Probabilistic Approach to Evaluating Critical Tensile Strength of Bituminous Surface Courses

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The strength of the upper layer (surface course) is one of the important indices of pavement serviceability. The serviceability of the pavement will be compromised when the stresses due to axle loads exceed the strength of the upper layer. The probability of failure of a structure is a function of the load effect and the resistance of the structure. In this paper, the load effect on the pavement structure is defined as the radial stress induced by traffic loading, and the resistance is defined as the layer strength. Therefore failure of the surface course is defined as occurring when the radial horizontal stress exceeds the horizontal tensile strength of the surface course. Basic variables for reliability analysis are introduced along 'th methods for determining the probability distributions of , basic variables. Monte Carlo simulation is used to determine the values for each variable for calculating radial stress using Boussinesq one-layer theory. First-order, second-moment probabilistic methods are used to determine the reliability index and critical tensile strength of the surface course. An example reliability study, using data for I-85 in South Carolina, is presented. Reliability and critical tensile strength of the surface course are obtained for the highway. The critical strength value is evaluated, on the basis of field conditions, to verify its usefulness. Pavement data for rutting and stripping are compared with tensile strength data for the upper pavement layer to evaluate the critical tensile strength value.

The upper layer (surface course) of flexible pavements is in the most severe stress condition because of direct contact of traffic loading. The surface course should be stronger than the other pavement layers. Analyses of field cores taken from I-85 in South Carolina, however, indicated that the tensile strength of the surface course was the lowest of all layers. This study was therefore intended to introduce an approach, using probabilistic concepts, to determine an acceptable value of tensile strength for bituminous surface courses.

Probabilistic design concepts and a general procedure for reliability analysis are first introduced. A first-order, second-moment probabilistic method is used for the reliability analysis 2). An example reliability analysis that uses field data ‹ained from I-85 is presented. The results of the analysis are evaluated by comparison with current field conditions.

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PROBABILISTIC DESIGN CONCEPTS

The probability of failure of a structure is a function of load effect (S) applied to the structure and resistance (R) of the structure, where S and R are random variables. If $R - S$ is defined as safety margin, then $P(R - S > 0)$ is the probability that the structure will remain safe (*1*, *2*). If the means and standard deviations of R and S are known, then a function, $Y = R - S$, can be defined with mean $Ym = Rm - Sm$ and standard deviation $\sigma_y = (\sigma_r^2 + \sigma_s^2)^{1/2}$ where Rm, Sm, σ_r, and σ_s are the means and standard deviations of R and S, respectively. In Figure 1, probability of failure is then defined as

$$P_f = Pr(R - S < 0) = Pr(Y < 0) \quad (1)$$

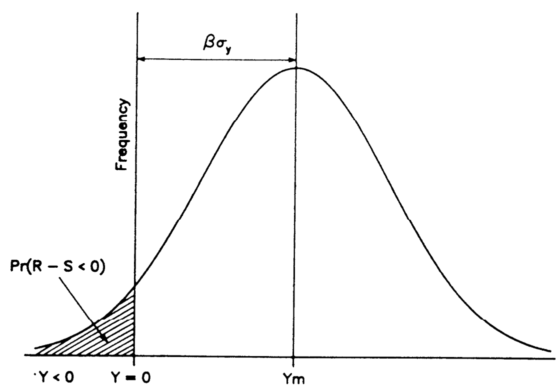


FIGURE 1 Definition of probability of failure.

Subtracting Ym from both sides of the inequality in Equation 1 and also dividing both sides by σ_y results in

$$Pr(Y < 0) = Pr[(Y - Ym)/\sigma_y < -Y_m/\sigma_y] = Pr(u < -Ym/\sigma_y) = F_u(Ym/\sigma_y) \quad (2)$$

where $u = (Y - Ym)/\sigma_y$, and F_u is the cumulative distribution function for the standardized variable (u) with mean $u_m = 0$ and $\sigma_u = 1$. Therefore, probability of failure is

$$P_f = F_u[(Rm - Sm)/(\sigma_r^2 + \sigma_s^2)^{1/2}] \quad (3)$$

In Equation 3, $[(Rm - Sm)/(\sigma_r^2 + \sigma_s^2)^{1/2}]$ is referred to as β, the safety index or reliability index:

$$\beta = (Rm - Sm)/(\sigma_r^2 + \sigma_s^2)^{1/2} \qquad (4)$$

Equation 4, introduced in first-order, second-moment probabilistic design concepts (*1*, *2*), can be used for calculating the numerical value of reliability for normal variates. If R and S are log-normal variates, using the mean ratio of the natural logarithm $Ym = [\ln(R/S)]_m$ and the standard deviation $\sigma_y = \sigma_{\ln(R/S)}$, β is defined as $\ln(R/S)]_m/\sigma_{ln(R/S)}$. Using mean and small variance approximations,

$$\beta = \ln(Rm/Sm)/(V_r^2 + V_s^2 + V_r^2V_s^2)^{1/2} \qquad (5)$$

where V_r and V_s are coefficients of variation (COV) of the load effect and the resistance, respectively, and

$$(V_r^2 + V_s^2 + V_r^2V_s^2)^{1/2}$$

represents the uncertainty associated with load and resistance. If V_r and V_s are smaller than 0.3, the $V_r^2V_s^2$ term is generally ignored because less than 5 percent error is introduced by doing so (*2*, *3*). Then Equation 5 becomes

$$\beta = \ln(Rm/Sm)/(V_r^2 = + V_s^2)^{1/2} \qquad (6)$$

If β is increased with a constant σ_y in Figure 1, then the P_f (shaded area) is reduced. Thus β is a measure of the reliability of a structural member.

Rewriting Equation 5 leads to the following equations:

$$Rm = Sm \text{ Exp } [\beta(V_r^2 + V_s^2 + V_r^2V_s^2)^{1/2}] = Sm\ \theta \qquad (7)$$

where

$$\theta = \text{Exp } [\beta(V_r^2 + V_s^2 + V_r^2V_s^2)^{1/2}] \qquad (8)$$

β is defined as the central safety factor and is used as a safety parameter for member strength (Figure 2).

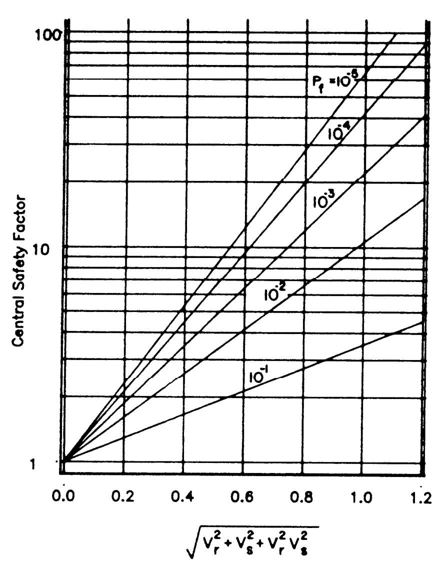


FIGURE 2 Central safety factor.

To apply these concepts of probability of failure to the flexible pavement system, the following assumptions were made in this study: The failure of any layer in a flexible pavement system will cause a functional failure of the pavement structure. Among other factors, failure of the pavement layer is a function of layer strength (resistance) and the stress (load effect) applied to the layer by vehicle loading. Therefore failure of the surface course was defined in this study as occurring when the radial horizontal stress due to traffic loading exceeded the horizontal tensile strength of the surface course.

Reliability Analyses

Data for basic variables for reliability analysis must be collected either from the field or from the literature. Probability distributions for those variables can be determined by a goodness of fit test at a certain level of significance (*4*). On the basis of the established probability distributions, values for each variable can be simulated by computer. The probability distribution for tensile strengths can be used for simulation of layer strength, and the probability distributions for layer thickness and axle load can be used for simulation of applied radial stresses on the layer.

Reliability in this study represents the probability that the surface course will not fail under the current traffic load. The reliability of the surface course can be obtained by comparing that layer's tensile strength with the radial horizontal stress. A critical value of tensile strength for the surface course for a certain level of reliability can then be obtained for the given traffic condition.

Basic Variables

The basic variables for which statistical data must be determined include thicknesses of the pavement layers, tensile strength of the surface course, axle loads, and radial stresses. Data on strength and thickness of the pavement layers and on axle loads of vehicles can be collected from field cores and from the literature to establish appropriate probability distributions.

Layer Thickness

The thickness of the pavement layer is a random variable, differing from one location to another. Data on layer thicknesses can be obtained from field cores. The major portion of the top layer of the flexible pavement in a highway is surface course; the second layer is binder course. Because the intent is to analyze the reliability of the surface course, only the thickness of the first layer is needed to determine probability distributions for layer thickness.

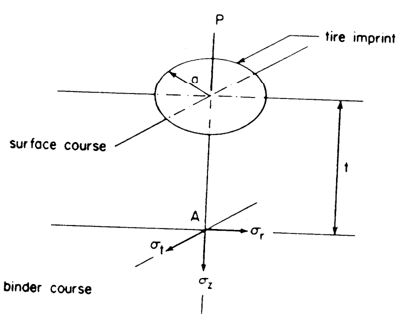
Axle Load

Traffic data on axle loads can be obtained from the state highway department or from truck weight data published by FHWA, or both (*5*, *6*). The probability distribution of weights on a wheel for light vehicles (pickup trucks, passenger cars, and vans) and heavier vehicles must be determined. The probability distribution for weights on a wheel for combined light and heavy vehicles can be determined by combining the two distributions with the appropriate ratio of heavy vehicles to light vehicles.

The weight on a wheel is determined by dividing an axle load (obtained from references) by the number of wheels on the axle. Sixty percent of gross weight is, on average, allocated to the main axle for light vehicles.

Radial Horizontal Stress

Radial stress is a function of axle load and layer thickness, both of which are random variables. Therefore radial stress can be considered a random variable. The stress mechanism under an axle load at the surface course of the pavement is shown in the Figure 3 (*7*). In Figure 3, the actual value of the radius (a) of tire contact area depends on the magnitude of axle load, and 3 to 6 in. can be used.



t = Thicknesses of surface course
P = Load on a wheel
a = Radius of tire imprint
σ_r = Radial stress in point A
σ_t = Tangential stress in point A
σ_z = Vertical stress in point A

FIGURE 3 Stresses at bottom of surface course under tire loading.

Radial horizontal stress (RHS) in the surface course can be calculated on the basis of Boussinesq one-layer theory. The load at the surface of the flexible pavement is assumed to be distributed over a circular area of tire contact. Because the depth at which the radial stress is measured is less than one-half of the clear distance between tire edges of dual wheels, the equivalent single wheel load (ESWL) concept need not be applied. That is, the maximum thickness of surface course in most highways is less than 5 in., which is approximately one-half the clear distance between tire edges of dual tires for most heavy traffic.

In the original Boussinesq equations, the pavement is considered homogeneous, isotropic, and elastic (*7*). The following Boussinesq equation can be used to obtain radial horizontal stress:

$$Sr = p[2 \mu A + C + (1 - 2\mu)F] \quad (9)$$

where

Sr = RHS,
p = pressure at tire-pavement contact,
μ = Poisson's ratio, and
A, C, and F = one-layer elastic function values.

The value for p can be obtained by dividing the weight on a wheel by contact area. An appropriate value for Poisson's ratio can be obtained from experiment or from the literature. The values of A, C, and F are tabulated by Yoder and Witczak (*7*) as functions of depth and offset distance in radii (z/a and r/a).

Maximum radial stress due to a single wheel occurs at a point along the vertical line beneath the geometric center of the tire imprint (load point). Maximum radial stress due to dual wheels occurs either at a point beneath the load point or at a point beneath the point halfway between the two tires. At a point beneath the midpoint of the two tires of dual wheels, radial stress is duplicated by the loads of the two wheels. Radial stress at this point is sometimes greater than the stress at a point vertically beneath the load point when the depth is greater than the clear distance between the two tires. Because the depth of the surface course is generally less than one-half the clear distance of two tires, maximum stress does not occur at this point. Many trucks are equipped with tandem axles. Because the distance between tandem gears is at least 40 in., however, no stress duplication occurs between tandem wheels at the surface course.

Tensile Strength

Tensile strength of the pavement layer can be used for layer strength (resistance) in reliability evaluation. Tensile strength can be measured for cored specimens by the indirect tensile strength test. The tensile strength of the layer can be considered a random variable that varies from one location to another.

The indirect tensile strength (ITS) test is one type of tensile strength test used for stabilized materials. This test involves loading a cylindrical specimen with a compressive load along the diameter of the specimen. This results in a relatively uniform tensile stress acting perpendicular to and along the diametral plane of the applied load, which results in a splitting failure generally occurring along the diametral plane. For most engineering materials, initial failure occurs by tensile splitting in accordance with the following equation (*7*).

$$\text{Tensile strength} = 2P/(\pi d t) \quad (10)$$

where

P = load applied to the specimen,
t = thickness of the specimen, and
d = diameter of the specimen.

For the study discussed in this paper, the indirect tension test was conducted on the cored specimens after the cores were sliced by layer. The testing temperature was 77°F, and load was applied vertically using a Marshall testing machine at a rate of 2 in./min through 0.5-in.-wide curved metal strips on the top and bottom of the specimen.

Determination of Reliability Index and Critical Strength

Probability distributions for resistance (tensile strength) and load effect (radial stress) can be obtained as described previously. The reliability index (β) can be calculated using Equation 4, if both tensile stress and radial stress follow normal distributions, or Equation 5 or 6, if they follow log-normal distributions. Probability of failure (P_f) or reliability ($1 - P_f$) can be obtained from the value of β. If resistance and load effect do not both follow a normal or log-normal distribution, Monte Carlo simulation can be used to obtain an estimated probability of failure.

Given coefficient of variation (COV) values for load and resistance, and target reliability, the central safety factor (β) can be obtained from Equation 8 or Figure 2 and used as a parameter for structural design. The central safety factor represents numerically how many times the resistance should be stronger than the load effect. If a central safety factor is specified on the basis of a target reliability and COV for load and resistance, a minimum or critical value of resistance can be determined.

EXAMPLE STUDY OF RELIABILITY ANALYSIS METHOD

A reliability study was conducted using data from I-85 in South Carolina and is presented in this paper for illustration. The probability distributions for each variable used for the study were determined using a computer program for goodness of fit testing. The program tested the data with *K-S* and χ^2 tests at the 5 percent level of significance (*4, 8*). The probability distribution for weight on a wheel based on traffic data (*5, 6*) is shown in Figure 4. The probability distribution for the tensile strength values is shown in Figure 5. Radial stress was obtained using Equation 10 with a Poisson's ratio of 0.35, which is a widely accepted value for asphaltic concrete.

The probability distributions (*9*) selected for each variable are given in Table 1. Because both radial stress and tensile strength data were found to follow log-normal distributions, $\ln(R_t)$ and $\ln(S_r)$ follow normal distributions, where R_t and S_r are tensile strength and radial stress, respectively. Because

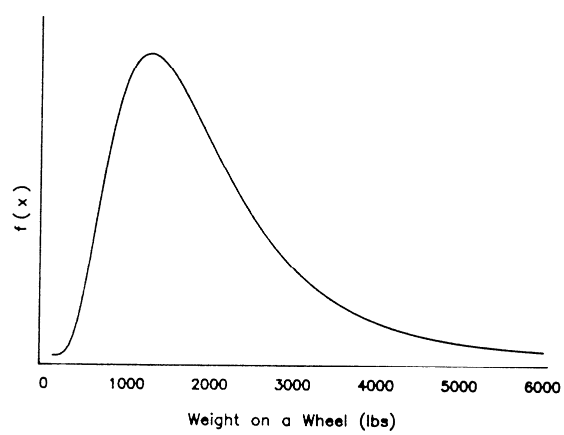


FIGURE 4 Probability distribution for weight on a wheel (total vehicles).

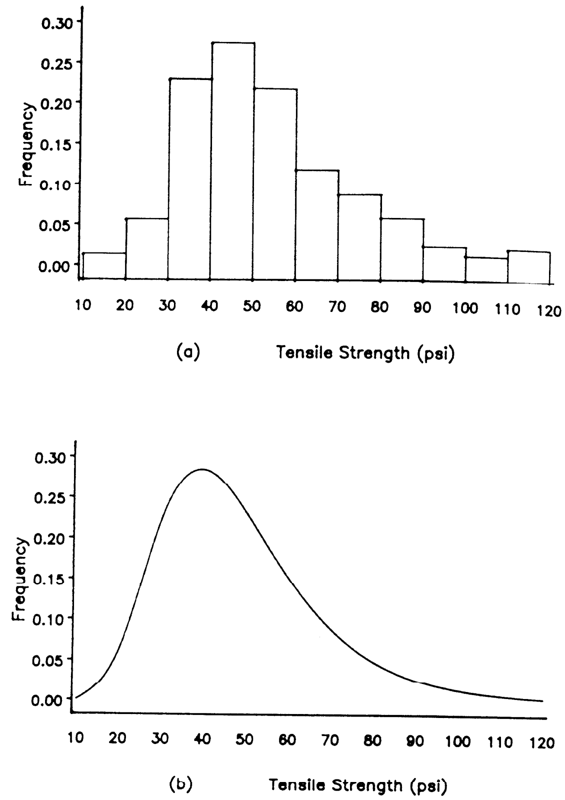


FIGURE 5 (a) Histogram and (b) PDF for tensile strength.

these are random variables, probability of failure can be defined as shown in Figure 6 (*10, 11*). The reliability index for a given strength value can therefore be obtained from Equation 5 (V_r and V_s are greater than 0.3 as given in Table 2).

The calculated values for β, θ, and reliability, based on the data, are given in Table 2. Therefore a tensile strength of 53.4 psi has a probability of failure (P_f) of 0.015. The safety level, however, for functional failure for most engineering problems requires a probability of failure of less than 0.01 (*10*). The value of tensile strength to satisfy this requirement is 15.25 θ = 61 psi, where $\theta = 3.99$ for $P_f = 0.01$ and $V_r^2 + V_s^2 + V_r^2 V_s^2) = 0.594$. According to the central safety factor in this case, average tensile strength needs to be approximately four times greater than the average radial stress induced by traffic loading. Several example values of tensile strength and their associated P_f-values are given in Table 3. The relationship between reliability and tensile strength is shown in Figure 7.

EVALUATION OF TENSILE STRENGTH BASED ON FIELD CONDITIONS

It was found from coring that pavement strength was related to pavement condition in the field. Surface rutting and stripping were the most significant distress mechanisms that were correlated with low-strength pavements. On the other hand, most of the cores from sites where cracks had developed on the surface

TABLE 1 PROBABILITY DISTRIBUTIONS AND THEIR PARAMETERS

Variable Name	Probability Distribution	Parameters
Thickness	Log-Normal	μ = 1.29 inch σ = 0.24 inch
Weight on a Wheel		
Light Vehicles	Weibull*	a = 0 b = 1225 lbs c = 3.69
Heavy Vehicles	Weibull*	a = 0 b = 2784 lbs c = 2.69
Total Vehicles	Log-normal	μ = 1598 lbs σ = 1011 lbs
Radial Stress	Log-normal	μ = 15.25 psi σ = 6.72 psi
Tensile Strength	Log-normal	μ = 53.4 psi σ = 19.49 psi

* Parameters for Wiebull distribution:
a= location factor, b= scale factor, c= shape factor (8,9)

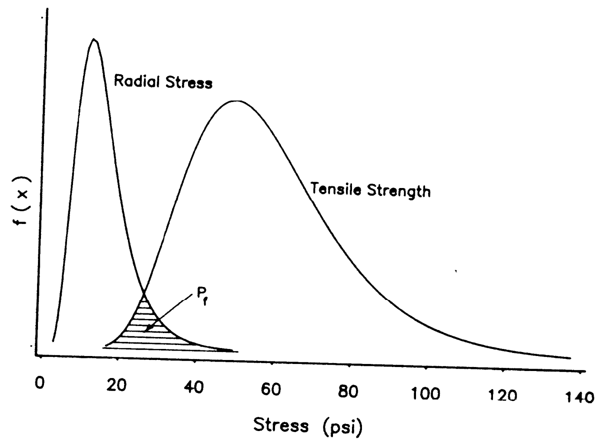


FIGURE 6 Probability of failure for surface course.

showed vertical or horizontal cracks, or both, throughout the cores. Most of them fell apart while being removed from the pavement. Even when the cores were removed without breaking, they were usually so weak that tensile strength values could not be measured. Therefore tensile strength values used in this study did not include values from cracked cores.

Because rutting is caused by consolidation or lateral movement of the materials due to traffic loading, rutting results in permanent deformation of one or more of the pavement layers or in the subgrade. This deformation causes a loss of strength in the mixture, leading to major structural failure of the pavement (*12*). Average tensile strength for the surface layer at sites that were free from rutting was approximately 81 psi. Average tensile strength of the surface layer decreased as surface rutting increased. Following the ITS test on each specimen, visual stripping rate (VSR) was measured on the broken faces of the specimen by the method developed by the Georgia Department of Transportation. Moisture-damaged (stripped) mixtures generally produced lower strengths in ITS tests than did undamaged mixes.

The relationship of rutting and visual stripping ratio with tensile strength values was statistically analyzed using the general linear model (GLM) procedure in the Statistical Analysis System (SAS) (*13, 14*). *F*-tests at the $\alpha = 0.05$ level of significance were conducted in the analyses of variances of rut depth and VSR. Mean values for tensile strength in various conditions were compared using the least square difference (LSD) method. The average tensile strength value for mixtures that were free from both stripping and rutting was greater than 90 psi. However, the average tensile strength for mixtures that

TABLE 2 CALCULATED RELIABILITY

Strength (psi)		Stress (psi)		β	θ	Reliability
R_t	V_r	S_r	V_s			
53.40	0.365	15.25	0.440	2.107	3.456	0.9743

TABLE 3 TENSILE STRENGTH VALUES AND PROBABILITIES OF FAILURE

Tensile Strength (psi)	β	P_f
40	1.623	0.0530
50	1.999	0.0233
60	2.306	0.0110
70	2.566	0.0051
80	2.790	0.0026
90	2.989	0.0014
100	3.166	0.0008
110	3.326	0.0004
120	3.473	0.0003

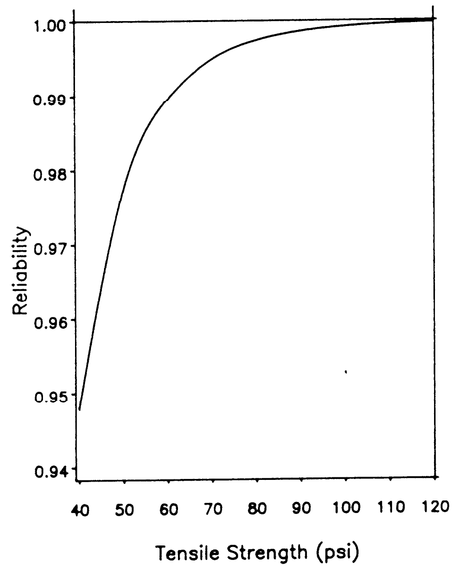


FIGURE 7 Reliability of tensile strength.

were stripped, for example, 10 to 40 percent in either the fine or the coarse aggregates, and also rutted at least 0.5 in. on the pavement surface, was approximately 50 psi.

The results of the analyses, given in Tables 4–6 and shown in Figure 8, reveal that tensile strength values generally decreased as rutting or stripping increased. The tensile strength values for specimens from the distressed pavement area were significantly below (at $\alpha = 0.05$) the tensile strength values from distress-free pavement sections. The combined effect (interaction) of both types of distress on tensile strength, however, was not significant at the same level of α.

Most of the tensile strength values for mixtures from nearly distress-free conditions were above 65 psi, which is 4 psi higher than the critical strength obtained in this study for the 0.01 probability of failure. Because level of probability for serviceability failure should be less than 0.01, the value of 65 psi for which P_f is 0.0073 is acceptable for critical tensile strength. Some of the specimens from areas of minor distress had tensile strength values of approximately 60 psi (for example, rutting = A and VSR = 1.0, rutting = A and VSR = 1.5 in Figure 8). Otherwise, almost all average tensile strength values from distressed conditions were below 60 psi, and many were below 50 psi. Therefore, for the surface course to perform satisfactorily under current traffic loading, a tensile strength of approximately 65 psi for field mixtures appeared to be needed for this section of Interstate highway.

SUMMARY AND CONCLUSIONS

A reliability study, based on first-order, second-moment probabilistic concepts, was conducted to develop a measure of an acceptable value of tensile strength for bituminous surface courses. Probabilistic design concepts and a general procedure for reliability analysis were first introduced, and basic variables to be used in the reliability study were defined. On the basis of the reliability concepts, and using test data from cores drilled from a portion of I-85 in South Carolina, an example reliability study was conducted for bituminous surface courses. Probability distributions for pavement layer strength, layer thickness, axle load, and radial stress were determined from field data and from the literature. The Monte Carlo method was used to determine values for the variables by computer simulation procedures. The reliability of the strength of the surface course

TABLE 4 ANALYSIS OF STRENGTH (tensile strength versus VSR)

		VISUAL STRIPPING RATIO			
		0	1	1.5	2
TENSILE STRENGTH (psi)	MEAN	56.22	48.98	48.27	59.77
	STD.	22.74	15.69	15.96	15.22

Legend for Visual Stripping Ratio

0: Almost no Stripping
1.0: Stripping < 10 %
1.5: 10 % < Stripping < 40 %
2.0: Stripping > 40 %

TABLE 5 ANALYSIS OF STRENGTH (tensile strength versus rutting)

		RUTTING					
		A	B	C	D	E	F
TENSILE STRENGTH (psi)	MEAN	80.98	51.69	51.11	45.46	41.12	45.31
	STD	34.62	14.80	18.73	13.54	17.93	11.98

Legend for Rutting

A: Rutting = 0 B: 0 < Rutting < 0.25 inch
C: 0.25 < Rutting < 0.5 D: 0.5< Rutting < 0.75 inch
E: 0.75 < Rutting < 1.0 F: Rutting > 1.0 inch

TABLE 6 ANALYSIS OF VARIANCE (based on GLM)

SOURCE	DF	SUM OF SQUARES	F VALUE	PR > F
RUTTING	5	12528.3121	8.07	0.0001 *
VSR	3	5022.8908	5.39	0.0012 *
VSR*RUTTING	9	4909.7989	1.76	0.0755
ERROR	334	103712.5993		
CORRECTED TOTAL	351	126173.60112781		

* Significant at α = 0.01

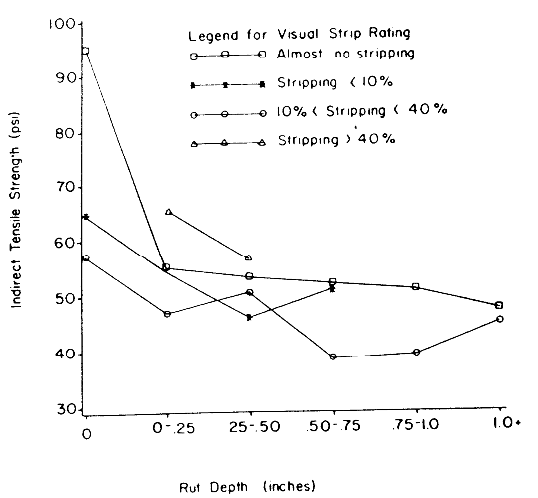


FIGURE 8 Analysis of tensile strength.

was calculated by comparing the tensile strength of the pavement with the load-induced radial stress. A critical value of tensile strength for the surface course was developed from the field data and the results of the simulation procedure.

On the basis of the results of tests on field cores, traffic data obtained from the literature, simulation analyses, and reliability analysis, a critical tensile strength value of 65 psi measured at 77°F for field-cored mixtures was identified as necessary to sustain a given traffic loading for a given section of Interstate highway. The value is based on the assumptions made with respect to traffic volume and make-up. The accuracy of the value is therefore dependent on the correctness of the assumptions made. A more appropriate value could be obtained if traffic studies were conducted to obtain the actual traffic loads carried by the section of pavement.

The procedure and methodology that are presented in this paper can be applied to any surface course and traffic condition. Cores can be drilled and tested to obtain estimated distributions for thickness and tensile strength. Similarly, traffic data can be obtained to determine the appropriate load distribution for the pavement section. Depending on the distributions identified, either simulation or analytical methods can be used to determine the critical tensile strength values for selected levels of reliability.

Although it is recognized that estimation of the potential performance of a pavement is a complex problem and that there are many possible failure mechanisms to consider, the procedures presented in this paper are a first attempt at developing a method for determining a minimum acceptable tensile strength for bituminous surface courses.

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