CRACK DEVELOPMENT IN PAVEMENTS

Joseph E. Soussou and Fred Moavenzadeh, Massachusetts Institute of Technology

This paper discusses methods of analyzing crack development in pavements subjected to random repetitions of various loads in variable operating conditions. The approaches based on Miner's law present the advantage of simplicity. Their principal limitations are that the predicted fatigue life is generally a function of geometry and loading conditions. The data obtained in the laboratory constitute a good approximation to limited types of pavement systems. This law can be modified to account for temperature variations and variations in loading spectrum and material properties. The approaches based on the concepts of fracture mechanics can be viewed as a generalization of Miner's law. Although the latter bases the prediction of crack development on the initial configuration of the system, the fracture mechanics approach incorporates the changes of configuration (i.e., geometry). The application of this approach to linear elastic systems can also be viewed as a special case of a general approach including time-dependent and nonlinear material properties. A simplified extension to viscoelastic systems is also discussed. The modified Miner's law is preferred in the short range because of its simplicity and the availability of data, whereas the approaches based on fracture mechanics concepts are preferred in the long range because of their completeness.

THE structural damage in pavement systems has two principal indicators: cracking and permanent deformation. The AASHO Road Test (1) indicates that the present serviceability index is not very sensitive to the degree of cracking of the pavement. Cracking may have, however, indirect effects on the accumulation of permanent deformation. These indirect effects are generally ignored in the mathematical models for pavement analyses.

Cracking results from load repetitions or environmental factors. Cracking due to repeated loading occurs primarily as a result of bending deflections. This type of failure is often designated as "fatigue failure." Fatigue in bituminous paving materials has been shown to be a progressive process. Cracks propagate from small flaws inherent in the material until ultimately the amount of cracking reaches an unacceptable level or the remaining section becomes so weak that catastrophic failure occurs.

This study reviews the principal approaches for the analysis of cracking in pavements. These methods are discussed, and two models are proposed.

THEORIES AND LAWS FOR FATIGUE CRACK ANALYSIS

A general law of fatigue crack propagation should take into account the five following factors (2):

1. Geometry (dimension of structure and shape and size of crack),
2. Loads (magnitude and location),
3. Material properties (constitutive equation),
4. Time (number of cycles, duration of loads, and time intervals between loads), and
5. Environment (temperature, moisture, pressure, and surrounding media).

These various factors are aggregated to different degrees depending on the method of analysis. Two broad types of theories are recognized: phenomenological and crack

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propagation theories. The labeling and distinction between these two classes of approaches is rather arbitrary. The phenomenological theories include the approaches that measure a degree of damage for a homogeneous material without any description of the number, length, and width of cracks. The homogeneous material is assumed to deteriorate uniformly until total failure. These theories focus principally on the life duration of the structure rather than on the description of progressive changes of the damage indicators.

The crack propagation theories recognize the presence of flaws in the material and can describe the number and size of cracks as time elapses. Some of the crack propagation theories could be labeled as phenomenological theories because they try to fit some mathematical model to the macroscopic behavior of the material. However, the preceding definition will be utilized in this text.

PHENOMENOLOGICAL THEORIES

Review of Literature

The phenomenological theories generally assume a failure criterion for a homogeneous material. A general type of failure criterion would predict the failure under any type of loading history. Such a unified criterion is not known for any material. Instead different criteria are developed to account for various classes of loading histories. This approach is used in a number of engineering practices. For example, a maximum shear criterion is used for parts of a structure subjected to a given stress field, whereas a maximum tensile stress criterion is used for other parts of the same structure. Following a rational mechanics approach, it seems that a general criterion should involve combinations of stress or strain tensor invariants. Many analysts (3) use the form

$$F(I_1, I_2, I_3) > F_c \tag{1}$$

where I_1, I_2, and I_3 are stress invariants. Novozhilov (4) has given physical interpretation to these invariants. In a more general case, the failure envelope should also contain the time dependence (e.g., for viscoelastic materials). Thus, it could be written as

$$F[I_1(\tau), I_2(\tau), I_3(\tau), t, \text{temperature, etc.}] \geq K \tag{2}$$

Many simplifications are necessary in order to give this relation a tractable form. In particular, the important simplification of neglecting the influence of the past histories of I_1, I_2, and I_3 may be achieved. However, general failure envelopes allowing for stress multiaxiality are not known yet. Most of the failure envelopes determined for time-dependent materials correspond to uniaxial loadings. These envelopes are characterized by an indetermination in their values; i.e., statistical variations are the rule rather than the exception. Also, failure envelopes corresponding to long periods of loading (creep type) generally show more variations than failure envelopes corresponding to short times of loading.

Examples of such envelopes are a log stress at failure versus log strain at failure envelope suggested by Smith (5) for constant rate of strain loading histories. This concept, however, applies better to monotonic loading histories than to cycling or otherwise varying loading histories (3, 6). These criteria apply best when the material can be considered continuous (that is, it does not include flaws or other stress concentrators) and when it is subjected to a homogeneous state of stress.

When failure occurs under a large number of repetitions of the load, it is caused by a smaller load level than what would produce failure under a single load application. The failure is classified then as a fatigue failure and is analyzed by special methods. Generally, the main variable is the number of cycles, and it is assumed that every application of the load increases the amount of damage done to the material. The best known phenomenological theory for fatigue is Miner's law (7). This law states that, under constant stress amplitude, the increment of damage per cycle is constant and that this rate is a function of the load level. The law is written in the form

$$\sum_{i=1}^{m} \frac{n_i}{N_i(\sigma_i)} = 1 \tag{3}$$

where N_i is the number of cycles to failure at stress level σ_i and n_i is the actual number of applied cycles at this stress level. It can be seen readily that a major shortcoming of this law is that it does not account for sequence effects. Nonlinear forms of this law were derived to gain accuracy. These modifications, however, tend to impair the simplicity of the law and do not offer much accuracy and rigor. Indeed, by increasing the number of parameters necessary in a law, one can eventually describe any experimental set of data.

A generalized form of Miner's law is given by Freudenthal (8). The increment of damage F per cycle is assumed to be a function of the number of N or prior load applications:

$$\frac{dF}{dN} = f(N)_s \tag{4}$$

where the function f(N) is also a function of the stress level. Thus, after N cycles, the amount of damage is given by the integral

$$F = \int_{N_{0s}}^{N} f(N)_s dN \tag{5}$$

where N_{0s} is the incubation period when the damage is assumed to be zero. This form could account for sequence effects.

Freudenthal and Heller (9) modified it in order to include the interactions between different stress levels. They introduced in Miner's law a stress interaction factor that is a function of the frequency distribution of the random loading. In its modified form, Miner's law is written as follows:

$$\sum_i \frac{n_i}{\omega_i N_i(\sigma_i)} = 1$$

where ω_i are the interaction factors. Another specialized form of Miner's law proposed by Corten and Dolan (10) attempts to account for the effects of prior history.

These different forms of Miner's law have been mainly applied to rate-insensitive materials and result in the determination of an S-N envelope or stress level versus number of cycles to failure at this level.

Williams (11) adapted the concept of Miner's law to rate-sensitive material by replacing the cycle ratio $\frac{n_i}{N_i}$ by a time ratio $\frac{t_i}{T_i}$; t_i is the elapsed time from the start of the test at the strain rate R_i, and T_i is the time to failure under the constant rate of strain R_i. It was observed that Williams' law could be combined with the failure envelope of Smith (5) to relate the results of different types of loading histories. However, its application is mostly successful for monotonic types of loading histories and should not be applied for cyclic loading (fatigue) where the amount of dissipated energy is large compared to the amount of strain energy.

Another phenomenological model was derived by Dong (26). It assumes that the damage function or damage index can be expressed as a nonlinear functional of the stress and strain tensor functions. Dong then proceeds to expand this nonlinear functional into a series of multiple integrals. This model yields Miner's law and Williams' law as particular cases. His model is very general but not very tractable. It can account for the sequence effects and the interactions of different stress levels; however, as mentioned previously, a model can account for any type of effect when the number of required parameters is increased indefinitely.

Proposed Model

With some assumptions of continuity, the damage functional F(t) can be expanded into a series of multiple integrals (26):

$$F(t) = \int_0^t K_1(t, s)V(s)ds + \int\int_0^t K_2(t, s_1, s_2)V(s_1)V(s_2)ds_1ds_2 + \ldots$$

$$+ \int \cdots \int_0^t K_n(t, s_1, \ldots, s_n)V(s_1)\ldots V(s_n)ds_1\ldots ds_n \tag{6}$$

The measure of damage F(t) is not, however, uniquely defined. The damage may be measured by the density of cracking, or by the value of dynamic modulus of the layer materials at a given frequency, because this modulus decreases as the density of cracking increases (8). The creep compliance of the material can be used as a measure of crack propagation (12) or the number or remaining cycles before complete failure under a given mode of loading can be used for this purpose (7). Any of these measures can be used, and it is convenient to normalize them so that the damage functional equals 0 when the material is intact and increases to 1 at failure. In the preceding equation, V(s) is a function involving stress or strain invariants or both, and s is an arbitrary parameter that may have a meaning of time or cycles. This representation of the damage functional is general and accounts for accumulation of damage, recovery processes such as healing, and aging effects.

The review of literature showed that, for various asphaltic and bituminous mixtures, failure envelopes were related to a strain measure. In the general case of triaxial loading conditions, the strain measure should be expressed as a combination of invariants. In the absence of results of triaxial tests, we will use the derivative of the major principal strain as a strain measure in the damage functional. Thus,

$$F(T) = \int_0^t K_1(t, s)\dot{\epsilon}(s)ds + \int\int_0^t K_2(t, s_1, s_2)\dot{\epsilon}(s_1)\dot{\epsilon}(s_2)ds_1ds_2 + \ldots \tag{7}$$

where the symobol with a dot over it represents a differentiation with respect to the argument. When s has a time meaning, expansion is similar to the representation of the time response of a nonlinear viscoelastic material. When s has a cycle meaning, it may be related to the dynamic representation of a nonlinear viscoelastic material and may be determined as a transfer function of a system subjected to a cyclic loading.

In order to simplify this expansion, we assumed that three different damage processes may be recognized: a damage process depending on the number and amplitude of cycles, a healing process (or a recovery process) depending on the elapsed time since the damage was created, and an aging process where the materials properties are changing with time. The damage functional may now be written as

$$F(t) = \int_0^t K_1(s, t - s)\epsilon(s)ds + \int\int_0^t K_2(s_1, t - s_1, s_2, t - s_2)\epsilon(s_1)\epsilon(s_2)ds_1ds_2 + \ldots \tag{8}$$

This equation implies that the kernels are functions of the running time s (cumulative and aging processes) and of the lapse of time t - s (recovery process).

In a first approach to the problem, the second and higher order kernels will be neglected in the damage expression. We will further assume that the first-order kernel may be factorized:

$$K_1(s, t - s) = K_{cum}(s)\, K_{rec}(t - s, s) \tag{9}$$

i.e., the cumulative and recovery processes are independent. The aging process is included in both K_{cum} and K_{rec} through the dependency on the time s.

Determination of these kernels depends on choosing a measure for the damage and normalizing it as mentioned previously. Let N be the number of cycles to failure (i.e., inadmissible density of cracking) under a given type of random load during relatively short time (no aging or recovery takes place). A damaged material will undergo only N′ cycles under the same conditions before failing. The amount of damage is represented by $\frac{N - N'}{N}$. N and N′ can be measured on control specimens. Note that in this case F(t) is not a measure of the amount of cracking but is a function of it.

Cumulative Kernel—For a small period of time over which there is neither aging nor damage recovery, we have for the increment of damage $\Delta F(\tau)$ an expression such as

$$\Delta F(\tau) = \sum_{i=1}^{m} \frac{dn_i}{\omega_i N_i[\Delta\epsilon(\tau), \tau]} \tag{10}$$

where τ indicates that the number of cycles to failure may vary because of aging and that the envelope is to be determined for different values of τ. The increment of damage is also a function of the average strain amplitude applied during the increment of time τ.

Recovery Kernel—The recovery kernel $K_{rec}(t - \tau, t)$ is a function of the time $(t - \tau)$ elapsed since the application of the damage increment and of the age of the material. It is apparent that healing requires the presence of a minimum compressive stress (13). Thus, we will assume that the argument $(t - \tau)$ can be replaced by $(t^* - \tau)$ and

$$t^* = \int_{\tau}^{t} H[\sigma(s) - \sigma_{min}]ds \tag{11}$$

where H[ ] is the Heaviside step function that is equal to 1 when its argument is positive and zero elsewhere, and σ_{min} is the minimum compressive stress that triggers healing. Thus $(t^* - \tau)$ is the accumulated time during which a minimum compressive stress is present.

To determine $K_{rec}(T)$, two identical specimens (or sets of specimens) should be given the same amount of damage F. F is determined by testing one of the two specimens (control specimen) and measuring the amount of damage that should be applied to cause the specimen to fail. The second specimen is left to rest for a time T and then caused to fail to determine the amount of recovery $K_{rec}(T)$.

Aging—Aging is accounted for through changes in the characteristics of the constitutive equation and in the cumulative and recovery kernels.

Thus, the complete expression can take the form

$$F(t) = \int_{0}^{t} \left\{ \sum_{i=1}^{m} \frac{dn_i}{\omega_i N_i[\Delta\epsilon(\tau), \tau]} \right\} K_{rec}(t - \tau, \tau)d\tau \tag{12}$$

CRACK PROPAGATION THEORIES

Review of Literature

A different method of approach is based on the concepts introduced in fracture mechanics. The formulations are based on the balance of energy release and energy accumulation at the crack tip. Generally, in these formulations the damage index is a crack length. The rate of crack length increase per cycle is given by a relation such as

$$dc/dN = f(\sigma, c, C_i) \tag{13}$$

where C_i are some material properties. The form of this equation describing the crack growth per cycle of loading is derived from a dimensional analysis, from a work-hardening model, from equations relating the growth rate to the crack opening displacement, or from a molecular theory. The basis of the law could also be empirical.

These approaches are essentially based on the energy balance concept introduced by Griffith (14) for brittle materials. He postulated that a crack will propagate when the rate of release of the strain energy becomes greater than the rate of creating a new surface, i.e., rate of increase of surface energy. Orowan (15) extended this theory to ductile materials by adding to the surface free energy the plastic energy of deformations. Rivlin and Thomas (16) generalized this theory by writing that the release of strain energy does not go entirely to produce new surfaces (surface free energy). In this way, they were able to apply the Griffith theory to rubbers.

The Griffith theory leads to the determination of a critical stress that is given in the following form:

$$\sigma_{cr} = K\ E \sqrt{T_i/c} \tag{14}$$

where K is a constant of the geometry, E is the material modulus, and c is the crack length. Williams (11) postulates that, to generalize this equation, one may use for the dissipated energy T_i the sum of energies dissipated in the brittle, ductile, and viscoelastic dissipation processes. A better way of extending the Griffith energy balance concept is to use the so-called general power equation (11). This equation may be written as

$$\int_{C_1} T_i \dot{u}_i d_s = \dot{D} + \dot{U} + \dot{\Gamma}_c \tag{15}$$

where T_i and u_i are the surface traction and displacements of a contour C_1 that encloses the crack C, the dots represent a time derivative, U is the free energy (strain energy), D is the energy dissipated in the form of heat, and Γ is the specific surface energy. One can also include in this equation other types of energies such as the kinetic energy as proposed by Blatz (17). There are different methods of applying the power equation to determine the law of crack propagation.

However, as Griffith (14) and Orowan (15) have pointed out, the Griffith energy balance criterion is equivalent to the attainment of specific stresses at the crack tip. In other words, it is the local state of stress at the crack tip that controls cracking. Following this idea, one computes the local stresses ahead of a crack using the results of the theory of elasticity. These local stresses are given in the form of stress intensity factors K_1, K_2, and K_3, which correspond respectively to the opening, in-plane sliding, and tearing modes. K_1, K_2, and K_3 are determined by the theory of elasticity as shown by Inglis (18) and Sneddon and Lowengrub (19). Lee (20) showed that, for the case of stress boundary conditions, the elastic constants do not appear in the expression of the stress distribution; thus, the stress distribution is the same for a viscoelastic material as it is for an elastic material. Therefore, Williams (21) solved the problem of the growth of a crack in a viscoelastic material subjected to stress boundary conditions by using the elastic values of the stress intensity factors. The viscoelastic properties are used to deduce the strains at the crack tip. An essential advantage in this approach is that it concentrates on stress or strain criteria rather than energies, and stresses can be superposed whereas energies cannot.

Most of the crack propagation laws obtained for rate-insensitive materials are of the form

$$dc/dN = C\ \sigma^m c^n \tag{16}$$

where C, m, and n depend on the material properties, the geometry, and the method used for the derivation of the law (i.e., dimensional analysis or work-hardening model).

Proposed Model

It was observed for pavement structures that cracks propagate under the influence of repeated loading rather than under the influence of monotonically varying loading histories. Using these assumptions, one can restrict the study of crack growth in pavement structures to the study of crack growth under the influence of repeated loadings (fatigue).

For fatigue cracking, it is generally convenient to give the probability of crack growth per cycle rather than crack growth per time unit. A cycle is the interval separating two successive troughs or two successive peaks of the load. It is readily seen that, although this representation will be very convenient for sinusoidal loadings, it will not be very useful for some types of random loadings. We will adopt this representation because it is a convenient one for the case of pavement systems where we have loads of different magnitudes that are applied for different durations at various intervals of time.

The use of the number of applied cycles instead of the time increments can be mainly justified when the measure of damage (crack accumulation in this case) varies slowly as a function of the number of applied cycles n. In this case it can be differentiated with respect to n. The probability of a given crack growth per cycle μ_c can be related to the probability of crack growth per unit time μ_t by the expression (22):

$$\mu_c = \mu_t/\omega \tag{17}$$

where ω is the angular frequency.

It appears, however, that, for many materials, the dependence on the past history of the local stress tensor at the tip of the crack can be restricted to the last cycle of a local stress measure. In other words, the crack propagation occurring during the last cycle of load application is exclusively a function of the local stress changes resulting from the last cycle of loading. This is a simplifying assumption, and its degree of validity depends on the type of material.

Because the local stress at the tip of the crack is a function of the nominal stress σ and the crack size c, the rate of crack propagation can be written generally as

$$dc/dn = f(\sigma, c, M_i)_{\text{last cycle}} \tag{18}$$

where M_i are the materials properties; σ and c can also be combined in the stress intensity factor K:

$$dc/dn = f(K, M_i)_{\text{last cycle}} \tag{19}$$

The best method of approach for the determination of the functional f is to develop a good understanding of the micromechanisms of crack propagation. Otherwise one can only make some educated guesses and check them experimentally. For instance, for non-rate-sensitive materials, one can assume that

$$dc/dn = f[K_n, K_{n-1/2}, K_{n-1}, M_i] \tag{20}$$

where K_n is the last peak value of the stress intensity factor at the tip of the crack, $K_{n-1/2}$ is the last trough, and K_{n-1} is the value of the previous peak. It appears logical to express these three independent variables by another set of independent variables (Fig. 1),

$$\left.\begin{aligned} K &= \tfrac{1}{3}(K_{n-1} + K_{n-1/2} + K_n) \\ \Delta_1 K &= K_n - K_{n-1/2} \\ \Delta_2 K &= K_n - K_{n-1} \end{aligned}\right\} \tag{21}$$

and to expand f in terms of K, $\Delta_1 K$, and $\Delta_2 K$, and the different combinations of their powers. The influence of K is known to be negligible for non-rate-sensitive materials, $\Delta_1 K$ intervenes by its fourth power (23), and the influence of $\Delta_2 K$ is not well known. It is only known that, when $\Delta_2 K$ is positive, the rate of crack propagation is not much affected. Thus, the rate of crack propagation for many rate-insensitive materials can be written as

$$dc/dn = A(\Delta_2 K) \times \Delta_1 K^m \tag{22}$$

where m is assumed to have the value of 4, and $A(\Delta_2 K)$ is a material property. For rate-sensitive materials, the coefficient A may also become a function of rate and temperature. In this crack law, A and m may become dependent on the structure and the type of loading history. For instance, Ramsamooj (24) finds that it applied better to beams or slabs that were elastically supported than to the same structures with weakened supports. This fact suggests that, for cases where the viscoelastic effects are important, a more complete law of crack propagation will be required. Following the same reasoning to simplify the rate propagation law, we may conclude that, if the rate of crack propagation can be considered a function of the changes in the stress intensity factor during the last cycle, then

$$dc/dn = f(K, \Delta_1 K, \Delta_2 K, t, \Delta_1 t, \Delta_2 t) \tag{23}$$

where K, $\Delta_1 K$, and $\Delta_2 K$ have been defined previously, t is the time corresponding to K_n, and $\Delta_1 t$ is the time lag between K_{n-1} and K_n. Being the parameter accounting mostly for aging and creep rupture, $\Delta_1 t$ represents the time of loading; $\Delta_2 t$ represents the rest period and could correspond to the healing effects. With these assumptions, we may assume a general law of crack propagation for rate-sensitive materials that isolate the most important parameters. This law can be expanded into different forms according to the experimental results that are found. If a simple expansion can result, this relation will be useful. If a complicated expansion is needed, it is preferable to use a more fundamental approach such as the power law described previously. We will assume that such a simplified expression can be found for paving materials:

$$dc/dn = f[(K, t), (\Delta_1 K, \Delta_1 t), (\Delta_2 K, \Delta_2 t)] \tag{24}$$

Such a law can be used to account for random loadings.

For computation purposes, we will use the special case of this propagation law:

$$dc/dn = A(\Delta_1 t, \text{temp})\, \Delta_1 K^4 - CH(\Delta_2 t, \text{temp}) \tag{25}$$

where the loading time $\Delta_1 t$ represents the frequency dependence of the coefficient A. More accurate laws will undoubtedly be found when more is known about the fracture of paving materials.

EXAMPLES OF APPLICATIONS

Phenomenological Law

Typical values for the failure envelope were taken from the literature (25). These envelopes were obtained from bending tests under constant stress levels. More accurate failure envelopes should be obtained by trying to simulate typical histories of the triaxial state of stresses that develop at the critical points in the pavements. In the example that was studied, the failure envelope was given by

$$N = K\left(\frac{1}{\Delta\epsilon}\right)^n$$

where $K = 5.10^{-4}$ and $n = 4.5$, and $\Delta\epsilon$ is the strain amplitude and is measured as half the difference between two consecutive peaks and troughs in the strain function. For tem-

Figure 1. Time variations of the stress intensity factor.

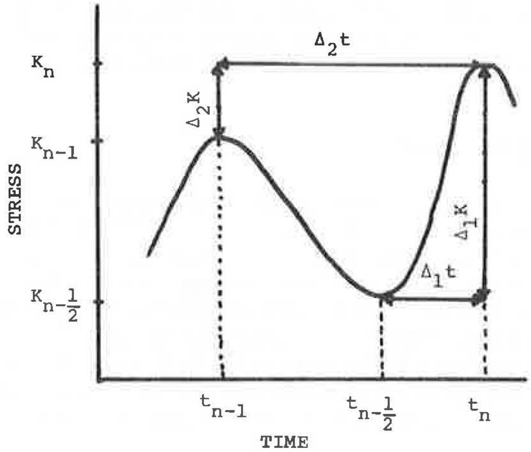


Figure 2. Ratio of tensile strength for damaged and undamaged materials (13).

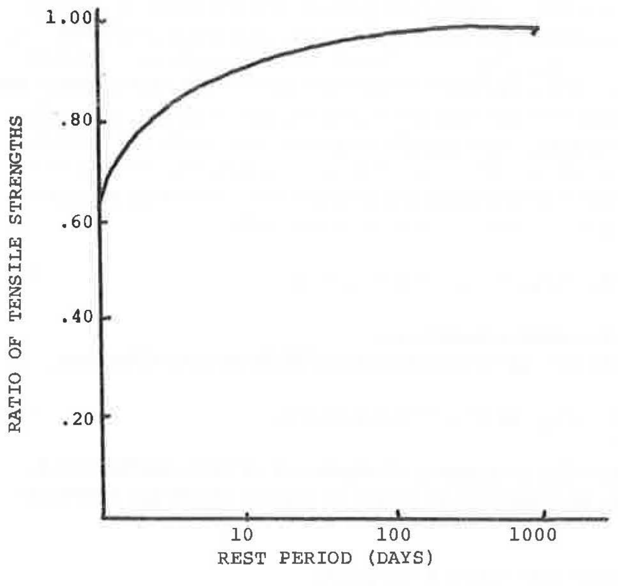


Figure 3. Typical recovery function for a fatigued dense bituminous mix (13).

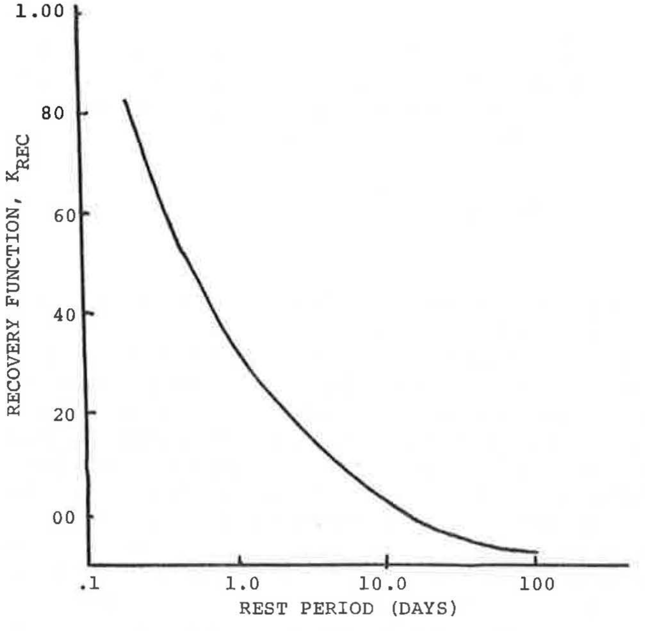
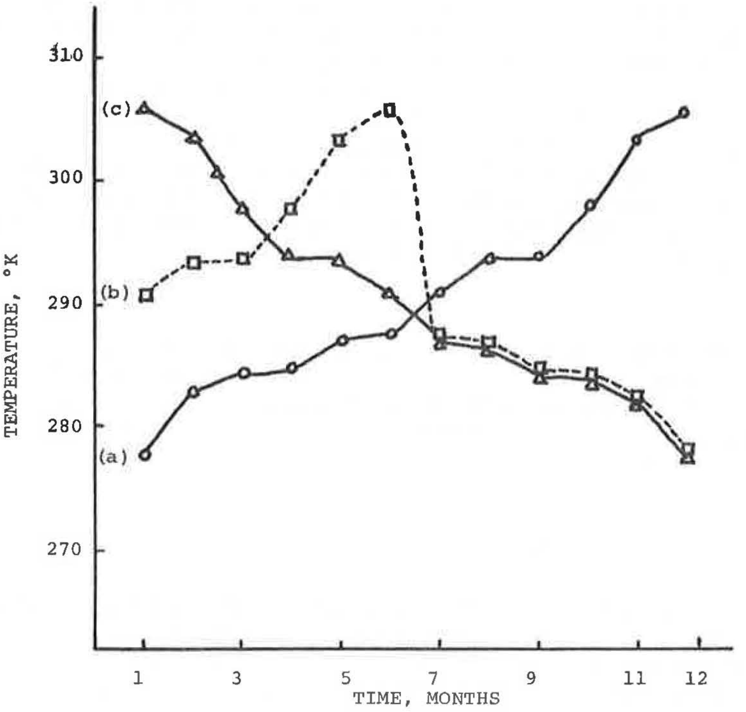


Figure 4. Temperature variations.



peratures above 22 C, K was increased by a function of temperature to account for the fact that, at higher temperatures, very large strains occur but do not contribute to cracking.

The recovery kernel is obtained by curve fitting with a series of exponential experimental results such as shown in Figures 2 and 3 (13). These figures show that, for broken and fatigued bituminous mixes, a complete recovery is obtainable after 3 months at 10 C. This recovery is also a function of temperature. In the example that was treated, the temperature dependence of this function was simplified to

$$K_{reo}(t) = 0.2\left(1 + \sum_{i=1}^{5} e^{-\alpha_i t^*}\right)$$

where t^* = total amount of time during which no load is applied and the temperature is above 22 C; α_i were chosen so that the recovery is completed in a period of 3 months.

Twelve different temperatures were generated by the random number generator. These temperatures were arranged by increasing order, then by decreasing order, and finally by a successively increasing and decreasing order [Fig. 4, (a), (b), and (c) respectively]. These series were used as inputs to the analysis, and the number of load applications was assumed constant and equal to 15,000 loads. The resulting residual strains are shown in Figure 5. These residual strains at the first interface were used in lieu of the residual deflections at the surface because they present the same type of behavior and permit the qualitative study of the trends of this behavior. Figure 5 shows that the residual strains after a period of 12 months were almost equal for the series (b) and (c) because the temperatures corresponding to the last 5 months of these series were identical. The effect of the difference of temperature history in the first months was negligible. The residual strains due to the increasing order series (a) were a little different. On the whole, for the assumed materials properties, the residual strains were not very sensitive to changes in the sequence of temperatures.

Figure 5 shows also the strain amplitudes corresponding to each basic unit of time (month). These amplitudes were essentially functions of the present temperatures. Hence, their variations are directly related to the temperature changes.

Figure 5 shows also the damage function, F(t), for the three sequences of temperatures. The irregularities in the shape of F(t) result from the strong nonlinearities arbitrarily introduced by the data used in the formulations. It can be seen, however, that the model accounts for the differences in temperature sequences and that the sequences are more important for the determination of the damage function F(t) than they are for the determination of the residual strains. It is not possible to derive more conclusions from the computed behavior because the data are not real.

Crack Propagation Law

An approximation is used to apply this law. The stress distribution is computed at different temperatures in a multilayered homogeneous viscoelastic system (Fig. 6). The stress intensity factors are approximated by those obtained for simple tension by considering a small volume at the interface between the first and second layer. We find for a penny-shaped crack that

$$K = \frac{2.05}{\sqrt{\pi}} \sqrt{a}\,\sigma$$

where σ is the nominal stress (Fig. 6) and a is the crack length. The crack is assumed to start at the interface and to propagate toward the surface. The initial flaw size a_o is given from experimental data, and the final crack size a_f is chosen to be equal to half the thickness of the first layer. Using the Paris (23) crack propagation law, we obtain

$$N_f = \int_{a_o}^{a_f} \frac{1}{A(\Delta K^4)}\, da = \frac{\pi^2}{A(2.05)^4} \int_{a_o}^{a_f} \frac{d_a}{\sigma^4(c)dc}$$

Figure 5. Comparison of different sequences of temperatures.

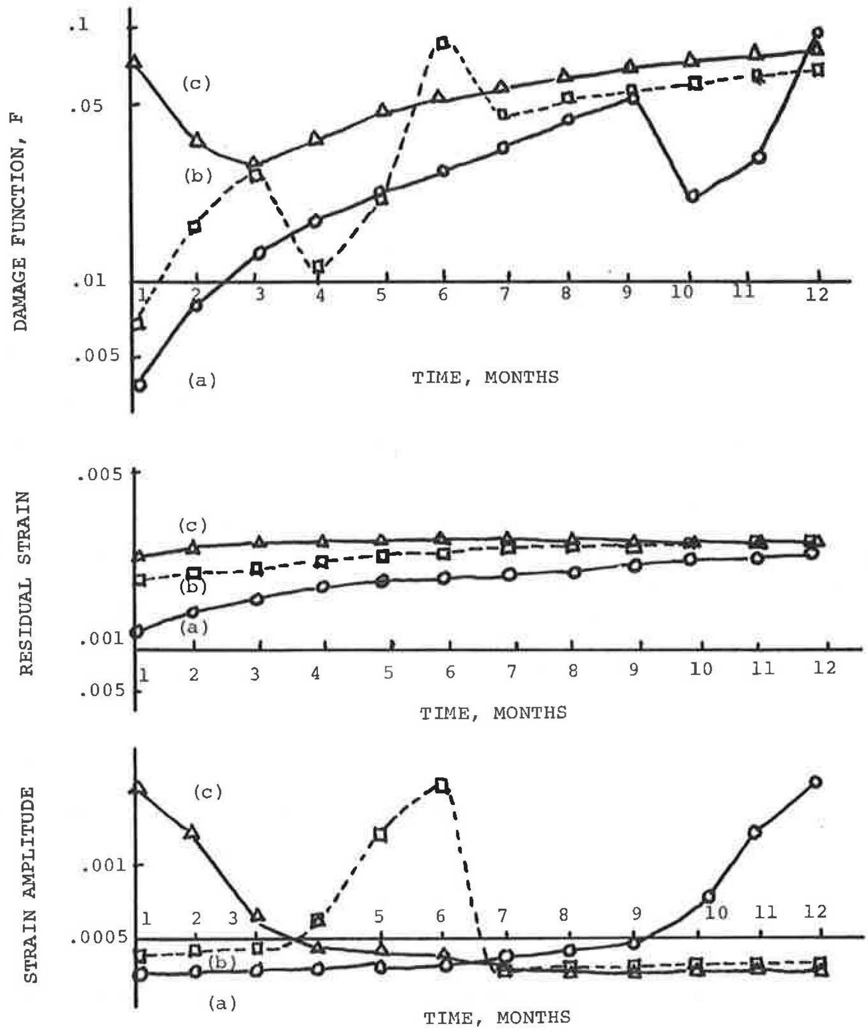
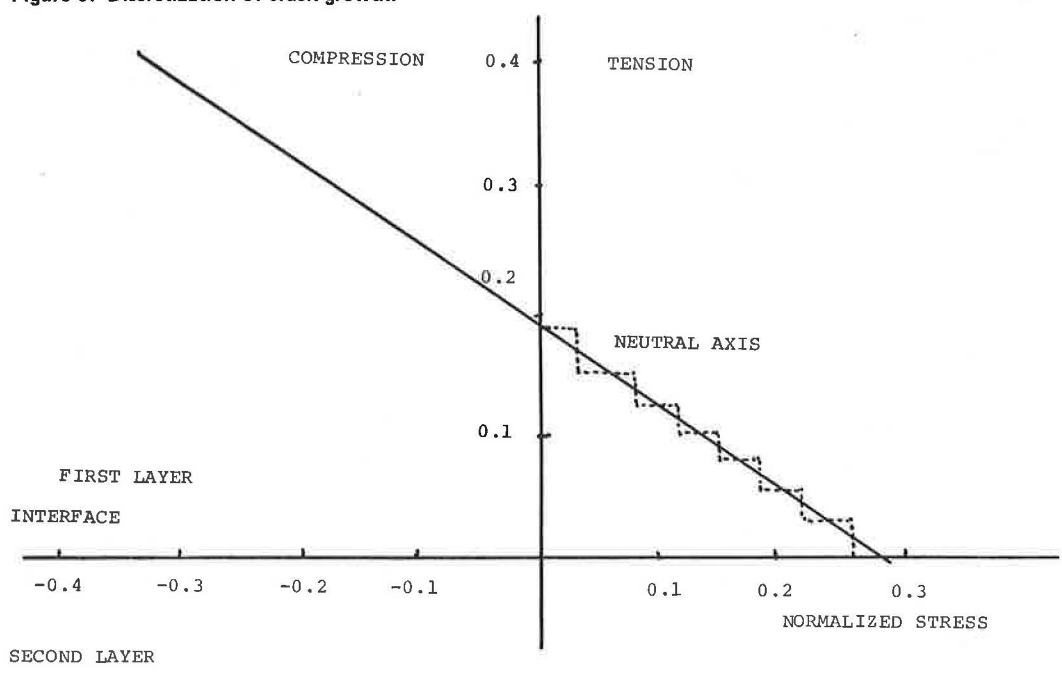


Figure 6. Discretization of crack growth.



where A is a material property.

This integral is broken into a summation of elements representing a small crack propagation during which σ may be assumed to be constant:

$$N_f = \frac{\pi^2}{A(2.05)^2} \sum_{i=1}^{m} \frac{1}{\sigma^4\left(\frac{a_i + a_{i-1}}{2}\right)} \left(\frac{1}{a_{i-1}} - \frac{1}{a_i}\right)$$

CONCLUSIONS

The approach based on Miner's law presents the advantage of simplicity. Also, experimental data are readily available for its use. Its principal shortcomings are that it does not account properly for geometry effects and it does not give a physical description of the cracking of the pavement.

The approach based on the crack propagation laws can be viewed as a generalization of Miner's law. The latter bases the prediction of crack development on the initial configuration of the system, and the fracture mechanics approach incorporates changes in configuration. It can be used in a simplified form as shown previously. It is preferred in the long range because it accounts for the five factors previously described (geometry, loads, material properties, time, and environment). It also provides a physical description of the cracking of the system.

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