# RELATIVE EFFECTS OF STRUCTURAL VARIABLES ON THE PERFORMANCE OF CONTINUOUS PAVEMENTS

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This paper describes a sensitivity analysis performed to establish the relative importance of structural variables on the performance of continuously reinforced concrete pavements. The experiment design for this study consisted of three basic variables: slab bending stiffness, subgrade modulus, and crack spacing. The discrete-element method of slab analysis was the mechanistic tool applied to obtain slab responses, i.e., deflections, principal moments, and stresses. For the range of variables studied, the analysis of variance showed that the most significant variables, which explained about 90 percent of the variation in deflection and principal moment (stress) responses, were slab bending stiffness and modulus of subgrade reaction. Although the first variable made a higher contribution to principal moments than to deflections, subgrade modulus had a contrasting effect. The orthogonal polynomial breakdown indicated that in a logarithmic model the linear effect of both subgrade modulus and slab bending stiffness is highly significant. Furthermore, interactions between these two design variables do occur, indicating that variations in deflections and principal moments are not defined by the main effect of design variables alone. The comparison between slab responses for an uncracked slab and a slab with 90 and 100 percent reduction in bending stiffness at crack locations indicated the importance of cracks and crack width on the behavior of continuously reinforced concrete pavements. As cracks widen to approach the hinge case, slab deflections increase significantly, but no appreciable drop is experienced in the principal moments. Indeed, cracks as narrow as possible are desirable for the successful performance of continuous pavements.

•A GENERAL discrete-element method for solution of discontinuous plates and slabs has been developed by Hudson and Matlock (1) and Stelzer and Hudson (2). The method is based on a physical model representation of a plate or slab by bars, springs, and torsion bars that are grouped in a system of orthogonal beams. Computer programs developed for the method are designated by the acronym SLAB. These programs can handle complex problems with combinations of load and a variety of discontinuities (cracks and joints) and support conditions.

Extensive use of SLAB programs has been made with two-way floor slabs that are continuous over many supports (3) and in the analysis of rigid pavements (4, 5). This paper describes the application of SLAB methods in a study of the relative importance of the structural variables associated with the design of continuous pavements. These variables include slab thickness, concrete modulus, modulus of subgrade reaction, loading position, and crack spacing.

## FACTORIAL DESIGN EXPERIMENT

A sensitivity analysis is a procedure to determine the change in a dependent variable due to a unit change in an independent variable. It can be used to evaluate the effect of

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a certain number of variables in the system and the interactions among them. In this research, a full factorial of the variables (Fig. 1) was evaluated. Both maximum deflections and principal moments were computed for variations in each variable in each block of the factorial (Fig. 1). Two solutions were made for each block: one for the loads on a crack and the second for loads between cracks. Figure 2 shows the loading and crack-spacing pattern for the 4- and 10-ft cases only. This work was performed for a 24- by 40-ft slab size and for two 9,000-lb wheel loads located at 2 and 8 ft from the slab edge respectively.

The number of slab problems to be solved can be decreased by combining the modulus of elasticity of concrete E and the slab thickness t into the bending stiffness factor, namely $\frac{Et^3}{12(1 - \mu^2)}$. From the combination of the low, medium, and high levels of both E and t, the values shown in Figure 1 resulted for a Poisson's ratio of 0.20.

## RESULTS AND ANALYSIS

Continuously reinforced concrete pavement is a type of pavement that takes care of volume change stresses by developing a regular pattern of very fine transverse cracks. These cracks or discontinuities have been analyzed theoretically by Abou-Ayyash, Hudson, and Treybig (4). It has been shown that the bending stiffness at cracked sections is reduced by 80 to 93 percent of the uncracked stiffness value. In this study, for the closed-crack case, a 90 percent reduction in bending stiffness is applied at crack locations, which corresponds to a 0.55 percent longitudinal reinforcement. The results from the 90 percent reduction were compared with the uncracked and hinge cases also.

### Case 1: SLAB Program Results of Loads on the Crack

Maximum values of slab downward deflections and principal moments are shown in Figure 3.

The location of the maximum deflection was dependent on the relative magnitudes of the slab stiffness and the subgrade modulus. For the low level of stiffness, maximum deflection occurred 2 ft from the pavement edge, i.e., directly under the exterior 9,000-lb load. As stiffness increased, the maximum deflection was at the edge of the slab for low values of subgrade modulus. The effect of the subgrade was even more significant on high values of stiffness, as shown in Figure 3, where the maximum deflection occurred at the pavement edge for the low and medium levels of the subgrade modulus. So far as the principal moments are concerned, the maximum value was always under the interior load, which was 8 ft from the pavement edge.

Effect of Crack Spacing—Transverse cracks in continuously reinforced concrete pavements occur randomly, and in most cases they extend the whole width of the pavement. One of the principles of design of this pavement type is to provide sufficient reinforcement to keep the cracks tightly closed. In this study it was assumed that a very slight curvature is needed to bring the two parts of the slab in touch and hence allow the transfer of bending.

In light of this behavior, the variation in maximum deflection and principal moment with crack spacing and slab bending stiffness for a subgrade modulus of 40 lb/in.$^3$ is shown in Figure 4. As noted, there is a slight change in both responses as the crack spacing increases over the range studied. However, it is worthwhile to note that these results were based on the same value of bending stiffness reduction at the crack location, whereas in reality the stiffness should vary with crack width. As is demonstrated later in this study, as crack width increases, and hence the reduction in stiffness increases, the influence of crack spacing becomes more important. Similar results were obtained for the other levels of the modulus of subgrade reaction.

Effect of Modulus of Subgrade Reaction—Modulus of subgrade reaction, as defined by Westergaard and others, plays an important role in the evaluation of deflections and stresses in pavement slabs and plates resting on soils. In light of the very small effect of crack spacing, nine deflection values were determined by averaging the deflection values corresponding to the three levels of stiffness and the three levels of subgrade

**Figure 1. Factorial variables.**

| Modulus of subgrade reaction, lb/in²/in. | Crack Spacing, ft | Slab bending stiffness, (lb-in²)/in.: $20 \times 10^6$ | $150 \times 10^6$ | $1125 \times 10^6$ |
|---|---|---|---|---|
| 40 | 4 | × | × | × |
| | 6 | × | × | × |
| | 8 | × | × | × |
| | 10 | × | × | × |
| 200 | 4 | × | × | × |
| | 6 | × | × | × |
| | 8 | × | × | × |
| | 10 | × | × | × |
| 1000 | 4 | × | × | × |
| | 6 | × | × | × |
| | 8 | × | × | × |
| | 10 | × | × | × |

**Figure 2. Pavement loading and crack spacing pattern.**

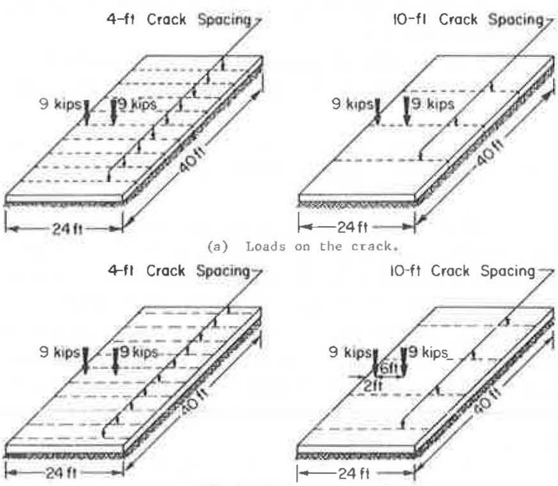


**Figure 3. Maximum values of deflection (90 percent reduction, loads on the crack).**

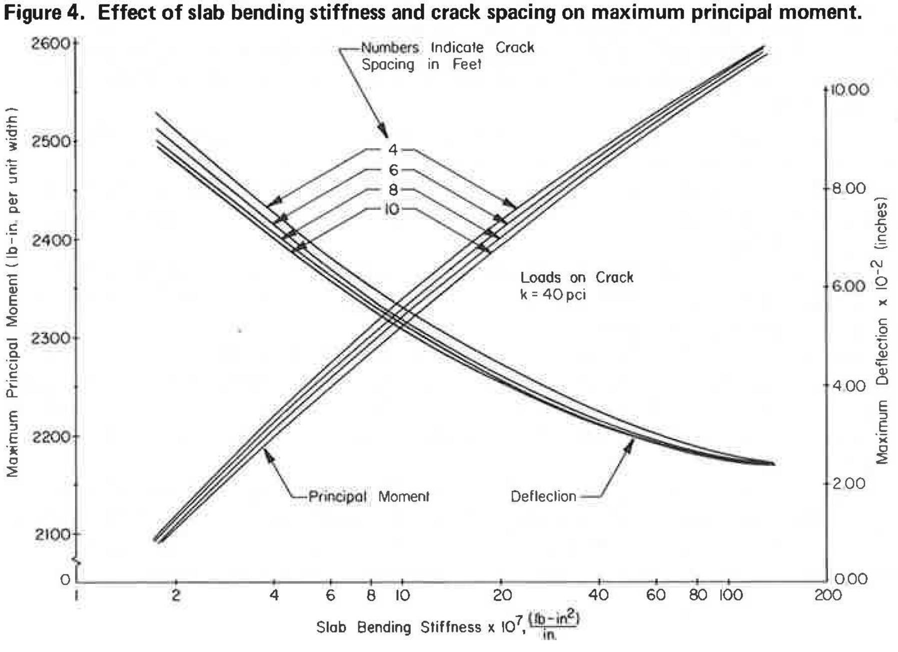
| Modulus of subgrade reaction, lb/in²/in. | Crack Spacing, ft | Slab bending stiffness, (lb-in²)/in.: 20 x 10^6 | 150 x 10^6 | 1125 x 10^6 |
|---|---|---|---|---|
| 40 | 4 | 0.0934<br>2117 | 0.0490*<br>2384 | 0.0248*<br>2587 |
| | 6 | 0.0886<br>2103 | 0.0485*<br>2374 | 0.0246*<br>2583 |
| | 8 | 0.0864<br>2099 | 0.0475*<br>2364 | 0.0245*<br>2582 |
| | 10 | 0.0859<br>2099 | 0.0464*<br>2357 | 0.0245*<br>2578 |
| 200 | 4 | 0.0330<br>1839 | 0.0166<br>2182 | 0.0086*<br>2430 |
| | 6 | 0.0321<br>1835 | 0.0158<br>2165 | 0.0085*<br>2432 |
| | 8 | 0.0320<br>1835 | 0.0153<br>2159 | 0.0084*<br>2413 |
| | 10 | 0.0320<br>1835 | 0.0151<br>2158 | 0.0083*<br>2406 |
| 1000 | 4 | 0.0131<br>1436 | 0.0057<br>1920 | 0.0029<br>2240 |
| | 6 | 0.0131<br>1436 | 0.0055<br>1914 | 0.0028<br>2223 |
| | 8 | 0.0131<br>1436 | 0.0054<br>1914 | 0.0027<br>2215 |
| | 10 | 0.0131<br>1436 | 0.0054<br>1914 | 0.0026<br>2213 |

+ All values occurred 8 feet from the pavement edge.

* Maximum deflection was at the pavement edge; otherwise, it was under the load, 2 feet from the edge.

NOTE: The upper value in each block is the deflection and the lower value is the principal moment.

**Figure 4. Effect of slab bending stiffness and crack spacing on maximum principal moment.**



modulus. In other words, an average value of deflection was obtained corresponding to an average value of crack spacing.

The average values of maximum principal moment were determined on the same basis; the logarithmic influence of subgrade modulus on deflections and moments is shown in Figure 5. As shown, the effect of the subgrade modulus on deflection is higher than on principal moments. Slab deflection experiences an important and significant drop as subgrade modulus increases from low to medium levels. However, this deflection decrease tends to level off as the subgrade reaction exceeds the medium level and approaches the high side. This implies that, for the loading condition studied, moderate values of subgrade modulus are quite satisfactory. Furthermore, there is about a 10 percent drop in the value of the principal moment as k varies from one level to another.

Effect of Slab Bending Stiffness—As mentioned previously, the slab bending stiffness per unit width is defined by $\frac{Et^3}{12(1 - \mu^2)}$. Obviously, the contribution of the thickness t to the magnitude of the stiffness term is more than the concrete modulus E.

The effect of bending stiffness on deflections and principal moments is shown in Figure 5. For the whole range of subgrade moduli, it can be seen that the influence of stiffness on principal moment is greater than on deflection. It is worthwhile to note that, for low values of subgrade modulus, the decrease in deflection as stiffness increases is highly significant and that, as the subgrade modulus increases, the influence of stiffness levels off. The logarithmic effect of the stiffness term is shown in a way similar to that for subgrade modulus.

## Case 2: SLAB Program Results of Loads Between Cracks

In the case when the loads acted between cracks, the effect of crack spacing was greater than when the loads were on the crack. Except for this, results for these two load placements were quite similar. As expected, deflection values were lower for the case when the loads were between cracks, whereas higher values were obtained for principal moments.

Figure 6 shows the maximum values of deflections and principal moments respectively. Values of principal moment were 2 ft from the pavement edge for all ranges in the pavement design variables encountered. As was the case when the loads were on the crack, maximum deflection occurred either under the exterior load or at the pavement edge, depending on the relative values of the slab stiffness and the subgrade modulus.

Effect of Crack Spacing—The influence of crack spacing and slab stiffness on maximum deflections and principal moments for subgrade moduli of 40 and 200 lb/in.$^3$ is shown in Figures 7 and 8. The effect of crack spacing on deflections is slight or practically negligible, whereas changes in principal moment for the low and medium levels of k are quite considerable.

Furthermore, this change in principal moment due to the spacing of the cracks increases with an increase in stiffness. The percentage of increase in the maximum moment between the 10- and 4-ft crack spacing (based on the 10-ft value) is about 18 percent. By comparing Figures 7 and 8, it can be seen that this percentage increase in the moment drops as k increases.

Effect of Subgrade Modulus and Bending Stiffness—For the purpose of demonstrating the effect of bending stiffness and subgrade modulus, average values of deflections and principal moments are shown in Figure 9.

The essential importance of the subgrade modulus in determining the amount of slab deflection is very well illustrated. Similar to the case where the loads are on the crack, the rate of change in deflection decreases as the subgrade modulus increases. About a 20 percent drop is experienced in the magnitude of the principal moment as k increases from one level to the next.

Although the subgrade modulus shows a higher contribution in the determination of deflection than principal moment, slab bending stiffness possesses a contrasting effect except for its effect on deflections for low values of subgrade modulus. Again, the logarithmic effect of stiffness as well as subgrade modulus is demonstrated.

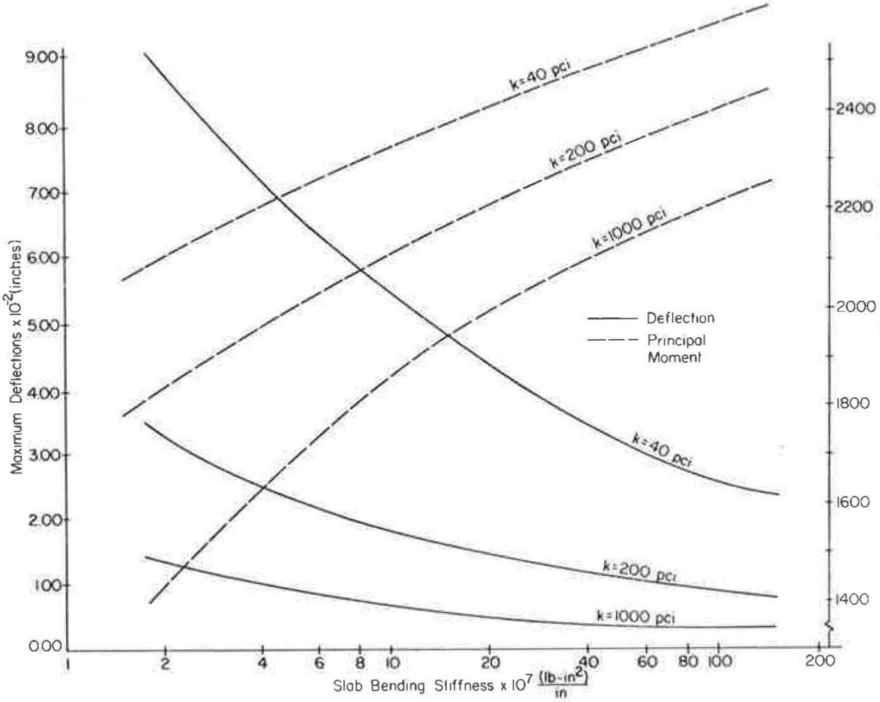



**Figure 5. Influence of slab bending stiffness and subgrade modulus on maximum deflections and principal moments for loads on the crack.**

**Figure 6. Maximum values of deflection (90 percent reduction, loads between the cracks).**

| Modulus of subgrade reaction, lb/in²/in. | Crack Spacing, ft | Slab bending stiffness, (lb-in²)/in.: 20 x $10^6$ | 150 x $10^6$ | 1125 x $10^6$ |
|---|---|---|---|---|
| 40 | 4 | 0.0606<br>2239 | 0.0383*<br>2791 | 0.0223*<br>3585 |
| | 6 | 0.0589<br>2406 | 0.0354*<br>3049 | 0.0210*<br>3901 |
| | 8 | 0.0600<br>2427 | 0.0344*<br>3226 | 0.0203*<br>4130 |
| | 10 | 0.0604<br>2391 | 0.0345*<br>3320 | 0.0198*<br>4297 |
| 200 | 4 | 0.0220<br>1826 | 0.0109<br>2338 | 0.0070*<br>2940 |
| | 6 | 0.0225<br>1898 | 0.0104<br>2541 | 0.0064*<br>3206 |
| | 8 | 0.0223<br>1853 | 0.0105<br>2603 | 0.0062*<br>3398 |
| | 10 | 0.0220<br>1835 | 0.0106<br>2581 | 0.0062*<br>3518 |
| 1000 | 4 | 0.0098<br>1444 | 0.0038<br>1993 | 0.0020*<br>2441 |
| | 6 | 0.0097<br>1401 | 0.0038<br>2037 | 0.0019*<br>2666 |
| | 8 | 0.0096<br>1395 | 0.0038<br>1992 | 0.0019*<br>2767 |
| | 10 | 0.0096<br>1396 | 0.0037<br>1962 | 0.0019*<br>2771 |

+ All values occurred 2 feet from the pavement edge.

* Maximum deflection was at the pavement edge; otherwise, it was under the load, 2 feet from the edge.

NOTE: The upper value in each block is the deflection and the lower value is the principal moment.

**Figure 7. Effect of slab bending stiffness and crack spacing on deflections and principal moments (k = 40 lb/in.$^3$).**

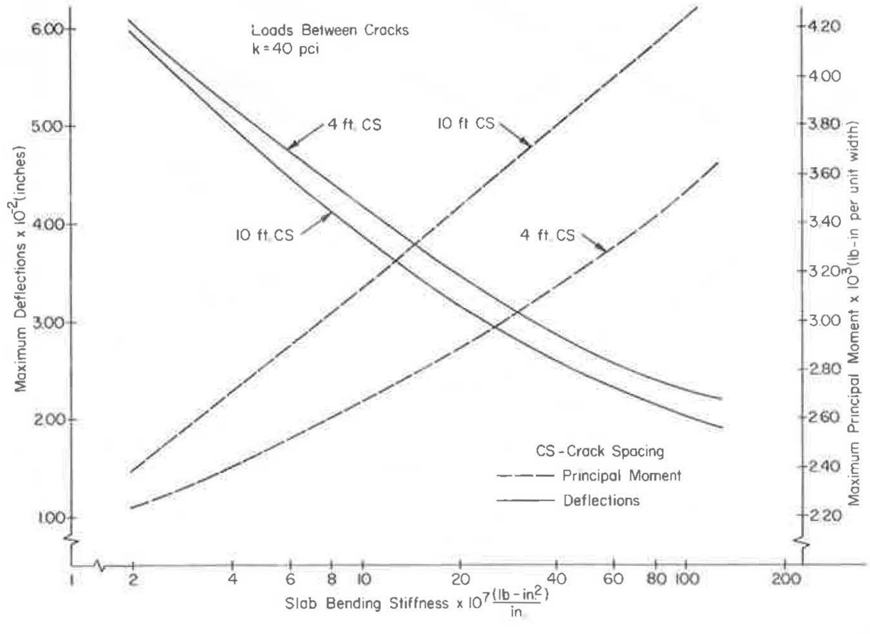
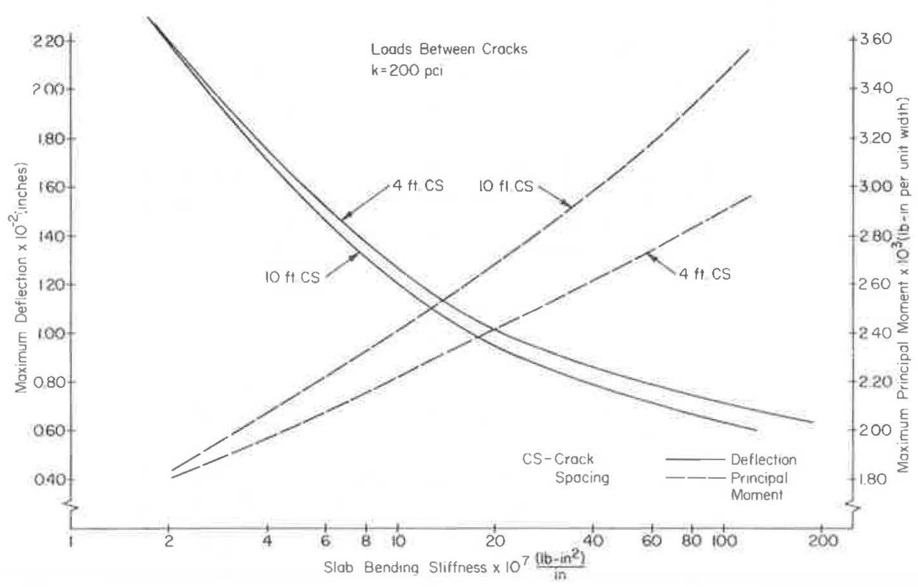



**Figure 8. Effect of slab bending stiffness and crack spacing on deflections and principal moments (k = 200 lb/in.$^3$).**



## Analysis of Variance

To determine the sensitivity of the rigid pavement design variables, we made an analysis of variance (ANOVA) on the maximum values of deflections and principal moments for both load positions. The ANOVA considered the three design variables encountered, namely, slab bending stiffness, crack spacing, and subgrade modulus.

Table 1 gives the average contribution of the main effects of each variable and their interactions on deflections and principal moments for the loads on the crack. For the levels given to subgrade modulus, slab bending stiffness, and crack spacing, the highest average contribution in the variation in deflections (58.88 percent) was due to the main effect of the subgrade modulus. This was followed by bending stiffness, which accounted for 27.74 percent of the variation, and interaction of subgrade modulus and stiffness ($k \times D$), which accounted for 13.26 percent. The amount that the crack spacing contributed was negligible.

For the variation of principal moment, the main effect of slab stiffness was the largest (59.35 percent), and the main effect of the subgrade modulus was next. The effect of crack spacing was slight.

Results of the ANOVA for the loads between cracks were similar to the case when loads were on the crack (Table 2). It is worthwhile to note that the effect of crack spacing was higher when loads were between cracks. However, over the whole range of the variables studied, neither the main effect of crack spacing nor its interaction with either k or D nor both were highly significant.

## ANOVA—Orthogonal Polynomial Breakdown

In a design experiment where the levels of factors are quantitative, it is often possible to extract more information on how the response variable might vary with the changing levels of the quantitative factor, e.g., how deflection varies with the modulus of subgrade reaction and whether or not there is a linear relation between subgrade modulus and deflection.

The use of orthogonal polynomials makes the analysis rather simple, provided the experiment is designed with equispaced quantitative levels. In the design experiment studied (Fig. 1), the levels of slab stiffness, as well as subgrade modulus, are equispaced logarithmically. That is, each level is obtained from the preceding one by a constant multiplier, which means that the levels progress geometrically. The multiplier was 7.5 for slab stiffness and 5.0 for subgrade modulus. The levels of crack spacing are also equispaced but constitute an arithmetic progression.

Loads on the Crack—Table 3 gives the ANOVA orthogonal breakdown for the deflection response for loads on the crack. In tabulating the orthogonal breakdown, effects that contributed less than 1 percent in response variation were neglected.

It is worthwhile to note that the levels of subgrade modulus are equispaced in the logarithm and that the linear effect refers to the deflection variation associated with log k and not k. Likewise, the quadratic effect is associated with $(\log k)^2$. Similar statements can be made concerning the stiffness term.

General ANOVA in Table 1 gives the average contribution of subgrade modulus to deflection as 58.88 percent. When this total effect is broken into its linear and quadratic log portions, it is seen that 54.40 percent of the deflection variation was due to the log linear effect and only 4.48 percent to the quadratic effect. In addition, the log linear effect of stiffness and the log linear interaction of k and D explain a substantial amount of the deflection response.

In the case of principal moments given in Table 3, the logarithmic linear effects of stiffness and subgrade modulus contributed around 97 percent. None of the quadratic log effects entered into the picture, and the first order interaction of D and k was not so high as in the deflection case.

Loads Between Cracks—Similar results were obtained for the case with loads between cracks. Table 4 gives the orthogonal breakdown of the pavement variables for deflections and principal moments. In both deflection and principal moment responses for loads on the crack, the effect of crack spacing was less than 1 percent, but this was true only for the deflection response when the loads were between cracks; on prin-

**Figure 9. Influence of slab bending stiffness and subgrade modulus on maximum deflections and principal moments for the loads between cracks.**

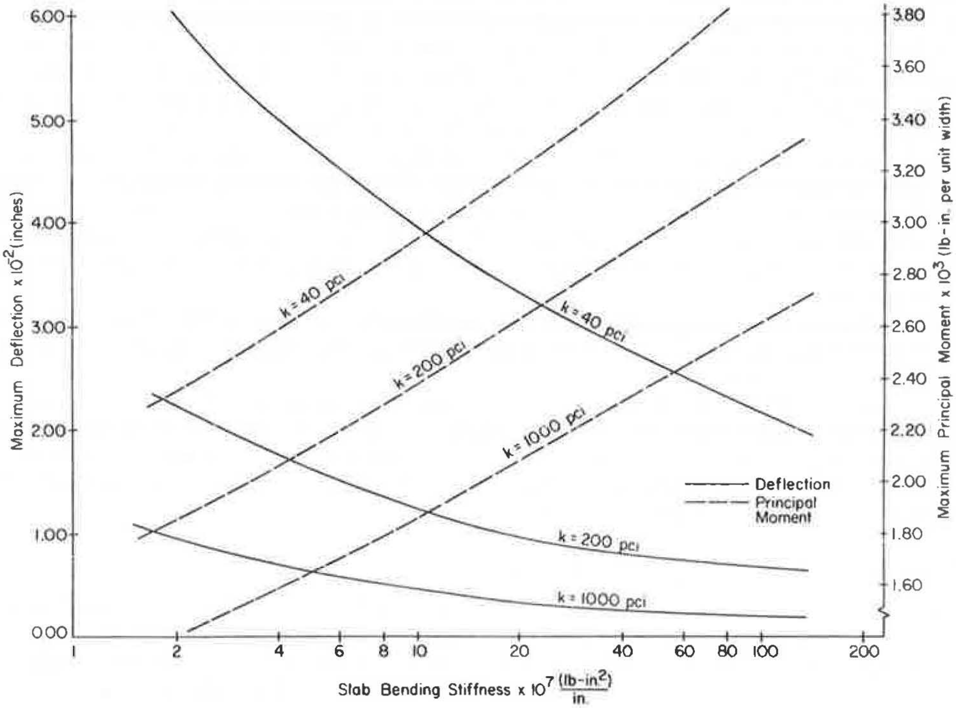



**Table 1. General ANOVA for loads on crack.**

| Source of Variation | Degrees of Freedom | Sum of Squares x $10^3$ | Mean of Squares x $10^3$ | Average Contribution[a] (percent) |
|---|---|---|---|---|
| Maximum Deflections | | | | |
| Log (subgrade modulus), log k | 2 | 14.089 | 7.045 | 58.88 |
| Crack spacing, CS | 3 | 0.012 | 0.004 | 0.05 |
| Log (bending stiffness), log D | 2 | 6.637 | 3.318 | 27.74 |
| Log k × CS interaction | 6 | 0.010 | 0.001 | 0.04 |
| Log k × log D interaction | 4 | 3.173 | 0.793 | 13.26 |
| CS × log D interaction | 6 | 0.007 | 0.001 | 0.03 |
| Log k × CS × log D interaction | 12 | – | – | – |
| Total | 35 | 23.930 | | 100.00 |
| Maximum Principal Moments | | | | |
| Log (subgrade modulus), log k | 2 | 1,459.43 | 729.71 | 37.84 |
| Crack spacing, CS | 3 | 1.16 | 0.39 | 0.03 |
| Log (bending stiffness), log D | 2 | 2,288.75 | 1,144.38 | 59.35 |
| Log k × CS interaction | 6 | 0.20 | 0.03 | – |
| Log k × log D interaction | 4 | 105.68 | 26.42 | 2.74 |
| CS × log D interaction | 6 | 0.28 | 0.04 | – |
| Log k × CS × log D interaction | 12 | 0.59 | 0.04 | – |
| Total | 35 | 3,856.03 | | 99.96 |

[a]Based on sum of squares.

**Table 2. General ANOVA for loads between cracks.**

| Source of Variation | Degrees of Freedom | Sum of Squares × $10^3$ | Mean of Squares × $10^3$ | Average Contribution[a] (percent) |
|---|---|---|---|---|
| **Maximum Deflections** | | | | |
| Log (subgrade modulus), log k | 2 | 7.473 | 3.736 | 66.37 |
| Crack spacing, CS | 3 | 0.004 | 0.001 | 0.03 |
| Log (bending stiffness), log D | 2 | 2.705 | 1.352 | 24.04 |
| Log k × CS interaction | 6 | 0.006 | 0.001 | 0.05 |
| Log k × log D interaction | 4 | 1.069 | 0.267 | 9.50 |
| CS × log D interaction | 6 | 0.006 | 0.001 | 0.01 |
| Log k × CS × log D interaction | 12 | — | — | — |
| Total | 35 | 11.263 | | 100.00 |
| **Maximum Principal Moments** | | | | |
| Log (subgrade modulus), log k | 2 | 7,599.56 | 3,799.78 | 36.82 |
| Crack spacing, CS | 3 | 384.46 | 128.15 | 1.86 |
| Log (bending stiffness), log D | 2 | 12,121.70 | 6,060.87 | 58.73 |
| Log k × CS interaction | 6 | 123.98 | 20.66 | 0.60 |
| Log k × log D interaction | 4 | 133.45 | 33.36 | 0.65 |
| CS × log D interaction | 6 | 239.32 | 39.87 | 1.16 |
| Log k × CS × log D interaction | 12 | 36.02 | 3.00 | 0.18 |
| Total | 35 | 20,638.52 | | 100.00 |

[a]Based on sum of squares.

**Table 3. ANOVA orthogonal polynomial breakdown for loads on crack.**

| Source of Variation | Degrees of Freedom | | Sum of Squares × $10^3$ | | Mean of Squares × $10^3$ | Contribution (percent) |
|---|---|---|---|---|---|---|
| **Maximum Deflections** | | | | | | |
| Log (subgrade modulus), log k | 2 | | 14.089 | | | |
| Linear | | 1 | | 13.016 | 13.016 | 54.40 |
| Quadratic | | 1 | | 1.073 | 1.073 | 4.48 |
| Log (bending stiffness), log D | 2 | | 6.637 | | | |
| Linear | | 1 | | 6.415 | 6.415 | 26.77 |
| Log k × log D interaction | 4 | | 3.173 | | | |
| Linear × linear | | 1 | | 2.881 | 2.881 | 12.02 |
| Linear × quadratic | | 1 | | 0.238 | 0.238 | 1.00 |
| Residual | | 30 | | 0.307 | 0.011 | 1.33 |
| Total | | 35 | | 23.930 | 23.634 | 100.00 |
| **Maximum Principal Moments** | | | | | | |
| Log (subgrade modulus), log k | 2 | | 1,459.43 | | | |
| Linear | | 1 | | 1,448.94 | 1,448.94 | 37.57 |
| Log (bending stiffness), log D | 2 | | 2,288.75 | | | |
| Linear | | 1 | | 2,269.96 | 2,269.96 | 58.87 |
| Log k × log D interaction | 4 | | 105.68 | | | |
| Linear × linear | | 1 | | 96.25 | 96.25 | 2.50 |
| Residual | | 32 | | 40.88 | 1.28 | 1.06 |
| Total | | 35 | | 3,856.03 | 3,816.43 | 100.00 |

**Table 4. ANOVA orthogonal polynomial breakdown for loads between cracks.**

| Source of Variation | Degrees of Freedom | | Sum of Squares × $10^3$ | | Mean of Squares × $10^3$ | Contribution (percent) |
|---|---|---|---|---|---|---|
| **Maximum Deflections** | | | | | | |
| Log (subgrade modulus), log k | 2 | | 7.473 | | | |
| Linear | | 1 | | 6.837 | 6.837 | 60.72 |
| Quadratic | | 1 | | 0.636 | 0.636 | 5.60 |
| Log (bending stiffness), log D | 2 | | 2.705 | | | |
| Linear | | 1 | | 2.606 | 2.606 | 23.17 |
| Log k × log D interaction | 4 | | 1.069 | | | |
| Linear × linear | | 1 | | 0.981 | 0.981 | 8.70 |
| Residual | | 31 | | 0.203 | 0.006 | 1.81 |
| Total | | 35 | | 11.263 | 11.066 | 100.00 |
| **Maximum Principal Moments** | | | | | | |
| Log (subgrade modulus), log k | 2 | | 7,599.56 | | | |
| Linear | | 1 | | 7,590.38 | 7,590.38 | 36.77 |
| Crack spacing, CS | 4 | | 384.46 | | | |
| Linear | | 1 | | 346.55 | 346.55 | 1.68 |
| Log (bending stiffness), log D | 2 | | 12,121.70 | | | |
| Linear | | 1 | | 12,096.98 | 12,096.98 | 58.62 |
| Residual | | 32 | | 604.61 | 18.89 | 2.93 |
| Total | | 35 | | 20,638.52 | 20,052.80 | 100.00 |

cipal moments, the linear effect of crack spacing was 1.68 percent of the total contribution, which still is not highly significant.

## Comparison Between 90 Percent Stiffness Reduction and Full Slab

The effect of crack formation on structural members is an increase in the flexibility of the system. Generally, this will produce an increase in deflections and a decrease in moments or stresses. Because no contraction joints are provided in continuously reinforced concrete pavements, volume change stresses will cause random transverse cracks to develop. The influence of these transverse cracks on maximum deflection and principal moments is shown in Figures 10 and 11.

Values of maximum deflections and principal moments for the full slab case were compared with those of the 90 percent reduction in bending stiffness at the cracks for the two load placements. For low and medium levels of subgrade modulus, the increase in deflection when loads are on cracks is quite significant (Fig. 10). This indicates the detrimental effect of these transverse discontinuities on the pavement slab. It is worthwhile to note that the difference in principal moments between the cracked and uncracked slab increases as the relative stiffness of the slab to that of the subgrade increases (Fig. 11).

## Comparison Between 90 and 100 Percent Reduction (Hinge) in Bending Stiffness

In this study, the cracks were analyzed as if they were completely closed. Deformations in the slab are thus resisted by some degree of moment transfer across the cracks as is customary in normal structural concrete analysis. In practice, however, slabs have crack openings of a finite width that varies primarily because of volume changes. Considering these finite crack widths would thus involve the nonlinear relation of no bending resistance until the crack closes at the top, at which time some bending resistance would then be felt. In this section, however, comparisons of deflections and principal moments are made between the two extreme cases: partial (closed crack) and zero (open crack) bending transfer across the transverse discontinuities.

Values of deflections and principal moments for the hinge case when the loads are on the crack are shown in Figure 12. A graphic representation is used to compare the ratios of deflections and principal moments in the hinge case with those in the 90 percent reduction case.

Figure 13 shows the change in deflections (expressed as a ratio of the values for the 100 to the 90 percent reductions) with the change in radius of relative stiffness for different crack-spacing patterns for loads on the crack. It is seen that, for high values of the radius of relative stiffness, as crack spacing decreases, changes in maximum deflections are highly significant. Obviously, this will emphasize the effect of the width of the crack on the behavior of the pavement structure. Deflections increase at a significant rate as the crack width increases. Hence, crack width should be given special consideration, and narrow cracks are indeed the desirable objective for successful performance of a continuously reinforced concrete pavement.

No significant difference in principal moments was evident between the 100 and 90 percent reductions when the loads were acting on the cracks. The same thing applies to deflections when loads were acting between cracks (Figs. 6 and 14). Comparing the values of principal moment for the loads between cracks (Figs. 6 and 14), certain differences resulted for high values of radius of relative stiffness (Fig. 15). The ratio of the two moment values approaches unity as crack spacing increases, and practically no difference is encountered in the range of 8 to 10 ft.

Therefore, the structural behavior of CRC pavement shows that crack width has a very important effect on the performance of such pavement. Because of the volume changes in the concrete mix, there is a direct relation between crack width and spacing. As crack spacing increases, crack width increases, which in turn causes significant changes in the pavement structure. Deflections increase at a high rate as crack width increases, causing several modes of distress. Furthermore, as shown in Figure 15, no significant drop is encountered in the principal moments between the partial and hinge cases for the 8- and 10-ft crack spacing.

**Figure 10. Influence of bending stiffness and subgrade modulus on deflection for cracked and full (uncracked) slabs.**

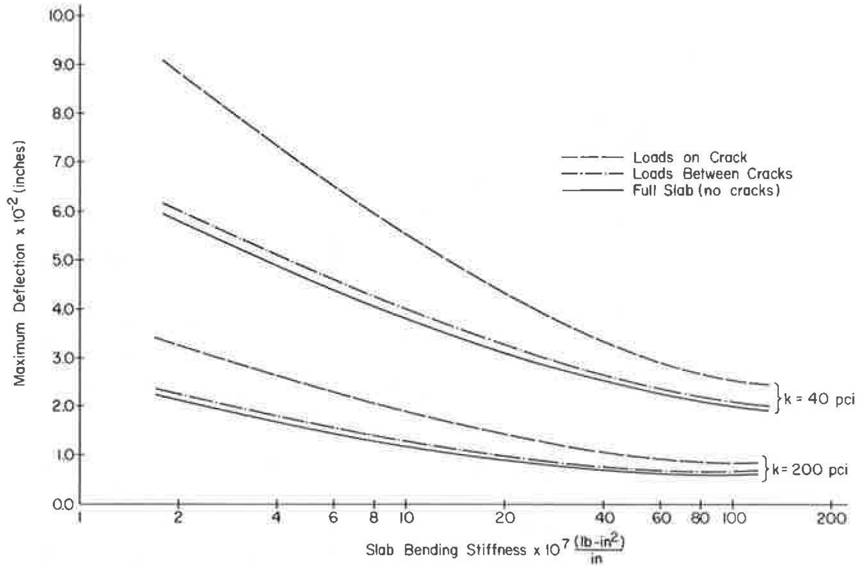



**Figure 11. Influence of radius of relative stiffness on principal moments for cracked and uncracked slabs for loading (Fig. 5).**

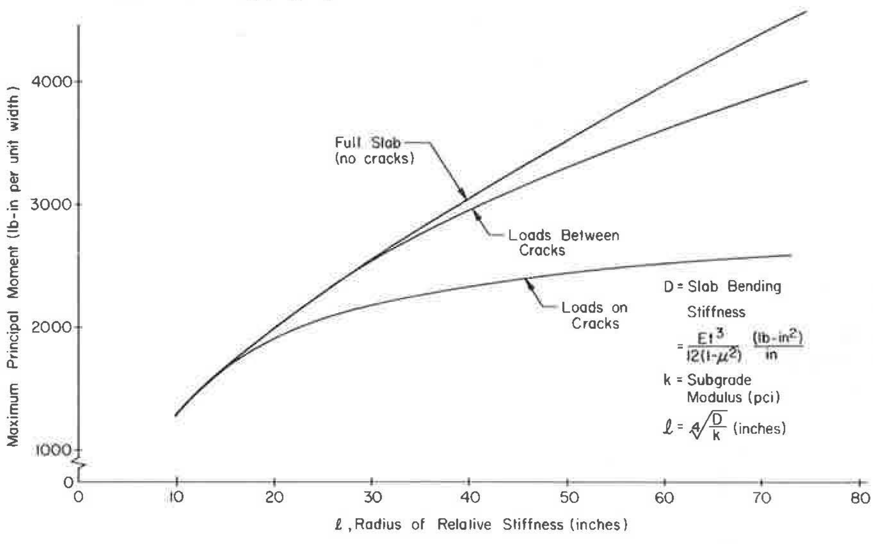


**Figure 12. Maximum values of deflection (100 percent reduction, loads on the crack).**

| Modulus of subgrade reaction, $lb/in^2/in.$ | Crack Spacing, ft | Slab bending stiffness, $(lb\text{-}in^2)/in.$: $20 \times 10^6$ | $150 \times 10^6$ | $1125 \times 10^6$ |
|---|---|---|---|---|
| 40 | 4 | 0.1277<br>2096 | 0.0905*<br>2354 | 0.0643*<br>2554 |
| | 6 | 0.1087<br>2083 | 0.0741*<br>2331 | 0.0515*<br>2543 |
| | 8 | 0.1036<br>2079 | 0.0671*<br>2318 | 0.0438*<br>2535 |
| | 10 | 0.1030<br>2078 | 0.0598*<br>2312 | 0.0388*<br>2532 |
| 200 | 4 | 0.0380<br>1838 | 0.0246<br>2156 | 0.0176*<br>2402 |
| | 6 | 0.0362<br>1834 | 0.0202<br>2138 | 0.0141*<br>2380 |
| | 8 | 0.0362<br>1834 | 0.0188<br>2132 | 0.0121*<br>2365 |
| | 10 | 0.0362<br>1834 | 0.0185<br>2131 | 0.0111*<br>2358 |
| 1000 | 4 | 0.0139<br>1438 | 0.0068<br>1916 | 0.0046<br>2211 |
| | 6 | 0.0139<br>1438 | 0.0063<br>1909 | 0.0037<br>2190 |
| | 8 | 0.0139<br>1438 | 0.0063<br>1909 | 0.0034<br>2182 |
| | 10 | 0.0139<br>1438 | 0.0063<br>1909 | 0.0033<br>2180 |

+ All values occurred 8 feet from the pavement edge.

* Maximum deflection was at the pavement edge; otherwise, it was under the load, 2 feet from the egde.

NOTE: The upper value in each block is the deflection and the lower value is the principal moment.

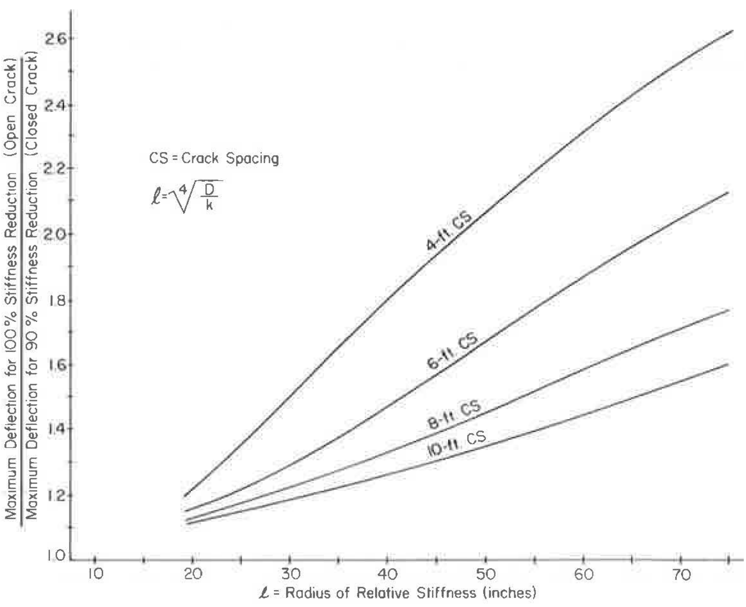



**Figure 13. Comparison of maximum deflections between the 100 and 90 percent reductions in bending stiffness at crack locations for loads on cracks.**

**Figure 14. Values of maximum deflection (100 percent reduction, loads between cracks).**

| Modulus of subgrade reaction, lb/in²/in. | Crack Spacing, ft | Slab bending stiffness, (lb-in²)/in. 20 x $10^6$ | 150 x $10^6$ | 1125 x $10^6$ |
|---|---|---|---|---|
| 40 | 4 | 0.0626<br>2159 | 0.0457*<br>2311 | 0.0330*<br>2430 |
| | 6 | 0.0594<br>2446 | 0.0375*<br>2828 | 0.0261*<br>3042 |
| | 8 | 0.0613<br>2487 | 0.0349*<br>3198 | 0.0225*<br>3558 |
| | 10 | 0.0619<br>2427 | 0.0350*<br>3398 | 0.0207*<br>3971 |
| 200 | 4 | 0.0221<br>1919 | 0.0116<br>2196 | 0.0087*<br>2332 |
| | 6 | 0.0228<br>1925 | 0.0105<br>2561 | 0.0070*<br>2876 |
| | 8 | 0.0226<br>1861 | 0.0107<br>2676 | 0.0064*<br>3294 |
| | 10 | 0.0221<br>1834 | 0.0109<br>2641 | 0.0063*<br>3560 |
| 1000 | 4 | 0.0099<br>1458 | 0.0037<br>2005 | 0.0022*<br>2226 |
| | 6 | 0.0097<br>1402 | 0.0038<br>2077 | 0.0019*<br>2647 |
| | 8 | 0.0096<br>1394 | 0.0038<br>2010 | 0.0019*<br>2840 |
| | 10 | 0.0096<br>1396 | 0.0038<br>1964 | 0.0020*<br>2852 |

+ All values occurred 2 feet from the pavement edge.

* Maximum deflection was at the pavement edge; otherwise, it was under the load, 2 feet from the edge.

NOTE: The upper value in each block is the deflection and the lower value is the principal moment.

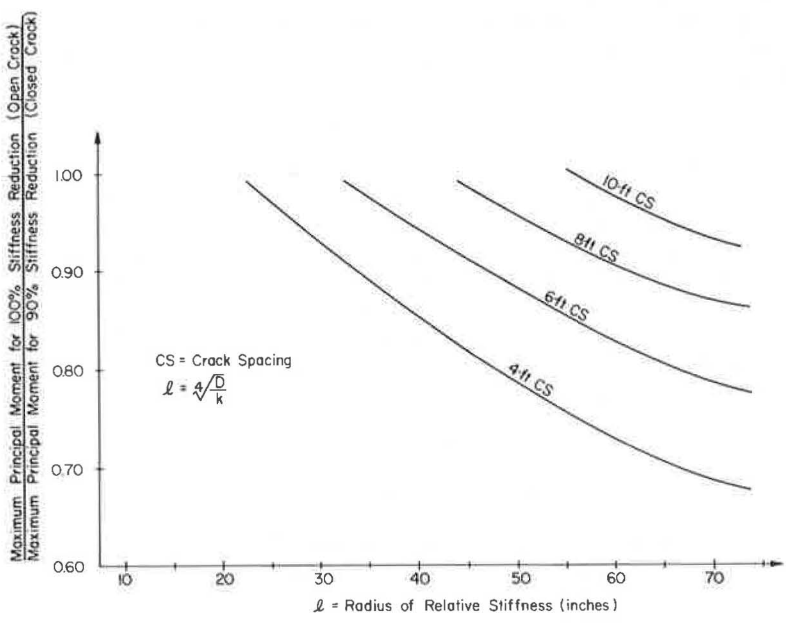



**Figure 15. Comparison of maximum principal moment between the 100 and 90 percent reductions in bending stiffness at crack locations for loads between cracks.**

Discussion of Results

Transverse cracks are characteristic of continuously reinforced concrete pavements and significantly influence the behavior and performance of this pavement type. Comparison between the cracked and uncracked slab (Figs. 10 and 11) indicates that, when loads were on the crack, there was a substantial increase in slab deflections, whereas the drop in principal moment that occurred for loads between cracks was not highly significant. This illustrates the detrimental effects of these transverse discontinuities on the pavement structure.

According to SLAB program results, the modulus of subgrade reaction, as defined by Westergaard and others, plays an important role in the determination of deflections and principal moments (Figs. 5, 7, 8, and 9). Pavement deflections decrease at a significant rate as subgrade modulus increases. Furthermore, there is about a 10 percent decrease in the value of principal moment as k varies from one level to the next when loads are on the crack and about a 20 percent decrease for the case when loads are between cracks.

Although the subgrade modulus showed a higher contribution in the determination of deflections than principal moments, slab bending stiffness possessed a contrasting effect (Figs. 5, 7, 8, and 9). Hence, if the most important design criteria are pavement stresses, the thickness of the slab is the factor that requires the greatest consideration, and it is followed in importance by the modulus of subgrade reaction.

For the case when loads were on the crack, as crack spacing varied over the range studied (4 to 10 ft), a small change was encountered in deflections and principal moments (Fig. 4). For the second load placement investigated (i.e., loads between cracks), the effect of crack spacing on deflections was also slight or practically negligible, whereas changes in principal moments for the low and medium levels of k were significant (Figs. 7 and 8).

The ANOVA performed on the SLAB results has indicated that the main effects of slab bending stiffness and subgrade modulus contributed around 90 percent to the variation in each of the deflection and principal moment responses (Tables 1 and 2). It also showed the minor effect of crack spacing on the pavement behavior.

The polynomial orthogonal breakdown yielded similar results for the two load placements. The logarithmic linear effect of subgrade modulus and bending stiffness was highly significant and explained most of the variations in the pavement responses, deflections, and principal moments.

The comparison between the 90 percent (closed crack) and 100 percent (open crack) reduction in bending stiffness at the crack location indicated the importance of crack width on the behavior of continuously reinforced concrete pavements. Slab deflections increase at a high rate as crack width increases (Fig. 13), whereas no significant drop is encountered in principal moments.

## CONCLUSIONS

This investigation was conducted to determine, by use of the discrete-element slab model, the sensitivity of pavement deflection and principal moment (or stress) to changes in design parameters. The conclusions are limited to the range of variables studied. These findings, however, can provide reasonable information to use in design, for selecting those variables that require the most intensive consideration and those that will yield the best results.

Based on changes in deflections and principal moments, the following conclusions have been drawn:

1. Higher principal moments or stresses are produced when loads are located between cracks than when loads are at the crack. The reverse is true for the deflection response.
2. The effect of the modulus of subgrade reaction on slab deflection is highly significant.
3. Principal moments or stresses are mainly dependent on, first, the stiffness of the slab and, second, the subgrade modulus.

4. As crack spacing increases, principal moment values for the loads placed between cracks approach those of the full slab case.

5. The effect of crack spacing on deflections and principal moments was greater for the case of 100 percent reduction in bending stiffness at crack location than it was for the 90 percent case.

6. The width of the crack has a big influence on the performance of continuously reinforced concrete pavements. The reduction of the bending rigidity of the slab and the consequent increase in the slab deflection as a result of an increase in crack width are important. Perhaps the requirement most necessary to the success of continuously reinforced concrete pavement is that the steel reinforcement hold transverse cracks as tightly as possible.

7. For the increments given to subgrade modulus, slab bending stiffness, and crack spacing, the analysis of variance and its orthogonal polynomial breakdown showed that (a) a definite logarithmic linear trend of subgrade modulus with deflection is observed as well as a tendency toward a logarithmic quadratic relation, (b) the linear effect of the log of bending stiffness on principal moments and deflections is quite significant, and (c) interactions do occur among design variables, indicating that the effect of any one design variable on deflections and principal moments is dependent on levels of the other two design variables.

## RECOMMENDATIONS

Based on this investigation, the following recommendations are made:

1. Pavement design procedures should include greater consideration of the modulus of subgrade reaction because of its influence on deflections and stresses.

2. Stress criteria in present design procedures should be coupled with deflection criteria, which will enable the designer to ensure a pavement deflection less than the desired maximum.

3. Transverse cracks should be maintained very narrow in order to (a) prevent progressive infiltration of incompressible materials such as soil, which eventually might cause excessive compressive stress to develop in the pavement and thus produce blowups; (b) prevent appreciable amounts of surface water from reaching the subgrade; and (c) maintain effective aggregate interlock between the crack interfaces.

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