PROBABILITY MODEL FOR JOINT DETERIORATION

Lawrence Holbrook, Michigan Department of State Highways

Based on condition surveys at 5-, 10-, and 15-year intervals, joints from 43 post-World War II pavement construction projects in Michigan were grouped into 4 classes of deterioration depending on the extent of damage. These classes were then considered as states through which a joint could progress as it deteriorated in time. It was further assumed that the probability of a joint passing from a given state to the next higher state depended only on which state the joint was in and not on the previous deterioration history. This assumption allowed joint deterioration to be modeled as a continuous-time Markov process that specified the probability of a joint being in each state for any time during service life. A nonlinear least squares estimation technique was used to estimate 4 parameters governing the process for each of the 43 construction projects. The model's fit to field data for each of these projects was quite good, thereby suggesting that similar procedures might be used for a large variety of structural deterioration problems.

• BECAUSE joint deterioration is a serious problem from both a roughness and a maintenance point of view, to model the deterioration process for predictive purposes is desirable. A successful model would forecast problem occurrences such as blowups and thereby afford an opportunity for preventive maintenance. It is unlikely that the large number of variables that affect joint deterioration would be tractable enough to allow an extract (deterministic) formulation of the problem. Under such circumstances, one often resorts to the prediction of probabilities, provided one can reasonably define states for the process. This type of predictive model is called stochastic and often fits the real world quite well. For the case of joint deterioration, the first problem encountered in developing the model occurred in the measurement of deterioration. The general index approach to deterioration measurement is given in an earlier paper (1).

THE DATA

Because all post-World War II state trunk-line concrete pavement construction was examined in Michigan by condition survey, it was possible to record joint condition at survey intervals of 5, 10, and 15 years of service life (Fig. 1). Tabulation and analysis of these data were made possible through the financial support of the Federal Highway Administration. Deterioration, usually joint spalls and slab cracks, is visually recorded on survey sheets more or less to scale. One might suppose that a good measure of joint deterioration would be spall count. However, as the number of spalls increases, they tend to merge; hence, more advanced deterioration, namely the blowup, is qualitatively different from spalls and is therefore not amenable to this scaling procedure.

To measure all types of joint deterioration in proportion to their seriousness, a more comprehensive scaling method had to be developed. Consequently, it was decided to measure joint deterioration as the percentage of the total transverse joint length affected by all kinds of concrete failure. Mathematical modeling was facilitated by grouping these percentages into 4 categories or states as follows:

Publication of this paper sponsored by Committee on Pavement Condition Evaluation.

State Defined	Percentages
1	0 to 25
2	26 to 50
3	51 to 75
4	76 to 100

State 1 consists mostly of external corner spalls, and state 4 consists mostly of blowups—rarely would a joint be spalled 75 to 100 percent of its transverse length. Usually state 4 joints were seen by the investigator as full slab width patches that by their extent and character suggested that a blowup occurred. These definitions of joint deterioration were used to classify each joint and to chart its progression from state to state with each subsequent condition survey.

DEVELOPMENT OF THE MODEL

It seems reasonable to assume that the state of current joint deterioration essentially determines the probability of progression to the next higher state. This is tantamount to assuming that the time history of deterioration is irrelevant as far as future behavior is concerned and that all one needs to know is the present state and the probabilities of further deterioration associated with each state. From a stochastic process point of view, this assumption of lack of system memory is called the "Markov property" and, if applicable, suggests that the deterioration process may be considered as a Markov chain (2, 3, 4). Crucial to the concept of a Markov process are the following assumptions.

1. For each pair of state E_j, E_k, for $j \neq k$, there exists the continuous function $\lambda_{ik}(t)$ such that

$$\frac{P_{kk}(t,\ t + h)}{h} \rightarrow \lambda_k(t)$$

as $h \rightarrow 0$. Moreover, the above limit is uniform in t and uniform in j for fixed k. $\lambda_{jk}(t)$ defines the time rate of change that governs the passage from state E_j to state E_k (Fig. 2).

2. For each state E_k there exists a continuous function $\lambda_k(t) \geq 0$ such that

$$\frac{1 - P_{kk}(t,\ t + h)}{h} \rightarrow \lambda_{kk}(t)$$

uniformly in t, as $h \rightarrow 0$. $\lambda_k(t)$ defines the time rate of change that governs the passage out of state E_{jk} and must equal the sum of the particular passages, $\sum_{k \neq 1} \lambda_{k1}(t)$. Now the Markov assumption of the independence of past and future events allows the Chapman-Kolmogorov equation for continuous time.

$$P_{ik}(\tau,\ t + h) = \sum_j P_{ij}(\tau,\ t)\, P_{jk}(t,\ t + h)$$

where τ, t, and $h = 0$. This equation states that the probability of a transition from state i to state k during the time interval $(\tau,\ t + h)$ is equal to the probability of transition from state i to some intervening state j during the time interval $(\tau,\ t)$ multiplied by the probability of transition from state j to state k during the time interval $(t,\ t + h)$ summed over all intervening states j. This equation allows us to compose

$$\frac{P_{ik}(\tau,\ t + h) - P_{ik}(\tau,\ t)}{h} = -\frac{1}{h} P_{ik}(\tau,\ t) + \frac{1}{h} \sum_j P_{ij}(\tau,\ t)\, P_{ik}(t,\ t + h)$$

or by extracting the j = k term from the sum

$$\frac{P_{ik}(\tau, t+h) - P_{ik}(\tau, t)}{h} = -\frac{1}{h}P_{ik}(\tau, t) + \frac{1}{h}P_{ik}(\tau, t)\,P_{kk}(t, t+h) + \frac{1}{h}\sum_{j \neq k} P_{ij}(\tau, t)\,P_{jk}(t, t+h) \quad (1)$$

By assumptions 1 and 2, we have, by letting $h \to 0$ in Eq. 1,

$$\frac{\partial P_{ik}(\tau, t)}{\partial t} = -P_{ik}(\tau, t)\,\lambda_k(t) + \sum_{j \neq k} P_{ij}(\tau, t)\,\lambda_{jk}(t) \quad (2)$$

Now, by definition, we have

$$\sum_{l} P_{kl}(\tau, t) = 1$$

and

$$\frac{\partial}{\partial t}\sum_{l} P_{kl}(\tau, t) = 0 = \sum_{l} \lambda_{kl}(t)$$

Further, noting Eq. 2, we have

$$\lambda_k(t) + \lambda_{kk}(t) = 0$$

or

$$\lambda_k(t) = -\lambda_{kk}(t)$$

If we let

$$P(\tau, t) = [P_{ij}(\tau, t)]_{ij}$$

and

$$\Lambda(t) = [\lambda_{ij}(t)]_{ij}$$

then Eq. 2 becomes

$$\frac{\partial}{\partial t}P(\tau, t) = P(\tau, t)\,\Lambda(t) \quad (3)$$

If the $\{\lambda_{ij}(t)\}$ are constant for all time, the process is said to be time-homogeneous. If the $\{\lambda_{ij}(t)\}$ change over time, the process is said to be non-time-homogeneous. Because joints like other physical structures age, it seems unlikely that joint deterioration would be time-homogeneous. Notice also that it is impossible for joints to pass from a given state to a lower state. This requires that the matrices $P(\tau, t)$ and $\Lambda(t)$ be upper triangular—a feature that makes it possible to solve the system of differential equations generated by Eq. 3. Now let us require that in the case of joint deterioration it is reasonable to specify that a joint cannot progress to an advanced state of deterioration without passing through each intervening state. Consequently, all transition probability rates for which $j > i + 1$ must be 0. Thus $\Lambda(t)$ now becomes

$$\Lambda(t) = \begin{bmatrix} -\lambda_{12}(t) & \lambda_{12}(t) & 0 & \ldots & 0 \\ 0 & -\lambda_{23}(t) & \lambda_{23}(t) & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & \ldots & \ldots & \ldots & 0 \end{bmatrix} \tag{4}$$

because $\sum_k \lambda_{jk}(t) = 0$ as discussed earlier. It was decided to classify joint condition into 4 classes and thereby to limit Eq. 4 to a 4×4 matrix. All of these considerations define a special case of Eq. 3 that generates the system of differential equations:

$$\frac{\partial}{\partial t} P_{11}(\tau, t) = - P_{11}(\tau, t)\lambda_{12}(t)$$

$$\frac{\partial}{\partial t} P_{12}(\tau, t) = P_{11}(\tau, t)\lambda_{12}(t) - P_{12}(\tau, t)\lambda_{23}(t)$$

$$\frac{\partial}{\partial t} P_{22}(\tau, t) = - P_{22}(\tau, t)\lambda_{23}(t)$$

$$\frac{\partial}{\partial t} P_{23}(\tau, t) = P_{22}(\tau, t)\lambda_{23}(t) - P_{23}(\tau, t)\lambda_{34}(t)$$

$$\frac{\partial}{\partial t} P_{33}(\tau, t) = - P_{33}(\tau, t)\lambda_{34}(t)$$

$$\frac{\partial}{\partial t} P_{13}(\tau, t) = P_{12}(\tau, t)\lambda_{23}(t) - P_{13}(\tau, t)\lambda_{34}(t)$$

The $P_{j4}(\tau, t)$ are known because $\sum_k P_{jk} = 1$.

The preceding development does not specify the way in which the transition rates $\lambda_{i,i+1}(t)$ vary with time. As mentioned before, aging very likely increases the probability that a joint in a given state of deterioration will pass to a higher state for the same time interval. Therefore, it would seem plausible that the $\lambda_{i,i+1}(t)$ would increase in time. A simple formulation that fits graphical plots of the data is

$$\lambda_{12}(t) = \alpha t^{\phi} \tag{5a}$$

$$\lambda_{23}(t) = \beta t^{\phi} \tag{5b}$$

$$\lambda_{34}(t) = \gamma t^{\phi} \tag{5c}$$

where α, β, γ are scaling coefficients and ϕ is a parameter that indicates the degree of time nonhomogeneity. If $\phi = 0$, the process is time-homogeneous; if $\phi \neq 0$, the process is non-time-homogeneous. Furthermore, we would expect that $\alpha < \beta < \gamma$ because a joint already in a highly deteriorated state can be expected to decay more rapidly to the next higher state. This specification of $\lambda_{i,i+1}(t)$ together with the initial conditions, $P_{11}(\tau, \tau) = P_{22}(\tau, \tau) = P_{33}(\tau, \tau) = 1$, yields the following solutions to the system of equations:

$$P_{ii}(\tau, t) = e^{-\theta_i \int_\tau^t x^{\phi} dx} = e^{-\theta_i \left(\frac{t^{\phi+1} - \tau^{\phi+1}}{\phi + 1}\right)}$$

where

$\theta_1 = \alpha$,
$\theta_2 = \beta$, and
$\theta_3 = \gamma$.

Also,

$$P_{12}(\tau,\ t) = c_1(\tau)e^{-\beta\int_\tau^t x^\phi dx} + \text{particular solution}$$

$$\text{Particular solution} = \alpha e^{-\beta\int_\tau^t x^\phi dx} \int_\tau^t e^{\beta\int_\tau^y z^\phi dz}\, y^\phi P_{11}(\tau,\ y)\,dy$$

$$= \alpha e^{-\beta\left(\frac{t^{\phi+1}-\tau^{\phi+1}}{\phi+1}\right)} \int_\tau^t y^\phi e^{\frac{(\beta-\alpha)(y^{\phi+1}-\tau^{\phi+1})}{\phi+1}}\,dy$$

$$= \alpha e^{\frac{-\beta(t^{\phi+1}+\alpha\tau^{\phi+1})}{\phi+1}} \int_\tau^t y^\phi e^{\frac{(\beta-\alpha)y^{\phi+1}}{\phi+1}}\,dy$$

$$= \frac{\alpha}{\beta-\alpha}\left[e^{-\frac{\alpha(t^{\phi+1}-\tau^{\phi+1})}{\phi+1}} - e^{-\frac{\beta(t^{\phi+1}-\tau^{\phi+1})}{\phi+1}}\right]$$

Because the initial condition $P_{12}(\tau,\ \tau) = 0$, $c(\tau) = 0$. Therefore,

$$P_{12}(\tau,\ t) = \frac{\alpha}{\beta-\alpha}\left[e^{-\frac{\alpha(t^{\phi+1}-\tau^{\phi+1})}{\phi+1}} - e^{-\frac{\beta(t^{\phi+1}-\tau^{\phi+1})}{\phi+1}}\right]$$

where τ specifies the time at which the process starts. Similarly,

$$P_{23}(\tau,\ t) = \frac{\beta}{\gamma-\beta}\left[e^{-\frac{\beta(t^{\phi+1}-\tau^{\phi+1})}{\phi+1}} - e^{-\frac{\gamma(t^{\phi+1}-\tau^{\phi+1})}{\phi+1}}\right]$$

Now,

$$P_{13}(\tau,\ t) = c_2(\tau)e^{-\gamma\int_\tau^t x^\phi dx} + \text{particular solution}$$

$$\text{Particular solution} = e^{-\gamma\int_\tau^t x^\phi dx} \int_\tau^t e^{\gamma\int_\tau^t z^\phi dz}\, \beta y^\phi P_{12}(\tau,\ y)\,dy$$

$$= \frac{\alpha\beta}{(\beta-\gamma)(\alpha-\gamma)} e^{-\frac{\gamma(t^{\phi+1}-\tau^{\phi+1})}{\phi+1}} + \frac{\alpha\beta}{(\gamma-\beta)(\alpha-\beta)} e^{-\frac{\beta(t^{\phi+1}-\tau^{\phi+1})}{\phi+1}}$$

$$+ \frac{\alpha\beta}{(\beta-\alpha)(\gamma-\alpha)} e^{-\frac{\alpha(t^{\phi+1}-\tau^{\phi+1})}{\phi+1}}$$

Because the initial conditions are $P_{13}(\tau,\ \tau) = 0$, $c_2(\tau) = 0$. Therefore,

$$P_{13}(\tau,\ t) = \alpha\beta\left[\frac{e^{-\frac{\alpha(t^{\phi+1}-\tau^{\phi+1})}{\phi+1}}}{(\beta-\alpha)(\gamma-\alpha)} + \frac{e^{-\frac{\beta(t^{\phi+1}-\tau^{\phi+1})}{\phi+1}}}{(\gamma-\beta)(\alpha-\beta)} + \frac{e^{-\frac{\gamma(t^{\phi+1}-\tau^{\phi+1})}{\phi+1}}}{(\beta-\gamma)(\alpha-\gamma)}\right]$$

ESTIMATION OF PARAMETERS

The problem now arises as to how to estimate α, β, γ, and ϕ. Because the expressions for $P_{ii}(\tau, t)$ and $P_{ij}(\tau, t)$ are nonlinear, classical least squares techniques are not helpful. In the present case, a computer optimization procedure containing a modification of the steepest method was used (7, 8, 9). The procedure minimized the expression where the $P_{ij}(\tau, t)$ were proportions computed directly from the survey data. In addition, each residual was weighted in proportion to the number of joints entering into the probability determination.

RESULTS WITH FIELD DATA

An attempt was made to estimate the model's 4 parameters by the nonlinear least squares procedure for all the 43 construction projects surveyed at 5-, 10-, and 15-year intervals. Except for a small number of extremely good projects for which there was no appreciable deterioration at 15 years, estimates for α, β, γ, and ϕ converge rapidly. In all cases, the model's fit was within ±0.10 for the 4 probabilities × 3 survey years × 43 projects = 516 data points (Figs. 3, 4, 5, and 6). Examples of state probability history curves for several particularly good and poor projects are shown in Figures 7 and 8. Shown in Figure 9 are the expected (average) state histories for the same 2 projects. The probabilities for states 2 and 3 of the poorly performing projects peak at about 11 and 13 years and then decline. This is because joints are not entering states 2 and 3 so fast as they are leaving these states for state 4. State 4 is a terminal or absorbing state and naturally captures more joints with time until all joints are finally in this state.

Also of interest is the finding that $\hat{\alpha} < \hat{\beta} < \hat{\gamma}$. Thus, a joint is more likely to deteriorate to the next highest state if it is already in a deteriorated condition. Distributions of $\hat{\beta}/\hat{\alpha}$ and $\hat{\gamma}/\hat{\alpha}$ are shown in Figure 10. Attempts were made to quantify these relations, thereby reducing the number of model parameters, but the models did not fit the data satisfactorily (residuals as high as ±0.30 were encountered). Because $\hat{\gamma}$ was generally greater than $\hat{\alpha}$ and $\hat{\beta}$, one would presume that state 3 joints would be the ones most likely to progress to state 4. Therefore, if state 4 (mostly blowups) prediction is desired, a good strategy would be to look for joints in state 3. Because the model will predict the probability of state 4, given state 3 for any elapsed time, one can compute state 4 probability curves once α, β, γ, and ϕ have been estimated from earlier performance data (or possibly environmental and materials variables).

Figure 11 shows for an arbitrary construction project the cumulative probability of state 4 occurring given that a joint was in state 3 at the selected τ times of 1, 11, and 15 years. Notice the rapid rate of increase in probability as τ increases. For example, if a joint is in state 3 at 1 year ($\tau = 1$), it takes just over 12 years before the potential occurrence of state 4 has reached a probability of 0.50. However, if the joint is in state 3 at 11 years ($\tau = 11$), it takes only 3 years for the probable occurrence of state 4 to reach 0.50. These curves will not give good forecasts of blowup probability unless α, β, γ, and ϕ are reliably estimated from early performance data for each project itself or from a group of relevant causal variables.

As discussed earlier, ϕ is a measure of the non-time-homogeneity of the process. Figure 12 shows the frequency distribution of $\hat{\phi}$ for the 40 projects for which ϕ could be estimated. $\hat{\phi}$ varies from about 0.20 to 5.83 with a median value of about 2.3. Thus, our hypothesis concerning non-time-homogeneity is tenable, particularly because most $\hat{\phi}$ are significantly greater than 0.0 (α level = 0.05, as tested by a linearized t test).

CONCLUSION

Condition survey data were used to define 4 joint conditions in terms of the percentage of transverse joint length deteriorated. Progressive deterioration of a joint was considered as the passage from a given state to the next higher state. This process over time appeared to embody the Markov assumption, which requires that only the current state determine the probability of passage to another state. The Markov assumption was used to design a continuous time, non-time-homogeneous Markov process

Figure 1. Survey data on joint condition.

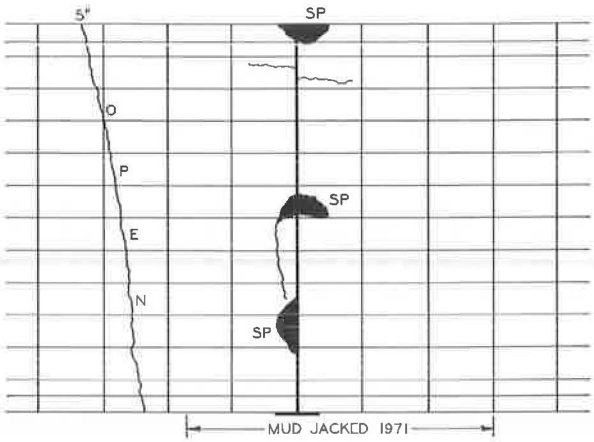


Figure 2. Probability transition from state i to state j as a function of time.

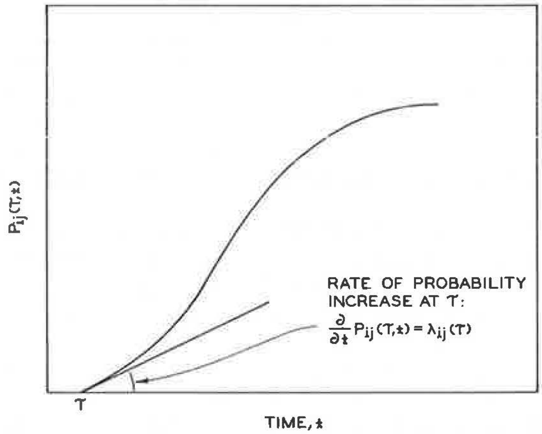


Figure 3. Estimated versus actual probability of a joint in state 1 for 5, 10, and 15 years.

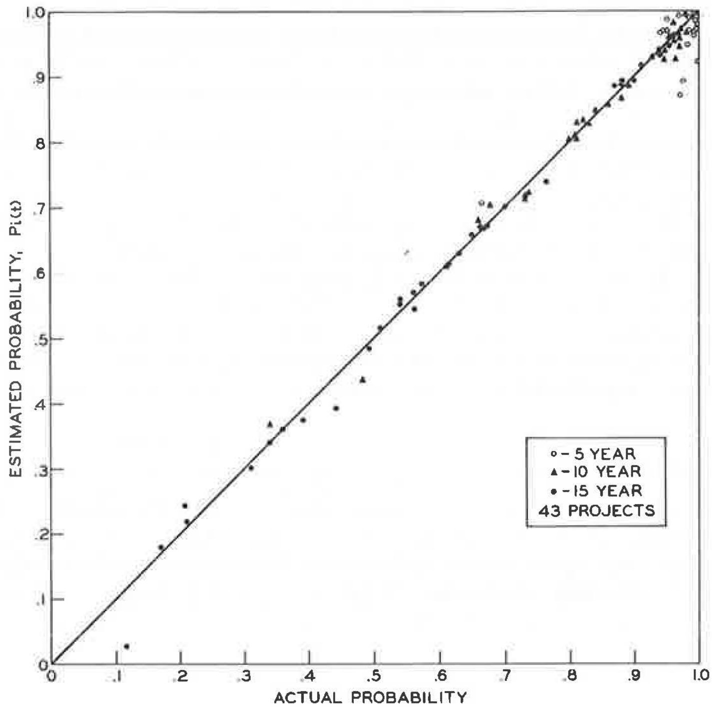


Figure 4. Estimated versus actual probability of a joint progressing from state 1 to state 2 within 5, 10, and 15 years.

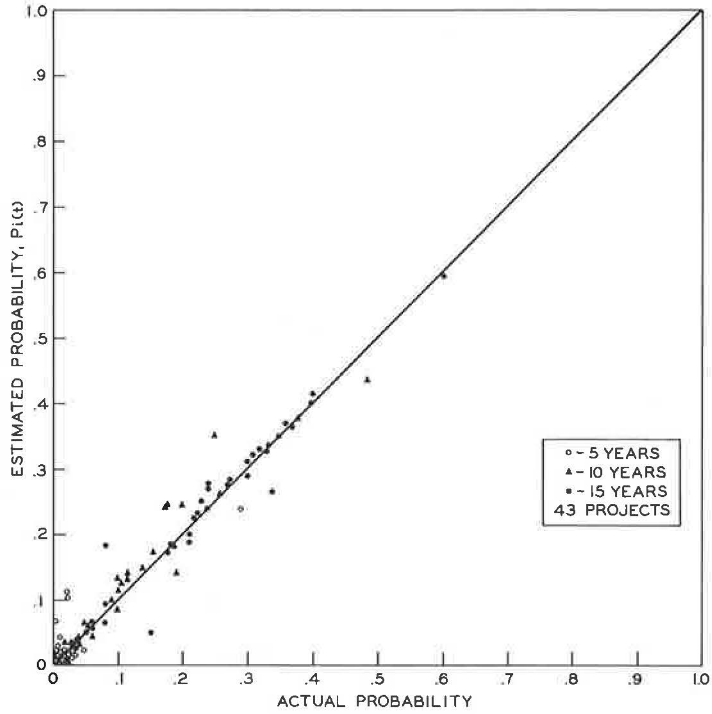


Figure 5. Estimated versus actual probability of a joint progressing from state 1 to state 3 within 5, 10, and 15 years.

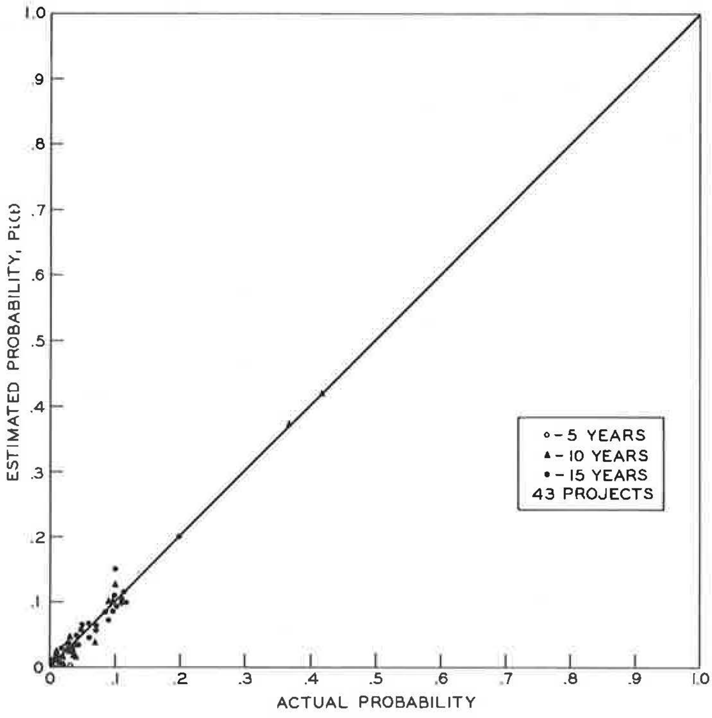


Figure 6. Estimated versus actual probability of a joint progressing from state 1 to state 4 within 5, 10, and 15 years.

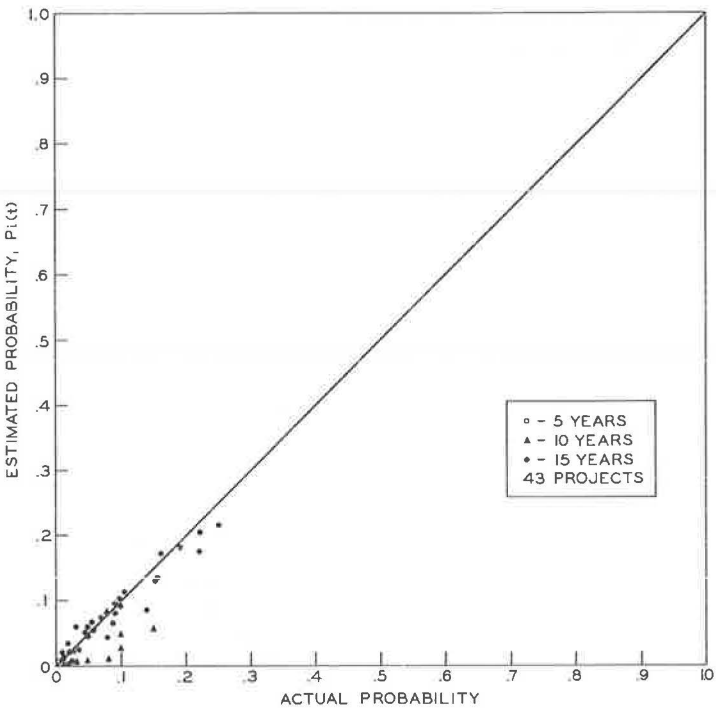


Figure 7. Estimated probability of a joint from a good project being in a state during a 15-year service life.

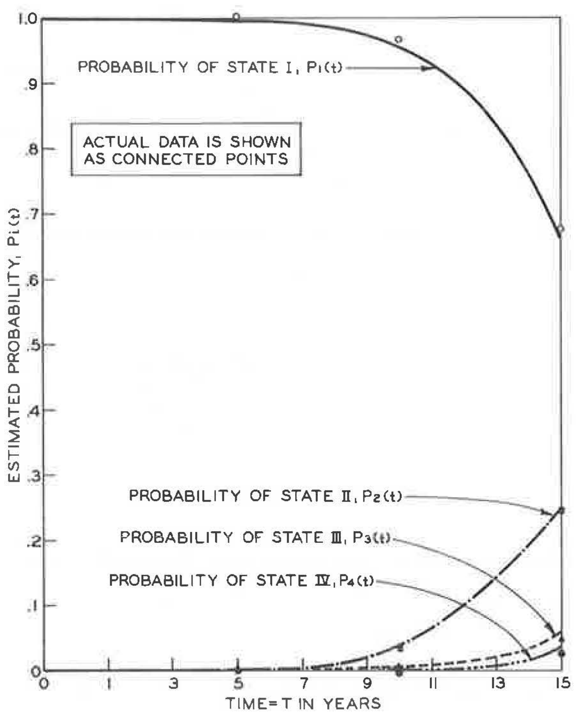


Figure 8. Estimated probability of a joint from a poor project being in a state during a 15-year service life.

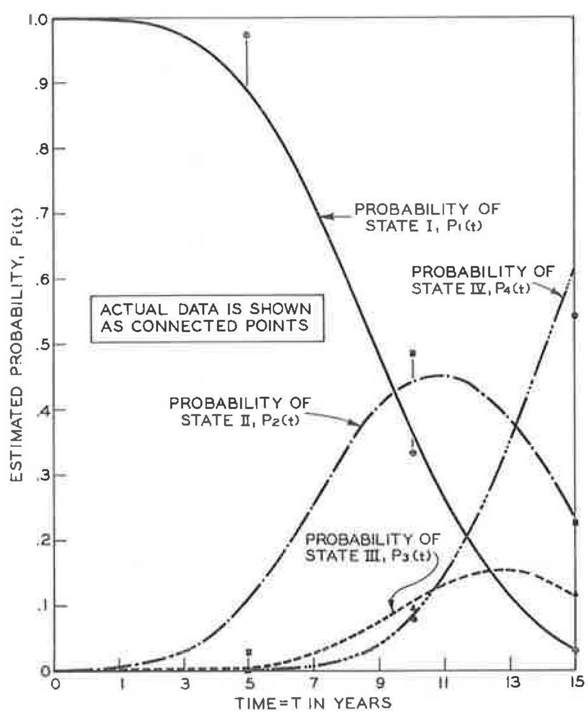


Figure 9. Estimated expected state for projects within 15-year service life.

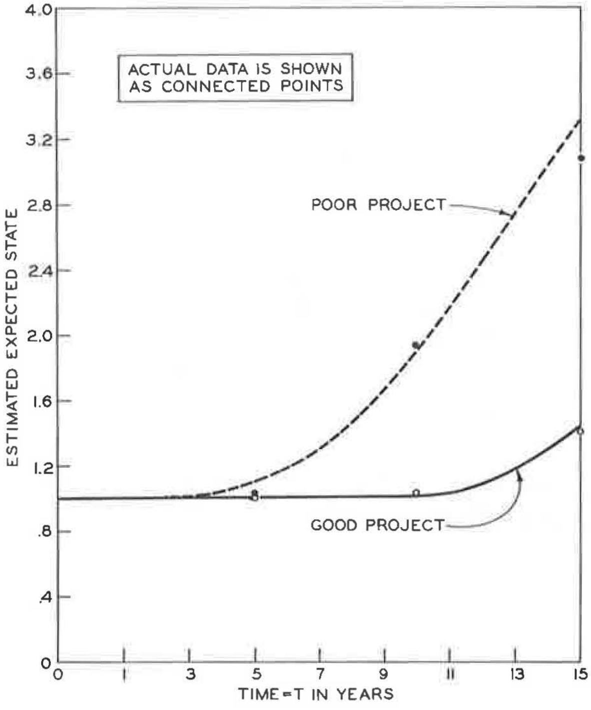


Figure 10. Cumulative ratios for coefficients plotted on normal probability paper.

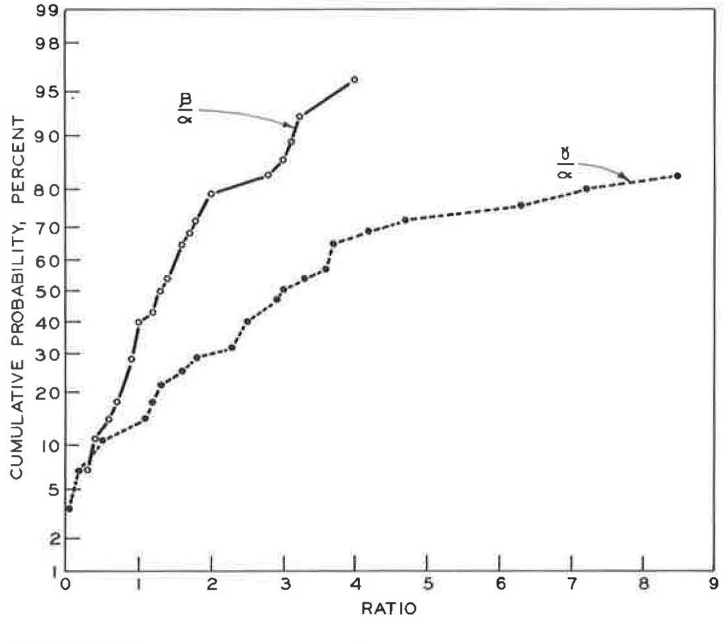


Figure 11. Cumulative probability of a joint progressing from state 3 at 1, 11, and 15 years to state 4 at t.

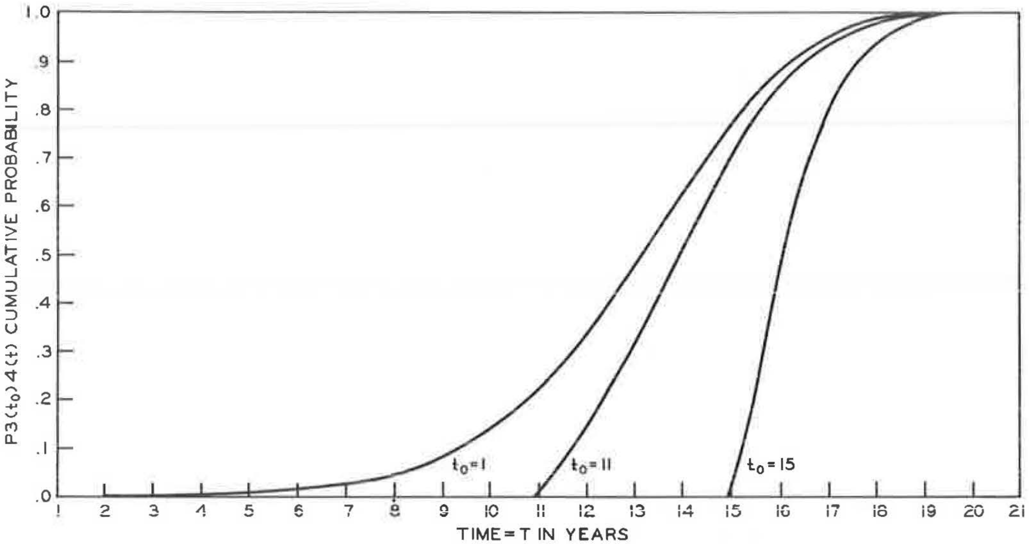
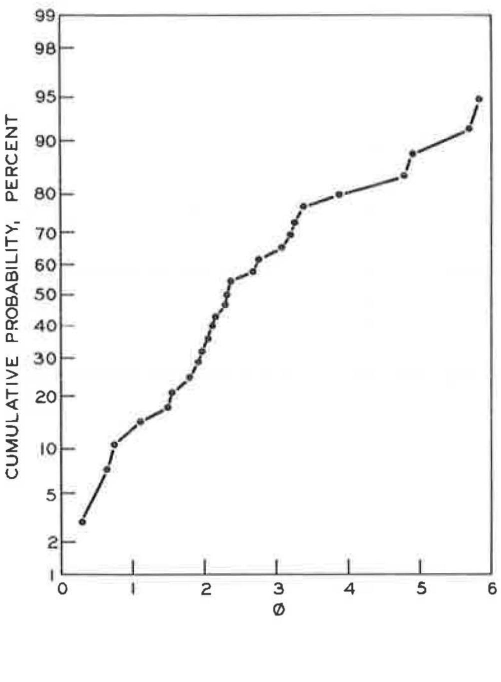


Figure 12. Cumulative distribution of ϕ.



to model joint deterioration. The particular model chosen required the nonlinear estimation of 4 fitting parameters to predict 27 probabilities of joint condition measured at the 5-, 10-, and 15-year periods of service life. This procedure proved very satisfactory except for projects showing practically no deterioration, For these, parameter estimation was not possible because of computer overflow problems. Generally excellent fits of estimated and actual data were obtained for the 43 projects examined. Based on these results the procedure looks quite promising for structural deterioration modeling in general.

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