load applications W that a pavement will sustain:

$$\log W = 7.35 \log (D + 1) + 0.05782 + \frac{G}{\beta} \pm \Delta \tag{9}$$

where

$G = \log \frac{P1 - P2}{P1 - 1.5}$,

$\beta = 1 + \frac{16.196 \times 10^6}{(D + 1)^{8.46}}$,

D = concrete thickness (in inches),

P1 = initial serviceability index, and

P2 = serviceability index of the pavement after sustaining W applications of 18-kip single-axle load applications.

The term Δ associated with the prediction Eq. 9 is called the "lack-of-fit" error. It is defined here as the error produced by the prediction equation not containing all the necessary parameters of design because of a lack of data or because the equation has not been fitted properly through the available data. Some of the causes of this lack of fit are very obvious. An excellent example is the lack of data that resulted in showing that the effects of subbase thickness and amount of reinforcement were insignificant, and therefore these terms should be ignored. Other causes of this lack of fit are drastic linearizations, extrapolations, and assumptions used for the sake of achieving simplicity in data analysis. The errors also arise because of material variability, which will be dealt with in later sections. Errors arising because of material variabilities will be excluded from the definition of Δ.

To design rigid pavements with materials and conditions appreciably different from those that existed during the Road Test, an AASHO subcommittee on design (16) developed an additive term to Eq. 9 to account for different physical properties of pavement materials. The modified number of 18-kip single-axle applications, W_m, are given as

$$\log W_m = \log W + \log\left(\frac{f\delta}{215.625\ J} \times \frac{D^{0.75} - 1.1326}{\delta^{0.25}D^{0.75} - 18.423}\right)^b \tag{10}$$

where

$\delta = E/k$,
E = modulus of elasticity of concrete (in psi),
k = gross modulus of foundation reaction (in lb/in.^{3}),
f = flexural strength of concrete (in psi),
J = joint or crack efficiency coefficient, and
$b = 4.22 - 0.32\ P2$.

It should be noted that Eq. 10 has an additional factor J termed as joint or crack efficiency coefficient. In the AASHO Guide design equation, this factor was eliminated by using a value of 3.2. This led to the major restriction on the use of the AASHO Guide that only pavements similar to AASHO Road Test pavements (jointed, free corners with no load transfer devices) can be designed by the AASHO Guide. In this paper, this factor will be treated as a variable so that other kinds of pavements can also be designed.

VARIANCE MODEL

Variance in $\log W_m$ can be predicted in terms of the variances of individual variables affecting $\log W_m$. The general form of the variance model is given in the following paragraphs.

If x is a function of a series of variables $y_1, y_2, y_3, \ldots, y_n$, the variance V_x of response variable x can be written in the following general form in terms of individual variances $V_{y_1}, V_{y_2}, V_{y_3}, \ldots, V_{y_n}$:

$$V_x \approx \sum_{i=1}^{i=n}\left[\left(\frac{\partial x}{\partial y_i}\right)^2 V_{y_i}\right] \tag{11}$$

By applying the general Eq. 11 to the AASHO Guide model represented by Eq. 10, we obtain the following expression:

$$\begin{aligned} V_{\log W_m} = {} & \left(\frac{\partial \log W_m}{\partial f}\right)^2 V_f + \left(\frac{\partial \log W_m}{\partial J}\right)^2 V_J + \left(\frac{\partial \log W_m}{\partial P1}\right)^2 V_{P1} \\ & + \left(\frac{\partial \log W_m}{\partial E}\right)^2 V_E + \left(\frac{\partial \log W_m}{\partial k}\right)^2 V_k + \left(\frac{\partial \log W_m}{\partial D}\right)^2 V_D \\ & + \left(\frac{\partial \log W_m}{\partial \Delta}\right)^2 V_\Delta \end{aligned} \tag{12}$$

Each term in Eq. 12 represents the variance in $\log W_m$ contributed by the variable involved in that term. For example, variance in $\log W_m$ contributed by parameter f as denoted by C_f can be given as follows using Eq. 10:

$$C_f = \left(\frac{\partial \log W_m}{\partial f}\right)^2 V_f = \left(\frac{b \log_{10} e}{f}\right)^2 V_f \tag{13}$$

Similarly, variances in $\log W_m$ contributed by parameters J, P1, E, k, D, and Δ as denoted by C_J, C_{P1}, C_E, C_k, C_D, and C_Δ respectively can be given as follows:

$$C_J = \left(\frac{-b \log_{10} e}{J}\right)^2 V_J \tag{14}$$

$$C_{P1} = \left(\frac{\log_{10} e}{\beta}\right)^2 \left(\frac{1}{P1 - P2} - \frac{1}{P1 - 1.5}\right)^2 V_{P1} \tag{15}$$

$$C_E = \left(\frac{b \log_{10} e}{4}\right)^2 \left[\frac{1}{E} - \frac{D^{0.75}}{(\delta^{0.25} D^{0.75} - 18.423)\, k^{0.25} E^{0.75}}\right]^2 V_E \tag{16}$$

$$C_k = \left(\frac{b \log_{10} e}{4}\right)^2 \left[\frac{E^{0.25} D^{0.75}}{(\delta^{0.25} D^{0.75} - 18.423)\, k^{1.25}} - \frac{1}{k}\right]^2 V_k \tag{17}$$

$$C_D = \left[\frac{7.35 \log_{10} e}{D + 1} + \frac{(1.3739 \times 10^8) G}{\beta^2 (D + 1)^{9.46}} + \frac{0.75 b \log_{10} e}{(D^{0.75} - 1.1326)\, D^{0.25}} - \frac{0.75\, \delta^{0.25}\, b \log_{10} e}{D^{0.25} (\delta^{0.25} D^{0.75} - 18.423)}\right]^2 V_D \tag{18}$$

$$C_\Delta = V_\Delta \tag{19}$$

The total variance in $\log W_m$ (adding Eqs. 13 through 19) can be given as

$$V_{\log W_m} = C_f + C_J + C_{P1} + C_E + C_k + C_D + C_\Delta \tag{20}$$

Variances V_f, V_J, V_{P1}, V_E, V_k, V_D, and V_Δ are the squares of the respective standard deviations s_f, s_J, s_{P1}, s_E, s_k, s_D, and s_Δ associated with the variables. The standard deviation in log N as used in Eq. 3 can now be given as

$$s_{\log N} = \sqrt{V_{\log W_m}} \tag{21}$$

VARIABILITY CHARACTERIZATION

The probabilistic design approach requires estimates of the variations associated with the design parameters. Results have been obtained from actual concrete pavement projects to establish estimates of these variations. The design engineer should consider these as general values and should estimate the standard deviations of the design parameters for the specific project that is being designed. This section briefly summarizes available data of variations associated with concrete properties, joint load transfer, thickness, serviceability, subgrade support, lack of fit of design equation, and traffic forecasting.

Concrete Properties

The variations of concrete strength and modulus of elasticity have been measured in numerous field and laboratory studies in the past. The causes of these variations are attributed to two major factors: nonhomogeneous ingredients and nonuniform concrete production and placing. Property variations due to ingredients arise from changes in types and quantities of aggregates, cement, and water during concrete pavement construction. Variations due to concrete production occur during batching, mixing, transporting, placing, finishing, and curing of concrete.

Flexural strength data were obtained from 15 projects from the files of the Texas Highway Department and other sources. A plot of mean flexural strength versus standard deviation for each project is shown in Figure 1. An overall coefficient of variation of 10.7 percent was obtained from these data, a value that is very close to that obtained from the analysis of compressive strength data on 56 concrete projects throughout the United States. A typical histogram of the flexural strength data for one project is shown in Figure 2. The frequency distributions for most of the projects showed the flexural strength to be approximately normally distributed.

The modulus of elasticity variations were obtained from laboratory studies as well as from AASHO Road Test data. The standard deviation was found to increase with the mean as for compressive and flexural strengths. An overall coefficient of variation of 8.6 percent was obtained for the laboratory data and 12.8 percent for the AASHO Road Test concrete slabs.

Joint and Crack Continuity Coefficients

Structural efficiency of joints and cracks in providing deflection and stress relief at these locations in concrete pavements is a very critical factor in concrete pavement design. There are essentially three types of concrete pavements in wide use today: continuously reinforced pavements, jointed pavements without load transfer devices, and jointed pavements with load transfer devices. An empirical value of 3.2 was assigned to the J-term in the AASHO Guide for jointed pavements without load transfer units. This value was originally assigned by Spangler (17) in his empirical formula that represented the stress at a free corner. The stresses computed with this value matched closely with the stresses observed in warped corners at the Arlington Road Test. A value of 2.2 for this factor was suggested by Hudson and McCullough for continuously reinforced pavements (18). The J-term is not a measurable property but empirically represents the structural capability of a joint or a crack to transfer loads across them. The standard deviations were estimated by Treybig, McCullough, and Hudson (19) based on variability of deflections measured across joints. The resulting values are as follows:

Value of J	Description	Standard Deviation of J
3.2	Jointed pavement without load transfer units	0.13
2.2	Continuously reinforced pavements	0.19

Slab Thickness

Because of construction variations, the thickness of a concrete slab has always been found to vary throughout a project. Thickness of concrete is usually measured on construction projects for quality control purposes. Variability of the slab thickness is important in that localized, premature failures may occur, causing loss of serviceability and increase in pavement roughness.

Data showing the variation of concrete slab thickness were obtained for 27 pavement projects in four states, and the variances within the projects were pooled to obtain an overall average standard deviation for nominal pavement thicknesses. These standard deviations are given as follows:

Nominal Concrete Pavement (in.)	Standard Deviation	Number of Projects
8	0.32	14
9	0.29	8
10	0.29	5

As can be noted, the data do not indicate the standard deviation of concrete pavement thickness to be dependent on the average thickness for the range of thicknesses in common practice for concrete pavement construction.

An example of distribution of concrete slab thickness for an actual pavement as measured from cores taken from the pavement is shown in Figure 3. A normal distribution curve shown in this figure shows that the pavement thickness is approximately normally distributed along the project length.

Initial Serviceability Index

The initial serviceability index is a direct function of the smoothness of a pavement immediately after construction. A histogram showing the distribution of the initial serviceability indexes of 224 test sections is shown in Figure 4. The data approximately follow a normal distribution curve. Results show a standard deviation of 0.14, which may be considered a minimum value or the "best" obtainable value in the field because the Road Test pavements were constructed under controlled conditions. Normal pavement projects may be expected to have twice as great a standard deviation—about 0.3, as was measured for a newly constructed concrete pavement in Texas.

Foundation Modulus

Foundation support is represented by the modulus of reaction, k, in the AASHO Guide. This factor probably has the greatest variation because of the nonuniformity of soil support along and across a typical pavement. Any change in type of soil, compaction, moisture, and factors such as loss of support, erosion, and pumping causes variations in foundation support along a project and during numerous seasonal changes during the design life of a pavement.

Estimates of possible variation of the foundation modulus were obtained in rigid pavement projects in New York (20) and from the AASHO Road Test (15). The data show a general increase in standard deviation with increasing value of mean k and an overall coefficient of variation of 35 percent.

Lack of Fit of Design Equation

The basic design equation derived empirically from the AASHO Road Test data does not predict the exact life of all Road Test sections and therefore has a lack-of-fit error. This scattering of data is caused by the lack of fit of the equation (i.e., the equation does not contain all the necessary parameters or it is not in a proper form) and nonuniformity of design parameters in the Road Test pavements. The latter error is also called replicate error, which is the difference in pavement life between two replicate sections or pavements.

The total variation about the fitted design equation is given in the AASHO Road Test Report 5 (15). Total variance of errors in actual and predicted log of load applications is given as $V_{\Delta t}$.

$$V_{\Delta t} = (s_{\Delta t})^2 = (0.22)^2 = 0.0484$$

This total variance is made up of replication and lack-of-fit components:

$$V_{\Delta t} = V_{\Delta} + V_{\Delta(\text{replicates})} \tag{22}$$

The variance contributed only by lack of fit of the equation was determined by subtracting, from the total variance, the variance of error due to nonuniformity of design parameters at the Road Test. An analysis was made of 36 pairs of replicate sections used at the AASHO Road Test. The replicate error was estimated by taking the mean squared difference in log N of the replicate pairs at serviceability indexes of 3.5, 3.0, 2.5, and 2.0. Variance due to replicate errors was found as follows:

$$V_{\Delta(\text{replicates})} = 0.0131$$

Figure 1. Average flexural strength of concrete versus standard deviation for various projects.

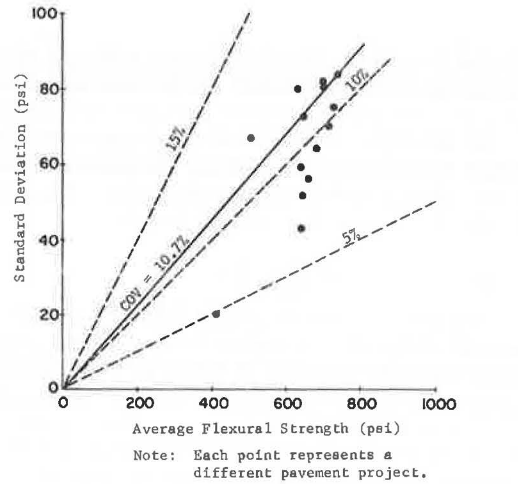


Figure 2. Frequency distribution of flexural strength for a concrete pavement project.

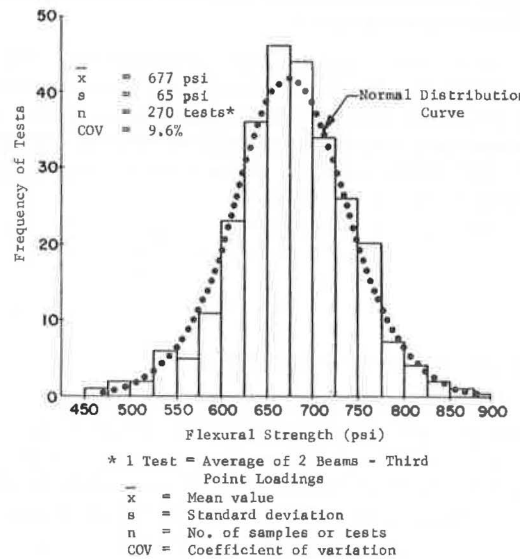
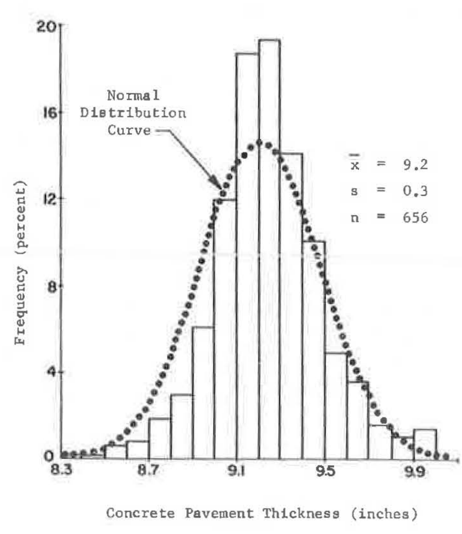


Figure 3. Frequency distribution of concrete slab thickness (22).



Therefore, the lack-of-fit variance was estimated as

$$V_{\Delta} = 0.0484 - 0.0131 = 0.0353 \tag{23}$$

It will be advantageous at this time to clarify the purpose of the preceding analysis, which in turn will clarify the overall purpose of this study. The objective is to determine for a new project the total variance of log N, which will consist of (a) total variance of error about the fitted design equation for the AASHO Road Test data (b) minus the variance due to nonuniformity of design parameters at the Road Test (c) plus the variance due to nonuniformity of design parameters for a new project.

Equation 23 encompasses items a and b, whereas item c is estimated by Eqs. 13 through 18.

Traffic Estimation

The AASHO Guide requires an estimate of the number of equivalent 18-kip single-axle load applications that the pavement will carry during its design life. There are many available methods to estimate this parameter, but each has considerable uncertainties associated with it. Basically, there are three types of uncertainties associated with forecasting this parameter:

1. Uncertainties involved in the estimation of total number of axles during pavement life, axle configurations, and axle weight distributions;
2. Uncertainties involved in equivalency factors used in the conversion of mixed traffic to equivalent applications of 18-kip single-axle loads; and
3. Uncertainties involved because of directional and lane distribution of traffic, lateral placement of loads, axle growth rate, and other unforeseen traffic increases during the life of a pavement.

A procedure has been developed (9) that gives an estimate of this error for the Texas Highway Department. Kentucky (21) analyzed the accuracy of its equivalent wheel load forecasting procedures and concluded the following: "...in some instances the actual accumulation may be somewhere between half and twice the predicted value but in the majority of cases will conform much closer." For general use, the following method is suggested to give an approximate estimate of the variance involved.

For a specific project, the average, maximum, and minimum number of 18-kip load applications that could pass during the pavement life should be estimated. This range should be selected to ensure that 95 percent of the time the values will fall between the maximum and minimum estimates. A conservative estimate of maximum and minimum equivalent applications could be approximately twice and half the average prediction respectively. The variance may then be calculated as follows (assuming n as log normally distributed):

$$V_{\log n} = \left[\frac{\log(\text{maximum applications}) - \log(\text{minimum applications})}{4}\right]^2 \tag{24}$$

SENSITIVITY ANALYSIS OF AASHO INTERIM GUIDE MODEL

Variance models developed in Eqs. 13 through 19 provide an excellent method for conducting a significance study to determine the relative effects of different input factors on the response of the model. The technique consists of determining the value of variance contributed by a variation in any input factor. This individual variance due to a factor, when computed as a percentage of the total variance contributed by all the factors, gives an estimate of the significance of that factor relative to the significance of the other factors.

The individual percentage-of-significance values for the factors were computed for 32 representative problems. Table 1 gives the percentage-of-significance value of each factor for each problem, the significance of a variable averaged over all the problems, and the range of such significance for each factor. The table demonstrates that the lack of fit of the AASHO data is the most important factor in the design model,

Figure 4. Frequency distribution of serviceability index after construction of AASHO Road Test pavements (23).

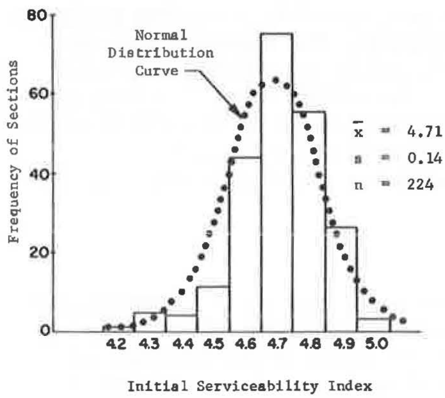


Table 1. Percentage-of-significance values.

Problem Number	Flexural Strength	Concrete Modulus	Concrete Thickness	Foundation Modulus	Continuity Coefficients	Initial Serviceability Index	Lack of Fit
1	24.1	0.8	14.2	2.1	19.9	0.3	38.6
2	26.0	0.4	8.1	1.1	21.5	1.4	41.6
3	26.0	0.3	8.5	0.8	21.5	1.4	41.6
4	23.0	3.7	6.8	10.3	19.0	0.3	36.9
5	23.8	2.3	9.5	6.4	19.7	0.3	38.0
6	25.7	1.1	6.7	3.0	21.2	1.3	41.1
7	25.8	0.7	7.4	2.0	21.4	1.3	41.3
8	27.0	1.3	14.3	3.5	10.5	0.3	43.2
9	27.0	0.9	15.8	2.4	10.5	0.3	43.1
10	29.3	0.5	9.2	1.3	11.4	1.5	46.9
11	29.3	0.3	9.6	0.9	11.5	1.5	46.9
12	25.6	4.1	7.6	11.4	10.0	0.3	41.0
13	26.5	2.6	10.6	7.2	10.4	0.3	42.4
14	28.9	1.2	7.6	3.3	11.3	1.5	46.2
15	29.1	0.8	8.3	2.3	11.4	1.5	46.6
16	23.9	1.1	13.7	3.1	19.8	0.1	38.3
17	23.9	0.8	15.1	2.1	19.8	0.1	38.3
18	26.2	0.4	8.3	1.1	21.6	0.5	41.9
19	26.2	0.3	8.6	0.8	21.6	0.5	41.9
20	22.9	3.7	7.5	10.2	18.9	0.1	36.7
21	23.6	2.3	10.3	6.4	19.5	0.1	37.8
22	25.9	1.1	6.8	3.0	21.4	0.5	41.4
23	26.0	0.7	7.5	2.0	21.5	0.5	41.7
24	26.7	1.2	15.3	3.5	10.4	0.1	42.8
25	26.7	0.8	16.8	2.3	10.4	0.1	42.7
26	29.5	0.5	9.3	1.3	11.5	0.6	47.3
27	29.6	0.3	9.7	0.9	11.6	0.6	47.3
28	25.5	4.1	8.3	11.3	9.9	0.1	40.7
29	26.3	2.6	11.4	7.1	10.3	0.1	42.1
30	29.1	1.2	7.7	3.4	11.4	0.6	46.6
31	29.4	0.8	8.4	2.3	11.5	0.6	47.0
32	24.1	1.1	12.8	3.1	19.9	0.3	38.6
Average	26.3	1.4	10.0	3.8	15.7	0.6	42.1
Range	22.9 to 29.6	0.3 to 4.1	6.8 to 16.8	0.8 to 11.4	9.9 to 21.6	0.1 to 1.5	36.7 to 47.3

followed by concrete flexural strength, continuity coefficient, concrete thickness, and foundation modulus.

EFFECTS OF QUALITY CONTROL

Quality control has always been a matter of great concern to the engineers supervising any construction project. The probabilistic analysis developed in this study made it possible to investigate the effects of varying amounts of quality control exercised in the construction of concrete pavements.

The effects of quality control are illustrated here for two areas of major concern: quality of concrete production and quality of pavement construction. The quality of concrete is represented, in the model, by its flexural strength and modulus of elasticity; the quality of pavement construction is represented by the three factors: concrete thickness, joint construction, and initial serviceability index (or smoothness of the pavement).

Based on the variability characterizations described in a previous section, the values of standard deviations were selected for each variable to represent poor, average, and good conditions of quality control. A 7-in. jointed pavement without load transfer devices was selected to illustrate the quality control analysis. It was found that this pavement could carry about 3.51 million 18-kip single-axle applications if no safety factor was used in the AASHO Guide. This, according to the variance analysis, corresponds to a reliability of 50 percent. With the help of the modified AASHO Guide design model, pavement thicknesses were computed that will carry 3.51 million applications at various levels of reliability up to 99.99 percent. The results obtained for the two examples are shown in Figure 5. The relative concrete thicknesses required for various levels of reliability are presented for poor, average, and good quality control conditions. The figure lists the standard deviations used for each variable to represent good and poor quality control with respect to concrete production and pavement construction. The figure also gives the standard deviations assigned to each variable to represent an average quality control.

It can be noted from Figure 5 that poor quality control requires higher pavement thickness for the same level of reliability than that required when an average quality control is exercised. Similarly, a good quality control can lead to a significant reduction in the required concrete thickness, or, in other words, a good quality control for a fixed pavement thickness can lead to having a higher level of reliability. Figure 5 also demonstrates that the effect on thickness of poor and good quality control varies with the level of required reliability.

Though the two examples previously given are studied in terms of required pavement thicknesses, monetary values can be assigned to these thicknesses, thus providing (for the first time in pavement design and construction) a powerful tool to quantitatively study the economics of quality control relative to pavement performance.

EFFECTS OF LACK-OF-FIT ERROR

Statistically derived relations always possess a certain lack-of-fit error because of the scatter of data around the developed regression equation. Pavement engineers have always used numerous empirical equations for the design of pavement structures but have never considered the lack-of-fit errors. Rather, they have adopted a tendency to consider these equations as completely deterministic. This has led to an inadequate designing process.

Lack-of-fit error associated with the AASHO Guide model has already been described in an earlier section. For the sake of demonstrating the significance of this error, Figure 6 is presented in which the required pavement thicknesses have been determined with and without the consideration of the lack-of-fit error. The example data and procedure of design are the same as those used for Figure 5 for the average quality control. As can be noted, significant differences in required thicknesses result for the pavements designed at various reliability levels, with and without lack-of-fit error.

Figure 5. Relative thicknesses required at various confidence levels for poor, good, and average quality control for concrete and construction quality.

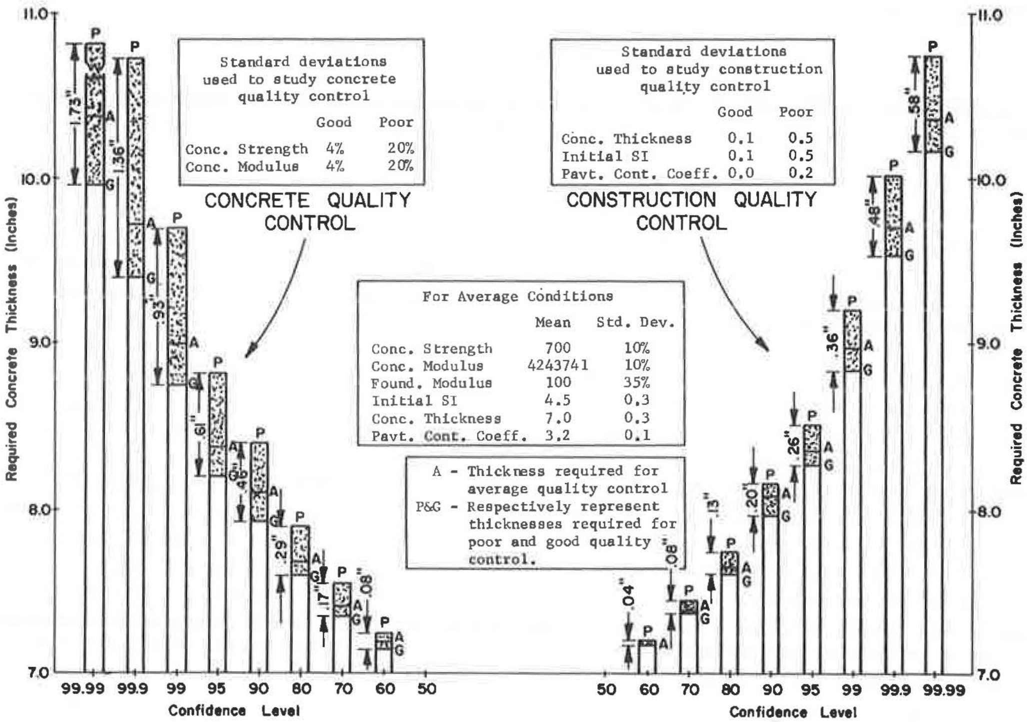
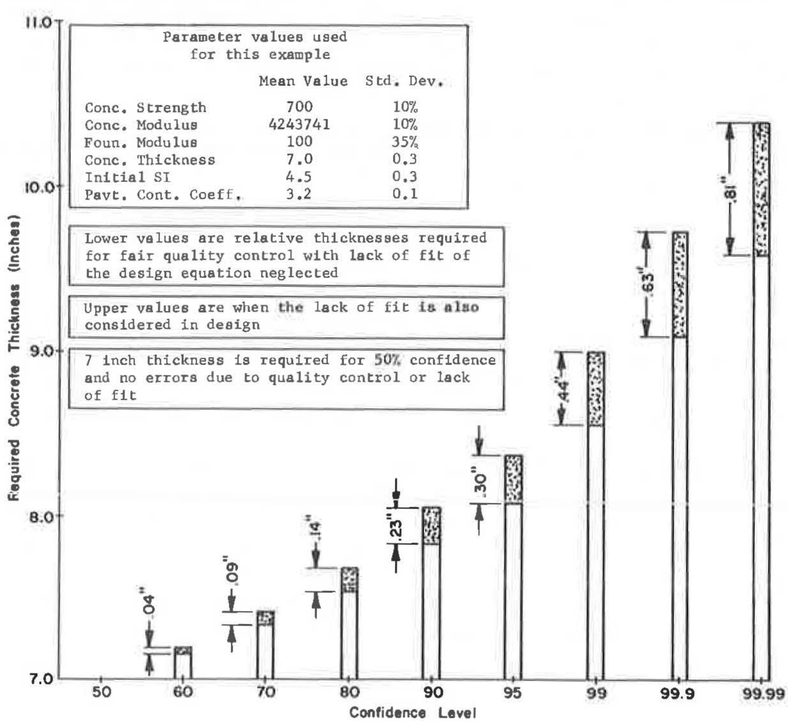


Figure 6. Relative thicknesses required at various confidence levels with and without the consideration of lack-of-fit error.



REVISED NOMOGRAPH

Based on the variance model developed in this research study, the AASHO Guide nomograph is modified (Fig. 7). The nomograph makes it possible to design a pavement thickness at any level of reliability taking into account the uncertainties associated with various parameters. This is achieved by two scales shown in the nomograph, a variance scale and a reliability scale. The two scales are combined in such a way that the designed thickness will stand a good chance of lasting the required number of applications with a reliability for which the pavement is designed.

Variance (excluding the variance due to traffic) can be theoretically computed by using Eqs. 13 to 20. However, Figure 8 has been developed so that the value of variance can readily be obtained. This figure has been developed using variability characterization and judgment factors and represents average conditions of scatter in material properties and other parameters. The following values of variability have been used to develop this figure:

1. Flexural strength, coefficient of variation = 10 percent;
2. Concrete modulus, coefficient of variation = 10 percent;
3. Concrete thickness, standard deviation = 0.3 in.;
4. Foundation modulus, coefficient of variation = 35 percent;
5. Initial serviceability index, standard deviation = 0.3; and
6. Continuity coefficient, standard deviation = 0.1 for JCP without load transfer units and 0.2 for CRCP and JCP with load transfer units.

The figure does not contain initial serviceability index and concrete modulus as variables because the sensitivity analysis showed that the effects of variabilities in these parameters are insignificant. Minimum serviceability index is a basic design criterion, and therefore no variation in this factor is considered.

Although the figure is developed from the best available data in connection with parameter variability, and therefore can be effectively used for design, the designers are encouraged to develop similar figures by using Eqs. 13 to 20 to suit their own construction conditions and quality controls.

The reliability scale presents 50 to 99.99 percent reliability. The designer can use any reliability in design. It has been observed during the use of this and other similar systems that the designers prefer to select this number based on their own judgments and the importance of highway facility under design. A thorough investigation of the old practice of using working stress as 0.75 times the flexural strength demonstrated that it corresponded to having reliability levels between 90 to 95 percent with the modified nomograph.

USE OF THE NOMOGRAPH

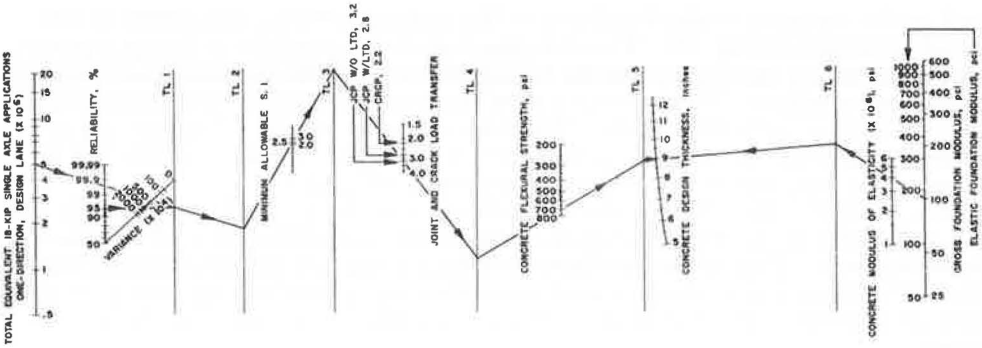
Concrete thickness can be designed with the use of the revised nomograph (Fig. 7) by going through the following steps:

1. Determine the overall variance by either of the following methods: Estimate standard deviation associated with each design factor, determine total variance according to Eq. 20, and add to it the variance due to error in traffic prediction; or estimate total variance by Figure 8 (an initial estimate of the required thickness will be needed to use this table), and add to it the variance due to error in traffic prediction using Eq. 24.

2. Estimate the design reliability level based on experience and judgment. The design reliability should depend on the "consequence of failure" to provide an adequate performance throughout the design period. The consequence of failure should be judged by user delay and accident costs during rehabilitation operations and other socioeconomic and political effects. Thus, design reliability levels should be selected based on consideration of all these factors and not only the initial construction cost.

3. Select concrete thickness from nomograph in the following manner: (a) Join the reliability and variance to intersect at the TL 1; (b) draw a line through traffic and the point already established on TL 1 in step a to intersect TL 2; (c) go to TL 3 from TL 2

Figure 7. Nomograph for concrete pavement design at desired reliability level.



EXAMPLE PROBLEM

Traffic = 5,000,000 single axle equivalent 18-kip applications
Variance = 1,000 (corresponds to average quality control)
Minimum Allowable Serviceability Index = 2.5
Joint and Crack Load Transfer Coefficient = 3.2
(JCP w/o load transfer device - LTD)
Concrete Flexural Strength = 700 psi
Concrete Modulus of Elasticity = 4,000,000 psi
Gross Foundation Modulus = 100 pci

REQUIRED CONCRETE THICKNESS

Reliability	90	95	99	99.9	99.99
Thickness, inches	8.6	8.9	9.7	10.4	11.5

Concrete thickness required by original interim design guide using working flexural stress of .75 × 700 = 8.75 inches (corresponds to 92.5 percent reliability)

Figure 8. Variances (10^4) for use in modified AASHO Guide nomograph.

Concrete Flexural strength, psi	Foundation Modulus, pci	JCP Without Load Transfer Devices				JCP With Load Transfer Devices				CRCP			
Concrete Thickness, inches →		6	8	10	12	6	8	10	12	6	8	10	12
600	25	775	718	684	661	866	809	775	752	935	879	845	822
	100	779	722	688	664	870	813	779	755	940	883	849	825
	300	821	742	700	673	912	833	791	764	982	903	861	834
	600	918	777	719	685	1009	868	810	776	1078	938	880	846
700	25	775	718	684	661	866	809	775	752	936	879	845	821
	100	778	721	687	664	869	812	778	755	938	882	848	824
	300	810	737	698	671	901	828	788	762	971	898	858	832
	600	886	766	714	682	977	857	805	773	1047	927	874	843
800	25	775	718	684	661	866	808	775	752	936	879	845	821
	100	776	720	687	663	867	811	778	754	937	881	847	824
	300	803	734	696	670	894	825	787	761	964	895	856	830
	600	865	759	710	679	956	850	801	770	1026	920	870	840
900	25	776	718	684	661	867	809	775	752	937	879	845	821
	100	776	720	686	663	867	811	777	754	936	881	847	824
	300	797	731	694	669	888	823	785	760	958	892	855	829
	600	849	753	706	677	940	844	797	768	1010	914	867	838

through minimum serviceability level; (d) go to TL 4 from TL 3 through joint and crack load transfer coefficient; (e) go to TL 5 from TL 4 through average concrete flexural strength (do not use any safety factor); (f) start now on the extreme right-hand side of the nomograph, and draw a line through foundation modulus and concrete modulus of elasticity to intersect TL 6; and (g) join the two points established in steps e and f on TL 5 and TL 6 respectively (this joining line will pass through concrete thickness scale and will intersect it at the required design concrete thickness).

EXAMPLE PROBLEM

An example problem solved for an actual project is demonstrated on the nomograph (Fig. 7). A jointed concrete pavement without load transfer devices has been designed to carry an expected 5 million equivalent 18-kip axle applications. Other pertinent input data are shown in the figure. The total overall variance used for this project is estimated to be 0.1, and a reliability level of 95 percent is used for the design. The required concrete thickness was estimated to be 8.9 in.

Concrete thicknesses required for reliability levels of 90, 99, 99.9 and 99.99 were also computed as shown in Figure 8. The respective thickness values were 8.6, 9.7, 10.4, and 11.5 in. The design was compared with the original method by designing the pavement using working stress equal to 0.75 times the flexural strength and the original nomograph. The required thickness was computed as 8.75 in., which corresponded to a reliability level of 92.5 percent for the new method.

CONCLUSIONS AND RECOMMENDATIONS

As concisely stated by Finn (24): "It is the role of research to improve and quantify and control the reliability factor in order to provide the most economic balance between performance requirements and costs." The procedure presented in this paper has attempted to further knowledge in achieving this overall goal. The reliability method of design has shown excellent promise, and it appears possible to incorporate the concepts of reliability into any design model as well as to conduct a significance study of the parameters of the model and to study the effects of quality control on pavement performance.

It is recommended that the modified nomograph be used for design so that it will be possible to consider variabilities associated with the design parameters and the lack-of-fit errors associated with the AASHO design equation.

ACKNOWLEDGMENTS

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The opinions, findings, and conclusions expressed in this publication are those of the authors and not necessarily those of the Federal Highway Administration.

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DISCUSSION

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In their report the authors deal with an extremely important problem: the reliability of the design and thus the behavior of highway pavements at an adequate serviceability index.

It should be appreciated that the study presented in this report was generally well conducted, and the results are very interesting from both theoretical and experimental points of view. It should also be noted that this study is one of the first done in the area of the stochastic design of highway pavements.

Although the study is remarkably accurate in all aspects, I would like to express some reservations and make some suggestions.

The authors assumed that both total traffic during the design period and forecasting error are log normally distributed, but it is noted that this assumption is valid only for a rather small number of cases.

It should also be stated that, if the variables follow the normal distribution, it is not theoretically possible to also follow the log-normal distribution. From a practical point of view, one can accept this approach only if the range of the values is very small.

The variance model that was adopted can be generally used only if either all the exponents of the variables are equal to one or there are no more than two variables with exponents of two. Usually, when the exponents of the variables are greater than one, the principles and methods of numerical calculus are used, which leads to quite different variance models.

The preceding statements are true only if all the variables are independent of one another. If not (and this is the case of Young's modulus that is related to Poisson's ratio or to the deviator stress for granular materials), only a step-by-step approach using the techniques of numerical calculus could be used to reach a solution.

Finally, the variance model differs among design methods, and only very general principles can be used.