use of CATS original capacity-restraint function provides an assignment slightly closer to actual counts, but the results are not significantly better than the remaining two assignments. All three assignments tend to overpredict traffic on low-volume links, partially because the local street network over which the beginning and ending segments of trips travel is incomplete. Comparison of the second and third assignments shows that the effect of the adjustment to the FHWA curves is almost negligible.

The changes that do occur, however, are in the desired direction, which indicates that some control over the assignment can be exerted through capacity-restraint functions. Since the equilibrium-assignment algorithm produces a convergent series of assignments, it should be possible to calibrate these functions according to route type or location in an urban area.

CONCLUSIONS

Although our experience with applications of equilibrium assignment to large-scale, congested networks is still limited, we believe that the results reported in this paper provide convincing evidence that equilibrium assignment should always be preferred to FHWA iterative assignment for congested networks. We reach this conclusion for three reasons:

1. Equilibrium assignment provides a better assignment in terms of the overall objective of equal travel times over all paths used between each origin and destination pair,
2. The computational effort is similar and may be less in some cases in which the equilibrium algorithm converges quickly, and
3. Equilibrium assignment can be readily incorporated into FHWA's PLANPAC battery; moreover, it is already available in UTPS.

The preliminary results we have presented concerning the ability of equilibrium assignment to reproduce observed 24-h flows are not as convincing. There are two reasons for this result. First, the capacity-restraint functions are probably too crude. This problem has been explored slightly here, but more study and experimentation are needed. Second, the use of equilibrium assignment to produce 24-h assignments may be inappropriate in that only the peak periods have truly congested flow. All-or-nothing assignment may be sufficient for off-peak periods. Additional study of this question is needed to determine the actual cause of these apparent differences between ground counts and assigned flows.

ACKNOWLEDGMENT

The basic research on equilibrium assignment, on which this paper is based, has been conducted by many individuals during the past 10 years. Since it was not our purpose to review the development of equilibrium-assignment methods, we have not referred to this literature, except in the few cases in which it was directly pertinent. We are grateful for the advice and encouragement that we received from David Gendell of the Federal Highway Administration and Thomas Hillegass of the Urban Mass Transportation Administration.

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Equilibration Properties of Logit Models

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Despite the importance of supply-demand equilibration in travel-demand forecasting and urban planning, no attention has been paid to the equilibration properties of logit models of travel demand and residential mobility. The preponderance of logit models in travel demand and related fields suggests that these properties are worth examining if these models are to become useful forecasting tools. This paper demonstrates the basic price equilibration properties of logit models for simplified versions of six typical problems encountered in travel-demand and residential-location forecasting. Measures of the differential price of any two alternatives are derived in closed form and shown to reflect the well-known logit property of the independence from irrelevant alternatives as long as the population of travelers and households is one homogeneous group. It is shown that

this property is lost when the population consists of several segments that have distinct preferences. In such cases closed-form solutions are not possible and numerical procedures are necessary.

Many problems in transportation systems and urban planning require an equilibrium relation between demand and supply in order to measure or evaluate system performance. The crucial steps for the planner or system analyst are (a) the estimation of demand functions, (b) the estimation of supply functions, and (c) the performance of a consistent forecast for a future state by equilibrating demand and supply.

In recent years economists, transportation planners, and systems analysts have contributed to the development and empirical estimation of a class of demand functions based on the logit and related models of discrete choice. Logit models have been applied widely in travel-demand and modal-choice analysis and to a lesser extent in the related areas of housing-market and residential-location analysis. The best-known works on the subject are those of McFadden (1) and Domencich and McFadden (2). Despite the preponderance of logit models as tools of demand analysis, no attention has been paid to the equilibration properties of these models. This issue finds brief mention in the recent book by Domencich and McFadden (2). As they put it:

> If the travel-demand function is structured so that all of the decisions incorporated within it are allowed to be responsive to the performance of the transportation system, then provisions must be made to equilibrate demand and the performance of the transport system to estimate properly the effects of changes in the transportation system on trip interchanges. It is not the purpose of this study to analyze or develop equilibration procedures, but the implications of a policy-sensitive demand model on other modeling requirements should be noted.
>
> . . . failure to equilibrate demand and system performance properly could result in substantial error in estimating the expected impact of a facility change on travel volumes and service levels.

In many instances it is realistic to assume that supply or capacity will be inelastic, at least in the short run. For such cases, an equilibration problem determines price adjustments that clear the market by matching demand and supply for each alternative in the market. From the practical point of view, the importance of price adjustments in forecasting may be demonstrated by the following scenario. Suppose that a logit model of residential location has been estimated for a city by using data from 1975. It is now desired to use this model to forecast residential-location patterns for 1980 under the assumption that transportation services to subarea A of the city will be much improved between 1975 and 1980. In the meantime, let us assume that the housing stock in the same area remains approximately constant due to such factors as zoning, unavailability of vacant land, and high costs of redevelopment. The 1980 travel improvements will strengthen the demand for subarea A. If the forecasting procedure assumes that housing prices will remain unchanged between 1975 and 1980, the demand for housing in subarea A may well exceed the supply of housing units there, assuming other subareas receive comparatively minor travel improvements. In other areas demand may be found to be below the supply. To correct this mismatch, housing prices should increase in those zones where demand exceeds supply and should decrease in those zones where supply exceeds demand. The housing market is equilibrated when a new set of housing prices is found such that demand is less than or equal to supply in every zone.

TRANSPORTATION ANALYSIS AND URBAN PLANNING PROBLEMS

In this paper several fundamental equilibration properties of logit models are demonstrated within the context of six specific problems that are typical in transportation analysis and urban planning. Problems A through E are united by the assumption that there is one homogeneous population of commuters or households, and this assumption enables closed-form solutions. Several properties of the logit-demand structure are reflected in these solutions:

1. Price differentials (or the relative prices of alternatives) are unique, although the level of prices is nonunique up to the arbitrary specification of any one price;
2. The well-known logit property of the independence from irrelevant alternatives (IIA) implies that the relative prices of two alternatives (or locations) are determined independently of the information for all other alternatives (or locations); and
3. Price adjustments tend to absorb advantages that result from travel improvements so that they are reflections of these.

Later we relax the assumption of a homogeneous population and introduce several population segments that have different utility functions and choice behavior. It is shown that the IIA property no longer applies to the relative prices of two locations. Closed-form solutions are not possible, but I have developed and tested a numerical solution method (3).

Problem A: Parking Fees and Bus Fares

Suppose that a city's downtown receives commuters from a suburb through two travel modes. One is automobile, which requires parking in public lots operated by the city. The other alternative is to take the bus, which is also operated by the city. Each commuter pays a parking fee or a bus fare. The city operates a rush-hour bus capacity of S_B seats and maintains exactly S_A parking spaces. It receives N suburban commuters daily and we assume that there is no carpooling; that is, each automobile commuter drives alone. We also assume that $S_A + S_B = N$. What should be the parking fare and what should be the price of a two-way bus trip, assuming that both modes operate without congestion?

Suppose that each commuter decides whether to be a bus rider or a driver in such a way that aggregate demand is logistic and given by

$$f_i = \exp(\alpha P_i + K_i) \Big/ \sum_{j=1}^{2} \exp(\alpha P_j + K_j) \qquad i = 1,2;\ \alpha < 0 \tag{1}$$

where

f_1 and f_2 = the proportion of commuters that take automobile and bus, respectively,
P_1 and P_2 = the parking fee and two-way bus fare, and
$K_i \equiv \sum_{n=1} \beta_n Q_{in}$ = an abbreviation for the remaining utility terms.

P_1 and P_2 are the unknowns to be determined by the city, which seeks

$$Nf_1(P_1, P_2) = S_A \tag{2a}$$

and

$$Nf_2(P_1,P_2) = S_B \tag{2b}$$

which, by using Equation (1), can be rewritten as

$$N/\{1 + \exp[\alpha(P_2 - P_1) + (K_2 - K_1)]\} = S_A \tag{3a}$$

and

$$N/\{1 + \exp[\alpha(P_1 - P_2) + (K_1 - K_2)]\} = S_B \tag{3b}$$

By rearranging either Equation 3a or 3b we get,

$$P_1 - P_2 = [(1/\alpha)(K_2 - K_1)] - [(1/\alpha)\ell n(S_B/S_A)] \tag{4}$$

The right-hand side of Equation 4 is the amount by which the prices should differ so that the number of drivers exactly matches the number of parking spaces and the number of riders exactly matches the number of bus seats. From Equation 4 we note several properties. First, the equilibrium prices are nonunique: any two prices that have the same difference (P_1-P_2) will do. Second, take the case where $S_B = S_A$. In this case we have $P_1 - P_2 = (1/\alpha)(K_2 - K_1)$, from which we know that if $K_2 > K_1$ then $P_2 > P_1$—the less attractive mode is priced lower. Third, suppose that more buses are added and an equal number of parking spaces is closed. From Equation 4 this would require increasing P_1 (the price of parking, which is now scarcer) or decreasing P_2 (the price of a bus trip, which is more available). Next, suppose that a third mode (train) is introduced with the number of seats (S_T) such that $S_A + S_B + S_T = N$, with P_3, the two-way train fare, and K_3, the remaining utility. Then, the above derivation can be repeated to derive Equation 4, but also

$$P_1 - P_3 = (1/\alpha)(K_3 - K_1) - (1/\alpha)\ell n(S_T/S_A) \tag{4a}$$

and

$$P_2 - P_3 = (1/\alpha)(K_3 - K_2) - (1/\alpha)\ell n(S_T/S_B) \tag{4b}$$

Note that Equations 4 and 4a will satisfy

$$N/\{1 + \exp[\alpha(P_2 - P_1) + (K_2 - K_1)] + \exp[\alpha(P_3 - P_1) + (K_3 - K_1)]\} = S_A \tag{5}$$

Equations 4a and 4b are of the same form as Equation 4, and any one of these is a direct reflection of the property of IIA—the price difference for any two modes is independent of any other mode. Given an arbitrary price for any one mode, Equations 4, 4a, and 4b can be used to make a unique determination of all the other prices. But how should this price level be determined? It seems reasonable to assume that the city should set these prices so as to cover the cost of operating the modes net of any subsidies from other sources (assumed to be zero here). Let the total costs be given by $C = C(S_A, S_B, S_T)$. Total daily revenues are $R = P_1S_A + P_2S_B + P_3S_T$. Setting $R = C$ we can substitute from Equations 4, 4a, and 4b for any two of the prices and solve for the third, thus determining the break-even price level.

Problem B: Supply of Buses and Parking Spaces

In problem A we assumed that the supply of parking spaces and bus seats is fixed. In this problem we allow the public authority to determine jointly both the price levels (P_1 and P_2) and also the market size of each mode (S_A and S_B) such that $S_A + S_B = N$. This problem may be posed as follows: The city contracts with a bus company that supplies buses and another firm that supplies parking space. Each of these firms operates under regular, upward sloping supply functions such that $S_A = F_1(P_1)$ and $S_B = F_2(P_2)$. The public authority must determine the regulated prices (P_1 and P_2) under which the two firms should operate. By using the similarity with problem A we know that P_1 and P_2 should satisfy

$$F_1(P_1) + F_2(P_2) - N = 0 \tag{6}$$

and

$$P_1 - P_2 = (1/\alpha)(K_2 - K_1) - (1/\alpha)\ell n[F_2(P_2)/F_1(P_1)] \tag{7}$$

where Equation 7 is a restatement of Equation 4 and assures that $Nf_i(P_1, P_2) = F_i(P_i)$ for each i. If N is fixed, P_1 and P_2 can be found from Equations 6 and 7. Alternatively, if N is considered flexible another relationship is needed to replace Equation 6. This may be

$$(P_1 - c_1)F_1(P_1) + (P_2 - c_2)F_2(P_2) = 0 \tag{8}$$

where c_1 and c_2 are the costs of supplying a marginal capacity. Equation 8 states that both operations taken jointly break even. This may happen in two ways. Either $P_1 = c_1$ and $P_2 = c_2$ or $P_1 > c_1$ and $P_2 < c_2$ (or equivalently $P_1 < c_1$ and $P_2 > c_2$), but $(P_1 - c_1)F_1(P_1) = -(P_2 - c_2)F_2(P_2)$. This means that mode 1 produces a surplus of $\tau_1 = (P_1 - c_1)F_1(P_1)$ and mode 2 needs a subsidy of $\sigma_2 = (c_2 - P_2)F_2(P_2)$. Equation 8 assures that $\tau_1 = \sigma_2$ and thus both modes are kept in operation, by taxing mode 1 and by subsidizing mode 2.

Problem C: Demand for Housing and Location Rents

Logit models estimated by Quigley (4), Lerman (5), and Anas (6) are intended to capture the demand for residential location or type of housing. Typically, this problem may be stated as follows: Suppose that there are i = 1. . . I distinct zones, each of which contains S_i identical housing units. Then, the demand for zone (location) i can be expressed by the following logit model with grouped alternatives,

$$f_i = S_i\exp(U_i)\Big/\sum_j S_j\exp(U_j) \qquad i = 1\ldots I; \quad \sum_i f_i = 1 \tag{9}$$

If we also assume that each household rents one housing unit and that the number of housing units in the rental market is equal to the number of households (N) then $N = \sum_j S_j$. This means that each housing unit will be occupied. In the short run the supplies (S_i) are assumed fixed for each i. Thus

$$Nf_i = S_i \text{ for each } i = 1\ldots I \tag{10}$$

From Equations 9 and 10 we can write

$$Nf_i/Nf_j = S_i\exp(U_i)/S_j\exp(U_j) = S_i/S_j \tag{11}$$

We can now examine the implication of Equation 11 for rent adjustments if we first specify the utility function. Suppose it is given as

$$U_i = \alpha R_i + \beta T_i + K_i \qquad \alpha,\beta < 0 \tag{12}$$

where K_i is an abbreviation of terms such that $K_i \equiv \sum_{n=1} \gamma_n Q_{in}$ with Q_{in} a measure of the n^{th} characteristic of zone i and γ_n the corresponding utility parameter. R_i is the rent (price) of a housing unit in zone i and T_i is the generalized travel cost associated with zone i.

Equation 11 will hold only if

$$U_i = U_j \tag{13}$$

From this we derive

$$R_i - R_j = (\beta/\alpha)(T_j - T_i) + (1/\alpha)(K_j - K_i) \tag{14}$$

This result is analogous to our previous result in problem A. Suppose that the two zones are identical in all characteristics except transportation costs, then $K_i = K_j$ and the rent differential reflects the transport cost differential. The nonuniqueness and other considerations noted in problem A apply to Equation 14 as well.

Several variants of Equation 14 are worth noting. Suppose that the utility function was specified as follows, where Y represents household income,

$$U_i = \ln\{R_i^\alpha T_i^\beta K_i\} \quad \alpha,\beta < 0,\ K_i \equiv \prod_{n=1} Q_{in}^{\gamma_n} \tag{15}$$

or

$$U_i = \alpha[Y - R_i - T_i] + K_i \quad \alpha > 0,\ K_i \equiv \sum_{n=1} \gamma_n Q_{in} \tag{16}$$

or

$$U_i = \ln\{[Y - R_i - T_i]^\alpha K_i\} \quad \alpha > 0,\ K_i \equiv \prod_{n=1} Q_{in}^{\gamma_n} \tag{17}$$

By using Equation 13, Equations 15-17 will lead to the following,

$$R_i/R_j = (K_j T_j^\beta / K_i T_i^\beta)^{1/\alpha} \text{ for Equation 15} \tag{18}$$

$$R_i - R_j = (1/\alpha)(K_j - K_i) + (T_j - T_i) \text{ for Equation 16} \tag{19}$$

$$R_i - R_j(K_j/K_i)^{1/\alpha} = (Y - T_i) - (Y - T_j)(K_j/K_i)^{1/\alpha} \text{ for Equation 17} \tag{20}$$

The nonuniqueness argument applies to these as well. The IIA property of logit comes through in every case as the relative rents do not depend on any zone other than the two we are concerned with. Let $K_i = K_j$, then Equations 14 and 18-20 reduce to the following,

$$R_i - R_j = (\beta/\alpha)(T_j - T_i) \tag{21}$$

$$R_i/R_j = (T_j/T_i)^{\beta/\alpha} \tag{22}$$

$$R_i - R_j = T_j - T_i \text{ for both Equations 19 and 20} \tag{23}$$

The last of these is reminiscent of the early location rent model developed by Wingo (7) who assumed, rather arbitrarily, that rent plus transportation costs add up to the same constant at every location, namely $R_i + T_i$ = constant for every i.

In Equations 14 and 18-20, if the rent of any one zone is arbitrarily fixed, then the location rents of all other zones are uniquely determined.

Problem D: Impact of a New Travel Mode on Differential Location Rents

Assume that two locations i and j are identical in all respects and each is served by the same travel mode—automobile. Let R_i represent location rent for zone i, as before, and also let T_{i1} and T_{j1} represent travel costs by automobile to zone i and zone j. If we assume that demand is given by a logit model of joint location and mode choice

$$f_{im} = S_i \exp(U_{im}) / \sum_i \sum_k S_j \exp(U_{jk}) \qquad \sum_i \sum_m f_{im} = 1 \tag{24}$$

where U_{im} is the utility of choosing zone i and mode m for commuting to zone i. Now suppose that a new travel mode, transit, is introduced but serves only zone j and has travel cost $T_{j2} \neq T_{j1}$. Let the utility function be $U_{im} = \alpha R_i + \beta T_{im}$; then with condition $\sum_i S_i = N$ we can repeat our previous derivations in slightly different form, namely

$$Nf_{i1}/(Nf_{j1} + Nf_{j2}) = S_i \exp(U_{i1})/[S_j \exp(U_{j1}) + S_j \exp(U_{j2})] = S_i/S_j \tag{25}$$

By mutiplying Equation 25 by S_j/S_i we get

$$\exp(U_{i1}) = \exp(U_{j1}) + \exp(U_{j2}) \tag{26}$$

which implies

$$\exp(\alpha R_i)\exp(\beta T_{i1}) = \exp(\alpha R_j)[\exp(\beta T_{j1}) + \exp(\beta T_{j2})] \tag{27}$$

and

$$R_i - R_j = (1/\alpha)\ln[\exp(\beta T_{j1}) + \exp(\beta T_{j2})] - (\beta/\alpha)T_{i1} \tag{28}$$

If we also assume that $T_{j1} = T_{i1}$, that is, that the automobile costs of the two zones are identical, then the differential rent $R_i - R_j$ is attributable purely to the impact of transit. Thus, if we let $T_{i1} = T_{j1} \equiv T_1$ we have

$$R_i - R_j = (1/\alpha)\ln[\exp(\beta T_1) + \exp(\beta T_{j2})] - (\beta/\alpha)T_1 \tag{29}$$

Let us now take this one step further. Suppose that the introduction of transit does not create any real advantage. This would be the case if $T_{j2} = T_1$, which would reduce Equation 20 further to

$$R_i - R_j = (1/\alpha)\ln 2 \tag{30}$$

Since $\alpha < 0$, Equation 30 implies that $R_j = R_i + |(1/\alpha)\ln 2|$ where $|\cdot|$ measures the rent increase in zone j attributable to the presence of a new mode identical in transport cost to the existing mode. Note that, although the two zones are indistinguishable in terms of travel cost and all other characteristics, zone j still has a higher rent than does zone i. Intuitively, this seeming paradox is clarified as follows: Suppose that initially $R_i = R_j$ for these two zones. This would imply that $f_{j1} = f_{j2} = f_{i1}$ and thus $Nf_{j1} + Nf_{j2} = 2Nf_{i1}$. In other words, twice as many households choose zone j. Clearly then, to properly reallocate this demand and assure $Nf_{j1} + Nf_{j2} = Nf_{i1}$ rents in zone j must be higher.

We must also note that, if x new travel modes with equal transportation costs are introduced into zone j, then $R_j = R_i + |(1/\alpha)\ln(x+1)|$.

Next, suppose that the utility function includes a mode-specific dummy variable so that $U_{i1} = \alpha R_i + \beta T_{i1}$ and $U_{j1} = \alpha R_j + \beta T_{j1}$ but $U_{j2} = \alpha R_j + \beta T_{j2} + \gamma_2$ where γ_2 measures the bias due to mode 2. From this we obtain the equivalent of Equation 28,

$$R_i - R_j = (1/\alpha)\ln[\exp(\beta T_{j1}) + \exp(\beta T_{j2} + \gamma_2)] - (\beta/\alpha)T_{i1} \tag{31}$$

When $T_{j2} = T_{j1} = T_{i1} \equiv T_1$ we get

$$R_i - R_j = (1/\alpha)\ln[1 + \exp(\gamma_2)] \tag{32}$$

Finally, if x new modes are introduced, each with equal transport costs, the equivalent of Equation 32 is

$$R_i - R_j = (1/\alpha)\ln[1 + \sum_{n=2}^{x} \exp(\gamma_n)] \tag{33}$$

Problem E: Before-and-After Differential Rent Due to a Transportation Improvement

We now return to model Equation 9 of problem C. Let U_{ib} be the utility before a transportation improvement takes place and let U_{ia} be the utility after a transportation improvement. Assume that $U_{ib} = \alpha R_{ib} + \beta T_{ib}$ and $U_{ia} = \alpha R_{ia} + \beta T_{ia}$ with $T_{ia} < T_{ib}$, then what is the relationship between R_{ib} and R_{ia}? Note that Equation 9 can be written as

$$f_{ib} = 1\Big/\Big[1 + \sum_{j \neq i} (S_j/S_i)\exp(U_{jb} - U_{ib})\Big] \qquad i = 1 \ldots I \qquad (34a)$$

and

$$f_{ia} = 1\Big/\Big[1 + \sum_{j \neq i} (S_j/S_i)\exp(U_{ja} - U_{ia})\Big] \qquad i = 1 \ldots I \qquad (34b)$$

Again, assume that $\sum_j S_j = N$, what is $R_{ia} - R_{ib}$ in zone i, if this is the only zone affected by the transport improvement, that is, $T_{ia} < T_{ib}$ and $T_{ja} = T_{jb}$ for all $j \neq i$?

Note that

$$Nf_{ib}/Nf_{ia} = \Big[1 + \sum_{j \neq i} (S_j/S_i)\exp(U_j - U_{ia})\Big] \div \Big[1 + \sum_{j \neq i} (S_j/S_i)\exp(U_j - U_{ib})\Big] = 1 \qquad (35)$$

where $U_j \equiv U_{ja} = U_{jb}$ for $j \neq i$. The above equality can be maintained only if $U_{ia} = U_{ib}$. By using the definition of utility this requires that

$$R_{ia} - R_{ib} = (\beta/\alpha)(T_{ib} - T_{ia}) \qquad (36)$$

Thus, the rent increase must be such that the utility level before and after the investment remains the same, assuming that the utility level remains unchanged in all other zones not affected by the transportation improvement. For this to occur it is only necessary that the rent of any one zone unaffected by the investment remain unchanged before and after the investment. The above readily generalizes to the case of a transportation improvement that affects more than one zone—if utility remains unchanged before and after the improvement $U_{ia} = U_{ib}$ for each i, then the market is cleared before and after the improvement and Equation 36 holds for each zone i.

Next, consider the possibility that a new travel mode is introduced to every zone. In this case we are dealing with a model such as that of problem D (see Equation 23). The market will clear before and after the investment if

$$\exp(U_{i1b}) = \exp(U_{i1a}) + \exp(U_{i2a}) \qquad (37)$$

where 1 denotes automobile and 2 the new mode, say transit. Assuming that automobile characteristics remain the same before and after the transit investment, the three utility functions are $U_{i1b} = \alpha R_{ib} + \beta T_{i1}$, $U_{i1a} = \alpha R_{ia} + \beta T_{i1}$, and $U_{i2a} = \alpha R_{ia} + \beta T_{i2}$. In this way, Equation 36 becomes

$$R_{ib} - R_{ia} = (1/\alpha)\ln\{1 + \exp[\beta(T_{i2} - T_{i1})]\} \qquad (38)$$

Since $\alpha < 0$ this implies $R_{ia} > R_{ib}$. Note that as the transit improvement worsens the rent increase vanishes (recall $\beta < 0$):

$$\lim_{T_{i2}\to\infty} R_{ib} - R_{ia} = (1/\alpha)\ln\{1 + \exp[\beta(T_{i2} - T_{i1})]\} \qquad (39)$$

Problem F: Traffic Congestion

A common equilibration problem of a different nature is that of capacity-constrained traffic flow, where the travel times or generalized costs on a network's links depend on the traffic-flow capacity of the link and the volume (number of passengers) that use the link. Unlike the destination- and housing-choice problems considered in this paper, traffic-flow equilibration is highly network sensitive, and problems can quickly become complicated beyond the reach of analytical solutions. Still, the basic nature of the problem can be illustrated for the simplest of all networks: two highway routes that connect an origin-destination pair used by a homogeneous population of drivers (N). In this case, let the proportion of drivers that use route i be logistic. Then

$$f_i = \exp(\alpha t_i + K_i)\Big/\sum_{j=1}^{2} \exp(\alpha t_j + K_j) \qquad i = 1, 2 \qquad (40)$$

where K_i is an abbreviation of the utility due to other (fixed) characteristics of the route i. Let the travel time (t_i) be given via a simple volume-delay function, namely,

$$t_i = t_{oi} + A_i(Nf_i/C_i)^g \qquad i = 1, 2 \qquad (41)$$

where

t_{oi} = the free-flow link travel time,
A_i = a link-specific parameter,
C_i = the link capacity,
Nf_i = the volume that uses link i, and
g = a parameter ($g > 0$).

By abbreviating $A_i N^g / C_i^g$ as b_i and substituting Equation 41 into Equation 40 we obtain

$$f_i = [\exp(\alpha t_{oi} + \alpha b_i f_i^g + K_i)]\Big/\Big[\sum_{j=1}^{2} \exp(\alpha t_{oj} + \alpha b_j f_j^g + K_j)\Big] \qquad i = 1, 2 \qquad (42)$$

Either one of these two equations can be written as

$$(f_1 - 1)\exp(\alpha t_{o1} + \alpha b_1 f_1^g + K_1) + f_1\exp(\alpha t_{o2} + \alpha b_2 f_2^g + K_2) = 0 \qquad (43a)$$

or

$$\ln(f_2/f_1) = \alpha(t_{02} - t_{01}) + \alpha(b_2 f_2^g - b_1 f_1^g) + (K_2 - K_1) \qquad (43b)$$

and should be solved for equilibrium-flow proportions f_1^*, f_2^* by using an iterative procedure.

EXCESS CAPACITY

Since the assumption that aggregate supply equals aggregate demand is somewhat unrealistic, we will examine the implications of relaxing it. In problems A and B this is achieved by assuming $S_A + S_B \geq N$ and in problems C, D, and E we must assume $\sum_i S_i \geq N$. Thus, some parking spaces or bus seats can remain unused or some dwellings can remain unoccupied. Since the residential location problem (C) is typical of the remaining problems, we will examine the implication of $\sum_i S_i \geq N$.

Suppose that we introduce a new set of nonnegative variables (v_1, v_2, . . . , v_I) that measure the number of vacant dwelling units in each zone. Then, we can write

$$\sum_i S_i - \sum_i v_i = N \quad (44)$$

The problem can now be restated as

$$Nf_i = S_i - v_i \qquad i = 1 \ldots I \quad (45)$$

and more precisely as

$$NS_i\exp(\alpha R_i + \beta T_i + K_i) = (S_i - v_i) \sum_j S_j\exp(\alpha R_j + \beta T_j + K_j)$$
$$i = 1 \ldots I. \quad (46)$$

Equations 44 and 46 are I + 1 equations in the 2I unknowns, which are the rents and the vacancies. The sytem is underdetermined: Given the vacancy levels for all but any one of the zones, Equations 44 and 46 become I + 1 equations, with the rents and the remaining vacancy as the unknowns. If we fix vacancies as $\bar{v}_1, \ldots, \bar{v}_I$ so that these satisfy Equation 44 we can state

$$Nf_i/Nf_j = S_i\exp(U_i)/S_j\exp(U_j)$$
$$= (S_i - \bar{v}_i)/(S_j - \bar{v}_i) \quad (47)$$

From which we note that

$$\exp(U_i - U_j) = S_j(S_i - \bar{v}_i)/S_i(S_j - \bar{v}_j) \quad (48)$$

and

$$R_i - R_j = (1/\alpha)\ell n[S_j(S_i - \bar{v}_i)/S_i(S_j - \bar{v}_j)] + (\beta/\alpha)(T_j - T_i)$$
$$+ (1/\alpha)(K_j - K_i) \quad (49)$$

which reduces to Equation 14 if $\bar{v}_i = \bar{v}_j = 0$.

Since a unique set of vacancies cannot be determined without specifying additional relationships, the effect of vacancies is to introduce a new source of nonuniqueness in the determination of market prices and to increase the uncertainty in the prediction of these prices. It has been shown in Anas (8) that one way that market prices can be determined is by specifying certain additional conditions of competitive-pricing behavior, such as profit maximization, and deriving an equilibrium set of market-clearing prices.

INTERACTION DUE TO SEVERAL CONSUMER TYPES

In each of the problems the entire population of consumers (travelers or households) were assumed to have the same utility function and choice behavior. This is a strong assumption and may not always be appropriate in practice. It is, therefore, fruitful to examine several consumer types, each with a different utility function and choice behavior. We do this for problem C. Suppose that the population of households is segmented into h = 1...H segments according to certain socioeconomic criteria and the work places of the household heads. Then, let the behavior of each segment be logistic according to

$$f_i^h = S_i\exp(U_i^h)\Big/\sum_j S_j\exp(U_j^h) \qquad \sum_i f_i^h = 1, h = 1 \ldots H \quad (50)$$

with the utility function given as $U_i^h = \alpha_h R_i + \beta_h T_i^h + K_i^h$

where

R_i = the rent of location (zone) i,
T_i^h = the cost of commuting to zone i from the workplace of a type h household, and
K_i^h = the part of the utility function due to other characteristics of zone i.

Let N_h represent the number of households of type h and impose $\sum_h N_h = \sum_j S_j$. Now we must solve

$$\sum_h N_h f_i^h(R_1, R_2, \ldots, R_I) = S_i \qquad i = 1 \ldots I \quad (51)$$

by finding $R_1, R_2, \ldots, R_I$. This is a system of I simultaneous, nonlinear equalities in I unknowns but cannot be solved in closed form. To see this, we may follow a procedure similar to that of problem C. Doing so for the case h = 1, 2

$$S_i/S_j = (N_1 f_i^1 + N_2 f_i^2)/(N_1 f_j^1 + N_2 f_j^2)$$
$$= \{[N_1 G_2 S_i\exp(U_i^1) + N_2 G_1 S_i\exp(U_i^2)]/G_1 G_2\}$$
$$\div \{[N_1 G_2 S_j\exp(U_j^1) + N_2 G_1 S_j\exp(U_j^2)]/G_1 G_2\} \quad (52)$$

where

$$G_h \equiv \sum_i S_i\exp(U_i^h) \qquad h = 1, 2 \quad (53)$$

From Equation 52 we get,

$$N_1 G_2 [\exp(U_j^1) - \exp(U_i^1)] = N_2 G_1 [\exp(U_i^2) - \exp(U_j^2)] \quad (54)$$

Equation 54 shows that we cannot establish a simple relation for differential rent $(R_i - R_j)$. It is also seen that the IIA property no longer holds. The competition of the two household types for the housing supply in all zones establishes an interactive effect and the relative rents of i and j depend on characteristics of all the zones. A unique solution need not exist. It is true, in general, that many rent vectors will satisfy the simultaneous equations (Equation 51). Solutions can be obtained via special numerical techniques. One such application will be found in Anas (3), where problem E is solved for a 60-zone, five-household-segment spatial system for the case of a transit investment and excess capacity in housing.

CONCLUSIONS

The problems solved here and the more complex problems hinted at in the preceding part of the paper are a sample of a large number of supply and demand equilibration issues that form the basis of policy evaluation and planning analysis in transportation and related areas in urban planning. To date, most of the work dealing with logit models has confined itself to parameter estimation and crude forecasting. These forecasting exercises suffer from a serious weakness to the extent that the relevant equilibration issues are ignored, and thus the forecasts obtained are ultimately inconsistent. This paper has shown that these inconsistencies are readily rectifiable. Because of the complexity of problems that can be approached in this way, our objective has been to select simple, yet typical, problems of policy interest and to demonstrate the necessary manipulations and results for these problems. More complex problems can be solved by developing appropriate numerical simulation methods (3) or by specifying the nature of competitive pricing (8). My other work has shown that even for these problems, which involve several consumer groups and excess supply, the market-clearing distribution of prices is well behaved, even though it may not be possible to express it analytically.

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Validation and Application of an Equilibrium-Based Two-Mode Urban Transportation Planning Method (EMME)

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The purpose of this paper is to report on the validation and application of the two-mode urban transportation planning technique called EMME. This method may be characterized as an integrated two-mode traffic equilibrium method. Roughly speaking, this method combines a zonal aggregate-demand model with an equilibrium-type road assignment and a transit-assignment method. We describe the validation and application of the model by using data from the city of Winnipeg, Manitoba, Canada.

The purpose of this paper is to report on the validation and application of the two-mode urban transportation planning technique équilibre multimodal-multimodal equilibrium (EMME). This method may be characterized as an integrated two-mode traffic equilibrium method. It was suggested by Florian (1). Roughly speaking, this method combines a zonal aggregate-demand model (which may be a direct-demand model or an origin-destination table coupled with a suitable modal-split function) with an equilibrium-type road assignment and a transit-assignment method. The method has been described previously (2) and some of its theoretical properties have been studied by Fisk and Nguyen (3). The model was validated by using data from the city of Winnipeg, Manitoba, Canada. The equilibrium-type route-choice model for travel by private automobiles in congested urban areas was validated by Florian and Nguyen (4) in the Winnipeg road network. The transit-assignment model is essentially a shortest-route choice coupled with the diversion mechanism among sections served by common lines, which was devised by Chriqui and Robillard (5).

For the purpose of transportation planning, the city of Winnipeg is subdivided into 147 zones. The road network has 1040 nodes and 2836 one-way lines; observed link flows and link times were available for most of the links. The transit network has 56 lines, 1755 line segments, 500 egress-access links, and 800 nodes; 575 of the road network nodes are used in the coding of the transit network as well.

In the summer of 1976 the city of Winnipeg performed a speed-delay study, which consisted of measuring link volumes and link automobile travel times for 80-90 percent of the street system. In addition, bus travel times were measured for 446 transit line sections. These data served to recalibrate the volume-delay curves that were used in the road assignment and to calibrate the bus-automobile travel-time relationship required by EMME.

Since the city of Winnipeg had not previously used a transit-assignment model, the transit network was coded according to the EMME specifications, described by Achim and Chapleau (6), that permit the interface between the road and transit networks.

During the summer of 1976, the city of Winnipeg also performed an origin-destination survey of trips taken from home to work. A 17 percent sample of households was sampled and a separate survey of 23 percent of student trips was performed at about the same time. Since all of the analysis is done for the 7:30-8:30 a.m. peak hour, one of the first tasks considered was to define the departure codes, that is, the starting time of trips that will be using the road and transit networks during the peak hour. The departure codes were determined by the city of Winnipeg staff and were specified by origin,