THE PRINCIPLE OF SUPERPOSITION IN PAVEMENT ANALYSIS

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The principle of superposition was used in analysis of instrumentation data obtained in test sections constructed at the Waterways Experiment Station. Included were homogeneous sand and clayey silt test sections under plate loads and the multiple-wheel heavy gear load flexible pavement test sections under single- and multiple-wheel loads. It is found that the principle of superposition is approximately valid, indicating linear theory is not unreasonable in application to pavement analysis, although laboratory tests have definitely proved pavement materials behave nonlinearly under loads. The authors cannot answer the problem; the central purpose of this paper is to stimulate discussions by other researchers that will aid in pavement analysis.

•IN dealing with the stress and deformation of continuous media caused by loads in mechanics problems, it is frequently convenient to consider the loads to be composed of two or more systems of loads and to assume that each system produces stresses and deformations independently, as though it were the only system of loads acting on the body. The actual effect is then considered to be the resultant of the effects of the two systems of loads. The method of obtaining the actual effect as a resultant effect, by adding or combining independent partial effects, is called the method of superposition. The method is applicable only if a linear relation exists between the loads and the effect they produce (1).

The method of superposition has been used very frequently by engineers. The method used is based on the assumption that the small displacements in the deformation do not affect substantially the action of the external forces; otherwise the justification of superposition principle fails. This method of superposition provides the backbone for the mathematical theory of linear elasticity. Because the stress-strain relation is linear, the deformation is a linear function of the load, regardless of the order in which the loads are applied; also the material constants are the same for compression and for tension and are invariant relative to the state of the stress.

Although the principle of superposition possesses overwhelming advantage in its simplicity of application, it is the principle itself that so drastically limits the application of the theory to the field of pavement design. Laboratory tests indicate that pavement materials generally do not exhibit linear stress-strain behavior and that the elastic moduli of such materials vary with the state of the stresses (except perhaps under very small stresses and strains). In this study, the principle of superposition was used in the analysis of instrument-measured data from homogeneous sand and clayey silt test sections under plate loads (2, 3) and from the multiple-wheel heavy gear load (MWHGL) flexible pavement test sections under single- and multiple-wheel loads (4). The results and measured values are presented and followed by discussions.

SUPERPOSITION OF MEASURED STRESSES AND DEFLECTION

In spite of nonlinearity of the soil, the principle of superposition was found to be reasonable in homogeneous test sections (2, 3) where stresses and deflections were

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measured at different locations under single and dual plates of various sizes and load intensities. The plotted points shown in Figure 1 represent deflections in the sand test section measured beneath dual plates, which were 1,000 in.^{2} in size and were spaced 4.5 ft center to center. Curves were developed by superposing ordinates from smooth curves drawn through points representing deflections measured beneath single-plate loads. Figure 2 shows relations for vertical stresses developed in the same manner as shown in Figure 1. It is seen that the principle of superposition is more valid for stress than for displacement.

Figures 3 and 4 show data from the clayey silt test sections and are similar to Figures 1 and 2 respectively. The dual plates were 500 in.^{2} in size and were spaced 3.0 ft center to center Measurements of single-plate loads smaller than 500 in. were not available (2). For the purpose of this study, measurements at the outside region of dual-plate loads spaced at 7.5 ft center to center were considered to be the same as measurements of single-plate loads. The readings so obtained are slightly greater than actual single-plate loads would be, but the differences are believed to be negligibly small. The plots strongly indicate that the principle of superposition is more valid in clay than in sand.

Results shown in Figures 1 through 4 are for homogeneous soil masses. The principle of superposition was also applied to instrumentation data obtained in test item 3 of the MWHGL test section (4). The item consisted of a 3-in. asphaltic concrete surface, a 6-in. graded crushed-stone base, a 24-in. gravelly sand subbase, and a 4-CBR heavy clay subgrade soil. WES pressure cells (3, p. 19), LVDT, and other instruments were embedded in the pavement at different depths up to 12 ft. Figures 5 and 6 show the results of deflection measurements at various depths for loading by one twin-tandem component of a Boeing 747 wheel assembly (120 kip) and a C-5A 12-wheel gear assembly (360 kip) respectively. The gear configurations are shown in Figure 7. The plots in Figures 5 and 6 strongly indicate that the principle of superposition is reasonably valid for flexible pavements except at depths near the surface. It appears that large stress intensities involved near the surface have stressed the material into its nonlinear range, whereas materials below the surface are still within or near their linear ranges. It should be noted that the analysis presented in this paper is based on heavy aircraft loads and relatively thin pavement; the principle of superposition should be more valid for light highway loads.

IMPLICATION OF THE PRINCIPLE OF SUPERPOSITION

In the application of linear theory of elasticity to compute the maximum stresses and displacements in a pavement structure under multiple-wheel loads, the computations are carried out for each individual load, and the results are linearly summed up for all loads. The principle of superposition is used in the Corps of Engineers flexible pavement method of design to estimate the equivalent single-wheel load (ESWL), which is defined as the load on a single tire of an assembly that produces the same vertical deflection of the supporting medium as that particular multiple-wheel assembly. The computations are usually carried out by constructing the deflection-offset curve under a single-wheel load and then summing the ordinates at the proper offsets, which gives the deflection due to the multiple-wheel load.

For dual wheels, it has been found that the use of the superposition principle gives results that are within the acceptable limit of error. For multiple-wheel heavy gear loads, however, such as Boeing 747 and C-5A, the computed ESWL becomes so large that the current criterion is too conservative (4). The error is not directly caused by the use of superposition principle but by many other inadequate assumptions of the linear theory of elasticity. This can best be explained by the MWHGL test results (4).

Figure 8 shows measured and computed deflection basins at the 12-ft depth of item 3 of the MWHGL test section. The deflections were induced by a single-wheel load (30 kip) and by the C-5A 12-wheel gear assembly (360 kip). The computed deflections are plotted in percentage of the maximum values. In both cases, the basin shape is quite different between computed and measured values. However, when the superposition principle is used on the measured single-wheel deflection basin (instead of the

Figure 1. Superposition of deflections measured in sand test sections.

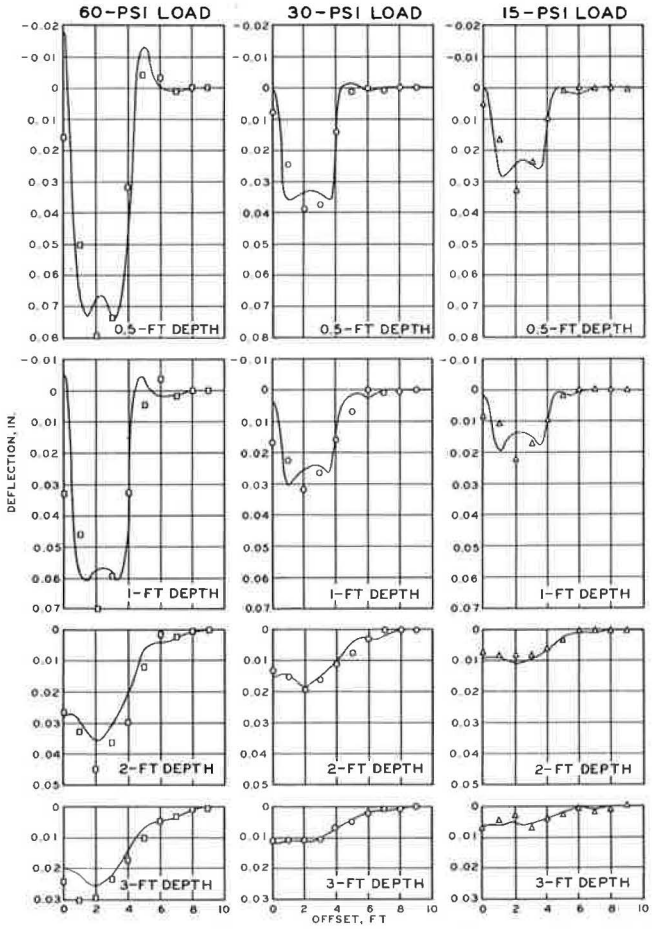


Figure 2. Superposition of stresses measured in sand test sections.

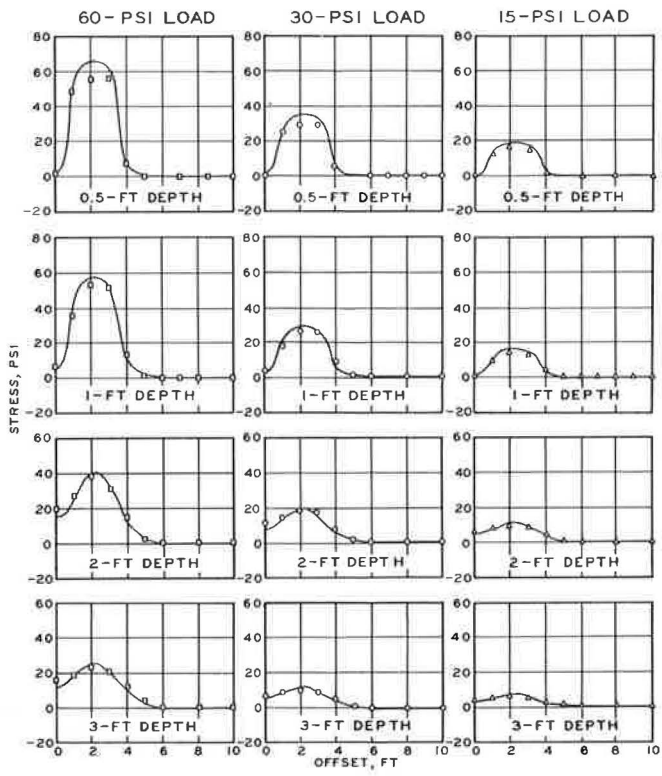


Figure 3. Superposition of deflections measured in clayey silt test sections.

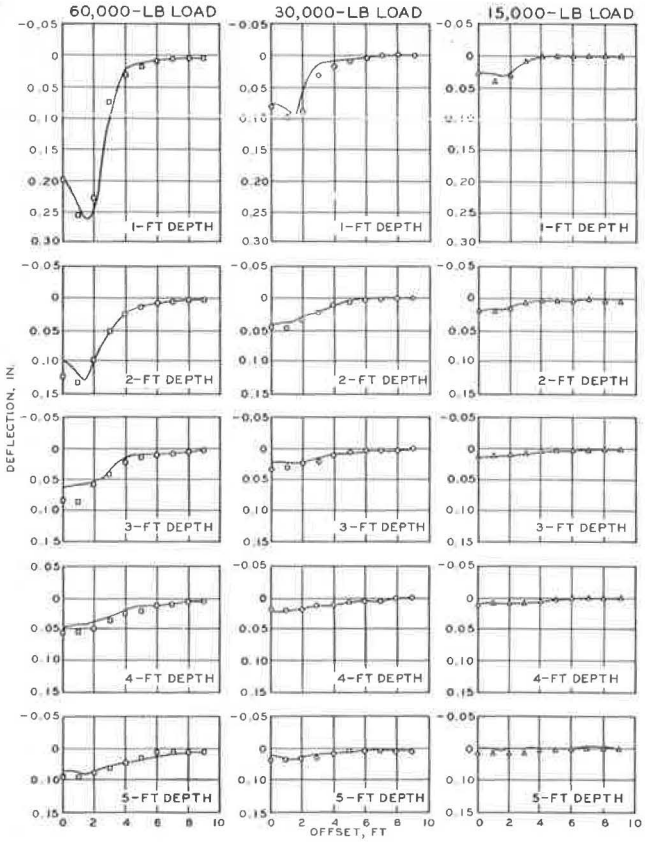


Figure 4. Superposition of stresses measured in clayey silt test sections.

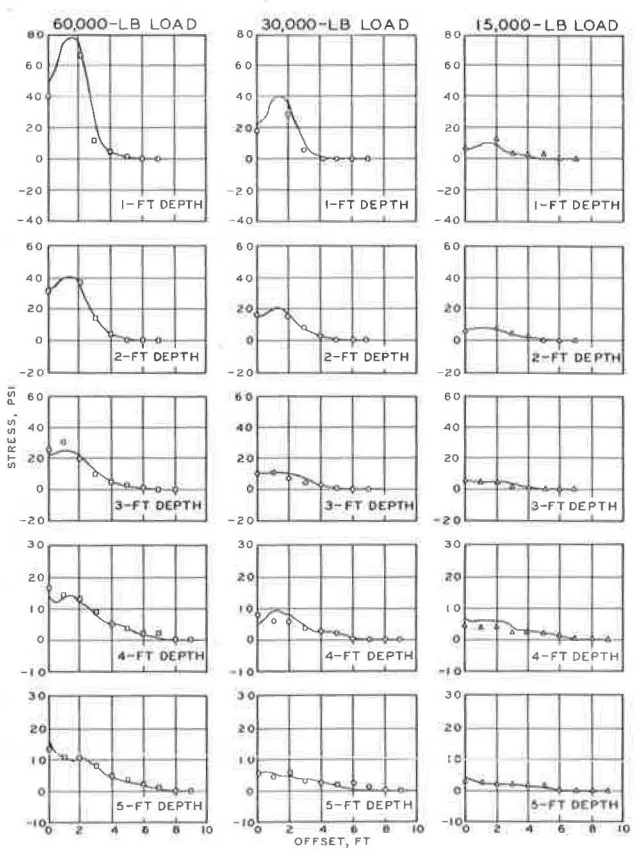
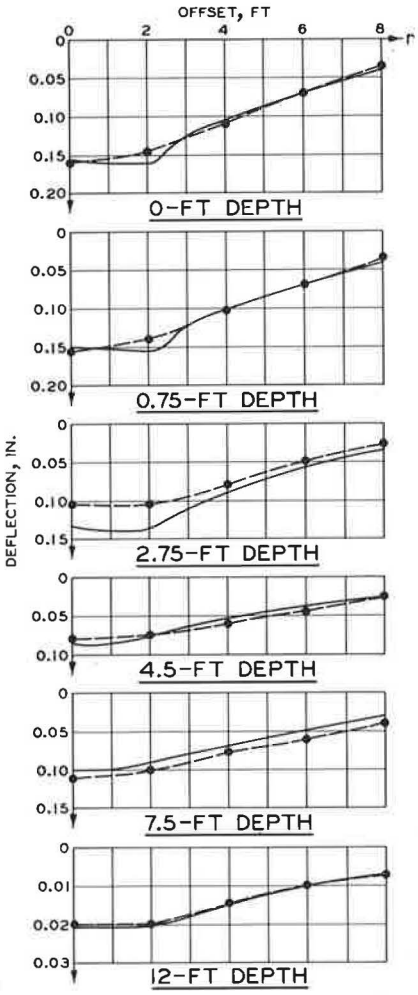
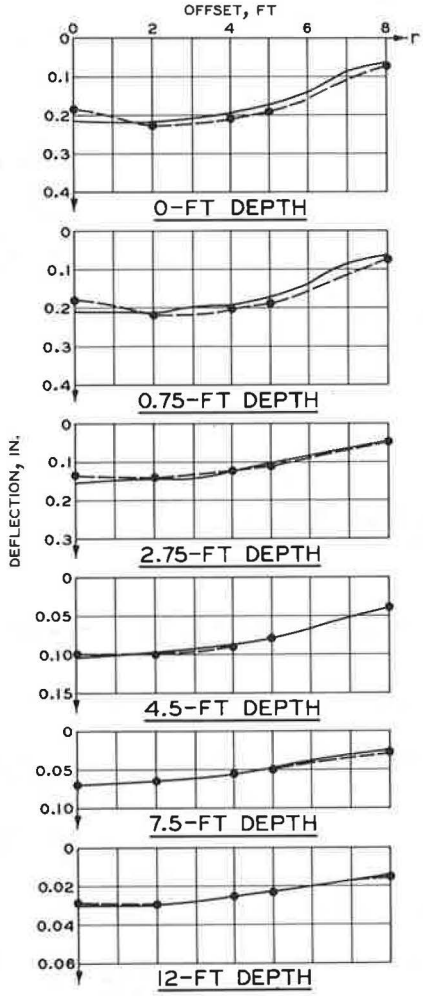


Figure 5. Superposition of deflections measured under loading by twin-tandems component, Boeing 747 assembly.



NOTE: OFFSET DISTANCES MEASURED ALONG r-AXIS SHOWN IN FIG. 7.

Figure 6. Superposition of deflections measured under loading by a 12-wheel gear, C-5A assembly.



NOTE: OFFSET DISTANCES MEASURED ALONG r-AXIS SHOWN IN FIG. 7.

Figure 7. Gear configurations of C-5A and Boeing 747 test assemblies.

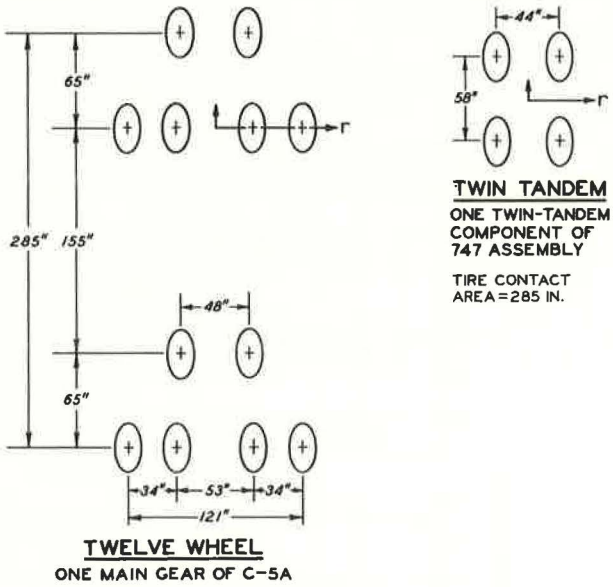
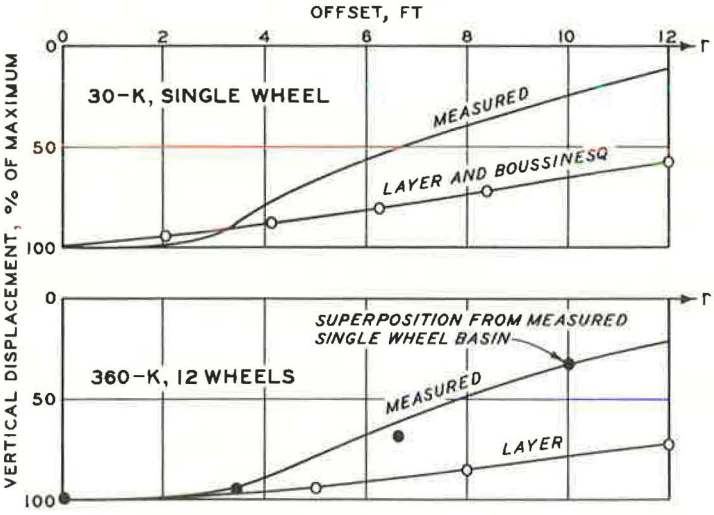


Figure 8. Theoretical and measured deflection basins.



theoretical one) to obtain that of the 12-wheel basin, the resulting basin value is astonishingly close to the measured one of the 12-wheel basin. These are shown by the dots along the measured 12-wheel basin. Similar relations were also found for deflections measured at other depths. Test results shown in Figure 8 strongly verified the remark made in a previous paragraph regarding the source of error in the ESWL derived for multiple-wheel heavy gear loads.

As discussed earlier, the principle of superposition provides the backbone of the linear matematical theory of elasticity. The question promptly arises as to why the superposition principle is approximately valid for pavement problems but the linear theory of elasticity is not.

Elasticity implies that the material resumes its initial form completely after removal of loads and that the stress path is independent. Linearity further assumes the validity of the principle of superposition; i.e., the strain is linearly proportional to the stress, and the tension and compression properties are identical. For instance, when a triaxial sample is subjected to an axial compressive stress of 30 psi, if the material is linear, the sample would behave identically when the load is replaced by an axial compressive stress of 60 psi and an axial tensile stress of 30 psi. Certainly the assumption of linear elasticity is not true for soils unless the stresses and strains are very small.

Under pure shear, a linear isotropic material should suffer no volume change because the contraction in one direction is compensated for by an expansion of equal magnitude in the perpendicular direction. Consequently, the change in volume is the effect of hydrostatic forces, and the change in shape is the effect of shear forces. Because any state of stresses can be resolved into a mean normal stress and a pure shear, an ideally linear material, in which the principle of superposition is valid, must satisfy the following requirements: The change in volume resulting from the mean normal stress must be independent of the pure shear, and the change in shape resulting from the pure shear must be independent of the mean normal stress. Consequently, a stress tensor can be separated into two independent parts, one of which is the hydrostatic forces and the other is shear forces. The shear forces govern distortion, and the hydrostatic forces control dilatation.

Laboratory tests have definitely shown that the stress-strain relations for pavement materials are not linear because the shear strain depends not only on the shear stress but also on the mean normal stress, and the volume change depends on the mean normal stress as well as the shear stress. The dependence of shear strain on the mean normal stress naturally leads to the conclusion that material properties are different for tension and for compression.

The analysis presented in Figures 1 through 8 demonstrates that the principle of superposition as applied to pavement design is approximately valid. The stresses and deflections under multiple-wheel loads can be obtained from the correct stress and deflection basins of single-wheel load by the use of superposition principle. The implication is that the stresses and strains in the pavement, except near the surface, are so small that materials are stressed within or near their linear ranges; hence, linear analysis may not be the most critical factor in the disagreement between measurements and predictions. Evidently, there is a strong contradiction between prototype field measurements and laboratory findings in material behaviors. The writers cannot answer this problem; the central purpose of this paper is to stimulate discussions by other researchers that will aid in pavement analysis. It is also our intent to propose that, because the superposition principle has been proved valid (at least experimentally), efforts should be concentrated for refining technology in the nonlinear finite-element analysis method under single-wheel loads and that second priority should be given to efforts for developing a computer program for multiple-wheel loads, at least at the present time. This recommendation is made mainly because of the excessive computer time involved in multiple-wheel load analysis.

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DISCUSSION

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The authors of this paper deal with a fundamental problem concerning the structural design of pavements.

Their results are very important because they demonstrate experimentally that, despite the nonlinear behavior of the highway construction materials, under current loading highway pavements behave almost like linear-elastic solids.

It must be emphasized again that the real problem we deal with is the highway itself and not the theories and that experience can be the origin of knowledge. It also follows that every theoretical development should be verified experimentally. It is very difficult to do this because of the variance in properties of materials we use and the variance of the thickness of the layers. But, despite these difficulties, there is a method that can be used to eliminate any lack of fit between theory and practice.

Let us define the standard deviation of the characteristics of the materials by σ_j^m and the standard deviation of the thicknesses by σ_j^t. Let us also define the factors that are used in the design formula by P and a particular factor by P_i.

The particular factor P_i can be calculated in relation to the others (P_i^c):

$$P_i^c = f\ (P) \tag{1}$$

or it can be measured (P_i^m).

The function f in Eq. 1 may be either theoretical or empirical. Either one is useful, provided the computed results agree with the measured values.

Let us now define the control factor of the parameter P_i by X_i,

$$X_i = \frac{P_i^m}{P_i^c} \tag{2}$$

The perfect situation would be when, having P_i^c's as the abscissa and P_i^m's as the ordinate, one would get for X_i's a straight line (Fig. 9).

Unfortunately, this situation never occurs. If it were to take place, it could be represented from a statistical point of view (Fig. 10).

Generally, one gets more or less normal distribution values (Fig. 11).

The fact that the mode M is not at $X_i = 1$ indicates that the theoretical model is not perfect.

The quality of the model can be appreciated by the obliquity of the distribution of X_i. If one knows the standard deviation of the distribution in X_i (σ_{xi}), one can define the obliquity (O_i) as being

$$O_i = \frac{M - 1}{\sigma_{xi}} \tag{3}$$

Figure 9. Perfect correlation between measured and calculated values.

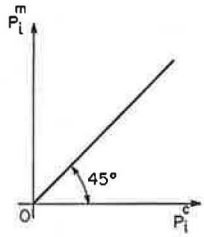


Figure 10. Perfect statistical correlation.

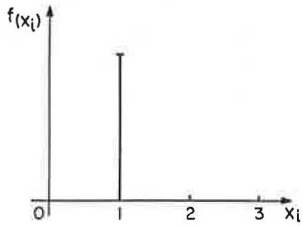


Figure 11. Normal distribution with obliquity.

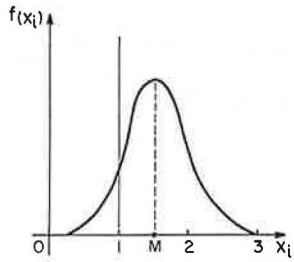
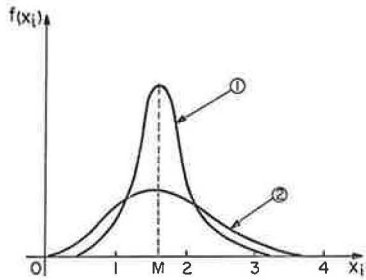


Figure 12. Two normal distributions with different standard deviations.



An obliquity rating scale has been proposed by the author as follows:

Obliquity	Rating
0 to 0.3	Perfect
0.3 to 0.8	Excellent
0.8 to 0.15	Very good
0.15 to 0.32	Good
0.32 to 0.50	Acceptable
More than 0.50	Not acceptable

But the distribution of X_i can be a sharp or a flat one (Fig. 12). Which is better?

To answer this question, let us consider that, starting from the design formula, one can develop a variance model that predicts a theoretical standard deviation of X_i ($\sigma^c_{x_i}$). Let us now consider that the actual standard deviation of k_i is $\sigma^m_{x_i}$.

One can state that the design formula is good if

$$\frac{\sigma^m_{x_i}}{\sigma^c_{x_i}} \approx 1 \tag{4}$$

The author considers that a design formula is good if, and only if, the obliquity is low and the relation given in Eq. 4 is observed for each factor P_i that occurs in the design method.

If the principle of superposition really works, even for flexible pavements, the way for elastic methods is open indeed. However, all of the preceding considerations will have to be accounted for to get a consistent and sensitive method of design.