Hypernetworks and Supply-Demand Equilibrium Obtained With Disaggregate Demand Models

Yosef Sheffi, Department of Civil Engineering, Massachusetts Institute of Technology, Cambridge
Carlos F. Daganzo, Department of Civil Engineering, University of California, Berkeley

This paper presents a framework for discussing many transportation demand and supply-demand equilibrium problems. It regards the sequence of choices an individual faces when he or she is about to make a travel (or not-to-travel) decision as a case choice of a route on an abstract network (hypernetwork). Hypernetworks are intimately related to the multinomial probit (MNP) model of travel choice. For instance, the multivariate normal distribution underlying this model enables one to represent processes of travel choice as route choices on networks and to use the networks as visual aids in conceptualizing the specification of covariance matrices for MNP choice models. Hypernetworks enable us to carry out supply-demand equilibrium analyses with disaggregate demand models on a mathematically consistent basis (heuristic equilibration techniques based on feedback loops do not necessarily converge, as shown with a simple counterexample). This greatly enhances the potential of probabilistic discrete-choice disaggregate demand models, since it is now possible to avoid their mispredictions when applied to congested transportation systems.

This paper departs from the main line of thought in previous transportation planning research. It suggests that most transportation forecasting problems can be regarded as network problems and that by doing so it is possible to address several outstanding and seemingly unrelated problems in a unified way. Although the urban passenger transportation planning process is used throughout this paper as an example, it should be noted that the concepts introduced here are applicable to small-scale problems and sketch-planning issues as well. In fact, if one wanted to adapt our approach to the very large models that are sometimes used in urban transportation planning, further research would be needed. A discussion and literature review of existing problems with the above-mentioned transportation planning process follows.

The most noticeable and widely used approach to transportation equilibrium analysis is the Urban Mass Transportation Administration's (UMTA's) Urban Transportation Planning System (UTPS) (1, 2, 3). The UTPS is a battery of computer programs designed to perform the above analyses, which include trip generation, trip distribution, modal split, and traffic assignment. Each phase in the process has a methodology of its own that has been extensively discussed in the literature (4, 5, 6, 7, 8, 9, 10, 11).

There are other computer packages that attempt to perform equilibrium analysis [such as Dodotrans (12), which, unlike the early versions of the UTPS, is an explicit equilibration package] and are based on similar ideas. A review of many of these packages can be found in a report by Peat, Marwick, Mitchell (13).

Although these transportation planning tools are commonly accepted among transportation planners, they have received severe criticism in the literature in the last several years.

Some of the criticism is general and points out the deficiencies of all large-scale models (14, 15, 16, 17, 18). Some of it is directed at specific models used in the planning process (19, 20, 21, 22, 23, 24). Yet others have based their criticism on a more fundamental issue that applies to small-scale problems as well—the behavioral assumptions that underlie this process (25, 26). The latter line of criticism led to the so-called disaggregate behavioral demand models (27, 28), which, by using individuals or households as the study units, attempted to capture choice-makers' behavior. Some of these models can be interpreted according to the economic principle of utility maximization, giving them a flavor of causality and behavioral realism.

Since the use of disaggregate data is more efficient—disaggregate models need less data than aggregate models to get a specific confidence level on the estimated coefficients (29)—and because the estimated coefficients are potentially independent of the distribution of the explanatory variables, those models have gained popularity among planners and are used increasingly in practice. Examples of applications and further developments of disaggregate demand models can be found elsewhere (30, 31, 32, 33, 34, 35).

These models, however, raised a new set of unresolved issues. The first of those difficulties is the aggregation problem (36, 37, 38, 39), i.e., how to use disaggregate demand models to predict the behavior of the population of choice makers. The second difficulty is in incorporating these models into a supply-demand equilibrium framework. The third difficulty is that most of the disaggregate modeling and estimation effort has been with models such as logit (40, 41) that involve assumptions that are sometimes unrealistic and often fail to capture reasonable user behavior. Obvious examples are the blue bus—red bus problem (42), the case of nested alternatives (43), and route-choice problems (44).

In addition, some of the issues that arise from the heuristic nature of the transportation planning process still remain, as happens for instance with its failure to naturally represent the intimate interdependencies among the various phases that comprise it, including the performance characteristics of the system (the "supply" side).

This is a problem with the traditional (sequential) planning process because, if, for example, modal split is performed before trip distribution, transit trips might be allocated to zone pairs not used by transit. If modal split is performed after trip distribution, too many trips might be allocated from, say, an origin with low car ownership to a destination connected to it by highway only. Although sequential models can avoid this problem by the use of accessibility measures (45, 46, 47, 48), the numerical values of these accessibility measures are not usually known a priori, which creates some problems. For instance, in congested systems, travel time (an explanatory variable appearing in trip generation, trip distribution, and modal split models) can only be determined after the traffic assignment step.

In order to circumvent such problems it is suggested that the model system be iterated several times in order to achieve a state of equilibrium, which, for the case of probabilistic and discrete choice demand models, has not been formally defined. However, due to the high computation costs involved, this is seldom done in practice.

Last, there are some aggregation problems that remain even if aggregate models are used. For instance, although market segmentation might enhance predictions (49, 50) and is commonly used in practice (trip generation and auto ownership studies are typical), no firm guidelines have been given in the literature as to the necessary extent of the segmentation. Similarly, in the traffic assignment step, no definite criteria exist on how to represent the network, i.e., how to locate the zone centroids, or on how to decide on the number and select the characteristics of dummy centroid connectors.

In summary, the following issues can be identified as those

1. Concerning disaggregate choice models in general: (a) the aggregation problem (including market segmentation), (b) incorporation in equilibrium analysis, and (c) alternatives to logit;
2. Concerning the traditional process in general: (a) equilibrium formulation and equilibration procedure, (b) consistency throughout the steps, and (c) network representation.

The objective of this research is to provide a framework within which some of these issues can be resolved. Of course, some of the problems have already been solved; numerical approaches to probit (51, 52) and generalized logit (53)—both reasonable alternatives to logit—have already been developed. Some of them, such as the aggregation problem (39), are partially solved, and yet some of them, such as efficient equilibration approaches with probabilistic choice models, remain unsolved.

This paper uses some of these results and some new ideas to formulate a solution (at least partial) to the above-mentioned problems. Mathematical formulations and algorithmic steps are not within the scope of this paper, which concentrates on the concepts and the application. A comprehensive treatment of our approach is included in Sheffi (54).

Several equilibrium models have been recently developed. The first ones dealt (rigorously) with route choice and network equilibrium only, by casting the problem as a mathematical program (55, 56). Ruiter and Ben-Akiva (57) developed a complete equilibrium-forecasting system incorporating an integrated set of production-oriented disaggregate demand models, and a conceptually similar model system is described by Jacobson (58). Neither of the last two methods, however, is guaranteed to produce the desired results in terms of convergence to a defined equilibrium. A formal solution to the equilibration problem over a transportation corridor, using disaggregate demand models, was obtained by Talvitie and Hasan (59). Their approach consists of formulating the equilibration as a fixed-point problem and solving it with the algorithm proposed by Scarf (60).

The approach taken in this research is to view and formulate all choice processes as route-choice processes on abstract networks, which we call hypernetworks, and to use an efficient procedure to analyze stochastic hypernetwork equilibrium problems. Although a numerical example is provided at the end, the emphasis of this paper is on presenting a concept rather than a new technique.

The following section, which discusses the idea of hypernetworks and relates it to existing approaches to forecasting, is followed by a section that shows how hypernetworks are related to MNP models and how they can be used to alleviate some of the above problems. Next the concept of equilibrium on a hypernetwork is defined and the possible failure of heuristic equilibrium approaches to converge is illustrated. The results obtained with a mathematical equilibration procedure that has been recently developed by the authors are also presented.

HYPERNETWORKS

We assume that the various alternatives open to travelers in choice situations (mode, route, destination, etc.) can be viewed as paths in a hypothetical network (a hypernetwork) made up of links characterized by disutilities. We also assume that, as in route-choice problems (44), people select the shortest route, that is, the alternative with the lowest disutility. This is consistent with the principle of utility maximization of choice theory.

Assume for instance that we are concerned with a modal split-route choice problem for one single origin-destination pair, and to further simplify matters assume that there are one transit mode and two automobile routes. Figure 1 represents a possible configuration of the hypernetwork for such a problem.

In the figure there are three hyperpaths corresponding to the three alternatives. Links OA and OB represent the inherent disutility of the two modes, fare and comfort, and links AD and BD represent the travel time characteristics of the three alternatives. Choice of a car implies that the shortest route in the hypernetwork consists of the car link and a route through the street network.

In the most general case, link disutilities may be flow dependent (e.g., travel time under congested conditions), fixed (e.g., fare on a transit line), and/or multiattributed if it is so desired. It should be noted at this point, however, that the algorithm described in the sequel requires the modeling of links that exhibit flow-dependent utility as single attributed (i.e., as multiattributed with fixed weights on the attributes). Disutilities are also assumed to be additive so that the disutility of an alternative (hyperpath) equals the sum of the disutilities of the links that make it up. Both of these assumptions are discussed by Sheffi (54).

By modifying the structure of a hypernetwork, one can affect the probabilistic structure of the corresponding choice problem. This will be seen in the next section as we show how the probability of choice is affected by network topology. For instance, Figure 2 displays an alternative representation of the problem in Figure 1 that, as will be seen later, would have approximately the independence of the irrelevant alternative (IIA) property.

Figure 3 demonstrates a more complicated choice problem that can be represented by a hypernetwork. It displays a hypernetwork for a combined modal split, route, and destination choice problem, where a fraction of the population does not have access to the car mode. The links of this network are of two types. The ones belonging to the street network are real links and are associated with travel time and travel cost. All other links are dummy links representing different dimensions of the problem. For instance, the links leading from the destinations to D represent the unattractiveness of the specific destinations and the links labeled car and transit represent the unattractiveness of the respective modes. Note that O_2 does not have access to the street network in order to represent market segments that do not own an automobile. The number of hyperpaths in this network is larger than in the preceding example; as a matter of fact, in real problems this number can be so large as to preclude enumeration of all possible hyperpaths.

These examples were intended to illustrate that it is possible to construct a hypernetwork for many choice problems and that different market segments can be

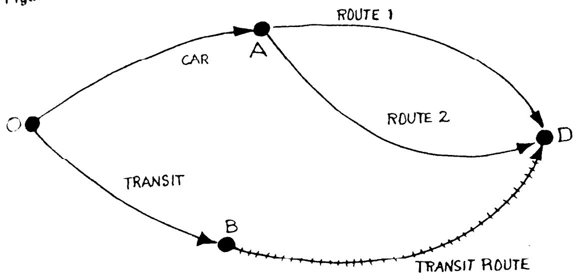


Figure 1. Simple hypernetwork example for mode and route choice.

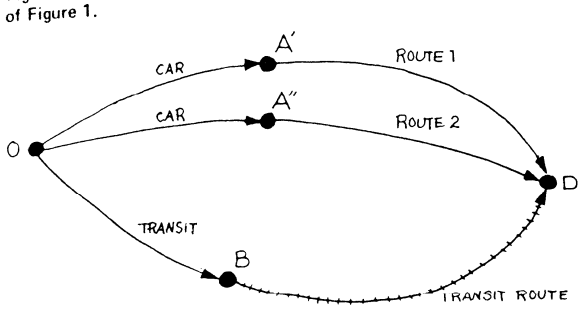


Figure 2. Independent utility representation of the hypernetwork of Figure 1.

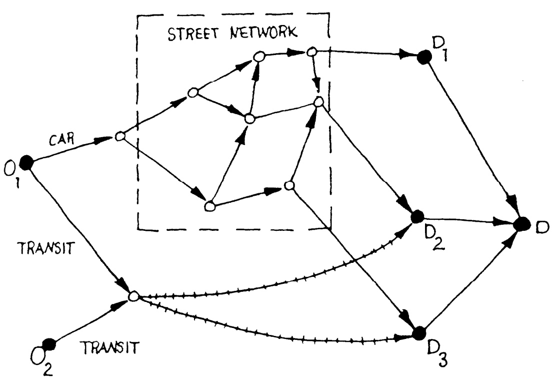


Figure 3. Hypernetwork example for mode, route, and destination choice with two market segments.

adequately handled by appropriate representation.

Before launching a more in-depth exploration of hypernetwork properties and applications, it is worthwhile to note that the idea of hypernetworks has been latent in the literature for some time. As early as 1972 at the Williamsburg Conference, Wilson (61) noted that

> It is tempting as computer capacity expands, to think of assigning on multimodal networks, in effect, possibly directly to routes on an abstract modal basis. . . . This is another example of a class of mathematical aggregation problems.

Manheim (62) formulated the transportation planning process as a network, using Dial's assignment method (63) to predict flow on the hypernetwork. However, since Dial's assignment algorithm is based on a logit formulation, it suffers from the IIA property, which, in route-choice problems, can be shown to produce unacceptable results (44, 64, 65, 66).

Dafermos suggested an integrated equilibrium flow model for transportation planning (67), based again on visualizing the whole transportation planning process as a solution to a network assignment problem. In her words,

> We adopt the natural behavioral assumption that each user chooses his origin, his destination, as well as his path as to minimize his "travel cost". Of course, "travel cost" should be interpreted in a very liberal fashion. In reality additional factors such as "attractiveness" of the origins (residential areas) and destinations (places of work) have to be taken into account but this can be incorporated into the model as "travel cost" by a straight-forward modification of the network. . . . Interestingly, we establish a mathematical equivalency which reduces integrated transportation problems for a network into assignment problems for a modified network.

Dafermos' model, although very similar to the hypernetwork concept, is not quite as general, because she was working exclusively with deterministic travel costs over the modified network. This explicitly excludes many demand models from the realm of application of her model, since, as it is assumed with deterministic equilibrium traffic assignment methods, users are identical (this excludes disaggregate demand models), fully informed (which excludes logit, probit, and other stochastic models) individuals making consistently perfect decisions.

The well-known elastic demand traffic assignment problem formulated by Beckmann, McGuire, and Winsten (68) can be solved with existing fixed demand traffic assignment problems on an expanded network (69). Such an expanded network can be viewed as a hypernetwork since it has dummy links going from each origin to each destination in order to represent the no-travel alternative.

It is also worth noting that traditional trip distribution models such as the entropy model can be cast as hypernetwork problems; Golob and Beckmann (70) showed the equivalence of entropy maximization and utility maximization over a hypernetwork similar to the one in Figure 3. Others (71, 72, 73, 74) contain mathematical programming formulations of the combined distribution-assignment problem that can also be regarded as hypernetwork problems.

The concept of a hypernetwork thus seems reasonable and flexible enough to be applied to many transportation forecasting problems. We shall now proceed to an analysis of the hyperpath choice process.

HYPERPATH CHOICE AND MULTINOMIAL PROBIT

In this section we consider the choice process to be hyperpath when the attributes characterizing the disutilities of each link are given. We thus concentrate on the choice process and leave the more complex network equilibration issues to the next section.

The behavioral basis of the approach presented in this section is the economic concept of random utility (75, 76, 77, 78, 79). Early applications of the concept of random utility were suggested by McFadden (80) and Kohn, Manshi, and Mundel (81).

It has long been recognized from empirical studies that highway route choice is not a deterministic proposition, as seen, for instance, in the early traffic diversion studies (82, 83, 84). This gave rise to traffic assignment methods that do not allocate all the traffic to the shortest route (63, 85, 86, 87). These methods, however, are deficient (44, 64, 65, 66, 88), and only recently has a general theory of route choice been developed (44).

Others (44) have presented an analytically consistent method that accounts successfully for the topology of the networks. It is based on a probit model of route choice and it admits a straightforward generalization for hypernetworks with multiattributed link disutilities. This is done below.

Assume a general hypernetwork composed of links representing various dimensions of travel choice. Every link, i, in the network is associated with some disutility, U_i' (we use primes for variables associated with links):

$$U_i' = V_i' + \xi_i' \tag{1}$$

where V_i' is the measured (known) disutility, and ξ_i' is a random error term distributed across the population. The vector of error terms, ξ', is assumed to be multivariate normally (MVN) distributed with zero mean and a known variance-covariance matrix Σ'. Thus, using vector notation, $U' \sim$ MVN (V', Σ').

As an aside, note that the structure of the variance-covariance matrix Σ' can only be decided on reasonableness grounds, as is the case with any specification decision. For instance, corr $(\xi_i'; \xi_j')$ can be set equal to zero if one can reasonably assume that links i and j are sufficiently unrelated as to be perceived independently by the decision maker. As argued by Daganzo and Sheffi (44), disutilities of links belonging to the street network can be considered independent. If this is not the case, there may be a representation of the problem that will admit independent link error terms. If the error terms on links OA' and OA" in Figure 2 are considered independent, the IIA property arises; however, the representation in Figure 1 overcomes this. Finding such a representation is equivalent to finding a reasonable specification of the problem.

The disutility minimization approach yields a multinomial probit (MNP) model of hyperpath choice, since the disutilities of hyperpaths are also MVN distributed. This can be seen by considering the hypernetwork link-route incidence matrix, $\Delta = \delta_{ik}$, where $\delta_{ik} = 1$, if link i belongs to hyperpath k, or 0, if otherwise. Letting U denote the vector of perceived hyperpath disutilities (the disutility vector of our choice model), we see that

$$U = U' \times \Delta \tag{2}$$

and $U \sim$ MVN (V, Σ) where $V = V' \times \Delta$ and $\Sigma = \Delta^T \times \Sigma' \times \Delta$ (T denotes the transportation operation).

Equation 2 implies that the covariance of two alternatives is heavily dictated by the amount of overlap of their corresponding hyperpaths. If Σ' is diagonal, the covariance of two hyperpaths is given by the sum of the variances of the common links. Consequently, the topology of the hypernetworks bears directly on the probabilistic structure of the corresponding choice model.

Heterogeneous populations are well handled by hypernetworks, for, if we assume that the vector of attributes entering the disutility functions is MVN distributed across the population and that Σ' is fixed, the choice process also follows an MNP law at the aggregated level (39). Non-normally distributed attributes can be handled by market segmentation (hypernetwork representation), as was done in Figure 3.

Of course, MNP models can be applied to choice problems without hypernetworks, but the graphical aid provided by visualizing the degree to which hyperpaths overlap helps to conceptualize reasonable parameters of the matrix Σ, especially for problems involving more than one stage of the traditional transportation planning process. Note, for instance, that the logit model arises from a random utility model with independent error terms and that it therefore corresponds approximately to hypernetworks with nonoverlapping routes. It should also be noted that, although many choice problems can be cast as hypernetwork problems, the use of MNP codes is basic to the analysis of general hypernetworks.

Hypernetworks thus present the same advantages and disadvantages of MNP models. Namely, MNP models and hypernetworks solve, or at least alleviate, the market segmentation problem, since an MNP model of individual choice is also MNP for the aggregate predictions (39). The step sequence issue (e.g., should mode choice be predicted before destination choice, after it, or simultaneously with it), posed by Ben-Akiva (89), who demonstrated the feasibility of a simultaneous approach, has already been addressed in this paper. Our approach is equivalent to a simultaneous MNP choice model whose covariance matrix can be studied visually.

Although hypernetworks enable us to visualize choice problems in a unified way and can help select appropriate probabilistic structures for choice models, their main advantage is that they will enable us to perform supply-demand equilibrium analysis on a mathematically consistent basis with disaggregate demand models. This subject is covered in the next section.

EQUILIBRIUM

Although most of the work related to transportation equilibrium has been done in the context of traffic assignment, equilibrium analyses should not be restricted to route-choice situations, especially since in problems dealing with congested transportation facilities the supply considerations can be a more important determinant of use than the demand function itself. For instance, what good does it do to have a sophisticated demand model that predicts a transit park-ride ridership larger than what the access parking lots can accommodate?

Although the equilibrium problem has been addressed in the literature in connection with the traditional planning process (and as was mentioned earlier there is a rich literature dealing with heuristic iterative schemes involving feedback loops and accessibility measures), there is no general definition of equilibrium in the transportation market when the demand side is modeled as a probabilistic proposition. Because of this, the following definition is proposed. The equilibrium criterion is the condition such that "at equilibrium no user perceives that he or she can decrease his or her disutility by unilaterally changing alternatives."

This is a generalization of the stochastic user equilibration principle (44) of traffic assignment, which, in turn, is a generalization of Wardrop's rule (90).

The equilibrium solution is obtained by the solution of two systems of equations representing the demand and supply relationships. For a hypernetwork with only one origin, demand is

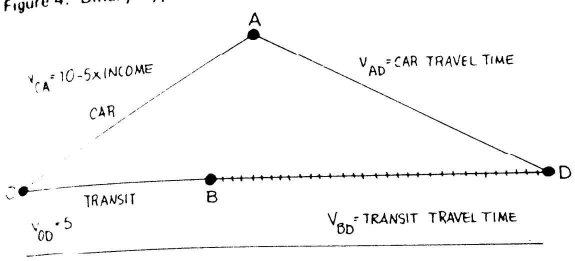
$$\Pr\{U_k \leq U_j; \forall j\} = X_k \Big/ \sum_{\forall j} X_j ; \forall k \tag{3}$$

where

X_k = number of users selecting alternative k,
ΣX_j = known fixed quantity (the population size), and
U_j = disutility of travel alternative j as perceived by an individual chosen at random from the population.

This equation is, of course, merely a statement of the weak law of large numbers. It states that the (predicted) market share of each alternative equals the choice probability for a randomly sampled choice maker, which is determined by the distribution function of the measured

Figure 4. Binary hypernetwork example.



$\xi_{OA} \sim N(0;50)$	$\xi_{AD} \sim N(0;0)$
$\xi_{OB} \sim N(0;75)$	$\xi_{BD} \sim N(0;0)$
ALL MUTUALLY INDEPENDENT	

disutility across a population.

If the latter distribution is approximately multivariate normal [Bouthelier and Daganzo (39) have identified some conditions for this], the distribution function is totally characterized by a vector of means V and a covariance matrix Σ, and in the general case one has the supply equations

$$V = V(X);\ \Sigma = \Sigma(X) \tag{4}$$

This equation states that the vector of mean disutilities and the corresponding covariance matrix are functions of the use on each one of the alternatives; i.e., the total link flows on the hypernetwork. In instances where Σ can be considered independent of X, supply modeling techniques can be used to determine Equation 4. Discussion of this point is, however, beyond the scope of this paper (54).

We now use a simple example with two alternatives and a known equilibrium solution to demonstrate how heuristic iterative techniques that alternatively solve Equations 3 and 4 fail to converge. The example is also used to demonstrate a new algorithm that is mathematically proved to converge.

Figure 4 displays a hypernetwork corresponding to an idealized modal split problem. The street network is represented by one link (AD); the inherent characteristics of the car mode are represented by a link leading into the street network (OA); and the transit alternative is similarly represented (links OB and BD). The measured link disutilities are written down by each link. The distribution of the link error terms is also specified in the figure.

The supply equations are defined for each link and expressed in travel time units: link OA = 10-5 income (independent of flow); link OD = 5 (independent of flow). Thus

$$\text{Car travel time} = 10/(1 - X_{AD}) \tag{5A}$$

where X_{AD} is the flow on link AD.

$$\text{Transit travel time} = 15 \qquad \text{(independent of flow)} \tag{5B}$$

Using Equation 2, the choice model corresponding to the hypernetwork turns out to be

$$U_{CAR} = 10 - 5\ \text{income} + \text{car travel time} + \xi_{CAR} \tag{6A}$$

$$U_{TRANSIT} = 5 + \text{transit travel time} + \xi_{TRANSIT} \tag{6B}$$

$$\text{with } \Sigma = \begin{Bmatrix} 50 & 0 \\ 0 & 75 \end{Bmatrix}$$

Equations 6A and 6B, however, are not ready for use because they do not represent the disutilities of a user sampled at random from the population. Thus, carrying out an aggregation procedure with, say, income being a normally distributed attribute with mean and variance equal to four yields, as the reader can check,

$$U_{CAR} = -10 + \text{car time} + \xi_{CAR} \tag{7A}$$

$$U_{TRANSIT} = 5 + \text{transit time} + \xi_{TRANSIT} \tag{7B}$$

$$\text{with } \Sigma = \begin{Bmatrix} 150 & 0 \\ 0 & 75 \end{Bmatrix}$$

If instead of a normal attribute, such as income, we had a zero-one variable, or market segmentation needed (39), one would have introduced additional origins representing the different market segments and would have connected them to A and B, as was done in Figure 3. In any case, Equations 7A and 7B yield for the probability of car choice:

$$P_{CAR} = \Pr\{U_{CAR} < U_{TRANSIT}\} = \Pr\{U_{CAR} - U_{TRANSIT} < 0\}$$

$$= \Phi[(15 + \text{transit time} - \text{car time})/(225)^{1/2}] \tag{8}$$

where Φ is the standard normal cumulative distribution curve. Equation 8 of our example corresponds to Equation 3, and Equations 5A and 5B correspond to Equation 4.

Assuming that we are studying one unit of population, ($\sum_{\forall k} X_k = 1$), $P_{CAR} = X_{AD}$, and Equation 8 reduces to

$$X_{AD} = \Phi\{2 - [\tfrac{2}{3}/(1 - X_{AD})]\} \tag{9}$$

Equation 9 can be solved graphically, as shown in Figure 5A, and yields $X_{AD} = 0.61$. Since in most instances it is impossible to reduce the original problem to a manageable set of equations (e.g., in multicommodity networks with just a few links), efficient numerical techniques must be sought, especially since heuristic iterative algorithms do not necessarily converge. This is shown below.

A typical heuristic procedure involving feedback loops consists of solving Equations 5A, 5B, and 8 alternatively, until a convergence criterion is met. In our case and to better illustrate the point, we select an initial value, $X_{AD} = 0.62$, very close to the equilibrium solution.

The table below displays the results obtained with the heuristic approach, which the reader can verify. The same iterative scheme could have been carried out graphically on Figure 5 to yield a familiar pattern known to economists as the cobweb model (Figure 6).

Iteration	X_{AD}	Iteration	X_{AD}
0	0.620	6	0.736
1	0.599	7	0.301
2	0.630	8	0.852
3	0.579	9	0.006
4	0.661	10	0.908
5	0.512	11	0

The approach developed to solve for the equilibrium in the transportation market has been formally described (54). It is computationally very efficient, can handle many origins on the hypernetwork, does not require that all the alternative routes be enumerated, and can take explicitly into account finite hypernetwork link capacities. This last feature proves to be invaluable for analysis of problems involving parking and other finite capacity transportation facilities.

Technically, the algorithm solves an associated disutility minimization program for the hypernetwork. The

Figure 5. Numerical solution of the hypernetwork example.

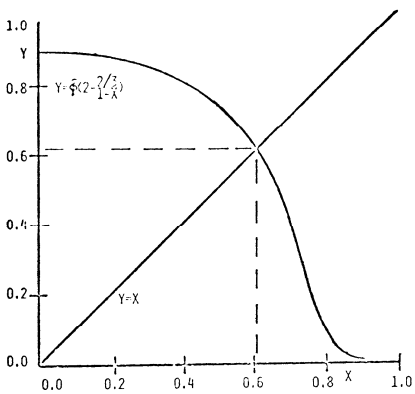
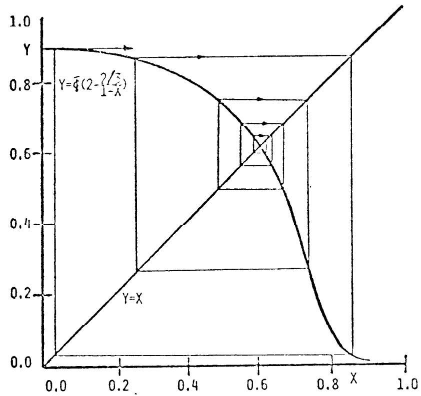


Figure 6. Divergence pattern of a naive iterative technique.



disutility additivity assumption mentioned in the preceding section is used to separate between the flow-dependent and flow-independent link disutilities using the optimality principle of dynamic programming. Then the assignment over the links associated with the flow-independent disutility is carried out by using analytical expressions for the flow allocation and the total disutility for all the populations of each origin zone. These quantities are computed at every iteration of the flow-dependent links equilibration to ensure a global minimum for the associated minimization program. A detailed description of the algorithm can be found in Sheffi (54) and a special application of it to the spatial aggregation problem of traffic assignment in Daganzo (91).

The major assumptions of the methodology include the above additivity of the flow-dependent component in all the disutility functions. For example, if car travel time over the street network is modeled as flow dependent, it has to enter the disutility functions in a generic linear-additive form. Furthermore, each hypernetwork link can be modeled as stochastic, as multiattributed, or as flow dependent (54). Another important limitation on our methodology is that the covariance matrix of the multivariate distribution underlying the MNP models involved has to be independent of the associated vector of means, otherwise Equation 3 does not follow an MNP model (39, 54).

For our particular example, with a relatively poor initial point $X_{AD} = 0.5$, the algorithm converges in six iterations. The table below displays the results.

Iteration	X_{AD}	Iteration	X_{AD}
0	0.5	4	0.637
1	0.748	5	0.571
2	0.664	6	0.610
3	0.555	7	0.610

DISCUSSION AND SUMMARY

This paper has presented a framework for carrying out supply-demand equilibration of a transportation market. It is argued that choice problems can be regarded as route-choice problems on abstract hypernetworks. This idea has been latent in the literature, and, as shown, it is intimately related to multinomial probit models. Hypernetworks may help us conceptualize useful parameters of MNP models, especially for the covariance matrix, with not too many additional parameters.

We addressed the important issue of equilibrium analysis by providing a formal definition of equilibrium. It is also shown, by means of a counterexample, that heuristic iterative algorithms do not necessarily converge to the equilibrium solution and that a newly developed, efficient mathematical algorithm does indeed converge.

The example and the paper do not include a specific discussion of the spatial aggregation problem, but, as shown elsewhere (39, 91), the intrazonal level-of-service attributes can be well approximated with MVN distributions, and therefore MNP models can be used to predict the loading of the network. This obviates the need for centroids and dummy links.

Although most of the emphasis of our discussion was on the urban transportation planning process, microscopic problems can also be represented as hypernetworks and handled with the above-mentioned equilibrium algorithm. Typical examples of applications would be finding equilibrium on a dial-a-ride market. Supply equations for these systems have recently been developed with queueing theory (92). Other examples are selection of parking lots in a downtown area as a function of their capacities and locations, airport selection as a function of an intercity origin-destination table, and the location and service characteristics of the individual airports.

In summary, hypernetwork theory seems to be a flexible and powerful methodology that enables us to directly address some problems that had not been previously tractable.

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Disaggregate Travel Demand Models for the San Francisco Bay Area

System Structure, Component Models, and Application Procedures

Earl R. Ruiter, Cambridge Systematics, Inc., Cambridge, Massachusetts
Moshe E. Ben-Akiva, Massachusetts Institute of Technology and Cambridge Systematics, Inc., Cambridge

Significant advances have recently been made in developing and applying disaggregate behavioral travel demand models to many aspects of urban travel decisions. What has not previously been developed is a full set of urban models integrated into a complete forecasting system for use by a metropolitan planning organization. The purpose of this paper is to describe the first such system, which was developed for the Metropolitan Transportation Commission, the designated metropolitan planning organization for the San Francisco area. First, the background of the current modeling project is briefly set out, followed by a description of the structure of the model system. The model development process—estimation, prediction testing, and validation—is described, and two computerized model application procedures—a regional network analysis system compatible with available urban transportation planning packages and a generalized policy analysis system based on random sample forecasting—are presented. Conclusions concerning the advantages and disadvantages of the system of disaggregate models are presented.

Significant advances have recently been made in developing and applying disaggregate behavioral travel demand models to many aspects of urban travel decisions (1, 2, 3, 4). What has not previously been done is the development of a full set of urban models and their integration into a complete forecasting system for use by a metropolitan planning organization. The purpose of this paper is to describe the first such system, which was developed for the Metropolitan Transportation Commission (MTC), the designated metropolitan planning organization for the San Francisco area. The models have been developed by the Travel Model Development Project, carried out by a consultant team consisting of the COMSIS Corporation, Cambridge Systematics, Inc., and Barton-Aschman Associates. Because of the number of models included in the system, this paper must be an overview of the system as a whole, rather than a detailed description of each individual model. Complete documentation of the project is available in a three-volume final report (5).

BACKGROUND

The MTC is the successor to the Bay Area Transportation Study Commission (BATSC), the original region-wide multimodal transportation planning agency in the Bay Area. Although BATSC carried out the traditional first steps of metropolitan transportation planning—collecting and analyzing land-use and travel data—neither it nor MTC was previously successful in developing an accepted complete land-use and travel modeling system that could be used to forecast future travel patterns. In cooperation with the Association of Bay Area Governments (ABAG), the projective land use model (PLUM) was developed to forecast future land use, employment location, residential location, and socioeconomic characteristics (6).

Although at least two generations of trip-making models that use these forecasts to predict future travel have been developed, both MTC and other Bay Area