NATIONAL COOPERATIVE HIGHWAY RESEARCH PROGRAM REPORT 116

STRUCTURAL ANALYSIS AND DESIGN OF PIPE CULVERTS

HIGHWAY RESEARCH BOARD
NATIONAL RESEARCH COUNCIL
NATIONAL ACADEMY OF SCIENCES—NATIONAL ACADEMY OF ENGINEERING

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STRUCTURAL ANALYSIS AND DESIGN OF PIPE CULVERTS

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AREAS OF INTEREST:

- BRIDGE DESIGN
- GENERAL MATERIALS
- MECHANICS (EARTH MASS)

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DIVISION OF ENGINEERING NATIONAL RESEARCH COUNCIL
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NATIONAL COOPERATIVE HIGHWAY RESEARCH PROGRAM

Systematic, well-designed research provides the most effective approach to the solution of many problems facing highway administrators and engineers. Often, highway problems are of local interest and can best be studied by highway departments individually or in cooperation with their state universities and others. However, the accelerating growth of highway transportation develops increasingly complex problems of wide interest to highway authorities. These problems are best studied through a coordinated program of cooperative research.

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FOREWORD

By Staff
Highway Research Board

This report examines previous and on-going research, design procedures, construction practices, and field performance related to pipe culverts. Based on a thorough evaluation of the state of knowledge, recommendations are made that are applicable to both (1) current design problems, and (2) future research needs. In view of the broad scope of the study, the report should be of value to highway design, materials, soils, and research engineers. Culvert pipe industry personnel and many members of the engineering academic community should also find that it contains much useful information.

Generally accepted methods for the structural design of pipe culverts require determination of the magnitude and distribution of loading and selection of a readily available rigid (concrete) or flexible (corrugated metal) culvert compatible with the loading. Although the Marston-Spangler and the more recently developed ring compression theories are currently being used extensively as a basis for designing buried conduits, a great deal of engineering judgment is involved in applying these load determination procedures, particularly in the case of rigid culverts. In addition, durability and handling problems, which are frequently critical in the case of flexible culverts, require the exercise of considerable engineering judgment.

One of the major uncertainties faced by the present-day designer is associated with the appropriate consideration of construction practices. This problem, together with the difficulty of specifying a generally acceptable failure criterion, makes the selection of a suitable safety factor extremely complicated. Perhaps the most important reasons which dictate the need for an evaluation of current design practices for both rigid and flexible culverts are the following:

1. There is serious concern about the extrapolation of currently used empirical relationships and field experience to the larger diameter pipes and higher fills coming into use.
2. With culvert-size highway drainage structures resulting in an expenditure of about $500,000,000 annually, the possibility of overconservatism in culvert design should be explored.
3. Current methods used in the design of pipe culverts fail to reflect in a rational way many of the factors that influence behavior in the field; for example, a better understanding of soil-pipe interaction is needed to further the development of intermediate-stiffness pipes made of different materials, such as plastics.

In view of the expressed purpose of the study to "survey and evaluate existing information and current research" and to "develop a design procedure for both flexible and rigid culverts based on the evaluation," the Northwestern University researchers first conducted a thorough review of literature pertaining to previous and current culvert research. Information on engineering practice with regard to design of culverts and practical field problems encountered during their installation was also collected. Although a large amount of information is available for evaluation, it was determined that sufficient information is not available at present for the development of completely new and more rational design procedures. Emphasis during the study was place on (1) identification of conditions for which currently used design procedures, with modifications and improvements, are satisfactory for continued judicious use; (2) improvements to methods for selecting some of the more important material properties used in existing design methods; (3) determination of conditions for which different approaches should be developed; and (4) recommendation of long-range research needs.

From the designers' standpoint, a most important finding is that durability, handling, and construction considerations are much more significant that structural design parameters when selecting a suitable generally available pipe culvert for cases involving small pipes to be placed under shallow-to-moderate fills. Currently used empirical design procedures appear adequate for the majority of these cases. More complex analysis and design procedures should be employed when large-diameter pipes are to be used and there is a vital need for extensive research in the area of extra large pipes, pipe arches, and other than round shapes. Also, there is a need to investigate more fully the effect of heavy construction loads on pipes under shallow fills.

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STRUCTURAL ANALYSIS AND DESIGN OF PIPE CULVERTS

SUMMARY

The objective of this work was to survey and evaluate current research and existing procedures for the structural analysis and design of pipe culverts and, on the basis of this evaluation, to suggest more rational procedures for accomplishing this task. The well-known work by Marston and Spangler has exerted a significant influence on virtually all currently used design procedures. This is because (1) on the whole, such procedures have worked reasonably well, and few, if any, failures can be attributed to the theory itself, and (2) no reliably superior theory has been produced. One of the criticisms most often leveled against the Marston-Spangler approach is its apparent conservatism, but this is extremely difficult to quantify. Other shortcomings include its consideration of only extreme (flexible and rigid) cases and its dependence on a variety of special parameters (such as settlement ratio and modulus of soil reaction) that apply only to the culvert problem. Nevertheless, the experience gleaned over the years cannot be discounted lightly, and a continued judicious use of this approach, together with recent improvements, is felt justified for the near term.

Although the findings of this work are general in nature, they do provide the background for several specific short-term recommendations for which there is felt to be sufficient research and field experience to justify immediate implementation in the majority of cases. In particular, for small-diameter pipes constructed of currently used materials and buried under moderate fill heights, existing design methods based on the various works of Marston, Spangler, White, and Watkins are generally satisfactory; in these cases durability and handling considerations frequently govern, and associated criteria are largely based on experience. For large-diameter pipes under shallow fills, the rigid design, which has thus far been dominant, is currently being challenged by more economical flexible designs, which use the interaction effect of the surrounding soil, and several of the latter type have been constructed recently; however, the influence of live loading and the possibility of buckling make this problem particularly complex, and extensive research is required in this area. For large-diameter pipes under high fills, the potential savings resulting from the use of a flexible design procedure are substantial, because the sections necessary to provide a rigid structure are very large. Although a variety of analytical and numerical approaches have been undertaken recently in an attempt to formulate a refined, general design method, these are still in the preliminary developmental stage, and none has been fully verified by field experience; one of the major problems associated with the use and evaluation of such approaches is the current inability to characterize quantitatively the compressibility of the compacted fill.

It appears increasingly evident that a substantially different approach to the soil-culvert problem is desirable in order to enhance the chances of establishing a

significant advancement in the current state of the art. Such an approach, an example of which is presented herein, would probably treat the soil surrounding the culvert as a continuum, and it would have the advantages that (1) the coupling or soil-culvert interaction effect is inherently taken into account, (2) input parameters would consist of more fundamental characterizations of the soil and culvert material behavior, and (3) pipes of intermediate stiffnesses can be handled. Although solutions of this type have previously been limited in their ability to include realistic bedding and backfill conditions, the advent of the high-speed digital computer has made possible the inclusion of these conditions, as well as nonlinear material behavioral characteristics. In testimonial to the versatility and advantages of this latter approach, the majority of current research effort seems to be following these lines.

Any advantages gained by virtue of improved methods for analysis and design can, and often are, cancelled by improper construction procedures, and, unfortunately, this frequently seems to be the case. Although considerable advancements have been made in construction techniques, the matter of providing high-quality inspection to ensure that the intended design is achieved in the field presents a serious problem. Despite a general recognition and extensive discussion of the problem, relatively little progress toward a solution has been made. Until adequate control is exercised over the culvert installation procedure, analytical advances will not attain their full potential. The foregoing conclusions can be reasonably well substantiated by the fact that virtually every culvert failure reported can be attributed to either improper construction procedure or the subsequent imposition of a condition more extreme than that on which the design was based.

The current concept of safety factor for a culvert is conducive to considerable misunderstanding and ambiguity and leads to a variety of interpretations that are sorely in need of reconciliation. An effort is made herein to present an organized and systematic interpretation of the safety factor of a culvert. The concept advanced consists of establishing a relationship between failure stresses and the stresses produced by a specified load distribution on the culvert. However, the specification of failure, which is a most important and highly controversial aspect of this problem, is left to the designer.

Although the primary emphasis in this work was directed toward the structural aspects of the culvert problem, several related topics were treated because they are considered to be directly or indirectly involved with the structural design. In particular, a simplified procedure is developed to predict approximately the camber of pipes resting on compressible foundation soils. Following a study of existing literature on the durability of metal pipes, a design procedure based on statistical evidence is suggested. In addition, the problems associated with construction and inspection are discussed, and a formulation of the economic considerations of a culvert installation is presented together with an identification of the parameters required for a meaningful evaluation. Finally, a survey and discussion of procedures for the analysis and/or design of culverts in Canada, Japan, and several European countries is given.

CHAPTER ONE

INTRODUCTION AND RESEARCH APPROACH

BACKGROUND OF CULVERT DESIGN

The analysis and design of pipe culverts is essentially a problem of soil-structure interaction, and it is necessary to give full cognizance to this fundamental coupling phenomenon when formulating or evaluating any specific procedures. In brief, this interaction aspect of the problem may be regarded as the predominant theme of this report. Historically, pipe culverts have been divided into two general categories—flexible and rigid—and independent analysis and design procedures have been developed for each. However, despite the individual design treatment given these extreme situations, the load acting on the culvert in each case is determined in basically the same manner; this, of course, is inconsistent with the interaction effect between the soil and the culvert. Of significant interest is the fact that no techniques are currently available for handling culvert pipes of intermediate stiffness.

The procedures that are prevalent today attempt to account for the relative stiffness between the soil and the pipe by a variety of parameters that are largely empirical in nature and associated specifically with the culvert problem. Although such parameters may achieve their intended goal when used with good engineering judgment within limited ranges of applicability for which experience is available, often their use cannot be easily extended or generalized. Also, the strong dependence of these empirical parameters on the exercise of engineering judgment is not especially desirable. In some cases, particularly for corrugated metal pipes, design tables based primarily on experience have been developed and modified over the years.

Although several failure modes in a culvert are possible, the quantification of "failure" is often arbitrary, and the specification of a safety factor for any given set of circumstances is ambiguous and subject to considerable misunderstanding; this becomes especially difficult when durability considerations are taken into account. Despite the well-recognized importance of the method of installation in the subsequent performance of the culvert, the inspection and control required to ensure that design conditions are achieved in the field are frequently inadequate; indeed, the majority of failures can be attributed to this particular shortcoming. Perhaps, in view of this brief background, the question of possible overconservatism in culvert design is one of the more significant ones to be considered.

OBJECTIVES

This research is directed toward the following major objectives:

1. To critically review and evaluate research efforts and current procedures for the analysis and design of both flexible and rigid pipe culverts.
2. Based on the foregoing evaluation, to suggest procedures, including a factor of safety determination, for analyzing and designing both flexible and rigid pipe culverts in the immediate future.
3. To provide direction for a research program to substantially improve the present state of the art for culvert analysis and design.

Various methods are currently being used in the design of pipe culverts, and considerable research by many organizations is in progress to examine and improve these methods. However, there is often a lack of agreement between theory and field experience because many current design methods fail to reflect in a satisfactory way some of the major aspects of soil-culvert interaction. In addition, there is need for a more accurate prediction of the factor of safety against each possible mode of failure from measurable properties of the soil-culvert system; this, in turn, requires a more accurate definition of the anticipated loadings, which are governed by the construction process, bedding condition, and fill material.

In accordance with the foregoing objectives, the findings of this research could be essentially classified into any of three categories. First, it may be concluded that one or more of the currently used and widely available design procedures are completely satisfactory in view of existing knowledge and therefore justify continued use for the indefinite future. Second, it may be concluded that current design procedures are basically sound, but in need of modification to enhance their applicability under present conditions. Or, third, it may be concluded that the current design procedures are unsatisfactory in the light of current knowledge and a completely different approach should be developed. If either the second or third conclusion is reached, there arises the related problem of what modifications should be made or what new design procedures should be developed.

As work progressed, it became evident that current culvert design procedures cannot be unequivocally classified into any of the categories described. It is simply not possible to state categorically that a given procedure is good or not good; each has its inherent advantages and disadvantages and must be considered in the light of available alternatives. In an effort to resolve this dilemma to some degree, it was decided to seek answers to the questions of what culvert design procedures should be used in the immediate future and what procedures should be exploited and developed for potential use in the more distant future. In keeping with the foregoing approach, the problem became much more tenable, and it was possible to make certain realistic evaluations.

RESEARCH APPROACH

In view of the one-year duration and the expressed purpose of this project to "survey and evaluate existing information and current research" and to "develop a design procedure for both flexible and rigid culverts based on the evaluation," it was mutually decided by the researchers and NCHRP that the course of the study should be directed primarily toward near-term solutions. Early in this investigation it became clear that the culvert field is one in which a considerable amount of work has been and is being done; however, the applicability and validity of the results depend strongly on the assumptions (soil behavior, pipe behavior, failure criterion, bedding condition, etc.) underlying the theoretical studies and the test conditions (gauges, instrumentation, techniques, etc.) in the case of experimental work. Hence, an effort is made herein to assess these two aspects of the problem. In both cases, of course, proper interpretation of the results is essential.

The task of surveying and evaluating current design procedures and research efforts was accomplished in large part by consultation with a variety of persons who are or have been directly involved with experiences related to culvert analysis, design, and installation. The number and variety of consultants who were involved with this problem are considered an extremely significant and important part of this study. It is felt that this approach is highly desirable because it takes full advantage of the knowledge and experience of those who have worked on culverts for long periods of time, and it attacks the aspects of the problem that are felt to be most in need of critical evaluation. The persons contacted included both practical and research-oriented structural and soils engineers, contractors, economists, and administrators; government representatives, both federal and state, were consulted, together with representatives from private industry, manufacturing, and universities. In all cases, an attempt was made to determine why a given procedure is followed and what is the basis for such a procedure.

ORGANIZATION OF THE REPORT

This report is organized into two parts. The first part contains a brief presentation of the essential findings of the study and the suggested procedures by which to approach the analysis and design of pipe culverts. The second part contains a fairly extensive synthesis of current knowledge on the subject of culvert design and the background information to support the recommendations advanced in the first part.

Although they are not explicitly stated in the title of this project or the statement of the research objectives, there are several items, such as durability, economy, and camber determinations, that may have a distinct bearing on the structural aspects of a particular culvert. Consequently, in an effort to consider this problem from a more general systems approach, a study of each of these aspects has been included. In addition, special sections on some failures of buried conduits and culvert practices in foreign countries have been prepared.

NOTATION

In view of the extensive and varied notation in this report and in the ensuing list, the following explanatory comments are offered. Insofar as possible, an effort was made to maintain in the various equations the symbols and terminology that have become identified with the development and use of these equations. In many cases, this practice led to the use of a given symbol for two or more different parameters and, alternatively, the designation of a given parameter by two or more different symbols. Although this may initially appear confusing and undesirable, careful thought indicated that the alternative option of changing many symbols to be consistent in this report, but inconsistent with commonly accepted use, would be more generally unacceptable. In addition, despite the extent of the following notation list, many symbols of a very specific nature and having a limited use in this report are not included; the identification of such symbols should be evident from the text immediately preceding or following them. The primary intent of this notation list is to provide the reader with an organized and readily available identification of the most commonly used terms in this report and to obviate the necessity of perusing other sections of the report to obtain their descriptions.

$A =$	coefficient	$(F^0L^0T^0)$
$A =$	transverse cross-sectional area of conduit wall	$(F^0L^2T^0)$
$A_c =$	coefficient parameter (Burns and Richard)	$(F^0L^0T^0)$
$A_l =$	cross-sectional area of conduit wall per unit length of conduit	$(F^0L\,T^0)$
$a_0{}^* =$	soil-conduit parameter (Burns and Richard)	$(F^0L^0T^0)$
$a_2{}^* =$	soil-conduit parameter (Burns and Richard)	$(F^0L^0T^0)$
$a_2{}^{**} =$	soil-conduit paramcter (Burns and Richard)	$(F^0L^0T^0)$
$B =$	coefficient of elastic support	$(F^0L^0T^0)$
$B =$	soil parameter (Burns and Richard)	$(F^0L^0T^0)$
$B_c =$	horizontal outside breadth of conduit	$(F^0L\,T^0)$
$B_d =$	horizontal width of ditch at top of culvert	$(F^0L\,T^0)$
$b =$	horizontal distance	$(F^0L\,T^0)$
$b_2{}^* =$	soil-conduit parameter (Burns and Richard)	$(F^0L^0T^0)$
$b_2{}^{**} =$	soil-conduit parameter (Burns and Richard)	$(F^0L^0T^0)$
$C =$	coefficient of subgrade reaction	$(F^0L^0T^0)$
$C =$	soil parameter (Burns and Richard)	$(F^0L^0T^0)$
$C_c =$	coefficient of compressibility for soil	$(F^0L^0T^0)$
$C_c =$	load coefficient for positive projecting conduit	$(F^0L^0T^0)$
$C_d =$	load coefficient for ditch conduit	$(F^0L^0T^0)$
$C_h =$	load coefficient	$(F^0L^0T^0)$
$C_n =$	load coefficient for negative projecting conduit	$(F^0L^0T^0)$
$C_o =$	bedding coefficient	$(F^0L^0T^0)$

$C_t =$ load coefficient for live load effect on conduits ($F^0L^0T^0$)
$C_v =$ load coefficient ($F^0L^0T^0$)
$C_w =$ Winkler's coefficient of proportionality ($F^0L^0T^0$)
$C_t' =$ load coefficient ($F^0L^0T^0$)
$C_t'' =$ load coefficient ($F^0L^0T^0$)
$C_u' =$ load coefficient ($F^0L^0T^0$)
$C_u'' =$ load coefficient ($F^0L^0T^0$)
$c =$ coefficient ($F^0L^0T^0$)

$D =$ thickness of consolidating clay layer below conduit ($F^0L\,T^0$)
$D =$ D-load in pounds per linear foot per foot of internal diameter ($F\,L^{-2}T^0$)
$D =$ depth of box culvert ($F^0L\,T^0$)
$D_l =$ deflection lag factor ($F^0L^0T^0$)
$D_{max} =$ maximum allowable grain size ($F^0L\,T^0$)
$d =$ nominal diameter of circular conduit ($F^0L\,T^0$)
$d =$ over-all height of rectangular conduit ($F^0L\,T^0$)
$d =$ thickness of relatively incompressible foundation soil layer ($F^0L\,T^0$)

$d_c =$ shortening of vertical height of conduit ($F^0L\,T^0$)
$d_i =$ inside diameter of circular conduit ($F^0L\,T^0$)
$d_m =$ average or mean diameter of circular conduit ($F^0L\,T^0$)
$d_o =$ outside diameter of circular conduit ($F^0L\,T^0$)

$E =$ modulus of elasticity of conduit material ($F\,L^{-2}T^0$)
$E_s =$ modulus of soil ($F\,L^{-2}T^0$)
$E' =$ modulus of soil reaction ($F\,L^{-2}T^0$)
$e =$ modulus of passive resistance of the surrounding soil ($F\,L^{-3}T^0$)
$e =$ void ratio of soil ($F^0L^0T^0$)
$e_o =$ natural void ratio ($F^0L^0T^0$)
$e_1 =$ initial void ratio ($F^0L^0T^0$)

$F =$ compressibility factor equal to $C_c/(1 + e_o)$ ($F^0L^0T^0$)
$F =$ safety factor ($F^0L^0T^0$)
$\bar{F} =$ mean value of F for N data points ($F^0L\,T^0$)
$F_{est} =$ estimated compressibility factor ($F^0L^0T^0$)
$F_e =$ safety factor on earth loads ($F^0L^0T^0$)
$F_w =$ safety factor on live loads ($F^0L^0T^0$)
$f =$ number of data points in an angular sector ($F^0L^0T^0$)
$f =$ modification factor ($F^0L^0T^0$)
$f =$ coefficient of friction between soil and pipe ($F^0L^0T^0$)
$f_b =$ buckling stress ($F\,L^{-2}T^0$)
$f_c =$ critical buckling stress ($F\,L^{-2}T^0$)
$f_y =$ yield strength of culvert material ($F\,L^{-2}T^0$)

$G =$ coefficient of subgrade reaction ($F^0L^0T^0$)
$G =$ force due to earth load ($F\,L^0T^0$)
$G_s =$ specific gravity of soil particles ($F^0L^0T^0$)
$g =$ horizontal moment arm ($F^0L\,T^0$)

$H =$ height of fill above top of conduit ($F^0L\,T^0$)
$H_c =$ vertical distance between surface of fill and plane of equal settlement ($F^0L\,T^0$)
$H_e =$ height of plane of equal settlement above top of conduit ($F^0L\,T^0$)
$H_e =$ height of fill equal to preconsolidation stress of foundation soil divided by unit weight of embankment soil ($F^0L\,T^0$)
$H_r =$ vertical height from any point to surface of fill ($F^0L\,T^0$)
$H_t =$ height of embankment above top of incompressible sublayer ($F^0L\,T^0$)
$h =$ vertical distance ($F^0L\,T^0$)

$I =$ moment of inertia of longitudinal cross section of conduit wall per unit length ($F^0L^3T^0$)
$I_c =$ impact factor ($F^0L^0T^0$)
$I_d =$ moment of inertia of pipe wall about the horizontal centroidal axis ($F^0L^4T^0$)
$I_f =$ impact factor ($F^0L^0T^0$)
$I_h =$ moment of inertia of conduit transverse cross section about horizontal centroidal axis ($F^0L^4T^0$)

$K =$ Rankine coefficient $\tan^2(45° \pm \phi/2)$ ($F^0L^0T^0$)
$K =$ bedding factor whose value depends on the bedding angle ($F^0L^0T^0$)
$K =$ coefficient of earth pressure ($F^0L^0T^0$)
$K_a =$ coefficient of active earth pressure ($F^0L^0T^0$)
$K_d =$ load coefficient ($F^0L^0T^0$)
$K_e =$ load coefficient ($F^0L^0T^0$)
$K_o =$ coefficient of earth pressure at rest ($F^0L^0T^0$)
$K_p =$ coefficient of passive earth pressure ($F^0L^0T^0$)
$K_\gamma =$ coefficient of earth pressure as determined from the theory of elasticity ($F^0L^0T^0$)
$K_d' =$ load coefficient ($F^0L^0T^0$)
$k =$ coefficient ($F^0L^0T^0$)
$k_i =$ nondimensional factor accounting for beam curvature ($F^0L^0T^0$)

$L =$ length of conduit section on which load is computed ($F^0L\,T^0$)
$L =$ load factor equal to $\log[(p_o + p)/p_o]$ ($F^0L^0T^0$)
$L =$ total length of culvert ($F^0L\,T^0$)
$L_f =$ load factor relating field strength to three-edge bearing strength ($F^0L^0T^0$)
$L_o =$ unit overload ($F\,L^{-2}T^0$)
$l =$ effective length of conduit ($F^0L\,T^0$)

$M =$ transverse bending moment in conduit wall per unit length ($F\,L^0T^0$)
$M_d =$ longitudinal bending moment in conduit wall per unit length ($F\,L\,T^0$)
$M^* =$ constrained modulus of soil ($F\,L^{-2}T^0$)
$m =$ fractional part of outside diameter of conduit over which lateral pressure is effective ($F^0L^0T^0$)
$m =$ coefficient determined by type of soil ($F^0L^0T^0$)

$N =$ a parameter that is a function of the distribution of the vertical load and vertical reaction ($F^0L^0T^0$)

n = coefficient determined by foundation conditions $(F^0L^0T^0)$
P = applied three-edge bearing test load $(F\,L^{-1}T^0)$
P = dead load per unit length of conduit $(F\,L^{-1}T^0)$
P' = reduced dead load per unit length of conduit $(F\,L^{-1}T^0)$
$P_{\min}$ = required minimum load-bearing capacity of conduit per unit length $(F\,L^{-1}T^0)$
p = free field vertical soil pressure at the level of the ring center $(F\,L^{-2}T^0)$
p = overpressure $(F\,L^{-2}T^0)$
p = dead load stresses in soil $(F\,L^{-2}T^0)$
p_a = uniformly distributed loading around the pipe $(F\,L^{-2}T^0)$
p_a = equivalent uniform vertical stress $(F\,L^{-2}T^0)$
p_b = equivalent uniform vertical reaction $(F\,L^{-2}T^0)$
p_f = free field vertical soil stress at level of the center of conduit $(F\,L^{-2}T^0)$
p_h = horizontal normal stress in soil $(F\,L^{-2}T^0)$
p_o = stress due to natural overburden of soil $(F\,L^{-2}T^0)$
p_r = projection ratio for positive projecting conduit $(F^0L^0T^0)$
p_r = radial component of earth pressure $(F\,L^{-2}T^0)$
p_t = tangential component of earth pressure $(F\,L^{-2}T^0)$
p_t = total stress $(F\,L^{-2}T^0)$
p_v = vertical normal stress in soil $(F\,L^{-2}T^0)$
p_{bec} = equivalent loading associated with maximum compressive stress $(F\,L^{-2}T^0)$
p_{bet} = equivalent loading associated with maximum tensile stress $(F\,L^{-2}T^0)$
p_1 = major principal stress in soil $(F\,L^{-2}T^0)$
p_2 = intermediate principal stress in soil $(F\,L^{-2}T^0)$
p_3 = minor principal stress in soil $(F\,L^{-2}T^0)$
$\overline{p_h}$ = horizontal effective stress in soil $(F\,L^{-2}T^0)$
$\overline{p_v}$ = vertical effective stress in soil $(F\,L^{-2}T^0)$
$\overline{p_1}$ = major effective stress in soil $(F\,L^{-2}T^0)$
$\overline{p_2}$ = intermediate effective stress in soil $(F\,L^{-2}T^0)$
$\overline{p_3}$ = minor effective stress in soil $(F\,L^{-2}T^0)$
p^* = uniform applied pressure required to cause buckling in a soil-surrounded tube $(F\,L^{-2}T^0)$
p_r' = projection ratio for negative projecting conduit $(F^0L^0T^0)$

Q = live load per unit length of conduit $(F\,L^{-1}T^0)$
Q_s = concentrated surface load $(F\,L^0\,T^0)$
q = stress at soil-structure interface $(F\,L^0\,T^0)$
q = ratio of total lateral pressure to total vertical load $(F^0L^0\,T^0)$
q = live load stress in soil $(FL^{-2}T^0)$
q_h = horizontal live load stress in soil $(FL^{-2}T^0)$
q_v = vertical live load stress in soil $(FL^{-2}T^0)$

R = radius of curvature of the longitudinal axis $(F^0L\,T^0)$
R = radial distance from surface load $(F^0L\,T^0)$
R_o = load concentration factor $(F^0L^0T^0)$
r = correlation coefficient $(F^0L^0T^0)$
r = nominal radius of circular conduit $(F^0L\,T^0)$
r_i = inside radius of circular conduit $(F^0L\,T^0)$
r_m = average or mean radius of circular conduit $(F^0L\,T^0)$
r_o = outside radius of circular conduit $(F^0L\,T^0)$
r_{sd} = settlement ratio $(F^0L^0T^0)$

S = settlement of embankment due to consolidation of underlying soil $(F^0L\,T^0)$
S = soil deformation $(F^0L\,T^0)$
$S_{F.e_o}$ = standard error of estimate of F on e_o $(F^0L^0T^0)$
S_m = deformation of compacted soil adjacent to conduit between natural ground surface and top of conduit $(F^0L\,T^0)$
s = coefficient depending on foundation under conduit $(F^0L^0T^0)$
s_d = deformation of fill between top of conduit and natural ground surface $(F^0L\,T^0)$
s_f = settlement of conduit into its foundation $(F^0L\,T^0)$
s_g = settlement of natural ground surface adjacent to conduit $(F^0L\,T^0)$

T = ring compression load in conduit wall per unit length $(F\,L^{-1}T^0)$
$T_{r\psi}$ = tangential shear stress at soil-conduit interface $(F\,L^{-2}T^0)$
t = thickness of slab for box culvert $(F^0L\,T^0)$
t = thickness of conduit wall $(F^0L\,T^0)$

UF = extensional flexibility ratio $(F^0L^0T^0)$
u = excess porewater pressure in soil $(F\,L^{-2}T^0)$

VF = bending flexibility ratio $(F^0L^0T^0)$
V_s = volume of solids in a soil sample $(F^0L^3T^0)$
V_v = volume of voids in a soil sample $(F^0L^3T^0)$

W = critical surface wheel load $(F\,L^0T^0)$
W = unit load of added layer $(F\,L^{-2}T^0)$
W_c = vertical load on conduit per unit length $(F\,L^{-1}T^0)$
W_s = dry weight of a soil sample $(F\,L^0T^0)$
W_t = average load per unit length on conduit due to wheel load $(F\,L^{-1}T^0)$
w = span of a box or "plate" culvert $(F^0L\,T^0)$
w = radial displacement of conduit wall $(F^0L\,T^0)$

X = tensile force acting on the culvert $(F\,L^0\,T^0)$
x = a parameter that is a function of the area of the vertical projection of the pipe on which the active lateral pressure of the fill material acts $(F^0L^0T^0)$
x' = x for Class A bedding $(F^0L^0T^0)$

Z = section modulus of pipe $(F^0L^2T^0)$
Z = section modulus $(F^0L^2T^0)$
z = vertical coordinate $(F^0L\,T^0)$

α = bedding angle $(F^0L^0T^0)$
α = side slope of compacted embankment $(F^0L^0T^0)$
α = ratio of inside radius to outside radius of concrete pipe $(F^0L^0T^0)$

β = stress factor ($F^0L^0T^0$)
β = coefficient ($F^0L^0T^0$)
γ = unit weight of soil ($F L^{-3}T^0$)
γ_b = submerged unit weight of foundation soil ($F L^{-3}T^0$)
γ_d = dry density of soil ($F L^{-3}T^0$)
γ_s = standard value for unit weight of soil ($F L^{-3}T^0$)
γ_w = unit weight of water ($F L^{-3}T^0$)
γ_{do} = initial dry density of soil ($F L^{-3}T^0$)
γ_{180} = dry density at AASHO compaction designation T 180 ($F L^{-3}T^0$)
Δ = maximum differential settlement of culvert ($F^0L T^0$)
Δ = horizontal displacement of culvert wall ($F^0L T^0$)
ΔH = difference between free field surface settlement and settlement of column of soil above culvert ($F^0L T^0$)
Δh = relative upward movement of conduit ($F^0L T^0$)
Δp = change in soil dead load stress ($F L^{-2}T^0$)
Δx = change in horizontal dimension of conduit ($F^0L T^0$)
Δy = change in vertical dimension of conduit ($F^0L T^0$)
Δx^* = change in horizontal dimension before modification for ring stiffness ($F^0L T^0$)
Δy^* = change in vertical dimension before modification for ring stiffness ($F^0L T^0$)
Δd_h = horizontal deformation of the diameter ($F^0L T^0$)
Δd_r = radial deformation ($F^0L T^0$)
Δd_v = vertical deformation of the diameter ($F^0L T^0$)
δ = settlement of point in soil mass ($F^0L T^0$)
ϵ = soil strain ($F^0L^0T^0$)
κ = ratio of horizontal earth pressure to vertical earth pressure ($F^0L^0T^0$)
λ = factor to account for stratification of foundation soils ($F^0L^0T^0$)
λ_r = pressure concentration ratio ($F^0L^0T^0$)
ν = Poisson's ratio for soil ($F^0L^0T^0$)
ν_c = Poisson's ratio for conduit material ($F^0L^0T^0$)
ν_s = Poisson's ratio for soil ($F^0L^0T^0$)
σ = total stress in soil ($F L^{-2}T^0$)
σ_l = longitudinal normal stress in conduit ($F L^{-2}T^0$)
σ_r = radial normal stress in conduit ($F L^{-2}T^0$)
σ_t = tangential normal stress in conduit ($F L^{-2}T^0$)
σ_1 = major principal stress in conduit ($F L^{-2}T^0$)
σ_2 = intermediate principal stress in conduit ($F L^{-2}T^0$)
σ_3 = minor principal stress in conduit ($F L^{-2}T^0$)
$\bar{\sigma}$ = effective stress in soil ($F L^{-2}T^0$)
τ = stress in an elastic ring ($F L^{-2}T^0$)
θ = angle between regression line and line passing through $F_{est} = 0$ ($F^0L^0T^0$)
ψ = angle relative to horizontal plane ($F^0L^0T^0$)
ϕ = angle of internal friction for fill material ($F^0L^0T^0$)
ϕ_s = standard value for angle of internal friction for fill material ($F^0L^0T^0$)
ϕ' = effective angle of internal friction for cohesive soils ($F^0L^0T^0$)
ϕ' = angle of friction between fill and sides of ditch for ditch conduits ($F^0L^0T^0$)
η = bedding characteristic ($F^0L^0T^0$)
ρ = radius of curvature of culvert wall ($F^0L T^0$)

CHAPTER TWO

FINDINGS

The findings of this study are presented under five general headings. The first section briefly summarizes the prevalent current procedures for the analysis and design of pipe culverts; the second is concerned with a discussion of alternative methods for determining the loads acting on buried conduits and the associated deformations. The third section proposes a technique for calculating the required camber at which to install a culvert, and the fourth section presents the conclusions reached from an evaluation of available literature on durability of metal culverts. The final section gives a comparative appraisal of results obtained by various design procedures for different installation conditions. There is no effort in this part of the report to present elaborate descriptions and justifications for the evaluations made; rather, there is a distinct attempt to concisely summarize the findings. Details substantiating these findings appear in the appendices.

CURRENT DESIGN METHODS

The design of pipe culverts currently follows one general path: (1) the loads acting on the culvert are determined by either the Marston-Spangler or the ring compression theory, and (2) a culvert section compatible with the determined loadings is selected. Except for special situations, local buckling stability is not considered. Because both the magnitude and distribution of earth loads on culverts are known to depend on the relative stiffness of the culvert and the soil, current design methods distinguish between a rigid (concrete or cast iron) culvert and a flexible (corrugated steel or aluminum) culvert, and these are treated separately with different parameters being used in each respective design procedure.

Rigid Culverts

The commonly used method for determining loads on rigid culverts can be attributed to the early work of Marston, Spangler, Schlick, and their colleagues (*1, 2, 3, 4, 5*). Because of the recognized effects of the soil surrounding the culvert and the method of installation on the load transmitted to the pipe, underground conduits have been classified into several groups and subgroups. The major groups are trench or ditch conduits and embankment conduits, and the latter are further subdivided into positive projecting, negative projecting, and imperfect trench subgroups. The development of the associated load determination procedures for each of these classes was accompanied by some full-scale studies. Because most highway culverts fall into the embankment conduit class, emphasis is directed toward this group.

The Marston theory of load determination gives only the total vertical earth load that is assumed to act on a circular pipe. This load is expressed as the product of the unit weight of the fill material, the square of a horizontal dimension (reflecting either the trench width at the top of the conduit for a trench or negative projecting conduit or the outside width of the conduit for a positive projecting conduit), and a load coefficient which is determined by the method of installation. The load coefficient depends on the geometry of the soil-culvert system and the physical properties of the fill and the culvert materials; it is usually expressed as a function of the height of fill above the culvert, the horizontal width of the culvert or trench, the coefficient of internal friction of the soil, the projection ratio, and the settlement ratio. The determined load is basically equal to the weight of the prism of soil above the culvert plus or minus the shearing resistance developed due to relative deformation of this prism and the adjacent columns of soil.

In brief, the assumptions and limitations of this load determination theory are:

1. Only the total vertical load, not its distribution, on the culvert is given directly; horizontal loads are handled indirectly in the design.

2. A soil element of width equal to that of the prism (or culvert diameter) is considered, and a uniform vertical pressure distribution is assumed to act across this width.

3. It is assumed that the differential movements between the soil prism above the culvert and the adjacent prisms are sufficiently large to fully mobilize the total soil shear strength.

4. The forces on the sides of the sliding element of soil are assumed equal to the product of a friction coefficient and the horizontal active soil pressure; however, because the value of the mobilized shear stress along the slip surfaces depends on the relative displacement, which is not constant with depth, it seems incorrect to consider the coefficient of friction as constant over the entire depth. In addition (according to arguments presented in Appendix B), the assumption that active earth pressure acts against the sliding element does not appear to be justified.

5. Although the assumption of vertical sliding surfaces may be acceptable for a trench conduit, provided the trench was cut through relatively unyielding material, it is very questionable for an embankment conduit. In particular, when vertical sliding surfaces are assumed, the assumption of active horizontal earth pressure becomes critical; in fact, different investigators have shown that required values for the coefficient of horizontal earth pressure may range from 1.0 to 2.5, and these are much larger than the active coefficient values (0.3 or 0.4) and even larger than at-rest coefficient values (0.5 or 0.6). Work by Voellmy (*6*), Terzaghi (*7*), and others indicates that the sliding surfaces for an embankment conduit may be better approximated by surfaces that are inclined to the vertical to form a wedge.

6. The effects of the deformation of the fill, the culvert, and the natural ground under the culvert due to the weight of the fill are handled by an abstract parameter termed the settlement ratio, which enters in determining the load coefficient previously discussed. Although quantitative values for this semi-empirical factor are based on an extremely small amount of experimental data obtained many years ago (see Appendix B), they continue to be used today with virtually no modification.

7. In the imperfect trench installation, the vertical load on the culvert is reduced by placing compressible material above the pipe to obtain an arching effect in the soil; this phenomenon will, however, increase the horizontal pressure on the culvert. The problem of reducing the vertical load while increasing the horizontal load to obtain an approximately uniform radial load on the culvert is extremely complex, and it is difficult to exercise adequate control over the construction procedure to ensure that desirable results are being achieved. In addition, even though Spangler reported 20 years ago that two highway departments had made extensive and advantageous use of the imperfect trench method, a need still exists for adequate, well-documented and reliable field observations to evaluate the time effects on such installations.

After the total vertical load acting on the pipe has been determined, a specific pipe is selected to satisfy the imposed strength requirements. The inherent strength of a concrete pipe is determined by conducting a standard three-edge bearing test: in this test the pipe is subjected to concentrated loads at the crown and invert, and the loads are increased until either a 0.01-in. crack has occurred throughout a length of 1 ft or until the ultimate strength load of the pipe has been reached. To relate the three-edge bearing strength to the field supporting strength, load factors have been developed, and tabulated for different classes of bedding; a more detailed discussion of the load factor concept is given in Appendix B. Because the in-place supporting strength of a rigid pipe depends largely on the installation conditions and local quality control, and because currently used quantitative definitions of failure are diverse and subject to much controversy, the particular value assigned to the factor of safety must be chosen with considerable discretion and based strongly on engineering judgment.

Flexible Culverts

In the case of flexible culverts, nonuniform stress distributions at the soil-culvert interface will tend to be redistributed as the culvert deforms, and the imposed loads will be resisted largely by membrane action in the culvert wall. For circular pipes, two major design procedures are commonly used; one is concerned with limiting culvert deformations, and the other is concerned with limiting the compressive load in the culvert wall. In general, design methods recommend selection of the wall thickness in accordance with the latter and a deflection check by use of the former. Local buckling stability is not considered to any great degree, except for pipes of very large diameter; this appears justifiable because no report of a buckling failure of an in-service circular culvert has been found unless the failure was preceded by excessive deflection.

The Spangler (*8*) approach asserts that deformations will usually control the design of flexible culverts. Seams or lap joints are designed to resist ring compression, but no apparent consideration is given to the compressive stress in the culvert wall. Based on the assumption that the horizontal deflection of the pipe is inversely proportional to the modulus of passive resistance of the soil, Spangler developed the well-known Iowa formula for computing the change in the horizontal diameter of the culvert; the vertical load is determined in a manner similar to that used for rigid culverts. The other parameters in the Iowa formula are a bedding constant, pipe diameter, modulus of elasticity of the pipe material, moment of inertia of the pipe wall, modulus of soil reaction, and a deflection lag factor. Of these, the modulus of soil reaction is the most controversial and has prompted the most research; currently recommended values for this parameter are 700 and 1,400 psi for good and excellent backfill, respectively, but experimentally determined values from a few hundred to more than 8,000 psi have been reported. Culvert deformations, as calculated by the Iowa formula, are normally limited for design purposes to 5 percent of the pipe diameter. Originally, design criteria for flexible culverts were empirically established by means of observational studies; based on these observations, gauge tables were prepared and continually revised in accordance with experience. During the course of such studies, it was noted that pipes deflected up to about 20 percent of their vertical diameter before failure occurred, whereupon the use of a so-called safety factor of 4 established the current commonly accepted design criterion of 5 percent deflection. Despite the qualified success of the Iowa formula, a critical evaluation of the assumed load distribution on which it is based certainly seems appropriate.

The ring compression theory, proposed by White and Layer (*9*), states that the culvert wall should be designed to resist the compressive stresses produced by a hydrostatic soil pressure equal in magnitude to the overburden pressure; a factor of safety of 4 is frequently applied. As a deflection criterion is not included with this method of design, it is presumed that the soil surrounding the culvert is well compacted and that the soil-culvert system works to carry the overburden load in the most desired manner.

ALTERNATIVE DESIGN SUGGESTIONS

To arrive at improved methods for determining the stresses and deformations in a soil-culvert system, it will be helpful first to examine the necessary components of an idealized design method, based on the premise that sufficient theory and analytical techniques are available. Then, an effective approach to an economical and practical design method may be obtained by systematically applying the simplifying approximations necessary to adapt the idealized situation to the currently available theory and analytical techniques.

Requirements for Ideal Design Method

The ultimate objective of any culvert design procedure is to predict stresses and deformations at any part of the system at any point in time for a given fill height. A rigorous theory should consider the soil and the culvert as a composite unit, so that the interaction effect is taken into account, and the system should be treated as a three-dimensional, nonhomogeneous continuum. The theory should be versatile in that it should cover all possible field situations, such as nonhomogeneous modulus distributions, various conduit shapes, height of cover ranging from shallow to very deep, conduit stiffnesses ranging from rigid to flexible, various soil types and behavior, and special loading conditions, including concentrated unsymmetric traffic loads and seepage forces. Both dilatational and deviatoric deformations in the soil should be taken into account, and time effects should be considered so that, in addition to elastic response, plastic and viscous phenomena can be handled. The independent parameters in the idealized theoretical formulation should be the geometry of the system and the material properties of both the culvert and the surrounding and underlying soils.

Despite the desirability of including in a design method all of the characteristics described previously, the present state of the art precludes such completeness, and some compromise must be made between the objectives of accuracy and versatility and the practicality of developing a workable procedure; the latter will, of course, be attained by neglecting factors and conditions that exert a small influence on the result and unnecessarily complicate the design method. If available theory and solutions are used, several approximations and limitations to the applicability of the design method are immediately introduced. Because appropriate solutions have not yet been obtained for cases where the medium surrounding the conduit is assumed to be viscoelastic or plastic, elastic solutions must be exploited. However, the reasonable past success of elastic theory in soil mechanics lends some justification for its use in the culvert problem; in particular, the use of elastic theory is probably more appropriate for compacted soils than for many other soils to which it is commonly applied. Although experience has indicated that time effects do play some part in the behavior of buried conduits, such effects are generally small, and it is usually justifiable to neglect them. More important may be the failure of elastic theory to account for deformations due to shear stresses that exceed the shear strength of the soil. Although the finite element method does provide a technique that has the capability of considering time effects and the nonlinear behavior of the components of the soil-culvert system, its use for investigating the culvert problem is presently in the preliminary stages of development.

Adaptation of Elastic Solution

Burns and Richard (*10*) have reported a plane strain solution for a circular conduit buried in an infinite, linearly elastic medium subjected to a uniformly distributed overpressure; the medium is considered to be weightless, homogeneous, and isotropic. In effect, this is the solution for a uniformly loaded linear elastic infinite plate that contains a circular hole tightly fitted with a ring having different properties. The following example, described in detail in Appendix C, illustrates the adaptation of this elastic solution to development of a design procedure for a certain class of culverts. As a result of the assumptions mentioned previously, any design method based on this solution may be subject to some or all of the following approximations and restrictions:

1. The stress-strain characteristics of the soil surrounding the culvert are assumed to be homogeneous, isotropic, and time-independent.
2. The soil cover is assumed to be sufficiently high so that the behavior will be the same as that for a conduit surrounded by an infinite medium.
3. Because the soil load on the culvert is applied in the form of an overpressure, the differential effect of the soil self-weight in the area of the culvert is neglected.
4. Deformations are assumed to occur only in the plane perpendicular to the culvert axis.
5. No consideration is given to changes in deformations and stresses over extended periods of time.

To adapt the Burns and Richard results to the determination of culvert behavior, it is convenient to convert their equations to graphical form. Provided the equations are simplified by neglecting any change in length of the culvert periphery, curves may be drawn to indicate relationships between deformation, bending moment, and circumferential thrust parameters, respectively, and a soil-culvert stiffness parameter; these curves are shown in Figures C-1, C-2, and C-3. Subject to the foregoing restrictions, conduit behavior determined from these graphs offers the following major advantages not available from current methods:

1. The soil-culvert interaction phenomenon is properly handled.
2. The behavior of the system is provided over a continuous range of soil-culvert stiffness ratios.
3. The material properties required as input are fundamental characteristics of the respective materials, not properties peculiar to the culvert problem.

The conditions at the soil-culvert interface are considered to some extent in that the solution treats both the full-slip and the no-slip cases; knowledge of the particular problem can be used as a guide for the selection of an optimum intermediate condition. Use of the constrained modulus in the solution is especially advantageous because there is evidence that suggests that, for the usual embankment soils, dry density may be used as an indicator of its value; in addition to facilitating the determination of immediate elastic deformations, the constrained modulus provides for the short-term time effects associated with the dissipation of porewater pressure. The choice of Poisson's ratio presents some difficulty because little information on which to base its selection is available, particularly for compacted fills. If conduit deformations are computed by incremental applications of this method in a manner simulating the building of the fill, the effects of a nonlinear soil modulus can be handled; however, for computation of circumferential thrust and bending moment, one application of the total load and the use of an average tangent constrained modulus is considered to be sufficient. All of these aspects are discussed in detail in Appendix C.

General application of this procedure is restricted to cases where it may be reasonably assumed that (1) the weight of the soil and the effect of surface loads may be applied to the culvert in the form of overpressure loading, and (2) the stress-strain characteristics for the soil surrounding the culvert are homogeneous and isotropic. For such cases the pressures at the soil-culvert interface may be approximated mathematically by the sum of two terms, one representing a uniformly distributed pressure and the other a sinusoidally distributed pressure. However, either low cover height conditions or nonuniformity of the surrounding soil may cause considerable variation from this simple peripheral pressure distribution. For the latter situations, the solution may require additional harmonic terms to adequately describe the actual pressure distribution at the soil-culvert interface; such a procedure is illustrated and discussed in Appendix E. However, at present sufficient experimental data do not exist to prescribe conclusively which terms are appropriate, and only assumed pressure distributions are available as a basis for comparison. As more reliable and conclusive experimental data become available, future research along these lines may provide a means for extending the applicability of such an elasticity approach.

Numerical Approaches

Many of the foregoing restrictions and approximations, which are necessary in applying to practice the solutions obtained from elastic theory, are no longer required when numerical techniques are used in conjunction with an electronic computer. Drawsky (*11*) has proposed a mathematical model based on a segmented ring surrounded by a system of radial springs whose response is designed to simulate that of the soil, and the complex problem of determining the response of this system to an applied load may be solved to any desired accuracy by an iterative numerical procedure. Although the basic premise of the method is open to question, if this premise is acceptable and provided some minimum height of cover is exceeded, a wide variety of situations may be handled, including various culvert shapes, distribution with depth of pressures acting on the culvert, variations in the soil moduli, and concentrated loads according to the Boussinesq solution. Versatility enhances the desirability of this method, but the availability and cost of computer time may restrict its direct application to certain conditions.

An alternative numerical procedure, having even greater versatility than that discussed previously, is the finite element approach. This method has been used widely in recent years to obtain solutions to boundary value problems, and more recently initial value problems, and it is most adaptable to the two-dimensional plane strain condition that approximates the buried conduit problem. The soil and the culvert walls are replaced by an assemblage of discrete elements interconnected at nodal points, and these elements are assigned the material properties of the original continuum, thereby comprising a stiffness matrix that is used to determine displacements at the various nodal points. Based on these displacements, associated stresses are calculated. Two important advantages of the finite element technique are the ease with which irregular boundaries can be handled and the ability to assign different mechanical properties to any region of the fill, the underlying soils, or the culvert.

Brown (*12, 13*) has developed finite element programs for both rigid and flexible culverts under high fills, but these programs have not yet been refined to the point where they can handle culverts of intermediate stiffnesses. Also, it appears that considerable time and effort may be required to adapt the programs to handle changes in boundary conditions. Although the case of a shallow conduit is solvable by this technique, no work along these lines has been published to date. As with the spring analog approach, the greatest value of the finite element method appears to lie in the possibility of providing graphical data for a wide range of conditions that may be applied to the general culvert problem. At present, however, its usefulness is restricted to the more specialized high-cost projects.

SAFETY FACTOR

The entire concept of safety factor in engineering design is currently being re-evaluated. In the past the application of some factor to either the loading on a structure or its computed dimensions was considered to provide an adequate margin of safety against possible serviceability breakdown. However, the selection of such a factor is almost entirely an art, and, except in rare circumstances, there is little to guide the designer other than tradition, experience, and intuition. The role of experience cannot be denied, and it is, in fact, an essential component of virtually any satisfactory engineering design; nevertheless, despite the fact that experience can provide some measure of confidence that a sufficient degree of safety is afforded by a given choice of safety factor, rarely are there enough cases of failure for any one design situation to provide a reliable basis for establishing an intuitive ability to select such factors. More importantly, even the most experienced designer has no rational means by which he can adjust the applicable safety factor to reflect the influence of site conditions and material costs.

The conventionally accepted definition of safety factor is that quantity that relates the failure load to the known or assumed service load. As such, the currently used methods for the design of pipe culverts do not provide for a realistic assessment of either the actual safety of the structure or the reliability of the culvert response under an assumed service loading. This situation is due largely to the inadequate definition of the supporting strength of a culvert and to our lack of knowledge of the exact nature of the loading on the system. The inconsistency of current design methods is illustrated by the fact that there are cases where the recommended value of the safety factor to be used in the design calculation is unity; by definition this would imply a state of imminent collapse when the service load is applied to the culvert. Because such a situation does not normally occur, the design engineer is confronted with a factor-of-safety use that is unrealistic, inconsistent, and meaningless.

In an effort to establish a rational basis for evaluating the safety factor of culverts, a new concept is presented and discussed in Appendix E. This concept is consistent with the general definition of safety factor, as understood by the structural engineering profession, and it is relatively easy to apply. Furthermore, it is valid for both rigid and flexible conduits and has the capability of determining the safety of a culvert against all possible modes of failure. The essence of this concept is that the safety factor of a conduit be defined as the ratio of the intensities of two normal (or radial) loadings, each of which produces only an axial stress resultant on each cross section of the culvert. Details for evaluating those two loading conditions are outlined in Appendix E, and an example that illustrates the application of the proposed concept is presented.

In a somewhat broader sense, the aspects that should be considered in providing some safety margin in the design of underground conduits are (1) the degree of certainty of the values for the material properties, (2) the degree of certainty of the load values, (3) the accuracy of the formulations used to determine the system response, (4) the cost of variations in the structure dimensions (or material property values), and (5) the cost (including intangibles) of a failure. The nature of these considerations suggests that probability theory be employed to study this problem. This approach has been used for a number of years in engineering design problems concerned with wind, rainfall, and earthquakes. Although it has not yet been applied specifically to the design of underground conduits, there is no reason why this cannot be done in a straightforward manner.

For any given design problem, the optimum design condition may be evaluated in terms of variations in the uncertainties associated with the design, as well as variations in the cost of material and the cost of possible failure. By treating a wide variety of hypothetical cases and determining for each the equivalent safety factor that would provide the optimum design, an empirical relationship between safety factor, uncertainties, and costs may be determined. Thus, the safety factor for any given conduit design problem could be readily determined. Another advantage of such an approach is that it would force the designer to consider formally such questions as "How accurately are the site conditions known?", "How much reliance can be placed on achieving the specified dry density of the fill?", "How accurate is the load estimate?", "What is the cost of pipe per foot?", and "What would be the cost of removing the pipe and replacing it if a failure occurred?"; most of these questions are presently given little, if any, formal attention in the design stage. Hence, a design procedure based on this approach to safety factor would shift the emphasis in the design process, and economic considerations would play a deservedly greater role.

CAMBER DESIGN

The weight of a highway embankment causes consolidation of underlying layers of compressible soil; because of the trapezoidal cross-sectional shape of highway embankments, the settlement is greatest beneath the central portion of the fill, decreasing appreciably toward the toes. If a culvert installed under such an embankment settles at any point along its length, the invert at that point may drop beneath its established grade; furthermore, differential settlements may cause longitudinal stresses in the culvert walls as a result of longitudinal beam action. The stresses caused by the beam action are at right angles to the stresses caused by the ring action, and the combination of longitudinal and ring stresses complicates the culvert design procedure. This beam action becomes highly significant when the culvert is rigid, less significant when the culvert is flexible, and much less significant when the culvert is of corrugated metal. Watkins (*14*) discusses the implication of the resultant stresses caused by beam action and ring action, together with their relative effect on flexible culverts; for practical design work, he suggests that beam stresses and ring stresses be analyzed separately.

To eliminate the source of such a condition, it is necessary to predict the settlement of the soil beneath the culvert. This may be accomplished by standard methods described in books on soil mechanics and foundation engineering, but such methods applied to culvert problems may lead to unjustifiable expenses and time delays. Hence, an approximate scheme for determining the anticipated consolidation settlements for different types of soil has been developed and is presented in Appendix F, together with several example problems. After the anticipated settlement profile is determined, economy and engineering judgment will dictate the adoption of one or more of the following steps:

1. Install the culvert with a camber so that settlement due to the load of the embankment will, in time, lower the culvert to approximately the desired grade.
2. Excavate some or all of the compressible soils and replace them with well-compacted soils.
3. Preload the area to induce the major portion of the settlement before the pipe is installed; tunneling may be considered under these conditions.
4. Select a culvert composed of short sections.
5. Maintain flexibility in the joint connections.

DURABILITY OF METAL CULVERTS

As discussed in Appendix G, no reliable means exists for accurately predicting the performance of a corrugated metal culvert in a given environment. The problem of corrosion of metal culverts is extremely complex and dependent on a variety of factors, and it is felt that existing studies that purport to have yielded correlations with these factors should be used only with extreme caution. Research directed at relating metal loss with the various physical and chemical properties of the soil at a given site has, to date, provided only very limited results. For example, although there appears to be sufficient evidence to suggest limiting cases for metal loss in culvert applications on the basis of pH and water velocity, it is felt that no sufficient correlation has been found for other properties, and for the present time there exists doubtful justification for their measurement on a design project.

The following suggested design method is based directly on a statistical evaluation of past performance. Prior to design, the engineer should obtain the culvert flow velocity at peak design flow, the pH of soil and water at the site under normal climatic conditions, the desired culvert life, and a percentage value that represents the degree of importance (based on economic and other considerations) that the culvert reach its desired life span. Then, the design shall take into account the following considerations:

1. Where the peak flow velocity is excessive and the water contains significant amounts of sediment, allowance should be made for abrasion.
2. Where the normal water pH is less than 4.5, concrete culverts should generally be used. Although asbestos-bonded and bituminous-coated steel and stainless steel have been known to show high resistance to corrosion, further field experience is necessary before their use can be recommended.
3. Where the water pH exceeds 4.5, it may be desirable to provide an additional metal thickness to allow for corrosion. In the absence of local information, the method described in "Durability of Corrugated Metal Culverts," by J. E. Haviland, P. I. Bellair, and V. D. Morrell, Department of Transportation, State of New York (1967) is recommended (see Fig. G-4).
4. Although long-term results are not available, short-term results indicate that aluminum culverts are suitable within the pH range of 4.5 to 9.
5. When concrete culverts are to be exposed to chlorides (deicing salts) and sulfates (coal-mine drainage), special considerations should be observed during pipe production, such as the use of air-entrained concrete and sulfate-resistant cement, respectively.

COMPARISON OF DESIGN PROCEDURES

In view of the varied design procedures currently available, it is of significant interest to compare the resulting designs that are obtained by application of these procedures to given sets of conditions. Such a comparison will facilitate a direct appraisal of (1) the differences between the various procedures available, and (2) the effect of various design parameters on the final design. As mentioned several times throughout this report, the selection of quantitative values for many of these design parameters is largely a matter of engineering judgment, and it is instructive to obtain an idea of their influence on the final product. Hence, the primary objective of this section is to determine for a range of conduit diameters and heights of fill whether or not any significant differences exist for the situations described previously. For example, if all of the available design procedures lead to the selection of a similar pipe for a given set of design conditions, the technical differences between the various methods can be judged to be largely academic.

If several different structural designs are deemed equally acceptable, the ultimate decision becomes one of economics, and, for this reason, it would be desirable to make these comparisons on an economic basis. However, as explained in Appendix I, such a comparison is not feasible at this time and would not, in the opinion of the researchers, be meaningful. For example, the possible savings achieved by using a lower-class reinforced concrete pipe would have to be balanced against the increased cost of a higher-class bedding. Or the savings in gauge thickness for a corrugated metal pipe would have to be compared with the cost of high-grade, well-compacted backfill. Although comparative pipe costs are readily obtained, realistic figures for the associated labor charges are not generally available; furthermore, the latter would be expected to vary considerably with locality and with time. For this reason, the basic comparative design conditions and products are presented in the following tables, and the actual economic interpretation is left to the individual design engineer who is most familiar with local conditions.

Table 1

The structural design of a reinforced concrete pipe culvert requires a determination of the probable maximum load acting on the pipe and its distribution around the pipe periphery. Once this is known, the pipe wall may be designed by means of normal reinforced concrete design procedures. However, most current design procedures consider only the magnitude of the vertical load and select the pipe in accordance with strength classes given by ASTM C 76-68, wherein the strength is specified in terms of a D-load (three-edge bearing test load, expressed in pounds per linear foot per foot of diameter, to produce a 0.01-in. crack in the pipe).

The determination of loads for the examples given in Table 1 is accomplished by two procedures. One follows the Marston-Spangler theory with assumed values of 0, 0.5, and 1.0 for the settlement ratio, r_{sd}. Although Spangler recommends the use of 0.33 for the lateral soil pressure coefficient, K, the results of computations for an additional K value of 0.50 are given to illustrate the role of this factor in the final design. The relative effects of Class A, B, and C bedding are also indicated.

The other design procedure given in Table 1 is based on Olander's pressure distribution (Fig. E-4), which is based on Spangler's experimental results (Fig. E-5). In this method, the stresses in the pipe are determined as a function of the bedding angle; Table 1 gives the results for bedding angles of 90°, 45°, and 30°, which correspond to some extent with bedding Classes A, B, and C, respectively. However, the correspondence between Class A bedding and a 90° bedding angle is questionable because the pressure distribution for a pipe bedded in a concrete cradle is considerably different from that assumed by Olander.

For pipe diameters of 2, 5, and 8 ft and heights of fill of 5, 25, and 50 ft, the required D-load strengths, based on a safety factor of 1.0 and using a fill unit weight of 120 lb per cubic foot, and the selected pipe classes are given in Table 1 for bedding Classes A, B, and C. Where the required D-load strength exceeds 3,000 lb per linear foot (Class V pipe), the symbol >V is used to indicate that a special pipe must be provided.

In using the Olander pressure distribution and its associated design procedure, it is assumed that the ratio of the inside diameter to the outside diameter of the pipe is constant and equal to an average value of 0.825; the actual range for practical pipe sizes was found to vary from 0.80 to 0.85. As a consequence of this assumption, the stresses in the pipe wall are independent of the pipe diameter. The maximum tensile stress, σ_{max}, in the pipe walls was computed, and, based on a safety factor of 1.0, the D-load strength was obtained by the relation σ_{max} (psi) $= 0.697$ D-load (plf), given in Appendix B. Although the Olander method does not allow for soil and conduit compressibility (r_{sd}) or variation in the lateral soil pressure (K), those soil and conduit parameters are incorporated in the assumed pressure distribution.

The following findings are drawn from the data given in Table 1:

1. For deeply buried pipes with a given class of bedding, the required D-load strength varies only slightly with the pipe diameter.
2. The required strength increases with an increase in r_{sd}, generally doubling as r_{sd} increases from zero to unity.
3. Increasing the assumed lateral earth pressure on the pipe by increasing K from 0.33 to 0.50 causes the required pipe strength to decrease by 10 to 30 percent. Because a concrete pipe is relatively rigid compared to the surrounding soil, the actual value of K may be greater than 0.5 for certain soils, thus reflecting some of the inherent conservatism in the Marston-Spangler method.
4. For low height of fill, Class I pipe seems to be adequate for a wide range of soil parameters, pipe diameters, and bedding classes or bedding angles. Hence, it appears that, except for live loads, refinement of the design procedure for pipes with low cover heights is not warranted.
5. The soil parameters and the bedding conditions have a considerable influence on the design of pipes under moderate and high fills.

TABLE 1

COMPARISON OF PROCEDURES AND INFLUENCE OF VARIABLES FOR CONCRETE PIPE DESIGN

Height of fill (ft)		5.0						25.0						50.0					
Lateral soil pressure coefficient, K		0.33			0.50			0.33			0.50			0.33			0.50		
bedding class/ bedding angle	pipe diam. (ft)	settlement ratio, r_{sd}						settlement ratio, r_{sd}						settlement ratio, r_{sd}					
		0.0	0.5	1.0	0.0	0.5	1.0	0.0	0.5	1.0	0.0	0.5	1.0	0.0	0.5	1.0	0.0	0.5	1.0
A	2	127/I	212/I	251/I	83/I	168/I	208/I	650/I	1089/III	1283/III	446/I	896/II	1096/III	1330/III	2237/V	2636/V	926/II	1803/IV	2203/V
	5	107/I	192/I	192/I	52/I	134/I	134/I	633/I	1053/III	1241/III	430/I	840/I	1026/III	1299/III	2169/V	2567/V	893/II	1737/IV	2133/V
	8	79/I	149/I	149/I	17/I	84/I	84/I	611/I	1012/III	1182/III	393/I	802/I	972/II	1359/III	2259/V	2597/V	853/II	1735/IV	2112/IV
90°	●	183/I						913/II						1825/IV					
B	2	232/I	349/I	410/I	197/I	318/I	375/I	1186/III	1800/IV	2069/V	1038/II	1659/IV	1936/IV	2390/V	3651/>V	4213/>V	2100/V	3338/>V	3890/>V
	5	219/I	339/I	339/I	180/I	297/I	297/I	1172/III	1765/IV	2026/V	1024/II	1603/IV	1874/IV	2372/V	3585/>V	4138/>V	2076/V	3266/>V	3825/>V
	8	194/I	290/I	290/I	149/I	242/I	242/I	1149/III	1713/IV	1953/IV	989/II	1558/IV	1800/IV	2343/V	3580/>V	4100/>V	2025/V	3262/>V	3787/>V
45°	●	247/I						1233/III						2465/V					
C	2	256/I	423/I	494/I	207/I	375/I	444/I	1333/III	2165/V	2521/V	1115/III	1945/IV	2290/V	2703/V	4318/>V	5055/>V	2256/V	3915/>V	4633/>V
	5	234/I	373/I	373/I	174/I	312/I	312/I	1322/III	2155/V	2465/V	1087/III	1939/IV	2243/V	2667/V	4324/>V	5010/>V	2231/V	3886/>V	4596/>V
	8	199/I	253/I	253/I	125/I	182/I	182/I	1283/III	1904/IV	2550/V	1037/III	1887/IV	2340/V	2622/V	4287/>V	4992/>V	2213/V	3857/>V	4538/>V
30°	●	278/I						1388/IV						2775/V					

Notes: Arabic numerals indicate the required pipe strength (D-load) in pounds per linear foot

Roman numerals indicate the required pipe class : I = 800 lbs/lin.ft ; II = 1000 lbs/lin.ft ; III = 1350 lbs/lin.ft ; IV = 2000 lbs/lin.ft ; V = 3000 lbs/lin.ft

● Values shown are based on Olander's pressure distribution and are independent of the pipe diameter.

Unit weight of fill equals 120 pounds per cubic foot

Table 2

The influence of various parameters on Spangler's deflection determination for corrugated metal culverts is given in Table 2. In the computations the deflection lag factor, D_l, is taken as unity in order to facilitate subsequent comparison with other design methods, and the modulus of soil reaction, E', is assigned values of 700, 1,400, and 7,000 psi. For culverts under high fills, an E' value of 700 psi is very low, even for soils placed at 85-percent relative compaction. Although a value of 1,400 psi is normally recommended for excellent compaction, much higher values can often be expected, especially under high overburden confining pressures, and a value of 7,000 psi is used to illustrate a possible field situation. The required gauges, which are presented only for comparison, are determined from the ring compression theory by using a yield strength of 33,000 psi and employing safety factors of 2 and 4.

The effects of different bedding conditions are not considered in this comparative study. Although it is probable that a uniform backfill around the pipe results in the most desirable pressure distribution, a bedding angle of 45° is most likely obtained for an average installation; hence, a bedding factor, K, of 0.1 was assumed in the computations.

Based on the results given in Table 2, the following conclusions can be advanced:

1. The modulus of soil reaction, E', substantially controls the conduit deflection because the stiffness of the pipe has little effect.
2. When the settlement ratio, r_{sd}, which reflects the relative soil and culvert compressibilities, is reduced from 0 to —0.5, the conduit deflection is reduced by about one-third.
3. There are several pipes that satisfy the 5-percent deflection criterion, but do not satisfy other essential criteria, such as handling.

Table 3

Table 3 gives an evaluation of some of the factors controlling the design of corrugated metal culverts; although deflection is certainly one criterion, this is treated in Table 2. The order of the analyses given in Table 3 is as follows:

1. The ring compression load is determined from the ring compression theory.
2. The required gauge of the conduit is determined by assuming a yield stress of 33,000 psi and employing safety factors of 2 and 4.
3. The handling criterion is checked by determining the flexibility factor; this factor should be less than some empirical value for a given pipe corrugation.
4. The buckling criterion is checked by comparing the compressive stress in the pipe walls for the appropriate factor of safety with the allowable buckling stress, determined in accordance with the Bureau of Public Roads' criteria (*42*) and based on a safety factor of 2.

A study of these data led to the following observations:

1. For small-diameter pipes under virtually all heights of cover, at least up to 50 ft, the smallest available gauge is satisfactory, and durability and handling criteria normally will control the design.
2. For the 5-ft-diameter pipes, handling and ring compression generally control the design; for example, in the case of a pipe with 2⅔ × ½-in. corrugations, handling alone requires at least 12-gauge thickness, as given in Table 4; Table 3 indicates that a 16-gauge thickness is satisfactory, except under 50 ft of fill and a safety factor of 4. Although Table 3 indicates that buckling is critical under greater fill heights, both experience and recent theoretical study (see Appendix C) do not substantiate this conclusion.
3. For an 8-ft-diameter, 3- × 1-in. corrugation pipe under 50 ft of fill, it is of interest to note that a 12-gauge thickness is required. However, neither the deflection nor the buckling requirements are satisfied for poorer quality fills, and it is essential that high standards of compaction be maintained.

Table 4

Table 4 has been prepared as a means for comparing the results of several different methods for designing circular flexible culvert pipes for a range of diameters and fill heights. Wall thickness in terms of pipe gauge and, where applicable, deflection (percent) were determined for each case in accordance with each method. Except for the use of a 16-gauge minimum wall thickness, no consideration was given to durability in the gauge selection. As nearly as possible, the same data were used for each design method. The pipe material selected for this comparison is corrugated steel with a yield stress of 33,000 psi, and the unit weight of the fill is taken as 120 lb per cubic foot. In cases where the computed pipe gauge was found to be less than that required by the handling criterion ($d^2/EI \leqq 0.0433$), the gauge in accordance with the latter was indicated in the table, and for cases where the required wall thickness was not available, either the next heavier available wall thickness was shown or no entry was made. Owing to the different theoretical bases for the various methods, it was necessary to make certain assumptions peculiar to each method; these are indicated briefly in the table, and they are discussed in greater detail, as follows:

1. As the Marston-Spangler method involves no circumferential stress failure criterion, the wall thickness was selected entirely by availability and the handling criterion ($d^2/EI \leqq 0.0433$). Although the establishment of this criterion is attributable to neither Marston nor Spangler, its use seems appropriate for this comparison in the absence of some other criterion. In the computation of deflection, the deflection lag factor, D_l, was taken as unity; the bedding constant, K, as 0.083; the vertical pressure, W_c, as the weight of the overburden, γHd; and the modulus of soil reaction, E', as 1,400 psi.
2. For the ring compression theory by White and Layer, two sets of computations have been made, one based on a safety factor of 4 and a second based on a safety factor of 2.
3. Use of the Meyerhof method requires the determination of a coefficient of soil reaction, e, and the particular equation to be used for this determination depends on

TABLE 2

EFFECT OF DESIGN VARIABLES ON DEFLECTION OF CORRUGATED STEEL PIPES (IOWA FORMULA)

Pipe diameter (ft)	Settlement ratio, r_{sd}	E' (psi)	Factor of Safety	Height of Fill, H (ft)								
				5			25			50		
				Corrugation (in)								
				2⅔ × ½	3 × 1	6 × 2	2⅔ × ½	3 × 1	6 × 2	2⅔ × ½	3 × 1	6 × 2
2	0.0	700	2	0.6 – 16●			2.8 – 16			5.5* – 16		
			4	0.6 – 16			2.8 – 16			5.5* – 16		
		1400	2	0.5 – 16			2.4 – 16			4.9 – 16		
			4	0.5 – 16			2.4 – 16			4.9 – 16		
		7000	2	0.1 – 16			1.5 – 16			1.0 – 16		
			4	0.1 – 16			1.5 – 16			1.0 – 16		
	-0.5	700	2	0.4 – 16			1.9 – 16			3.8 – 16		
			4	0.4 – 16			1.9 – 16			3.8 – 16		
		1400	2	0.4 – 16			1.7 – 16			3.4 – 16		
			4	0.4 – 16			1.7 – 16			3.4 – 16		
		7000	2	0.1 – 16			0.3 – 16			0.7 – 16		
			4	0.1 – 16			0.3 – 16			0.7 – 16		
5	0.0	700	2	0.9 – 16	0.7 – 16		4.5 – 16	3.5 —		9.0* – 16	7.0* – 16	3.8 —
			4	0.9 – 16	0.7 – 16		4.5 – 16	3.5 – 16		9.0* – 10	7.0* – 10	3.8 – 10
		1400	2	0.5 – 16	0.4 – 16		2.3 – 16	2.0 —		4.7 – 16	4.1 – 16	2.7 —
			4	0.5 – 16	0.4 – 16		2.3 – 16	2.0 – 16		4.7 – 10	4.1 – 10	2.7 – 10
		7000	2	0.1 – 16	0.1 – 16		0.5 – 16	0.5 —		1.0 – 16	0.9 – 16	0.8 —
			4	0.1 – 16	0.1 – 16		0.5 – 16	0.5 – 16		1.0 – 10	0.9 – 10	0.8 – 10
	-0.5	700	2	0.8 – 16	0.6 – 16		3.1 – 16	2.4 —		6.2* – 16	4.8 – 16	2.6 —
			4	0.8 – 16	0.6 – 16		3.1 – 16	2.4 – 16		6.2* – 10	4.8 – 10	2.6 – 10
		1400	2	0.4 – 16	0.4 – 16		1.6 – 16	1.4 —		3.2 – 16	2.8 – 16	1.9 —
			4	0.4 – 16	0.4 – 16		1.6 – 16	1.4 – 16		3.2 – 10	2.8 – 10	1.9 – 10
		7000	2	0.1 – 16	0.1 – 16		0.3 – 16	0.3 —		0.7 – 16	0.6 – 16	0.6 —
			4	0.1 – 16	0.1 – 16		0.3 – 16	0.3 – 16		0.7 – 10	0.6 – 10	0.6 – 10
8	0.0	700	2	0.9 – 16	0.9 – 16		4.7 – 16	4.5 – 16	3.2 —	9.3* – 12	9.0* – 12	6.4* – 12
			4	0.9 – 16	0.9 – 16		4.7 – 12	4.5 – 12	3.2 – 12	9.3* —	9.0* —	6.4* – 7
		1400	2	0.5 – 16	0.5 – 16		2.4 – 16	2.3 – 16	1.9 —	4.8 – 12	4.7 – 12	3.9 – 12
			4	0.5 – 16	0.5 – 16		2.4 – 12	2.3 – 12	1.9 – 12	4.8 —	4.7 —	3.9 – 7
		7000	2	0.1 – 16	0.1 – 16		0.5 – 16	0.5 – 16	0.5 —	1.0 – 12	1.0 – 12	0.9 – 12
			4	0.1 – 16	0.1 – 16		0.5 – 12	0.5 – 12	0.5 – 12	1.0 —	1.0 —	0.9 – 7
	-0.5	700	2	0.8 – 16	0.7 – 16		3.5 – 16	3.4 – 16	2.4 —	6.5* – 12	6.3* – 12	4.5 – 12
			4	0.8 – 16	0.7 – 16		3.5 – 12	3.4 – 12	2.4 – 12	6.5* —	6.3* —	4.5 – 7
		1400	2	0.4 – 16	0.4 – 16		1.8 – 16	1.7 – 16	1.4 —	3.3 – 12	3.2 – 12	2.7 – 12
			4	0.4 – 16	0.4 – 16		1.8 – 12	1.7 – 12	1.4 – 12	3.3 —	3.2 —	2.7 – 7
		7000	2	0.1 – 16	0.1 – 16		0.4 – 16	0.4 – 16	0.3 —	0.7 – 12	0.7 – 12	0.6 – 12
			4	0.1 – 16	0.1 – 16		0.4 – 12	0.4 – 12	0.3 – 12	0.7 —	0.7 —	0.6 – 7

Notes: 0.6 – 16● indicates the percent deflection and the required gage, respectively.
* resulting deflection exceeds the 5 percent criterion.
unit weight of fill equals 120 pounds per cubic foot.
blanks indicate not applicable.

TABLE 3

EFFECT OF VARIOUS DESIGN CRITERIA ON GAUGE SELECTION FOR CORRUGATED STEEL PIPES

Pipe diam. (ft)	Height of fill, H (ft)		5			25			50		
	Corrugation (in)		2⅔ × ½	3 × 1	6 × 2	2⅔ × ½	3 × 1	6 × 2	2⅔ × ½	3 × 1	6 × 2
2	Ring compression, T (lb/ft)		600			3000			6000		
	required gage	factor of safety 2	16			16			16		
		factor of safety 4	16			16			16		
	flexibility factor $d^2/E{\cdot}I$	factor of safety 2	0.010			0.010			0.010		
		factor of safety 4	0.010			0.010			0.010		
	compressive stress (psi)	factor of safety 2	774			3870			7740		
		factor of safety 4	774			3870			7740		
	allowable buckling stress (psi)	backfill good	16500			16500			16500		
		backfill excell.	16500			16500			16500		
5	Ring compression, T (lb/ft)		1500			7500			15000		
	required gage	factor of safety 2	16	16		16	16		16	16	
		factor of safety 4	16	16		16	16		8	10	10
	flexibility factor $d^2/E{\cdot}I$	factor of safety 2	0.064	0.014		0.064	0.014		0.064	0.014	
		factor of safety 4	0.064	0.014		0.064	0.014		0.027	0.006	0.002
	compressive stress (psi)	factor of safety 2	1933	1680		9680	8420		19330	16830	
		factor of safety 4	1933	1680		9680	8420		8600	7460	7490
	allowable buckling stress (psi)	backfill good	7750	16500		7750	16500		7750	16500	16500
		backfill excell.	16500	16500		16500	16500		16500	16500	16500
8	Ring compression, T (lb/ft)		2400			12000			24000		
	required gage	factor of safety 2	16	16		16	16		12	12	12
		factor of safety 4	16	16		12	12	12			7
	flexibility factor $d^2/E{\cdot}I$	factor of safety 2	0.163	0.035		0.163	0.035		0.090	0.020	0.005
		factor of safety 4	0.163	0.035		0.090	0.020	0.005			0.003
	compressive stress (psi)	factor of safety 2	3095	2680		15500	13500		17700	15400	15400
		factor of safety 4	3095	2680		8850	7700	7700			8760
	allowable buckling stress (psi)	backfill good	7000	**12000**		7000	12000	16500	7000	12000	16500
		backfill excell.	25000	16500		25000	16500	16500	25000	16500	16500

Notes: Factors of safety are based on yield stress of 33000 pounds per square inch.
Handling criterion is indicated by empirical maximum values for the flexibility factor,

corrugation :	2⅔ × ½	3 × 1	6 × 2	[in]
flexibility factor :	0.0433	0.0433	0.0200	$[in^2/(psi \cdot in^4/in)]$

Unit weight of fill equals 120 pounds per cubic foot.
Blanks indicate not applicable.

TABLE 4

GAUGES AND DEFLECTIONS OF CORRUGATED STEEL PIPES BASED ON VARIOUS DESIGN METHODS

	Design Method	Pipe Diam. (ft)	Height of Fill, H (ft): 5			25			50		
			Corrugation (in)								
			$2\frac{2}{3}\times\frac{1}{2}$	3×1	6×2	$2\frac{2}{3}\times\frac{1}{2}$	3×1	6×2	$2\frac{2}{3}\times\frac{1}{2}$	3×1	6×2
Wall Thickness (Gage)	Marston-Spangler handling criterion: $d^2/EI < 0.0433$	2	16			16			16		
		5	12	16	12	12	16	12	12	16	12
		8		16	12		16	12		16	12
	White and Layer $f_y = 33000$ psi factor of safety, F.S. = 4	2	16			16			16		
		5	12	16	12	12	14	12	8	10	10
		8		16	12		12	12			5
	White and Layer $f_y = 33000$ psi factor of safety, F.S. = 2	2	16			16			16		
		5	12	16	12	12	16	12	12	14	12
		8		16	12		16	12		12	12
	Meyerhof $f_y = 33000$ psi $e = 1500/(1.5\cdot r)$	2	16			16			16		
		5	12	16	12	12	16	12	12	12	12
		8		16	12		16	12			8
	Watkins $f_y = 33000$ psi factor of safety, F.S. = 2	2	16			16			16		
		5	12	16	12	12	16	12	12	12	12
		8		16	12		14	12		10	10
	Burns and Richard $f_y = 33000$ psi factor of safety, F.S. = 2	2	16			16			16		
		5	12	16	12	12	16	12	12	14	12
		8		16	12		16	12		12	12
Deformation (Percent)	Marston-Spangler $E' = 1400$ psi	2	0.2			1.0			2.1		
		5	0.4	0.3	0.2	1.9	1.6	1.1	3.9	3.4	2.3
		8		0.4	0.3		1.9	1.7		3.9	3.4
	Watkins $M^* = \frac{1400\cdot144}{1.5} = 134500$ psf $K_o = 0.5$	2	0.4			1.8			3.7		
		5	0.5	0.5	0.5	2.0	2.0	2.0	3.8	3.8	3.8
		8		0.6	0.6		2.1	2.1		3.8	3.8
	Burns and Richard $M^* = \frac{1400\cdot144}{1.5} = 134500$ psf $\nu_s = 0.4$; full slip	2	0.3			1.2			2.3		
		5	0.7	0.5	0.2	2.7	2.0	1.0	5.1	3.9	2.0
		8					2.8	2.1		5.2	4.0

Notes: Blank indicates not applicable

Unit weight of fill equals 120 pounds per cubic foot whether sand or clay is used for the fill material. In this case, a clay fill was selected, and e was determined by use of the equation $e = K_c/1.5r$, in which r is the culvert radius and K_c is a constant of soil reaction; a value of 1,500 psi was used for K_c. This method provides for a safety factor of 2 based on the yield stress.

4. In the Watkins approach, a safety factor of 2 was applied, and a K_o (ratio of free field horizontal stress to free field vertical stress) value of 0.5 was used. For computation of deformation, the constrained soil modulus was taken as 134,500 lb per square foot, the value equivalent to an E' value of 1,400 psi.

5. For the Burns and Richard approach, the constrained soil modulus was again taken as 134,500 lb per square foot, the full slip condition was assumed, Poisson's ratio for the soil was chosen to be 0.4, and a safety factor of 2 was used.

A review of the data in Table 4 indicates that the different design methods generally provide similar results except where the fill height is large. This similarity is not due entirely to the equivalence of the theoretically determined wall stresses; in fact, computations indicate a greater variation in stresses than is apparent from the table. The important conclusion that may be drawn, however, is that ring stresses, to a large extent, are overshadowed by other practical considerations such as the handling criterion. This is best seen by comparing the gauge determined from each method with those determined from the Marston-Spangler method, for which ring stresses are not considered and pipe gauges are based entirely on handling or availability considerations. Some difference in the required gauge did occur for the 8-ft-diameter pipe under the 50-ft height of fill. The greatest thickness was indicated when a safety factor of 4 was used in conjunction with the ring compression theory. In their original presentation of this theory, White and Layer recommended a safety factor of 2 for carefully controlled and well-engineered installations and 4 for average backfilling practice. The necessity for the more careful field control to enable the use of a lower factor of safety, particularly under these conditions, is aptly illustrated. Other deviations are found in the results from the Meyerhof approach, which includes some consideration of buckling (not considered by the other methods), and the Watkins method, which is more conservative in determining the circumferential thrust.

With regard to pipe deformation, it is interesting to note that, although good agreement is obtained among the various methods for the 5-ft-diameter 3- × 1-in. corrugation pipe at all fill heights, the influence of variations in pipe diameter and wall stiffness appears to follow different patterns for each of the three methods. However, the influence of these variables on the results is considerably less for the Watkins method than for the Spangler method, the former being almost negligible; on the other hand, the Burns and Richard method indicates a greater influence of pipe diameter and flexibility variation.

CASE STUDIES OF CONDUIT FAILURES

Twelve separate studies, presented in Appendix J, of actual failures of both rigid and flexible conduits provide considerable insight into possible weaknesses of the design and construction techniques that have been used for the past 20 years or so. These and similar studies indicate that most structural difficulties associated with buried conduits arise from the inability of the pipe foundation to furnish the desirable distribution of reaction forces; for example, either the proximity of rock or improper shaping of the bedding prior to placement of the pipe may lead to stresses or deformations that considerably exceed the design values. Also, in many instances there is considerable doubt regarding the adequacy of compaction to provide the necessary lateral resistance at the sides of the pipe. Although inadequate inspection and field control are largely responsible for this situation, the designer may share this responsibility in cases where specifications are inadequate or loosely written. In general, however, it appears that the principal reasons for culvert failures lay not in the inadequacy of the design procedures but in the improper coordination between the assumptions used in the design and the actual construction conditions and techniques.

CHAPTER THREE

INTERPRETATIONS AND RECOMMENDATIONS

This chapter is organized into three sections. The first presents the general observations that have been gleaned throughout the course of this investigation; these are not meant to be conclusions or suggestions, but simply objective observations gathered from reviewing the literature and from discussing culvert problems with a large variety of individuals. The second section gives a concise interpretation and appraisal of the current philosophy and prevailing trends underlying culvert design; in addition, several general suggestions are discussed. The final section contains a summary of recommendations that are advanced on the basis of this study and evaluation; these include short-term recommendations for immediate implementation and long-term recommendations directed toward guiding future research.

GENERAL OBSERVATIONS

Based on a literature review, extensive discussions, and a comprehensive study and evaluation of currently used procedures for the analysis, design and construction of pipe culverts, the following observations, most of which are substantiated in the appendices of this report, are advanced:

1. The fundamental work that was conducted 40 or 50 years ago at Iowa State University and that forms the basis for a large majority of present-day culvert design practice deserves a great deal of well-earned praise, especially when placed in context. Indeed, the theories developed have withstood the critical test of time and have accumulated a broad background of experience that cannot be readily discounted. On the other hand, the fact that no substantial changes have been made over this period may indicate that considerable change is overdue.
2. Despite the measure of success realized by use of these procedures, several of the assumptions underlying their formulation are subject to question; these include the assumption of vertical failure surfaces in the soil adjacent to the culvert, the use of active earth pressures and total shear stresses to calculate the portion of the overburden load carried by the soil columns adjacent to either side of the culvert, the application of the same procedure to calculate loads acting on both rigid and flexible culverts, and many others.
3. Although the design procedures in use today are claimed to be theoretically sound, they are nevertheless strongly empirical and depend extensively on experience and the exercise of engineering judgment.
4. Current procedures depend heavily on a variety of "intermediate" or "lumped" parameters, instead of fundamental properties of the soil-culvert system. Examples include the modulus of soil reaction, which cannot be evaluated directly, but only correlated with results of other tests, and the settlement ratio, which groups together the relative compressibilities of the individual components of the system. The selection of values for these parameters involves substantial engineering judgment and constitutes a disadvantage to the use of the associated theories.
5. Although the mechanical properties of culvert materials can be specified within reasonably small tolerances, the mechanical properties of the surrounding and underlying soils present a more formidable challenge; accordingly, the accuracy and reliability of any design procedure, however well founded, will be limited by the ability to characterize the soil stress-strain-time behavior.
6. In view of the scarcity of failures attributable to design shortcomings and the fact that failures do not normally entail loss of life, it appears that current safety factors can be reduced; however, the application of current procedures and criteria render it extremely difficult to quantify the degree of present conservatism.
7. It seems that most culvert design engineers do not appreciate the potential supporting capacity of the soil surrounding a pipe; if it is sufficiently flexible, the conduit itself simply does not have to be designed to carry a load greater than the free-field overburden stress. Some tunnel engineers are now recognizing this and are effecting considerable economic savings by appropriate design of tunnel linings.
8. Most current analysis and design procedures do not take full advantage of the greatly increased reservoir of analytical tools that are available. In this regard, it appears that the chances for achieving substantial improvements in the present state of the art lie in applying these analytical tools to the over-all soil-culvert interaction problem and not in modifying some parameters associated with the existing method.
9. Despite continued research leading to the improvement of criteria for selecting certain parameters in current design procedures, there is often a delay and even an apparent reluctance in some cases to incorporate this knowledge into actual practice.
10. No widespread acceptable procedure exists for the analysis and design of shallow culverts. This is especially significant in light of the fact that construction loads in many cases are the severest loads to which a culvert will be subjected during its service life. Although it may not be feasible to design the culvert for these construction loads, an acceptable criterion is required in order to impose realistic limitations on such loads. At present, empirical criteria are usually used, and generally the burden of this responsibility is passed on to the contractor.
11. There are currently a significant number of culvert design decisions that are based almost entirely on experience or policy, to the exclusion of any rational or theoretical justification. In general, such an approach is not inherently bad, but there is a distinct indication that such procedures are not subject to review very often.
12. Notwithstanding all efforts to the contrary, an uncomfortably large gap exists between the design of a culvert and the construction of the same culvert; despite admitted design deficiencies, adequate construction control is difficult to achieve in many cases.
13. Proper bedding and backfilling around the conduit are important to the response of soil-culvert systems; however, there are strong current opinions that bedding is more critical than backfilling in the case of a rigid pipe, and vice versa, in the case of a flexible pipe.
14. Small pipes are frequently installed with a minimum of design effort and construction control; in such cases local durability considerations often control the design.
15. There is an obvious lack of reliable, well-documented field data that can be used to validate new or existing theories. Most attempts to acquire such data are incomplete, improperly planned, or inadequately instrumented; frequently, for example, there is little or no record concerning the degree of compaction of the all-important fill surrounding the culvert or the condition of the underlying natural soils.
16. Although the three-edge bearing test offers a practical basis for determining the strength of reinforced concrete pipes, the desired criterion for field application should be based on a closer simulation of field loading.

17. Current safety factor concepts for both rigid and flexible pipes are confusing and ambiguous, generally inconsistent, and sometimes theoretically ill-founded; some of this misunderstanding arises from the inability to define "failure" in a completely acceptable manner. For example, in the case of concrete pipes under certain conditions, a safety factor of unity is used; this implies imminent failure, which is not the case. On the other hand, for flexible pipes a so-called safety factor of 4 is applied to limit the allowable deflection; however, owing to the nonlinear nature of the soil-culvert system at such large deflections, a simple linear interpolation is not theoretically correct.

18. As a result of contract bid procedures, especially such manipulations as unbalanced bids, and the difficulty of defining unambiguously the extent of a culvert installation, it is virtually impossible to readily obtain meaningful economic comparisons between various types of culverts.

INTERPRETATION AND APPRAISAL

Until recent years, reinforced concrete pipes generally have dominated the culvert industry. Owing to their rigidity, these pipes frequently have been subjected to loads considerably greater than the soil overburden, and the classical Marston theory was developed to calculate these loads. This theory gives the impression of utmost simplicity and logic when one is studying the highly idealized drawing that depicts the assumed failure mechanism; it is only when one attempts to apply the theory that major difficulties are encountered in evaluating in terms of standard test results some of the parameters that appear so straightforward on the diagram. As discussed elsewhere, a careful examination will lead to several objections in the theoretical derivation; but, despite these objections, this theory is used almost exclusively for the determination of loads acting on rigid culverts.

Load Redistribution by Conduit Deformation

Notwithstanding the popularly accepted concept of a rigid culvert, several engineers have questioned the necessity and desirability for providing rigidity in a culvert or similar structure and have suggested that it may be designed to deform, thereby effecting a favorable redistribution of loading so that it is resisted primarily by membrane stresses in the conduit wall. As an example, Terzaghi (*16*) proposed that a 20-ft-diameter tunnel in the Chicago subway system be constructed of an 8-in.-thick concrete shell lining instead of a more conventional 2- to 3-ft-thick structure. In addition, Lane (*17*) proposed a similar design concept after measuring a considerable reduction in structure loading following the installation of steel "hinges" in one of eight 30-ft-diameter reinforced concrete lined tunnels at Garrison Dam. Lum (*18*) discussed a possible design method and proposed the adoption of this flexibility concept to concrete culvert design.

At the opposite extreme, this advantage of load redistribution by conduit deflection has been exploited by the proponents of corrugated metal culverts. For example, on the basis of the results of extensive field tests, White and Layer (*9*) concluded that the circumferential thrust in a circular corrugated metal culvert may be determined by assuming that a hydrostatic load distribution, equal in magnitude to the free-field overburden stress, acts at the soil-culvert interface. Also, Appendix C shows that a similar result can be obtained theoretically for the case where a smooth-walled flexible conduit is buried in an infinite elastic medium.

As far as deformation is concerned, the major difference between the failure criteria for corrugated metal pipe and concrete pipe lies in the magnitude of the permissible deformation. For a corrugated metal pipe, a frequently used maximum deformation criterion is 5 percent of the conduit diameter, and this is related to the possible occurrence of failure by snap-through buckling, which is associated with excessive deformation. For a concrete pipe, buckling is not normally a consideration; a primary concern is the potential exposure of the reinforcing steel in the pipe, and the maximum deformation may be related to a maximum outer fiber strain. On the basis of a strain of 0.003 and other assumptions, Lum (*18*) found that the allowable conduit deformation is equal to the square of the pipe diameter divided by 1,200 times the pipe wall thickness. For the smaller diameter pipes with realistic wall thicknesses, the allowable deformations will be very small; for example, a 6-ft-diameter pipe with a 4-in. wall thickness would have a maximum allowable diameter change of 1.1 in., or about 1.5 percent; for the same diameter pipe with a 6-in. wall, the allowable diameter change would be 0.72 in., or 1.0 percent.

Load Redistribution by Use of Compressible Layer

A different approach to the redistribution of culvert loads was taken by Davis (*19*) and Davis and Bacher (*20*). Using field tests, they studied the effects on the load distribution of various arrangements of compressible materials adjacent to the culvert. By artificially inducing arch action, the loading was redistributed so that the ratio of the vertical load to the horizontal load was substantially reduced. Although such a redistribution should ideally produce membrane stresses in the conduit wall, experience has shown that considerable care is required in the design of the compressible layers if this desired loading is to be realized. In particular, some arrangements of compressible layers have led to serious load concentration problems. In addition, the compressible materials used for this purpose are often organic in nature, and they manifest a number of disadvantages, including the difficulty of specifying the compressibility characteristics and the possibility of decomposition, which may lead to uncertain and perhaps undesirable results. Some consideration is given in Appendix C to the design of an imperfect trench installation, and it is felt that the use of loose soil, rather than organic materials, though perhaps not so effective in reducing the load, would be more reliable in predicting performance, provided proper field control is maintained in the construction process.

Appraisal of Current Conservatism

When one is evaluating current design procedures, two of the most important aspects of the problem are economics and the likelihood of failure. It is noteworthy to observe that, in a field in which loss of human life is not at stake and in which heavy criticism may be justifiably aimed at the design procedures employed, few failures have occurred, and, where failures have occurred, they usually have been attributable to factors of an abnormal nature, rather than to deficiencies in design. One reason for this apparent conservatism stems from the difficulty in defining failure. For example, a 0.01-in. crack is often defined as failure for a reinforced concrete pipe, but this is a somewhat arbitrary criterion, and cracks of even 4 to 5 times that width do not normally lead to a failure of the soil-culvert system; in particular, the performance design criterion can normally far exceed this condition without serious consequences. In an analogous manner, it may be argued for a flexible pipe that stresses in excess of the yield stress of the pipe material are not a serious matter, but some design procedures consider such stresses as a failure criterion.

As previously discussed, any deformation that occurs, either tangentially in the culvert wall or in the form of a shape change, leads to a favorable change in the culvert loading, and any plastic deformation in the pipe is associated with an even greater reduction of loading. This means that the culvert load-deformation relationship determined in the laboratory is normally different from that manifested in the field, and this difference becomes considerably greater when the elastic limit is exceeded. Hence, the definition of failure based on the results obtained from a three-edge bearing test in the case of a rigid culvert or from a condition of yield in the case of a flexible metal culvert is questionable. This "fail-safe" type of phenomenon does not apply to buckling failure which, when applicable, may be of a catastrophic nature; however, as indicated in Appendix B, buckling is not usually a consideration in current practice.

Discussion of Design Considerations

Based on the foregoing arguments, it appears that culvert design has been extremely conservative and that considerable savings in cost may be possible. On the other hand, designers readily justify this conservatism by pointing out that proper field control of the culvert installation is rarely obtained. At this point, the question becomes one of obtaining an economical compromise. On one hand, it may be desirable to accept a field installation that is less than ideal and to overdesign the pipe accordingly; or, alternatively, it may be desirable to design the culvert under the assumption that a high degree of field control will be exercised to ensure proper construction. The choice between these two alternatives rests mainly on a comparison of the relative costs associated with each. For smaller diameter pipes under low fills, material savings probably will be small, and extensive field control probably will be relatively costly; however, improved construction techniques that may perhaps be specified by the designer may significantly alter this cost relationship. For example, one technique already used by contractors is to place the fill to an elevation above the top of the proposed pipe and then use a trenching machine with a suitable bucket width to excavate a trench for the pipe, which is installed in the appropriate manner. Another technique that exhibits excellent potential for the near future is to build the complete fill without the pipe and then install the pipe by means of a tunneling process; although this technique is currently available, it is probably too expensive to use except in unusual situations. However, with the widespread interest in tunneling machines, rapid developments may be expected. These procedures not only may mean a reduction in the cost of installation, but they also may permit a more economical design by virtue of providing a more reliable basis for load determination.

Although a considerable economic advantage can often be realized in cases where corrugated metal rather than concrete pipes are used, a major objection to the metal pipes has been the possibility of a shortened service life due to metallic corrosion and abrasion. Where any doubt existed, the aura of permanence associated with the reinforced concrete pipe frequently resulted in its preference. However, the use of various types of lining and paving has substantially improved the durability of metal pipes, and the introduction of aluminum culvert pipes has increased the use of flexible culverts somewhat. Hence, when selecting culvert materials, the designer should give serious consideration to corrugated metal for installations involving small pipes and low cover heights; provided durability criteria are satisfied, significant economy, even on a long-term basis, may result. If durability considerations indicate that corrugated metal is unacceptable, reinforced concrete is the normal alternative. Recently, synthetic pipe materials have become available, and these appear to offer attractive properties; however, their cost generally is high, and their durability properties are yet unproven. Provided these pipes become economically competitive and provided their durability properties are acceptable, they may be worthy of consideration. In order to promote the development of these materials, test installations are essential in field situations that are not critical; only in this manner can the effectiveness of such materials be quantitatively evaluated.

Despite the fact that current design methods certainly present some objections, they have long been compared with field results for small-diameter pipes under low fills, and it is difficult to reject established design procedures under these conditions. Yet, it is risky to use these same procedures for large culverts under very shallow or very deep fills. The appendices contain some suggestions for modifications to the parameters involved in these procedures; the use of these modifications in design may justify a possible reduction in safety factor. With the inevitable introduction of pipes of "intermediate" stiffness, the need arises for a design method that can handle a continuous range of pipe stiffnesses. One example of such a method, having a somewhat limited application, is given in Appendix C; more comprehensive and more versatile procedures can be developed by use of numerical techniques.

For culverts under high fills, the cost associated with a culvert installation justifies the use of a more sophisticated design procedure in addition to a high standard of field control. In this regard, it is notable that, if the soil exhibits some cohesion, tunnels that are sometimes constructed deep below the ground surface and through high fills can remain unsupported until weathering causes some breakdown in the soil structure. To avoid this in certain cases, a coating of lightly reinforced sprayed-on concrete has provided the necessary seal to ensure reasonable permanence of such a structure. These cases clearly illustrate the capacity of a competent compacted soil surrounding a nominally lined tunnel to support a large proportion, if not all, of the overburden weight. With the current rapid development of tunneling methods, it is conceivable that this approach may become feasible for normal culvert construction under high fills. Although stresses may be low initially, there is some possibility that creep may occur in the soil and cause these stresses to increase, with a corresponding increase in the ring stress in the pipe wall. Despite the fact that little is known about the relationship between the incidence of soil creep and the stress level in the soil for compacted fills, it is difficult to visualize how the stresses acting on the culvert could become very much greater than the associated free-field overburden stresses. Hence, unless laboratory tests on the fill provide some alternative data, the circumferential thrust can probably be determined with reasonable accuracy by use of the ring compression theory. However, as there is little likelihood that the stress will exceed far beyond this level, a very low safety factor seems permissible.

For culverts constructed in the conventional manner under high fills, a flexible corrugated metal structure may be entirely adequate; however, because of the higher total cost and the investment involved, the designer may more readily justify the need for the more durable reinforced concrete structure. If a corrugated metal pipe is used, the ring compression theory with a relatively small safety factor should be adequate for tangential thrust, but careful consideration should be given to deformation and buckling. It is believed that a stepwise use of the Iowa formula or the Burns and Richard solution, both of which are indicated in Appendix C, will provide a reasonable result, provided field control ensures that the assumed parameters are realized in the field. Excessive deformation may be prevented by using high-quality, low-compressibility fill adjacent to the culvert walls or by providing an imperfect trench installation, or both.

When a small-diameter reinforced concrete structure is used under a high fill, any consideration for reducing the pipe stiffness is usually unrealistic, and an imperfect trench installation at the present time cannot be constructed with sufficient reliability to design for membrane action. Hence, considerable bending resistance must be provided in the pipe, and the cost becomes correspondingly great. However, in many of these situations, the greater cost of a sufficiently rigid pipe may nevertheless be less than the cost of the labor involved in an imperfect trench installation, even if the latter could be reliably designed and constructed. Hopefully, in the near future, sufficient economy and control may be obtained from synthetic materials to justify their use where small-diameter pipes under high fills are required and corrugated metal structures are not permissible.

The experiences reported by Davis and Bacher (*20*) have indicated that serious problems may arise when one is using the imperfect trench for large-diameter reinforced concrete culverts under high fills. Also, Lane (*17*) has shown that small deformations may have a considerable influence on the load distribution at the soil-culvert interface. One major obstacle to this approach is that no established design method exists for such an "intermediate" stiffness problem. Although a very limited design procedure to handle such a problem is described and illustrated in Appendix C, it has not been substantiated by field evidence, it can be applied only to circular pipes, and it is not capable of considering the imperfect trench and its effect on the response. Alternatively, the design of such a structure may be accomplished by numerical techniques with either the spring analog (*11*) or the finite element method (*12, 13*). At present, neither method appears to have attained a sufficient degree of refinement, and considerable additional effort will be required to apply these methods to specific conditions. However, it may be possible to use thin-walled concrete structures in conjunction with the imperfect trench procedure and in so doing to account for inaccuracies of one design procedure by the advantages of the other. For example, an imperfect trench system may be designed to provide a uniform peripheral pressure while, at the same time, the culvert can be made sufficiently flexible to provide for possible nonuniformity.

The remaining case to be discussed is the large-diameter culvert under shallow fill. Little research has been done in the United States, and available data and design concepts are due largely to the efforts of Meyerhof (*21*) at Nova Scotia Technical College in Canada. This work, details of which are given in Appendix M, has been concerned almost exclusively with corrugated metal structures, and it has been found that, where large spans are involved, buckling stability becomes a major factor for consideration in design. Based largely on the results of model studies, formulae have been developed to predict allowable critical buckling stresses. Several impressive structures designed on the basis of this research have been completed—one a truncated ellipse spanning 40 ft. A considerable cost saving is possible by the use of corrugated metal in this manner, and additional research is likely to lead to broadened applications.

RECOMMENDATIONS AND FUTURE RESEARCH

Based on an extensive survey and evaluation of the theoretical and practical aspects associated with the structural analysis, design, and installation of pipe culverts, the following short-term and long-term recommendations are advanced. Short-term recommendations are interpreted to mean those for which there is felt to be sufficient research and experience to justify immediate implementation in the majority of situations; long-term recommendations consist of those that presently require additional basic research and experience before implementation, but which, in the opinion of the researchers, constitute the probable best approach to the culvert problem. Accordingly, included in this list are not only practical matters, but also suggestions for future research. Although these recommendations are considered to apply in the average case, undoubtedly special situations exist wherein they are decidedly inapplicable; such discretion can only be left to the exercise of good engineering judgment. This report and its appendices contain the explanations and justifications for these recommendations and several positive suggestions regarding their implementation.

Short Term

Durability, handling, and installation considerations play significant roles in the selection of culvert pipe materials, especially for smaller diameter pipes under low to moderate heights of fill; indeed, provided structural adequacy is assured, one or the other of these criteria will virtually control the design. Owing to the favorable redistribution of stresses, a flexible culvert normally will result in considerable cost savings, and this advantage will become greater with increased heights of fill. Although a flexible culvert may be constructed of either thin-wall concrete or corrugated metal, the former is yet to be developed fully and the latter is certainly much more common in current practice. However, corrugated metal pipes require more careful consideration of durability, and, in many cases, additional metal thickness or even protective coatings may be necessary. Alternatively, the use of metal may be entirely inadvisable, and the superior durability of concrete pipes may be needed. Frequently, the additional wall thickness required for the handling and installation of metal conduits provides sufficient metal (over and above that required for structural considerations) to satisfy durability requirements. If metal conduits are selected, it is recommended that design for corrosion resistance be in accordance with the statistical approach based on work by the New York State Highway Department. Under most commonly encountered conditions (see Appendix G), aluminum culverts seem to manifest durability characteristics which are superior to steel, and consideration of this aspect should be taken into account when making economic comparisons.

Smaller Diameter Pipes Under Moderate Heights of Fill

For smaller diameter pipes under moderate heights of fill, most currently used procedures (patterned after the work of Marston, Spangler, and White) are reasonably satisfactory, and their continued use with some modification to detail is considered justified where durability considerations permit. In computing the deformation of corrugated metal pipes by use of the Iowa formula, the use of more realistic higher values for the modulus of soil reaction is suggested, and certain experimental correlations of E' with other test parameters are presented in Appendix C. For cases where the cost of such auxiliary tests is not warranted, an approximate value for E' can be obtained from Figure C-4 with a knowledge of the as-compacted dry density; even though the validity of this correlation has not been conclusively substantiated, it seems to offer a considerable improvement over the arbitrary selection of values used currently. Although sound arguments may be presented to justify the use of a safety factor of 2 or less on the yield stress of a metal conduit, little cost advantage is realized by such a reduction because other considerations, such as handling, will normally control. In the case of rigid pipes, a wide range of values for the settlement ratio and the load factor does not change substantially the required pipe class, and further refinement of these parameters is unnecessary. However, in view of current conservatism in culvert design and the fact that backfill placement techniques have improved significantly in recent years, use of a higher value, at least 0.5, for the lateral earth pressure coefficient, K, is recommended for well-compacted backfill soils. Despite the fact that extraordinarily good field control is not normally required or economically justified for such installations, reasonable exercise of good construction practices, such as bedding and backfilling, should be required for both flexible and rigid pipes.

Larger Diameter Pipes Under Deep Fills

For larger diameter pipes under deep fills, a decision must be made between the use of existing design procedures and the application of numerical techniques; extrapolation of the former has not proven entirely satisfactory, and the latter are still in the preliminary stages of development and have not been fully verified. When such culverts are being designed, advantage should be taken of the potential supporting capacity of the soil surrounding the culvert, and either thin-shell lightly reinforced concrete or corrugated metal flexible structures, in conjunction with various combinations of imperfect trench and incompressible sidefill, should be considered instead of the massive reinforced concrete structures that result from conventional design procedures. Alternatively, normally reinforced concrete sections joined by "plastic hinges" may be contemplated, as indicated in Appendix K. In determining the deflections of such culverts, the nonlinearity of the soil modulus should be properly considered by the incremental application of either the elasticity approach or the Iowa deflection formula, both of which are discussed and illustrated in Appendix C.

For the larger diameter pipes, buckling stability becomes an important design criterion. In the past the relationship among pipe flexibility, pipe diameter, and modulus of the surrounding soil has been such that there was little need for consideration of buckling, but the increasing diameters of corrugated metal pipes have changed this situation. Luscher and Meyerhof have suggested means for determining the critical buckling stress in a soil-surrounded buried conduit, and these procedures are discussed in Appendices B and M. It is recommended that for large-diameter corrugated metal pipes buckling be considered by one of these methods.

Large-Diameter Shallow Conduits

Current design methods for large-diameter shallow conduits are generally unrefined and not well substantiated. Although the conventional design methods normally used for rigid concrete pipe frequently result in massive structures, no immediate modifications can be suggested. On the other hand, the economic advantage of flexible metal pipes is offset to some extent by the problems encountered in determining the load distribution and the nature of the response of the combined soil-culvert system. For the latter type of installation, buckling stability is a major consideration; largely on the basis of the results of model tests, Meyerhof has made considerable progress in both research and design, and it is recommended that these design methods, discussed in Appendix M, be used.

Longitudinal stresses in the conduit wall and possible joint separations should be minimized or eliminated by proper camber design, in addition to proper joint design. Accordingly, high-cost projects warrant a soils investigation in order to predict culvert settlements; for lower-cost projects the procedure outlined in Appendix F may be used.

Construction Practices

With regard to construction practices and control, both good bedding and good backfilling adjacent to the pipe are important to the performance of both flexible and rigid pipes; however, these requirements become more significant as the pipe diameter and height of fill increase to the point where structural considerations outweigh handling and durability considerations. For the more critical installations involving high fills, if the stiffness of the natural soil underlying a culvert varies substantially from that of the backfill material, such soils should be overexcavated to perhaps one-half the pipe diameter and replaced with compacted backfill material. For very stiff natural soils or rock, this requirement should be extended to about one pipe diameter. In such cases, bedding classes should be modified accordingly. It is admittedly good engineering practice to balance the economic advantages of weaker pipes with high-quality backfill versus stronger pipes with lower-quality backfill, but this compromise pertains to only the backfill material selection and the degree of compaction; it should not be construed to imply any reduction of construction control. If adequate field control is exercised to ensure that an intended design is being achieved in the field, lower safety factors may be justified. In light of currently available construction equipment, the following construction procedures may expedite the actual installation of a culvert while improving the reliability with which the loads acting on a culvert may be determined. As one possibility, consider placing the fill without the pipe to a level a few feet above the proposed crown of the pipe; then excavate a relatively narrow trench to receive the pipe, and properly place granular material around the sides of the pipe. Alternatively, consider completing the placement of the fill and installing the pipe by boring or tunneling through the compacted embankment; although this procedure is probably economically unfeasible at present, except for unusual situations, its effectiveness should increase rapidly in the next few years. For the larger diameter pipes under high fills, good construction control is an absolute necessity, and adequately paid, well-qualified, properly motivated inspectors offer the only reliable means currently available to ensure the implementation of this control.

Instrument Field Installations

To accumulate sufficient reliable data by which new or existing analysis and design procedures may be evaluated, it is essential that the following suggestions be implemented as soon as possible. First, invest the funds required to properly instrument and document several selected field installations, including some with large-diameter pipes of intermediate stiffness and some employing the imperfect trench concept; despite the high cost of such an activity and the general reluctance of a given highway department or agency to commit this investment, data acquired from this type of installation are necessary to substantially advance the current state of the art. In addition, more completely documented records of the actual field conditions, especially for the compacted soil adjacent to the culvert and the underlying natural soils, should be maintained for all culvert installations. Second, to obtain a rapid evaluation of new and promising synthetic materials, users must be sufficiently progressive to field-test these materials in noncritical situations under a variety of field conditions; this test program is particularly important in determining the long-term durability properties of new materials.

Long Term

Continuum Theories

The application of continuum theories, which handle in a unified manner the soil-culvert interaction effects, should be emphasized in future research, as opposed to the development or refinement of theories based on assumed failure mechanisms, such as sliding prisms or wedges. In addition to the foregoing advantage of treating the all-important coupling phenomenon, which implies an inherent ability to handle pipes of intermediate stiffnesses, such approaches have the advantage of including more fundamental characteristics of the various material properties. In addition, the continuum approach is more representative of the actual field situation. However, owing to mathematical complexities introduced by realistic physical considerations, such problems are generally not tractable in closed form at present, and recourse must be made to numerical methods of solution.

Numerical Procedures

The development of numerical procedures, especially the finite element approach, appears to offer considerable promise for the improvement of culvert analysis and design procedures. These methods should be refined to include pipe stiffness, nonhomogeneous soil conditions, and nonlinear soil behavior; the last will necessitate treatment in an incremental or stepwise manner. Although consideration of time effects introduces an additional refinement, it is felt that this aspect of the development is of lesser importance. In particular, numerical procedures may be expected to improve considerably the analysis and design of shallow culverts and deeply buried, large-diameter culverts. After adequately comprehensive computer programs to solve culvert problems are developed and verified by field evidence, they should be used to produce design graphs for a variety of commonly encountered field situations.

Acceptability Criteria

Increased efforts must be made to establish more realistic acceptability criteria for culvert installations. Although the three-edge bearing test may provide an adequate means for achieving quality control of reinforced concrete pipes, the 0.01-in. crack and the related load factors do not seem to offer a very desirable criterion by which such pipes can be designed. Instead, the actual load distribution on the pipe should be considered, and the design criterion should be related to the "limit of acceptability" of a field installation. Similarly, the reaching of the yield stress in a metal culvert does not necessarily imply "failure" of a pipe that is properly installed in an earth embankment. Testing should be performed in a manner that more closely simulates field conditions; then, based on the results of such experiments, it should be possible to define a realistic "limit of acceptability." The establishment of such a criterion is necessary to quantify more accurately the value assigned as a safety factor.

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APPENDIX A

SOIL-CULVERT INTERACTION

INTERACTION PHENOMENON

The loads on a buried conduit may stem from two primary sources—dead loads due to the earth overburden, and live loads due to the traffic passing over the conduit. The latter are significant only for shallow conduits with a height of cover of less than about 5 ft. The reaction to these loads is provided by means of arching in the soil and the resistance offered by the conduit. In a general way, arching in a soil-culvert system is the mechanism by which the free field stress in the soil surrounding a conduit is redistributed away from or onto the culvert. Depending on the relative compressibility of the culvert and the surrounding soil, positive or negative arching may occur. Positive arching results when the compressibility of the conduit is greater than that of the surrounding soil, and the load on the culvert is less than the free field load; negative arching occurs for the opposite conditions, and the culvert load is greater than the freefield load. Thus, the arching effects in the soil and the compressibility of the conduit are not independent of each other. In general, the response of the conduit depends on the characteristics (geometry, stiffness) of the pipe, the characteristics (geometry, order of placing, mechanical properties) of the adjacent and overlying compacted fill, and the characteristics (compressibility) of the in-situ soil under and adjacent to the conduit. The determination of the loads acting on a buried conduit is highly complex and depends on the interaction among the foregoing parameters. In addition, because some of the foregoing parameters are time-dependent, the magnitude and distribution of the load may vary over the life of the conduit, especially between the construction and in-service stages.

APPROACHES TO DESIGN

To design a buried conduit, one of two general approaches may be followed. The first entails determining the loads acting on the conduit and then choosing the conduit to resist these loads. One of the outstanding examples of this approach is the design procedure proposed by Marston and Spangler for both rigid and flexible conduits. Such an approach is normally associated with statically determinate problems and, to a large extent, ignores the fact that deformations in the structure and the surrounding soil may affect both the magnitude and distribution of the loads acting on the conduit. Because the behavior of the structure and the soil are coupled, loads simply cannot be determined correctly without giving due consideration to this interdependence. The second approach involves preselecting a conduit and analyzing the soil-culvert system as a composite problem. This is the approach followed in most elasticity solutions. Once the system is solved, stresses and deformations in the conduit may be checked against allowable values. If the result is unsatisfactory, a different conduit is selected and the process is repeated. Thus, such a design procedure may be described as one of trial and error.

METHODS OF ANALYSIS

The available methods for analyzing the loads imparted to buried conduits may be classified into two general categories—plastic analyses and elastic analyses. In general, these correspond to the two preceding design approaches, respectively. In addition, there are some methods that combine elements of both and are neither wholly elastic nor wholly plastic in nature; examples of such methods include Bull's analysis (*22*), Voellmy's solution for radial pressures (*6*), and the "relative yield theory" proposed by Smith (*17*) for use on the Garrison Dam outlet tunnels. Plastic analyses assume that sufficient deformation has occurred in the soil adjacent to a conduit to mobilize virtually the full shear strength of the soil on certain specific planes or in certain specific zones. Elastic analyses, on the other hand, assume that the soil surrounding the buried conduit has nowhere reached failure and that the induced stresses are sufficiently low that the soil may be considered as acting in a stress-strain range wherein it may be approximated by linear elastic behavior. The use of high-speed, large-capacity computers in conjunction with numerical procedures allows some nonlinear behavior to be taken into account.

Plastic Analysis

A plastic analysis is most appropriate when large differential movements, which are sufficiently large to mobilize virtually the full shear strength of the soil, occur in the soil adjacent to a conduit. As a result, solutions resulting from such analyses are often termed "limit solutions" and they usually have the disadvantages that stress paths in the soil are ignored and that the behavior of the pipe is not considered at deformation conditions that mobilize only a small portion of the full shear strength of the soil. One characteristic feature of many of these solutions is the concept of one "mass of soil" moving over another "mass of soil" and thereby imparting a load to the buried conduit. In the case of the Marston-Spangler approach for an embankment conduit, a prism of soil above the pipe is considered to move relative to the adjacent soil along vertical planes extending from the sides of pipe and, depending on the shear strength mobilized along these vertical planes, the magnitude, but not the distribution, of the load acting on the conduit is determined by considering the vertical equilibrium of a slice of the backfill. Although this concept of a prism of soil above the conduit sliding along vertical failure surfaces may be acceptable for a ditch conduit with relatively loose backfill, it appears to be inappropriate for a projecting conduit under an embankment. The Voellmy (*6*) approach is also characterized by this type of analysis, but it considers a wedge instead of a prism above the conduit.

Elastic Analysis

In contrast to a plastic analysis, an elastic analysis assumes that deformations are sufficiently small to preclude mobilization of the full shear strength of the soil. The soil and culvert materials are usually assumed to be linearly elastic, and several other idealized assumptions are required to obtain a solution. Although the assumption of a linearly elastic soil is often criticized as being unrealistic, it nevertheless provides a starting point that gives insight into the actual soil-culvert problem. In addition, a solution by this method offers several other advantages, among which are (1) the soil-culvert interaction effects are taken into account, (2) the necessity for determining the load distribution at the soil-culvert interface can be circumvented, and (3) the parameters required for the calculations are relatively basic soil and conduit properties that are not dependent on the culvert geometry and that may be determined with reasonable ease and reliability. One example of this type of analysis was presented by Burns and Richard (*10*); they considered the interaction of an elastic circular cylindrical shell embedded in an elastic medium that is loaded by a surface overpressure. Other works of this nature have been reported by Mindlin (*23*), Forrestal and Herrmann (*24*), Watkins (*25*), and others. The major disadvantage of the elastic approach is that unless numerical techniques are used such analyses generally are not adaptable to the variety of field conditions that may be encountered. For example, most solutions that are available assume that the conduit is surrounded by a homogeneous, isotropic medium of infinite extent and that the load is applied in the form of overpressure. These assumptions, of course, preclude consideration of the effects of construction procedures, various types of bedding, a compressible inclusion (imperfect trench), or the difference between the properties of embankment soil and the underlying natural foundation soil; in addition, if the height of cover is small relative to the conduit diameter, results obtained from use of these solutions will not be accurate.

Many of the foregoing shortcomings are eliminated if numerical techniques, such as finite elements (*12, 13*) or spring analogs (*11*), are used. The recent development of numerical procedures has made it possible to handle most of the preceding conditions, as well as some types of nonlinear (piecewise linear) material behavior, in the analysis. Nevertheless, the success of such numerical techniques, as well as any other approach, depends on the selection and determination of realistic material constants, the choice of representative boundary and interface conditions, and the consideration of the change in culvert geometry during the fill operation.

DESIRED STRUCTURAL STATE

From a design viewpoint, it is desirable to maintain a conduit in a state of compression with no flexural effects. This means that the line of thrust should coincide with the effective centroidal axis of the conduit wall. In the reinforced concrete culvert, this avoids shear associated with rate of change of bending moment and the associated possibility of fracture. In the flexible culvert the bending resistance is small and most of the loading has to be taken by a membrane action. To ensure small radial deformations, it is necessary that the initial loading condition be such as to minimize bending moment in the culvert wall. If this is not so, the culvert geometry will attempt to readjust itself to approach this condition, and such readjustments may be precursory to collapse. Therefore, it is desirable to try to arrange the pressure distribution on the culvert to ensure that the line of thrust coincides with the effective centroidal axis of the conduit wall.

Although the assumption of rigidity may appear reasonable in a reinforced concrete culvert, the work of Davis (*19*) would indicate that, although the material of the culvert is stiff compared to the fill, any sudden change of the culvert stiffness tends to alter the pressure magnitude. Specifically, he noted that shoulder cracks occurred in a reinforced concrete arch culvert at points where the reinforcing bars terminated. Although this observation was made on an arch culvert, it seems reasonable to expect that it would apply to pipe culverts. The geometry of a rigid culvert generally will alter little under increasing load unless fracture occurs. On the other hand, the flexible culvert has a geometry that changes with load in order that the loading may be accommodated largely by membrane action. In the early stages of fill placement, drastic alterations in culvert geometry may occur; however, once the fill is well above the crown, the culvert geometry usually will remain sensibly unaltered, although measured loads and deformations tend to increase approximately in proportion to the height of the overburden.

For the particular case of a circular conduit, the most desirable loading condition is hydrostatic, and various attempts have been made to achieve this type of loading. One of the well-known techniques is the imperfect trench installation, but care must be exercised so as not to decrease vertical loads and increase horizontal ones to such an extent that failure occurs. Newer and thus far unproven techniques include surrounding the conduit with a compressible material or cushion and using slip joints to relieve and redistribute large stresses at the periphery of the conduit. The objectives of analytical and experimental work may be regarded as threefold:

1. To determine the primary factors that affect the response of a soil-conduit system.
2. To determine how these factors affect the pressure distribution on the conduit.
3. To determine what variations in these factors are necessary to produce a desired pressure distribution and magnitude.

APPENDIX B

CURRENT DESIGN PROCEDURES

Current practice in culvert construction, for the most part, involves either of two materials—reinforced concrete or corrugated metal. Generally accepted design methods treat the first as a rigid structure and the second as a flexible structure, separate design procedures being available for each. Current design methods require (1) a determination of the magnitude and distribution of the loading, and (2) selection and analysis of a structure compatible with the determined loading. Recently, relatively thin-walled, large-diameter, unreinforced concrete pipe and reinforced plastic mortar pipe have been used; both have considerable bending resistance and a capacity for transmitting considerable load laterally to the soil without distress. Although a comprehensive design procedure covering the full range of pipe stiffnesses is desirable, only limited research has been directed toward this end and no procedure has yet reached the stage of acceptance in design practice.

LOAD DETERMINATION

Vertical Earth Loads by Marston-Spangler Theory

The commonly used methods for load analysis can be traced to the work of Marston, Schlick, and Spangler, (*1, 2, 3, 4, 5*). Because of the importance of recognizing installation conditions, when determining loads on culverts, underground conduits have been classified into several groups and subgroups. There are two major groups—trench or ditch conduits and embankment conduits; embankment conduits are further subdivided into positive projecting, negative projecting, and imperfect trench subgroups. These groups may be described as follows:

1. A ditch (or trench) conduit is one that is installed in a relatively narrow ditch excavated in undisturbed soil and then backfilled.
2. A positive projecting conduit is one that is installed on shallow bedding, with its top projecting above the natural ground, and then covered with embankment material.
3. A negative projecting conduit is one that is installed in a relatively narrow and shallow ditch, with its top below the adjacent natural ground, and then covered with embankment material.
4. An imperfect ditch conduit is one that is installed as a positive projecting conduit, with embankment material being placed until it covers the conduit by about one diameter; then a trench having a width equal to the outside diameter of the pipe is excavated and backfilled with compressible material.

The development of the associated theories for load determination was accompanied by some full-scale studies, and the assumptions incorporated into each theory, together with an evaluation of the method, are presented in the following sections.

The total vertical load that is assumed to act on the pipe is expressed by

$$W_c = C_i \gamma B_j^2 \tag{B-1}$$

in which W_c is the total vertical earth load on the pipe in pounds per linear foot; γ is the unit weight of the fill material in pounds per cubic foot; B_j is the horizontal dimension in feet (the subscript j is determined by the method of installation, B_c being the outside diameter of the pipe and B_d the trench width at the top of the conduit); and C_i is the load coefficient (the subscript i is determined by the method of installation; $i = c$, d, or n). The load coefficient C_i depends on the geometry of the soil-culvert system and the physical properties of the fill and the culvert materials; such physical factors include: (1) the ratio of the height of the fill, H, to the horizontal width, B_j, (2) the coefficient of internal friction of the soil, (3) the projection ratio, p, which is defined as the vertical distance between the top of the pipe and the natural ground surface divided by the outside horizontal diameter of the pipe, B_c, and (4) the settlement ratio, r_{sd}, which determines the direction of action of the frictional forces acting on the prism of earth above the conduit.

In the case of a ditch conduit, the vertical load, W_c, is basically equal to the weight of the prism of fill above the conduit minus the shearing resistance along the vertical sides of the trench of height H. This approach assumes that there are sufficiently large differential movements of the soil above the conduit to fully mobilize the shear strength of the soil. Eq. B-1 is obtained by integrating the forces acting on an element of fill of width B_d. The vertical pressure at any depth in the ditch backfill is assumed to be uniform across the width of the element.

For positive projecting conduits the sliding surfaces are assumed to be vertical planes extending upward from the sides of the conduits. The vertical load, W_c, is thus equal to the weight of the fill above the conduit plus or minus the vertical shearing forces generated from relative movements of the prism of soil above the conduit (interior prism) and the adjacent soil (exterior prisms). These shear forces are added to the weight of the interior prism when the exterior prisms settle more than the interior prism, and the shear forces are subtracted when the opposite condition exists. This relative settlement effect is handled by the selection of a rather abstract parameter known as the settlement ratio, r_{sd}, which enters into the determination of C_i in Eq. B-1. An evaluation of this settlement ratio is presented later.

In the Marston theory of load determination for trench conduits, the vertical shearing resistance along the side walls of the trench is computed with the assumption that the coefficient of friction is constant over the entire depth of the trench. Because the shearing stress depends on the relative displacement along the slip surfaces, the value of the mobilized shearing stress should increase from zero at the ground surface to a maximum value at some depth. If the assumption that horizontal active earth pressure acts against the sliding element within the trench is correct, there should be sufficient horizontal movement of the trench walls to develop active earth pressure. If the trench were cut through rock or relatively unyielding material, such as highly overconsolidated clay, little or no lateral strains would occur, and the "at-rest" lateral pressure may be assumed. However, the "at-rest" condition implies that there are zero shearing stresses along the vertical (principal) plane, and this is not the case when the backfill material displaces; hence, the "at-rest" condition does not appear to be applicable. On the other hand, the active pressure coefficient is equal to the ratio at failure of the horizontal minor principal stress to the vertical major principal stress. Therefore, the use of the active lateral pressure coefficient implies that the vertical sides of the trench are principal planes, whereas the failure planes are inclined to the vertical. This is contradictory to the assumption that a failure surface coincides with the vertical principal plane. If the failure surfaces are assumed vertical, the principal planes form some angle with the vertical, and the normal stresses on the failure surfaces are different from those determined by use of the active lateral pressure coefficient.

In the case of an embankment conduit, the assumption of vertical sliding surfaces is a critical one. Based on work by Voellmy (*6*) (see Terzaghi, *7*, and Moran et al., *26*), the failure surfaces formed by lowering a portion of a glasswalled box filled with sand were inclined to form a wedge. The resulting analysis assumed that, owing to differential movement between the wedge of soil above the conduit and the surrounding soil, the full shear strength of the soil was mobilized on the failure surfaces. With regard to the assumption of vertical failure planes, Terzaghi (*7*) stated: "Fortunately the sources of error associated with this assumption are clearly visible. In spite of the errors the final results are fairly compatible with the existing experimental data."

Luscher and Höeg (*27*) stated that the main uncertainty in the vertical-sliding-surface analysis is the magnitude of the lateral pressure acting normal to the sliding surfaces. Specifically, they say that most investigators seem to assign values of K_o (e.g., Newmark, *28*), or K_a (e.g., Spangler, *29*), where K_o and K_a are the at-rest and active pressure coefficients, respectively. The reasoning behind these assumptions does not seem to reflect the action taking place in the soil-culvert system. In the classical experiments by Terzaghi (*30*), the value of the horizontal to vertical pressure increased with height above the trap door from 1.0 to 1.6 and then decreased toward K_o at a height of about 2.5 times the width of the trapdoor. Ang and Newmark (*31*) present results from trapdoor tests performed with wood toothpicks as "soil" material; by applying the sliding-surface analysis to their data, the average value of K over the depth was backfigured to be 1.5. Similarly, data from the buried dome tests by Whitman et al. (*32*) indicate a value of 1.2. This generally leads to the conclusion that there is a need to replace the vertical sliding surfaces by some other concept.

As mentioned previously, the Marston-Spangler theory of load determination considers the effect of settlement of the interior prism of soil relative to that of the exterior prism; the relatively abstract parameter that is intended to account for this effect is called the settlement ratio, r_{sd}. The load coefficient, C_c, is, in turn, dependent on r_{sd}. For the positive projecting conduit, r_{sd} is defined as

$$r_{sd} = \frac{(S_w + S_g) - (S_f + d_c)}{S_m} \tag{B-2}$$

in which S_m is the shortening of the side columns of soil; S_g is the settlement of the natural ground surface adjacent to the conduit; d_c is the shortening of the vertical height of the conduit; and S_f is the settlement of the conduit into its foundation. The ratio of the distance from the natural ground to the top of the conduit to the diameter of the conduit is called the projective ratio, p_r, and the horizontal plane through the top of the conduit is called the "critical plane." The total load, W_c, on the conduit is greater than the weight of soil directly above it if the critical plane settles more than the top of the conduit and r_{sd} is considered positive. In this case, the exterior prisms move downward relative to the interior prism, and downward shearing forces are generated. On the other hand, if the critical plane settles less than the top of the conduit, r_{sd} is negative, the shearing forces are directed upward, and part of the weight of the soil above the conduit is relieved. The plane of equal settlement defines a height above the conduit at which the accumulated settlement in the exterior prisms plus the settlement of the critical plane just equal the settlement of the interior prism plus the settlement of the top of the conduit. If the settlement ratio equals zero, or the projection ratio equals zero, the plane of equal settlement coincides with the critical plane, and the load on the culvert is equal to the free field load, γHd.

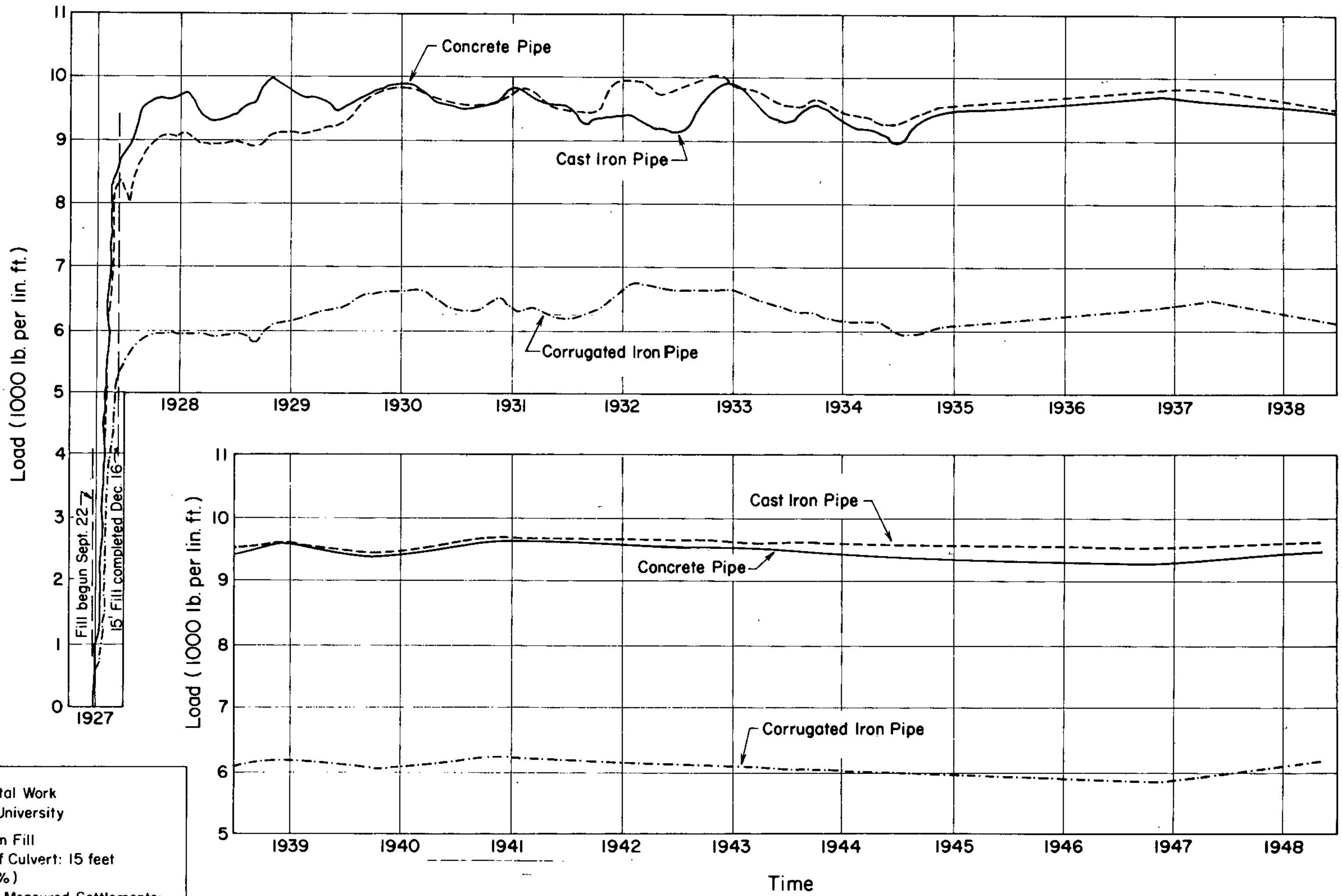
The settlement ratio is a semi-empirical factor that is based on an extremely small amount of experimental data reported in 1950. Field observations were made on 22 actual conduit installations: 15 concrete box culverts, 2 concrete arch culverts, 1 concrete pipe culvert, and 4 corrugated metal pipe culverts. Figure B-1 shows some typical results of the experimental work at Iowa State University. Based on these investigations, recommended values of the settlement ratio have been presented by Spangler (*29*), and these values continue to be used today with virtually no modification.

Imperfect Ditch Conduit

The imperfect ditch method of construction was proposed by Marston (*2*). If rigid culverts are installed at or near the natural ground surface and covered with earth fill, the prisms of soil at the sides of the pipe will tend to settle more than the prism of soil directly above. This differential movement between soil prisms will increase the load on the culvert. Spangler (*33*) stated that, in some of Marston's early experiments, the actual measured loads on rigid conduits in this type of installation were 90 to 95 percent greater than the weight of the soil directly above the conduit. An attempt to avoid this increase in load on the conduit led to the development of the imperfect ditch method of construction.

As defined earlier, in the imperfect ditch installation, the conduit is installed as a positive projecting conduit; then, the earth fill is thoroughly compacted at the side of the pipe to about one diameter above the top. Next, a trench having a width equal to that of the pipe is excavated directly over and down to the top of the conduit. This trench is refilled with loose, highly compressible soil, straw, hay, or cornstalks, and the embankment construction continues in a normal manner. The purpose of the compressible material is to ensure that the prism of soil directly above the culvert will settle more than the adjacent soil, thereby reducing the load on the conduit. The amount of the differential movement of the soil prisms and the resulting reduction in conduit load will depend on the depth of the imperfect trench and the compressibility of the refilling material.

In the Marston-Spangler method for load determination, the effects of horizontal earth pressures are generally ignored. For example, although the use of compressible material above the conduit may reduce the vertical load on the pipe, its effect on the lateral pressures was not mentioned. However, the analysis by Spangler (*34*) for a negative projecting conduit, which applies for an imperfect ditch installation, assumes that the shearing forces from the interior prism distribute over exterior prisms of the same width. This, however approximate, would increase the lateral pressures on the conduit. The primary objective of



Experimental Work
Iowa State University

Soil Type: Silty Clay Loam Fill
Height of Fill Above Top of Culvert: 15 feet
Projection: Positive (90%)
Settlement Ratio Based on Measured Settlements:
Concrete Pipe: +0.92
Cast Iron Pipe: +0.87
Corrugated Iron Pipe: −0.09

Figure B-1. Variation of culvert load with time.

any modified installation procedure should be to approach the desired state of loading discussed in Appendix A.

In addition, redistribution of the loads on a conduit by means of an imperfect ditch installation is intimately related to the invariance of the refill material. Although, for example, it may be possible to modify the initial load distribution on a conduit by use of hay as a refill material, this material may decompose over a period of time and alter its stiffness such that an unsatisfactory load distribution may be imposed on the conduit. Brown et al. (*12, 13, 35*) showed that, when the stiffness of an organic refill material becomes negligible, the crown pressure reduced to nearly zero while the pressure away from the crown of a rigid arch remains essentially unchanged or increased slightly. This may cause unexpected bending moments in the conduit wall. The long-term serviceability of imperfect ditch installations presents a largely unanswered question.

When the compressible material is loose soil, consolidation under the overlying weight of fill will occur. Owing to the soil arching phenomena, the weight of the fill will be partly supported by the adjoining soil. The stresses at the arch support will be higher than the free field stresses, and a zone of maximum shear stress will develop. Again, the load distribution on the conduit will vary as the soil approaches a state of equilibrium. Depending on its inherent strength, the soil in the zone of higher shear stresses may yield with time, and this will have an adverse effect on the conduit. On the other hand, if equilibrium is reached, the arch will be permanent. Only field observations will provide the information required to evaluate the time effects of an imperfect ditch culvert installation.

Horizontal Earth Loads

Although the Marston-Spangler analysis is concerned directly with only the determination of vertical loads, horizontal soil pressures are indirectly taken into account in the design procedure. Because the supporting strength of a culvert is dependent on the load distribution, it is necessary to know the conditions of both vertical and horizontal loading. Horizontal earth pressures are, of course, intimately related to the yielding of the structure.

Spangler (*36*) stated that rigid culverts are capable of using only the active earth pressures because they do not distort materially under vertical load and the sides of the pipe do not move outward enough to produce any appreciable passive pressure. He further stated (*36*) "that it is *safe* to assume that active horizontal pressures about equal to those calculated by Rankine's formula may be considered to act against those portions of (rigid) pipe culverts which project above the surface of the natural ground adjacent." This concept of active pressure acting against "rigid" conduits does not seem entirely reasonable, and it appears that a lateral pressure closer to the "at-rest" condition may be more realistic. However, Spangler stated that his conclusion was based on observations and experience obtained during many years of experimental work. Nevertheless, it is possible that Spangler's findings may be explained in part by either the effect of measurement technique or the assumptions incorporated into the vertical load determination.

In the case of flexible culverts, the vertical load, as determined by Marston's theory, is assumed distributed uniformly over the breadth of the pipe (*3*). The vertical reaction at the bottom of the pipe is equal to the vertical load and uniformly distributed over the bedding width of the pipe. According to Spangler (*8*), the magnitude and distribution of the horizontal pressures developed in response to the outward movements of the sides of a flexible pipe are a function of the density of the sidefill material. Experimental evidence has indicated that, under vertical loads, the ratio of the horizontal pressure to the horizontal diameter change is practically constant, regardless of the height of fill, and it was assumed constant at any point along the side of the pipe. This ratio, called the modulus of passive resistance, e, of the fill material, is higher for dense granular fill than for silt, and increases with increased degree of compaction. The shape of the deflected pipe was assumed elliptical, with the same peripheral length as the original circular pipe. The horizontal movements of the different points on the circular pipe were described by a parabola that extended over the middle 100° (plus or minus 50° from the horizontal). Because horizontal pressures were found to be proportional to horizontal movements, the horizontal pressures on each side of the pipe are distributed parabolically over the same middle 100° with a maximum unit pressure, occurring at the springline, equal to the product of the modulus of passive resistance and one-half of the horizontal deflection of the pipe.

The modulus of passive resistance, e, was later found by Watkins and Spangler (*37*) to be not constant, but rather inversely proportional to the radius, r, of the conduit. A parameter termed the modulus of the soil reaction, E', which is equal to er, was introduced and some values based on measured deflections were presented. Despite the widespread use of the assumed Spangler load distribution, shown in Figure B-5, results are often suspect because of the inability to determine reliably a numerical value for E'. This is discussed later in more detail.

Traffic Loads

Traffic loads applied at the surface of a highway embankment are transmitted through the soil to underground structures, and the resulting stress distribution with depth is usually obtained by use of the Boussinesq solution for a semi-infinite elastic solid. To standardize the design of highway culverts, the American Association of State Highway Officials has adopted "Standard Vehicles" that produce representative live loadings on the surface of an embankment. For example, an H-20 truck loading consists of two 16,000-lb loads applied to two 18- by 20-in. areas. One of these loads is placed over the point in question; the other is 6 ft away. The Boussinesq equation for the vertical stress, q, at a point in an elastic soil medium due to a concentrated point load, Q_s, is

$$q = \frac{3Q_sH^3}{2\pi R^5} \tag{B-3}$$

in which H and R are the vertical depth and radial distance, respectively, from the surface load to the point in question. The vertical stress produced by an H-20 live load versus the height of cover above a culvert is shown in Figure B-2. As can be seen, the effect of a surface load decreases rapidly with increasing height of cover. If the vertical earth load on a culvert is assumed equal to the weight of the soil directly above, the vertical stress versus height of cover relationship for a soil with a density of 120 lb per cubic foot is also shown in Figure B-2. If the live load and the dead load are added, the total load, shown in Figure B-2, is minimum at about 4 ft of cover. For depths of cover greater than 4 ft, the vertical stress due to a live load decreases very rapidly, approaching zero at about 20 ft; on the other hand, for depths of cover less than about 2 ft, the total load consists almost entirely of live load. It is in this area of culverts with shallow cover that current theories are most deficient; this is especially important in view of the heavy loads being imposed on culverts during the construction phase.

In current practice, charts similar to that shown in Figure B-2 are generally used to determine live loads on culverts. However, in the preparation of Figure B-2, no consideration was given to the size of the culvert—that is, the height-of-cover to diameter-of-pipe ratio. In addition, the stress variation shown in Figure B-2 does not indicate the distribution across the width of the culvert.

Spangler (*29*) proposed the following formula to determine the live loads on underground conduits:

$$W_t = \frac{1}{l} I_c C_t Q_s \tag{B-4}$$

in which W_t is the average load, in pounds per lineal foot, on the conduit due to wheel load; l is the length (or effective length) of the conduit; I_c is an impact factor; C_t is a load coefficient; and Q_s is a concentrated truck-wheel load, in pounds, on the surface of the fill. For a precast segmented culvert section which is 3 ft or less in length, l is the actual length. For continuous conduits or those constructed of segmented sections more than 3 ft in length, l is the "effective length," which is defined as the length of culvert over which the *average* live load produces the same effect or stress or deflection as does the actual load, which is of varying intensity along the pipe. The impact factor, I_c, depends on the speed of the vehicle, the axle load, the roughness of the road surface, and the type of pavement. The load coefficient, C_t, is the influence value that depends on the length and width of the section of conduit under consideration, the depth of the conduit below the road surface, and the position of the point of application of the wheel load. This approach may be used to develop charts for typical highway truck loadings, similar to that shown in Figure B-2 but showing the effect of conduit size.

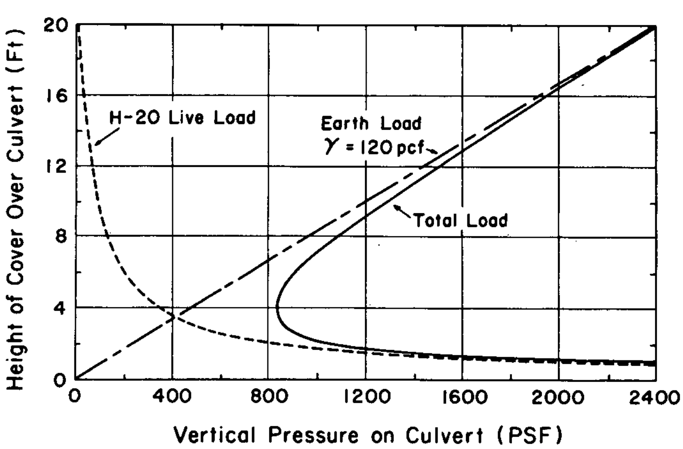


Figure B-2. Vertical pressure on culvert versus height of cover.

RIGID CONDUITS

The design procedure commonly used for rigid culverts is based largely on research conducted at Iowa State University by Marston, Schlick, and Spangler. The total vertical load, W_c, which is assumed to act on the pipe, is obtained by use of Eq. B-1, and a variety of design charts are available to evaluate the appropriate C_i as a function of H/B_j, r_{sd}, and p_r for different values of the coefficient of friction of the soil.

The standard method used to determine the inherent strength of a concrete pipe is to conduct a three-edge bearing test (ASTM C 497-65T). Under the three-edge bearing method of loading, the pipe is subjected to concentrated loads at the crown and invert. Loading is applied until either a 0.01-in. crack has occurred throughout a length of 1 ft or more or until the ultimate strength load of the pipe has been reached. The three-edge bearing strength of a pipe is expressed in pounds per linear foot. Another means of expressing reinforced concrete pipe strength is in terms of D-load, which is the three-edge bearing strength (pounds per foot) per foot of inside diameter of the pipe. The D-load concept enables strength classification of pipe independent of pipe diameter. ASTM C 76-66T for culvert pipe describes five strength classes based on D-load at 0.01-in. crack and/or ultimate load.

The maximum elastic normal stress in pounds per square inch produced in a pipe by the three-edge bearing test is

$$\sigma_{\max} = 0.00332\, k_i \left(\frac{1+a}{1-a}\right)^2 \frac{P}{r} \tag{B-5}$$

in which k_i is a nondimensional factor that is a function of a and accounts for the effect of bending in a curved beam (*38*); a is the ratio of the inside radius to the outside radius of the pipe; P is the applied test load in pounds per foot; and r is the mean radius of the pipe in feet. When expressed in terms of the D-load, Eq. B-5 becomes

$$\sigma_{\max} = 0.01325\, k_i \frac{a(1+a)}{(1-a)^2} D \tag{B-6}$$

For the usual sizes of pipe used as conduits, the average value of $\sigma_{\max}$ is 0.697 D.

The three-edge bearing test is probably the most severe loading to which any pipe will be subjected. Concentrated vertical loads are applied and there is no lateral support for the pipe, as is provided under actual field conditions. To relate the three-edge bearing strength to the in-place supporting strength, load factors have been developed (*29*). The load factor, L_f, is defined as the ratio of the supporting strength of a pipe under any stated condition of loading in the field to the supporting strength of a similar pipe as determined in the three-edge bearing test. Load factors for trench installations have been determined experimentally for four classes of bedding. Load factors for embankment installations are based on equivalent uniformly distributed lateral earth pressures acting on the pipe, as shown in Figure B-3.

The parameter m designates the fractional part of the outside diameter of the conduit over which lateral pressure is effective. It is important to note that m may or may not be the same as p_r. The nondimensional factor q, as shown in Figure B-3, is the ratio of the total lateral pressure to the total vertical load on the pipe. An expression for q is obtained with the aid of Rankine's formula:

$$\frac{qW_c}{mB_c} = \gamma H_r K \tag{B-7}$$

in which H_r is the vertical height from any point to the upper surface of the fill; and K is Rankine's lateral pressure coefficient. Substitution of Eq. B-1 into Eq. B-7 yields

$$q = \frac{mKH}{C_c B_c} \tag{B-8}$$

In deriving the general expressions for load factor, it was assumed that the wall thickness of the pipe is equal to 15 percent of the mean radius of the pipe. If the maximum normal stress at the invert of the pipe, as produced by the loading condition shown in Figure B-3, is equated to the maximum normal stress at the same point when the pipe is subjected to the loading condition of the three-edge bearing test, the ratio of the total vertical load, W_c, to the applied test load, P, is formed, and the ratio yields the following design equation for evaluating the load factor:

$$L_f = \frac{1.431}{N - xq} \tag{B-9}$$

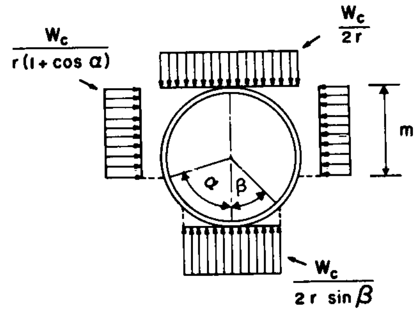
in which N is a parameter that depends on the distribution of the vertical load and vertical reaction; N is a function of the bedding angle, and it assumes the values given in Table B-1 for conditions of Class B, C, and D bedding. The factor x in Eq. B-9 is a function of the area of the vertical projection of the pipe on which the active lateral pressure of the fill material acts (i.e., mB_c). Appropriate values for this factor have been determined (*36*) and are tabulated for use in design (*39*).

Equation B-9 usually applies for cases of field loading in which the pipes initially crack at the invert. When the load and reaction are such that the pipe cracks first at the crown, which is usually the case when pipes are bedded in a concrete cradle (Class A bedding), the expression for the load factor becomes

$$L_f = \frac{1.431}{0.505 - x'q} \tag{B-10}$$

in which x is replaced by x' for Class A bedding. Values of x' have been established and are tabulated.

To study the effectiveness of Eq. B-9 for evaluating a load factor, consider the distribution of earth pressures and reactions as developed by Olander of the Bureau of Reclamation (*40*). The bedding angle for this distribution is 90°.



$$q = \frac{mK}{C_c}\left[\frac{H}{B_c} + \frac{m}{2}\right]$$

Figure B-3. Equivalent uniformly distributed lateral earth pressure for load factor calculation.

By use of the thrust and moment coefficients, it is possible to calculate the maximum normal stress at the invert. If this value is equated to the maximum stress obtained in the three-edge bearing test, as given by Eq. B-5, it is possible to evaluate the load factor for this pressure distribution as

$$(L_f)_{\text{exact}} = \frac{2.95}{1.167 - \dfrac{1-\alpha}{k_i(1+\alpha)}} \tag{B-11}$$

Thus, the load factor is a function of α, which, in turn, reflects the stiffness of the pipe. For the average size pipe in use today, Eq. B-11 gives the exact value of 2.74 for the load factor.

On the other hand, the load factor evaluated by Eq. B-9 depends on the class of bedding and the height of cover. If the appropriate projection ratio for the Olander loading is used, and if K and r_{sd} are assumed equal to 0.33 and 0.5, respectively, load factors for both B and C classes of bedding can be evaluated from Eqs. B-8 and B-9, and the results are given in Table B-2. The discrepancies between the values of Table B-2 and the exact value of 2.74 are due to the assumptions that were made in the derivation of Eq. B-9.

The safe supporting strength of a structure is equal to the in-place supporting strength divided by an appropriate factor of safety. According to current practice, the in-place supporting strength of a rigid conduit is equal to the three-edge bearing strength multiplied by the load factor. Hence,

TABLE B-1

VALUES OF N FOR VARIOUS CLASSES OF BEDDING

BEDDING CLASS	BEDDING ANGLE (°)	N
B	90	0.707
C	60	0.840
D	0 (point load)	1.310

TABLE B-2

LOAD FACTOR AS A FUNCTION OF BEDDING CLASS AND HEIGHT OF COVER

	L_f	
H/B	CLASS B BEDDING	CLASS C BEDDING
2	2.41	2.08
4	2.38	2.04
8	2.36	2.02
10	2.35	2.02

the safe supporting strength of a rigid pipe is the three-edge bearing strength times a load factor divided by a factor of safety. Because the in-place supporting strength of a conduit depends to a large degree on the installation conditions and local quality control, the particular value assigned to the factor of safety must be based on engineering judgment. This is discussed in more detail in Appendix E.

FLEXIBLE CONDUITS

The predominant source of supporting strength for a flexible conduit is the lateral pressure of the soil at the sides of the pipe. The pipe itself has relatively little inherent bending strength, and a large part of its ability to support vertical loads must be derived from the passive pressures induced as the sides move outward against the soil.

Some of the original design criteria for flexible culverts are empirical in nature and were established by means of observational study. Because the deflection of a flexible pipe varies directly as some power of the fill height, H, and the diameter, d, and inversely as the wall thickness, t, a formula expressing deflection was written in the form

$$\Delta y = k \frac{H^m d^n}{t^s} \qquad \text{(B-12)}$$

To determine values for the exponents m, n, and s and the constant k, measurements were made on corrugated metal pipe of various diameters under different heights of fill. The deflection data from the American Railway Engineering Association investigation of 1926 (*41*) were most useful in establishing the original values of these factors.

The maximum deflection before failure was investigated by inspecting numerous large-diameter pipe installations, and the average safe maximum deflection was determined to be 20 percent of the vertical diameter. The use of conservative "factor of safety" of 4 established the design deflection at 5 percent over 40 years ago, and gauge tables based on the empirical equation and use of a 5-percent deflection criterion were prepared. As more installations were studied, the equation was revised, and the gauge tables reflected these changes. Thus, the published gauge tables were, in effect, "experience tables." As an example, the data from two editions of these tables for corrugated steel pipe are plotted in Figure B-4.

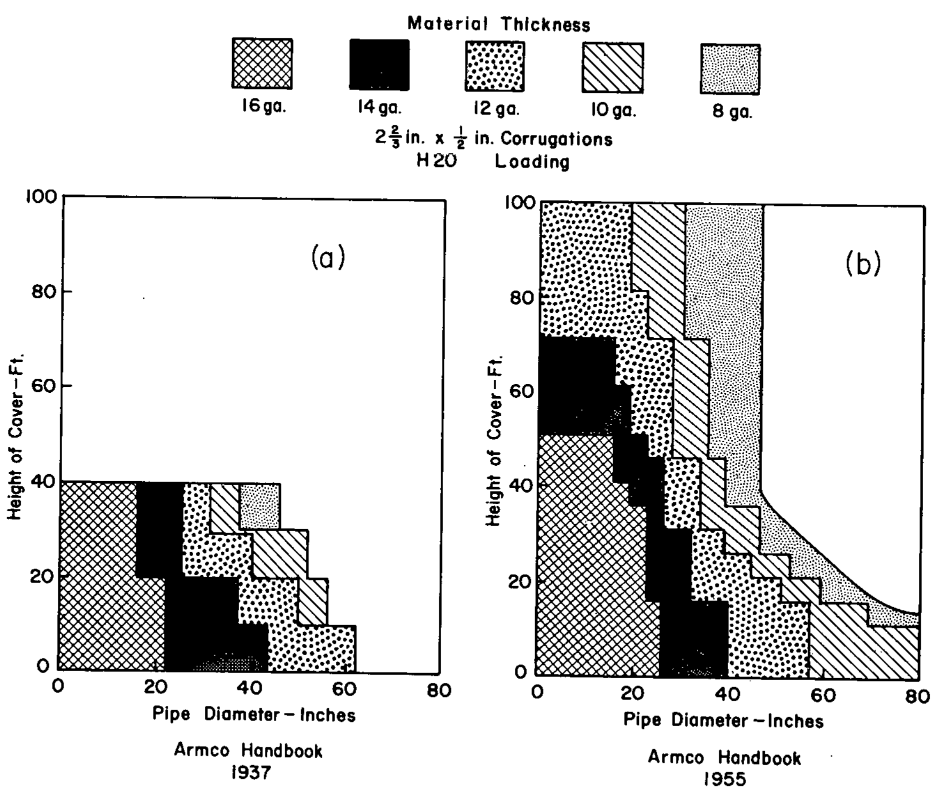


Figure B-4. Typical examples of variation in gauge tables with time and experience.

Marston-Spangler Theory

During experiments on circular flexible conduits, it was observed that, as soil loads were applied, the pipe deformed from a circular to an elliptical shape, with the minor (vertical) axis of the ellipse less than the diameter of the circle by the amount of the vertical deflection and the major (horizontal) axis greater than the diameter by the amount of the horizontal deflection. On the basis of this behavior, Spangler (*8*) assumed the pressure distribution shown in Figure B-5 and developed the following equation, which has come to be known as the Iowa Deflection Formula, for predicting the deflection of buried flexible conduits:

$$\Delta x = D_l \frac{KW_c r^3}{EI + 0.061E'r^3} \qquad \text{(B-13)}$$

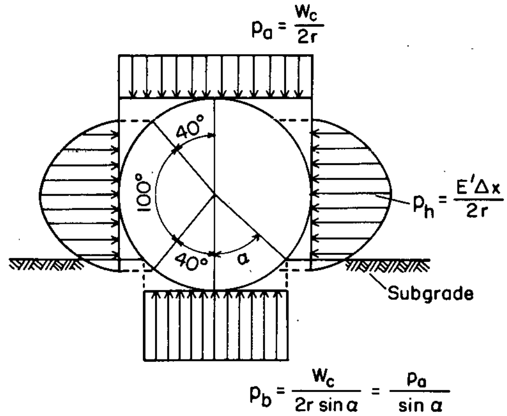


Figure B-5. Spangler assumption for pressure distribution on a flexible pipe.

in which Δx is the horizontal deflection of the pipe; D_l is the deflection lag factor; K is a bedding constant whose value depends on the bedding angle (values have been calculated and are tabulated); W_c is the Marston load (see Eq. B-1); r is the mean radius of the pipe; E is the modulus of elasticity of the pipe material; I is the moment of inertia per unit length of cross section of the pipe wall; and E' is the modulus of soil reaction. The deflection lag factor is introduced to account for time-dependent changes in deflection. Suggested values for design purposes normally range from 1.25 to 1.50.

Little is known about the exact nature of the modulus of soil reaction, E'. Data from actual installations indicate that E' varies widely; values from 230 psi to 8,000 psi have been reported and it is probable that these limits have been exceeded in other instances. Spangler (*29*) has recommended that a value of 700 psi be used in design if the side-fill soil is compacted to at least 90 percent of the standard Proctor density for a distance of two pipe diameters on each side of the pipe. The Bureau of Public Roads' design criteria (*42*) have used values of $D_l = 1.5$ and $E' = 700$ psi for good backfill at 85 percent of standard Proctor density, and $D_l = 1.25$ and $E' = 1{,}400$ psi for excellent backfill at 95 percent of standard Proctor density. It is recommended (*29*) that the deflection of a corrugated metal culvert should not exceed about 5 percent of the nominal pipe diameter.

Ring Compression Theory

For conduits with sufficient height of cover and surrounded by well-compacted fill, White and Layer (*9*) have suggested that the ring compression load can be best approximated by considering the circular conduit to be loaded uniformly by a load equivalent in magnitude to the overburden pressure, γH, in which γ is the unit weight of the soil above the pipe and H is the distance from the top of the pipe to the surface of the fill. For the case of a cylinder under uniform radial pressure, the ring compression load per lineal foot, T, is given by

$$T = r\gamma H \qquad \text{(B-14)}$$

In the United States, a safety factor of 4 is normally used in conjunction with design by this method.

A basic difference exists in the philosophy behind the Marston-Spangler and the ring compression design methods. The Marston-Spangler approach asserts that culvert deformation will usually control design; seams or lap joints are designed to resist ring compression, but no consideration is given to the compressive stress in the pipe wall, and exceeding the compressive yield stress of the steel is not considered serious. On the other hand, White and Layer contend that the ring compression load is a vital consideration for determining the thickness of the pipe wall.

In general, design manuals in the United States recommend both a deflection check in accordance with the Marston-Spangler approach and selection of the plate thickness in accordance with the ring compression theory. As mentioned previously, deformation is limited to 5 percent of the culvert diameter, and a safety factor of 4 is applied to the ring compression load for comparison with the yield stress of the plate and the laboratory measured ultimate strength of the seams and lap joints. Data from two recent sets of gauge tables for corrugated steel pipe are plotted in Figure B-6.

Buckling

Buckling has heretofore not been considered to a great extent by designers for two main reasons: (1) a failure that could be attributed to buckling of an in-service circular culvert has never been reported, except where excessive deformation preceded the failure, and (2) until recently no suitable theory has been available for analyzing the buckling response of an embedded conduit.

Luscher (*43*) developed the following expression for the critical uniform applied pressure, p^*, required to cause buckling in a soil-surrounded tube:

$$p^* = 1.73 \sqrt{\frac{EIBM^*}{r^3}} \qquad \text{(B-15)}$$

in which E is the modulus of elasticity of the pipe material; I is the moment of inertia of the longitudinal cross section of the conduit wall per unit length; B is a coefficient of elastic support; M^* is the constrained modulus of the soil; and r is the nominal radius of the tube. Luscher found

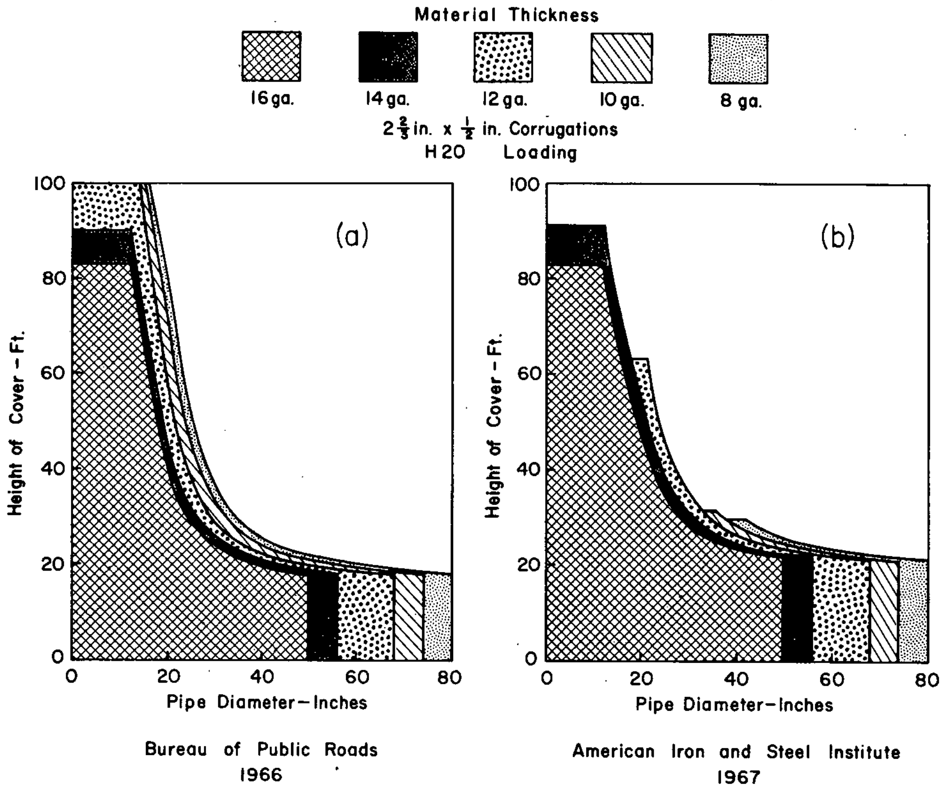


Figure B-6. Typical examples of recent gauge tables by different agencies.

close correlation between pressures calculated by Eq. B-15 and those measured in laboratory tests, at least for one type of soil (a dense-to-medium loose coarse sand). If the criterion described previously is applied to a 10-gauge (0.141-in.) standard corrugation (2⅔-in. by ½-in.) metal pipe embedded in the same soil, buckling is found to be critical for diameters of 30 ft or greater. However, this value would require modification for any change in curvature within permissible deformations and for possible nonuniformity in the load distribution. Current practice normally involves the use of 6-in. by 2-in. corrugated sectional plate pipe for culvert diameters in excess of 8 ft; for such a plate culvert of 10-gauge material the corresponding diameter above which buckling is theoretically critical is 148 ft. These results suggest that, for the type of construction currently being undertaken in the United States, buckling could be neglected, as the experience of both Spangler and White has indicated. Nevertheless, it is extremely important to point out that the neglect of buckling presumes (1) a properly compacted fill, (2) a limited deformation, (3) a sufficient height of cover, and (4) a structure geometry within the limits of present practice; if any of these conditions is violated, a check for buckling may be necessary.

Sectional Plate Pipe-Arches

White and Layer (*9*) have proposed that the ring compression load in a pipe-arch be determined in a manner similar to that for the circular flexible culvert. In this case, the formula for the ring compressive load per unit length, T, becomes

$$T = 0.5 B_c \gamma H \tag{B-16}$$

in which B_c is the outside width of the structure. They proposed further that the pressure distribution varies inversely with the curvature of the culvert wall, and at any point

$$p = \frac{T}{\rho} \tag{B-17}$$

in which p is the soil pressure; and ρ is the radius of curvature of the culvert wall. At locations where radius of curvature is small, it is essential that the soil provide a high resistance to deformation.

Because the base of the pipe-arch normally has a large radius of curvature, it is a possible location for a buckling failure and it should be checked by a suitable formula, such as the one previously described (*43*). If a pipe-arch is constructed under a low fill, deformation of the structure is usually small, provided adequate compaction of the fill has been achieved. Under high fills, where deformation may be considerable, no accurate means are available for calculating the deformation. However, a knowledge of this deformation is essential and a design of this type should be treated with extreme caution, as changes in culvert width are likely to produce large changes in the radius of curvature of the bottom surface, and buckling failure may follow. Special attention to bedding is necessary in the case of the pipe-arch. A high resistance to deformation is essential in the vicinity of the haunches where the radius is small and the pressure is high.

Recently Developed Design Procedure

In an effort to circumvent some of the difficulties attributed to inadequate consideration of material properties in the older methods of culvert design and at the same time to provide for a wide range of flexibility or stiffness of the pipe material, Watkins (*25*) developed a design method that applies to circular culverts only. This method is largely theoretical in nature and contains the basic assumptions that (1) the compacted fill and the natural ground behave as a homogeneous, linearly elastic medium of infinite extent with an inclusion (the culvert); (2) the response of the soil-culvert system can be analyzed by first evaluating the response of a perfectly flexible ring and then applying a modification factor to account for the ring stiffness; (3) the deformed shape of the ring is elliptical; and (4) no friction exists between the ring and the soil. As shown by Watkins, the culvert deformations before accounting for bending resistance of the ring are

$$\frac{\Delta x^*}{d}=\frac{p/M^*}{1-p/M^*}\left[\frac{2-p/M^*+K_o(1-p/M^*)^3}{2+M^*d/EA_l}-K_o(1-p/M^*)^3\right] \tag{B-18}$$

and

$$\frac{\Delta y^*}{d}=\frac{p}{M^*}-\frac{p/M^*}{1+p/M^*}\left[\frac{2-p/M^*+K_o(1-p/M^*)^3}{2+M^*d/EA_l}-1\right] \tag{B-19}$$

in which Δx^* and Δy^* are the changes in the horizontal and vertical diameters, respectively, before modification for ring stiffness; d is the original ring diameter; p is the free field vertical soil pressure at the level of the ring center; M^* is the constrained soil modulus; K_o is the coefficient of earth pressure at rest; E is the modulus of elasticity of ring material; and A_l is the cross-sectional area of the conduit wall per unit length of conduit. Adjustment for the bending resistance of the ring is made by use of the empirical formula

$$f=e^{-2EI/M^*d^3} \tag{B-20}$$

in which f is a modification factor and I is the moment of inertia of the longitudinal cross section of the conduit wall per unit length. The actual deformations are then determined from

$$\frac{\Delta x}{d}=\frac{\Delta x^*}{d}\cdot f \tag{B-21}$$

and

$$\frac{\Delta y}{d}=\frac{\Delta y^*}{d}\cdot f \tag{B-22}$$

in which Δx and Δy are the changes in the horizontal and vertical diameters, respectively. The ring compression load per unit length, T, is given by

$$T=\frac{pd}{2}\left[\frac{2-p/M^*+K(1-p/M^*)^3}{(1\quad p/M^*)(2+dM^*/A_lE)}\right] \tag{B-23}$$

The preceding relationships are presented by Watkins in the form of curves to facilitate their use.

Comparison of Design Methods

Because the Marston-Spangler formula given by Eq. B-13 can be rewritten in a form similar to Eq. B-21, it is possible to compare the deformation resistance due to the soil and the deformation resistance due to the ring stiffness as computed by each method. Rearranging Eq. B-13 gives

$$\frac{\Delta x}{d}=\frac{D_lKW_c}{0.122E'r}\left[\frac{1}{\frac{EI}{0.061E'r^3}+1}\right] \tag{B-24}$$

If the bending resistance is not taken into account, Eq. B-24 becomes

$$\frac{\Delta x^*}{d}=\frac{D_lKW_c}{0.122E'r} \tag{B-25}$$

If E′ is taken as $1.5M^*$, as shown by Nielson (*44*), and if D_l, K, and W_c are taken as 1.0, 0.083 (corresponding to the maximum bedding angle), and $2rp$, respectively, for a flexible culvert, Eq. B-25 may be rewritten as

$$\frac{\Delta x^*}{d}=0.91\frac{p}{M^*} \tag{B-26}$$

If the coefficient of earth pressure is taken as 0.5 and the ring is assumed to be incompressible, Watkins' formula, given by Eq. B-18, becomes

$$\frac{\Delta x^*}{d}=\frac{p/M^*}{2(1-p/M^*)}[2-p/M^*+0.5(1-p/M^*)^3-(1-p/M^*)^3] \tag{B-27}$$

Computed values for the percentage deformation, $(\Delta x^*/d)\cdot 100$, versus the load-soil stiffness ratio, p/M^*, for both methods are shown in Figure B-7a. The modification factor for the Marston-Spangler theory can be found from Eq. B-24 to be

$$f=\frac{1}{\frac{EI}{0.061E'r^3}+1}=\frac{1}{\frac{87.5EI}{M^*d^3}+1} \tag{B-28}$$

Comparison of the foregoing relation with that given by Watkins in Eq. B-20 can be seen in Figure B-7b, which shows the modification factor, f, plotted against Md^3/EI, a term referred to by Watkins as the stiffness ratio. These comparisons indicate that the two methods show good agreement for the part played by the soil in resisting deformation (Fig. B-7a), but considerable variation for the part played by the stiffness of the ring (Fig. B-7b). However, this latter variation is of little consequence for larger culverts; this is indicated in Figure B-7b for stiffness ratios greater than 500. As an example, for an 8-ft-diameter, 10-gauge, standard-corrugation metal pipe the bending re-

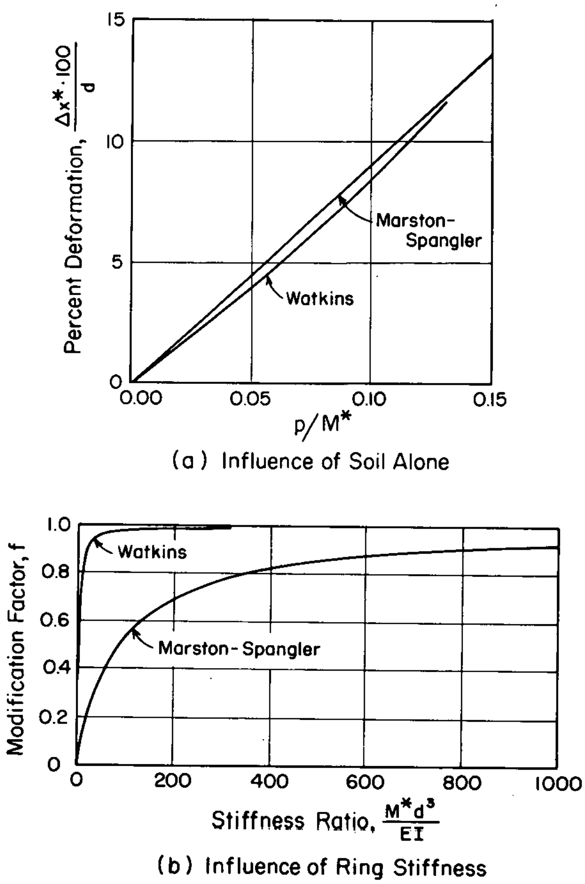


Figure B-7. Comparison of Marston-Spangler and Watkins procedures for design of flexible culverts.

sistance affects the deformation by only 3 percent for E' equal to 700 psi and by only 1.5 percent for E' equal to 1,400 psi. This computation is based on the Marston-Spangler method, which shows the bending resistance of the ring to have greater effect.

A further comparison between the ring compression theory and Watkins' formula, given by Eq. B-23, can be made for the ring compression load. By assuming an incompressible ring and K_o equal to 0.5, and by neglecting higher powers of p/M^*, Eq. B-23 can be rewritten as

$$\frac{2T}{pd} = 1.25 \qquad \text{(B-29)}$$

whereas, for the ring compression theory, the equivalent expression is

$$\frac{2T}{pd} = 1.00 \qquad \text{(B-30)}$$

The observations of Meyerhof and Fisher (*45*), who state:

> Field experiments on corrugated steel culverts under fills of sand, silt, and clay with heights exceeding the pipe diameter have shown that the vertical soil pressures vary between about 50 and 90 percent of the overburden pressure at the top of the culvert, . . .

and White and Layer, who gave consideration to field measurements in proposing the ring compression theory, tend to indicate that Watkins' theoretical approach for determining the ring compression load produces an overly conservative relationship.

In summary, current pipe culvert design practices in the United States may be divided into two categories—rigid and flexible. Rigid pipe culverts are designed almost entirely according to methods developed by Marston and Spangler, whereas flexible pipe culverts are generally designed by determining deformations from the Iowa formula (*29*) and compressive forces in the wall from the ring compression theory. An alternative design method for both deformation and ring compression load has been developed by Watkins (*25*). Deformations calculated from the two methods are similar, provided the bending resistance of the pipe wall is small. However, comparison of ring compression loads indicates that those computed from Watkins' work are somewhat higher than those computed from the ring compression theory.

The Bureau of Public Roads' manuals (*39, 42*), prepared by Townsend, are widely used in current design practice for both rigid and flexible culverts. The manual for rigid culverts is based on the work of Marston and Spangler, whereas the manual for flexible culverts is based largely on the works of Spangler, Watkins, and White. Both provide construction recommendations and charts to facilitate design.

APPENDIX C

CURRENT RESEARCH

In an effort to improve the currently popular and commonly used design and analysis procedure described in Appendix B, considerable research is presently under way to supplement the reservoir of knowledge and experience available to the highway engineer concerned with soil-culvert systems. Most of this work is directed toward the deep-culvert problem, although a limited amount of shallow-culvert research is being conducted. In general, current research, instead of seeking modifications to currently used theories, is based on elastic continuum theories with considerable emphasis on computer-oriented approaches. The one notable exception to this is the work being performed to better identify and quantify the settlement ratio and modulus of soil reaction parameters that are inherent components in currently used procedures. Several specific items of current research are described and evaluated in the following sections.

ANALYTICAL STUDIES

Owing to the great expense associated with conducting experimental studies, especially full-scale investigations, a large proportion of current research activities are of an analytical nature. However, lest there be any misunderstanding or overenthusiastic tendency to accept any of the relatively sophisticated approaches described subsequently, either in this report or in future research efforts, it must be remembered that, from an engineering point of view, any and all analytical studies must serve only as a guide toward anticipating the actual behavior of an installed soil-culvert system, and the results of such investigations must not be construed as design procedures until they are substantiated by adequate field evidence.

Elastic Continuum Approach

Abundant analytic solutions to a variety of buried conduit problems may be found in the literature, but very often the conditions assumed to render such problems tractable preclude any meaningful application to a soil-culvert system. Nevertheless, some solutions are available that can be interpreted to provide insight into the soil-culvert problem. One example of such an application is the work of Mindlin (*23*), as discussed by Moran et al. (*26*). As discussed previously, solutions based on an elastic continuum approach, as well as those obtained by finite element techniques (discussed subsequently), have the advantages that (1) the effect of the soil-culvert interaction is automatically taken into account, (2) relatively basic material parameters are used, and (3) conduits of intermediate stiffnesses can be considered. At present, the interaction effect is taken into account in an empirical manner, "intermediate" and unmeasureable material parameters (r_{sd} and E') are employed, and design methods treat rigid conduits and flexible conduits separately, with no design method available for intermediate cases.

With regard to the latter situation, Terzaghi (*16*) suggested for tunnel linings in the Chicago subway the use of a thin concrete shell, 8 in. thick for the 20-ft diameter, instead of the 2- to 3-ft-thick more conventional structure. More recently, Lane (*17*) measured a considerable reduction in the structure loading after installing steel "hinges" in one of eight 30-ft-diameter reinforced concrete lined tunnels at Garrison Dam. With the advent of plastic, bituminous fiber, and asbestos cement pipes, certainly a need exists for a design method that has the potential for considering the intermediate range of stiffness, while, if possible, providing better results for the limiting cases of the rigid and flexible conduits.

Adaptation of Solution by Burns and Richard

One example of such an approach can be developed from the theoretical work of Burns and Richard (*10*), wherein a circular conduit of an elastic material is buried deeply in a weightless, homogeneous, isotropic, linearly elastic soil and the response (stresses and deformations) of the system is determined for loads applied in the form of a surface overpressure; the nature of this overpressure is discussed subsequently. Subject to the foregoing conditions, the following equations have been obtained (*10*) for the limiting cases of full-slip and no-slip at the soil-culvert interface:

Full-slip:

$$w = \frac{pr}{M^*} \tfrac{1}{2} \left\{ UF[1 - a_0^*] - \frac{2}{3} VF[1 + 3a_2^{**} - 4b_2^{**}]\cos 2\psi \right\} \tag{C-1}$$

$$T = pr \left\{ B[1 - a_0^*] + \frac{C}{3}[1 + 3a_2^{**} - 4b_2^{**}]\cos 2\psi \right\} \tag{C-2}$$

$$M = pr^2 \left\{ \frac{C}{6}\frac{UF}{VF}[1 - a_0^*] + \frac{C}{3}[1 + 3a_2^{**} - 4b_2^{**}]\cos 2\psi \right\} \tag{C-3}$$

$$p_r = p\{B[1 - a_0^*] - C[1 + 3a_2^{**} - 4b_2^{**}]\cos 2\psi\} \tag{C-4}$$

No-slip:

$$w = \frac{pr}{M^*} \tfrac{1}{2} \{UF[1 - a_0^*] - VF[1 - a_2^* - 2b_2^*]\cos 2\psi\} \tag{C-5}$$

$$T = pr\{B[1 - a_0^*] + C[1 + a_2^*]\cos 2\psi\} \tag{C-6}$$

$$M = pr^2 \left\{ \frac{UF}{VF}[1 - a_0^*] + \frac{C}{2}[1 - a_2^* - 2b_2^*]\cos 2\psi \right\} \tag{C-7}$$

$$p_r = p\{B[1 - a_0^*] - C[1 - 3a_2^* - 4b_2^*]\cos 2\psi\} \tag{C-8}$$

in which

w = radial displacement of the conduit wall;
T = ring compression load per unit length;
M = bending moment in the conduit wall per unit length;
p_r = radial pressure at the soil-conduit interface;
p = overpressure;
r = mean radius of the conduit;
M^* = constrained soil modulus;
ψ = angle relative to the horizontal;
$UF = \dfrac{2BM^*r}{EA}$, extensional flexibility ratio;

$$VF = \frac{2CM^*r^3}{6EI}, \text{ bending flexibility ratio;}$$

$$B = \frac{1}{2}\frac{1}{1-\nu}, \text{ soil parameter;}$$

$$C = \frac{1}{2}\frac{1-2\nu}{1-\nu}, \text{ soil parameter;}$$

$$a_0^* = \frac{UF-1}{UF+B/C}, \text{ soil-conduit parameter;}$$

$$a_2^* = \frac{C(1-UF)VF + 2B - (C/2)(C/B)UF}{(1+B+CUF)VF + 2(1+C) + (1+C/2)(C/B)UF},$$

soil-conduit parameter;

$$b_2^* = \frac{(B+CUF)VF - 2B - (C/2)UF}{(1+B+CUF)VF + 2(1+C) + (1+C/2)(C/B)UF},$$

soil-conduit parameter;

$$a_2^{**} = \frac{(2VF-1+1/B)}{(2VF-1+3/B)}, \text{ soil-conduit parameter;}$$

$$b_2^{**} = \frac{(2VF-1)}{(2VF-1+3/B)}, \text{ soil-conduit parameter;}$$

EA = circumferential extensional stiffness per unit length;

EI = circumferential bending stiffness per unit length; and

ν = Poisson's ratio for soil.

Investigation of the Burns and Richard equations produces some interesting results when the equations are interpreted in terms of the soil-culvert interaction problem. For example, when the shear stress between the soil and the conduit wall is considered to be zero (full-slip case), the radial pressure, P_r, on the conduit wall is given by Eq. C-4. If the bending resistance of the conduit is considered to be zero ($VF = \infty$) and the resistance to circumferential stress is considered to be infinite ($UF = 0$), Eq. C-4 reduces to $p_r = p$. Hence, for a flexible conduit where no shear stresses occur at the soil-culvert interface, the radial soil pressure on the conduit wall is hydrostatic and equal in magnitude to the weight of the overburden. This relationship was postulated in the ring compression theory by White and Layer (*9*) purely on the basis of field measurements. To justify the preceding assumption that the resistance to circumferential stress is infinite, consider the full-slip case studied previously; substitution of $VF = \infty$ alone into Eq. C-4 leads to

$$p_r = \frac{p}{2C\dfrac{M^*r}{EA} + 1} \tag{C-9}$$

If one considers the special case of a 10-ft-diameter 10-gauge corrugated metal conduit together with the conditions $\nu = 0.4$, $M^* = 2 \times 10^5$ psf, $A = 0.1454$ in.²/in., and $E = 30 \times 10^6$ psi, the radial pressure, as determined by Eq. C-9, may be written

$$p_r = \frac{p}{\dfrac{2 \times 0.167 \times 2 \times 10^5 \times 5}{30 \times 10^6 \times 0.1454 \times 12} + 1} = 0.994p \tag{C-10}$$

A comparable result is obtained for the no-slip case. Because this case represents an extreme as far as the effect of circumferential stress is concerned, it may be reasonably concluded that the effects of circumferential stress on the radial pressure may be neglected under normal circumstances.

The circumferential thrust in the wall of a rigid conduit for the full-slip case may be considered by use of

$$T = pr\{B[1 - a_0^*] + C/3[1 + 3a_2^{**} - 4b_2^{**}]\cos 2\psi\} \tag{C-11}$$

in which T equals circumferential thrust per unit length. As UF and VF vanish in the limit and for the case where ν equals 0.4, Eq. C-11 reduces to

$$T = pr(1 + 0.25\cos 2\psi) \tag{C-12}$$

Specifically, one has $T = 1.15\ pr$ at the springline ($\psi = 0$), $N = pr$ at $\psi = 45°$, and $T = 0.85\ pr$ at the crown ($\psi = 90°$). Similarly, for the no-slip case the corresponding values are $T = 1.29\ pr$ at the springline ($\psi = 0$), $T = pr$ at $\psi = 45°$, and $T = 0.71\ pr$ at the crown ($\psi = 90°$).

For the perfectly flexible conduits, the deformation equations for the full-slip case reduce to

$$\frac{wM^*}{pr} = \frac{1}{B} \tag{C-13}$$

and for the no-slip case to

$$\frac{wM^*}{pr} = \frac{2}{1+B} \tag{C-14}$$

For the particular case where Poisson's ratio, ν, is taken as 0.4, Eqs. C-13 and C-14 become

$$\frac{wM^*}{pr} = 1.21 \text{ for full-slip} \tag{C-15}$$

and

$$\frac{wM^*}{pr} = 1.09 \text{ for no-slip} \tag{C-16}$$

The foregoing results may be readily compared with the Iowa formula (*29*)

$$\Delta x = \frac{D_l K W_c r^3}{EI + 0.061E'r^3} \tag{C-17}$$

in which Δx equals change in diameter; D_l equals deflection lag factor; K equals bedding factor; W_c equals vertical load on conduit per unit length; and E' equals modulus of soil reaction, by assuming that $D_l = 1$, $K = 0.083$, $w = \Delta x/2$, and $E' = 1.5M^*$ (*44*); then, Eq. C-17 reduces to

$$\frac{wM^*}{pr} = \frac{0.083}{\dfrac{1}{\dfrac{M^*r^3}{EI}} + 0.0915} \tag{C-18}$$

or for $EI = 0$, it becomes

$$\frac{wM^*}{pr} = 0.91 \tag{C-19}$$

The foregoing comparisons indicate that the application of the work of Burns and Richard to some of the special cases already treated by well-tried design methods produces results of similar magnitude. Further evidence of these similarities will be given in the design examples at the end of this section. Although this limited evidence certainly cannot be regarded as proof of the validity of the application suggested, it provides, in conjunction with the theoretical justification, a basis for some confidence in the development of a design procedure founded on these techniques.

Development of Design Curves

The use of $UF = 0$ in conjunction with Eqs. C-1 through C-3 and C-5 through C-7 allows dimensionless parameters to be written as follows for the deformation, thrust, and moment at the spring- and crownlines ($\psi = 0°$ and $90°$, respectively):

Full-slip:

$$\frac{wM^*}{rp} = \pm \frac{1}{3} VF[1 + 3a_2^{**} - 4b_2^{**}] \tag{C-20}$$

$$T/pr = 1 \pm \frac{C}{3}[+ 3a_2^{**} - 4b_2^{**}] \tag{C-21}$$

$$\frac{M}{pr^2} = \pm \frac{C}{3}[+ 3a_2^{**} - 4b_2^{**}] \tag{C-22}$$

No-slip:

$$\frac{wM^*}{pr} = \pm \frac{VF}{2}[1 - a_2^* - 2b_2^*] \tag{C-23}$$

$$\frac{T}{pr} = 1 \pm C[1 + a_2^*] \tag{C-24}$$

$$\frac{M}{pr^2} = \pm \frac{C}{2}[1 - a_2^* - 2b_2^*] \tag{C-25}$$

It is shown in the foregoing that the effect of UF on the radial pressure is negligible for practical cases. Similarly, it can be shown that its effect on deformation, thrust, and moment is likewise negligible, and the assumption $UF = 0$ is felt to be justified. Although the substitution $UF = 0$ is theoretically valid for the deformation and thrust equations over the entire range of VF values from zero to infinity, the moment equations (Eqs. C-22 and C-25) do not necessarily apply for the case where $VF = \infty$, because the limits of Eqs. C-3 and C-7 are indeterminate as UF approaches zero and VF approaches infinity. However, this condition can be shown to be purely a theoretical consideration because trial substitutions show Eqs. C-22 and C-25 to be applicable for the range of values found in practice. Because the independent variables in Eqs. C-20 through C-25 are only Poisson's ratio, ν, and the bending flexibility parameter, M^*r^3/EI, curves may be calculated and plotted to relate the deformation, thrust, and moment dimensionless parameters, respectively, to M^*r^3/EI for various values of ν; such curves are shown in Figures C-1, C-2, and C-3.

Discussion of Assumptions

To summarize, the theory on which the curves in Figures C-1, C-2, and C-3 are based involves the assumptions that (1) the conduit material is elastic, homogeneous, and isotropic, (2) the medium is elastic, homogeneous, isotropic, and infinite in extent, and (3) the response of the system is due to a unidirectional uniformly distributed overpressure acting at an infinite distance from the conduit. Certainly any error concerning the conduit material assumption will be well within the accuracy of this soil-culvert interaction problem. The assumption of elastic

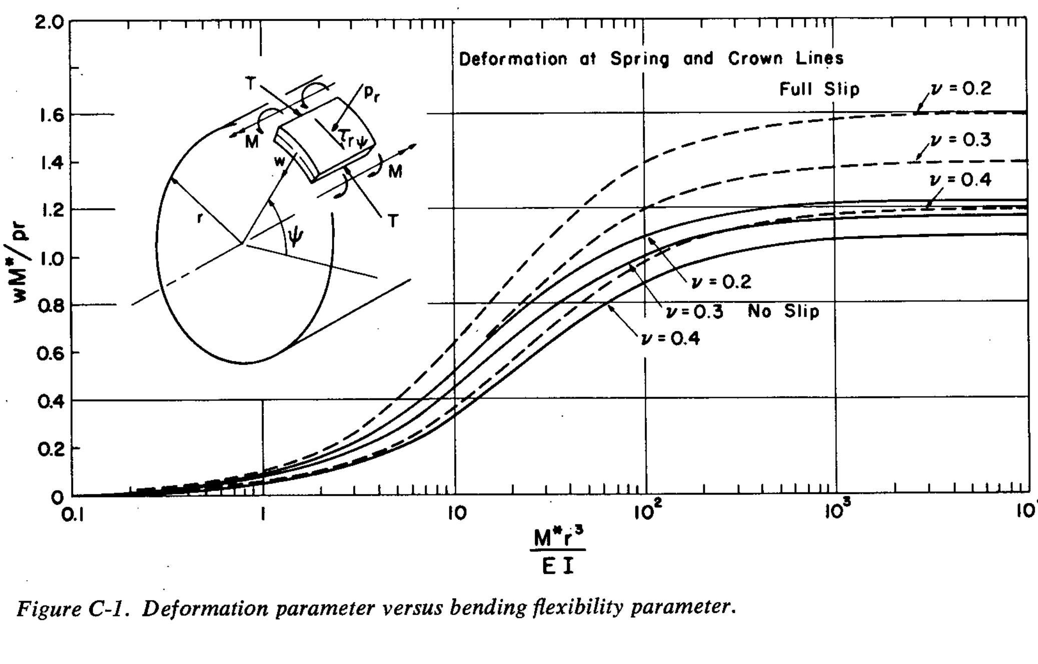


Figure C-1. Deformation parameter versus bending flexibility parameter.

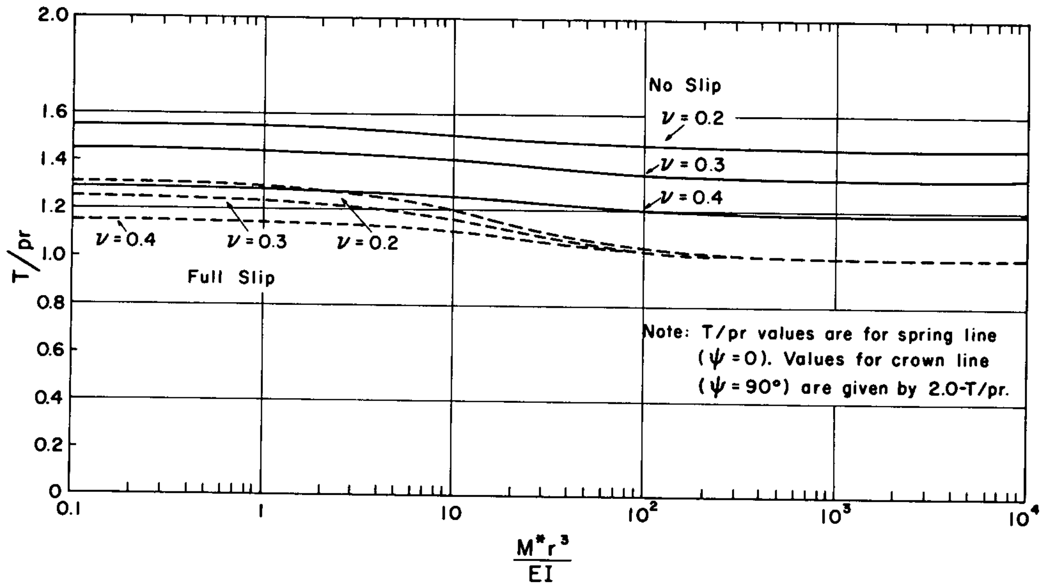


Figure C-2. Thrust parameter versus bending flexibility parameter.

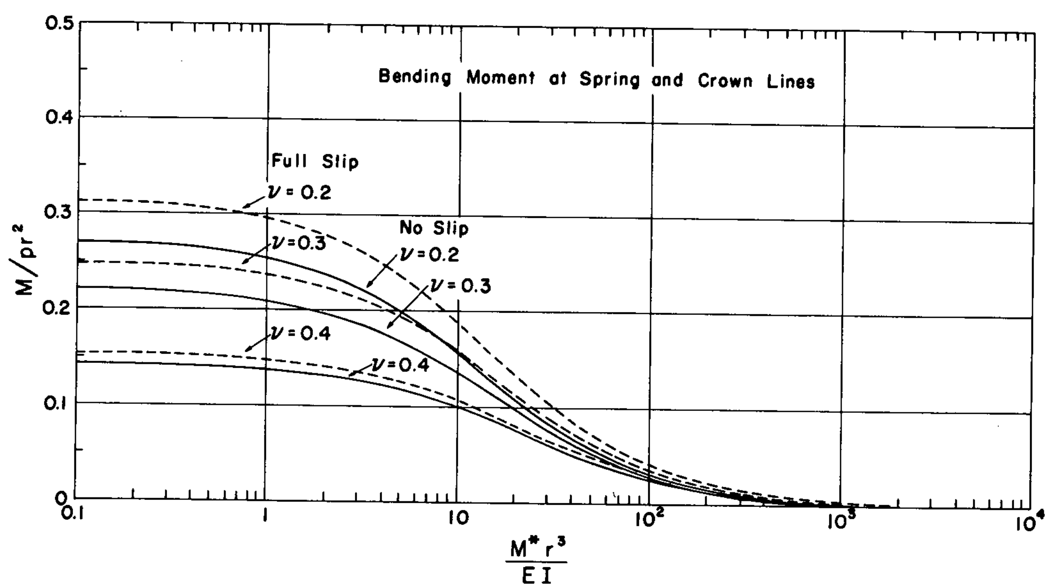


Figure C-3. Moment parameter versus bending flexibility parameter.

behavior for the soil, however, is open to question. At points of high stress, the shear strength of the soil may be exceeded and plastic flow may occur; in addition, creep may occur at stresses below the soil shear strength. On the positive side, modern high-density compacted fills may probably be approximated by elastic behavior better than many natural soils. At any rate, solutions based on elastic theory are the best available at present.

The homogeneity of the fill may also be questioned. For the compacted fill itself, the field control requirements probably ensure that, relative to the geometrical scale of the conduit, the uniformity is reasonable. However, the underlying soil may have a higher or lower modulus than the fill. The conduit bedding should be designed to avoid any large difference in soil stiffness, because a reasonable degree of homogeneity is highly desirable. In general, a softer foundation will lead to difficulties in controlling the culvert elevation, whereas a harder foundation will lead to undesirable nonuniform deformations and excessive loads in the conduit. The appropriate bedding conditions can be obtained by good construction control in overexcavating and replacement of the foundation soils. The applicability of this theory will be considerably enhanced if this construction practice is followed. Alternatively, if the bedding is not appropriate, some adjustment must be made in the values obtained.

Whether a compacted fill responds in a manner approximating an isotropic medium is uncertain. Little research can be found in the literature on this matter and it is expected that some difference may exist between the respective moduli in the vertical and horizontal directions. No consideration is given in this work to these effects.

The load application for the theoretical solution by Burns and Richard is assumed to be a unidirectional overpressure acting outside the zone of the culvert influence. This type of loading leads to two other considerations, the case of shallow fills and the nonlinearity of the embankment stress-strain relationship. Application to the practical case requires that, ideally, during the period of load addition the conduit is confined within a medium of infinite extent. Because loading of a culvert begins essentially when the fill level rises above the springline, the assumed confinement conditions are certainly not realized and computations based on the foregoing theory would not be correct. However, the conduit behavior during this fill period is largely controlled by construction practices. For this reason, only the response due to the addition of the fill above the crown is considered in computations for the deformation and moment. For the same reason, the response of the soil-culvert system due to the first few feet of fill above the crown cannot be accurately computed; therefore, although the over-all effect for deeper fills will be slight, accurate results cannot be expected for shallow cover conditions. Furthermore, because the stress-strain relationship is nonlinear, the total load cannot be considered as applied instantaneously. The constrained modulus of the soil will vary according to the stress condition at the conduit, and an incremental or stepwise calculation, as described later, is desirable to simulate the progress of loading in the field.

Constrained Soil Modulus

On the basis of the results of consolidation tests on a variety of compacted soils, Osterberg (*46*) suggested that, for many commonly encountered soils, the constrained modulus, M^*, may be uniquely determined from the compacted dry density. Curves based on this work (Fig. C-4) show the tangent constrained modulus versus soil stress for various values of dry density. Undoubtedly, there are many variables that can affect the compressibility of a compacted soil; among these, Lambe (*47*) lists temperature, soil composition, characteristics of permeant, void ratio, degree of saturation, and structure. Many of these variables are taken into account with dry density. Lambe suggests that, for clay samples compacted to the same dry density, one above and one below optimum moisture content, the one compacted at the lower moisture content will exhibit a more nearly linear void ratio-stress relationship; however, the degree of difference is not indicated. Osterberg's work appears to show that this difference is not sufficient in practice to prohibit the use of some average curve for design purposes. In addition, Lambe's observations apply primarily to clays, and the variation would be expected to be less for soils of a lower clay content. On the other hand, Osterberg's work does not appear to have been independently substantiated, and, for this reason, the relationship must be treated with caution, particularly where the less common embankment materials are concerned. For example, data from tests conducted by Dorris (*48*) on uniformly graded sand indicate moduli considerably higher than those shown by Osterberg's curves for a similar dry density.

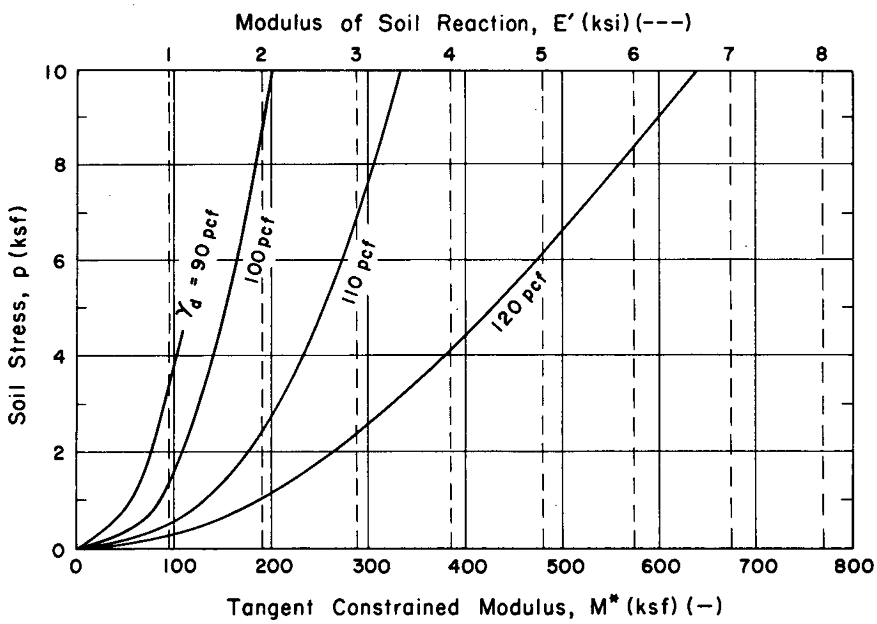


Figure C-4. Tangent constrained modulus and modulus of soil reaction versus stress level for various dry densities.

Nevertheless, in the methods currently used for design, selection of the soil modulus is extremely crude and it is believed that the use of Osterberg's curves interpreted in conjunction with sound engineering judgment offers considerable improvement. Additional research may lead to further refinement of the curves, including some provision for a wider range of materials.

Shear Failure at Soil-Culvert Interface

The two cases considered by Burns and Richard, the case of full-slip for which the shear stress at the soil-culvert interface is assumed zero, and the case of no-slip for which the shear strength at the same interface is assumed to always exceed the shear stress, are shown on the response graphs. In practice, high values of shear stress at the soil-culvert interface may result in shear failure, and the effect on the over-all response, referring to the curves of Figures C-1, C-2, and C-3, will be to move away from the no-slip curves and toward the full-slip curves. The relationship between the theoretical shear stress, $T_{r\psi}$, and the theoretical overpressure for the no-slip case may be studied from the Burns and Richard equation

$$T_{r\psi} = p\{C[1 + 3a_2{}^* + 2b_2{}^*]\sin 2\psi\} \qquad \text{(C-26)}$$

The shear stress distribution is theoretically sinusoidal, ranging from zero at the spring and crown lines to a maximum at $\psi = 45°$. The graphical representation of the variation of shear stress with flexibility at $\psi = 45°$ for $UF = 0$ is shown in Figure C-5. It may be seen that, for a Poisson's ratio of 0.4, the theoretical shear stress may range from 29 percent of the overpressure for rigid conduits to 36 percent of the overpressure for flexible conduits. At $\psi = 45°$ the radial pressure is theoretically equal to the overpressure, and a range of 25° to 35° for angles of internal friction for the fill would indicate a shear strength of $0.5p$ to $0.7p$. These computations appear to indicate that the possibility of shear failure under most circumstances is slight, and that response values nearer the no-slip curves should be used. However, the possibility of creep at stress levels below the shear strength of the soil should be considered. It is suggested that the computations be made for both the full-slip and no-slip cases. In many cases the differences will be small and a decision between the two will not be critical. Where the difference becomes important, however, consideration in the light of the foregoing discussion should enable a sound estimate to be made.

TABLE C-1

TYPICAL VALUES FOR POISSON'S RATIO FOR SOILS

INVESTIGATOR	SOIL TYPE	POISSON'S RATIO
Terzaghi	Sand	0.3 [a]
	Clay	0.41-0.43 [a]
Pokrovsky	Clay	0.38-0.40 [a]
Ramspeck	Moist clay	0.5
	Loess	0.44
	Sandy soils	0.42-0.47
Tsytovick	Sandy soils	0.15-0.25
	Clay with some sand and silt	0.30-0.35
	Clays	0.35-0.40
Katsenelenbogen	Pure clay	0.50
	Clay with 30% sand	0.42

[a] Computed by Barkan from K_o values reported by these authors.

Values for Poisson's Ratio

The selection of a value to be used for Poisson's ratio is difficult and little information is available to serve as a guide for its determination. Barkan (*49*) has summarized reported values in Table C-1. In obtaining some of the values in Table C-1 for cases where the investigators reported the coefficient of earth pressure at rest, Barkan used the theory of elasticity relationship

$$\nu = \frac{K_o}{1 + K_o} \qquad \text{(C-27)}$$

in which K_o is the coefficient of earth pressure at rest. On the basis of the data in this table, Barkan concludes that Poisson's ratio for clays is close to 0.5 and for sands is about 0.30 to 0.35.

Brooker and Ireland (*50*) have accumulated data from their own research and from that of others, and they have concluded that Jaky's relationship

$$K_o = 1 - \sin \phi \qquad \text{(C-28)}$$

is applicable to cohesionless soils; they propose a new relationship, given by

$$K_o = 0.95 - \sin \phi' \qquad \text{(C-29)}$$

in which ϕ' is the effective angle of internal friction for cohesive soils. If Eqs. C-27, C-28, and C-29 are applied over probable ranges of ϕ and ϕ', the following results are obtained:

Cohesionless soils	$\phi = 30°\text{-}35°$	$\nu = 0.33\text{-}0.30$
Cohesive soils	$\phi' = 10°\text{-}20°$	$\nu = 0.44\text{-}0.38$

These calculations do not apply specifically to compacted fills, but, in the absence of data for soil under the appropriate conditions, they may be used as a guide for the selection of reasonable values.

Possible Design Procedure

The general form of a possible design procedure is as follows:

1. Select a trial thickness for the conduit wall cross section.
2. Divide the fill above the conduit into a convenient number of layers.
3. Determine the tangent constrained modulus, M^*, for the first layer from Figure C-4 in accordance with the appropriate compacted dry density, γ_d, of the fill and the average stress level in the soil at the conduit springline during the construction of the layer.

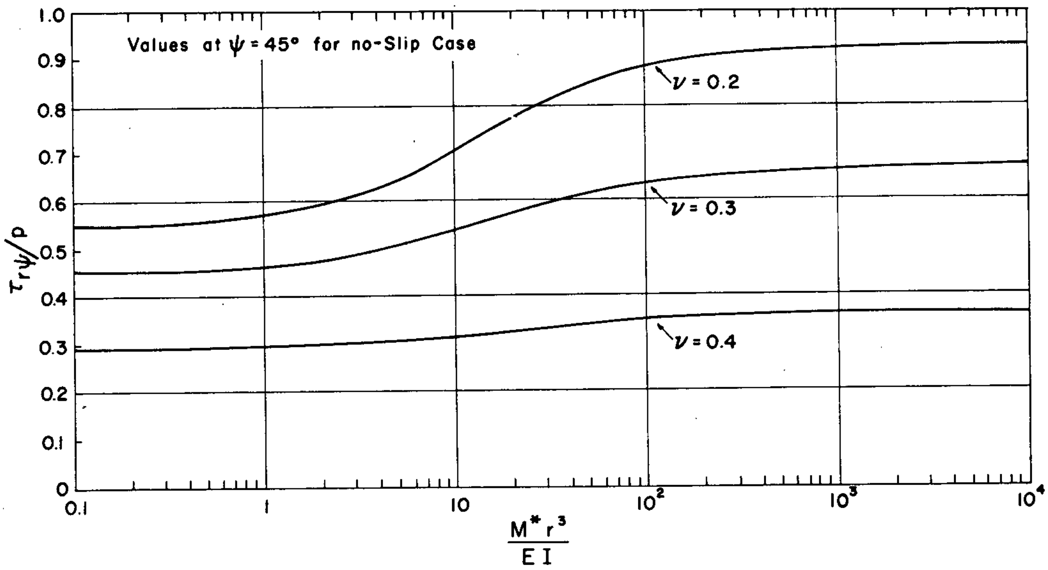


Figure C-5. Tangential shear stress at soil-conduit interface.

4. Determine the dimensionless flexibility parameter, M^*r^3/EI.

5. On the basis of an assumed value for Poisson's ratio, obtain wM^*/pr from Figure C-1 for both the full-slip and the no-slip conditions.

6. Determine the deformation for the first layer.

7. Repeat steps 3, 4, 5, and 6 for the remaining layers.

8. Obtain the total deformation by summing the deformation values for the individual layers.

9. Because the thrust and moment in the conduit wall are relatively insensitive to load path, it is sufficient when determining these values to use the modulus associated with the stress level existing at the mid-height of the fill; by use of this value for M^*, M^*r^3/EI is calculated and the parameters N/pr, M/pr^2 are read from Figures C-2 and C-3, respectively.

10. The total thrust and moment are determined by use of the maximum soil stress at the springline of the conduit.

11. The response of the assumed conduit section is then considered in the light of appropriate failure criteria and safety factors; if necessary, the section is revised and the process is repeated.

Sample Problems

Example 1.—This example has been used previously by Spangler (*51*):

> A 60-inch, 10-gauge corrugated metal pipe (standard corrugations, ½-inch deep at 2⅔-inch centers) is to be installed as a projecting conduit with 60-degree bedding (bedding angle=30 degrees) and covered with an embankment 20 feet high. Assume the projection ratio equals 0.7, the settlement ratio equals 0, the unit weight of soil equals 120 psf, and the value of E′ is 700 psi. Determine the long-time deflection of the pipe with the deflection lag factor of 1.25.

The answer according to Iowa formula is 2.68 in.; calculations according to the proposed method are given in Table C-2.

Example 2.—This example also has been used by Spangler (*51*):

> A 60-inch reinforced concrete culvert pipe having 6-inch sidewalls is to be installed as a projecting conduit in a Class C bedding and covered with an embankment 24 feet high. Assume the projection ratio is 0.7, the settlement ratio is 0.7, and the unit weight of the soil is 120 pcf. Also, assume that lateral earth pressure *K* equal to 0.33 is effective over the full projection of the pipe, *m* equals 0.7. Determine the required strength of pipe which will not develop a crack wider than 0.01 inch.

Assume Poisson's ratio, ν, equals 0.3, and from γ equals 120 pcf, assume a dry density, γ_d, of 100 pcf.

At the mid-height of the fill, $p = 13.25 \times 120 = 1{,}590$ psf; from Figure C-4, $M^* = 95{,}000$ psf; whereupon

$$\frac{M^*r^3}{EI} = \frac{95{,}000 \times 2.5^3 \times 12^2}{3 \times 10^6 \times 12 \times 6^3} = 0.0275.$$

By use of Figure C-3, one gets

Full-slip case: $M/pr^2 = 0.25$

No-slip case: $M/pr^2 = 0.225$

from which

Full-slip case: $M = 24 \times 120 \times 2.5^2 \times 0.25$
$= 4{,}690$ ft-lb/ft

No-slip case: $M = 24 \times 120 \times 2.5^2 \times 0.225$
$= 4{,}230$ ft-lb/ft

TABLE C-2

CALCULATIONS FOR EXAMPLE PROBLEM

DEFORMATION — $\gamma_T = 120$ PSF — ASSUME $\gamma_c = 100$ PSF — ASSUME $\nu = 0.3$

LAYER NO.	LAYER THICKNESS (FT)	LAYER LOAD (PSF)	AVERAGE STRESS AT SPRINGLINE (PSF)	M^* (PSF)	$\frac{M^* r^3}{EI} = M^* \times 0.0111$	FULL-SLIP $\frac{wM^*}{pr}$	FULL-SLIP DEFORMATION, w (FT)	NO-SLIP $\frac{wM^*}{pr}$	NO-SLIP DEFORMATION, w (FT)
1	5	600	600	70,000	777	1.37	$\frac{600\times2.5\times1.37}{70,000}=0.030$	1.15	$\frac{600\times2.5\times1.15}{70,000}=0.024$
2	5	600	1,200	90,000	999	1.375	$\frac{600\times2.5\times1.375}{90,000}=0.023$	1.15	$\frac{600\times2.5\times1.15}{90,000}=0.019$
3	5	600	1,800	102,000	1,132	1.38	$\frac{600\times2.5\times1.38}{102,000}=0.020$	1.15	$\frac{600\times2.5\times1.15}{102,000}=0.017$
4	5	600	2,400	114,000	1,265	1.38	$\frac{600\times2.5\times1.38}{114,000}=0.018$	1.15	$\frac{600\times2.5\times1.15}{114,000}=0.015$
Total							0.091 Change in diameter = 2.2 in.		0.075 = 1.81 in.

THRUST	NO-SLIP	FULL-SLIP
Average stress $=\frac{22.5}{2}\times120=1,350$ psf	$T/(pr)=1.325$	$T/(pr)=1$
$M^*=93,000$ psf	$T=2.5\times22.5\times120\times1.325=8,930$ lb/ft	$T=22.5\times120\times2.5=6,740$ lb/ft
$\frac{M^*r^3}{EI}=93,000\times0.0111=1,032$		

The three-edge bearing test moment at the springline is given by

$$M = 0.182\, P_c r \qquad \text{(C-30)}$$

in which P_c is the concentrated load at the crown. Therefore, the equivalent three-edge bearing test loads required to produce the calculated bending moments are:

$$\text{Full-slip case: } P_c = \frac{M}{0.182r} = \frac{4,690}{0.182 \times 2.5} = 10,300 \text{ lb/ft}$$

$$\text{No-slip case: } P_c = \frac{M}{0.182r} = \frac{4,230}{0.182 \times 2.5} = 9,300 \text{ lb/ft}$$

From ASTM C 76-66T,

Class IV pipe: 0.01-in. crack at 2,000 d = 10,000 lb/ft

Class V pipe: 0.01-in. crack at 3,000 d = 15,000 lb/ft

Hence, use Class V pipe, and the safety factors against a 0.01-in. crack are

$$\text{Full-slip case: SF} = \frac{15,000}{10,300} = 1.46$$

$$\text{No-slip case: SF} = \frac{15,000}{9,300} = 1.62$$

Note that the preceding computation assumes a uniform condition surrounding the culvert; unless field control ensures that this condition is satisfied, stress conditions may be less favorable.

Example 3.—A 10-ft-diameter circular reinforced concrete conduit having a 6-in. wall thickness is to be installed beneath 100 ft of fill having a compacted unit weight of 130 pcf and a dry density of 110 pcf. Analyze the response of the system.

Table C-3 gives the computations for w, T, and M. If one assumes the no-slip case, $w = 2.0$ in., $T = 50,500$ lb/ft, and $M = 24,200$ lb/ft. On the basis of the moment-area method, the hoop deformation corresponding to a maximum strain of 0.003 is given by $\Delta x = d^2/1,200t = \frac{100 \times 12}{1,200 \times 0.5} = 2$ in. Therefore, the design is satisfactory for deformation. The concrete section may be analyzed in accordance with the following approach (i.e., ultimate design of eccentrically loaded columns failing in tension); the notation is shown in FigureC-6.

$$p_u = 50,500 \text{ lb/ft}$$

$$e' = \frac{24,200}{50,500} \times 12 = 5.8 \text{ in.}$$

$$t = 6 \text{ in.}$$

$$d = 4\tfrac{1}{2} \text{ in.} \qquad d' = 1\tfrac{1}{2} \text{ in.}$$

$$f_y = 50,000 \text{ psi}$$

$$f'_c = 6,500 \text{ psi}$$

$$A_s = A_s' = 2.355 \text{ in.}^2/\text{ft } (\#8 \text{ at } 4 \text{ in.})$$

$$A_s = A_s' \qquad T = C_s \qquad C_c = P_u$$

$$C_s(d - d') = P_u z \qquad z = e' - 0.5t + 0.5a$$

$$T = A_s f_y = 2.355 \times 50,000 = 118,000 \text{ lb} = C_s$$

$$z = 5.8 - 3 + a/2 = 2.8 + a/2$$

$$C_s(d - d') = 118,000 \times 3 = 354,000 \text{ in.-lb}$$

TABLE C-3

CALCULATIONS FOR EXAMPLE PROBLEM

DEFORMATION — ASSUME $\nu = 0.3$

LAYER NO.	LAYER THICKNESS (FT)	LAYER UNIT LOAD (PSF)	AVERAGE STRESS AT SPRINGLINE (PSF)	M^* (PSF)	$\frac{M^* r^3}{EI} = M^* \times 28 \times 10^{-6}$	FULL-SLIP $\frac{wM^*}{pr}$	FULL-SLIP DEFORMATION, w (FT)	NO-SLIP $\frac{wM^*}{pr}$	NO-SLIP DEFORMATION, w (FT)
1	20	2,600	1,950	172,000	4.8	0.31	$\frac{0.31 \times 2600 \times 5}{172,000} = 0.024$	0.26	$\frac{0.26 \times 2600 \times 5}{172,000} = 0.20$
2	20	2,600	4,550	245,000	6.9	0.40	$\frac{0.40 \times 2600 \times 5}{245,000} = 0.021$	0.34	$\frac{0.34 \times 2600 \times 5}{245,000} = 0.018$
3	20	2,600	7,150	292,000	7.2	0.42	$\frac{0.42 \times 2600 \times 5}{292,000} = 0.019$	0.35	$\frac{0.35 \times 2600 \times 5}{292,000} = 0.016$
4	20	2,600	9,750	331,000	8.1	0.45	$\frac{0.45 \times 2600 \times 5}{331,000} = 0.018$	0.39	$\frac{0.39 \times 2600 \times 5}{331,000} = 0.015$
5	20	2,600	12,350	350,000	8.6	0.47	$\frac{0.47 \times 2600 \times 5}{350,000} = 0.017$	0.41	$\frac{0.41 \times 2600 \times 5}{350,000} = 0.015$
Total							0.099		0.084
						Change in radius	= 1.19 in.		= 1.01 in.
						Change in diameter	= 2.4 in.		= 2.0 in.

Thrust $P_{av} = 55 \times 130 = 7,150$ psf, $M^* = 292,000$ psf, $\frac{M^* r^3}{EI} = 7.2$

From Fig. C-2 $T/pr = 1.17$ at springline
and 0.83 at crown

$pr = 55 \times 130 \times 5 = 35,800$
$\therefore T = 41,800$ lb/ft at springline
and 29,750 lb/ft at crown

Full-slip

= 1.41 at springline
and 0.59 at crown
= 50,500 lb/ft at springline
and 21,150 lb/ft at crown

No-slip

Moment

From Fig. C-3 $M/pr^2 = 0.172$
$pr^2 = 50 \times 130 \times 5^2 = 162,500$
$M = 27,970$ ft-lb/ft

Full-slip

= 0.149
= 24,200 ft-lb/ft

No-slip

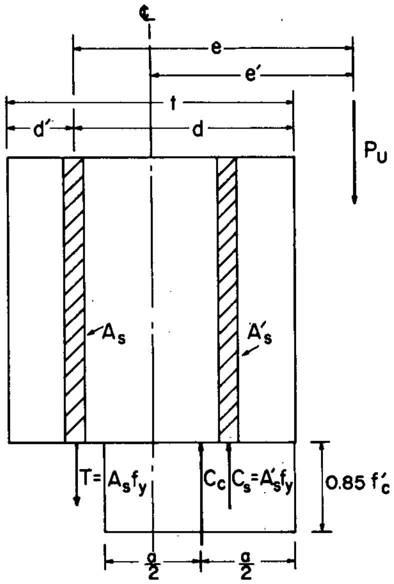


Figure C-6. Notation for eccentrically loaded column section.

Try $a = 2$ in.: $z = 2.8 + 1 = 3.8$

$$\therefore P_u = 354{,}000/3.8 = 93{,}100 \text{ lb}$$

$$\therefore a = P_u/0.85\, f_c'b$$

$$= \frac{93{,}100}{0.85 \times 6{,}500 \times 12} = 1.4 \text{ in.}$$

Try $a = 1.5$ in.: $z = 2.8 + 0.75 = 3.55$

$$\therefore P_u = 354{,}000/3.55 = 99{,}800 \text{ lb}$$

$$\therefore a = \frac{99{,}800}{0.85 \times 6{,}500 \times 12} = 1.51 \text{ in.}$$

$$\text{Load factor} = \frac{99{,}800}{50{,}500} = 1.98 \approx 2$$

Therefore, the design is satisfactory. The preceding approach is highly simplified owing to the assumption that $T = C_s$; a more accurate method (see 1963, *ACI Building Code*) indicates a load factor of 1.39.

Finite Element Method

Many of the limitations of a continuum approach are removed by breaking up the continuum into a series of discrete elements and using numerical analysis. One versatile and convenient numerical technique for evaluating the distribution of stresses and strains within an elastic medium is the finite element method, which is ideally suited for obtaining the solution by means of a digital computer. The finite element approximation of a complex continuum is obtained by dividing it into a network of imaginary lines or surfaces. The elements are assumed to be interconnected only at a discrete number of nodal points that are situated on their boundaries, and it is normally the displacements of these nodal points that are the unknown parameters in the formulation. A displacement configuration is assumed for each of the elements, and the state of displacement within an element is expressed in terms of its nodal displacements. Thus, the state of strain within an element can be obtained in terms of the nodal displacements; the resulting state of stress throughout the element is then defined with the aid of the elastic stress-strain properties of the material.

Fictitious concentrated forces are introduced at the nodal points to represent any distributed loads and/or stresses acting on the element boundaries. Consideration of equilibrium and displacement compatibility between adjacent elements allows the formulation of a system of simultaneous equations that relate the nodal displacements and the nodal forces. Thus, the finite element idealization reduces an elastic continuum problem to a standard structural problem that is amenable to rapid solution on a digital computer.

The material properties of each element within a system need not be constant, but can vary from element to element; hence, the finite element method provides a convenient and simple approach for dealing with problems consisting of nonhomogeneous and/or anisotropic materials. By use of an incremental or step-by-step technique, it is possible to extend the solutions of the linear elastic problem into the range in which nonlinearities are introduced through material properties or through large deformations and geometrical changes in the structure and the elements. An additional advantage of the finite element method is that the size and shape of each element can be graded and so chosen as to follow arbitrary boundaries and to allow for a more detailed study of a desired function in regions where rapid variations are expected.

Recognizing the complexity of the soil-culvert interaction problem, Brown used the finite element method of plane elasticity in an attempt to determine the forces on rigid culverts under high fills (*12, 35*) and flexible culverts under high fills (*13, 52*). The following factors were recognized to affect the magnitude and distribution of loads on culverts: (1) method of fill placement, (2) movement of the embankment, (3) boundary conditions at the interfaces between the embankment and the natural ground and between the embankment and the culvert barrel, (4) deformation and change of shape of the culvert due to fill placement, (5) radical alteration due to placement of soft material and addition of fill in the imperfect trench method of construction, and (6) time dependency of the stress-strain properties of the material within the imperfect trench. As a part of the present study, Brown has provided the basis for the following discussion on the factors that affect the analytical procedure used for evaluating the soil-culvert interaction problem.

Geometry

Existing analytical procedures tend to yield plane solutions in which a precise statement of boundary geometry is required. The finite element solutions of plane elasticity have proved satisfactory in accounting for the geometry of the earth foundation soil, embankment, and culvert. However, in this respect, the finite element method has no advantage over the finite difference method. The effects of longitudinal stretching and culvert bending have been considered separately, but no procedures to yield three-dimensional solutions are yet available.

Fill Sequence

An esoteric analytical approach (*53*) has proved valuable in modeling the fill sequence by incremental procedures. In order to consider the manner of filling, a dislocation or incompatibility tensor must occur in the finished solution; if this is not present, the linear elastic solution will be the same as when the inertial effects are applied like an external load. In fill problems the incompatibility tensor results from a Somigliana dislocation, which occurs when a layer of material is applied to the partially completed body. The normal stress, $\overline{\sigma}_{nn}$, and shearing stress, $\overline{\sigma}_{nt}$, boundary conditions specified on this interface for the incremental boundary value problem are for a level horizontal layer

$$\overline{\sigma}_{nn} = -\gamma \Delta h \qquad \text{(C-31)}$$

$$\overline{\sigma}_{nt} = k\gamma \Delta h \qquad \text{(C-32)}$$

in which γ is the material weight density which could be a space function; Δh is the added material thickness; and k a factor depending on the frictional characteristics of the soil. Currently available solutions have been based on the assumptions that k equals zero and that free tangential motion is allowed on the interface. Although these conditions are seldom true, any value of k must, by equilibrium, satisfy

$$\int_A k dA = 0 \qquad \text{(C-32)}$$

in which A and dA are the total area and an elemental area, respectively, of the interface. This restriction on k likens it to the extension by Goodman and Keer (*54*) of the Hertz problem where it was shown that a tangential incremental boundary condition caused little change from the Hertzian response. With this in mind, it may be reasonable to continue to employ k equal to zero. In any case, there appears to be no difficulty in trying other self-equilibrating values of k and seeing what change occurs in the final solutions; it is believed that such changes will be trivial.

Fill Properties

Apart from the tendency of the incremental approach to replicate the actual physical events (*53*), it also makes the gravity solutions easy and allows the simple incorporation of varying fill density. This may be done by multiplying the influence density by the actual density before integration (*12, 35*). Changes in the fill stiffness can also be included in the finite element technique (an advantage over finite difference), but the more interesting problem is the possibility of changing the local fill stiffness with the local state of strain. This requires a combination of a finite element and an incremental solution, but it is not clear to what extent this dependence is important. One approach is to examine the deviatoric part of a linear elastic solution; if, in fact, the deviatoric energy is small compared with the dilatational energy in a given region, then it would seem that consideration of this feature is not of primary importance. It is possible that consideration of this aspect is important only in the region immediately surrounding the culvert.

The use of organic inclusions in the fill appears to be the dominant factor affecting the pressure distribution on the culvert. Although it is believed that this condition can be properly handled analytically (*52*) by including the deterioration of the material, it would be more satisfactory if an independent formulation could confirm the present methods. The main problem of handling organic materials in a computer program arises in the description of their stiffness characteristics. First, little information is available on the in-situ properties of organic materials; second, any description of material properties considers the problem from a continuum viewpoint. However, for computer solution of a finite element formulation based on incremental concepts, the interest lies in chord properties of loading and unloading for small load changes. It would seem possible to provide such information and to gear tests automatically to program the information. The methods of King (*55*) appear reasonable in the determination of creep effects.

When an equilibrium analysis is completed for the metal culvert, a state of pressure at all levels of fill is described; this is the initial information in a stability study. Generally, it would seem that the stability of the culvert under deep fills is enhanced by the surrounding fill (*43*). Instability considerations may be important for shallow fills with earth-moving equipment on the construction surfaces. To determine the alteration of pressure due to a change in the water level, it is necessary to determine the phreatic surface. The simplest way to do this employs the finite element method for nonhomogeneous bodies (*56*).

Culvert Properties

Some efforts (*13, 52*) have been made to include the effects of the stiffness of a flexible culvert. In the finite element formulation the consequences of an orthogonal pair of forces and a moment are included at each node of a curved member. No difficulty is apparent in applying this technique to a reinforced concrete culvert. With this application the change in stiffness due to dropping reinforcing bars may be investigated; in this way the comments of Davis (*19*) with regard to the effect of changing the reinforcing in a culvert section could be examined. For the case of a so-called rigid culvert, available analysis (*13, 52*) should be adequate; however, the flexible culvert case does need to have the geometry of the culvert at each load increment included in the analysis. It is believed that this ability to follow the geometry of the culvert is absolutely necessary.

A key question to be answered concerns the actual interface condition between the fill and the culvert. Does it mean a great loss of accuracy in critical regions to employ the simplest condition (normal stress continuous, and shearing stress equal to zero)? In this respect, the arguments to use this condition do not seem very respectable (*12, 35*).

Foundation Material

Although the inclusion of the effect of the earth's crust has been described analytically (*12, 35*), no experiments are available to check the formulation, and this effect has never proved important. The use of a definite block size of earth can be argued nicely, but these arguments have not been subjected to critical tests. It is believed that model studies could give a reasonably accurate picture of the way in which the earth's crust affects pressures and stresses on buried conduits. Also, such studies could indicate how a proper analytic method could be formulated. Not only is the block size important, but the support conditions on the block also are of interest. Clearly, if no great advantage is realized by a complex analytical model, the simple approach described previously (*12, 35*) can be justified.

Davis and Bacher (*20*) have indicated that the interface condition between the fill and the foundation soil may be of importance in determining the pressures on the culvert. To date, little information on the actual interface conditions is available, and the same questions raised about the fill-to-culvert effect must be faced. From the work of Davis (*19*), it would seem necessary to include a very accurate statement regarding the properties of the foundation material. Only then can the critical effects of differential motion be included.

DYNAMIC ALTERATION

The effects of earthquakes on dams have been studied recently (*57, 58*). It would seem that the approach used lends itself to considering the changes in pressures on culverts due to earthquakes.

Spring Analog

An interesting design method based on a numerical technique and utilizing the computer has been proposed by Drawsky (*11*) for flexible culverts. The conduit is modeled by a series of rigid straight segments supported at junction points by radially directed springs, each spring representing the influence of the soil on the conduit. The method provides for treatment of a wide variety of field situations, including (1) various conduit shapes (circular, elliptical, arch, etc.); (2) various load types (uniform soil load, H20 live load, E72 live load, and appropriate combinations of these); (3) any specified conduit cross section, including the effect of variable stiffness; and (4) variable soil properties, such as unit weight, coefficient of horizontal earth pressure, and soil modulus (soil modulus may be nonlinear and may vary from spring to spring).

This numerical technique handles simultaneously two major problems; one is a function of the mathematical model postulated and involves the solution of the associated statically indeterminate system, and the other is basic to the soil-culvert interaction problem and involves the interdependence of the deformations, stresses, and moduli of the system. A more detailed description of the method is given in the following abbreviated computer program sequence:

1. Initially the program selects a subroutine dictated by the conduit geometry.
2. Control then passes to a subroutine that generates the flexibility matrix for the applicable conduit cross section.
3. Entry is made to another subroutine that (1) converts the soil modulus characteristics into equivalent nonlinear spring characteristics, and (2) generates an initial set of spring values for the zero deflection condition.
4. Based on the overburden pressure and an assumed coefficient of soil reaction, another subroutine computes the vertical and horizontal pressures at each point around the culvert perimeter.
5. By use of the Boussinesq equation, another subroutine computes the influence of the appropriate live load at each point around the culvert perimeter.
6. A load subroutine converts unit pressures into spring force components.
7. Control passes to a subroutine that calculates the force transformation matrix for the particular culvert section; bending moments and spring forces for the statically determinate base structure are computed for unit loads applied at each point in the horizontal and vertical directions and also for unit loads applied in the direction of the redundants of the system.
8. The program then passes to two subroutines that exercise the sequence of matrix operations for the statically indeterminate analysis.
9. Following the analysis of the structure on the basis of its initial geometry and values of spring stiffness and force, these parameters (including the culvert geometry) are revised in accordance with the computed values and the process is repeated.
10. After four such cycles, the factor of safety for the stability of the assumed system is determined on the basis of the relationship:

$$FS = \frac{d_n - d_{n-1}}{d_{n+1} - d_n} \tag{C-34}$$

in which d_n is the crown deformation at iteration n; if this value is less than 3, the structural section is revised to the next larger section and the process is repeated.

11. Iterations are then continued until convergence to the desired accuracy is obtained.
12. A subroutine then computes the direct stresses and the bending stresses at each point and compares each with the allowable values; if the allowable value is exceeded, the next larger section is selected and the process is repeated.

The basic assumption of this method is that the system of linked, straight segments and springs responds in a manner similar to the real soil-culvert system. Because the soil response for the more general deeply buried condition involving properly placed fills is thought to be one of volumetric compression rather than one of plastic flow, a reasonably accurate representation should be possible by the procedure described previously. However, although the influence of culvert deformation on the response is taken into account, the relative interaction effects of the springs on one another is not considered and it is believed that the arching phenomenon in soil is not properly represented. Also, as a result of independent spring action, the influence of shear resistance between the soil and the conduit is not considered; possibly this effect could be handled by the addition of tangential springs at the radial spring connection points.

The use of this mathematical model to investigate buckling stability appears to offer interesting possibilities. The close relationship between a buckling stability criterion and variations of the system geometry with load make such an analysis highly desirable. Furthermore, the method is capable of considering concentric and eccentric concentrated loads. Currently available methods for considering buckling are incapable of taking any of these factors into account. The application of a higher safety factor (3 is suggested by the researcher) to buckling stability than to ultimate joint strength (1.5 to 2) is a sound innovation because failure from elastic buckling may be catastrophic, whereas compression yield or excessive joint deformation are usually associated with a relief of the culvert load.

Although this approach has some shortcomings, it is felt that these are relatively minor and, provided their possible influence is realized, the applicability of this algorithm to a wide range of field conditions makes it extremely valuable, particularly for the more complex situations. In addition, some of the details of the method suggest avenues for future research.

Arching Analysis

According to Nielson *59*),

> . . . recent observations on buried structures made in the laboratory have led to the conclusion that one is not justified in using the classical Marston theory indiscriminately for loads on underground pipe. To allow for pressure redistribution across the top of a buried flat-roofed structure, a different differential element must be assumed. . . . There is no physical justification for assuming that the arch extends only across the prism of soil directly above the buried structure.

This latter assumption is, of course, employed in the Marston theory and, although it may be reasonably acceptable for pipes in trenches with relatively rigid sides backfilled with loose soil, it does not appear applicable to positively projecting culverts under compacted embankments. Accordingly, Nielson suggested a differential element in the shape of a circular arch, one of which is assumed to act on top of another. A free-body diagram for determining loads on a buried conduit by means of such an arching analysis is shown in Figure C-7. The pressure transmitted away from the buried structure is the pressure that acts at the differential arch support. The suggested locations of the soil arch supports are taken as those regions of maximum shear stress as determined by an elastic analysis before any movement occurs—that is, by ignoring the effect of redistribution of stresses as the soil moves. For the soil arch to form, it is assumed that most of the strain or movement within the soil above the conduit occurs between regions of maximum shear stress and that the movements of soil between the regions of maximum shear stress is downward. Nielson compared the results of this approach with studies by Watkins (*60*), Watkins and Nielson (*61*), and Koepf (*62*), and it was concluded that agreement was sufficiently close to justify the approach and the assumptions made and that the procedure seems adequate for design purposes, including both deflections and pressures transmitted to buried pipes.

In this analysis, which is based on the soil arch concept, the support pressure or stress at the arch support is difficult to determine, and various assumptions (*59*) with varying degrees of uncertainty were introduced by Nielson in an effort to obtain a relationship that was consistent with the physical characteristics of the system. Ultimately, the method used to evaluate this support pressure incorporated

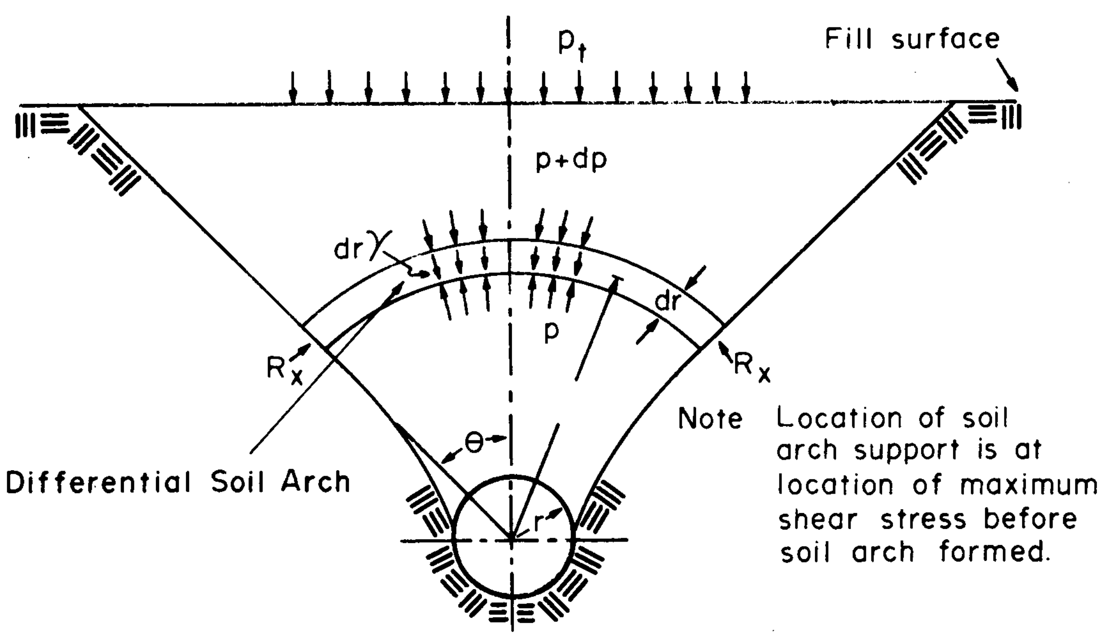


Figure C-7. Free body diagram for determining loads on buried conduits by an arching analysis.

an adaptation of the Spangler deflection equation. Despite the apparently good agreement between this theory and the limited experimental data studied, it seems that the problem of evaluating the stress at the arch support is worthy of additional investigation. Nevertheless, this method manifests an interesting deviation from the classical Marston procedure.

Inverted Settlement Approach

As an alternative to the more usual approach to the determination of loads on buried rigid structures, the following method, based on the compressibility characteristics of the soil rather than the concept of shear stresses on vertical planes, is offered for consideration. It is a strongly engineering-oriented approach and contains approximations and assumptions that may well be improved by further study; however, presentation of the method is felt justified by the fact that it suggests a new concept to the problem of determining total loads on buried rigid structures. For the examples given, this method yields results consistent with experimental measurements and the Marston theory.

The basis of this method rests on the analogy that may be drawn between a deep underground structure toward which the soil moves as the soil load is increased (Fig. C-8a) and a surface structure that moves into the soil as the structure load is increased (Fig. C-8b). As is shown elsewhere in this report, the concept of shear stresses acting on vertical planes above the structure has not been completely acceptable and, on occasion, has led to considerable difficulty in design. Because shear failure or plastic flow of the soil is normally limited to the region in the immediate vicinity of the corners of the structure (particularly for the high shear strengths associated with modern embankments), the process of soil-culvert interaction may be considered basically one of compression, provided there is a sufficient height of cover over the structure. If the cover height is low, such as is represented by plane DE in Figure C-8a, the compression of the overlying material may be of little importance, and the excess loading may be governed largely by the shear strength of the material in the vicinity of F and G. Certainly, some minimum cover height exists below which deformations along shear zones have considerable effect in controlling the load concentration on the structure. Although some consideration is given subsequently to low cover heights, the following development requires that the height of cover exceed this minimum, which is probably about two or three times the width of the structure. This restriction will enhance the use of the approximation, often used for surface structures, that the vertical stress distribution is given by the Boussinesq solution for a homogeneous, isotropic, linearly elastic, weightless half-space. Further simplifications of the Boussinesq distribution are made to simplify the application of this procedure.

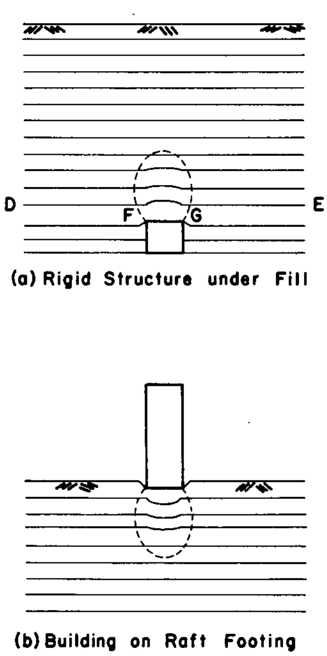


Figure C-8. Schematic diagrams for inverted settlement approach.

If the compressibility characteristics of the soil are known, the relationship between load and deformation for a given set of conditions can be determined. Whereas for the surface structure the deformation is determined under a given load, for the buried structure the load may be determined for a given deformation. This method becomes especially convenient when applied in conjunction with the results of work by Osterberg (*46*) which indicates that the compressibility characteristics of many embankment soils can be uniquely described by dry density; this work is discussed earlier in this appendix, and the resulting curves are shown in Figure C-4. As Figure C-4 shows, the constrained modulus is greatly influenced by the soil stress; at a low stress level, a given deformation will cause a lesser load increase than at a high stress level. This nonlinear behavior means that simplified approaches, such as that of considering the entire fill to be placed instantaneously, may lead to considerable error. It therefore becomes necessary to consider the load as being placed in stages in a manner similar to field placement.

To simplify calculations, the Boussinesq vertical stress distribution is approximated as shown in Figure C-9; the stress is considered to extend for a distance b at a constant intensity of $0.7q$, where q is the stress at the soil-structure interface, and for a further distance $2b$ at a constant intensity of $0.3q$. The effect of stress at a distance greater than $3b$ is neglected.

Development of Procedure

If one now considers, as shown in Figure C-10, a rectangular structure that is resting on a foundation soil with a compressibility similar to that of the fill, the increase in the load on the structure resulting from an increase in the height of fill may be determined by the following process; note that the horizontal mid-plane of the structure may be regarded as a plane of symmetry for this problem.

1. Determine the deformation, δ_p, of point P by first

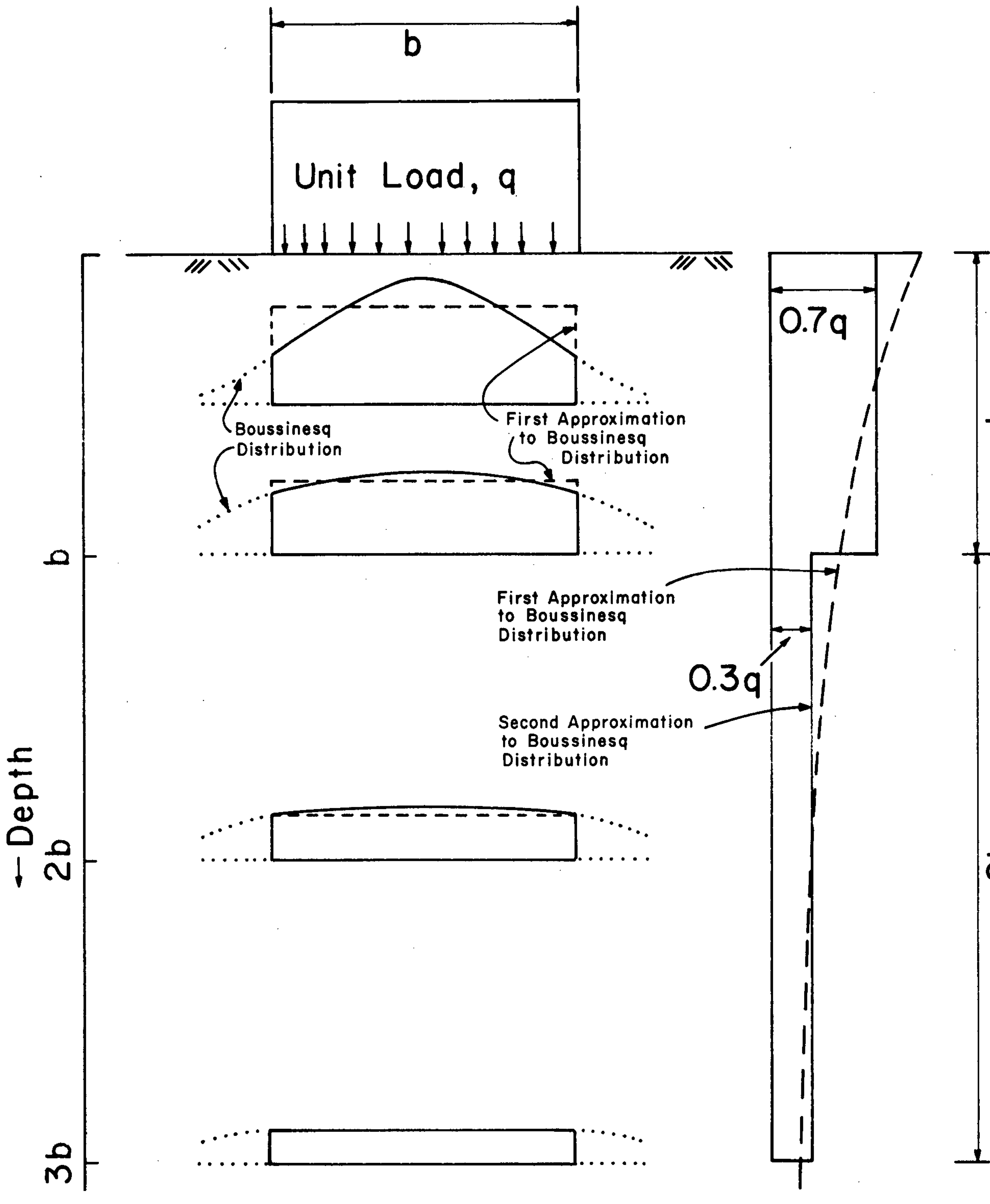
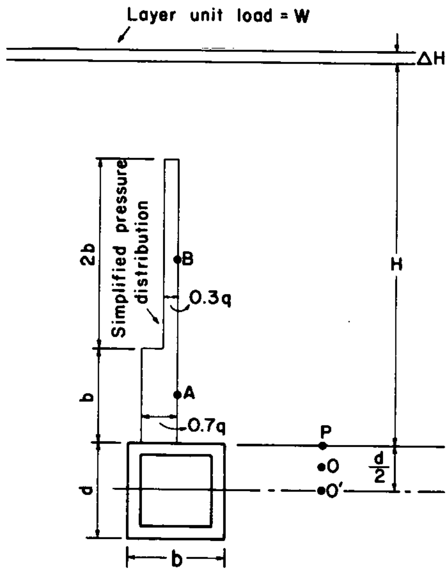


Figure C-9. Approximation for stress distribution due to strip loading.

obtaining from the curves of Figure C-4 the tangent constrained modulus, $M_0{}^*$, corresponding to the soil stress, p_o, at point 0 and the dry density, γ_d. The soil stress, p_o, is given by

$$p_o = (H + \Delta H/2 + d/4)\gamma \qquad \text{(C-35)}$$

in which γ is the total weight density and the other notation is given in Figure C-10; $\Delta H/2$ is used instead of ΔH to better approximate the gradual application of a soil layer of thickness ΔH. The deformation, δ_p, at P may then be determined from



Compacted unit weight of fill = γ

Dry density of fill = γ_d

Figure C-10. Extra-overburden stress distribution above rigid culvert in homogeneous soil.

$$\delta_p = \frac{\gamma \Delta H d}{2M_o^*} \qquad \text{(C-36)}$$

and δ_p will be equivalent to the relative displacement between the structure and the soil mass.

2. Determine the tangent constrained modulus, M_A^*, at A corresponding to the vertical stress, p_A, which will be the sum of the stress, p_{A1}, due to the weight of the column of soil above A and the stress, p_{A2}, due to the arching effect of the fill up to height H. The stress, p_{A1}, may be calculated directly by the relation

$$p_{A1} = \left(H + \frac{\Delta H}{2} - \frac{b}{2}\right)\gamma \qquad \text{(C-37)}$$

but the computation of the stress, p_{A2}, requires a preliminary estimate of a stress concentration ratio, R_o, which is defined as the ratio of the actual load on the structure to the weight of the soil above; determination of R_o is discussed later. Based on the assumptions stated thus far, the stress at A due to arching, p_{A2}, may be determined from

$$p_{A2} = 0.7\left(H + \frac{\Delta H}{2}\right)(R_o - 1)\gamma \qquad \text{(C-38)}$$

3. Determine the tangent constrained modulus, M_B^*, at B in a similar manner by summing p_{B1} and p_{B2}, as calculated from

$$p_{B1} = (H + \Delta H/2 - 2b)\gamma \qquad \text{(C-39)}$$

and

$$p_{B2} = 0.3(H + \Delta H/2)(R_o - 1)\gamma \qquad \text{(C-40)}$$

4. If the unit overload (caused by the weight of the added layer) acting on the structure due to stress concentration is designated as S, the deformation, δ_A, within the zone of extent b is given by

$$\delta_A = 0.7\frac{Sb}{M_A^*} \qquad \text{(C-41)}$$

Note that, at this point, the parameter S is an unknown. Similarly, the deformation, δ_B, in the zone between distance b and $3b$ from the structure is given by

$$\delta_B = \frac{0.3S(2b)}{M_B^*} = 0.6\frac{Sb}{M_B^*} \qquad \text{(C-42)}$$

Because δ_p is the sum of δ_A and δ_B,

$$\delta_p = \delta_A + \delta_B = \frac{0.7Sb}{M_A^*} + \frac{0.6Sb}{M_B^*} = Sb\left(\frac{0.7}{M_A^*} + \frac{0.6}{M_B^*}\right) = \frac{\gamma\Delta H d}{2M_o^*} \qquad \text{(C-43)}$$

from which

$$S = \frac{\gamma\Delta H}{M_o^*}\frac{d}{b}\frac{1}{\frac{1.4}{M_A^*} + \frac{1.2}{M_B^*}} \qquad \text{(C-44)}$$

The load concentration factor, R_o, is then given by

$$R_o = \frac{\gamma\Delta H + S}{\gamma\Delta H} \qquad \text{(C-45)}$$

Although specific values for R_o are not known a priori for use in Eq. C-38, values can be estimated for use in Eq. C-38 and then checked for agreement by Eq. C-45; if agreement is not satisfactory, a revised estimate for R_o can be used in Eq. C-38 and the procedure repeated. However, the final value for R_o will, in general, not be very sensitive to variations in the original estimate, and normally it will not be necessary to repeat the calculation with a corrected value.

If the structure is founded on an incompressible foundation, the horizontal mid-plane of the structure is no longer a plane of symmetry. Accordingly, the preceding equations must be revised as follows. Eq. C-35 becomes

$$p_o' = (H + \Delta H/2 + d/2)\gamma \qquad \text{(C-46)}$$

the equations for p_{A1}, p_{A2}, p_{B1}, and p_{B2} are unchanged, and Eq. C-44 becomes

$$S = \frac{\gamma\Delta H\mathrm{d}}{M_{o'}^* b}\left[\frac{1}{\frac{0.7}{M_A^*} + \frac{0.6}{M_B^*}}\right] \qquad \text{(C-47)}$$

If compressibility characteristics of the foundation soil are known, a similar form of pressure distribution may be considered below the structure, and, in addition to Eqs. C-37 through C-40 and C-45, the following equations may be used:

$$p_{C1} = (H + \Delta H/2 + b/2)\gamma \qquad \text{(C-48)}$$

$$p_{C2} = 0.7\,(H + \Delta H/2)(R_o - 1)\gamma \qquad \text{(C-49)}$$

$$p_{D1} = (H + \Delta H/2 + 2b)\gamma \quad \text{(C-50)}$$

$$p_{D2} = 0.3(H + \Delta H/2)(R_o - 1)\gamma \quad \text{(C-51)}$$

and

$$S = \frac{\gamma \Delta H d}{M_{o'}{}^{*} b}\left[\frac{1}{\frac{0.7}{M_A{}^{*}} + \frac{0.6}{M_B{}^{*}} + \frac{0.6}{M_C{}^{*}} + \frac{0.7}{M_D{}^{*}}}\right] \quad \text{(C-52)}$$

in which $M_C{}^{*}$ and $M_D{}^{*}$ are tangent constrained moduli at distances $b/2$ and $2b$, respectively, below the structure.

It will be found that, if other computations are made for other values of H in the fill, there will be very little change in the value of R_o. This agrees with field experience, because many load measurements have indicated a linear relationship between load and fill height. Therefore, if a load concentration factor is determined for a 1-ft layer at the center of the fill, it is reasonable to expect that the same factor may be applied to the entire fill to determine the load on the structure.

The foregoing procedure has been developed for a rectangular rigid structure. The case of a circular rigid structure, such as a culvert pipe, would introduce further complexities. However, preliminary considerations indicate that the differences are self-compensating to some extent, and it seems reasonable to treat the pipe as a square section having sides equal to the diameter.

Consideration of Low Cover Heights

Although the preceding approach is not strictly applicable to shallow buried structures, some preliminary thoughts regarding treatment of such cases may be advanced; however, it must be emphasized that these comments represent only preliminary thoughts, and considerable revision may be in order.

As previously indicated, the load concentration factor, R_o, for low heights of cover is governed largely by the shear strength of the soil. If one considers only the first thin layer placed over the structure, it is apparent that, as only a small area is available to resist shear stress, the effect in producing load concentration is slight. As the fill height is increased, R_o increases in accordance with the layer being placed and will probably approach a constant value at a height of about $3b$ above the structure.

One crude approach to the evaluation of culvert load at low cover heights may be to assume a linear variation in R_o from a value of zero at the soil-culvert interface to a value that remains essentially constant for heights of cover above two or three structure widths. Then, for a given situation where low cover is concerned, R_o may be evaluated for the deeply buried structure condition and modified by multiplication of $(R_o - 1)$ by the ratio of cover height to $2b$ or $3b$.

Sample Problem.—A 6-ft-diameter concrete culvert is placed under 40 ft of fill on "ordinary bedding" underlain by (1) stiff soil, and (2) rock. The placement of the fill material is well controlled, and an average compacted unit weight of 130 pcf and a dry density of 110 pcf are obtained. Determine the probable vertical loading on the culvert.

1. For the stiff foundation soil, consider the culvert to move equally into the fill and into the subgrade. Then, determine R_o (for purposes of calculation, assume R_o equal to 1.3) for a 1-ft layer added at H equal to 20 ft.

$$p_o = (H + \Delta H/2 + d/4)\gamma$$
$$= (20 + 0.5 + 1.5)130 = 2{,}860 \text{ psf}$$

From Figure C-4, $M_o{}^{*} = 203{,}000$ psf.

$$p_{A1} = (H + \Delta H/2 - d/2)\gamma$$
$$= (20 + 0.5 - 3)130 = 2{,}280 \text{ psf}$$
$$p_{A2} = 0.7(H + \Delta H/2)(R_o - 1)\gamma$$
$$= (20 + 0.5)(1.3 - 1)130 = 558 \text{ psf}$$
$$p_{A1} + p_{A2} = 2{,}840 \text{ psf}$$

From Figure C-4, $M_A{}^{*} = 202{,}000$ psf.

$$p_{B1} = (H + \Delta H/2 - 2b)\gamma$$
$$= (20 + 0.5 - 6)130 = 1{,}885 \text{ psf}$$
$$p_{B2} = 0.3(H + \Delta H/2)(R_o - 1)\gamma$$
$$= 0.3(20 + 0.5)(1.3 - 1)130 = 240 \text{ psf}$$
$$p_{B1} + p_{B2} = 2{,}130 \text{ psf}$$

From Figure C-4, $M_B{}^{*} = 180{,}000$ psf.

$$S = \frac{\gamma \Delta H}{M^{*}}\left[\frac{1}{\frac{1.4}{M_A{}^{*}} + \frac{1.2}{M_B{}^{*}}}\right]$$
$$= \frac{130}{203{,}000}\left[\frac{1}{\frac{1.4}{202{,}000} + \frac{1.2}{180{,}000}}\right] = 47 \text{ psf}$$

Hence, the load concentration factor is found to be

$$R_o = \frac{130 + 47}{130} = 1.36$$

Because this R_o value of 1.36 agrees well with the assumed value of 1.3, there is no need to repeat the computation, and the culvert load may be determined as $40 \times 130 \times 1.36 = 7{,}060$ psf.

2. For the rock foundation condition, consider no compression into the foundation, and assume a preliminary R_o value of 2.0. Then, one obtains:

$$p_{o'} = (H + \Delta H/2 + d/2)\gamma$$
$$= (20 + 0.5 + 3)130 = 3{,}060 \text{ psf}$$

From Figure C-4, $M_{o'}{}^{*} = 208{,}000$ psf.

$$p_{A1} = (H + \Delta H/2 - d/2)\gamma$$
$$= (20 + 0.5 - 3)130 = 2{,}280 \text{ psf}$$
$$p_{A2} = 0.7(H + \Delta H/2)(R_o - 1)\gamma$$
$$= 0.7(20 + 0.5)(2 - 1)130 = 1{,}860 \text{ psf}$$
$$p_{A1} + p_{A2} = 4{,}140 \text{ psf}$$

From Figure C-4, $M_A{}^{*} = 236{,}000$ psf.

$$p_{B1} = (H + \Delta H/2 - 2b)\gamma$$
$$= (20 + 0.5 - 6)130 = 1{,}885 \text{ psf}$$
$$p_{B2} = 0.3(H + \Delta H/2)(R_o - 1)\gamma$$
$$= 0.3(20 + 0.5)(2 - 1)130 = 800 \text{ psf}$$
$$p_{B1} + p_{B2} = 2{,}680 \text{ psf}$$

From Figure C-4, $M_B^* = 198{,}000$ psf.

$$S = \frac{\gamma \Delta H}{M_{o'}^*} \frac{1}{\dfrac{1.4}{M_A^*} + \dfrac{1.2}{M_B^*}}$$

$$= \frac{130}{208{,}000}\left[\frac{1}{\dfrac{0.7}{236{,}000} + \dfrac{0.6}{198{,}000}}\right] = 106 \text{ psf}$$

Hence, the load concentration factor is found to be

$$R_o = \frac{130 + 106}{130} = 1.81$$

which agrees well with the assumed 2.0; therefore, the culvert load is given by $40 \times 130 \times 1.80 = 9{,}380$ psf.

Application to Imperfect Trench Installation

As an alternative to the procedure developed by Spangler (*34*) for determining the effectiveness of an imperfect trench installation, the following approach, based on the relationship reported by Osterberg (*46*) between the compressibility and the dry density of a compacted fill, is presented. This method is, in effect, a special case of the preceding approach and is described with reference to Figure C-11. As a first step, the free field deformation of the embankment between A and B may be determined as follows:

1. Obtain from Figure C-12 the total average strain, ϵ_1, for the distance between A and B by use of the average vertical stress, $(H - T + \text{AB}/2)\gamma$, at the midpoint of layer AB and the compacted dry density of the fill.

2. Determine from Figure C-12 the average strain, ϵ_2, in the layer AB for the case in which the fill is completed to elevation A only; for this determination use the average vertical stress, $(\text{AB}/2)\gamma$, at the midpoint of layer AB.

3. Because the construction of the excavation for the compressible fill will obviate consideration of the effect of ϵ_2 on the culvert, the net effective average strain affecting the load on the culvert is $\epsilon_1 - \epsilon_2$, and the net effective deformation, δ, is given by

$$\delta = \text{AB}(\epsilon_1 - \epsilon_2) \qquad \text{(C-53)}$$

If the deformation, δ, is considered to take place entirely within the compressible layer, the average strain, ϵ_3, in the compressible layer of thickness, T, is given by

$$\epsilon_3 = \frac{\delta}{T} = \frac{\text{AB}}{T}(\epsilon_2 - \epsilon_1) \qquad \text{(C-54)}$$

From the compressibility characteristics of the compressible fill, the soil stress associated with ϵ_3 may be found; this represents the average vertical stress on the structure. It is important that field control of the compressible fill be sufficiently good to ensure that the compressibility characteristics used in design are obtained in the field. One possible approach is to place the compressible material in a loose, but controlled, manner at one of the lower dry densities represented by curves in Figure C-12. Although these curves were determined from data on soils compacted according to standard procedures, they should, in the absence of more accurate data, give reasonable estimates for very loosely compacted soils.

Sample Problem.—For a 6-ft-diameter rigid pipe under a 40-ft fill with a total density of 130 pcf and a dry density of 110 pcf, determine the effect on the culvert load due to an imperfect trench installation. The imperfect trench should be square in cross section and have a side equal in length to the outside pipe diameter; the soil in the trench should be placed at a dry density of 90 psf.

The solution may be obtained by first determining the average stresses at the midpoint of layer AB due to the total fill and due to the fill completed only to elevation A; these

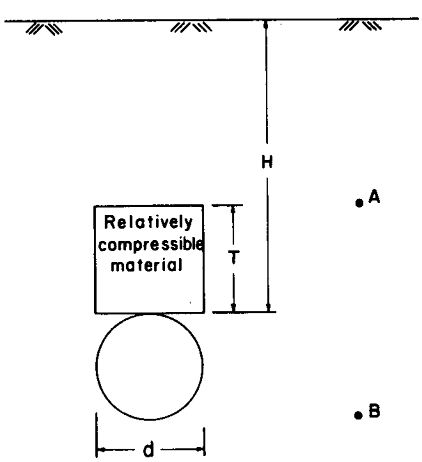


Figure C-11. Schematic diagram for imperfect trench installation.

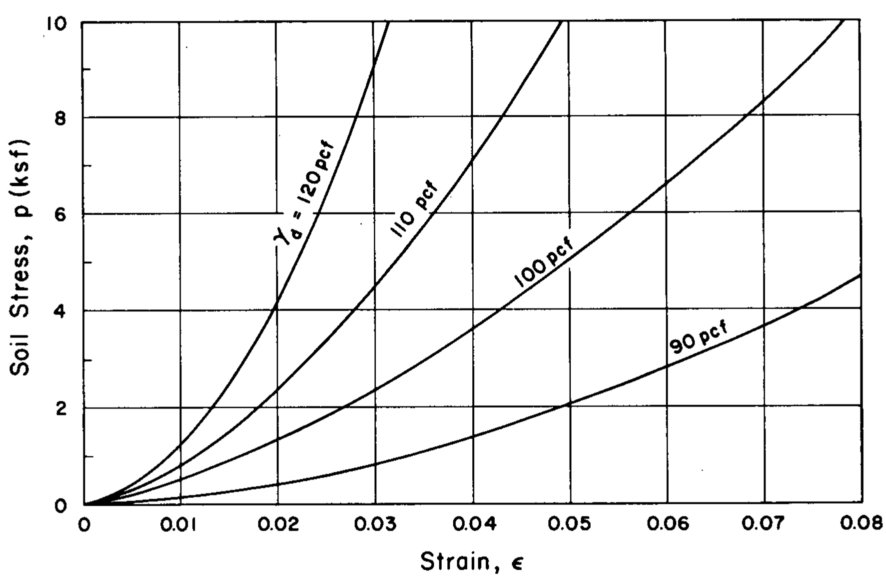


Figure C-12. Stress versus strain of soils compacted at various dry densities.

stresses are $p_1 = (H - T + \mathrm{AB}/2)\gamma = \left(40 - 6 + \frac{12}{2}\right)$ $130 = 5{,}200$ psf and $p_2 = (\mathrm{AB}/2)\gamma = (12/2)\,130 =$ 780 psf.

Use of these stresses in conjunction with the curves of Figure C-12 and a dry density of 110 pcf allows the determination of the corresponding strains as $\epsilon_1 = 0.0363$ and $\epsilon_2 = 0.0100$, whereupon the deformation becomes $\delta = \mathrm{AB}(\epsilon_1 - \epsilon_2) = 12(0.0363 - 0.010) = 0.312$ ft.

Hence, the average strain in the compressible layer is $\epsilon_3 = \delta/T = 0.312/6 = 0.052$.

Using Figure C-12 and γ_d equal to 90 pcf, it can be determined that the average vertical stress acting on the culvert is approximately 2,200 psf; this average stress may be compared with the value of 7,060 psf which was determined previously for the same culvert surrounded by a homogeneous soil.

EXPERIMENTAL STUDIES

Attempts to actually measure the soil pressures acting on a buried structure by means of mechanical or electrical sensors and transducers, either buried in the soil mass or embedded in the culvert at its interface with the soil, have always been handicapped by the difficulty of matching the mechanical stiffness or compliance of the transducer with that of the soil. Also, transducers placed in the soil will provide response measurements only at discrete points, which may or may not be representative of average conditions over a given area. Calibrated tapes have been used by Spangler and others to overcome this objection. Nevertheless, valid experimental data are extremely difficult to obtain, and a considerable portion of the data reported in the literature must be viewed with caution.

Analytical-Experimental Approach

To circumvent some of the shortcomings indicated previously, Gabriel and Dabaghian (*63*) have suggested that the culvert itself will act as the most faithful sensing device; that is, the culvert may be used as a transducer. This is possible because the elastic deformation response of a circular culvert can be uniquely related to a load distribution acting on its outer surface.

In their work, they assumed that the culvert responds in a linear elastic manner while interacting with the surrounding inelastic, nonlinear soil. Further, they considered only the central interior portion of the culvert, far from its ends, so that the behavior may be reasonably approximated by plane strain conditions. An additional simplification was incorporated initially by assuming that the load distribution was symmetric about a vertical axis; however, subsequent work on the problem has removed this restriction.

The theoretical development of this analysis showed that it is possible to determine the load distribution on the outer boundary of a rigid culvert if the displacements at the inner boundary were known. The radial and tangential motions of a number of points on the interior boundary of the culvert were expressed in terms of converging sine and cosine series. For the circular rigid culvert, initial polar symmetry, coupled with the assumed symmetry about the vertical axis, resulted in a cosine series for the radial displacements and a sine series for the tangential displacements. The coefficients of these series could be calculated from the experimentally determined displacements by means of a regression analysis.

The unknown culvert loading was then described by a similar series representation. By use of the same argument of symmetry described previously, the normal pressure is described by cosine functions only while the shear stresses at the interface are expressed in terms of sine functions only. The unknowns, therefore, are the amplitudes of these functions. By use of the stress-strain relation for the culvert material, plane strain conditions, and appropriate boundary conditions, expressions for the loads acting on the outer boundary of the culvert may be obtained in terms of the inner boundary displacements. The amplitudes of the load harmonics are then evaluated in terms of the experimentally determined inner boundary displacements. Like any other experimental technique, the success of this analysis depends largely on the ability to measure accurately the required displacements, and this is by no means a simple matter. Some suggestions and recommendations regarding this problem may be found in a report by Gabriel (*64*).

At present, only one test installation has been instrumented in an attempt to obtain data necessary to evaluate the loads on the culvert by means of the proposed theory. Unfortunately, the test was beset by instrumentation difficulties that invalidated the data, and thus the theory is still untried.

Utah Test Program

An extensive series of load tests on full-scale corrugated metal culverts is presently in progress at Utah State University, Logan, Utah. The project, sponsored by the American Iron and Steel Institute, is under the direction of Dr. Reynold Watkins. The culvert sections are placed on the ground at the base of a test stand, and standard backfilling procedures are used to construct the embankment over the culvert until the height of cover reaches the elevation of the rig that supports the hydraulic pistons. A uniform load is applied to the surface of the fill by means of the hydraulic pistons, and this load is increased until the culvert fails or the capacity of the loading system is reached. Measurements are being taken on the culvert during the loading process, and tests are also being performed on the compacted fill surrounding the culvert. Although they are not yet available, the results of these tests should provide valuable information concerning the response of flexible culverts to superimposed uniformly distributed loads.

Ohio Studies

There is in progress at Ohio State University a research program directed principally toward studying the effect of soil properties, such as inelastic behavior and failure strength, on the behavior of buried flexible pipes. Nonlinearity of structural behavior due to large deflections may also be examined. The means being employed to accomplish the established goal of this work are threefold; these are (1) an analysis of the soil-culvert interaction problem by plane strain finite element methods, (2) a study of the response of small-scale laboratory tests, and (3) an investigation of large-scale laboratory tests. Hopefully, this study will also provide the scaling laws required for the extrapolation of results from small-scale laboratory tests to full-size field structures and from small overburden pressures to large overburden pressures. Because the general analytical approach to this work is similar to that employed by Brown, his computer programs are being used with modifications. Although standard Ottawa sand is being used as the soil for the laboratory model tests, it is suggested that the method of analysis to be developed would have general applicability and may later be extended to the analysis of pipes in consolidating clay embankments.

On the basis of a series of meetings between representatives of the Ohio Department of Highways, the U.S. Bureau of Public Roads, the Ohio Concrete Pipe Manufacturers Association, and the American Concrete Pipe Association, the Ohio State University has been requested to formulate a research program aimed at exploring the soil-structure interaction in reinforced concrete culverts under highways. This work would recognize that, under a large range of pipe diameters and shell thicknesses, concrete pipes may manifest substantial deformations under load, and, as such, they may be capable of developing soil-structure interaction of a nature similar to that found in flexible pipes. Consideration would be given to the development of a satisfactory analytical technique to adequately approximate the stress distribution in the pipe, and analytical results for the overall response would be evaluated by means of large-scale laboratory or field tests.

Kentucky Performance Survey

In 1959 the Bureau of Public Roads initiated a performance survey of reinforced concrete pipe culverts, and this study was undertaken by the Kentucky Department of Highways. The reason for the survey was to evaluate the BPR design and installation criterion (*36*) that was developed in 1957 in cooperation with M. G. Spangler and the American Concrete Pipe Association. It was requested that a number of reinforced concrete pipe culverts, designed and installed in accordance with this criterion (revised and updated: *Reinforced Concrete Pipe Culverts, Criteria for Structural Design and Installation,* U.S. Department of Commerce, Bureau of Public Roads, Aug. 1963), be inspected periodically and reported at the end of each calendar year. In response to this request, the Kentucky Department of Highways early in 1960 selected a group of 113 reinforced concrete pipe culverts, and each culvert was inspected once each summer during the five summers from 1960 through 1964. The locations and other details of these culverts, together with the data recorded, may be found in the Kentucky report (*65*). All of the 113 culverts had Class B bedding with a maximum permissible height of fill determined by use of the design curves in Chart II of the report by Townsend (*39*). However, for fill heights in excess of the maximum permissible for Class B bedding, the imperfect trench method of construction was specified and the Class B bedding was then designated as B_1 bedding.

The purpose of this discussion is to describe the results obtained from this study and to outline the pertinent parameters required to render these results more useful. The data reported for the different culverts included the pipe diameter, strength class, bedding class, conduit classification (positive or negative projecting), embankment height, angle of skew, embankment material (classified as soil, rock, or combination), and factor of safety as constructed (related to the design factor of safety by the ratio of the maximum allowable height of fill, corresponding to the design factor of safety, to the actual height of fill as constructed). Other data, such as culvert length, number of sections, and the grade, were also included. Observed signs of distress, as well as changes and/or developments in these signs, were noted during the various inspection periods, and these are shown in the report (*65*) by different colors corresponding to the five surveys. The various notations used to express the different types of distress include hairline crack, crack, shear failure, spalling, broken, mortar missing, steel exposed, faulted, section settled, buckling, mortared, patched, and joint separated. Changes in original conditions were indicated as hairline crack changed to crack, crack or cracks to shear, mortar or patch out, steel exposed through patch, hairline crack changed to shear, hairline crack through patch, and crack through patch.

Although the data reported in this survey are unquestionably essential, other equally important data are not included; examples are complete soil data and a "during construction" inspection report. The required soil data include (1) classification of the natural soils and the fill materials in accordance with some well-known system, (2) the density, moisture content, strength, and compressibility of the natural soils to a sufficient depth (perhaps equal to the height of the embankment) and of the fill material, and (3) groundwater conditions. When such a performance study is to be made on in-service culverts, continuous inspection should be performed by a competent resident engineer, and any changes in plans should be recorded. The construction of the culvert should not be entrusted solely to the contractor, and compliance with written specifications should be checked more thoroughly than usual. An after-construction survey of the culvert and the embankment should be made; settlements of the fill above and to the sides of the pipe, the elevation of the culvert invert, and changes in the culvert shape should be surveyed periodically until equilibrium is reached. These latter data, in addition to those reported by the Kentucky Department of Highways, are believed necessary to evaluate adequately the design and installation practices used.

SPECIAL STUDY

In 1962 the consulting firm of Moran, Proctor, Mueser, and Rutledge published a report (*26*) that evaluated methods for determining earth loads on buried concrete pipes. This study was sponsored by the American Concrete Pipe Association, and one of its specific aims was to determine if current widely used design procedures, that originated 40 or 50 years ago, could be improved in the light of current knowledge and experience. Although this investigation is concerned primarily with earth loads acting on concrete pipes, some results for relatively flexible pipes or for excavated tunnels are of secondary importance and are included to clarify the principles involved. In addition to the Marston-Spangler theory, several other methods for determining earth loads on conduits are discussed in this report; among these are the Voellmy solution (*6*), the Mindlin solution (*23*), Bull's analysis (*22*), Voellmy's solution for radial pressures, and the "relative yield theory" (*66*), proposed for use on the Garrison Dam outlet tunnels. The basic assumptions and inherent criticisms of these methods are treated in detail in the original report (*26*) and are only briefly summarized herein; included also is a brief summary of the findings and conclusions advanced in the original report.

Voellmy Analysis

Based on the assumption that (1) the soil surrounding the culvert is homogeneous, (2) the top of the culvert settles more than the adjacent soil, (3) failure surfaces are formed at an angle, β, with the vertical, as determined by maximizing the load on the culvert, (4) full friction is mobilized on the inclined failure planes, and (5) horizontal pressures on the failure wedge are given by Rankine active values, Voellmy (*6*) suggested that the design vertical load for a flexible pipe be the maximum load as obtained by selecting various values for β. In a comparable analysis for a relatively rigid pipe, Voellmy (*6*) has computed the load that will be effective when failure surfaces are inclined at the friction angle with the vertical and resultant forces on the failure surfaces are horizontal. Although these analyses have the merit of avoiding the unlikely occurrence of vertical failure surfaces in the embankment above the buried pipe, they are both plastic analyses that suffer from the inherent erroneous assumption that full friction is mobilized on the failure surfaces.

In contrast to the foregoing plastic analyses, Voellmy (*6*) presented an essentially elastic-type solution that gives the radial pressures acting on relatively flexible pipes or tunnel linings. Initial horizontal pressures are assumed to be active values, and initial vertical pressures are assumed to be given by the weight of the overburden. Based on these initial pressure values, pipe deformations are computed and, by use of radial displacements and a modulus of soil compressibility, increments of radial pressures are determined. These incremental values are added to or subtracted from the initial assumed pressures, depending on whether the deflection of the pipe wall at a specific point is outward or inward. This procedure could be iterated to achieve greater accuracy. Provided that a modulus of radial compressibility for the soil could be determined at all, it would probably apply only to outward-directed deformations, because an inward deformation of the pipe does not imply an expansion of the surrounding soil.

Bull Analysis

Bull (*22*) proposed a radial pressure analysis similar to that of Voellmy (*6*), except that different assumed initial pressures are used, no subtraction of a decrement is made where inward deformations occur, and an iterative scheme is suggested.

Mindlin Analysis

Using three different assumptions for the initial ratio of horizontal to vertical stresses that exist before making a self-supporting cylindrical hole in a semi-infinite elastic solid, Mindlin (*23*) reported a solution for computing the tangential stresses around the periphery of the opening. No radial pressure or shear forces are assumed to exist on the boundary of the circular hole, and the boundary of the hole is assumed to move inward until elastic equilibrium is reached. Computed results show a strong dependence on the assumed initial pressure ratio. In view of currently available finite element techniques, the restrictive assumptions described previously can be partly eliminated by attributing certain stiffness characteristics to the pipe.

Smith Relative Yield Theory

According to the relative yield theory developed by Smith (*66*) for determining vertical loads on tunnels or pipes, the values for the side shears are computed by considering the relative moduli of compressibility for the pipe structure and for the adjacent soil, and it is not assumed that the deformations are sufficient to develop the full shear strength of the overlying soil. Perhaps the most difficult aspect of this analysis involves determining the width of the adjacent soil affected by the deformations. This method, similar in some ways to the Marston theory, assumes vertical planes along which the resisting shear forces act.

Experimental Observations

A summary of observations of vertical load and radial pressures on buried pipe and tunnels, together with a discussion of test setup, measurement methods, possible errors, and conclusions, has been reported by Moran, Proctor, Mueser, and Rutledge (*26*) for several test series and tunnel installations. The pipes were all in embankments of sandy soils, whereas the tunnels were in clayey cohesive soils with various degrees of preconsolidation. Some of the general conclusions obtained from these observations are:

1. Pipe rigidity influences both the total vertical load and the radial pressure distribution. For rigid pipes vertical loads are generally 120 to 150 percent of the overburden weight; for flexible pipes vertical loads are usually between 60 and 90 percent of the overburden weight, with the percentage increasing as the height of fill increases. The more rigid the pipe, the larger the vertical pressures at the crown and invert, and the greater the difference between these vertical pressures and the horizontal pressures at the springline. For rigid pipes crown pressures may be 150 to 200 percent of the overburden pressure, whereas springline pressures may range from 50 to 90 percent; for flexible pipes crown pressures may range from 80 to 140 percent of the overburden pressure, whereas springline pressures are generally slightly less than the overburden pressure.
2. Bedding and backfilling conditions below the springline exert a marked influence on the pressure distribution.

With regard to the tests on buried pipes, none of the information is recent and all of it contains some uncertainty introduced by movements required for load measurement.

The tests are of relatively short duration and do not reflect possible long-term changes in loading due to gradual deformation of the surrounding soil. In general, the available test data do not provide adequate information to establish a conclusive evaluation of earth loads on buried pipe. For the tunnel observations, the vertical earth loads as a percentage of the overburden are greater for tunnels in softer clays than in stiffer soils. For tunnels in the softer clays, where the tunnel lining is much stiffer than an equal height of the surrounding clay, the vertical load approaches 100 percent of the overburden with time. Horizontal pressure tended to approach the overburden pressure for flexible steel linings in the softer soils, whereas they equalled approximately two-thirds of the overburden pressure for concrete linings. For tunnels in very stiff clays or clay shales, vertical loads are significantly less than the overburden pressure.

Performance of Various Soil Types

Two distinctly different types of soil performance, representing the behavioral range of engineering soils, must be considered; these are an ideal plastic, cohesive clay and an ideal clean, coarse-grained, cohesionless sand. The major difference between these materials lies not primarily in the ultimate vertical shear forces available, but rather on their stress-strain-time characteristics for loads sustained over long periods of time. Loads on clay soils will ordinarily cause continual volume decreases, and mobilized shear strength may relax due to creep. The ideal cohesionless soil usually exhibits a relatively low compressibility under added loads, and it responds with little time delay. Cohesionless soils tend to develop and maintain a specific shear strength where differential movements occur. In general, the time-dependent stress-strain characteristics of clayey soils make it difficult for them to support permanently a load increment shifted from the pipe; consequently, it is probable that vertical earth loads on even relatively flexible structures will ultimately reach 100 percent of the overburden pressure with the passage of time. No specific observations are available to demonstrate the degree to which cohesionless soils can permanently sustain loads transferred from the pipe.

TABLE C-4

EQUIVALENT ELASTIC MODULUS FOR DIFFERENT SOILS

SOIL TYPE	EQUIVALENT ELASTIC MODULUS (TONS/SQ FT)
Very soft and compressible organic silts and clays	5-10
Ordinary clays of medium consistency	20-50
Hard clays or relatively soft uncemented clay shales	200-500
Very firm, sound, dense partially cemented clay shales	2,000
Sand and gravel mixtures (modulus depends on gradation, density, and confining pressure)	100-300
Heavily compacted clayey or coarse-grained soils under low-to-moderate overburden	100-500

Relative Rigidity of Pipe and Soil

Both the pipe load tests and the tunnel case histories indicate that the conduit rigidity relative to that of the surrounding soil is of primary importance in determining earth loads. It is important to note that rigidity and flexibility are relative terms; that is, a pipe that is termed "flexible" under given conditions may well be "rigid" under other conditions. The one-dimensional consolidation test is most appropriate for evaluating the modulus of deformation for the soil; field plate bearing tests and laboratory triaxial tests probably involve too large a proportion of shear strain to simulate soil behavior adjacent to the pipe. If an "equivalent elastic modulus" is defined as the average applied vertical pressure divided by the average vertical strain of the pipe, tunnel, or block of soil, the approximate range of this parameter for different soils may be as given in Table C-4. Most soils tend to become stiffer with increasing loads; however, for clays the modulus depends strongly on consolidation history. The "equivalent elastic modulus" for a conduit is influenced by the ratio between the vertical and horizontal pressures acting on the pipe. For a flexible conduit, the modulus becomes greater with increasing load, whereas for a rigid conduit the ratio of horizontal to vertical pressures tends to remain more nearly constant and hence the modulus is essentially constant as the load increases. Table C-5 gives typical values for the "equivalent elastic modulus" of various pipes under different conditions; these values were computed from deformation and pressures observed in the pipe load tests.

Beam on Elastic Foundation

To determine vertical loads on conduits, Moran, Proctor, Mueser, and Rutledge (*26*) have suggested a procedure based on considering the system to be analogous to a beam on an elastic foundation, for which an analytical solution has been presented by Hetenyi (*67*). The elastic foundation is considered to be the material below the top of the pipe, and this foundation is considered to have a discontinuity, represented by the conduit, in its elastic properties. The embankment material overlying the pipe is considered to be the beam, and the "beam" material is assumed to obey a linearly elastic stress-strain law. The most serious difficulty associated with applying this method lies in the selection of an appropriate beam height, which will exclude the material wherein the deformation of the beam will produce tensile strains that cannot be sustained by the soil. The manner in which this selection affects the result is thus far undetermined. Carried to completion, the method yields distribution of the contact pressures acting on the plane at the top of the pipe. The researchers suggest that this method has considerable promise for determining embankment loads on buried pipes.

BUCKLING AND SHALLOW CONDUITS

Earlier investigators gave little attention to buckling as a design criterion for corrugated metal culverts. Provided a sufficiently high standard of fill placement was obtained in the vicinity of the culvert, either deformation or ring stress criteria were considered sufficient to ensure the safety of the structure. Field performance, in general, has confirmed the validity of this approach, at least for the more common field conditions and design assumptions. In recent years a general increase in maximum structure size and loading and the associated requirements for refinement of the design methods have led to considerable progress in research on buckling.

Watkins (*14*) indicated the analogy between the buckling of buried tubes and the buckling of columns. The buckling stress for both depends on the flexural rigidity, as well as an additional length parameter, and the curves indicating these relationships are similar, both approaching asymptotically the material yield strength at high values of flexural rigidity and low values of the length parameter. In a more recent paper (*25*) Watkins indicated a variation, depending on the properties of the surrounding soil, in the form of such a conduit buckling stress curve. The curves indicate limits represented by the fluid state, where no shear stresses exist in the medium, and the rigid state, where no deformation is possible. Clearly, the resistance of the medium to deformation has a significant effect on the failure load. Although under a given loading condition there may be a tendency for catastrophic failure to occur at a low buckling mode for a fluid-type medium surrounding the culvert, similar conditions with a compacted soil surrounding the culvert lead to failure at a higher stress level and at a higher mode, and the failure is less likely to be catastrophic. The reason for this difference is that, whereas the deformations associated with buckling lead to little, if any, change in the normal pressure on the conduit wall surrounded by a fluid-type medium, deformations associated with buckling of soil-surrounded pipes lead to significant normal pressure changes. Outward-moving lobes are subject to a pressure increase, and inward-moving lobes are subject to a pressure reduction; this effect retards the progression of failure. Nevertheless, failure may occur at some higher buckling mode, provided the necessary higher stress level is reached.

In adapting the theory from Timoshenko and Gere (*68*) to curved plates, Meyerhof and Baikie (*69*) used data from laboratory experiments to derive the following relationship for the buckling stress, f_b:

$$f_b = \frac{2}{A}\sqrt{\frac{eEI}{1-\nu_c^2}} \tag{C-55}$$

in which A is the cross-sectional area of the conduit wall per unit length; e is the modulus of passive resistance of the surrounding soil; EI is the flexural rigidity of the conduit per unit length; and ν_c is Poisson's ratio. This relationship is subject to the requirement that $r/L \geq 2$, in which r is the conduit radius and L is the relative stiffness of the culvert given by

$$L = \sqrt[4]{\frac{EI}{(1-\nu_c^2)e}} \tag{C-56}$$

TABLE C-5

EQUIVALENT ELASTIC MODULUS AS A FUNCTION OF CULVERT TYPE AND MATERIAL

TYPE OF CONDUIT	EQUIVALENT ELASTIC MODULUS (TONS/SQ FT)
Smooth steel pipe (d=30 in.; t=0.349 in.) under low embankment	75
Ordinary corrugated metal pipe (d=2 to 4 ft) under low embankment; modulus increases with increase of load and deformation of pipe	50-300
Cast iron pipe (d=3 to 4 ft; t=1 in.) under low embankment	200-300
Unreinforced concrete pipe (d=3 to 4 ft; t=3 to 4 in.) under low embankment	2,000
Heavily reinforced concrete tunnel lining (1:12 ratio for wall thickness to diameter)	5,000-7,000

For properly constructed culverts, this condition is generally satisfied. Allowance for accidental eccentricities and imperfections in practice leads to the consideration of a critical stress, f_c, given by

$$f_c = \frac{f_y}{1 + f_y/f_b} \tag{C-57}$$

in which f_y is the yield strength of the material. A graphical representation of these formulae is shown in Figure M-1.

By considering the soil surrounding a conduit to be representable by a system of radial springs, Luscher (*43*) derived the following relationship for p^*, the critical pressure causing buckling:

$$p^* = 1.73\sqrt{\frac{EIBM^*}{r^3}} \tag{C-58}$$

Good agreement is obtained between this relationship and the results of laboratory tests on dense to medium loose Ottawa sand. After comparing buckling loads at EI/r^3 values of 0.1 and 1.0 (as determined by Eq. C-58 and formulae obtained by Dorris, Bulson, and Chelepati), Allgood (*70*) concludes that Luscher's results can probably be extended to practical soil cylinder systems; Table C-6 gives this comparison.

Both Meyerhof and Luscher agree that the buckling strength of a buried circular conduit is proportional to the square root of a compressibility parameter (such as M^* or E_s) and is inversely proportional to the square root of the radius. The application of these formulae is restricted to circular conduits having an approximately uniform load distribution. Such a requirement implies a certain minimum depth of cover which, according to Meyerhof, is approximately one diameter. For cases where the cover

TABLE C-6

COMPARISON OF BUCKLING LOADS

INVESTIGATOR	EQUATION	$(EI)/r^3$	0.1	1.0
Luscher	$556\left[\frac{EI}{r^3}\right]^{0.8}$	$B=2/3$	81	556
Dorris	$75\sqrt{\frac{EI}{r^3}}$	—	24	75
Bulson	$\frac{2.34}{t/r}\left[\frac{EI}{r^3}\right]$	$\frac{t}{r}=3.68\times10^{-3}$	64	635
Chelepati	$1.5\sqrt{\frac{K_sEI}{r^3}}$	—	90	620

height is less than one diameter, Eq. C-55 is modified as follows:

$$f_b = \sqrt{\left[1-\left(\frac{r}{r+H}\right)^2\right]\frac{eEI}{1-\nu^2}\cdot\frac{H}{2r}} \qquad \text{(C-59)}$$

in which H is the height of cover. For shallow cover conditions subject to concentrated loads, laboratory tests in dense sand have indicated buckling failure at strengths 10 to 20 percent of those for similar uniform loads. The results of tests with eccentric concentrated loads have indicated a further reduction of one-half in the buckling strength.

One approach to a method for a concentrated load analysis is given by Watkins, Ghavami, and Longhurst (*71*), who present curves based on the results of model tests. The model consists of a smooth-walled tube buried in sand of carefully controlled density; a loaded rubber-tired wheel, simulating a Caterpillar 651 carryall loader, is drawn over the sand surface. An effort is made to produce a realistic stress distribution due to the weight of the soil by drawing air downward through the system at a predetermined velocity.

By an analysis of the possible variables, it is found that the pertinent dimensionless terms are (1) the load term, W/M^*d^2, in which W is the critical surface wheel load at which the conduit begins to fail; M^* is the soil modulus or the average slope of the one-dimensional stress-strain diagram; and d is the conduit diameter; (2) the cover term, Z/d, in which Z is the height of the fill over the top of the conduit; and (3) the stiffness ratio, EI/M^*d^3, in which EI is the flexural rigidity per unit length of conduit wall. The assumption is made that any other variables are either accounted for or may be neglected, and the objective of the experimental work is to determine the relationship

$$W/M^*d^2 = f[(Z/d), (EI/M^*d^3)] \qquad \text{(C-60)}$$

The tube diameter and soil modulus are maintained constant while the tube thickness and wheel load are varied, and the corresponding cover height producing failure was found in each case. Curves are plotted to relate the three dimensionless parameters mentioned, and the corresponding equation is found to be

$$W/M^*d^2 = 160\left(\frac{EI}{M^*d^3}\right)^{0.5}[0.0071(Z/d)^2 + 0.0014] \qquad \text{(C-61)}$$

For model studies in soil mechanics there always arises the question of whether the scaling factors for all properties are correctly taken into account. The self-weight of the soil is a common problem, particularly for shallow structures, and the air flow method of simulating a real pressure gradient is novel. The presence of the tube in the system must have some effect on the flow, but the researchers believe that this effect is not particularly significant. Meyerhof (*72*) has had difficulty in correlating the results of model tests for concentrated loads (continuous over the conduit length) with theory; he suggests, as an explanation, that the soil modulus undergoes local changes as the failure load is approached. A similar process may affect Watkins' work. Furthermore, if it is difficult to achieve correlation between theory and model test results for the simple case studied by Meyerhof, then the effectiveness of the scaling process for similar, but more complex, models must be viewed with some doubt. Certainly, some full-scale testing is necessary for verification. Nevertheless, only the crudest rules of thumb are available as an alternative at the present time to predict the response of a shallow culvert due to a single-wheel construction load, and it is probable that the model test results will provide a reasonable indication of the effect of the stiffness ratio and the height of cover on the buckling load for culverts surrounded by granular soils.

Buckling of buried conduits is controlled largely by the flexural rigidity of the pipe, the radius of the pipe, the magnitude and uniformity of soil resistance to deformation, and the magnitude and uniformity of the loading on the culvert. In general, only materials such as smooth metal and corrugated metal are subject to buckling instability in normal culvert construction; for currently used combinations of pipe diameter and flexural rigidity, it is unlikely that earth loads can cause failure of the buckling instability type, provided the height of cover is sufficient and the deformation is limited. However, low cover heights can combine the undesirable conditions of nonuniform resistance to deformation, nonuniform soil loading, and concentrated live loading, and these effects are accentuated as the pipe diameter increases. Buckling may also be associated with cases where the compacted fill has an insufficient soil modulus to prevent excessive deformation, but in many of these cases the large deformation will serve as warning of an impending failure.

IMPROVEMENTS TO MARSTON-SPANGLER EMPIRICAL PARAMETERS

Use of the Marston-Spangler approach for the determination of culvert loads and deflections requires the selection of several rather empirical parameters; these include the deflection lag factor, D_l, the settlement ratio, r_{sd}, and the modulus of soil reaction, E'. Considerable difficulty has been encountered over the years in the quantitative evaluation of these parameters, and considerable research effort has been directed toward improving means to quantify them. From an over-all point of view, such research implies a general acceptance of the Marston-Spangler approach to the design and analysis of buried conduits, and indicates that one of the principal needs lies in modifying and improving the individual components of the theory. The following discussion summarizes briefly the recent work on the determination of the settlement ratio and, more especially, the modulus of soil resistance.

Settlement Ratio

Application of the Marston-Spangler theory to determine the total load acting on a buried conduit requires the selection of a parameter termed the settlement ratio, r_{sd}; this selection is based largely on empirical values that have been deduced from field observations. Recent work by Nielson and Koo (*73*) presents a solution for the settlement ratio in terms of the pipe diameter, modulus of elasticity for the pipe material, moment of inertia and cross-sectional area of the pipe, and the modulus of soil reaction. Their development uses the work of Burns and Richard (*10*), wherein a conduit is assumed to be embedded in a homogeneous elastic medium, and the theoretical results, which agree well with measured values, are shown in Figure C-13 for the limiting cases of no-slip and full-slip at the soil-culvert interface. Actual field conditions are probably closer to the no-slip case, and the intermediate curve shown in Figure C-13 is arbitrarily suggested for practical use. One of the chief difficulties in applying the results of this work lies in the need to know the modulus of soil reaction.

Modulus of Soil Reaction

In deriving an equation for the deformation of a circular flexible culvert, Spangler (*8*) included a parameter termed the modulus of passive resistance, *e*, and defined as the ratio of the unit load to the deformation for a long, narrow, horizontally loaded area at some depth below the surface. For the case of a flexible culvert, the horizontally applied pressure was assumed to be parabolically distributed over a 100° arc with a maximum ordinate at the springline. It later became evident (*37*) that the modulus of passive resistance was not a basic soil property, but depended on the dimensions of the loaded area. At this time a new parameter, termed the modulus of soil reaction, E', and given by the product of the modulus of passive resistance, *e*, and the culvert radius, *r*, was introduced.

Since its introduction, various methods have been proposed to quantitatively determine E'. Watkins and Neilson (*74*) developed the Modpares device, which simulated a pipe being forced into the side of a fill. Nielson (*44, 75*) has developed various correlations between E' and the slope of the stress-strain curve from a triaxial test, the CBR value, the Hveem stabilometer *R*-value, and the dry density; these are discussed subsequently. Spangler (*29*) has reported tests in which E' varied from 239 psi to 7,980 psi, and he suggested the use of a value of 700 psi for designs in which the soil is compacted to 90 percent or more of standard Proctor density for two diameters to either side of the pipe. Although it is widely recognized that these low values may be seriously in error, current design manuals provide for alternatives of 700 psi and 1,400 psi, depending on the quality of compaction adjacent to the pipe. It has long been

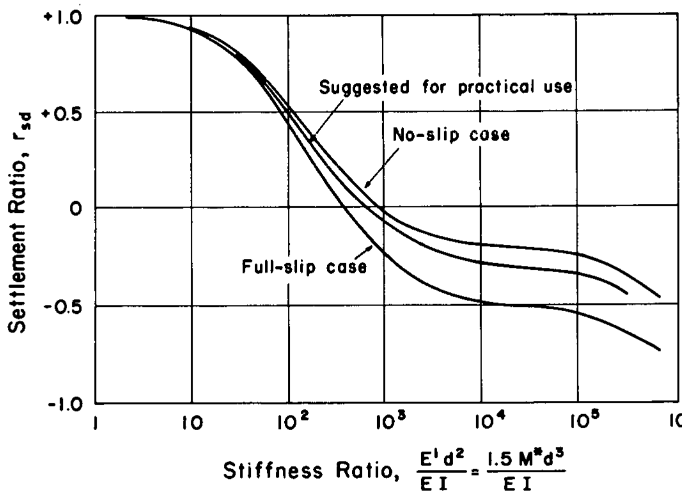


Figure C-13. Settlement ratio versus soil-culvert stiffness ratio.

recognized that the value of E' varied with confining pressure, and this has proven to be one of the difficulties in quantifying it.

Largely on the basis of the theory of elasticity, Nielson (*44*) has developed a method for determining E' from the slope of the stress-strain curve in a triaxial test. Beginning with the relationship

$$E' = p_h \Big/ \frac{\Delta x}{d} \qquad \text{(C-62)}$$

in which p_h is the pressure at the side of the pipe caused by forcing the side of the pipe into the fill, expressions by Burns and Richard (*10*), relating the loads, stresses, deflections, and soil and structure material properties for a cylinder buried in an infinite elastic medium, are substituted to obtain a highly complex relationship of the form

$$E' = f(M^*, \nu_s, r, E, A, I) \qquad \text{(C-63)}$$

in which E' is the modulus of soil reaction; M^* is the constrained soil modulus; ν_s is Poisson's ratio for the soil; r is the cylinder radius; E is the modulus of elasticity of the cylinder material; A is the cross-section area of the cylinder wall per unit length; and I is the moment of inertia of the wall cross section per unit length. For each of a series of constrained soil moduli, M^*, the modulus of soil reaction, E', was determined over a wide range of values for the other variables. Results indicated that a good approximation to the more complex relationship of Eq. C-63 was

$$E' = 1.5M^* \qquad \text{(C-64)}$$

By use of the relationship

$$M^* = \frac{E_s(1 - \nu_s)}{(1 + \nu_s)(1 - 2\nu_s)} \qquad \text{(C-65)}$$

in which E_s is the modulus of elasticity for the soil, Eq. C-64 was transformed to

$$E' = \frac{1.5E_s(1-\nu_s)}{(1+\nu_s)(1-2\nu_s)} \quad \text{(C-66)}$$

Nielson suggested that a value of 0.25 be used for ν_s and that a value for E_s be determined from the slope of the stress-strain curve in the triaxial test (presumably the S test) using a chamber pressure of ⅝ times the total overburden pressure at the culvert level.

One of the principal values of this work lies not so much in providing a useful tool for engineering design, but in illustrating some of the more important considerations controlling E'. Soil-structure response in terms of the constrained soil modulus, M^*, has been studied previously; in fact, Watkins (*25*) and Luscher (*43*) considered M^* as the basic and logical modulus to be used for the soil-structure interaction problem. Also, Burns and Richard (*10*) found its use convenient in their theoretical treatment of a cylinder buried in an idealized infinite elastic medium. The fact that M^* can be closely related to E' by a simple constant is not particularly surprising. From the soil mechanics point of view, M^* is a more basic soil property, and it should probably replace E' in the Iowa formula.

Use of the simplification that Poisson's ratio, ν_s, for the soil could be considered constant is questionable; in addition the relationship given by Eq. C-65 between E_s and M^* may be challenged. Also, Nielson recognized the fact that E' varied with pressure, and, on the basis of a theoretical evaluation that indicated that the ratio of horizontal stress to vertical stress was 0.75, he recommended the determination of an average E' by use of a triaxial cell pressure equal to ⅝ times the overburden pressure. Although theory may indicate a ratio of 0.75, no experimental confirmation appears to exist. In addition, there is evidence that the relationship between the modulus of soil reaction and the overburden pressure is nonlinear; consequently, use of these numbers for engineering design does not appear justified. On the other hand, Nielson's results appear to indicate that the simple relationship between E' and M^* is largely independent of ν_s. This would suggest that the relationship between E' and soil pressure can be obtained directly from the curve relating M^* to the soil pressure.

Nielson, Bahndhausavee, and Yeb (*75*) attempt to develop a convenient and reliable means for determining E' from the California Bearing Ratio, Hveem's stabilometer test, and the compacted soil density, as originally proposed by Watkins and Nielson (*74*). From the solution for the displacement of a rigid die into a semi-infinite elastic solid (*76*), one obtains

$$E_s = \frac{\pi a(1-\nu_s^2)p}{2\delta} \quad \text{(C-67)}$$

in which a is the die radius; p is the average unit load on the die; and δ is the displacement. By definition, the CBR value at a specified displacement is given by

$$\text{CBR}\ (\%) = \frac{\text{unit load on plunger}}{\text{standard unit load}} \times 100 \quad \text{(C-68)}$$

Combination of Eqs. C-66 and C-67 yields

$$E' = \frac{0.75\pi a(1-\nu_s)^2}{(1-2\nu_s)} \cdot 100 \cdot \text{CBR}\ (\%) \quad \text{(C-69)}$$

which, when the assumptions of ν_s equal to 0.25 and a equal to 0.975 in. are madc, bccomcs

$$E' = 260\ \text{CBR}\ (\%) \quad \text{(C-70)}$$

When the foregoing theory was compared with results obtained from soil tests using the Modpares device, it was found that, for a strain corresponding to 5-percent deformation of the pipe, the Modpares tests indicated the relationship

$$E' = 312\ \text{CBR}\ (\%) \quad \text{(C-71)}$$

The discrepancy between Eqs. C-70 and C-71 may possibly be attributed to the fact that either too low a value was assumed for Poisson's ratio, or the standard 0.1-in. penetration for the CBR test was too great. The suggested family of curves representing the relationship between the California Bearing Ratio and the modulus of soil reaction is shown in Figure C-14.

To relate the Hveem stabilometer test values to E', a similar procedure is adopted. E' is first related to the plate load test through the theory of elasticity; then, an empirical relationship between the R-value from the Hveem stabilometer test and the k-value from the plate bearing test is used to yield the relationship

$$E' = \frac{6a(1-\nu_s)^2}{1-2\nu_s} (-0.401 + 2.646R - 0.042R^2 + 0.0008R^3) \quad \text{(C-72)}$$

Reasonable correlation is indicated between E' values computed on the basis of Eq. C-72 and those obtained from the Modpares device. Figure C-15 shows the suggested correlation between the R-value and the modulus of soil reaction.

The modulus of soil reaction is further related in Figure C-16 with percent compaction on the basis of the ratio γ_d/γ_{180}, in which γ_d is the dry density of the soil used in the experimental test and γ_{180} is the dry density of the soil according to AASHO compaction designation T180; this latter reference was chosen arbitrarily. The two curves in Figure C-16 correspond to percentage deformations of 3 and 5 percent; however, because there is little difference between the curves, it seems reasonable to simply use an average value. Certainly this average value will be well within the accuracy of the experimental measurement. The researchers suggest that E' values based on this method be used only with the knowledge that an "error as much as 100 percent may exist."

All of the foregoing correlations are highly dependent for evaluation of their worth on test results obtained from the Modpares device. In fact, in the case of the dry density correlation, the Modpares results are the only means of obtaining a relationship. Therefore, it is of interest to give some consideration to the validity of values obtained from this device. In the original paper, Watkins and Nielson (*74*) well recognize the variation of E' with overburden pressure; however, owing to experimental difficulties, they are unable to vary this parameter and, consequently, results from their tests provide a modulus of soil reaction curve for some given overburden pressure. The authors admit

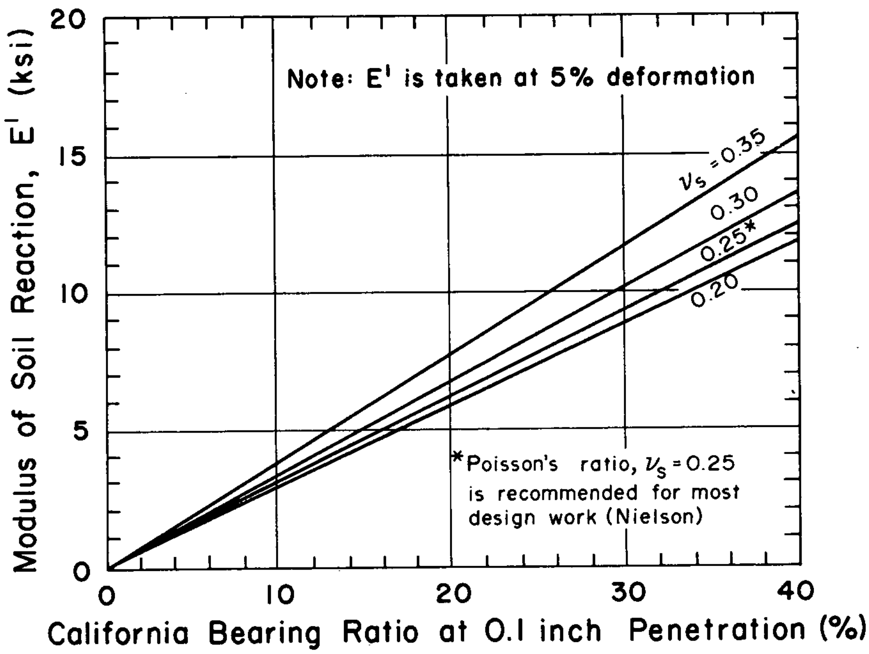


Figure C-14. Relationship between California Bearing Ratio and the modulus of soil reaction.

that there is insufficient evidence to establish a "rational relationship between E' as determined on the Modpares device and the E' that must be used in the Iowa formula." Furthermore, they report that evaluation of the method is made only on the basis of model tests and that "before the Modpares method can be used to predict a soil modulus E', it must be correlated with actual field measurements." According to the knowledge of the current researchers, no such correlation has been made.

With regard to the details of these correlations, it seems reasonable that a correlation should exist between E' and a CBR value. Also, some empirical relation may exist for the Hveem stabilometer test, but the development of a theoretical relationship that depends on several empirical relationships between somewhat dissimilar tests would appear to be undesirable. To achieve a high degree of validity, it is essential that studies of this nature be performed in conjunction with test results actually measured in the field. The CBR test and the Hveem stabilometer test are convenient for use in conjunction with tests on highway fills, and they may prove to be of considerable value in selecting parameters for culvert design. However, both of these tests were originally designed as index tests and their usefulness for the purpose indicated must be substantiated by direct field measurements.

Because approaches that require special soil-testing procedures are not received with enthusiasm by highway design engineers, it would be highly desirable to be able to determine E' from the result of some standard soil test performed during construction of the embankment. In keeping with this philosophy, it appears that work by Osterberg (*46*) offers an interesting approach. As previously discussed, Figure C-12 shows that a relationship, dependent

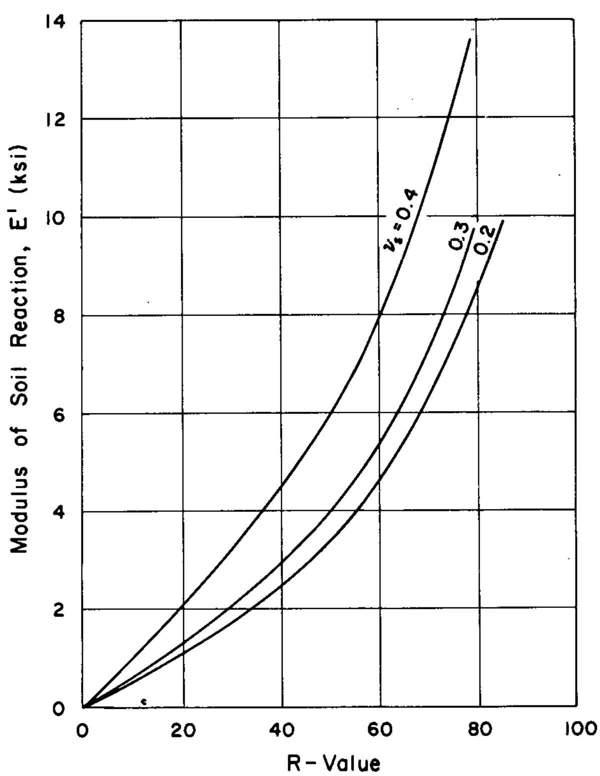


Figure C-15. Relationship between Hveem stabilometer **R**-*value and the modulus of soil reaction.*

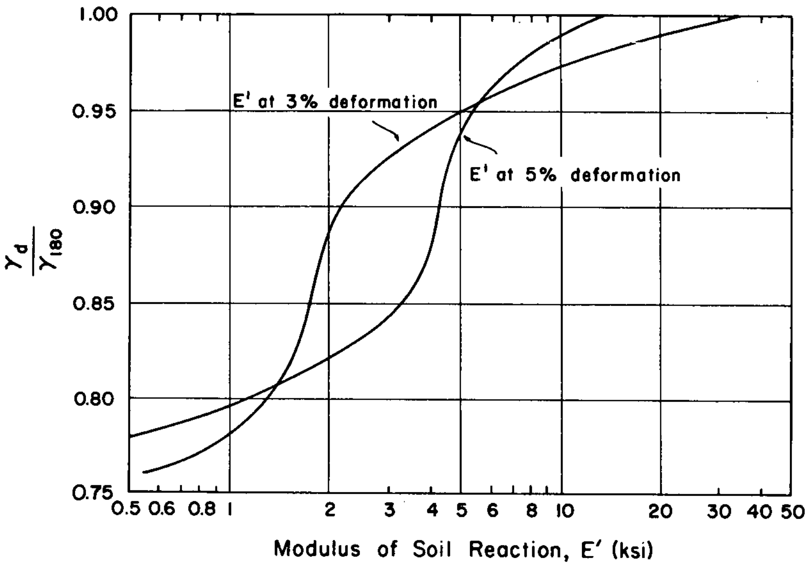


Figure C-16. Relationship between percent compaction and the modulus of soil reaction.

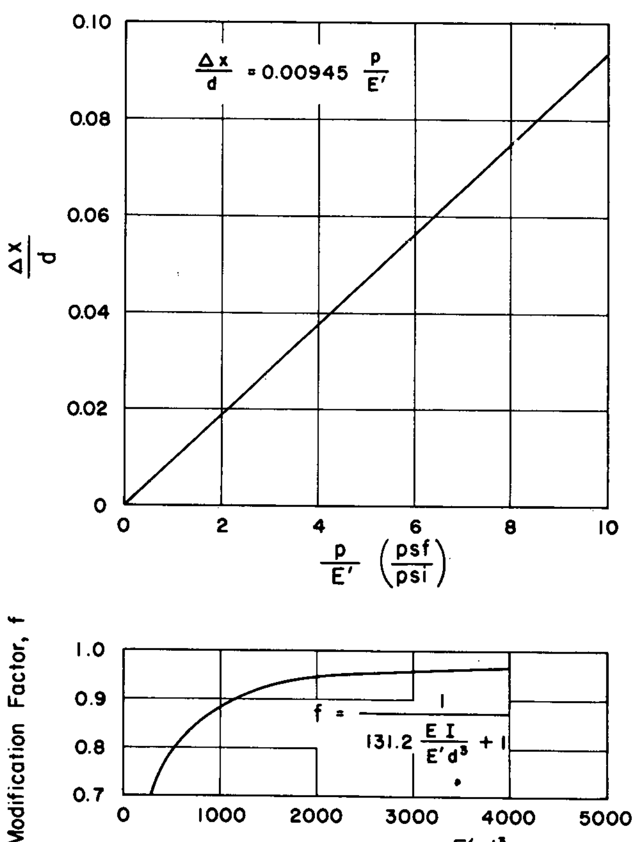


Figure C-17. Graphical representation of the Iowa formula.

primarily on dry density and largely independent of other soil properties over a wide range of soil types, exists between stress and uniaxial strain. Based on this observation and Eq. C-64, a relationship between E' and the vertical soil pressure for various dry densities can be readily developed; the resulting curves are shown in Figure C-4.

In properly applying the values of E' to design, consideration should be given to the culvert deformation history, and a design procedure is suggested to accomplish this. As a by-product, a method is developed to compute culvert deformations for cases where vertical strutting is used. For the case where strutting is used and the culvert is allowed to deform after the fill has been completed, deformation will take place in accordance with the E' value corresponding to the average vertical stress at the culvert springline. Experience has shown that this deformation is considerably less than the summation of the incremental deformations that occur continually throughout the construction process when strutting is not used. The major reason for this difference is the nonlinearity between E' and the stress level.

Until the fill height reaches the top of the culvert, deformation is largely a function of construction practice, and it is usually convenient to consider the shape of the culvert at this point as the datum. With the addition of the first layer of fill above the culvert, deformation will take place in accordance with the E' corresponding to the unit weight of this added layer plus the previously existing vertical stress at the horizontal mid-plane of the culvert. Because E' will be relatively low at this stage of construction, the culvert deformation will be relatively large. With the addition of a second layer, deformation will take place in accordance with the E' corresponding to the unit weight of the second layer plus the previously existing vertical stress at the horizontal mid-plane of the culvert. Because the total stress at the springline of the culvert is greater in the second situation than in the first, E' will be greater and the associated deformation will be smaller for an equal increment of applied load. Each succeeding layer will similarly cause a deformation corresponding to an increasing value of E', and the total deformation is the sum of the incremental deformations due to each layer. Because the lower values of E' will produce proportionately greater deformations, this sum will be somewhat greater than the deformation corresponding to the situation where the total load is applied instantaneously, as when struts are removed, and the deformation occurs in accordance with the final value of E'.

Because the use of a variable E' will require a number of applications of the Iowa formula, considerable work can be saved by converting this procedure to a graphical form. By considering a 90° bedding angle, a time lag factor of unity, and a hydrostatic pressure condition, the Iowa formula may be rewritten as

$$\Delta x/d = 0.00945 p/E' \left[\frac{1}{\dfrac{131.2EI}{E'd^3} + 1} \right] \quad \text{(C-73)}$$

in which $\Delta x/d$ is the percentage diameter change; Δx is the change in diameter; d is the culvert diameter; p is the pressure change; E' is the modulus of soil reaction; E is the modulus of elasticity of the culvert material; and I is the moment of inertia of the culvert wall. A linear relationship, shown in the top of Figure C-17, may be used to give the deformation for a perfectly flexible culvert (that is, $EI = 0$), and this deformation can be adjusted by use of a modification factor, shown in the bottom of Figure C-17, determined by the stiffness ratio between the culvert ring and the surrounding soil.

In summary, because laboratory experiments appear to show that the stress response in a uniaxial strain condition can be predicted for a wide range of soil types from a knowledge of the compacted dry density, a modulus of soil reaction can be predicted for use in conjunction with the Iowa formula to provide a reliable computation of the culvert deformation without recourse to either an arbitrary selection of E' or special soil testing. Only the dry density value, normally determined for the purpose of controlling the embankment construction, is required. The computation requires, however, that the deformation history of the culvert be taken into account, because the modulus of soil reaction varies with confining pressure.

Sample Problem

In proposing the Iowa formula for flexible culvert design, Spangler (*8*) provided a considerable amount of measured field data associated with flexible culverts. The largest installation investigated consisted of a 60-in. standard corrugation 12-gauge pipe constructed beneath 15 ft of fill. The wet and dry unit weights of the fill were 121.6 pcf and 110 pcf, respectively.

According to the proposed modified method, the design is performed in stages (three are taken for this case) to account for the nonlinearity of the soil modulus; calculations are given in Table C-7. Although it is not really practical for the conditions involved, the example is extended to illustrate the effect of strutting, had it been used, on the deflection. The calculated horizontal deformation (without strutting) was 0.72 in., compared to 0.87 in. calculated by Spangler and 1.01 in. actually measured. The calculated deformation where strutting is used was 0.53 in., a reduction of approximately 28 percent. It is possible that the discrepancy between calculated and measured deformation values may be less for conditions of higher fill, as some deformation, not taken into account in the calculations, probably occurs in the initial stages of loading.

TABLE C-7

CALCULATIONS FOR EXAMPLE PROBLEM

$$\frac{d^3}{EI} = \frac{60^3}{29 \times 10^6 \times 0.00345} = 2.16 \qquad \gamma_D = 110 \text{ pcf}$$

AVERAGE STRESS AT CULVERT CENTER, $h\gamma_w$ (PSF)	SOIL MODULUS, E' (FIG. C-4) (PSI)	p/E'	DEFORMATION (FIG. C-17)	$E'd^3/EI = 2.16E'$ (PSI)	MODIFICATION FACTOR (FIG. C-17)	DEFORMATION (IN.)
			(*a*) Case 1. No strutting			
$5 \times 121.6 = 608$	1,050	$608/1{,}050 = 0.58$	0.0055	$2.16 \times 1{,}050 = 2{,}270$	0.95	$0.0055 \times 0.95 \times 60 = 0.31$
$10 \times 121.6 = 1{,}216$	1,600	$608/1{,}600 = 0.38$	0.0036	$2.16 \times 1{,}600 = 3{,}460$	0.95	$0.0036 \times 0.96 \times 60 = 0.21$
$15 \times 121.6 = 1{,}913$	1,820	$608/1{,}820 = 0.33$	0.0032	$2.16 \times 1{,}823 = 3{,}940$	0.96	$0.0032 \times 0.96 \times 60 = 0.19$
						Total deformation = 0.71 in.
			(*b*) Case 2. Strutting used			
$17.5 \times 121.6 = 2{,}125$	1,950	$1{,}821/1{,}950 = 0.935$	0.0091	$2.06 \times 1{,}950 = 4{,}020$	0.97	$0.0091 \times 0.97 \times 60 = 0.53$
						Total deformation = 0.53 in.

APPENDIX D

CONSTRUCTION CONSIDERATIONS

The completion of a satisfactory structure depends not only on the preparation of an adequate engineering design but also on the accuracy with which the design is executed during the construction phase. Working drawings and specifications should be sufficiently thorough to require a performance consistent with the assumptions made in the design, but at the same time they should be sufficiently flexible to permit the use of ingenuity and engineering judgment by the field engineer and the contractor. Field inspection should be competent enough to ensure that, as an absolute minimum, the standards specified in the contract documents are satisfied in the field. The following brief discussion includes some construction considerations that are extremely important to the satisfactory performance of a culvert installation.

SITE PREPARATION

Because in most cases there is little or no choice in selecting a site for a culvert installation, attention must be directed toward adequate site preparation. One important aspect of site preparation is concerned with the minimization of settlements. Because culverts are usually located in stream beds, there is frequently an accumulation of soft compressible sediments and concentrations of vegetation. Complete removal of vegetation is essential before construction begins, and the removal of compressible stream-bed material is highly desirable. Every effort should be made to obtain as high a degree of homogeneity as possible between the culvert foundation soil and the adjacent compacted embankment soil. Also, in areas where large settlements are expected it is often desirable to maximize the pipe gradient, so that a reverse gradient of the pipe is avoided as settlement occurs.

It is common practice during highway soils explorations to locate borings where problem areas are likely to occur, and frequently borings are made in gullies or stream beds. A review of these borings in conjunction with an evaluation of the surface conditions at the site normally will provide sufficient information for design; alternatively, the necessity for a more detailed soils exploration may be indicated. Where removal of highly compressible material is not feasible, settlements may be tolerated and/or the culvert may be cambered. The acceptability of such a situation depends on the provisions that (1) a reasonable prediction of the settlement magnitude can be made, (2) the settlement can be expected to effectively cease after a reasonable time period, and (3) serious differential settlements do not occur along the longitudinal axis of the culvert. Although some uniform settlements of a culvert can be advantageous, because they may lead to a reduction in vertical load, differential settlements are undesirable.

In general, differential settlements are reduced when the compressibility of the foundation soil under a culvert is reasonably uniform throughout the length of the structure. This is one of the reasons why the line of the culvert frequently follows the old stream bed. It is highly undesirable to have one portion of the culvert founded on stream sediments while another portion rests on the less compressible soils of the adjacent stream bank. Total and differential settlements may also be reduced to some extent by using the culvert bedding to spread the load. To account for the differential settlement caused by the trapezoidally shaped embankment load, the culvert must be cambered; this is discussed in detail elsewhere in this report.

BEDDING

The main objective in providing pipe bedding is to produce as nearly as possible a uniform distribution of loading (or reaction to loading) over the area of the pipe surface that cannot be reached during the fill compaction process. The importance of proper bedding in the construction of a culvert cannot be overemphasized, because experience has indicated that inadequate bedding is one of the major contributors to culvert problems. In general, a uniform load distribution is most desirable, as it permits optimum structural performance of the pipe. Commonly found undesirable situations are (1) a rock surface in close proximity to the underside of the culvert (in the case of a concrete pipe this is likely to produce undesirable loading conditions approaching those of the three-edge bearing tests, whereas for a flexible pipe it may lead to excessive deformations), and (2) air voids or soft pockets in the vicinity of the pipe wall (this is a common occurrence where a pipe is placed on a relatively flat soil surface and an attempt is made to obtain the bed by shoveling bedding material under the lower areas of the pipe).

Although a material of uniform compressibility preformed to the shape of the pipe is generally ideal, some difference exists in the requirements for rigid and flexible culverts. For concrete pipes, where the structural deformations are small, a bedding material having a uniformly low compressibility is ideal; in such a case uniform conditions are more reliably obtained, and total settlements are minimized. Although concrete provides excellent bedding for a rigid pipe, it is not generally used. On the other hand, for flexible culverts a bedding material with extremely low compressibility may not deform in accordance with the pipe deformations produced by the compacted fill and, as a result, load concentrations or shape discontinuities may occur at the upper level of the bedding. Therefore, the bedding for flexible conduits should ideally be of a compressibility similar to the compacted backfill surrounding the upper portions of the pipe. With regard to the shaping and the nature of the bedding material for flexible pipes, Peck and Peck (*77*) have stated:

> Because of the difficulty of obtaining adequate compac-

tion from the bottom to about the lower third point of the height of the culverts, it would appear advisable to prepare a bed of compacted fill and to trim it to the contour of the lower part of the culvert with the aid of a template.

FILL CONSTRUCTION IN CULVERT VICINITY

Concrete pipe normally has sufficient strength to withstand loads from compaction equipment used in the adjacent fills. However, considerably more care is required in constructing the fills adjacent to a corrugated metal pipe. Possible excessive distortion of the structure may negate the use of heavy equipment close to the pipe wall. This problem, in addition to the possibility of damage to the structures by the less controllable heavy equipment, normally necessitates the use of hand compaction methods near the pipe wall. Because the performance of flexible culverts depends largely on the passive resistance provided by the fill within a distance of one pipe diameter, good compaction of the soil in this area is absolutely essential, particularly if the fill is high or the borrow material is poor.

A close control of the diameter of large flexible culverts as the fill increases is recommended. Should distortion exceed a few percent of the diameter, horizontal strutting may be necessary. However, under no conditions should the attainment of proper compaction be sacrificed. Special precautions must be taken to place the compacted fill reasonably symmetrically on both sides of the conduit; this is especially necessary in the case of flexible pipes. Also, care must be taken to ascertain that the pipe, especially a flexible one, does not rise as the fill is being compacted below the springline.

COMPACTION PROCEDURES

It is not the intent herein to specify any particular type of compaction equipment or any special compaction procedure. However, certain suggestions are made, and the implementation of these suggestions is left to the discretion of the engineer-in-charge. In general, regardless of the type of specifications used for a particular project, it is felt that an end-product type of specification is desirable for the soil compaction within approximately one diameter of the culvert wall. This is normally a very critical area where large motorized compactors cannot operate effectively; hence, any method-type specifications that may apply to the project in general would not be applicable in this area. Because it is often desired to surround the conduit by a homogeneous soil, the required degree of compaction for the soil adjacent to the conduit walls should ideally be the same as the rest of the fill. This is desirable not only from the standpoint of culvert performance, but also, in the case of low cover heights, from the standpoint of preventing differential settlements in the road surface above the conduit area. Another reason for the suggested use of end-product type of specifications for the compacted fill adjacent to the pipe wall is that there are big differences in the effectiveness of commercially available compaction equipment. It would be very difficult to provide a suitable method-type specification for the hand-operated equipment to correspond with that for the larger motorized equipment. Moisture content control is usually very essential if required densities are to be achieved; in addition, lift thicknesses should be controlled in accordance with the capacity of the compaction equipment.

STRUTTING

When calculations indicate that flexible culvert deformations due to the fill are likely to be excessive, considerable reduction in the ultimate deformation may be obtained either by elongating the conduit in the vertical direction or by maintaining the conduit shape during the construction process and releasing it after the fill has reached a certain height. This latter objective is commonly achieved by installing closely spaced vertical wooden struts throughout the length of the pipe; normally a small amount of deformation is permitted by providing compressible soft wood blocks at the top of the struts. Load concentrations from the struts on the wall of the metal pipe are highly undesirable and provisions should be made to spread the load both longitudinally and laterally. Because considerable structural support for a flexible conduit is obtained by allowing the conduit to increase its horizontal dimension, thereby increasing the horizontal reaction supplied by the adjacent soil, care must be taken to remove the struts at an appropriate point in the construction process. To a large degree the advantages of strutting stem from the nonlinearity of the stress-strain relationship of the soil. Figure C-4 shows the variation in constrained modulus with stress for soils of various dry densities. If the constrained modulus were the same at all stress levels, it would be doubtful that much, if any, advantage would be obtained by strutting. However, because the soil modulus increases significantly as the stress level increases from 0 to 2,000 or 3,000 psf, whereas beyond this stress level the change is relatively minor, it seems logical to remove the struts after the fill has reached approximately 20 to 30 ft above the culvert level, corresponding to approximately 2,000 to 3,000 psf.

SEQUENCE OF OPERATIONS

In some cases involving smaller diameter conduits (less than 4 or 5 ft), contractors have indicated a preference to complete the compacted embankment without the pipe to an elevation 1 or 2 ft higher than the top of pipe, and then excavate a trench in order to install the pipe. As long as acceptable procedures are followed in the bedding and backfill operations, this technique is acceptable and may provide some advantages. The backfill on either side of the pipe should not be constructed in such a manner as to produce significant unsymmetrical loads on the pipe; likewise, the quality of the compaction must be uniform on both sides of the pipe.

JOINTS

The performance of the transverse joints in a conduit is vitally important to the conduit's successful service. In the case of rigid pipes these joints must permit limited longitudinal and rotational movements to account for shrinkage, differential settlements, etc.; otherwise, induced shear and/or tensile stresses will fracture the pipe. If a pipe fractures or if there is a separation at the joint, the backfill soil around the periphery of the pipe may gradually pass through this opening and thereby eventually cause a complete failure of the pipe or excessive settlements of the pavement surface over the pipe; this is especially problematic if the soil around the pipe is a fine sand or silt. For corrugated metal pipes the corrugations ensure sufficient flexibility in the longitudinal direction to prevent such a problem. An investigation of the advantages and disadvantages of various transverse joint connections, as well as longitudinal seam connections, for flexible pipes recently has been completed by the Ohio Department of Highways. Although a study of joints is not within the scope of this report, the designer must be keenly aware of their role in the over-all design of a culvert; failure of a joint may lead to failure of the entire culvert.

APPENDIX E

A FACTOR OF SAFETY CONCEPT

The concept of safety factor is probably one of the most commonly misunderstood concepts in the field of engineering. This is perhaps because it is so closely associated with the nebulous notion of failure. Often, failure of an engineering structure is a condition that many are prone to consider as obvious until they are asked to be specific and quantitative; only then is it realized that a precise definition of failure is not easily formulated. Without belaboring the point, one must always be aware that a quantitative value for a factor of safety is completely meaningless without a clear and unambiguous understanding of the conditions under which it was obtained.

Because it is difficult, if not impossible, to quantify many of the parameters (such as definition of failure, applicability of method of analysis, material properties, and magnitude and distribution of applied loads, as well as sociological, psychological, and political considerations, which enter into the determination of a factor of safety for a given set of circumstances) there is an increasing tendency for engineers to interpret such a "factor" as a factor of ignorance rather than a factor of safety, because a knowledge of the boundary between safe and unsafe is extremely vague. The choice of a particular desired value for the factor of safety requires the exercise of good engineering judgment. No improved methods of analysis and no sophisticated descriptions for factor of safety will obviate the need for good engineering judgment in the foreseeable future.

The factor of safety concept proposed herein is based on a comparison of loads, as opposed to deflections; the magnitude and distribution of the loads acting on the periphery of the conduit are required as input information, and this approach is sufficiently flexible to permit ready adaptation to any assumed, controlled, or measured loading. This seems to be a significant feature in favor of the approach at this stage of development of culvert design procedures, because it may be anticipated that one of the most probable improvements in culvert design, installation, and performance will be brought about by improved installation technique whereby the actual pressure distribution on the culvert will be controlled to a greater extent. As explained previously, the most desirable pressure distribution is one in which flexural stresses are avoided in the conduit wall; for a circular conduit this desired loading would consist of uniformly (or hydrostatically) distributed radial pressures on the culvert wall. Better techniques are continually becoming available to measure actual pressure distributions, and this approach provides a consistent framework within which to interpret and compare different sets of conditions, as well as the various commonly used load distributions.

STATE OF THE ART

In the design of conventional engineering structures, the factor of safety is usually defined as the ratio of the minimum strength or resistance capacity of the structural system to the probable or specified set of loads acting on the structure. Quite different types of failure might govern the design, and the functional or collapse mode, which is associated with the minimum strength of the structure, is the critical one to be used for evaluating the safety of the structure. The possible failure modes might include crushing, fracture, fatigue, buckling, a specific state of stress, or the magnitude of elastic and/or plastic deformations. The establishment of realistic values for the magnitude and distribution of the applied loads is not a trivial matter for even the simplest structure. Very often a characteristic load pattern is specified by the appropriate building code, standard, or specification that governs the design of the structure. The relationship of these specified loads to the actual loading conditions is a problem of load analysis and should be considered in terms of probability theory.

In many instances, the design engineer may be able to adjust the morphology of the structural system so as to control the maximum service load that will be applied to the structure. This adjustment of the loading will make it possible for the designer to use a more economical structure, while still maintaining the desired factor of safety. The exact magnitude of the factor of safety to be used for a given structural system is a matter of engineering judgment; however, the designer knows that a safety factor of unity implies that the assumed failure mode should occur when the service loading is applied to the structure.

The conventionally accepted definition of factor of safety is that quantity that relates failure load to the known or assumed service load, and this concept has the virtue of being consistent and generally applicable to a wide variety of structures. This simple definition provides the design engineer with a measure of the safety of the structure, provided that the failure load and the service load have been evaluated in a deterministic manner. Although a more rational index of the safety of the structure would be based on a probabilistic approach (*78, 79, 80*), this discussion is confined to the more traditional engineering definition.

Commonly used current (1969) standard design methods for pipe culverts do not provide for a realistic assessment of the safety or reliability of the culvert under an assumed service loading. This situation is due in part to the definition of supporting strength of the culverts and our inadequate knowledge of the exact nature of the loading on the system.

In the case of rigid conduits, the supporting strength of the pipe is evaluated in terms of the cracking or ultimate load as obtained from a three-edge bearing test conducted according to ASTM C 76. This test produces the most severe type of loading to which the pipe will be subjected, but it does not reflect the service loads on the pipe when it is installed in the ground. Empirical load factors have been developed to relate the three-edge bearing strength to the in-place supporting strength (see Appendix A). Because of the assumed conservative nature of these load factors, the suggested factor of safety to be used for the design of reinforced concrete pipe varies from 1.0 (*29*) to 1.5 (*39*).

One of the critical failure criteria for circular flexible pipe is deflection. Experience has shown that failure by deflection will normally not occur until the vertical diameter is decreased by about 20 percent from the circular shape. It is suggested that designs based on the Iowa Deflection Formula (*29*) be restricted to a maximum deflection of 5 percent of the nominal pipe diameter, thus providing a "factor of safety" of approximately 4. This type of calculation disregards the nonlinear behavior of a structural system that is subjected to such large deformations. Two additional types of failure that must be considered in the design of flexible culverts are buckling and seam strength. The expressions that are presently in use for evaluating the strength of the pipe to resist these modes of failure have been obtained from empirical considerations and are based on the premise that the only loading on the conduit is a uniform circumferential thrust. In contrast, the Iowa deflection formula assumes that the principal components of the load system acting on the flexible culvert are in the vertical direction (*8*). The commonly accepted factors of safety for these modes of failure vary from 2.0 to 4.0 (*42, 81*).

As seen previously, there is no consistency in applying a factor of safety concept to the design of pipe culverts. This situation is of little or no concern for the majority of culverts that are being designed today, because most of the empirical constants and values listed in standard height-of-fill tables have been adjusted to yield conservative designs. These data are based on the experience that various agencies and manufacturers have obtained concerning the design, installation, serviceability, and maintenance of typical culvert projects. Although this type of background information and experience may be adequate for certain circumstances, it will not, in general, suffice to cope with the design of culverts for the future. For example, the design of culverts under very high fills requires a more consistent and rational basis for the evaluation of safety in order that the magnitude and distribution of the loading that acts on the culvert may be adjusted or controlled to obtain greater efficiency and economy in the culvert system.

PROPOSED CONCEPT

The following discussion presents a new concept for evaluating the factor of safety of culverts. This concept is consistent with the general definition of safety as understood by a structural engineer, and it is easy to apply. It is valid for both rigid and flexible conduits, and has the capability of determining the safety of a culvert against all possible modes of failure. It is proposed that the factor of safety, F.S., of a conduit be defined as the ratio of the intensities of two normal loadings on the conduit, each of which produces only an axial stress resultant on each cross section of the culvert; thus,

$$\text{F.S.} = \frac{p_{\text{failure}}}{p_{\text{maximum}}} \tag{E-1}$$

in which p_{failure} is the magnitude, at a given point on the culvert, of the normal loading, which will produce within the structure an axial stress resultant which, in turn, will induce at the critical section a stress intensity that is equal to the maximum stress at that point when the culvert reaches the stipulated failure condition; and p_{maximum} is the magnitude, at the same point on the culvert, of the equivalent normal loading that produces within the structure an axial stress resultant that is consistent with, and represents the effect of, the assumed in-situ load distribution acting on the conduit. One important advantage of this concept is that the factor of safety of the conduit is evaluated by comparing like quantities. For the special case of circular culverts, the quantities p_{failure} and p_{maximum} represent the intensities of loadings that are uniformly distributed around the circumference of the conduit.

EQUIVALENT LOADING FOR THE IN-SITU CONDITION

One possible distribution, $p(\theta)$, of an in-situ normal loading around a circular culvert is shown in Figure E-1. This loading condition may be assumed according to some appropriate authority, or based on test data taken from a field installation. The actual loading can be resolved into two principal components, a uniformly distributed loading, p_a, which produces only an axial stress resultant in the culvert,

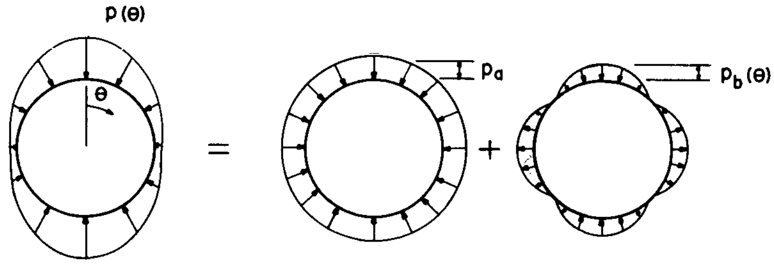


Figure E-1. Decomposition of loading diagram.

and a second component, $p_b(\theta)$, which produces shear and flexural stress resultants around the circumference of the culvert. This decomposition of the loading is shown in Figure E-1.

Next, the effect of the flexural loading, $p_b(\theta)$, on the culvert must be represented by a uniformly distributed normal loading. Because this type of loading will produce both tensile and compressive stresses in the culvert, there must be a set of two uniform loadings that produce the same maximum stresses as the flexural loading; let p_{bec} be the equivalent loading associated with the maximum compressive stress and p_{bet} the equivalent loading associated with the maximum tensile stress. It is important to note that p_{bec} is not necessarily equal in magnitude to p_{bet}. This decomposition of the flexural loading is shown in Figure E-2.

Based on the preceding arguments, it is now possible to define the appropriate value or values of p_{maximum} that represent the effect of the in-situ loading on the culvert. For the case of normal loading that will produce a critical state of compressive stress, the value of p_{maximum} is given by

$$p_{\text{max compression}} = p_a + p_{\text{bec}} \tag{E-2}$$

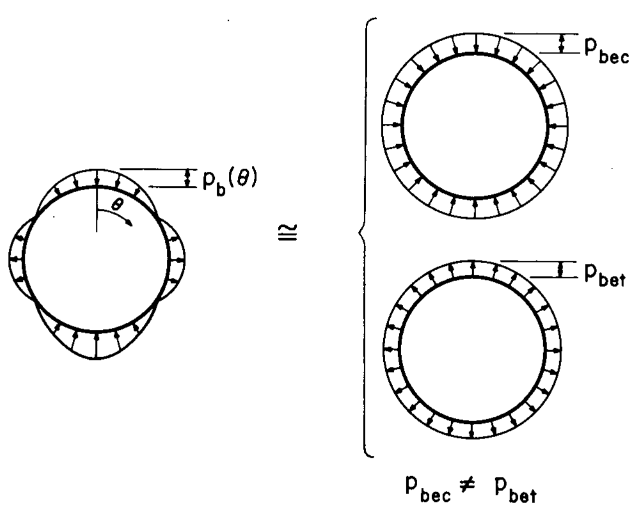


Figure E-2. Equivalent uniform loadings that produce the same maximum stress as the flexural loading.

Provided that $|p_{\text{bet}}|$ is greater than $|p_a|$, there will be another value of p_{maximum} that must be considered—namely, the value associated with a critical state of tensile stress,

$$p_{\text{max tension}} = p_a - p_{\text{bet}} \tag{E-3}$$

The actual decomposition of the in-situ loading can most conveniently be accomplished by means of a cosine series approximation of the original loading distribution. This procedure implies (1) that no shear tractions are present, so that the loading is normal to the surface of the culvert, and (2) that the loading is symmetric with respect to the vertical axis of the culvert. In addition, in order that the subsequent analysis will be applicable, the response of the structural system to the applied loading must be linear, so that the principle of superposition is valid. Within the limits of these assumptions, the in-situ loading on a circular culvert can be approximated by the cosine series:

$$p(\theta) = \sum_{n=0}^{\infty} p_n \cos n\theta \tag{E-4}$$

and the following analysis will be valid.

The first six terms of this approximation are shown in Figure E-3. It should be noted that each of the components is in equilibrium, with the exception of the loading for the case of $n = 1$; therefore, this component is of secondary importance when compared with the other flexural components. From Figures E-1 and E-3 it can be seen that

$$p_a = p_o \tag{E-5a}$$

and

$$p_b(\theta) = \sum_{n=1}^{\infty} p_n \cos n\theta \tag{E-5b}$$

The p_n amplitudes for a given loading condition may be conveniently evaluated by means of a standard regression analysis program on an electronic digital computer. These quantities may also be evaluated manually by means of the harmonic analysis technique (*82*). To illustrate the mechanics of this method, several different loading conditions are analyzed and presented herein. In each case a computer program was used to provide the best fit, in a statistical sense, of the given data. The program was arranged so that six different approximations were obtained for each loading condition.

For the initial solution, the p_o component was specified and the most suitable component, which, when in combina-

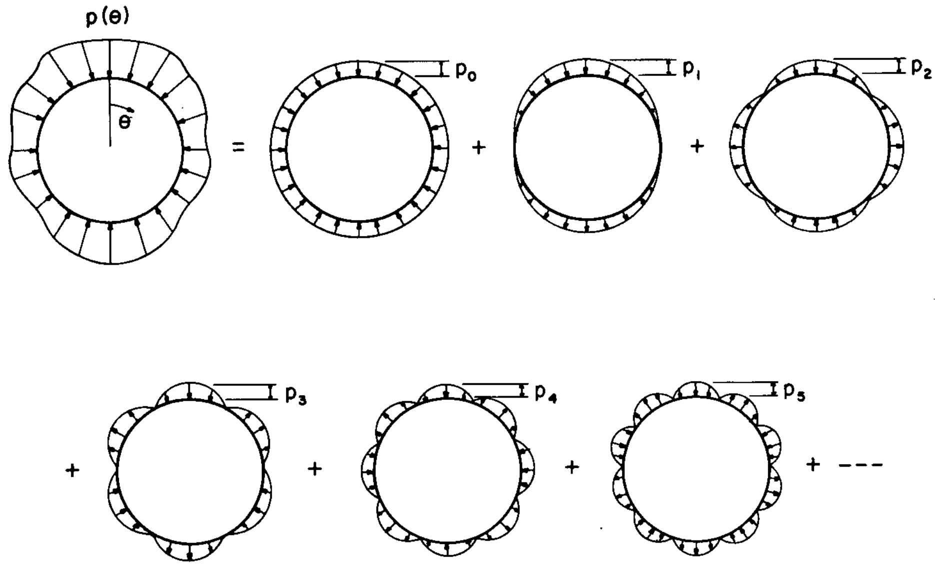


Figure E-3. Approximation of in-situ loading by cosine series.

tion with the p_0 term, produced the best fit of the data, was then selected from the $p_i \cos i\,\theta$ $(i = 1, 6)$ terms. Thus, the first approximation of the loading distribution consisted of the appropriate values of the magnitudes of these two terms. In most of the cases that were considered, the first approximation consisted of the terms p_0 and $p_2 \cos 2\theta$. The next and each succeeding approximation was constructed from a combination of the configurations of the previous approximation together with the most suitable remaining component in the p_1 to p_6 set. In arriving at the values for a new solution, it was possible that the magnitudes of the configurations used in the previous approximations could be altered to achieve the best fit for the new combination of components.

Results obtained by applying this series approximation technique to two loading distributions contained in the literature are given in Table E-1 and are shown in Figures E-4 and E-5. The first example is the analytic distribution developed by Olander (*40*), and the second is the symmetrized pressure data measured and reported by Spangler (*36*). It is easy to see that the accuracy of the series solution increases with each succeeding step of the approximation, and that a rather good description of the given loading function is achieved with only six terms of the cosine series.

The symmetrized data of three distributions of soil pressures on concrete pipe, as measured in tests at the U.S. Bureau of Reclamation (*83, 84*), have been approximated by the cosine series technique; the results are summarized in Table E-2. The three embankment conditions of the test setups are shown in Figure E-6. An assumed loading distribution is shown in Figure E-7a, and the cosine series approximations for various values of the parameters K and H/B are given in Table E-3. The distribution of loading on a flexible culvert, as assumed by Spangler (*8*), is shown in Figure E-7b, and the cosine series approximations for various values of the parameters ψ and ξ are given in Table E-4.

The process of converting those components of the in-situ loading diagram that reflect the desired degree of accuracy of solution into an equivalent normal loading by means of Eqs. E-2 and E-3 will depend on the material properties, together with the size and shape of the culvert. The evaluation of p_{maximum} for the case of rigid circular culverts is presented in the next section, and that discussion provides the basis for applying the basic concept to other types of systems.

TABLE E-1

RESULTS BASED ON OLANDER AND SPANGLER LOAD DISTRIBUTIONS

ORDINATE	APPROXIMATION NUMBER 1	2	3	4	5	6
	(*a*) Olander loading (see Fig. E-4)					
p_0	0.436	0.433	0.439	0.440	0.440	0.439
p_1					0.019	0.019
p_2	0.271	0.271	0.257	0.259	0.259	0.258
p_3		−0.224	−0.228	−0.217	−0.217	−0.217
p_4			0.143	0.144	0.143	0.143
p_5				−0.067	−0.067	−0.067
p_6						0.007
	(*b*) Spangler loading (see Fig. E-5)					
p_0	4.170	4.170	4.018	4.018	4.018	4.023
p_1					−0.789	−0.789
p_2	3.428	3.428	3.119	3.119	3.119	3.135
p_3		−1.871	−1.871	−1.620	−1.484	−1.484
p_4			1.682	1.682	1.682	1.698
p_5				−1.270	−1.139	−1.139
p_6						−0.089

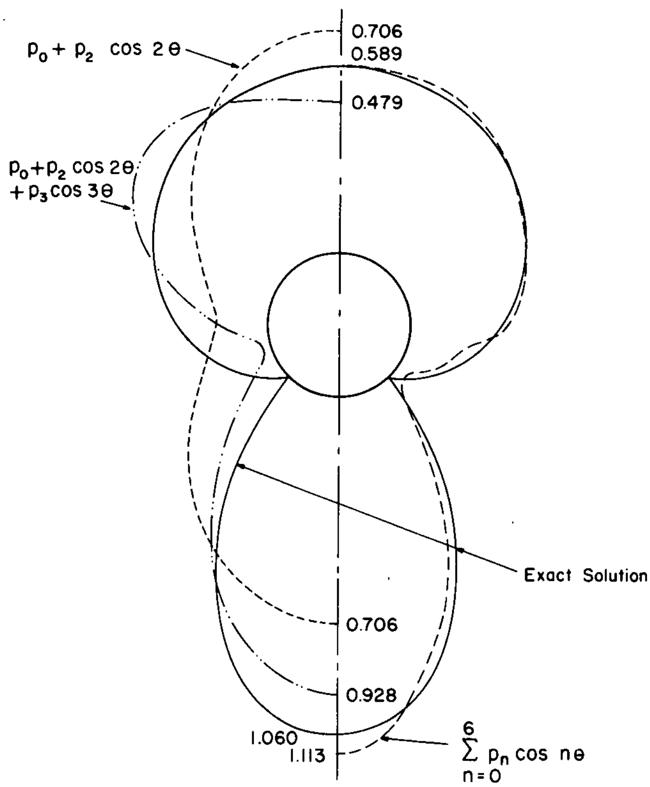


Figure E-4. Olander load distribution.

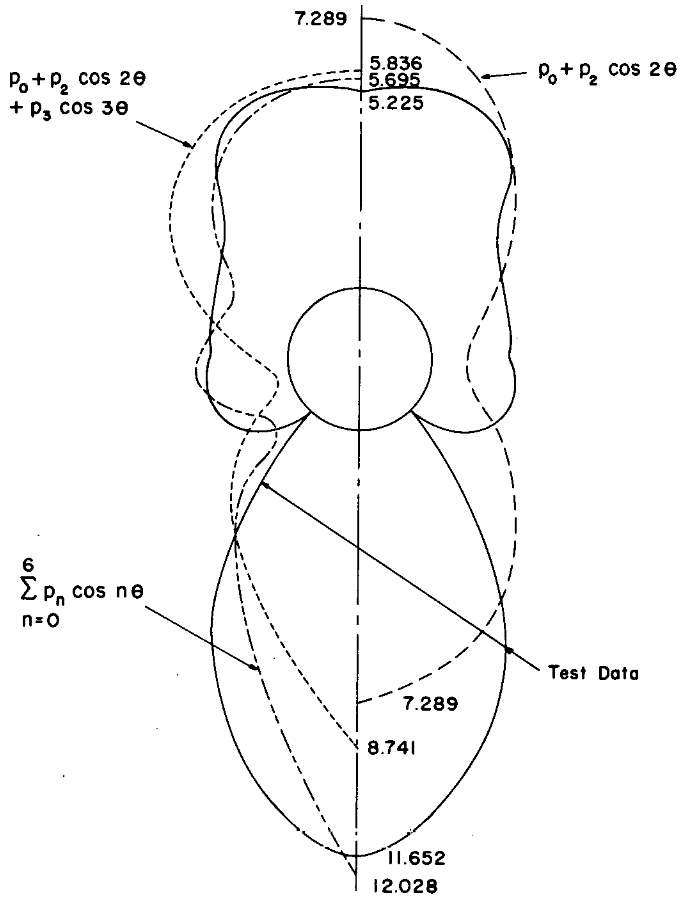


Figure E-5. Spangler experimental load distribution.

RIGID CULVERTS

General expressions for the tangential normal stresses in an elastic ring, which is subjected to external normal loading having a cosine type of distribution around the circumference of the ring, can be obtained as a plane strain solution from the theory of elasticity. The expressions of interest for use in the present analysis of rigid circular culverts have been developed by Gabriel (*63*) and in their most general form become:

$$\tau_{\theta\theta}(r) = \frac{p_o}{1 - \alpha^2} [1 + \alpha^2 a_r^{-2}] \tag{E-6}$$

$$\tau_{\theta\theta}(r, \theta) = p_1 a_r \left[\frac{3 + \alpha^4 a_r^{-4}}{1 + \alpha^4} \right] \cos \theta \tag{E-7}$$

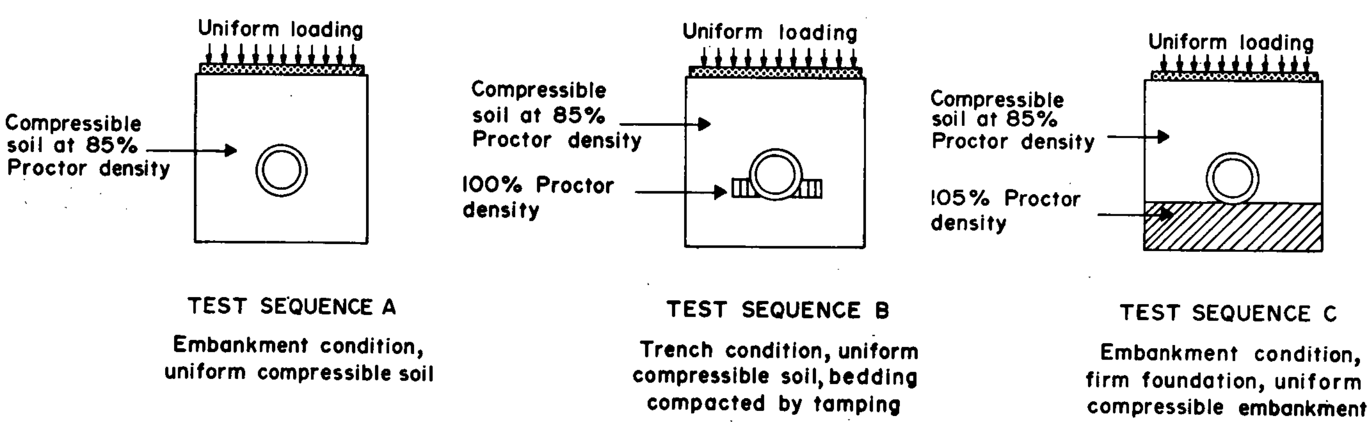


Figure E-6. Embankment conditions for Bureau of Reclamation tests.

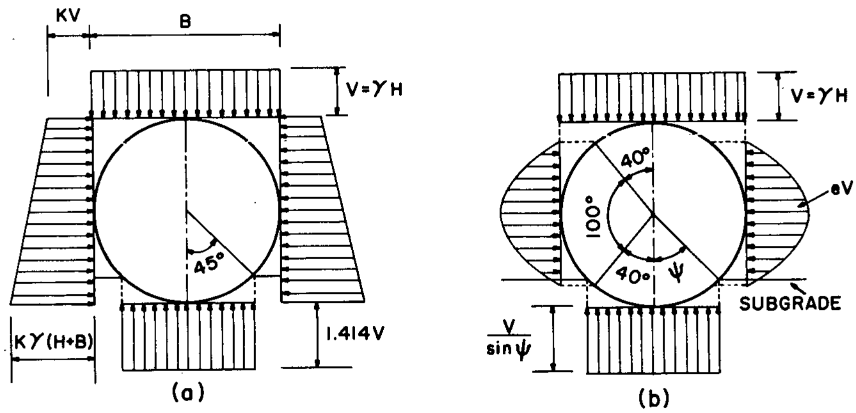


Figure E-7. Typical assumed load distributions on buried conduits.

$$\tau_{\theta\theta}(r,\theta) = \frac{p_n}{2D}[n\alpha_r^{n-2}(n-1+\alpha^{-2n}-n\alpha^2) + n\alpha_r^{-n-2}(n+1-\alpha^{2n}-n\alpha^2) + (n+2)\alpha_r^{n}(n+1-\alpha^{-2n}-n\alpha^{-2}) + (n-2)\alpha_r^{-n}(n-1+\alpha^{2n} - n\alpha^{-2})]\cos n\theta \quad \text{(E-8)}$$

when $n \geqq 2$;

in which

$$D = \alpha^{-2n} + \alpha^{2n} - n^2(\alpha^{-2}+\alpha^2) + 2(n^2-1) \quad \text{(E-9)}$$

α equals r_i/r_o; r_i and r_o are the inner and outer radii, respectively, of the ring; r is the coordinate distance to the point of interest $(r_i \leq r \leq r_o)$; $\alpha_r = r/r_o (\alpha \leq \alpha_r \leq 1.0)$; and p_n is the magnitude of the normal loading applied to the circumference of the ring and distributed around the ring in the configuration $\cos n\theta$. The stress at any point in the ring can be expressed as the sum of Eqs. E-6 and E-8. It should be noted that these expressions are not valid for shell-type structures; therefore, they cannot be used for evaluating stresses in flexible culverts.

In the present discussion, attention is confined to the maximum normal stresses that occur at the extreme fibers of the ring. Consequently, the stresses at the inner boundary $(r = r_i)$ become:

$$\tau_{\theta\theta}(r_i) = p_o\left[\frac{2}{1-\alpha^2}\right] \quad \text{(E-10)}$$

$$\tau_{\theta\theta}(r_i,\theta) = p_1\left[\frac{4}{1-\alpha^4}\right]\cos\theta \quad \text{(E-11)}$$

and

$$\tau_{\theta\theta}(r_i,\theta) = p_n\left[\frac{2n(\alpha^{-2}-1)(\alpha^n-\alpha^{-n})}{\alpha^{2n}+\alpha^{-2n}-n^2(\alpha^2+\alpha^{-2})+2(n^2-1)}\right]\cos n\theta \quad \text{(E-12)}$$

when $n \geqq 2$ and the stresses at the outer boundary $(r = r_o)$ are:

$$\tau_{\theta\theta}(r_o) = p_o\left[\frac{1+\alpha^2}{1-\alpha^2}\right] \quad \text{(E-13)}$$

$$\tau_{\theta\theta}(r_o,\theta) = p_1\left[\frac{3+\alpha^4}{1-\alpha^4}\right]\cos\theta \quad \text{(E-14)}$$

and

$$\tau_{\theta\theta}(r_o,\theta) = p_n\left[\frac{\alpha^{2n}+\alpha^{-2n}+n^2(\alpha^2+\alpha^{-2})-2(n^2+1)}{\alpha^{2n}+\alpha^{-2n}-n^2(\alpha^2-\alpha^{-2})+2(n^2-1)}\right]\cos n\theta \quad \text{(E-15)}$$

when $n \geqq 2$.

TABLE E-2

RESULTS BASED ON BUREAU OF RECLAMATION TESTS

ORDINATE	APPROXIMATION NUMBER 1	2	3	4	5	6
		(*a*) Bureau of Reclamation Test A				
p_0	12.962	13.005	13.005	13.005	13.026	13.026
p_1			0.844	0.844	0.844	0.844
p_2	7.005	7.101	7.101	7.101	7.065	7.065
p_3				−0.416	−0.416	−0.387
p_4					0.372	0.372
p_5						−0.158
p_6		−1.012	−1.011	−1.012	−1.047	−1.047
		(*b*) Bureau of Reclamation Test B				
p_0	12.300	12.301	12.301	12.300	12.258	12.300
p_1			2.199	2.198	2.199	2.199
p_2	5.800	5.801	5.801	5.800	5.705	5.634
p_3		2.971	2.972	3.340	3.341	3.341
p_4						7.411
p_5				−2.033	−2.032	−2.032
p_6					−0.999	−0.929
		(*c*) Bureau of Reclamation Test C				
p_0	18.026	18.025	18.024	18.115	18.175	18.175
p_1						−0.406
p_2	12.675	12.672	12.670	12.489	12.599	12.599
p_3		−7.186	−6.709	−6.708	−6.709	−6.709
p_4				1.725	1.826	1.826
p_5			−2.631	−2.630	−2.631	−2.631
p_6					−1.272	−1.272

TABLE E-3

RESULTS FOR VARIOUS VALUES OF K AND H/B

ORDINATE	APPROXIMATION NUMBER 1	2	3	4	5	6
	(*a*) $K=0.33$, $H/B=2.0$					
p_0	0.458	0.458	0.458	0.460	0.460	0.458
p_1			−0.068	−0.068	−0.068	−0.068
p_2	0.223	0.223	0.223	0.229	0.229	0.232
p_3		−0.081	−0.081	−0.081	−0.091	−0.091
p_4						−0.036
p_5					0.053	0.053
p_6				−0.058	−0.058	−0.055
	(*b*) $K=0.33$, $H/B=4.0$					
p_0	0.442	0.442	0.442	0.444	0.444	0.444
p_1			−0.058	−0.058	−0.058	−0.058
p_2	0.234	0.234	0.234	0.240	0.240	0.243
p_3		−0.088	−0.088	−0.088	−0.097	−0.097
p_4						−0.034
p_5					−0.052	0.052
p_6				−0.057	−0.057	−0.054
	(*c*) $K=0.33$, $H/B=10$					
p_0	0.437	0.437	0.440	0.440	0.440	0.438
p_1				−0.054	−0.054	−0.054
p_2	0.238	0.238	0.244	0.244	0.244	0.247
p_3		−0.091	−0.091	−0.091	−0.100	−0.100
p_4						−0.034
p_5					0.050	0.050
p_6			−0.057	−0.057	−0.057	−0.057
	(*d*) $K=0.50$, $H/B=2.0$					
p_0	0.525	0.525	0.525	0.528	0.528	0.525
p_1		−0.077	−0.077	−0.077	−0.077	−0.077
p_2	0.176	0.176	0.176	0.182	0.182	0.187
p_3			−0.075	−0.076	−0.085	−0.085
p_4						−0.046
p_5					0.055	0.055
p_6				−0.064	−0.064	−0.060
	(*e*) $K=0.50$, $H/B=5.0$					
p_0	0.503	0.503	0.505	0.505	0.505	0.503
p_1				−0.059	−0.060	−0.059
p_2	0.193	0.193	0.199	0.199	0.199	0.203
p_3		−0.087	−0.087	−0.087	−0.096	−0.096
p_4						−0.045
p_5					0.050	0.050
p_6			−0.064	−0.064	−0.064	−0.060
	(*f*) $K=0.50$, $H/B=10.0$					
p_0	0.493	0.493	0.496	0.496	0.496	0.493
p_1				−0.056	−0.056	−0.056
p_2	0.198	0.198	0.204	0.204	0.204	0.208
p_3		−0.089	−0.089	−0.089	−0.099	−0.099
p_4						−0.041
p_5					0.051	0.051
p_6			−0.061	−0.061	−0.061	−0.057
	(*g*) $K=1.0$, $H/B=2.0$					
p_0	0.754	0.750	0.752	0.752	0.752	0.752
p_1	−0.126	−0.126	−0.126	−0.126	−0.126	−0.126
p_2						−0.009
p_3					−0.016	−0.016
p_4		−0.082	−0.078	−0.078	−0.078	−0.078
p_5				0.020	0.023	0.023
p_6			−0.047	−0.047	−0.047	−0.048
	(*h*) $K=1.0$, $H/B=4.0$					
p_0	0.696	0.693	0.693	0.690	0.693	0.693
p_1	−0.086	−0.086	−0.086	−0.086	−0.086	−0.086
p_2				0.053	0.058	0.058
p_3			−0.051	−0.051	−0.051	−0.057
p_4		−0.070	−0.070	−0.074	−0.070	−0.070
p_5						0.032
p_6					−0.055	−0.055
	(*i*) $K=1.0$, $H/B=10.0$					
p_0	0.663	0.660	0.656	0.659	0.659	0.659
p_1					−0.062	−0.062
p_2		0.080	0.088	0.094	0.094	0.094
p_3	−0.086	−0.086	−0.086	−0.086	−0.086	−0.095
p_4			−0.071	−0.065	−0.065	−0.065
p_5						0.052
p_6				−0.070	−0.070	−0.070

TABLE E-4

RESULTS FOR VARIOUS VALUES OF ψ AND ξ

ORDINATE	APPROXIMATION NUMBER 1	2	3	4	5	6
	(*a*) $\psi=90°$, $\xi=0.0$					
p_0	0.313	0.311	0.310	0.310	0.310	0.310
p_1				−0.009	−0.009	−0.009
p_2	0.201	0.205	0.203	0.203	0.203	0.203
p_3					−0.006	−0.006
p_4		−0.042	−0.044	−0.044	−0.044	−0.044
p_5						0.001
p_6			0.028	0.028	0.028	0.028
	(*b*) $\psi=90°$, $\xi=2.0$					
p_0	0.632	0.639	0.638	0.638	0.638	0.638
p_1					−0.002	−0.002
p_2	−0.271	−0.286	−0.287	−0.287	−0.287	−0.287
p_3						0.002
p_4		0.154	0.152	0.152	0.152	0.152
p_5				0.002	0.002	0.002
p_6			0.021	0.021	0.021	0.021
	(*c*) $\psi=45°$, $\xi=0.0$					
p_0	0.300	0.299	0.300	0.302	0.302	0.302
p_1						−0.012
p_2	0.230	0.320	0.323	0.320	0.320	0.320
p_3		−0.132	−0.132	−0.133	−0.136	−0.135
p_4				0.033	0.033	0.033
p_5					0.016	0.016
p_6			−0.044	−0.048	−0.047	−0.046
	(*d*) $\psi=45°$, $\xi=0.5$					
p_0	0.379	0.378	0.381	0.382	0.382	0.382
p_1						−0.013
p_2	0.199	0.199	0.192	0.195	0.195	0.195
p_3		−0.131	−0.133	−0.132	−0.135	−0.135
p_4			0.077	0.082	0.082	0.082
p_5					0.017	0.017
p_6				−0.048	−0.047	−0.047
	(*e*) $\psi=45°$, $\xi=2.0$					
p_0	0.613	0.624	0.623	0.624	0.624	0.624
p_1						−0.016
p_2		−0.182	−0.182	−0.179	−0.179	−0.180
p_3			−0.131	−0.131	−0.134	−0.134
p_4	0.205	0.220	0.223	0.228	0.227	0.227
p_5					0.020	0.020
p_6				−0.048	−0.047	−0.046

For the standard sizes of concrete pipe being produced today (i.e., 24 in. to 144 in.), the parameter α is confined to a rather limited range; therefore, the subsequent discussion and development are based on the assumption that $0.80 \leqq \alpha \leqq 0.85$.

The relationships between an applied uniformly distributed loading and the resulting normal stresses at the boundaries are given by Eqs. E-10 and E-13. These two expressions are plotted in Figure E-8 in dimensionless form as a function of the geometric parameter α. This chart can be used to evaluate p_{failure} for the critical value of the normal stress that is produced in the culvert at a prescribed failure condition.

To evaluate the equivalent uniform loading that will produce within the culvert the same maximum intensity of normal stress as that produced by the flexural loading components, Eq. E-10 is equated to Eqs. E-11 and E-12, and Eq. E-13 is equated to Eqs. E-14 and E-15 for the case where $\theta = 0$. This procedure yields ratios of the two magnitudes of loading as a function of the geometric parameter α. These ratios are denoted by the symbols β_{ni} and β_{no}, in which the first subscript refers to the component of the cosine series, and the second subscript indicates whether the ratio is for an equivalent stress condition at the inner (i) or outer (o) boundary of the culvert. The resulting expressions are:

$$\beta_{1i} = \frac{(p_o)_1}{p_1} = \frac{2\alpha}{1+\alpha^2} \tag{E-16}$$

$$\beta_{ni} = \frac{(p_o)_n}{p_n} = \frac{n(\alpha^{-2}-1)(\alpha^{-n}-\alpha^n)(1-\alpha^2)}{\alpha^{2n}+\alpha^{-2n}-n^2(\alpha^2+\alpha^{-2})+2(n^2-1)}$$

when $n \geqq 2$ (E-17)

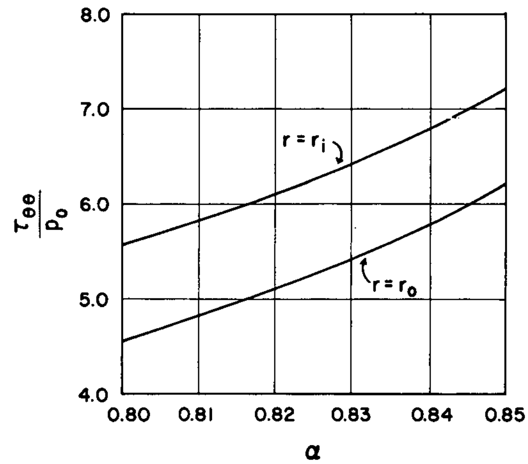


Figure E-8. Relationship between applied uniformly distributed loading and resulting normal stresses at the pipe boundaries.

$$\beta_{1o} = \frac{(p_o)_1}{p_1} = \frac{3+\alpha^4}{(1+\alpha^2)^2} \tag{E-18}$$

and

$$\beta_{no} = \frac{(p_o)_n}{p_n} = \left[\frac{1-\alpha^2}{1+\alpha^2}\right] \left[\frac{\alpha^{2n}+\alpha^{-2n}+n^2(\alpha^2+\alpha^{-2})-2(n^2+1)}{\alpha^{2n}+\alpha^{-2n}-n^2(\alpha^2+\alpha^{-2})+2(n^2-1)}\right]$$

when $n \geqq 2$. (E-19)

These relationships are shown graphically in Figures E-9 and E-10 for the cases of n ranging from 1 to 4. These

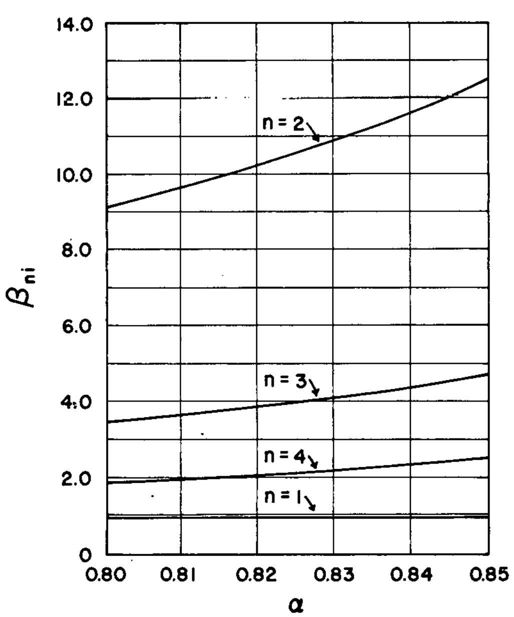


Figure E-9. Values of β_{ni} as a function of α and n.

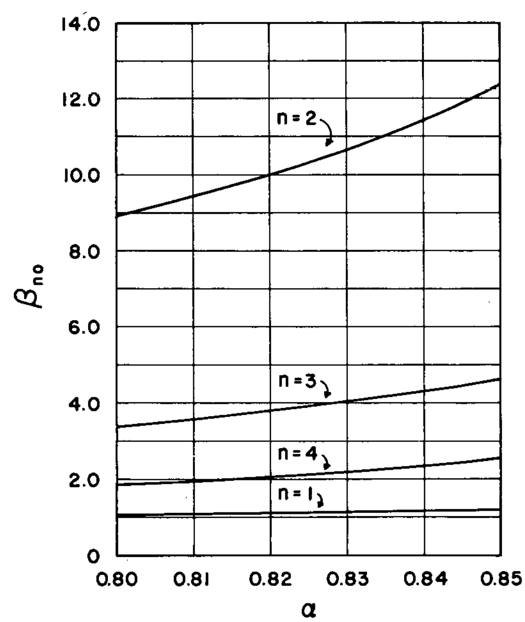


Figure E-10. Values of β_{no} as a function of α and n.

figures show that the second component is the most influential, and that the ratio of the loading magnitudes increases with increasing values of α; however, this ratio decreases as the order of the loading component increases above the case where $n = 2$. The values obtained from Eq. E-17 are always greater than those evaluated from Eq. E-19.

The graph of Figure E-9 could be used for evaluating the equivalent uniform loading, p_{bet}, of Eq. E-3, and the graph of Figure E-10 could be used for evaluating the p_{bec} loading of Eq. E-2. However, the calculational procedure can be simplified on the basis of some additional observations concerning the behavior of rigid culverts and some interrelationships among Eqs. E-16 through E-19. Because of the nature of the bedding conditions of the conduit (i.e., the vertical loading is distributed over the entire width of the cross section at the top of the section, whereas at the bottom of the conduit the same magnitude of vertical load is assumed to be distributed over a distance that is less than the total width of the section), the critical stress condition will occur at the bottom of the cross section; that is, at $\theta = 180°$. From Figure E-3 it can be seen that all of the loading distributions of the cosine series have their maximum values at this location in the cross section. Therefore, it is not necessary to consider the angular dependence of the stress equations (Eqs. E-11, E-12, E-14, and E-15), and the equivalent uniform loadings can be evaluated by a simple addition of the transformed magnitude of the loading components. For example, from Eq. E-3,

$$p_{\max\ \text{tensile}} = p_o + \sum_{n=1}^{N} \beta_{ni} p_n \qquad \text{(E-20)}$$

where the appropriate signs are assumed for each value of β_{ni} and p_n.

An examination of the ratios β_{ni}/β_{2i} and $\beta_{no}/\beta_{2o} (3 \leqq n \leqq 6)$ from Eqs. E-17 and E-19, respectively, over the range of α being considered reveals that each of these ratios is constant, and also that they are equal for the same value of n. In addition, it was determined that for practical purposes the values of β_{1i} and β_{1o} from Eqs. E-16 and E-18, respectively, and the ratio β_{2i}/β_{2o} can be assumed to be constants over the range of α considered. Based on these observations, it is possible to express Eqs. E-2 and E-3 in the form

$$p_{\max\ \text{tension}} = p_o + 0.980 p_1 - \lambda_t(\alpha) \rho(p_n) \qquad \text{(E-21)}$$

and

$$p_{\max\ \text{compression}} = p_o - 1.225 p_1 + 0.982 \lambda_t(\alpha) \rho(p_n) \qquad \text{(E-22)}$$

in which

$$\lambda_t(\alpha) = \beta_{2i}(\alpha) \qquad \text{(E-23}a\text{)}$$

and

$$\rho(p_n) = p_2 - 0.377 p_3 + 0.203 p_4 - 0.128 p_5 + 0.088 p_6 \qquad \text{(E-23}b\text{)}$$

The values of $\lambda_t(\alpha)$ are given in Table E-5.

To illustrate the application of Eqs. E-21 and E-22 and to evaluate the accuracy of the proposed method, consider the loading distribution presented by Olander (*40*) and shown in Figure E-4. For the case where α equals 0.80, the maximum normal stresses due to thrust and moment are evaluated by means of the charts (*40*) and found to be $\tau_{\theta\theta}(r_i, \pi) = 15.07$ psi tension and $\tau_{\theta\theta}(r_o, \pi) = 15.68$ psi compression. These stresses are converted to equivalent uniformly distributed loadings by means of Figure E-8, giving $(p_o)_{\text{tension}} = 2.71$ psi and $(p_o)_{\text{compression}} = 3.44$ psi. These will be considered as the exact values and will be used as a basis of comparison for the equivalent uniform loading as determined from Eqs. E-21 and E-22. The equivalent loadings are evaluated by substitution of the magnitudes of the components of the loading diagrams for each series approximation, as given in Table E-1.

The results of the foregoing evaluation are given in Table E-6, and indicate that the exact value is overestimated by a maximum of 11 percent for the cases in which at least three components are considered. The poorest estimate is obtained from the first approximation of the loading diagram when only the p_o and p_2 components are used. In addition, it should be noted that, for this particular example, the best accuracy is obtained for the values of the second approximation (i.e., p_o, p_2, and p_3).

TABLE E-5

VALUES OF λ_t AS A FUNCTION OF α

α	λ_t	α	λ_t	α	λ_t
0.800	9.111	0.817	10.030	0.834	11.139
0.801	9.161	0.818	10.089	0.835	11.211
0.802	9.211	0.819	10.149	0.836	11.284
0.803	9.262	0.820	10.210	0.837	11.359
0.804	9.313	0.821	10.271	0.838	11.434
0.805	9.364	0.822	10.334	0.839	11.510
0.806	9.417	0.823	10.397	0.840	11.587
0.807	9.470	0.824	10.460	0.841	11.665
0.808	9.523	0.825	10.524	0.842	11.744
0.809	9.577	0.826	10.590	0.843	11.824
0.810	9.631	0.827	10.655	0.844	11.905
0.811	9.686	0.829	10.722	0.845	11.828
0.812	9.742	0.829	10.789	0.946	12.070
0.813	9.798	0.830	10.858	0.847	12.155
0.814	9.855	0.831	10.927	0.848	12.240
0.815	9.913	0.832	10.996	0.849	12.327
0.816	9.971	0.833	11.067	0.850	12.414

TABLE E-6

EVALUATION OF EQUIVALENT UNIFORM LOADINGS

APPROXIMATE NO.	$p_{\max\ \text{tension}}$	$p_{\max\ \text{compression}}$
1	2.033	2.905
2	2.801	3.609
3	2.950	3.777
4	3.004	3.822
5	2.985	3.798
6	2.977	3.788
"Exact" solution	2.71	3.44

The load distribution reported in 1933 by Spangler (*36*) is shown in Figure E-5, and it is used to illustrate the application of the proposed method for evaluating the factor of safety of a conduit. This loading was obtained from the adjusted pressures, as measured when the height of fill was 8 ft above the top of the pipe, which is that height of fill at which the culvert cracked. Thus, this is the loading that produced a cracking "failure" of the pipe, so that the factor of safety of the conduit against tensile cracking is 1.0 for this distribution of loading.

The internal diameter of the pipe was 36 in. and the wall thickness was 4 in.; thus, $\alpha = \dfrac{36}{36 + (2 \times 4)} = 0.818$. According to Spangler (*85*), "the concrete of which they were made was mixed in the proportion of 1 part cement to 2 parts sand and 2 parts gravel. A single layer of mesh reinforcement was placed on the center line of the shell, merely to hold the sections together after cracking." Consequently, the conduit can be assumed to be made of unreinforced concrete.

The relationship between a uniform loading and the "failure" stress is obtained from Figure E-8 to be $p_{\text{failure}} = \tau_{\theta\theta}(r_i)/6.04$. The values of $p_{\text{max tension}}$ for the various approximations are evaluated by substituting the appropriate values from Tables E-1 and E-5 in Eq. E-21. Substitution of these data in the basic equation of the proposed method (i.e., Eq. E-1) yields the following expression for the tensile stress at cracking:

$$1.0 = \frac{p_{\text{failure}}}{p_{\text{maximum}}} = \frac{\tau_{\theta\theta}/6.04}{p_{\text{max tension}}}$$

or

$$\tau_{\theta\theta} = 6.04(p_{\text{failure tension}})$$

The stress values as determined for each of the approximations of the loading function are given in Table E-7. Once again, it is to be observed that the first approximation, $(p_0 + p_2)$, yields a result that is much lower than any of the others. There is only a slight variation in the value of the critical tensile stress as evaluated by the second through sixth approximations of the loading function. On the basis of the inadequate data concerning the concrete mix that was used for this culvert pipe (see previous quotation) it is not possible to check precisely the accuracy of these calculations. However, it would appear that a maximum tensile stress of approximately 230 psi would be a reasonable estimate of the ultimate strengths of the concrete that was in use 40 years ago.

To illustrate the application of the load ratio concept for evaluating the factor of safety (Eq. E-1), consider the loadings of Figure E-7a and Table E-3. Each of these loadings can be expressed as an equivalent uniform loading, $p_{\text{max tension}}$, by means of Eq. E-21. For purposes of this example, the unit weight of the soil is assumed to be 120 lb per cubic foot, and $\alpha = 0.825$. The factor of safety will be evaluated on the basis of a critical tensile stress, f_t', in the conduit. Therefore, the quantity p_{failure} is obtained from Eq. E-10, or the appropriate coefficient can be selected from the curve of Figure E-8. These load values are then substituted into Eq. E-1, so that the expression for the factor of safety for these loadings becomes:

$$\text{F.S.} = C_s \frac{f_t'}{H} \qquad \text{(E-24)}$$

in which the values of the coefficient C_s are given in Table E-8. The tensile failure stress, f_t', of the conduit material is expressed in psi whereas the height of cover, H, is in feet.

Because the conventional basis for the design and acceptance of rigid conduits is the D-line load strength, it would be convenient to express the present concept in terms of this quantity. A relationship between the uniform load that produces in the elastic conduit a stress that is equal to the maximum stress caused by a D-line load is obtained by equating Eqs. B-5 and E-10 with the result that

$$p_{\text{failure}} = 0.00662 k_i \frac{(1+\alpha)^2}{1-\alpha} D \qquad \text{(E-25a)}$$

or

$$p_{\text{failure}} = \delta(\alpha) D \qquad \text{(E-25b)}$$

in which the appropriate value of $\delta(\alpha)$ can be obtained from Table E-9. Substitution of these values into Eq. E-1 yields the following expression for the D-line load:

$$(D)_{\min} = \eta(\alpha)(\text{F.S.}) p_{\max} \qquad \text{(E-26)}$$

in which $\eta(\alpha)$ is obtained from Table E-9; $p_{\max}$ is evaluated by means of Eq. E-21 for the given or assumed loading condition; and F.S. is a specified factor of safety.

TABLE E-7

FAILURE STRESS FOR DIFFERENT SERIES APPROXIMATIONS

APPROXIMATION NO.	FAILURE STRESS (PSI)
1	184
2	226
3	230
4	234
5	235
6	236

TABLE E-8

VALUES OF C_s FOR DIFFERENT VALUES OF K AND H/B

	C_s		
H/B	$K=0.33$	$K=0.50$	$K=1.0$
2.0	0.0435	0.0588	[a]
5.0	0.0407	0.0517	[a]
10.0	0.0396	0.0495	0.194

[a] Because the equivalent uniform load is compression, no tensile failure is possible.

TABLE E-9

VALUES FOR δ AND η AS A FUNCTION OF α

α	$\delta(\alpha)$	$\eta(\alpha)$
0.80	0.093	10.75
0.81	0.100	10.00
0.82	0.107	9.35
0.83	0.116	8.62
9.84	0.125	8.00
0.85	0.136	7.35

SUMMARY

The foregoing discussion presents a new concept for evaluating the factor of safety of culverts. The proposed method is (1) consistent with the usual definition of structural safety, (2) independent of the kind of structure (flexible or rigid), geometry of the cross section, and conditions of bedding and backfill, and (3) convenient and easy to use. At present, the application of the concept is limited to circular rigid culverts whose response is in the elastic range. Extension of the concept to other shapes and materials and to the condition of inelastic response is limited only by the development of adequate analytical tools for evaluating the p_{failure} loading for these conditions, and work is now under way to derive the appropriate relationships.

APPENDIX F

CONSOLIDATION SETTLEMENTS UNDER CULVERTS

When an embankment is constructed, the foundation soil compresses under the weight of the fill material. The magnitude of the resulting settlement depends on both the compressibility of the foundation soil and the imposed weight of the embankment. Because of the trapezoidal shape of highway embankments, the settlement is greatest beneath the central portion of the fill, decreasing appreciably toward the side slopes and becoming relatively small under the toes. If the conduit is placed with the invert in a straight line, this settlement will cause a sag in the vertical alignment under the center of the embankment. Such a sag could pond water and decrease the drainage capacity of the culvert. In addition, the distortion due to this differential settlement will tend to lengthen the conduit, thereby inducing shear and tensile stresses which, if large enough, could fracture the pipe, shear the bolts, or cause a separation at the joints. Therefore, it is desirable to place the conduit at a proper camber to compensate for the expected settlements. Hence, this work is directed toward the development of an approximate procedure for estimating the total and differential consolidation settlements to be expected under such conditions.

SOIL COMPRESSIBILITY AND CONSOLIDATION

The application of a load to a soil will, in general, cause instantaneous and time-dependent deformations to occur. These deformations may consist of two parts; one part is due primarily to normal stresses and consists of volume changes, whereas the other part results from shear stresses under conditions of constant volume. Depending on the physical properties of the soil, the magnitudes and rates of these deformations may vary considerably. Although all types of deformation may be important in the design of embankments, the following analysis is limited to the time-dependent deformations due to volume changes or consolidation of the soil. In general, the foundation soil may be partially or fully saturated under natural conditions; however, for the relatively low-lying sites where culverts will be placed for the passage of water, the underlying soils will probably be fully saturated, or very nearly so. Thus, the assumption of complete saturation seems reasonable, and it is employed herein.

The total stress, σ, in a soil may be divided into two parts—an effective stress, $\bar{\sigma}$, which is that average intergranular stress acting between the soil particles, and an excess porewater pressure, u, which is that hydrostatic stress existing in the pore fluid; this well-known relationship may be written as

$$\sigma = \bar{\sigma} + u \qquad \text{(F-1)}$$

Under the assumption that the soil particles and pore fluid are incompressible, the volume reduction associated with consolidation must be accompanied by an expulsion of porewater from the voids and a corresponding decrease in the void ratio, e, of the soil, where void ratio is defined as the volume of voids, V_v, divided by the volume of solids, V_s, or

$$e = \frac{V_v}{V_s} \qquad \text{(F-2)}$$

Immediately after application of a given load increment, the induced stresses in the soil mass are reflected as increases in porewater pressure with no increase in the effective stress, and the initial volume of the soil is represented by an initial void ratio, e_1. As this excess porewater pressure dissipates through drainage of porewater from the drainage boundaries under the induced pressure gradients, the soil particles rearrange into a more closely packed system with a corresponding decrease in void ratio, Δe, and the induced stresses are gradually transferred to the soil skeleton as effective stresses. At the completion of this primary consolidation process (theoretically, at time equal to infinity, but empirical methods are applied to define "completion" practically), all of the induced stresses are assumed to be carried as effective stresses in the soil skeleton, and the final void ratio will be e_2. A typical plot of void ratio versus time for a constant applied stress is shown in Figure F-1a; with a proper scale, this plot can also represent settlement versus time. Laboratory specimens, on which consolidation tests are conducted, are usually loaded incrementally, and several graphs of the type shown in Figure F-1a are obtained, one graph for each load increment.

If the void ratios at the end of primary consolidation for each load increment of a typical consolidation test are plotted versus the corresponding effective stresses under which the sample was consolidated, a "quasi-equilibrium" soil compressibility relationship, as shown typically in Figure F-1b, will be obtained. The initial void ratio of the soil at the beginning of the test is designated e_o, and the slope of the normally straight-line portion of the virgin loading curve is called the coefficient of compressibility, or the compression index, C_c. As Figure F-1b shows

$$C_c = \frac{e_1 - e_2}{\log(\bar{\sigma}_2/\bar{\sigma}_1)} \tag{F-3}$$

From the phase diagram relationship shown as the inset to Figure F-1a, the change in height, ΔH, of a sample of initial height, H_1, and void ratio, e_1, is given by

$$\frac{\Delta H}{H_1} = \frac{\Delta e}{1 + e_1} \tag{F-4}$$

which, when combined with Eq. F-3, yields

$$\frac{\Delta H}{H_1} = \frac{C_c}{1 + e_1} \log \frac{\bar{\sigma}_2}{\bar{\sigma}_1} \tag{F-5}$$

For a layer of normally consolidated clay of thickness, D, natural void ratio, e_o, and natural overburden pressure, p_o, the settlement, S, under an applied load, Δp, may be expressed from Eq. F-5 as

$$\frac{S}{D} = \frac{C_c}{1 + e_o} \log \frac{p_o + \Delta p}{p_o} = FL \tag{F-6}$$

in which

$$\mathrm{F} = \frac{C_c}{1 + e_o} \tag{F-7}$$

and

$$L = \log \frac{p_o + \Delta p}{p_o} \tag{F-8}$$

The "loading factor," L, in Eq. F-8 includes the loading

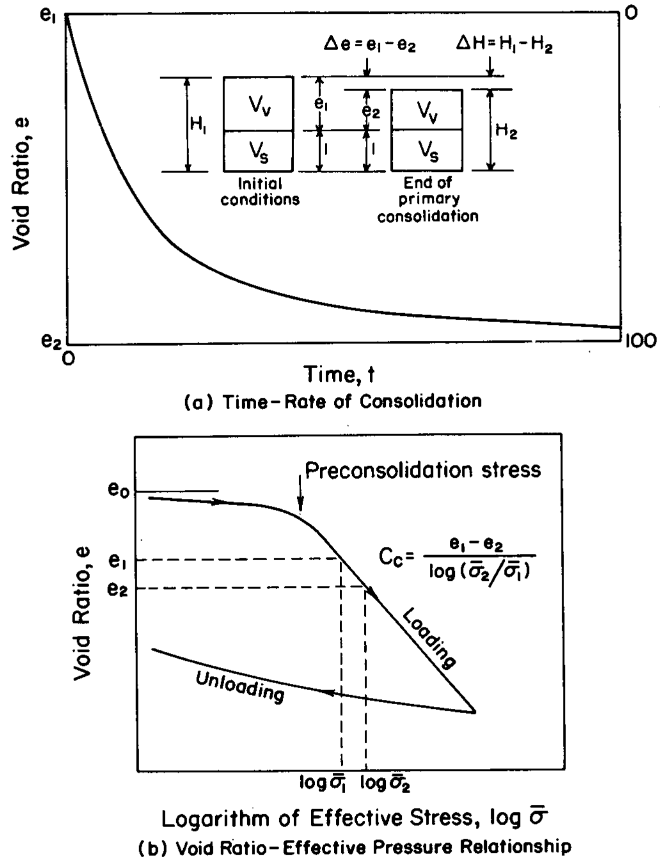


Figure F-1. Typical results of consolidation test.

effects, and its exact values depend on the actual configuration of the applied embankment load; the "compressibility factor," F, expresses the compressibility of the soil.

Because any disturbance or remolding of the soil will alter its in-situ or natural structure with resulting changes in its compressibility and consolidation characteristics, consolidation tests must be performed on essentially undisturbed soil samples. This, in turn, necessitates the use of good field sampling and laboratory preparation techniques.

METHOD OF ESTIMATING SETTLEMENT

Determination of Soil Compressibility

Because of the difficulties, expense, and time delay that normally accompany field sampling and laboratory consolidation tests, it is highly desirable to devise some approximate scheme for determining the soil parameters required to compute consolidation settlements for different types of soils. Rutledge (*86*) found some correlation between C_c and the natural water content, w_n, of various soils, and this was later confirmed by Peck and Reed (*87*) and Osterberg (*88*); Rutledge also found a correlation between C_c and the natural void ratio, e_o, of the different soils tested. Therefore, this idea was pursued herein, and data were collected from the literature. Approximately 300 data points, representing inorganic and organic clays and silty soils, were gathered, and a plot of F versus the initial void ratio, e_o, is shown in Figure F-2. For values of e_o less than approximately 2, there appears to be a reasonable correlation between F and e_o; for e_o greater than 2, parameters other than e_o obviously influence F to such an extent that any correlation between F and e_o is indistinguishable. As a result of this observation, the following development is *restricted to soils with an initial void ratio less than 2*. Soils with e_o greater than approximately 2 are highly compressible and are generally organic in nature, and a considerably greater problem exists; under such conditions, consultation with a soils engineer is advised.

Approximately 230 of the data points in Figure F-2 have an initial void ratio, e_o, less than 2, and the method of least squares was used to determine the "best fit" straight line that described these data. With F as the dependent variable and e_o as the independent variable, the regression line is given by the equation

$$F_{\text{est}} = 0.156e_o + 0.0107 \qquad \text{(F-9)}$$

To test the "goodness of fit" of Eq. F-9, the correlation coefficient, r, was computed by the equation

$$r = \sqrt{\frac{\Sigma(F_{\text{est}} - \bar{F})^2}{\Sigma(F - \bar{F})}} \qquad \text{(F-10)}$$

in which F_{est} represents the value of F, as estimated from Eq. F-9, for a given value of e_o; $\bar{F}$ is the mean value of F for N data points; and F is the observed value. The dimensionless correlation coefficient, r, measures the "goodness of fit" achieved by Eq. F-9, and it equals unity for perfect correlation; because the computed value of r is 0.93, a very high linear correlation is implied.

The standard error of estimate of F on e_o is given by the formula

$$S_{F\cdot e_o} = \sqrt{\frac{\sum_{i=1}^{N}(F^i - F_{\text{est}}{}^i)^2}{N}} \qquad \text{(F-11)}$$

For the data points considered, $S_{F\cdot e_o}$ was found to be 0.028, and the standard error of estimate for various subsets of N varied from 0.022 to 0.032. The standard deviation of F is equal to 0.0755, giving a variance of F of 0.0057, or $(0.0755)^2$.

To estimate the average percent error to be expected when using Eq. F-9, the following approach is used. Consider a straight line passing through the point $F_{\text{est}} = 0$ and making an angle of plus or minus θ with the regression line; the percent error in using the regression line will be the same for all data points lying on this line. More generally, the data points lying in the angular sectors, $\Delta\theta$, between two pairs of straight lines radiating from the point $F_{\text{est}} = 0$ at $\pm(\theta - \frac{1}{2}\Delta\theta)$ and $\pm(\theta + \frac{1}{2}\Delta\theta)$ to the regression line will have approximately the same percent deviation from the regression line. For any given angle, θ, the number, f, of data points in the particular angular sector, $\Delta\theta$, under consideration was counted and normalized with respect to the total number, N, of data points to give a parameter, f/N.

The values of f/N were plotted versus θ in Figure F-3, and from these data a probability density function can be defined; that is, the curve $\gamma = p(\theta)$ which, for a large sample of size N, approximates the frequency curve f/N. A normal distribution curve was found to describe these data very closely, as verified by a "goodness of fit" test. Approximately 69 percent of the data points lie between ±5° of the regression line, and 94 percent lie between ±10°. As the inset to Figure F-3 shows, the maximum expected error resulting from use of Eq. F-9 is 20 percent if 69 percent of the data are considered and 44 percent if 94 percent of the data are considered. The corresponding mean errors are 6 and 10 percent, respectively.

The preceding correlation between F and e_o indicates that soils with the same in-situ void ratio would consolidate approximately the same magnitude. Such a correlation, however, should not be used indiscriminately; rather, it must be applied in conjunction with sound engineering judgment in order to benefit from its full potential. In particular, it should be emphasized that the foregoing concept is valid only statistically, and its results, when applied to an individual situation, may be misleading. For example, as Figure F-2 shows, the compressibility factor may vary from the regression line by a factor of 2 or ½ for particular cases. To provide some suggested alternatives in applying the principles outlined, two additional lines at ±5° from the regression line are shown in Figure F-2; these lines may be regarded as upper and lower median estimates, F_{um} and F_{lm}, respectively. Under certain conditions, the use of one or the other of these lines will serve to place lower bounds on the expected error. Which of these lines best represents the characteristics of a given soil is left to engineering judgment; in the absence of any positive reason for using the F_{um} or the F_{lm} line, use of the F_m line is recommended. Extreme caution is advised against the overenthusiastic or indiscriminate use of this approach for determining expected settlements; remember that the procedure is a very approximate one and is intended for use only in situations where it is not necessary to determine precisely the magnitude of anticipated consolidation settlements. For structures that are sensitive to slight settlements a different approach should be used, and consolidation tests should be performed on good undisturbed samples to evaluate the consolidation characteristics of the soils underlying the specific structure.

The procedure suggested herein involves the determination of the natural void ratio of the soil. One convenient way of obtaining e_o is by use of the expression

$$e_o = G_s \frac{\gamma_w}{\gamma_d} - 1 \qquad \text{(F-12)}$$

in which γ_d is the dry density; γ_w is the density of water; and G_s is the specific gravity of the soil particles. Figure F-4 includes curves for γ_d versus e_o for different values of G_s, as well as the same series of lines shown in Figure F-2 and discussed previously. The dry density, γ_d, is defined as

$$\gamma_d = W_s / V_t \qquad \text{(F-13)}$$

in which V_t is the total volume (volume of solids plus volume of voids) of the sample; and W_s is the dry weight of the soil, which is determined by drying the wet sample in an oven at 110°C (ASTM D 2216-66). The use of the

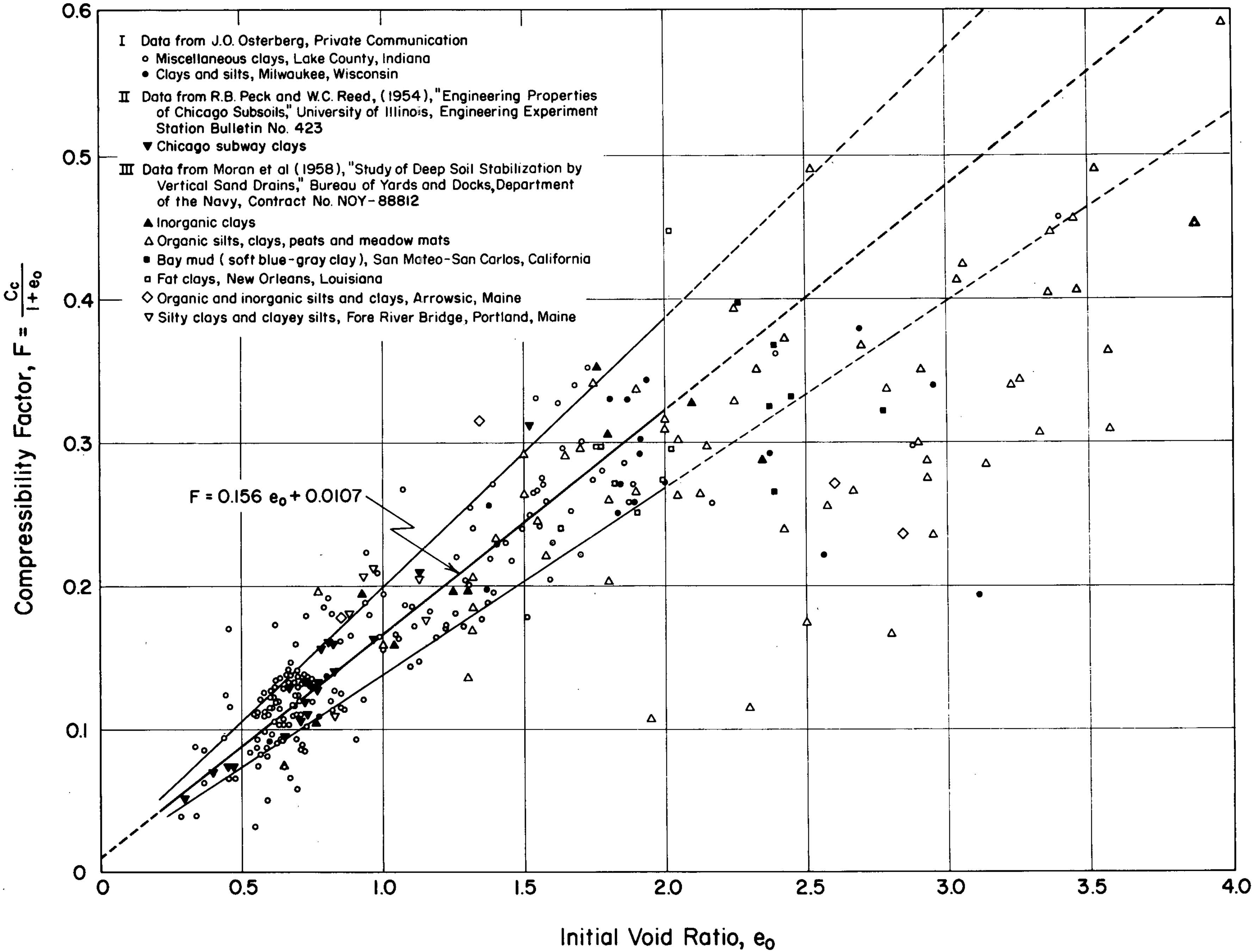


Figure F-2. Compressibility factor versus initial void ratio.

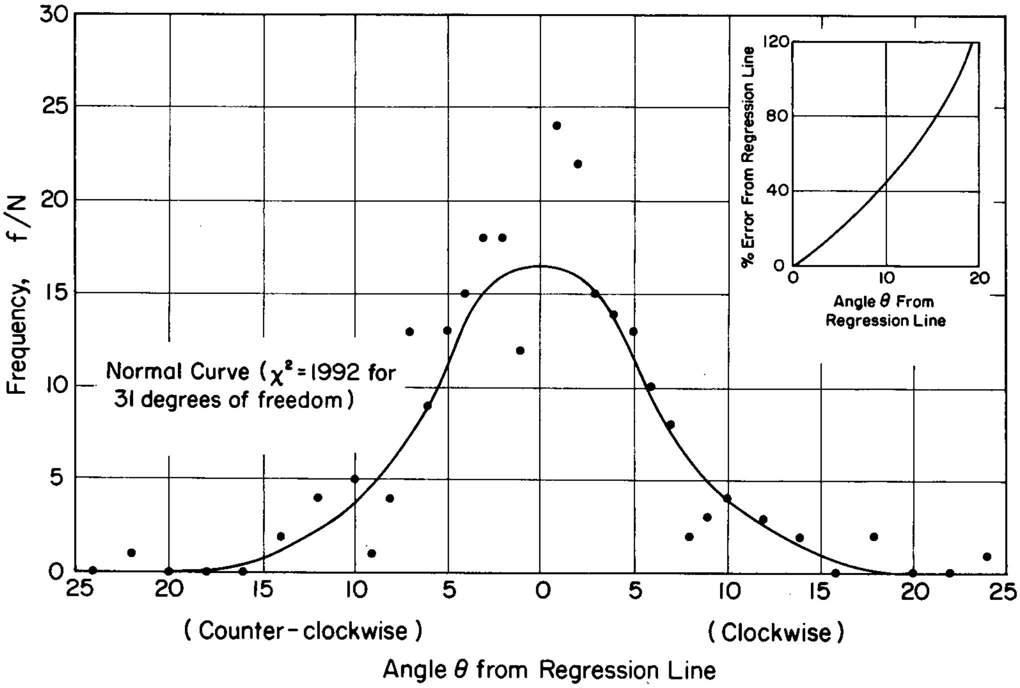


Figure F-3. Statistical distribution of compressibility factors.

chart in Figure F-4 is demonstrated by the following example:

Example 1:

Given: $\gamma_d = 90$ pcf; $G_s = 2.63$

Determine: 1. Under normal conditions, what is the mean value of the compressibility factor?

2. If visual inspection of the soil samples indicates some organic matter, which is evident by the dark color and the odor of

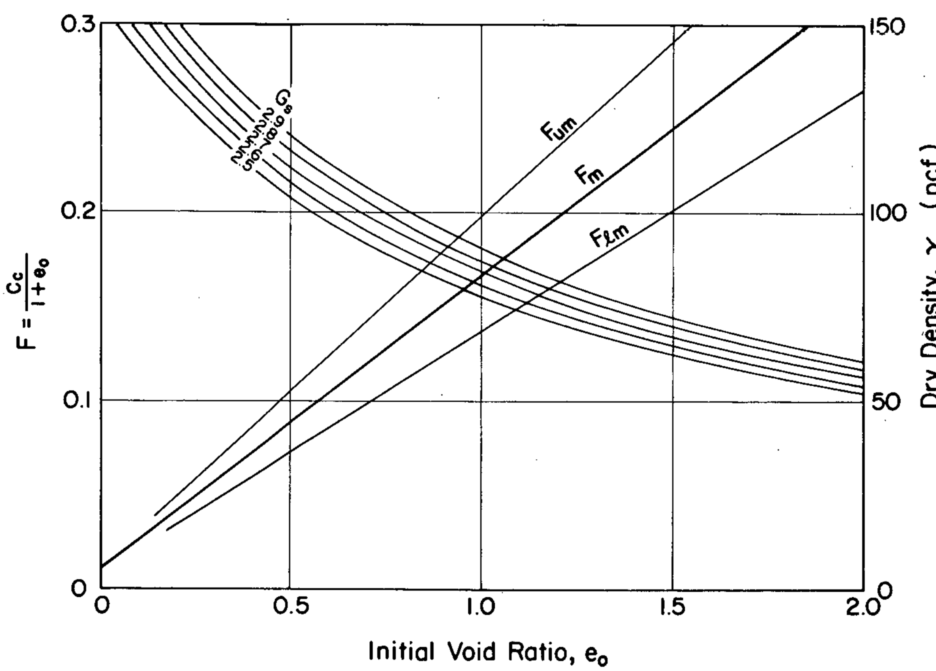


Figure F-4. Nomograph for compressibility factor.

the soil, what is the probable value for the compressibility factor?

3. What is the percent difference in F if the mean value, case (1), is used for the relatively more compressible soil, case (2)?

Solution: Referring to Figure F-4, and using the given values for γ_d and G_s, one obtains $e_o = 0.82$.

1. For $e_o = 0.82$, $F_m = 0.138$.
2. The soil in this case will probably be more compressible than average because of its content of organic matter; therefore, for $e = 0.82$, $F_{um} = 0.165$.
3. The percent different in F is

$$\frac{0.165 - 0.138}{0.165} \times 100 = \frac{0.027}{0.165} \times 100 = 16.4\%$$

Determination of Settlement

The upper sketch in Figure F-5 shows the cross section of a compacted fill embankment (soil 1) of top width, $2W$, height, H, and side slopes α to 1, underlain by a compressible normally consolidated soil layer (soil 2) of depth, D, resting on a relatively incompressible foundation soil. This general cross section is considered typical of situations in which culvert camber should be determined. If one assumes that the ground water surface is approximately the same as the surface of the compressible layer, the submerged unit weight, γ_{b2}, of the soil in the latter layer is

$$\gamma_{b2} = \gamma_2 - \gamma_w \qquad \text{(F-14)}$$

in which γ_2 is the total unit weight of the compressible soil; and γ_w is the unit weight of water. In addition, the total unit weight of the compacted fill may be designated as γ_1.

An approximation of the stress distribution in the compressible soil under and adjacent to the embankment may be obtained from Figure F-6, originally presented by Osterberg (*89*); with a knowledge of this stress distribution and the compressibility characteristics of the compressible soil, the settlement at any point A may be calculated from

$$S_A = \frac{C_c}{1 + e_o} \int_A^D \log \frac{p_o(z) + \Delta p(z)}{p_o(z)} dz \qquad \text{(F-15)}$$

in which $p_o(z)$ is the overburden stress distribution under point A; and $\Delta p(z)$ is the stress distribution under point A due to the embankment load. Although the application of Eq. F-15 is rather straightforward and is susceptible to the development of a family of curves, this degree of refinement is probably not justified for the type of problems considered herein. Alternatively, if one makes the simplifying assumption that the vertical stresses that exist at the midpoints of the compressible layer represent the average values of the vertical stresses in a vertical strip, the settlement at the top of the compressible layer may be written

$$S/D = \frac{C_c}{1 + e_o} \log \frac{p_o(D/2) + \Delta p(D/2)}{p_o(D/2)} \qquad \text{(F-16)}$$

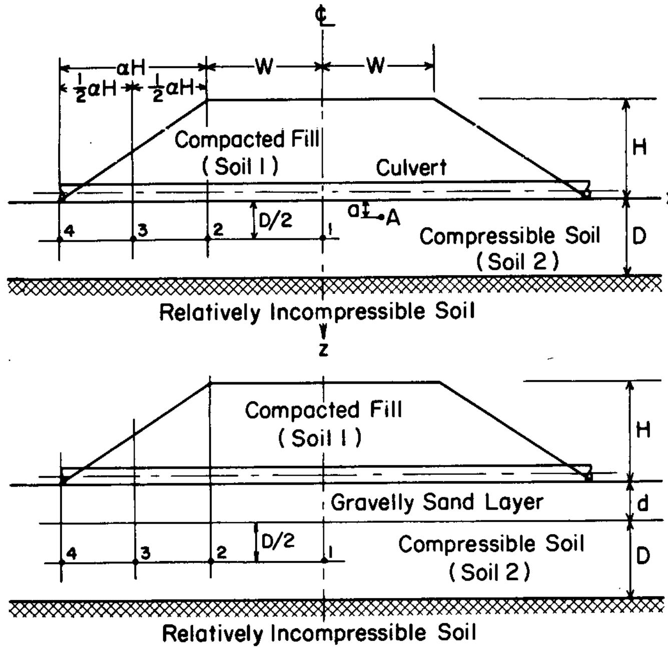


Figure F-5. Typical cross sections of compacted embankment and underlying soils.

in which the overburden effective stress, $p_o(D/2)$, is given by

$$p_o(D/2) = 0.5\gamma_{b2}D \qquad \text{(F-17)}$$

To obtain values for the induced vertical stresses, $\Delta p(D/2)$, and the associated settlements, S, consider four points 1, 2, 3, and 4, shown in Figure F-5. For point 1, which is under the center of the embankment, the weight of the embankment may be assumed as fully acting; hence, this gives

$$\Delta p_{(1)}(D/2) = \gamma_1 H \qquad \text{(F-18)}$$

If the further approximations are made that γ_{b2} equals 60 pcf and γ_1 equals 120 pcf, one has from Eq. F-8

$$L = \log\left(1 + 4\frac{H}{D}\right) \qquad \text{(F-19)}$$

A plot of L versus H/D is shown in Figure F-7; for convenience the scale of H/D is enlarged, as shown by the curve labeled $(H/D) \times 10^{-1}$, and it is also extended to cover larger values of H/D, as shown by the curve labeled $(H/D) \times 10$. To determine settlement at point 1 in Figure F-5, the solution of Eq. F-6 is shown in Figure F-7, where the settlement ratio, S/D, is plotted as a function of L and F for a given H/D ratio. The use of this chart is illustrated later.

The settlements at points 2, 3, and 4 in Figure F-5 may be determined from Figure F-7 by multiplying H/D by a factor, β. For points 2 and 3, β is on the order of 0.97 and 0.50, respectively. Values for β at point 4 are shown in Figure F-8 as a function of the embankment height, H, and the side slope, α.

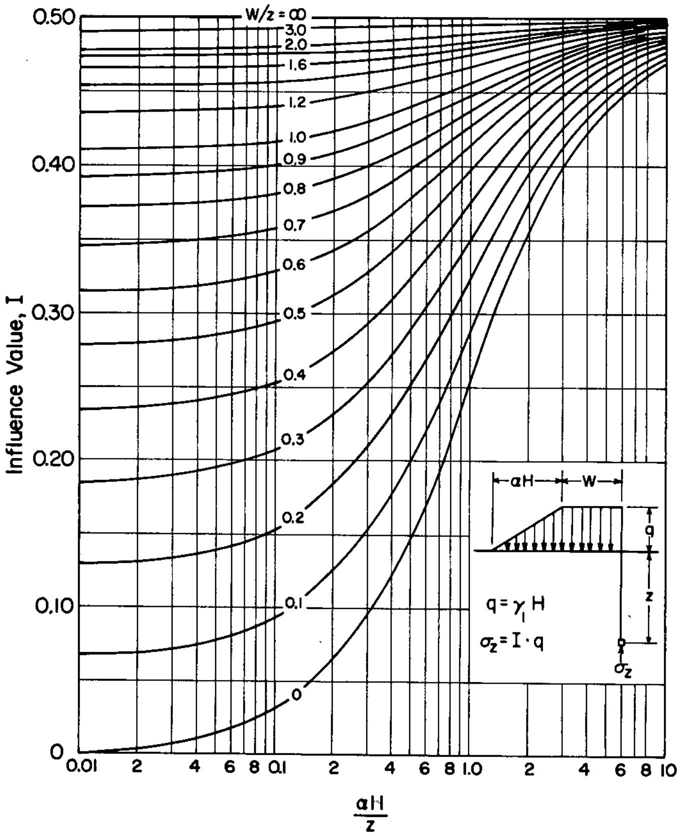


Figure F-6. Influence chart for vertical stresses under a trapezoidally distributed strip load.

For a case where the compressible layer of soil is covered by a relatively incompressible soil layer of thickness, d, as shown in the lower sketch of Figure F-5, the H/D values for any point under the embankment should be further adjusted by multiplying by λ where

$$\lambda = \frac{D}{D + 2d} \qquad \text{(F-20)}$$

For conditions other than those just discussed, Figure F-7 may still be used for settlement determinations, provided H/D is multiplied by the appropriate factor; suggested approximations are not given in such cases, and engineering judgment will play a large role in this choice. The following examples illustrate the proposed method:

Example 2:

Given: The embankment shown in Figure F-5 has a height, H, of 35 ft and a width, $2W$, of 40 ft, with side slopes of 2:1. The foundation soils below the original ground surface are silty clay to a depth of 15 ft, underlain by a thick stratum of gravelly sand.

Determine: What is the settlement profile of the original ground surface for the soils described in Example 1(1) and 1(2)?

Solution: For point 1 in Figure F-5, one has

$$H/D = 35/15 = 2.33$$

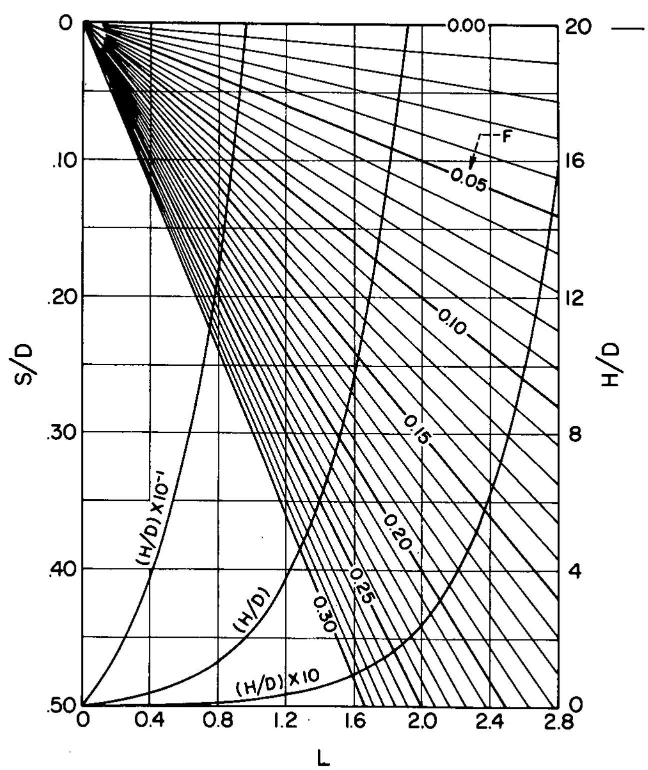


Figure F-7. Nomograph for determination of consolidation settlements.

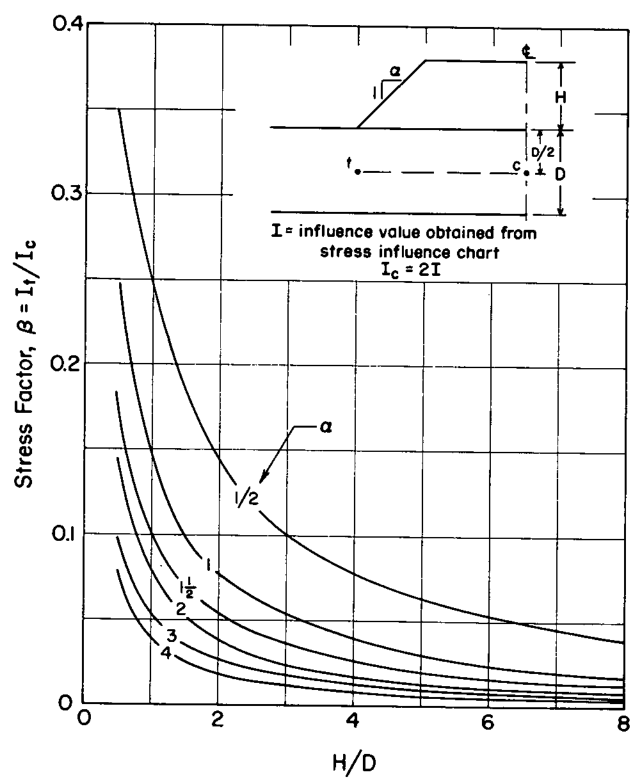


Figure F-8. Dependence of stress factor on side slope and height of compacted embankment.

which, for case (1), yields $F = 0.138$. Use of Figure F-7 gives $S/D = 0.140$ or

$$S = 0.14 \times 15 \times 12 = 25.2 \text{ in.}$$

For point 2,

$$H/D = 0.97 \times 2.33 = 2.26$$

which gives $S/D = 0.137$ or

$$S = 0.137 \times 15 \times 12 = 24.7 \text{ in.}$$

In similar manner, for point 3,

$$H/D = 0.5 \times 2.33 = 1.17$$

which gives $S/D = 0.104$ or

$$S = 0.104 \times 15 \times 12 = 18.7 \text{ in.}$$

Finally, at the toe, or at point 4, for $\alpha = 2$ and $H/D = 2.33$, one has from Figure F-8 that $\beta = 0.032$; hence, one gets

$$H/D = 0.032 \times 2.33 = 0.075$$

which corresponds to $S/D = 0.015$ or

$$S = 0.015 \times 15 \times 12 = 2.7 \text{ in.}$$

For case (2), $F = 0.165$, and the settlements at points 1, 2, 3 and 4 are, respectively,

at point 1, $S = 0.168 \times 15 \times 12 = 30.2$ in.,
at point 2, $S = 0.166 \times 15 \times 12 = 29.9$ in.,
at point 3, $S = 0.123 \times 15 \times 12 = 22.1$ in.,
at point 4, $S = 0.018 \times 15 \times 12 = 3.2$ in.

The predicted settlement profiles for the foregoing cases are shown in Figure F-9.

Example 3:

What are the predicted settlements at points 1 and 4 for case (1) of Example 2 if the silty clay layer were covered by 5 ft of dense sand? From Eq. F-20, one calculates

$$\lambda = \frac{15}{15 + 2 \times 5} = \frac{15}{25} = 0.6$$

At point 1, one has:

$$H/D = 2.33 \times 0.6 = 1.40$$

which for $F = 0.138$ yields $S/D = 0.113$ or

$$S = 0.113 \times 15 \times 12 = 20.3 \text{ in.}$$

At point 4, one has

$$H/D = (0.032 \times 2.33) \times 0.6 = 0.045$$

which gives $S/D = 0.010$ or

$$S = 0.10 \times 15 \times 12 = 1.8 \text{ in.}$$

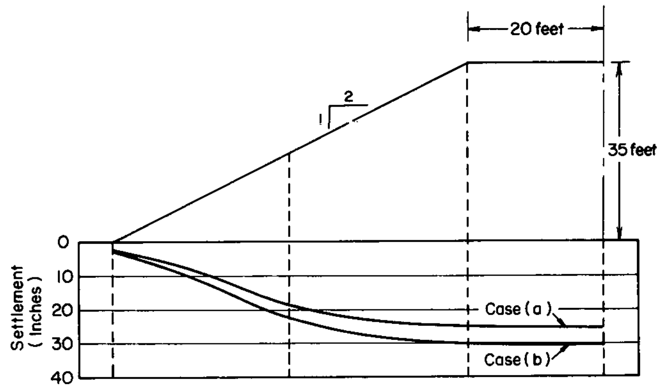


Figure F-9. Example of consolidation settlements under a culvert.

OVERCONSOLIDATED SOILS

All natural soil deposits have undergone consolidation under their own weight and, after a sufficiently long period of time, a soil element reaches a state of equilibrium under the weight of the column of soil above it. A soil in this condition is referred to as "normally consolidated." If part of this normally consolidated soil has been removed, either by man or due to natural causes, the remaining part will then exist under some overburden stress that is less than the consolidation stress. A soil in this condition is termed "overconsolidated"; therefore, an overconsolidated soil is one that has been consolidated under stresses greater than its present overburden. Removal of soils, as mentioned previously, is only one possible cause of overconsolidation; desiccation, for example, can consolidate soils under the resulting capillary stresses that induce negative pore pressures. Another common cause, if not the most common cause, of overconsolidation is lowering of the groundwater table at some time in the past to levels below the present level, thereby increasing the effective stresses in a soil.

If soils are highly overconsolidated, it is unlikely that settlement problems will arise for the type of situations under consideration. When the weight of embankment and other loads, plus the existing overburden stress, is less than the preconsolidation stress of the soil at a given point, no appreciable settlement may be expected. On the other hand, if the stresses due to the proposed loads are higher than the preconsolidation stress, the soil will consolidate essentially under the stress in excess of the preconsolidation stress. Therefore, an evaluation of the preconsolidation stress is essential for reasonably accurate settlement predictions, and laboratory consolidation tests may be required. Although the method presented herein is not directly applicable to overconsolidated soils, it may be used with modification. If the stress in excess of the preconsolidation stress is denoted by p_c, an equivalent height of compacted fill, H_e, may be determined from the relation

$$H_e = \frac{p_c}{\gamma_1} \qquad \text{(F-21)}$$

If H_e is equal to or greater than the height of the embankment, H, settlement may be considered negligible. On the other hand, if H_e is equal to nH, in which n is less than unity, the method presented herein may be used with engineering judgment to predict settlements if H/D is reduced by a factor of $(1 - n)$. Such a procedure is particularly advantageous if p_c is known without the necessity of conducting laboratory tests.

APPENDIX G

DURABILITY CONSIDERATIONS

Although the principal emphasis of the work presented herein is concerned with the structural analysis and design of pipe culverts, durability is one of the important factors that must be considered in selecting a particular type of culvert pipe for a given situation. Because durability greatly influences the service life of a culvert, it often forms the basis for choosing a particular material, as well as the thickness of the material or the protective coating that should be applied; obviously, the design life of the structure plays a major role in this decision. Indeed, there are occasions where durability considerations, and not structural considerations, govern the final design; in addition, there is a multitude of cases in which structural adequacy and durability must be considered simultaneously.

After the culvert size and type have been selected, durability considerations influence the more detailed phases of design. For example, in corrugated metal culverts additional metal may be allowed, bituminous coatings may be applied, or culvert inverts may be paved; in concrete culverts a special type of cement may be necessary. Because, apart from durability considerations, an economic advantage can often be gained by using a corrugated metal pipe culvert, the problem of service life expectancy is a prime consideration, and metallic corrosion is, in turn, the major factor in service life expectancy of metal culverts.

As far as metallic corrosion is concerned, a variety of factors is believed to influence the performance of a given culvert; among these are:

1. Hydrogen ion concentration (pH—acidity or alkalinity).
2. Presence of various other ions (sulfides, sulfates, chlorides, nitrates, ammonia, ferrous iron, etc.).
3. Water hardness (amount of calcium carbonate present).
4. Electrical resistivity.
5. Flow velocity.
6. Temperature.
7. Oxygen concentration.
8. Sulfate-reducing bacteria.

These factors are enumerated primarily to emphasize the complexity of the problem associated with determining a service life expectancy for a culvert, and to serve as a warning against the overenthusiastic acceptance of any particular reported correlation of service life with any one or two of the foregoing factors without examining the remaining details of the study. Perhaps the strongest suggestion advanced from this study is concerned with this latter point; it appears that supposed correlations reported in the literature do not provide satisfactory results under general conditions largely because variables, which were not taken into account in the individual investigations, strongly influence the corrosion rate.

Following a brief discussion of durability problems with concrete, emphasis is placed on the nature of metallic corrosion, and summaries of several recent works are given. Based on these works some conclusions are reached, and a suggested design method is presented.

DURABILITY OF CONCRETE

Concrete is generally considered a highly durable material, and this is usually true for concrete culverts, provided good manufacturing and placing practices are followed. However, under certain circumstances durability problems may arise; ice or salt crystals may cause excessive pressures in the pores of the concrete, flowing water may leach some of the concrete components, and chemical reactions (either between the original constituents of the concrete or between the components and compounds brought into the system by natural waters) may lead to expansive reactions.

Freezing, Crystal Growth, and Leaching

Destruction of concrete may occur due to the physical change of water to ice. It is believed (Troxell, Davis, and Kelly, *90*) that the pressure developed by the volumetric expansion associated with this change of state (about 9 percent) is responsible. If alternate wetting and drying by alkaline waters occur, evaporation may lead to the growth of crystals from dissolved salts, and pressures sufficient to fracture the concrete may be developed. If water is able to leak through construction joints or through areas of segregated or porous concrete, various constituents, such as calcium hydroxide, may be dissolved, and disintegration of the concrete may result. Because these problems are associated primarily with the permeability of the concrete, good construction practices and high-quality concrete with a low water-cement ratio will considerably reduce the possibility of their occurrence. Air entrainment is effective in reducing the permeability of concrete; it allows a reduction in the water-cement ratio and produces a corresponding general improvement in durability properties of the concrete. Also, air entrainment improves the resistance of concrete to freeze-thaw breakdown because expansion relief is provided by the air bubbles.

Sulfate Reaction

In cases where concrete is exposed to soil or water containing sulfates—especially magnesium, sodium, or calcium—disintegration is possible. The sulfates react chemically with the hydrated lime and hydrated calcium aluminate in the cement paste to form calcium sulfate and calcium sulfo-aluminate, and these reactions are accompanied by considerable expansion and disruption of the concrete. Disintegration of this type is effectively reduced by the use of sulfate-resistant cement (ASTM Type V). Although chemical attack by sea water is generally resisted by high-density, impermeable concrete, the use of a moderate sulfate-resistant cement (ASTM Type II) is advisable.

Reactive Aggregates

Expansion reaction between certain types of aggregates and high-alkali cements has caused random cracking and disintegration in concrete structures. Some rock types, which are known to have such reactive properties, are opaline silica, siliceous limestone, chalcedony, some cherts, andesites, rhyolite, dacite, and certain phyllites. Possible means of combatting aggregate reaction are: (1) rejection of an aggregate having reactive components in favor of a non-reactive aggregate; (2) use of a cement having an alkali content below some critical value (a value of 0.6 percent is sometimes specified); and (3) addition of some compound that will react with the harmful components in such a way that, after the concrete has hardened, no further reaction will take place.

Standard tests have been developed for aggregates suspected of being reactive. ASTM C 227 requires the use of high-alkali cement with aggregates crushed to sand size; limits of expansion are 0.04 percent after 6 months and 0.10 percent after 1 year. Some success has been obtained in the development of a more rapid test by the U.S. Bureau of Reclamation. The degree of reactivity is determined from the reduction in alkalinity of a sodium hydroxide solution to which the pulverized aggregate has been added. A more detailed coverage of concrete durability problems may be found in publications by the American Society for Testing and Materials (*91*) and Woods (*92*).

NATURE OF METALLIC CORROSION

Uhlig (*93*) states, concerning electro-chemical metallic corrosion theory:

> This theory, now with overwhelming evidence in its support, proposes that corrosion of metals is largely accomplished by the action of a network of short-circuited electrolytic cells on the metal surface. Metal ions go into solution at the anodes of these cells in amounts chemically equivalent to the reaction at the cathodes.

For example, the following reaction takes place at the anode on an iron surface:

$$Fe \rightarrow Fe^{++} + 2e^-$$

while at the cathode two types of chemical reactions occur, as follows:

Type A: $H^+ \rightarrow \frac{1}{2} H_2 - e^-$

Type B: $2H^+ + \frac{1}{2} 0_2 \rightarrow H_2O - 2e^-$

In acids Type A is relatively rapid, whereas in alkaline or neutral media it is slow. Type B is determined by the availability of dissolved oxygen, and the rate of reaction is therefore controlled at the cathode. Frequently, the oxygen supply is impeded by the formation of an oxide of the metal at the cathode, and continuation of the corrosion process is dependent on the removal of this oxide.

Various natural phenomena are able to cause current flow and set up an electrolytic cell. Three of these, which are normally associated with culvert corrosion, are (1) the oxygen concentration or differential aeration cell, (2) the bimetallic cell, and (3) the salt concentration cell. The oxygen concentration cell is established when an electrolyte is in contact with a metal at two points having different concentrations of oxygen. The oxygen-deficient areas become anodes of an electrolytic cell and corrosion occurs at these locations. Consumption of oxygen at the cathode may return the system to equilibrium, but, if the oxygen difference is maintained, the process may continue. Oxygen-concentration-type corrosion is probably the corrosion process that operates at an air-water interface, and it is by far the most common in culverts. Therefore, the factors that control this process will control to a large extent the over-all corrosion of metal culverts and will be of considerable practical importance; these factors are: (1) the oxygen content of the water, (2) the ability of the flow to replace the oxygen-deficient water with oxygen-rich water, and (3) the ability of the flow to remove the corrosion products. Dissolved salts may precipitate at the cathode and reduce the rate of corrosion. For example, calcium carbonate, if highly concentrated, tends to precipitate in conjunction with corrosion of iron. However, the effect of calcium carbonate on aluminum may serve to increase the corrosion rate.

The bimetallic cell is set up when metals of different types are in mutual contact in an electrolyte. The metal higher in the galvanic series will become the anode, from which current will flow and metal loss will occur. At the cathode a tendency to corrode from other causes will be reduced. Use is made of this process in the form of "cathodic protection"; the zinc coating on galvanized steel and the cladding on alclad aluminum plate are used as sacrificial anodes in this way. The salt concentration cell is formed when a metal is in contact with an electrolyte whose concentration varies; the area in contact with the lower concentration medium becomes the anode and corrodes. These circumstances occur when a culvert carries water into a stagnant pond or into a saltwater inlet.

As discussed by Uhlig (*93*) and shown in Figure G-1, there is little variation in the corrosion rate of mild steel for pH values between 4 and 9.5 for the condition where temperature and oxygen concentration are held constant. Below pH values of about 4, rapid corrosion occurred and hydrogen evolved; at higher values, a protective layer of hydrous ferrous oxide formed and produced an artificial pH value of about 9.5 in the vicinity of the metal. The actual pH of the solution was effective only for pH values less than 4 or greater than 9.5.

Mears (*94*) describes the influence of pH on aluminum as follows:

> There is no general relationship between pH and rate of attack. The specific ions present largely influence the behavior. Thus most aluminum alloys are inert to strong nitric or acetic acid solutions, but are readily attacked in dilute nitric, sulfuric, or hydrochloric acid solutions. Similarly, solutions with a pH as high as 11.7 may not attack aluminum alloys provided silicates are present; but in the absence of silicates, attack may be appreciable at a pH as low as 9.0. In chloride-containing solutions

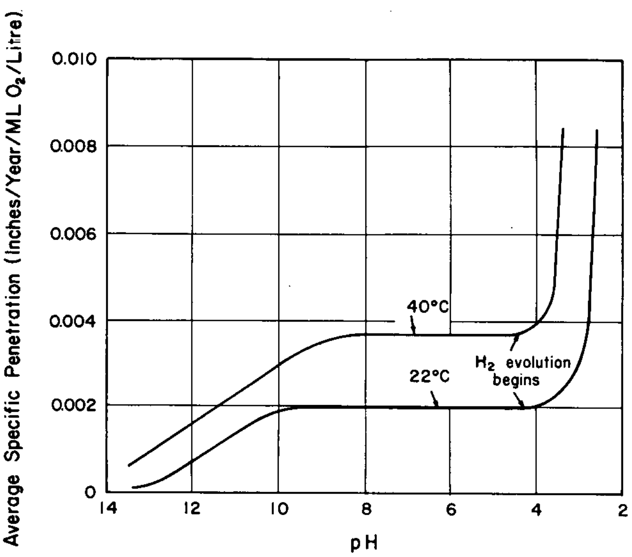


Figure G-1. Effect of pH on corrosion of steel (after Uhlig, 1948).

generally less action occurs in the near neutral pH range, say 5.5 to 8.5, than in either distinctly acid or distinctly alkaline solutions. However, the results obtained will vary somewhat, depending on the specific aluminum alloy under consideration.

If oxygen is virtually absent, an abnormally high rate of corrosion may occur as a result of the presence of sulfate-reducing bacteria (Uhlig, *95*). Under suitable conditions these bacteria may multiply in fresh water, brackish water, sea water, or soil, but they remain dormant where aeration occurs.

REVIEW OF RECENT CORROSION STUDIES

Within the last decade several studies have been undertaken in an effort to evaluate culvert durability or life expectancy. Although some of these studies have advanced to the point where procedures for the consideration of corrosion in design have been formulated, such procedures are not without serious limitations, and they are by no means generally accepted. With this caution in mind, some of the more extensive recent studies are discussed, as follows.

California Studies

A survey of approximately 7,000 corrugated metal culverts in one area of California was used by Beaton and Stratfull (*96*) to evaluate the effect of the various environmental influences on the service life of the structure. Statistical analyses indicated to Beaton and Stratfull that some relationship could be established among soil resistivity, pH, and years to perforation for a 16-gauge galvanized steel culvert. A trial-and-error procedure was used to obtain a graphical relationship, shown in Figure G-2, which gave a correlation coefficient of 0.344 and an 0.08 level of significance (a correlation coefficient of unity represents perfect correlation; a value of zero represents no correlation). The 11 California Highway Districts supplied data for a statewide check on the proposed method; on the basis of these data and the use of an averaging procedure, a correspondence was indicated between averages of results from the graph and averages of field inspection determinations. Hence, Beaton and Stratfull concluded that "a relatively accurate estimate of corrosion rate of galvanized metal pipe in a specified location can be made by using pH and resistivity values of the soils and the water."

The practice of providing for a metal loss due to corrosion over the life of the culvert is a good concept. If a

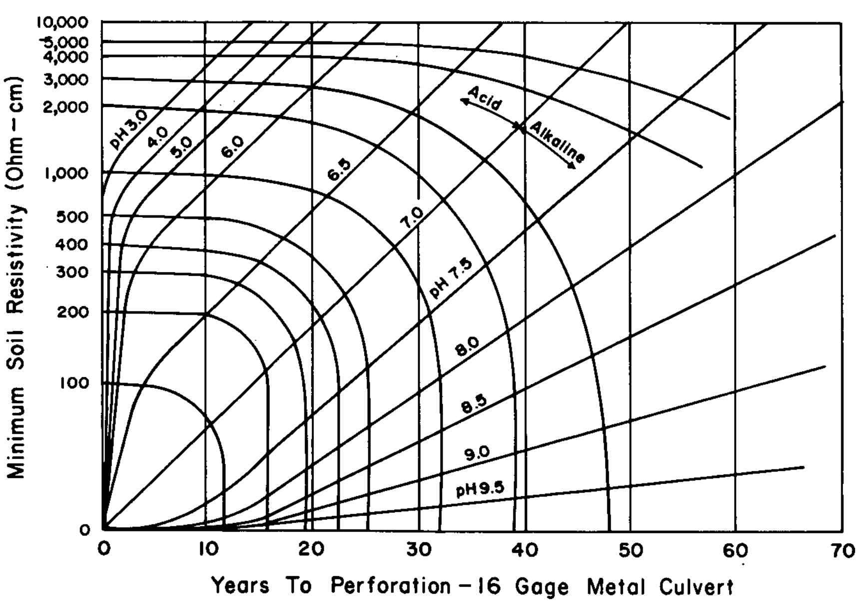


Figure G-2. Chart for estimating metal culvert corrosion rate (after Beaton and Stratfull, 1962).

large number of culverts, which are located in conditions generally similar to those in California, are designed with a corrosion allowance based on the California method, then the average actual corrosion rate should approximate the average calculated corrosion rate. Such a result is valuable and constitutes an advancement in durability design consideration. However, whether the corrosion rate is controlled by pH and resistivity is open to question. In the first place, a correlation coefficient of 0.344 is not good. The fact that some correlation exists may result from the inclusion of sites having pH values of 4 and less, and a high correlation for these sites alone would be expected. It would be interesting to evaluate the correlation coefficient for all of those sites having pH values greater than 5. In addition, much weight in confirming the method is apparently given to the statewide check. However, it is noted that, where site inspection indicated a culvert age in excess of 50 years, no comparison was made; the correlation between values for such sites is very poor. The fact that average service lives from the graph and average service lives from field inspections show good agreement for culverts having inspected perforation times less than 50 years, does not really validate the method. In order to confirm the influence of pH and resistivity in the manner shown by the graph, it would be necessary to provide some measure of the deviation of individual results from a norm. For example, two different sites in one of the highway districts may have pH and resistivity values of 6.5 and 3,000 ohm-cm, respectively, but site inspection may indicate that the service lives of culverts installed at these locations are 10 and 50 years. Evaluation from the graph shown in Figure G-2 indicates approximately 30 years to perforation for each culvert—a perfect correlation with the site average, but a poor correlation with the actual performance of each individual culvert. Obviously, the reporting of averages alone is insufficient and may even be seriously misleading.

In a study by Nordlin and Stratfull (*97*), similar aluminum and steel test culverts were installed at seven sites that were chosen for their highly corrosive or abrasive nature. On the basis of the results from these tests, they conclude that "For all seven comparative corrosion test culverts the field test data indicate that on the average the aluminum will be perforated by corrosion in less time than will galvanized steel." However, they later reported in an addendum to their original paper, representing similar but more recent data than those on which the foregoing conclusion was based, that a greater rate of metal loss is indicated at only four of the seven sites. At the sites where abrasion was a factor, steel showed considerably greater resistance to abrasion than did aluminum.

Various corrosion and abrasion tests were conducted in the laboratory to compare the relative performance of aluminum and steel; the independent variables were pH, resistivity, various combinations of added chemicals, and aeration of the solution. On the basis of these tests, Nordlin and Stratfull reached the conclusions that (1) aluminum is more resistant than steel in the pH range from 5.5 to 8.5, (2) there was no indication of any definite trend in the influence of resistivity on the rate of corrosion, and (3) in acquiescent solutions simulating bogs or marsh areas, the aluminum was attacked at the metal laps, scratches, etc., whereas the galvanized steel was untouched. These tests also indicated that aluminum can aggressively corrode in solutions with a pH beyond the limits of 4.3 and 9.0. No positive conclusions were drawn from the variation of added chemicals.

One general, but controversial, conclusion reached in this study was that aluminum culverts may have a service life of 25 years. Considerable objection was taken to this conclusion in several discussions of the paper. In particular, Koepf contends that, although the research data obtained are valuable, such extreme site conditions should not be used for evaluation of general culvert performance. He points out that, except for only one site, the measured values of pH and resistivity would have precluded the use of both aluminum and steel culverts according to current selection criteria.

Georgia Rating Chart

The State Highway Department of Georgia uses a method for the initial selection of culvert material based on pH, sodium chloride content, conductance, sulfate content, hardness (calcium), calcium carbonate (acidity), and sewage and industrial waste estimate. From the chart shown in Figure G-3, a rating point selection is made for each property and the rating points are added. If the rating determined in this manner exceeds 7, aluminum, coated steel, or concrete pipe is specified. Neither information used for the establishment of this chart nor data for sites where the chart has been used are available, so that a quantitative evaluation of its validity is not possible. Although this chart may work for conditions in the State of Georgia, a comparison of the results obtained therefrom and the results of other studies does not provide evidence that such a chart is suitable for general use.

Washington Study

As reported by Berg (*98*), a survey of more than 500 culverts was made for the purpose of establishing a performance record of the various materials used throughout the State of Washington in order to review existing design practices. In addition to the observance of the actual culvert condition, pH and resistivity values were measured for both soil and water samples taken from each site. For the data gathered on corrugated metal pipes, the projected service life of the culvert was determined by assuming a linear rate of deterioration and extrapolating from the measured metal loss up to that time. This value was compared with a year-to-perforation value based on pH and resistivity and computed according to the California Test Method 643B shown in Figure G-2, but the results gave no substantial correlation. Further efforts to correlate various combinations of resistivity and pH with projected service life met with little success. It was concluded that "There are seemingly far more variables which affect corrosion resistance than have been considered in present electrochemical tests." Because the pH values in these tests were generally between 5.5 and 6.5, the results do not imply independence of pH beyond this range.

Qualitative observations of the effectiveness of bitumi-

pH	8.5	8	7.5	7	6.5	6	5.5	5.25	5	4.75	4.5
Sodium Chloride NaCl (ppm)	5	6.5	8	9.5	11	12.5	14	15.5	17	18.5	20
Conductance (micromho)	Less Than 50	50	55	60	65	70	75	80	85	90	100
Sulphate, SO_4 (ppm)	Less Than 10	10	11	12	13	14	15	16	17	19	20
Hardness, Calcium (ppm)	Greater Than 120	110	100	90	80	70	60	50	40	30	20
Acidity $CaCO_3$ (ppm)	0	1	2.5	4	5.5	7	8.5	10	11.5	13	15
Sewage and Industrial Wastes			Light			Medium			Heavy		
Rating Points	0	1	2	3	4	5	6	7	8	9	10

Figure G-3. Georgia corrosion rating chart for steel.

nous coating led to the conclusion that, in general, such treatment does not substantially increase culvert life, because the coating in the pipe invert is usually worn away fairly rapidly. However, great benefit was observed for cases where the invert of the culvert had been asphalt-paved. The practice of paving 25 percent of the culvert was found to be insufficient in many instances, and it was recommended that half the circumference be paved; such an installation was considered to be satisfactory in all but highly corrosive areas. Performance of asbestos-protected corrugated metal pipe was judged far superior to installations having only ordinary bituminous coating, and asbestos-protected pipe in conjunction with invert paving is recommended for those areas of Washington that are highly corrosive.

Sixteen aluminum culverts, all of which had been installed since 1960, were inspected; of these, only one, a structure in a backfill composed of oyster shells, showed some evidence of corrosion. Owing to the lack of long-term data on any aluminum culverts, only tentative recommendations are advanced by Berg; these are: (1) under normal conditions, there appears to be little need for bituminous coating on aluminum culverts, and (2) until further data are available, bituminous coatings should be used where aluminum pipe is exposed to a marine environment.

Minnesota Study

The Minnesota study, according to Holt (*99*), indicated a "definite, exact, and understandable" correlation and established pattern between great soil groups and service life for corrugated steel pipes. The great soil group classification was originated by C. F. Marbut and was used for a corrosion prediction by the National Bureau of Standards in "Underground Corrosion," U.S. Department of Commerce Circular 579, 1957. Four of the great soil groups are found in Minnesota, and, based on these soil groups, the following guidelines are presented for culvert design:

1. Soil Group I (Podsol).—Plain galvanized corrugated steel pipe will not be used. When the use of corrugated steel is desired, it will be asbestos bonded and bituminous coated.
2. Soil Groups II and IV (Grey-brown Podsolic and Prairie).—Plain galvanized corrugated steel pipe may be used for installations that will drain dry. For wet installations, the soil pH is to be determined and, if it is 7 or greater, plain galvanized corrugated steel pipe may be used; if pH is lower than 7, any corrugated steel pipe used must be asbestos bonded and bituminous coated.
3. Soil Group V (Chernazem).—Plain galvanized corrugated steel pipe may be used for all installations without investigation.

Unfortunately, except for some isolated examples, Holt has not provided evidence of an exact correlation, and in view of the complexity of the corrosion problem such a correlation is considered unlikely. The corrosion study by the National Bureau of Standards was concerned with the corrosion of metals in direct contact with the soil. In general, the durability problem for corrugated metal culverts is one concerning corrosion at the air-water-metal interface on the inside of the pipe, not at the metal-soil interface on the outside of the pipe. Although the initial corrosion rate on the outside of the pipe may be high, continuation of the process is normally impeded by the accumulation of corrosion products; an exception to this is the case of anaerobic soils containing sulfate-reducing bacteria. Some relationship probably exists between the nature of stream water and the soil through which it flows. However, as no firm correlation between the properties of the flowing water and the corrosion rate has yet been established for normal conditions, the possibility of establishing a correlation between the nature of the soil and the corrosion rate seems slight.

New York Study

The New York study by Haviland, Bellair, and Morrell (*15*) is the outcome of two separate studies. One consists of a survey of 792 bituminous-coated and uncoated galvanized steel culverts installed between 1930 and 1963; a statistical evaluation of the measurable factors thought to control corrosion was made, and a design method was developed. The other is a comparative study of galvanized steel and alclad aluminum culvert exposed to similar conditions at 21 locations throughout the state.

Steel Culvert Survey

At each site pH, electrical resistivity, calcium carbonate content, and flow velocity were measured. The ranges of values encountered were as follows:

1. pH—varied from 3.8 to 9.4 with no apparent correlation between the values for water and soil at each site.
2. Resistivity—varied from 50 ohm-cm to 30,000 ohm-cm, with values for soil and water being fairly consistent at each site.
3. Calcium carbonate—qualitative determination at 148 sites indicated 76 saturated and 72 unsaturated conditions.
4. Flow velocity—of 291 sites tested, results indicated that 7 sites had a velocity of 5.0 to 7.9 fps (moderate); 113 sites, 2.0 to 4.9 fps (slow); and 171 sites, less than 2.0 fps (stagnant).

The distribution of surface treatment for the culverts was 111 uncoated, 238 bituminous coated, and 443 bituminous coated and paved.

Observations of the general condition of the culverts and of samples taken from the culverts indicated that (1) culvert extremities were far more distressed than were interior portions (end * conditions were not considered in the general evaluation), (2) metal loss consistently originated on the interior surface and progressed outward, (3) progressive corrosion was confined to the area below the waterline, and (4) there was little evidence that abrasion was more than a minor contributor. A statistical evaluation was made with the aid of an electronic computer on 146 installations for which complete data were available. The pH and resistivity of both soil and water and the age of each culvert were treated as independent variables, and metal loss in inches was treated as the dependent variable. A stepwise regression technique was used to analyze the effect of sequentially eliminating each independent variable, and culvert age was found to be the only statistically significant factor. It could, therefore, be concluded that, at least for the State of New York within the range of conditions tested, a culvert durability design based on the physical parameters measured at a particular site would be of little value. Apparently, other factors, such as oxygen concentration, temperature, and flow velocity, play a significant, but undetermined, role in the corrosion process. Unfortunately, the measurement of these parameters involves considerable difficulty, and prospects for including them in design criteria in the near future are small.

*p.10 of the N.Y report

However, because a large quantity of data was available, it was possible to determine the degree of variability from the average straight-line relationship between metal loss and age. A corrosion design method, based on the probability of exceeding any given rate of metal loss, is suggested, and curves are shown in Figure G-4 for the three cases of uncoated, coated, and coated/paved culverts. The following examples illustrate the use of these curves to determine the corrosion allowance to be made.

Example 1.—For a low-cover driveway pipe serving light traffic, a 30- to 40-percent probability of exceeding the determined corrosion rate is considered satisfactory. From the curve for uncoated pipe, one obtains a corrosion rate of approximately 0.0007 in./yr; if the required service life for the culvert is 25 years, the corrosion allowance should be $25 \times 0.0007 = 0.002$ in.

Example 2.—For a culvert under a high embankment on an Interstate Highway subjected to heavy traffic, it is desired to limit to 10 percent the probability of exceeding the determined corrosion rate. Use of the curve for coated pipe for a 50-year service life yields a calculated allowance for metal loss of $50 \times 0.0017 = 0.085$ in.

Comparison Survey

The second study involved a comparison of aluminum and steel culverts exposed to similar conditions. Because the aluminum culverts were installed between 1961 and 1964, the results obtained from this study are considered only preliminary. In a few cases the exposure time for the steel culvert was considerably greater than that of its aluminum counterpart. The ranges of the pH, electrical resistivity, and stream velocity values agreed, in general, with those of the previous study. Because the aluminum culverts showed no measurable metal loss, it is concluded from these limited results that bituminous coatings may be unnecessary for aluminum culverts "except in unusually aggressive chemical or abrasive environments." The performance of the steel culverts was consistent with the results of the previous statewide survey, and the average metal loss varied from zero to appreciable amounts.

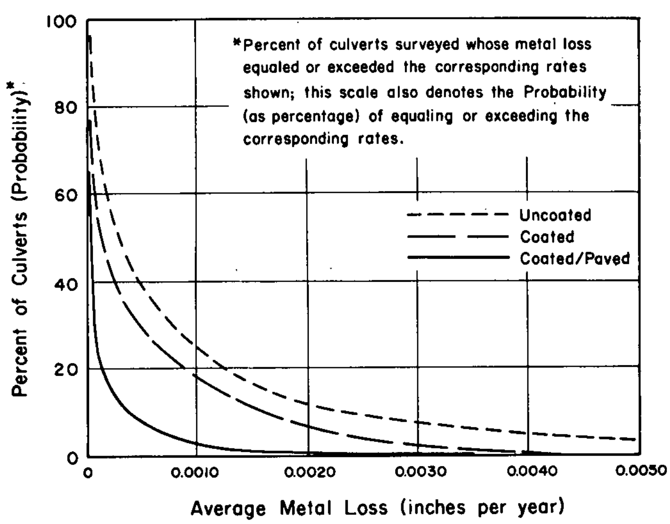


Figure G-4. Distribution of average metal loss for galvanized steel culverts (after Haviland, Bellair, and Morrell, 1967).

Other Studies

Other durability studies have been conducted by the States of Virginia, Tennessee, West Virginia, North Carolina, Kentucky, Alabama, and Idaho; these have been summarized in the New York report by Haviland, Bellair, and Morrell (*15*), and they are not discussed herein.

CONCLUSIONS FROM LITERATURE SURVEY

Three general approaches have been used in an effort to formulate a method for predicting the rate of culvert corrosion; these are (1) correlation between corrosion rate and environmental parameters, such as pH and resistivity of the soil and water and chemical constituents of the water, (2) correlation between corrosion rate and the great soil groups, and (3) use of a statistical average corrosion rate selected on the basis of importance that the structure reach a desired life. A great deal of testing has been done in an effort to relate the pH of the soil and water, the resistivity of the soil and water, and the dissolved salts to the rate of corrosion of culverts. The only well-defined result to emanate from such testing concerns pH; below a pH value of approximately 4, a high rate of metal loss will occur, and above a pH of 4, metal loss may or may not occur.

The original paper by Beaton and Stratfull (*96*) presents the California Test Method, but does not provide sufficient evidence to justify its use. Although several other states use the method or some variation thereof, no verification has come from these sources. On the other hand, New York and Washington have found that, within their areas, no useful correlation exists.

Although a correlation between corrosion rate and the great soil groups may be of some significance for pipelines and other subsurface structures where the metal surface is in direct contact with the soil, such a correlation seems less useful for culverts, because the air-water-metal surface inside the culvert normally presents the corrosion problem. Because efforts to find a correlation between the measurable properties of the water and soil and the corrosion rate have, in general, been unsuccessful, it is doubtful that a correlation can be found between soil type and corrosion rate. Local soil type can have, at best, only a partial influence on the water properties.

The use of a statistical average corrosion rate technique of the type suggested by Haviland, Bellair, and Morrell (*15*) for the State of New York appears to be the most useful approach available at present. If no substantial correlation between soil-water properties and corrosion rate exists (at least above some minimum pH value), then determination of a corrosion rate based on the importance of the structure reaching a desired life is logical, and the use of this procedure is recommended. For areas where conditions are more severe than those in the area where the method was developed, some additional considerations may be necessary, but these will be brought to the forefront only with further study. Finally, it appears to be generally accepted that abrasion is a relatively insignificant factor for conditions where flow velocities are less than approximately 8 fps during peak flow.

SUGGESTED DURABILITY DESIGN CRITERIA FOR METAL CULVERTS

It is evident from the foregoing that the culvert durability problem is a highly complex one that is a long way from being solved, and one for which any oversimplified approach seems liable to provide completely misleading results. However, for practical engineering problems when exact scientific answers do not exist, it is necessary to resort to the best available approximate approach to a solution. On the basis of this study, the following recommendations are offered as a set of guidelines for taking corrosion losses into account. They are to be used in conjunction with sound engineering judgment and should not be accepted as absolute standards.

1. Determine the water pH at the site under normal flow conditions.
2. Determine the flow velocity through the culvert at peak design flow.
3. Where the pH is less than 4.5, uncoated or bituminous-coated metal culverts should not be used; reinforced concrete culverts are very desirable for these conditions. Although asbestos-bonded bituminous-coated culverts with paved inverts have shown a high resistance to corrosive and abrasive environments, further study is necessary before their use can even be tentatively recommended.
4. Where the peak-flow velocity exceeds 8 fps, inverts should be paved.
5. Where the pH value exceeds 9, aluminum culverts should not be used.
6. Where the pH exceeds 4.5 for galvanized steel culverts, an allowance for metal loss may be made in conjunction with the design. This should be based on the rates of metal loss associated with the various probabilities from a statistical analysis of culvert performance for the area under consideration. If local data are not available, use of the New York curve (Fig. G-4) should give reasonable results. The provision of such an allowance should be considered in conjunction with the structural requirement for plate thickness. Unless some special load reduction technique is used (such as the imperfect trench), it is likely that loads may decrease from a maximum immediately after construction to considerably less after some years. It is believed that a more realistic design may involve application of the corrosion allowance to the structural thickness required to withstand the lesser loading.
7. Where the pH exceeds 4.5 and is less than 9, aluminum culverts may be used. Owing to the low age of most existing installations, data for a probability corrosion rate relationship are not available. However, it is believed that the use of the New York curve for bituminous-coated galvanized steel culverts for bare aluminum culverts would give conservative results until field data become available.

APPENDIX H

RELATED PROBLEMS

The design of a culvert must normally satisfy two major critera—hydraulic and structural. However, several other considerations must be taken into account if the most effective and efficient design is to be realized. At least two of these—durability and cost—are treated in some detail in other parts of this report, but others are discussed briefly in this section. Although these could probably be treated in a more general manner, individual treatment will emphasize their significance.

INSPECTION

The problem of providing adequate inspection is virtually a universal one in all areas of construction, and it is no less prevalent in the installation of highway culverts. Consultation with representatives of highway departments and other governmental agencies during the preparation of this report indicated, almost without exception, a wide dissatisfaction with current field inspection practice. Where major or minor failures did occur in conjunction with culvert construction, it was often found that the cause could be traced to inadequate or incompetent inspection. Although some failures have resulted from the exercise of poor engineering judgment, many more can be related to the fact that approval has been given to construction procedures (such as standard of bedding, compaction of backfill, and sequence of operations) that are in direct violation of specifications. This would seem to indicate that, although the technical capabilities of an inspector are very important, the present problem is closely associated with a lack of responsibility. It is also possible that inspectors are assigned duties so diverse that little time is available for proper attention to inspection. On the other hand, long periods of time may elapse between decisions, and the job may become boring; however, when a decision is required, competence and expediency are necessary. This gap between design and construction has long been recognized and has recently been termed by Spangler the "de-con gap."

The basic causes of this inspection problem seem to be threefold; because these are closely interrelated, the division is somewhat arbitrary, but perhaps helpful. First, the performance of good engineering inspection work is simply not accorded sufficient prestige by the profession. Very often an inspector is not technically qualified or is not at all knowledgeable about the design or intended results. In brief, the inspector is very often regarded as a second-rate engineer, if an engineer at all. Consequently, the position is not able to attract and hold competent personnel. Because this attitude pervades in significant proportions, many contractors often tend to develop an inattentive response to comments by the inspector. Second, all too often inspection is not given its proper place in the sequence of operations required to produce a good engineering job. Virtually everyone will recognize that good design and good construction are absolutely necessary ingredients to good engineering works, but too frequently the procedure of inspection, which is supposed to ascertain that the intent of the design is being properly executed in the construction process, is relegated to a position of virtual obscurity in the echelon of engineering operations. Finally, the third reason, intimately related to the previous two, is concerned with the general salary scale for inspectors. In short, the salary scale is very often simply too low to entertain any reasonable hope of improving the situation unless appropriate steps are taken to provide compensation commensurate with the responsibilities entailed.

Although the inspection problem has been confronting the profession for years, it should not be ignored. Adequate inspection is imperative, or else the full benefits of improved design procedures will almost certainly not be realized. Practical measures should be taken to minimize or eliminate the problem insofar as possible. In addition to the obvious solutions implied by the foregoing descriptions of the problems, several positive suggestions are offered for consideration. First, use engineers from the design staff to serve as inspectors on a periodic basis; such a procedure is being employed occasionally and has the following advantages:

1. The inspector will be a competent engineer who is reasonably knowledgeable and conversant about the design details of the project.
2. Valuable field experience will be provided for the design staff, and this will serve to increase its competence and effectiveness.
3. The design staff morale will be optimized by providing a change in routine and an opportunity for more extensive participation in a project.

Second, as a complement to the preceding suggestion, place field inspectors in the design office from time to time on a formal basis; the derived advantages will be similar to those just described. Third, send inspectors to periodic short courses, etc., to improve their technical competency. Fourth, perhaps a specified length of service as an inspector could be required of engineers-in-training or engineers with limited experience as a requirement for promotion. Finally, every effort must be made to upgrade inspection to a professional engineering status.

AVAILABILITY

The choice of a particular culvert material depends largely on availability and transportation convenience, and there is a distinct tendency to select culvert materials that are most readily available. Provided that careful scrutiny indicates such choices are economically justified, this procedure is, of course, in accordance with good engineering practice. In such instances structural design methods play a negligible role in selecting a particular type of culvert.

POLICIES

Very often the selection of a particular culvert type or material is based largely on the established policy of the agency involved. In other cases, the agency as such may not have an established policy, but the chief engineer or some similar individual may exercise a strong influence over the decisions within that agency. In still other instances, choices are affected largely by certain manufacturers or their representatives. The point to be emphasized in each of the foregoing instances is that structural design procedures may have little to do with the choice of a particular culvert material. It would be foolish to imply that all decisions based on the foregoing reasons are prejudiced and ill-founded; indeed, many of these decisions are well-founded and entirely warranted, as evidenced by a wealth of accumulated experience. However, in offices where such policies are prevalent, a thorough examination of the bases for their existence is essential from time to time. If they are found to be justified, they should be continued, by all means, and their bases should be documented, not entrusted to the memory of a specific individual; however, if no justification for their existence can be found, an objective re-evaluation should be made and rejection should be considered. Discussions regarding some of these policies lead to the suggestion that several of these policies seem to be based on over-generalizations of specific experiences. There seems to be a growing tendency to design and specify alternatives for culvert materials, with the choice resting with the contractor, and this certainly appears to offer economic advantages.

OTHER FACTORS

A variety of other factors influence the design and installation of a culvert, and most of these are extremely difficult to quantify in any meaningful manner. One of these is associated with the periodic incentives offered by some particular manufacturer in order to promote its product. On occasion, an unrealistic economic incentive is provided for purposes such as (1) introducing a product in some particular area, (2) advertising, (3) establishing a precedent for the use of a given product, and (4) preventing a competitor from obtaining a particular contract. Another somewhat intangible factor deals with the current work load of interested contractors; if construction activities, in general, are experiencing a lull, or if a particular contractor is seeking work, considerable economic advantages can be realized. However, it is very often difficult, if not impossible, to evaluate the cost of a specific culvert installation, because unbalanced bids are a common occurrence in the submission of proposals. Also, because culvert construction usually occurs as the fill is being placed, it is difficult to properly allocate costs to the culvert installation and the adjacent compacted embankment.

Another very significant factor in culvert installation is related to the time of the year when much culvert work is performed, particularly in areas where freezing winters are experienced. Because the installation of drainage structures is normally one of the early items to be completed on a project (this is, of course, the reason for the unbalanced bid), the contractor will usually try to get it completed as soon as possible so that the bulk of the construction work (such as cut-and-fill operations, and paving) can proceed as soon as good weather is available. Consequently, for projects started in the spring, culvert work is often performed when the soil is wet and/or partly frozen and generally not conducive to proper manipulation for culvert installation. Competent inspection is, of course, the answer to a large part of this problem. Although all of the preceding factors certainly affect a given culvert installation, as well as the over-all project, they are somewhat intangible and difficult to evaluate quantitatively; nevertheless, their consideration is essential. In fact, under certain circumstances one or two of these factors may override any determinations based on structural design considerations and may form the primary basis for major decisions on culvert material or type.

SYSTEMS APPROACH

Although many individual items are discussed in this report under individual headings, it is certainly evident to the highway engineer and administrator that these items should not and cannot be considered separately for any given situation. Each and every one of the factors treated herein, as well as the hydraulic requirements for a given culvert, exhibits a complicated interaction that cannot be determined by maximizing each aspect of the design. Rather, an effort, either formal or informal, must be made to maximize the system as a whole. Such a procedure usually will result in some compromise among the various component considerations, and it may very well be that the structural design, as such, is not the primary governing factor. In other words, maximizing the structural design of a culvert is not necessarily always the desired approach to a given problem.

APPENDIX I

ECONOMIC CONSIDERATIONS *

One of the principal differences between science and engineering is economics. Engineers have long been concerned with the realities of producing a technically satisfactory end product at a minimum cost. However, in the field of engineered construction, it is recognized that unit bid prices are often not a true reflection of the total cost of the completed system. The installed cost of an item neglects maintenance costs, service life considerations, and the possibility of future costs resulting from partial or complete replacement. The annual cost of an item is affected by many factors that are influenced by local conditions; these conditions will vary from job to job and depend on the nature of the soil, topology, weather conditions, wage scales, and construction practices. Although it is a relatively simple matter to compare the unit prices of two different pipes determined by the use of two different design procedures, this comparison will be of little value without due consideration of other aspects of the problem. The various factors discussed previously combine to pose a very complex problem for the engineer when he attempts to obtain a meaningful economic evaluation of various culvert designs.

Despite the extremely complicated nature of the economic aspects of the culvert design problem, it was deemed worthwhile to attempt to identify the relevant variables and to formulate a criterion on which an economically optimal decision could be based. With this formulation at his disposal, the engineer can more readily determine what information is required as input, what types of analyses are appropriate, and what conclusions can be reached. Although it will readily be seen that adequate input data are not generally available to permit the attainment of any well-founded conclusions, perhaps the most useful purpose of this formulation is to delineate more clearly which types of information are required to perform a competent economic analysis.

GENERAL APPROACH

There are two questions to be answered in any design problem; these are: (1) What are the technically feasible ways of accomplishing the task at hand? (2) Which of these is economically optimal in the given context? The specification of one or two alternate designs is not enough. Because resources are limited in the sense that funds that are not used for one project can be used productively elsewhere, all feasible design alternatives should be delineated. Once this is accomplished, an economic analysis should be done to determine which of these is economically optimal.

The principal purpose of this discussion is to formulate the economic problem faced by one who must choose between alternative designs for a particular culvert installation. In addition, an attempt is made to identify some of the more important aspects of the problem and to outline a general procedure for solving it. To accomplish this task one needs to understand a number of concepts and the general method of attack used by economists; these are introduced in the following section. After this, the implications of variation in design for the initial required investment are discussed. Next, the roles of costs that have to be incurred in the future are treated, and the final section provides a summary.

ECONOMIC CONCEPTS

If someone were asked to design a production process it would be necessary to know what product is to be produced by the process. Presumably the product can be described in terms of its characteristics, which are called the technical specifications of the product. In addition, it would be necessary to know the desired rate of production or the capacity of the process.

Given the technical specifications of the product and the capacity of the process, one must now search out the various ways or techniques that can be used to accomplish the task. This is essentially an engineering problem. The number of different available techniques depends on the state of existing technology and on the ability of the engineer to adapt this technology to the purpose at hand. The various possible engineering solutions constitute the set of feasible designs for the process. A feasible design specifies for each technique the inputs to be used and the quantities of each needed to satisfy the capacity requirement of the process. Inputs include the component parts of the product and the services of various resources, such as labor and machinery, needed in the process.

In a hypothetical problem, it can be supposed that there are five techniques available; these may be designated by the numbers 1 through 5. Furthermore, it can be assumed that the component parts used are the same for all techniques and that all techniques use the same types of labor and machinery. However, each technique will, in general, require different combinations of flows to produce 100 units of product per week; this latter figure is the required capacity of the process. Typical combinations of flows are given in Table I-1.

The relation that maps input combinations onto the rate of production is called the production function. A particular level curve in the input space or, more precisely, the set of input combinations that correspond to a given rate of output, is called an isoquant. Hence, Table I-1 is the relevant isoquant in this problem. The task now is to choose the least costly design from the set of feasible designs on the isoquant. To accomplish this, one must be able to relate the required inputs to the total cost; such a relationship is called the cost function. The set of input combinations that yields the same operating cost is called

* Prepared by D. I. Martensen, Department of Economics, Northwestern University, Evanston, Ill.

TABLE I-1

FEASIBLE DESIGNS FOR OUTPUT OF 1000 UNITS PER WEEK

	TECHNIQUE				
ITEM	#1	#2	#3	#4	#5
Machine-hours per week	10	20	30	40	50
Labor-hours per week	80	50	30	20	15

an iso-cost curve or an iso-cost set of input combinations. A further discussion of these concepts can be found in any standard price theory text, such as Stigler (*100*) or Leftwich (*101*).

As an illustration, it can be assumed that labor can be hired at an hourly wage, w, and that machine hours can be rented at an hourly rate, q. Then, if L represents man-hours per week and K denotes machine-hours per week, the cost function is

$$C = wL + qK \tag{I-1}$$

The weekly operating cost of any design can now be computed by substituting into this formula the input requirements of that design. Now that the set of technically feasible designs and the cost function have been specified, the task of finding that design which minimizes the weekly operating cost is simple. If w is \$3.00 per hour and q is \$4.00 per hour, the least costly design would be the one corresponding to technique #3 in Table I-1. It should be noted, however, that a different design could be optimal if the costs of input services were different.

In addition to operating costs, there may be certain set-up costs, and these may be different for different designs. If such is the case, these, as well as the weekly operating costs, must be taken into account. Because set-up costs are incurred prior to the production run, whereas operating costs are incurred during each and every week of the run, one must find some way to compare costs incurred at different points in time. The standard technique for handling such a problem is to discount those costs that have to be met after the process has been put into operation. A brief rationalization for this procedure follows, and a more complete discussion may be found in the book by Bavmol (*102*). Suppose that the firm in question has alternative uses for its funds and that the "best" of those yields a weekly net rate of return, r. The net rate of return is defined as the value of the service flows provided by the project per week less appropriate allowances for depreciation and maintenance. The "best" alternative is the one with the highest net rate of return.

Now, suppose the following question is asked: How much would have to be invested presently in the alternative project in order that the principal plus its compounded returns are just equal to the operating cost of a particular design in the tth week in the future? If C_t equals the operating costs in the tth week, the answer is $C_t/(1+r)^t$. This is true because every dollar invested now will equal $(1+r)$ dollars after the first week, $(1+r)^2$ dollars after the second week, etc., if both the principal plus returns in each week are reinvested in the next week. Therefore, if the operating costs are incurred at the end of the tth week, the amount one would have to invest now, C_t^o, in order that $C_t^o(1+r)^t$ equals C^t, is $C_t/(1+r)^t$.

The amount C_t^o is called the present value of the operating cost to be incurred in the tth week. In turn, the present value of all present and future costs implied by a particular design is defined by

$$C_o = I + \sum_{t=1}^{H} C_t^o = I + \sum_{t=1}^{H} \frac{C_t}{(1+r)^t} \tag{I-2}$$

in which H is the length of the planned production run in weeks, sometimes called the horizon; and I represents the initial set-up costs. C_o is the present value of all costs in the sense that, if C_o were invested in the alternative project, the principal invested plus compounded returns on it would just cover all of the costs implied by the design on the dates in which they have to be incurred. Alternatively, C_o is the investment that could be made and that would have yielded a time path of net income or profit just equivalent to the time path of costs implied by the design if the firm had not decided to set up the production process. Therefore, C_o reflects the opportunity cost of the firm's decision.

Clearly, the firm should choose from the set of feasible designs the design that minimizes C_o. Note that this criterion does not imply that the firm should necessarily use design #3, which has the lowest weekly operating cost. If the set-up costs for design #3 are very large relative to the set-up costs required by other designs, one of the others may be optimal. A conceptually similar problem arises in the design and construction of a culvert system.

CULVERT DESIGN AND THE INITIAL INVESTMENT

The minimization of cost would appear to be the appropriate criterion to use in choosing among feasible culvert system designs. However, the initial investment required to install a given system is certainly not the only cost to be considered. If a culvert in a given location is needed at all, it will probably be needed for a long time in the future, and maintenance, repairs, and possibly replacement may be necessary. Therefore, the implications of the various feasible designs for future service life should be taken into account. The problem of estimating future costs associated with a given design and the problem of comparing future cost with initial cost is left for consideration in the next section. This section is concerned with the problem of estimating the initial investment required by a given feasible design.

In any culvert design problem, certain technical specifications are predetermined by the function and location of the culvert. For example, the location of the culvert will determine its length and height of cover, hydraulic considerations will determine its diameter, and load estimates and installation procedure will determine its required strength. Given these specifications, standard design procedures can be used to determine which of the set of all possible designs satisfies the various criteria; those that do are included in the set of feasible designs.

A rational solution to the problem of selecting the least-cost design requires that the engineer specify as completely as possible the set of feasible designs. No amount of care in pursuing the rest of the problem can make up for the error of not including the least costly design in the feasible set. This situation emphasizes the need for effective, efficient, and accurate design procedures that are relatively easy to use and that conserve the time of an expensive and often overworked engineering staff.

To choose the least costly design from the feasible set, one must be able to relate the variations in design allowed by the feasible set to the cost of constructing the system. In the jargon of the economist, one must know the cost function. That is not to say that one must necessarily have a mathematical formula into which to substitute the values of the relevant variables. However, given the problem at hand, one must identify those factors that account for most of the differences in cost, and must estimate with reasonable accuracy how differences in these factors affect the cost of constructing the culvert system in question. Specifically, those designs that satisfy the technical specifications probably will vary with respect to the type and strength of the material used to construct the conduit itself and the quality of the foundation and/or backfill used. Because the conduit interacts with the adjacent soil to form a soil-culvert system, and because the strength of the system depends on this interaction, equivalent systems in terms of functional capacity and strength can be generated with different component parts. The economic problem is an implication of this fact.

For a given time and location, cost estimates for the different types of culvert material, such as steel and concrete, are relatively easy to obtain. For example, if a particular design calls for a steel conduit of a given diameter, span, and gauge, the cost of materials can be obtained from a price list. If the diameter of the culvert is in the appropriate range, the conduit may be field- or shop-assembled. To decide between these two alternatives, one must estimate the number of man-hours and the number of machine-hours, together with the representative cost of each, required for each type of assembly. Installation costs of the field-assembled and the shop-assembled conduit will also differ, in general. If the cost of materials, installation, and assembly for a field-assembled conduit are less than the price and installation of a shop-assembled conduit, the latter alternatives can be disregarded. Similar arguments apply to the choice of a concrete culvert and the comparison of steel and concrete culverts.

The cost of transporting the conduit to the site is often very important and should be taken into account. This cost can be estimated on the basis of the relevant distances and transportation rates. There are undoubtedly many locations where all feasible designs that call for one material type can be eliminated simply by comparing the transportation costs involved.

The problem of estimating the cost of constructing the foundation and backfill is especially complicated. To estimate this cost, one must know the type and quantity of all materials to be used for a foundation or backfill of a given type and quality, the types and quantities of labor services required, and the types and quantities of machine services needed. In addition, the prices of the various materials, the wage rates, and the hourly machine rates must be obtained. In other words, one must know the production function and the cost function for each type and quality of foundation and backfill called for by the set of feasible culvert designs.

The problem of costing earthwork cannot be done once and for all, because these costs vary from job to job, depend on the location of the job, and change with time and the situation. If the pertinent information for a foundation or backfill satisfying certain predetermined specifications did not vary significantly, one could use data collected from a sample of actual culvert installations to obtain estimates of the required input requirements; that is, the production function may be estimated on the basis of such a field study. The results of the study would simplify considerably the problem at hand. Economists have been estimating production functions of this type for some years now, as discussed by Walters (*103*).

Once the cost of installing each feasible design has been estimated, it is a simple matter to determine which one is least costly in this sense. However, the implications of the various feasible designs for service life have not yet been considered. Because the cost of maintaining a functional culvert system through time depends on these implications, a procedure for solving the design problem has not yet been completely specified.

DESIGNING FOR SERVICE LIFE

The problem of designing for service life is essentially economic. Suppose that a highway department or some other responsible agency wishes to install a culvert in a particular location and to maintain that culvert serviceable forever. Clearly, they would not be willing to invest their entire annual budget, though finite, in a culvert system designed to last forever, rather than invest less in a culvert system with a finite life. This is true, even though the latter system will have to be replaced an infinite number of times in the future. This rather extreme example serves to illustrate the fact that there is a trade-off between costs incurred in the future and current cost. In particular, if expenditures on a particular project can be deferred, there is an opportunity cost that must be accounted for if the department chooses not to defer them. The opportunity cost is the value of services that could be provided if the amount of the expenditure not deferred were invested in the best alternative use.

As an illustration of this point, consider the following problem. Denote by I the initial cost of constructing from the feasible set a particular design with a predicted service life of T years. Let there exist an alternative feasible design that has a predicted life of $T + 1$ years, but requires an additional expenditure of ΔI to construct. Finally, assume that a serviceable culvert must be maintained for $n(T+1)$ years; in other words, the horizon, H, is the end of the $n(T + 1)$th year. Given that one of these two designs will have to be used, is it worthwhile to incur the added cost of the design with the longer life?

If the design with the longer life is accepted, it will have to be replaced $n - 1$ times. Therefore, if the cost of replacement is equal to the initial construction cost, an expenditure of $I + \Delta I$ will have to be made $n - 1$ times at equally spaced dates during a future period of length $n(T + 1)$. Call this time pattern of expenditures the future cost flow implied by the design. On the other hand, if the original design is accepted, the system will have to be replaced more often, but the expenditure will be less each time. The number of times replacement will be required is equal to m, where m is the largest integer that satisfies the inequality

$$H/T = \frac{n(T+1)}{T} > m \qquad \text{(I-3)}$$

In this case the future cost flow is an amount I spent at dates equally spaced from the present if the cost of replacement equals the initial cost of construction.

Because the time paths of the future cost flows implied by the two designs differ, one must compare the present value of these flows in order to come to a decision. Presumably the department has alternative uses for the funds in its capital budget. Let r represent the annual net rate of return on the "best" alternative. Then, the department should decide to accept the design with the shorter life if the initial investment plus the present value of future replacement costs is less than the initial investment required to construct the design with the longer expected life plus the present value of future replacement costs implied by that design. It follows from the discussion in the section on economic concepts that this difference is given by

$$\begin{aligned} D = I + I \sum_{i=1}^{m} (1+r)^{-iT} - (I + \Delta I) - (I + \Delta I) \\ \sum_{i=1}^{n-1} (1+r)^{-i(T+1)} = I \sum_{i=0}^{m} (1+r)^{-iT} \\ - (I + \Delta I) \sum_{i=0}^{n-1} (1+r)^{-i(T+1)} \qquad \text{(I-4)} \end{aligned}$$

This is true because the ith replacement of the design with the shorter life will have to be made at the end of the iTth year, whereas the ith replacement for the design with the longer life is made at the end of the $i(T + 1)$th year. It has already been noted that the culvert has to be replaced m times in the former case and $n - 1$ times in the latter.

If the horizon is very short, it never pays to accept the design with the longer life. For example, if H equals T, then D equals $-\Delta I$, which is simply the difference in constructing the initial culvert. However, if the horizon is long enough, it may be optimal to use the design with the longer life because it need not be replaced so often. Suppose the horizon is infinite. Because n and m approach infinity as H approaches infinity, and because

$$\lim_{m \to \infty} \sum_{i=0}^{m} (1+r)^{-iT} = 1/[1 - (1+r)^{-T}] \qquad \text{(I-5}a\text{)}$$

and

$$\lim_{n \to \infty} \sum_{i=0}^{n-1} (1+r)^{-i(T+1)} = 1/[1 - (1+r)^{-(T+1)}] \qquad \text{(I-5}b\text{)}$$

D is negative if

$$\Delta I[(1+r)^{T+1} - 1] > r(I + \Delta I) \qquad \text{(I-5}c\text{)}$$

If the inequality does not hold, then the department is either indifferent to a choice between the two designs or it should accept the design with the longer life.

The inequality given by Eq. I-5c has an interesting interpretation. The term on the left is the realizable return at the end of a period of length $T + 1$ on an amount equal to ΔI invested in the alternative project. The term on the right side is the return obtainable if an amount $I + \Delta I$ were invested in the project for 1 year. If the design with the longer life is accepted, an extra amount equal to ΔI must be spent at the beginning of each period of length $T + 1$. The return to this extra investment is an extra year of life. Because the opportunity of investing an amount ΔI in the alternative project is lost if the design with the longer life is accepted, the opportunity cost of the decision is the term on the left side. The right side is the value of the extra year of life assigned by our decision procedure. In particular, the procedure assumes that the value of services per dollar of investment provided by the culvert system is at least as great as that for the best alternatives. Eq. I-5c can be interpreted in the following manner. The design with the longer life should be rejected if the opportunity cost is greater than the value of the extra year of life provided.

In the previous example problem, there are a number of simplifying assumptions that are neither realistic nor necessary. For example, it was assumed that each design could be replaced at some future date at the same cost as the cost of initial construction; this assumption will not be true, in general, for any one of the following reasons: (1) construction and material costs may be expected to increase during the interim, (2) replacement may require additional excavation not required at the time of the initial installation, and (3) during the time required for replacement, the services of the system of which the culvert is a part may have to be discontinued. It was also assumed that no expenditure for maintenance was required. Because a culvert will not attain its designed life unless a prescribed program of maintenance is performed, this is an unrealistic assumption.

Our decision rule can take these facts into account. Let R_t denote the estimated cost of replacement plus an estimate of the value of services that must be discontinued as a consequence of the replacement at a date t years in the future. Let M_t be the annual maintenance cost required by the design. Finally, let I denote the initial investment required to install a particular design from the feasible set plus the present value of all estimated future costs. If H is the horizon in years, r is the annual discount rate, and T is the expected length of life of the designed culvert, then

$$C = I + \sum_{i=1}^{m} R_{iT}(1+r)^{-iT} + \sum_{j=1}^{H} M_j(1+r)^{-j} \qquad \text{(I-6)}$$

in which m is the largest integer that satisfies

$$H/T > m \qquad \text{(I-7)}$$

The first term is the initial construction cost required by the design, the second term is the present value of future replacement costs, and the third is the present value of maintenance costs to be incurred in the future.

Eq. I-6 is of little use unless estimates of all the variables in it can be obtained for each design in the feasible set. The problem of estimating the cost of initial construction is discussed in the preceding section. The cost of replacement should be related to the initial cost, although it must be increased appropriately if construction costs are expected to inflate. For example, if no additional costs, implicit or explicit, are to be incurred at the time of replacement, but costs of inputs are expected to inflate at a rate of g percent per year, then the cost of replacement at date iT, R_{iT}, will be $I(1+g)^{iT}$. If there are other costs arising from one or more of the factors just discussed, these should be added. Finally, maintenance costs might be estimated from past experience with similar designs.

The problem of arriving at a reasonable number to use as the discount rate is very difficult. The reason for the difficulty arises from the fact that the services provided by projects and installations of a public agency, such as a state highway department, are typically not sold. There are, of course, exceptions, such as the case of a toll road. There appears to be no objective way of placing a valuation on the services of all alternative projects, let alone finding the alternative project with the highest net rate of return. Nevertheless, the difficulties inherent in the problem do not imply that the opportunity cost of using funds should be ignored. In fact, no truly rational economic decision can be made without taking them into account.

Finally, the decision problem outlined in this section is what economists call a planning problem; that is, the decision maker must decide on one among many alternative plans of present and future action. The one that will be chosen depends on how the decision maker expects the world to evolve in the future. In particular, the choice generated by the procedure outlined previously depends on what the decision maker expects concerning replacement and maintenance costs in the future, the service life of a given design, and the future rate of construction cost inflation. It may not be reasonable to expect a particular value of each of these variables to be realized at some future date when the decision is made. Instead, the decision maker may be uncertain, even with the best possible information at his disposal, about the actual values to use in the analysis. In such cases the uncertainty can and should be taken into account.

For example, it is unrealistic to assume that a given culvert design has a certain service life. In fact, there are field study data that suggest that the useful life of culvert structures in a sample having about the same design characteristics and service conditions varies widely. This information, where appropriate and available, can and should be used in the decision procedure. To illustrate how it can be used, suppose that records from a large sample of culverts with characteristics and service conditions similar to the one in question show that no structure in the sample was serviceable longer than k years. In addition, assume that the proportion of culverts that failed during the ith year after installation is known. That proportion, $q(i)$, is an estimate of the probability of failure in the ith year of life. Clearly then, $q(i) < 1$ for all $i < k$ and $q(k) = 1$, because by assumption at least one culvert in the sample had a service life of k years and no culvert functioned longer than k years. Given this information, the probability that the particular culvert under consideration will have to be replaced, if it were built according to the design in question, can be computed.

Let $p(t)$ denote the probability of having to replace the system in the tth year after the initial installation. The expected cost of replacing the culvert system at the end of the tth year is equal to the product of the probability that a replacement will be required in year t and the replacement cost in that year. Therefore, the sum of the initial investment plus the present value of all expected future costs is given by

$$C = I + \sum_{t=1}^{H} (1+r)^{-t} p(t) R_t + \sum_{t=1}^{H} (1+r)^{-t} M_t \tag{I-8}$$

The difference between Eqs. I-6 and I-8 is that in the former case it was assumed that replacement would be required with probability one at intervals of length T in the future.

The advantage of using Eq. I-8 rather than Eq. I-6 can be illustrated with an example. Suppose that there are two designs in the feasible set, each of which involves the same initial investment and the same future time path of replacement and maintenance costs. Suppose, as well, that the average service life was the same for both. Then, Eq. I-6 would imply that the two designs are economically equivalent. Suppose, however, that the range of the age distribution is the same, but that the mode life of one of the designs is longer than the other. I-8 would assign a lower cost to that design because replacement is more likely required later in each replacement cycle. Of course, this effect is less pronounced if inflation is expected.

SUMMARY AND SUGGESTIONS FOR FUTURE RESEARCH

This report is concerned principally with the identification and formulation of the economic aspects of the culvert design problem. As is shown, no rational economic decision can be made in a given instance unless (1) a criterion for the decision exists, and (2) the criterion is understood and properly defined. Although minimum cost seems an appropriate criterion, the definition of "cost" is no simple matter. In a problem such as this, where different designs imply different future costs as well as different initial construction expenditures and where these future costs and/or other relevant variables are uncertain, the problem of providing an appropriate definition is a rather complicated task. However, doing so is of paramount importance to determining the economically optimal design.

To an extent, some of the factors that appear to account for cost variation among designs have been suggested, but no attempt has been made either to list all the considerations that might be important or to rank in the order of their importance those factors that were discussed. In part, the failure to do so reflects the fact that little quantitative information exists concerning this aspect of the problem.

Although research in this area undoubtedly would be useful as a guide to those who are charged with the design of culvert systems, its value is limited. Because different installations have different technical specifications, the set of feasible designs for one problem is likely to be very different from that of another. Therefore, those factors that are responsible for explaining cost variations in one case may be quite different from those that account for cost variations in another case. Even if the feasible designs were the same in two different instances, differences in wage rates, material costs, and transportation costs may imply that the relative importance of the various factors is drastically different when the two jobs are separated either in space or in time.

The most fruitful research would be of the type that attempts to ascertain input and material requirements for the various components of the culvert system. For example, knowing what materials and inputs are required and how much time it takes to construct a backfill of a given quality and size would be very helpful to the designer. Presumably, these are technical relationships that do not vary significantly across jobs separated in space or time. With this information a designer need only find the input and materials costs that are relevant for his particular task in order to compute his cost estimate.

Another potentially fruitful study could be directed toward determining the service life distribution of different types of culverts under a variety of environmental conditions. The reasons why it is necessary to consider the uncertainty of the service life of a culvert is discussed elsewhere. The extent and structure of the uncertainty are important parameters in determining the economically optimal culvert system. In addition, any improvements in engineering design procedures would assist in solving the economic problem. For example, standardized definitions of component parts would be helpful, because past experience could then be more readily used as a guide for current decisions. Component parts of the system must be well defined and these definitions must be widely used in order to perform and apply the production function studies suggested previously. Finally, better design procedures, which assist the engineer in his effort to determine which of all possible designs are equivalent in a given instance, will also improve his ability to ascertain the least costly design for his purpose.

APPENDIX J

SOME FAILURES OF BURIED CONDUITS *

When a structure fails or shows signs of excessive structural distress, a great deal of valuable information may be gained by a thorough study and examination of the facts pertinent to its design and construction. This is particularly true with respect to underground conduits such as culverts and sewers, because of the intimate relationship between supporting strength of the finished structure and installation conditions and environmental details.

The researcher has had occasion during the past 20 years or so to investigate several dozen buried pipelines that have failed, and has attempted to identify, in each case, the cause or causes of the structural distress. These investigations have embraced a wide variety of pipelines, such as culverts, storm and sanitary sewers, water mains, and gas distribution mains and services. Materials involved have included plain and reinforced concrete pipes, monolithic reinforced concrete arches, burned clay pipes, corrugated steel pipes, and cast-iron pipes. A selected number of cases is presented herein for the purpose of illustrating the relationship between cause and structural effect in the hope that the whole process of design and installation of this type of structure may be upgraded.

The term "failure" is used herein in a broad sense to include a wide range of conditions, from complete failure or collapse necessitating replacement of the structure to relatively minor structural damage that could be successfully and economically repaired. It does not include cases of insignificant longitudinal cracks in reinforced concrete pipes, or cracks that are in the range of 0.01 in. or less in width, nor does it include cases of excessive deflection of flexible pipes that did not result in collapse or longitudinal seam failure.

Much of the ability of a buried pipeline to carry vertical load depends on details of the installation, such as width and quality of the pipe bedding, width of trench, and the presence or absence of circumstances that will enhance the development of lateral pressures against the sides of the pipe. Because most of these details are covered up and out of sight after a pipeline is backfilled, an investigator must depend not only on visual inspection of the line and examination of plans and specifications for the project but also on extensive interviews with resident engineers, inspectors, and construction personnel who actually saw the pipes installed. A great deal of engineering detective work is required to unearth all the facts that have contributed to the structural distress under investigation.

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Because of these conditions, it is obvious that a complete determination of all the pertinent facts relative to a failure situation may not always be possible to obtain. The following summaries of cases and the conclusions expressed relative to causes of failure represent the results of the researcher's investigations and his best judgment resulting therefrom.

CASE 1

Case 1 involved a study of six reinforced concrete pipe culverts under Interstate highway construction in one county of a midwestern state. The size of pipes, length of culverts, and maximum heights of fill over the pipes are as follows:

ITEM NO.	SIZE OF PIPE (IN.)	LENGTH OF CULVERT (FT)	HEIGHT OF FILL (FT)
1	60	216	25.4
2	60	218	25.5
3	66	400	24.0
4	72	320	26.3
5	84	287	27.6
6	84	187	30.5

All of the pipes were specified to be ASTM C 76 Class IV. There were no 3-edge bearing tests of the pipe available, but the quality of concrete was indicated by compression tests on cores taken from the pipe walls. These tests, though limited in number, indicated that the pipes were acceptable under the specification.

Visual examination of the interior of the culverts revealed a large number of longitudinal cracks in the crown ranging in width from hairline to 1/16 in. or more. It was difficult to inspect the pipe inverts because of water and ice of varying depths above the flow line, but a few cracks of damaging width in this region were observed. The most severe damage to the pipes was spalling of the concrete protective cover over the reinforcement, both in the crown and invert, as shown in Figure J-1. Also, tapping the walls with a ball peen hammer revealed extensive areas where the protective cover had separated from the pipe wall in these regions, indicating incipient spalling due to unequal distortion of the reinforcement and the concrete.

The resident engineer on the construction project described the pipe installation procedure as follows. First, the fairly stiff glacial till natural soil was bladed to an elevation approximately 1 in. below the bottom of the pipe. Then, a layer of pit-run fine sand was spread over the area and brought to the pipe grade. The pipes were installed on this sand without its being shaped to fit the contour of

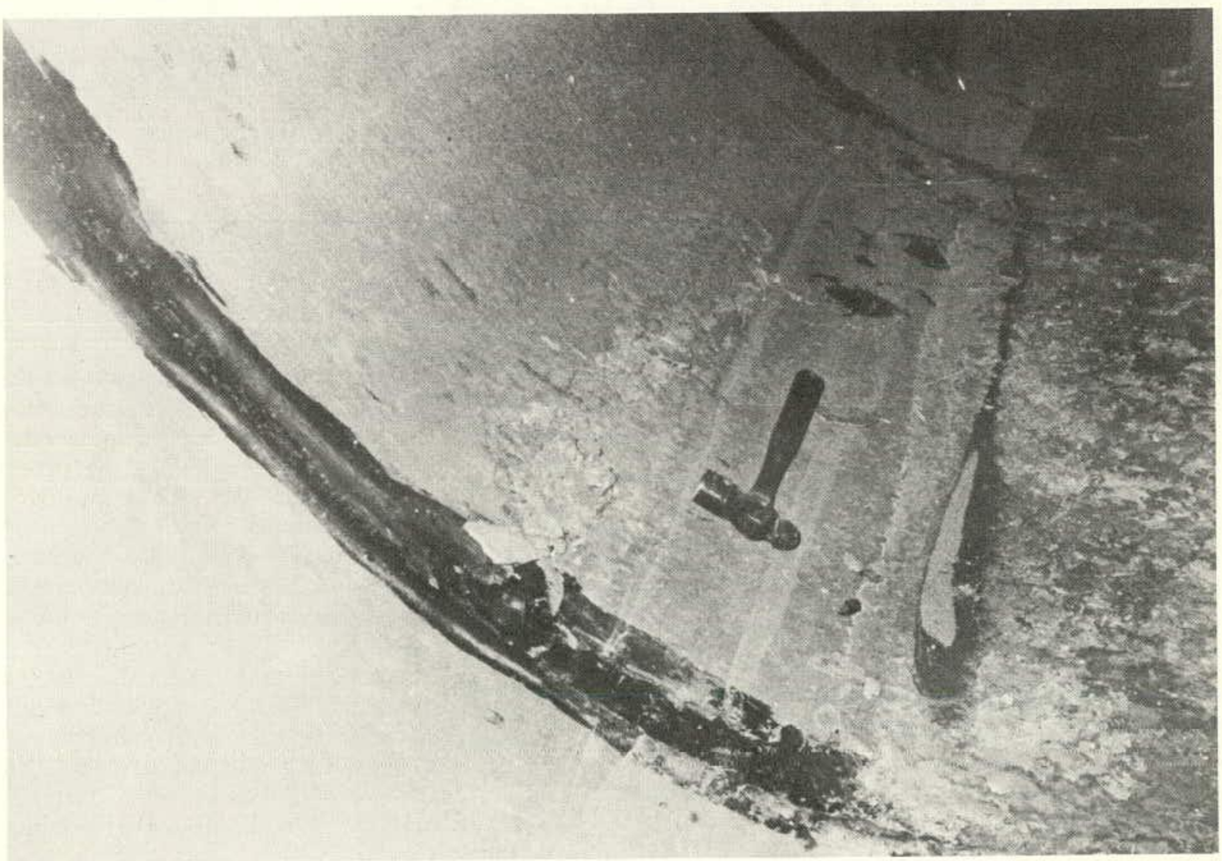

Figure J-1. Spalled concrete in invert (Case 1).

the pipe. Next, sand was placed alongside the pipe up to the springline and allowed to take its angle of repose. Job-site soil was bladed up at the sides adjacent to the sand and compacted by the wheels of a maintainer operated parallel to the pipeline. Laterally beyond the width of the maintainer the soil was compacted by a sheepsfoot roller.

It was concluded that the damage to the pipes was caused primarily by the fact that the pipes were placed on a fairly unyielding flat bed of soil that was not shaped to fit the contour of the pipe (Fig. J-2). This resulted in a high concentration of the bottom reaction as the pipes settled downward in the sand and approached contact with the stiff clay under-soil. This, plus the fact that the sand alongside the pipes was not compacted in the critical triangular areas below the lower quarter points, contributed to the development of excessively high bending moments in the pipe walls.

CASE 2

The structure in Case 2 was a 66-in.-diameter reinforced concrete pipe storm sewer located parallel to an Interstate highway in a southeastern city. The sewer in question was 288 ft long and located just beyond the toe of slope of the highway embankment. The pipe sections were 8 ft long and fabricated according to ASTM C 76, Class IV, Wall B. Three-edge bearing tests indicated that the pipes complied with the specification. Heights of fill over the pipes ranged from 13 to 21 ft. The pipes were installed in a trench that was 3 to 4 times as wide as the outside diameter of the pipes and were, therefore, classed as projecting conduits.

The soil in the area is sandy, but has enough cohesion to stand temporarily on a steep slope. The pipes were installed on a flat trench bottom that had not been shaped to fit the contour of the pipe. Backfill was placed under the haunches and up the sides of the pipe and compacted vertically with mechanical equipment having a tamping face approximately 12 × 18 in.

Examination of the pipeline approximately two months after completion of the backfill revealed relatively minor evidence of structural damage. Some pipe sections were not cracked; others showed longitudinal cracks up to 1/16 in. in width, and one section was spalled in the invert, as shown in Figure J-3. In several others, incipient spalling was indicated by a hollow sound under a ball peen hammer.

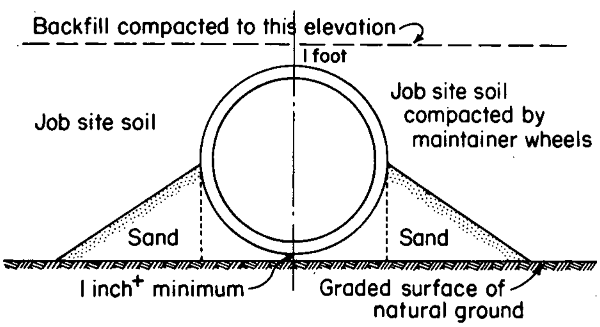

Figure J-2. Pipe bedding (Case 1).

Prior to the researcher's inspection, five of the pipe sections had been removed for examination of bedding and backfilling conditions. As the backfill was removed, a number of soil density measurements were made above the pipe, at the sides, and under the invert; the results are shown in Figure J-4. The measured densities indicated good compaction of the backfill in all regions except in the triangular spaces under the lower haunches of the pipe. Here the soil was so loose and friable that it was impossible to measure the density.

It appears that structural damage was caused by the fact that the bedding was not shaped to fit the contour of the pipe, with the result that the upward reaction on the pipe became highly concentrated as load developed. This caused a high bending moment to develop in the pipe wall. This case illustrates the fact that when soil is compacted vertically, densification does not extend laterally beyond the tamping face. This accounts for the fact that the soil in the critical areas below the lower haunches of the pipe was not densified in this installation..

CASE 3

Case 3 represents an investigation of four reinforced concrete pipe culverts under Interstate highway construction in a midwest state. These culverts suffered extensive structural damage, and were said by representatives of the highway department to be typical of the condition of an additional 20 similar structures in the same general region. Details of the culverts examined are as follows:

ITEM NO.	SIZE OF PIPE (IN.)	LENGTH OF CULVERT (FT)	HEIGHT OF FILL (FT)
1	60	348	65
2	72	128	25
3	54	388	37
4	42	96	23

The field examination revealed a widely varying extent of damage among the individual pipe sections, ranging from insignificant longitudinal cracking to very wide cracks accompanied by extensive spalling of the protective cover over the reinforcement. Some pipes showed gross diameter change, the worst example being that of one section in item 4 which had deflected nearly 6 in. Significantly, none of the pipe sections had collapsed, probably because of the development of passive soil pressures at the sides of the pipes.

Construction specifications under which the culverts were installed called for Class B beddings, but conversations with construction personnel lead the researcher to believe that the actual bedding achieved was no better than Class C. The pipe installations were designed with a factor of safety of 1.25 based on the ultimate three-edge bearing test strength of the pipes.

The soil profile in the region of these culverts reveals a

Figure J-3. Longitudinal cracks and spalled concrete in invert (Case 2).

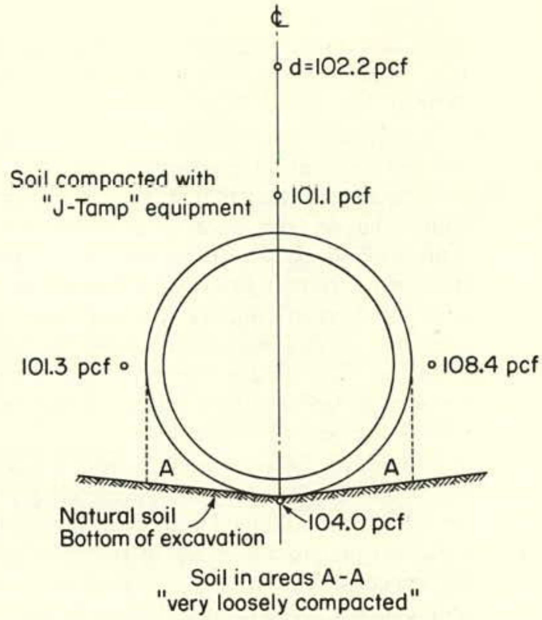

Figure J-4. Pipe bedding (Case 2).

relatively shallow mantle of soil overlying ledge rock. This mantle is thinnest in the draws where the culverts were constructed. An investigation to determine the thickness of soil bedding material between the bottom of the pipes and ledge rock was conducted by drilling through a number of pipe inverts and then probing with a steel rod. It was revealed that the distance from the bottom of the pipes to rock varied widely, but in general it was very shallow—as little as 4 in. in several instances.

It was concluded that the structural damage suffered by these culverts was attributable to three probable causes:

1. The thickness of soil bedding above ledge rock was too shallow.
2. The factor of safety was too low.
3. The quality of bedding was inferior to that assumed in the design.

The shallow thickness of bedding permitted the pipes to approach contact with ledge rock as they settled under load. This undoubtedly caused a concentration of the bottom reaction on the pipe, and, coupled wih a low factor of safety and relatively poor quality bedding, resulted in excessive bending moments and failure of the pipe walls.

CASE 4

A twin line of cooling water conduits at a power plant on the eastern seaboard consisted of reinforced concrete pipes 84 in. in diameter, increasing to 96 in. toward the outlet. The lines were installed in a trench wider than the transition width and were, therefore, classed as projecting conduits. The soil at the site was a fairly compact gravelly sand. The height of fill over the pipelines varied from 9 to 17 ft. The pipes were installed on a flat bed of the sand. Backfilling material was pushed into the trench, with little or no effort made to compact the soil under the lower haunches of the pipe, although compaction above the springline was satisfactory.

Shortly after backfilling, many of the pipe sections developed longitudinal cracks wider than 0.01 in. in the crown and the invert. In addition, there were extensive areas in which the protective cover over the reinforcement was spalled off; these occurred mostly in the invert region, but they also were found to some extent in the crown. Tapping with a ball peen hammer revealed many areas where the protective cover, though not visibly spalled, had separated from the body of the pipe wall. The separation, as indicated by a hollow sound under the hammer, appeared to extend over arcs of 60° to 90°.

The inherent strength quality of the pipes was poor. There was much evidence of improper placement of steel reinforcement and a limited number of three-edge bearing tests indicated the pipes were not as strong as might be reasonably expected of units made under ASTM C 76, Table II, although the specifications themselves did not, at the time these lines were constructed, indicate a required strength for sizes greater than 72 in. Also, inquiry revealed that some of the pipes actually contained only about one-half the amount of steel required, indicating poor quality control at the manufacturing plant.

Another fact of importance was that in some areas large quantities of earth from excavation for other facilities were piled on the ground surface over the pipelines under investigation. Obviously, this increased the load on the pipes above that which they normally would have had to carry. There was a definite relationship between the extent and location of pipe damage and the locations where this overloading had occurred.

In summary, it was concluded that damage to the pipelines resulted from five major factors, as follows:

1. The pipes were of inferior quality owing to inadequate control of steel placement.

2. Some pipes, which were purported to be fabricated in accordance with the provisions of ASTM C 76, Table II, actually contained only about 50 percent of the amount of steel specified.

3. The pipes were bedded on a flat surface of rather compact gravelly-sandy soil.

4. Attempts to compact the backfill soil beneath the lower haunches of the pipe were very meager and not successful.

5. The pipes were badly overloaded in certain areas by piling temporary spoil banks on the surface above the pipelines.

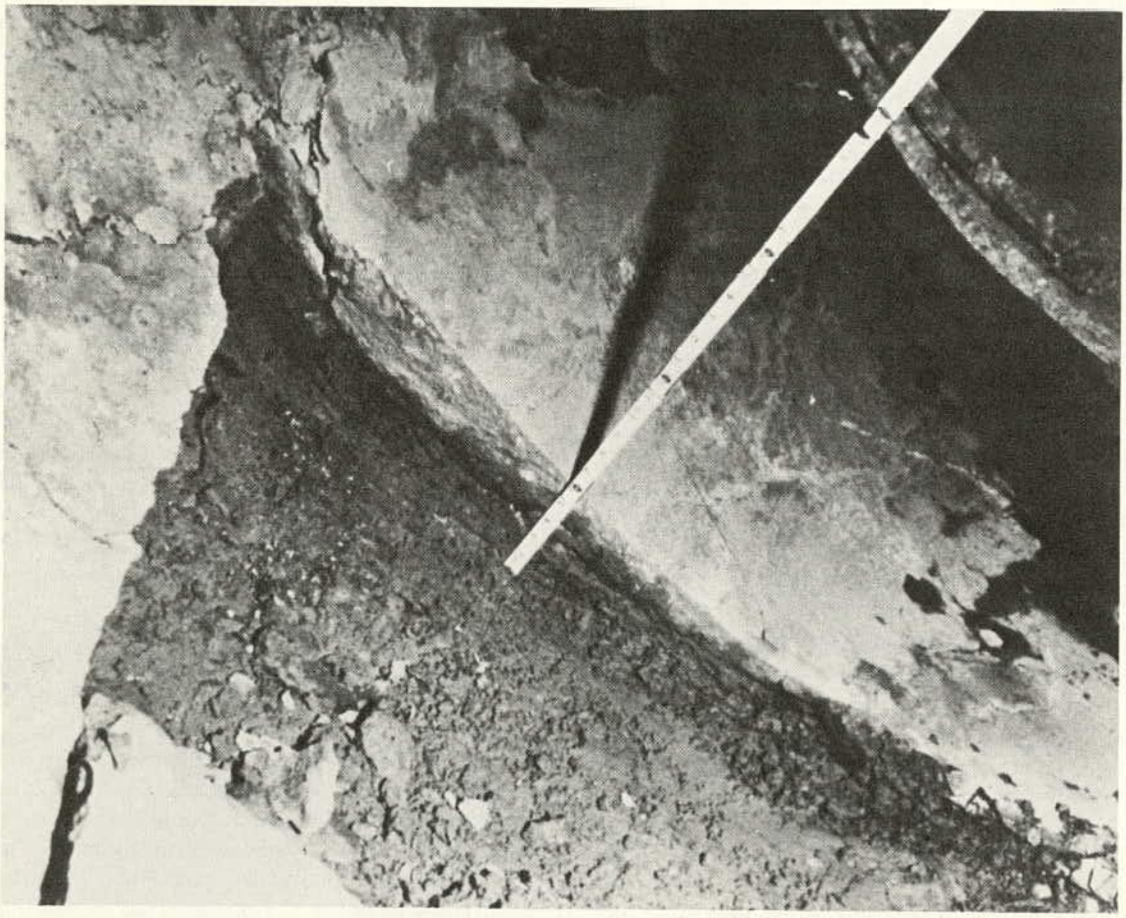

Figure J-5. Pipe removed to show distribution of grout (Case 4).

These pipelines were successfully repaired by pressure grouting through holes drilled at the lower quarter points, thus upgrading the bedding and backfill conditions. Some pipe sections were removed after grouting to observe the distribution of grout. One section is shown in Figure J-5. Subsequent to the grouting operation, the cracks were reamed out to the depth of the steel, all loose and broken concrete was removed with an air chisel, and the damaged areas were repaired with gunite concrete. The lines have served satisfactorily for a number of years and give promise of continued satisfactory service, just as though they had not suffered structural damage.

CASE 5

A 96-in.-diameter corrugated steel culvert was constructed at a location on the eastern seaboard. The planned height of fill was 28 ft, but, when the fill was about half completed, the pipe suffered excessive deflection and collapsed completely in a number of locations, as shown in Figure J-6. The pipe wall was of 8-gauge metal in standard corrugations (i.e., ½ in. in depth and spaced at 2⅔ in. center to center). The pipe was "shop-strutted" by means of ½-in. tie rods at the horizontal diameter, spaced 2 ft center to center. Each tie rod was equipped with a turnbuckle by means of which the horizontal diameter of the pipe could be decreased and the vertical diameter could be increased. When the pipe was installed, the vertical diameter of the pipe was approximately 3 percent greater than nominal.

The soil at the site was a yellow-brown clayey gravel (GC) having a plastic limit of 12 percent, plasticity index of 6 percent, modified AASHO density of 136.7 pcf, optimum moisture content of 7.0 percent, and soaked CBR of 16.4. It was very satisfactory soil for development of passive resistance pressure as the pipe deflected and pushed outward against it. The pipe was bedded in native soil shaped to fit the contour of the pipe. Job-site soil was compacted under the haunches of the pipe by pneumatic hand tampers, and up the sides of the pipe in layers about 6 to 12 in. in depth. The backfill was placed evenly and simultaneously on both sides of the pipe. During backfill operations, approximately 100 density measurements were made, and all but two of these indicated the density was equal to or greater than 95 percent of modified AASHO.

It is believed that the primary cause of failure of this culvert was the lack of stiffness of the pipe wall, as represented by the moment of inertia of its cross section. The 8-gauge standard corrugation wall has a moment of inertia of 0.0055 $in.^4$ per inch, and this is exceedingly light for a 96-in. pipe. After collapse, the structure was replaced with

Figure J-6. Deflection failure of 96-in.-diameter corrugated steel culvert (Case 5).

8-gauge structural plate pipe, which has corrugations 2 in. deep and spaced 6 in. center to center; the moment of inertia of this wall is 0.0961 in.[4] per inch, or approximately 17 times that of the 8-gauge standard plate. The replacement pipe has performed satisfactorily.

A contributing factor in the failure may have been the manner in which the shop struts were handled during placement of the fill. Ideally, these struts should be removed after the fill reaches a shallow height above the top of the pipe to allow the flexible ring to deform and develop the passive resistance pressures on which it depends so largely for structural strength. In this instance the tie rods were left in place and the turnbuckles were loosened gradually as the fill was raised. It is possible that the ties were kept too tight to allow normal deflection of the pipe, and this may have triggered the failure.

Figure J-7. Circumferential break in reinforced concrete pipe (Case 7). This crack was ⅝ to ¾ in. wide at the crown and ⅛ to ¼ in. wide at the invert.

CASE 6

Case 6 involves the excessive deflection of an 84-in. corrugated steel storm sewer in a midwestern city. The pipe wall was 8-gauge metal in standard corrugations (2⅔ in. by ½ in.). The project was designed and construction was supervised by a competent and widely known firm of consulting engineers. The excessive deflection of the pipe was the basis of controversy between the owner and contractor, and this controversy resulted in court action.

The specifications for installation of the pipeline called for the sidefill soils to be "thoroughly compacted," but no numerical degree of densification was specified, nor was the lateral extent of compacted soil specified. Both the consulting engineer's inspector and the contractor's foreman testified that they considered the soil at the sides of the pipe was thoroughly compacted. However, the court was critical of the fact that no definite minimum density was specified and that ". . . no tests for compaction were made, although there is a test which could have been used." The decision also stated:

> The court is further of the opinion that the supervising engineer did not exercise adequate supervision, especially in view of the fact that the specifications under the contract were not sufficiently precise and definite. The use of the term "thoroughly compacted" was so indefinite and broad in scope that he should have realized that he had a duty to carefully inspect the compaction of the soil as the sewer line progressed. If he had done so, and ordered corrective measures, it would have prevented the difficulties which ultimately resulted through his deficient supervision. . . .

CASE 7

A sanitary sewer constructed in the Pacific Coast region consisted of 27-in. and 36-in. reinforced concrete pipe sections that were 10 ft in length. The soil profile at the site showed a 6- to 8-ft stratum of fine-grained silty clay overlying a stratum of coarse gravel and cobbles. The grade of the pipeline was below the surface of the gravel stratum, and the specifications required that the trench be undercut and refilled with fine-grained soil to form the pipe bedding. Construction personnel stated that the thickness of this bedding beneath the pipe ranged from 4 to 12 in. The bedding was not shaped to fit the contour of the pipe.

Inspection of the line after completion revealed rather numerous longitudinal cracks ranging from hairline to a few that exceeded 0.01 in. in width. Of more serious import, however, were a number of pipe sections that were broken circumferentially at approximately the center, as shown in Figure J-7. These breaks ranged in width up to ⅝ or ¾ in. maximum, and in all cases were wider at the top than at the bottom of the pipe, indicating a concentrated upthrust of the reaction in the vicinity of the center of the section.

Some personnel on the project advanced the theory that, because the 10-ft-long sections were handled by a crane with a sling at the center, the weight of the pipe might have caused the breakage. However, bending moment calculations indicated a maximum outer fiber stress of only 17 psi, whereas 4,000-lb concrete probably has an ultimate tensile strength in the neighborhood of 475 psi. A more tenable hypothesis appears to be that the undercutting of the gravel and cobbles may not have been complete, and that a mound or pyramid of this material may have been left at a shallow depth in the vicinity of the center of some pipe sections. Then, as the pipe settled under the backfill load, a concentration of the reaction developed and broke the back of the pipe.

CASE 8

A 42-in. reinforced concrete interceptor sewer in a midwestern city was constructed in the floodplain of a small tributary to an aggraded river. The geological history of the river indicated a substantial rise in grade of the stream bed in postglacial time. After the sewer was completed and before it was accepted by the owner, it was discovered that the pipeline in a certain area was out of line laterally by as much as 33 in. and that it was 18 in. higher than the established grade. The pipes themselves were not structurally damaged.

Borings in the region of the displacement revealed that the sewer had been constructed through a prehistoric buried swamp. The soil in the vicinity of the pipeline was highly organic in character; it was as black as coal and gave off a strong odor of decomposition. It was classified as clayey silt (CL), it contained approximately 20 percent collodial material (less than 0.001 mm), and the natural moisture content of the soil was approximately equal to its liquid limit (36 percent). During the period between completion of the pipeline and the time of discovery of its displacement, the sewer was empty and in its maximum state of buoyancy.

During the investigation to determine the cause of the displacement, it was learned that a parking-lot pavement had been constructed in the area above the critical region of the pipeline. The pavement was constructed by placing a layer of crushed rock and then compacting it with heavy vibration equipment. Several layers of rock were placed and each layer was heavily vibrated. The pipeline construction foreman was in the sewer when the parking-lot pavement construction started. He said the vibration was strongly felt inside the line, although he did not believe the sewer moved while he was in it.

It is the researcher's opinion that this vibration caused the soil to liquefy, thus causing the partially buoyant pipeline to drift laterally and upward in the semi-fluid organic soil by which it was surrounded. This experience strongly indicates the need to locate a pipeline, or relocate it if necessary, to avoid an unstable environment for the structure.

CASE 9

A 60-in. reinforced concrete pipe outfall sewer was constructed in the floodplain of an alluvial river in a midsouthern city. A construction photograph of the line adjacent to a heavy junction box is shown in Figure J-8. A manhole riser (not shown) extended upward to the ground surface, which was approximately 27 ft above the top of the pipe. Particular attention is directed to pipe sections 2 and 3 upstream from the junction box in the lower center of Figure J-8.

The pipeline was constructed in a trench that was wider than the transition width; it was, therefore, a projecting conduit. The pipe was bedded in a concrete cradle (i.e., Class A bedding). The cradle was constructed in two lifts. First, a slab 7½ in. thick was poured; then, the pipe was set to line and grade, resting on a concrete block adjacent to the tongue and groove joint, and the balance of the cradle was poured under the pipe and up the sides. The bond between the first and second lifts of the cradle was very poor.

A few weeks after completion of the line and before acceptance by the owner, it was discovered that the manhole and junction box shown in Figure J-8 had settled 3 ft or more. Further investigation revealed that the pipeline in this vicinity had been dragged downward by this subsidence, and it was full of sand. Excavation revealed extensive damage to the pipes and the concrete cradle. As Figures J-9 and J-10 show, pipe sections 2 and 3 were completely crushed. Huge chunks of concrete were broken from the pipe walls and reinforcing steel was bent into an S shape at the top. They ruptured completely at the springline, and the steel bars were pulled in two. The upper pour of the cradle under the pipes in this vicinity appeared to be completely missing, but chunks of the cradle were dredged up from the adjacent soil, some of them from as far away as 6 to 10 ft from their original position. The condition of these pipes indicated that tremendous forces had been at work, much greater than could be accounted for by static load from the overburden soil.

Soil borings in the vicinity of the failure indicated that the junction box and adjacent pipes were founded on a 6- to 8-ft stratum of loose, poorly graded fine sand having an effective size of approximately 0.070 mm, a uniformity coefficient of 1.7, and a standard blow count as low as 11. The groundwater table was roughly 27 ft above the flow line of the pipe, but fluctuated widely, first due to pulling of the construction wellpoints, then later to the rise and fall of the stage of a nearby river. Construction records indicated that the upper 20 to 25 ft of the backfill around the manhole riser were placed in a very wet condition, with no attempt made to compact it.

A study of all available facts and data leads to the following hypothesis of the cause of this failure:

1. The stratum of fine sand on which the junction box rested was too low in density, probably below its critical density. Fluctuations of the groundwater table associated with removal of the construction well point system and with variations of stage of a nearby river probably caused the foundation soil to lose its stability and become quick, thus lowering its bearing capacity.

2. Consolidation of the wet, uncompacted backfill soil probably generated a down-drag shear on the manhole

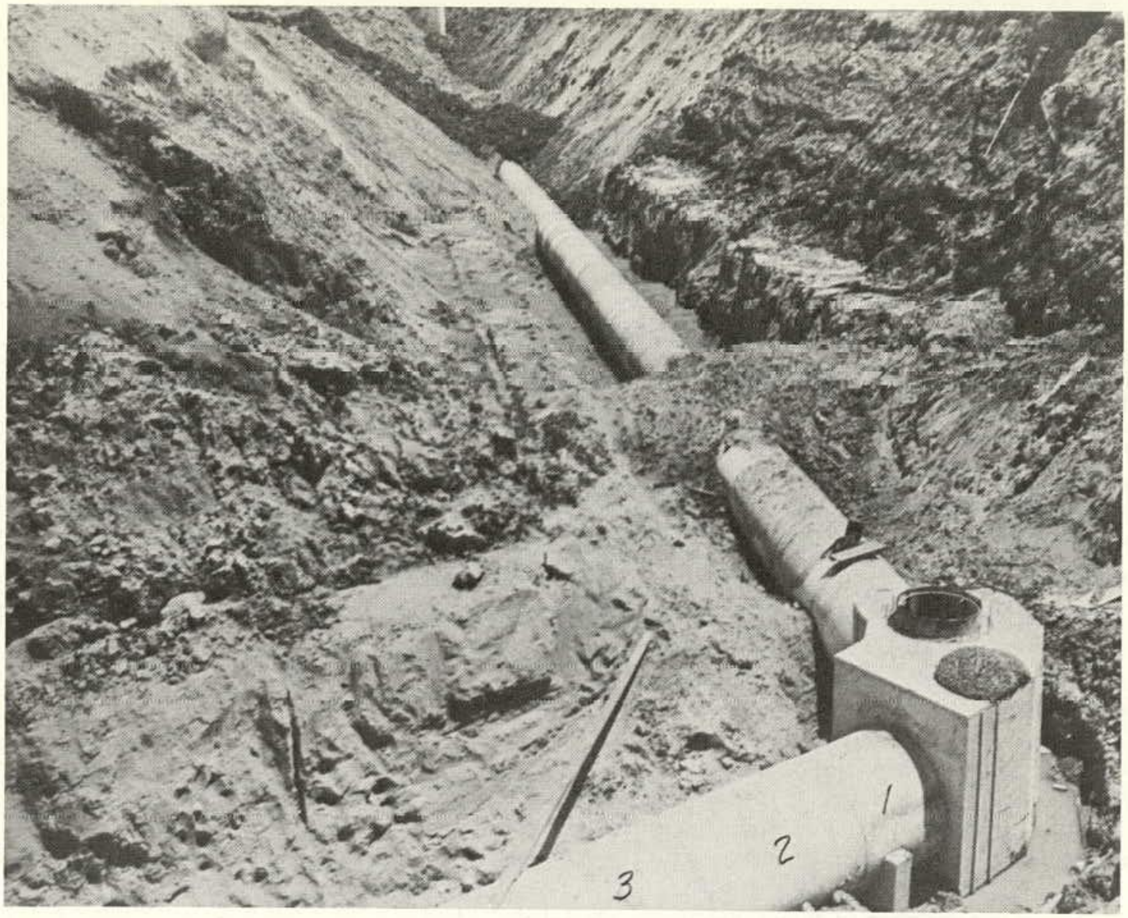

Figure J-8. Construction photograph of 60-in. pipeline and junction box (Case 9).

Figure J-9. Top of pipe after failure (Case 9).

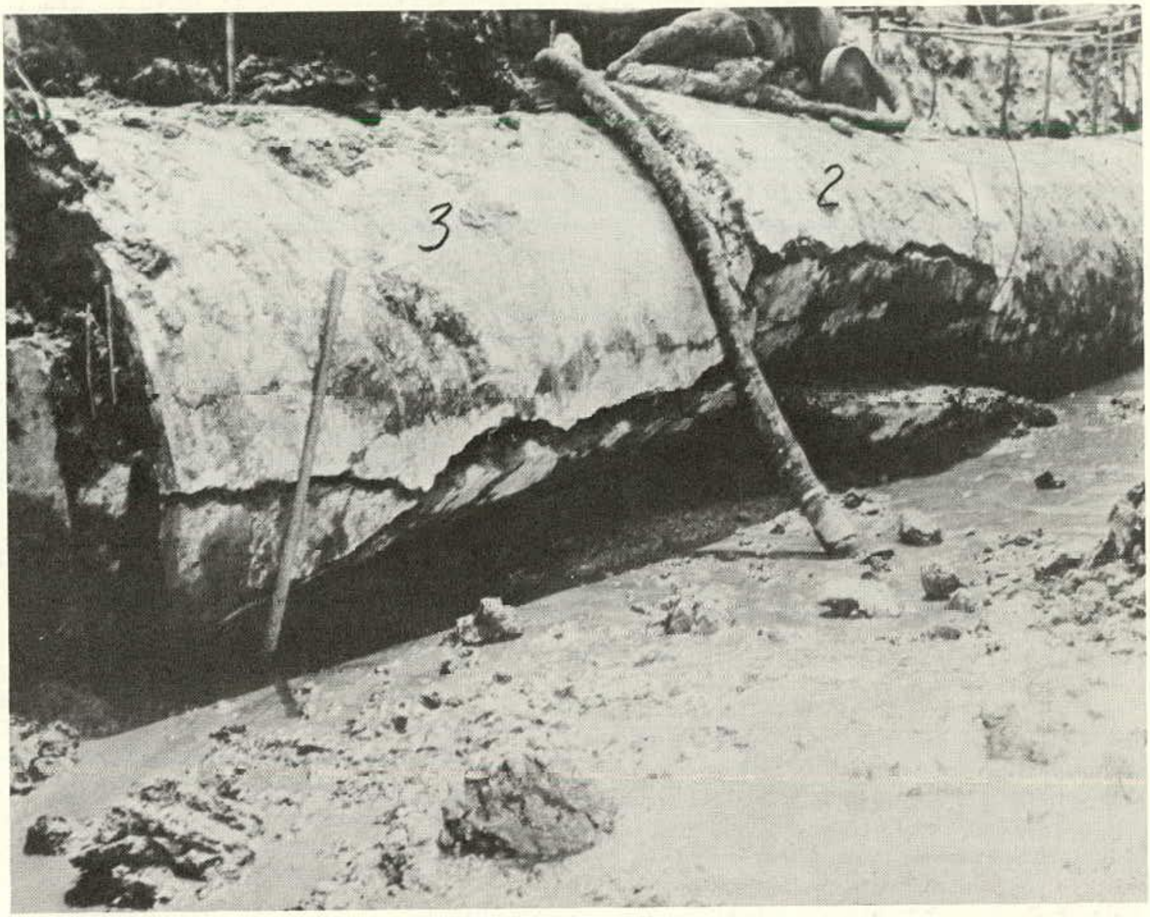

Figure J-10. Side view of pipe after failure (Case 9).

riser, which increased the footing pressure on the junction box.

3. This increased footing pressure and decreased bearing capacity of the foundation soil caused excessive settlement of the junction box.

4. Settlement of the junction box opened the joints at the bottom of the pipe between sections 1 and 2 on both the upstream and downstream sides of the manhole.

5. A differential head between water outside and inside the pipe caused water to flow through the opened joints with sufficient velocity to carry quantities of fine sand with it.

6. This action fed on itself; as more sand was carried into the pipe, greater settlements of the pipe and junction box and larger joint openings occurred, thereby letting in more water and sand.

7. As pipe sections 2 and 3 settled downward, the soil backfill overlying the pipes was supported temporarily by arch action.

8. A portion of this suspended soil suddenly gave way and crashed down onto sections 2 and 3, causing a tremendous dynamic load and causing the extensive damage shown in Figures J-9 and J-10.

9. The shock also shattered the upper portion of the concrete cradle under these sections and explosively forced fragments of it into the temporarily liquefied sand.

An alternative hypothesis is based on the fact that a mild earthquake occurred just eight days before discovery of the settlement of the manhole. The epicenter of this tremor was at a point about 40 miles downstream from the sewer. This earthquake was recorded at a seismograph station located about 100 miles on the opposite side of the sewer from the epicenter. The tremor had an intensity of III to IV on the Modified Mercalli scale. It is well within the realm of possibility that vibrations from this earthquake may have caused the foundation sand beneath the manhole and junction box to become quick and the structure to settle, thus initiating the events described in items 4 through 9 in the preceding paragraph.

CASE 10

An 18-in. vitrified clay pipe sewer constructed in an eastern seaboard city suffered extensive damage of an unusual nature, consisting of broken bells in addition to the more common longitudinal cracks and breaks. An investigation was made by excavating down to the pipe at a number of locations where breakage was most extensive; these areas were located by means of television cameras pulled through the pipelines.

Excavation was made well below the springline of the pipe; then, the bedding was probed with a steel rod alongside the pipes. It was discovered that ledge rock was present at an elevation only a few inches below the bottom of the pipes. In particular, some of the pipe bells had settled down very close to the rock, causing a high concentration of upward reaction on the bells. This was a very damaging situation, as shown in Figures J-11 and J-12. Recent unpublished studies by a private research agency have indicated that the load factor for a pipe that is supported primarily on the bells is in the neighborhood of 0.50 to 0.75. Load factor is defined as the ratio of the supporting strength of a pipe in the ground to its three-edge bearing strength. Thus, it is seen that the load-reaction system on this pipeline was very severe, and the need for good bedding, and especially the necessity for digging bell holes in the case of bell-and-spigot pipe, is emphasized.

CASE 11

Case 11 involves a 24-in. unreinforced bell-and-spigot concrete sewer pipeline in a midsouthern city. The line failed extensively and was reconstructed. Investigation revealed that the plans and specifications called for the width of ditch to be not greater than 45 in. However, the actual width of trench as constructed ranged from 54 to 60 in., and this, no doubt, seriously overloaded the pipe. An important feature of this investigation is that the consulting engineer who designed the project had an inspector constantly beside the ditch during construction and he himself visited the site almost daily. But neither the inspector nor the consulting engineer warned the contractor that his excavation was too wide.

The researcher did not see this pipeline before it was taken up and replaced. Study of the situation was made entirely by interviews with engineers, the contractor, and the contractor's foreman. There is strong evidence, from descriptions of the nature of the pipe failures and the somewhat haphazard excavation of bell holes, that these pipes developed a high concentration of reaction on the bells, somewhat similar to, but to a lesser degree than, those in Case 10. However, the evidence on this point is not conclusive.

CASE 12

A 120-in. reinforced concrete pipe storm sewer was constructed parallel to and just beyond the toe of the slope of an embankment of a highway in a midwestern city. The depth of cover over the pipe was 10 ft throughout most of the length, except that this depth was greatly exceeded at one end of the project where the grade of the parallel embankment ascended to provide an approach to a railroad crossing. The widened side slopes in this area increased the cover over the pipe to as much as 27 ft. Also, the investigation uncovered the fact that at the other end of the project there was a parallel line of 90-in. pipe within about 20 ft of the 120-in. line. This parallel line was constructed after the larger line was completed and backfilled. Because of limited right-of-way space, the spoil from the trench for the 90-in. line was piled temporarily over the 120-in. line. This temporary increase in height of fill was stated to be in the neighborhood of 15 ft.

The pipeline was visually examined by means of flashlights. There was about 18 in. of water in the line, so the condition of the invert had to be determined by feel. Numerous longitudinal cracks were observed in the crown, and much spalling of the protective cover over the steel was found in the invert. However, there was a distinct relationship between the depth of cover over the pipe, both where the increased depth was permanent and where it had been temporary. In those regions where the design height of fill was adhered to, the cracking was very light and well within acceptable tolerances.

Interviews with construction personnel indicated that the natural soil, which was a hard, stiff clay of glacial origin, was undercut about 4 to 6 in. and refilled with crushed slag to provide bedding for the pipe. This slag was not shaped to fit the pipe, but was screeded to a horizontal surface. The width of trench was held to 18 in. greater than the outside diameter of the pipe, leaving a space of only 9 in. on each side. After the pipe was set, crushed slag was placed and compacted up to the springline. However, it is doubtful whether the backfill in the triangular spaces below the lower haunches of the pipe could be effectively compacted because of the narrow side space. In spite of this inadequate bedding situation, the pipeline performed satisfactorily in those areas where the overfill did not exceed 10 ft, although probably with a reduced factor of safety. This may be accounted for by the probability that the strength of the pipes exceeded the minimum strength specification requirement, but this has not been definitely established.

CONCLUSIONS

The cases of pipeline failures just cited and other similar investigations lead the researcher to conclude that most structural difficulties experienced by structures of this kind arise from failure to provide a suitable bedding for the pipe. It is widely recognized in mechanics that a concentrated load applied to a simple beam causes a much greater bending moment than the same magnitude of load when it is distributed over a substantial length of the span. Exactly the same principle applies with reference to the bottom reaction on pipelines. If a pipe is installed on a flat bed of soil, the reaction on the pipe is highly concentrated along a narrow longitudinal element, and this causes a high bending moment in the pipe wall. Literally, if the full potential of the pipe to carry vertical load is to be realized, there is no substitute in pipe-laying practice for shaping the bedding to fit the contour of the pipe over a substantial width. Placing the pipe on a flat bed of soil does not provide for adequate lateral distributions of the bottom reaction, no matter how well the backfill may be compacted under and around the pipe, although, of course, such compaction is very desirable.

Closely allied with the necessity for shaping the bedding to fit the contour of a pipe is the need to provide bell holes

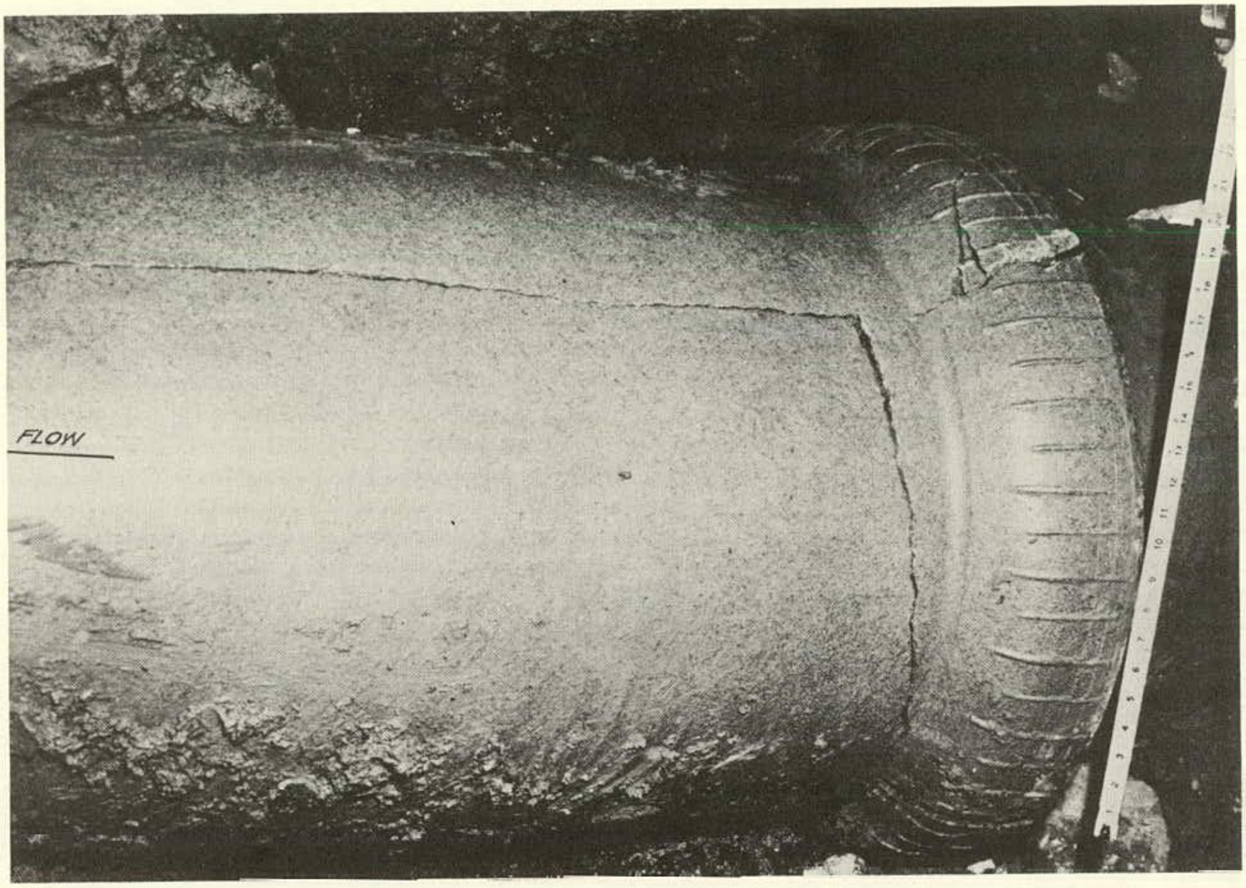

Figure J-11. Broken 18-in. clay pipe (Case 10).

Figure J-12. Broken 18-in. clay pipe (Case 10).

for bell-and-spigot pipe. The principal source of strength for this kind of pipe lies in the barrel, and serious damage can result if even a moderate amount of the total reaction is concentrated at the bells. The bell holes should be deep enough and wide enough to ensure that all the bottom reaction is carried by the pipe barrel.

Another circumstance that can cause a damaging concentration of the bottom reaction on a pipe is illustrated in Case 3, wherein the elevation of ledge rock was too close to the bottom of the pipe. The vertical strain in the supporting soil will always cause pipelines to settle under the influence of the earth load. If the pipe moves downward and approaches ledge rock or some other highly strain-resistant material, as in Case 7, the reaction forces will be concentrated and may result in severe damage.

Probably the most usual circumstance that results in an excessive load on a pipeline (that is, a load that exceeds the design load) is when the design width of a ditch is permitted to be exceeded in construction, as illustrated in Case 11. It is discouraging to note in this case that, although a maximum width of ditch was definitely specified, the contractor did not adhere to the specification and his violation went unheeded and unprotested by the inspector and the consulting engineer.

Cases 8 and 9 illustrate the age-old principle that "a structure is no better than the foundation on which it rests." The potentially dangerous character of the soil foundation materials at the site of these pipelines—a liquefiable muck in one case and potential quicksand in the other—illustrate the need for extensive soil exploration and study prior to the design of a pipeline. But even if such exploration is not available prior to design, all concerned should be alert during construction to identify unsuitable foundation soil conditions and either relocate the line to a better environment or remedy the unfavorable conditions encountered by removal and replacement with satisfactory material.

Cases 5 and 6 involving flexible metal pipelines illustrate the need for adequate stiffness of the pipe wall and the necessity for compaction of the soil at the sides of the pipe. The degree of compaction should be specified in specific numerical terms and not in indefinite expressions such as "well compacted" or "thoroughly compacted." Also, the lateral extent of the compaction should be specified. The researcher recommends that compaction be carried out for a minimum distance of two pipe diameters on each side of the pipeline. If shop struts are used to pre-deform flexible pipes, they should be removed when the height of fill is a few feet above the top of the pipe. Otherwise, they may inhibit deflection of the pipe and interfere with the normal outward movement of the sides against the enveloping soil, a process that is essential if the pipe is to develop strength from the passive resistance of the soil.

APPENDIX K

PRACTICES IN EUROPEAN COUNTRIES *

During the last two decades, researchers and practicing engineers in European countries have been directing more attention to different aspects of culvert design. The impetus for this increasing interest lies in the introduction of new materials, the complete change in traffic loads, the improvement of construction methods, the economic advantage of culverts as compared to bridges, and the large number of failures experienced. A number of European countries (including England, France, West and East Germany, the Soviet Union, and Poland) have started research work in this field and have developed various standards, specifications, and criteria for the design of culverts.

The purpose of this part of the study is to report the current state of the art and the prevalent trends in culvert design in these countries. Information has been collected from various publications, such as national standards, design codes, technical guidelines, standard projects, technical papers, and books. Although the principal emphasis is placed on present design practices abroad, certain research developments are also discussed, especially in connection with the theoretical aspects of culvert design and with the introduction of new materials.

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HISTORICAL DEVELOPMENT AND GENERAL TRENDS IN CULVERT DESIGN

According to the method of construction, culverts are often classified as (1) monolithic culverts or (2) culverts built of prefabricated elements (or so-called "plate culverts"). Plates of stone, and later reinforced concrete, were placed on two parallel vertical walls at a spacing of 1 m or less. The stream bed in the region of the culvert was covered with concrete, and the plates were covered with a layer of soil at least 0.4 m thick. The thickness, t, of the plates in meters was calculated from the relationships (*104*)

$$t = 0.10 + 0.20\,w, \text{ for } h < 1.5 \text{ m} \qquad \text{(K-1}a\text{)}$$

and

$$t = 0.12 + 0.24\,w, \text{ for } h \geqq 1.5 \text{ m} \qquad \text{(K-1}b\text{)}$$

in which w is the culvert span in meters. For larger spans or high embankments, arch culverts are preferred. Pipe culverts replaced plate culverts following the introduction of precast reinforced concrete pipes and centrifugal pipes; circular cross sections are most common for smaller installations. Figure K-1 shows typical cross sections of earlier plate culverts; Figure K-2 shows examples of more recent culverts built of concrete and reinforced concrete. Construction of cast-in-place pipe or framed structure culverts is usually limited to culverts with a width of 6.0 m or less and a height of 2.5 m or less. As far as protection against aggressive water is concerned, concrete culverts offered considerable improvement because it is much easier to apply a protective coating on the concrete surface.

With simplifications and standardization in construction, precast concrete, ceramic, steel, and plastic pipes were developed, and these gradually replaced the cast-in-place concrete pipes. The most common cross-sectional shapes are shown in Figure K-3. Attempts to drain larger and larger catchment areas with a single culvert led to very high design rates of flow which, in turn, required larger and larger culvert cross sections. The resulting precast sections were extremely heavy and required large equipment to hoist them into position; this situation led to the development of culvert sections constructed of prefabricated elements similar to those shown in Figure K-4. Prefabricated elements of corrugated steel, shown in Figure K-5, were subsequently introduced. At present, culverts constructed of prefabricated elements are common everywhere in Europe, and only in special situations are cast-in-place culverts used.

There is a distinct trend to construct single-pipe culverts, as contrasted to double- or triple-pipe culverts; the principal reason lies in the economy of construction. As Figure K-6 shows, the capacity of a culvert increases with diameter much more rapidly than does the cost (*105*). Thus, it is more economical to convey a given flow by a single, large-diameter culvert than to use two smaller-diameter culvert units. There are situations, however, when economy does not govern, and it is often necessary to construct a culvert of two or three small pipes because of technical reasons. For example, the construction of a single large-diameter culvert may not allow the required minimum thickness of soil between the crown of the culvert and the surface of the road.

SHAPES, DIMENSIONS, AND MATERIALS

In Germany, Italy, the Soviet Union, Czechoslovakia, Hungary, and many other countries the shapes and dimensions of culvert sections are specified in national standards. In Germany, for instance, the dimensions of circular, elliptic, egg-shaped, mouth, channel, and rectangular sections are given in DIN 4263 and DIN 19540. The circular cross section is most common; its hydraulic and structural characteristics offer many advantages, and the fabrication of circular pipes is simple compared to the fabrication of pipe sections of other shapes.

The egg-shaped cross section is rarely used in culvert

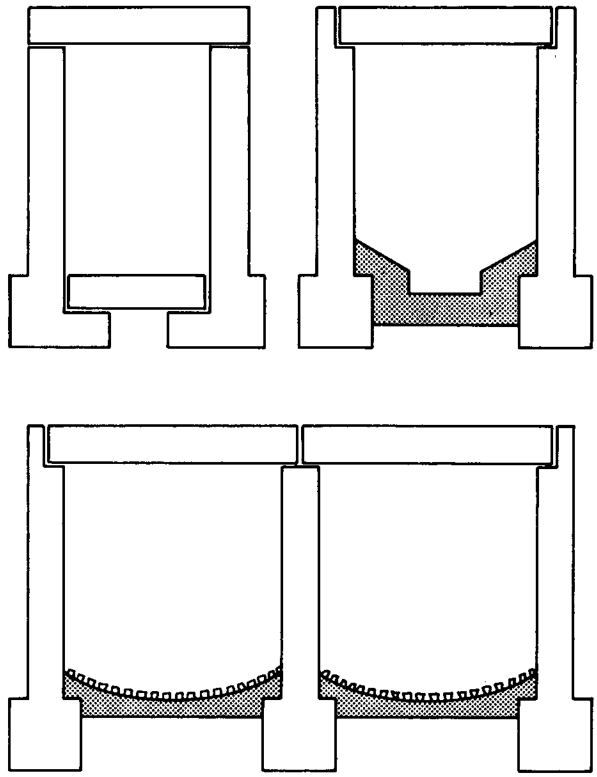


Figure K-1. Typical cross sections of box culverts.

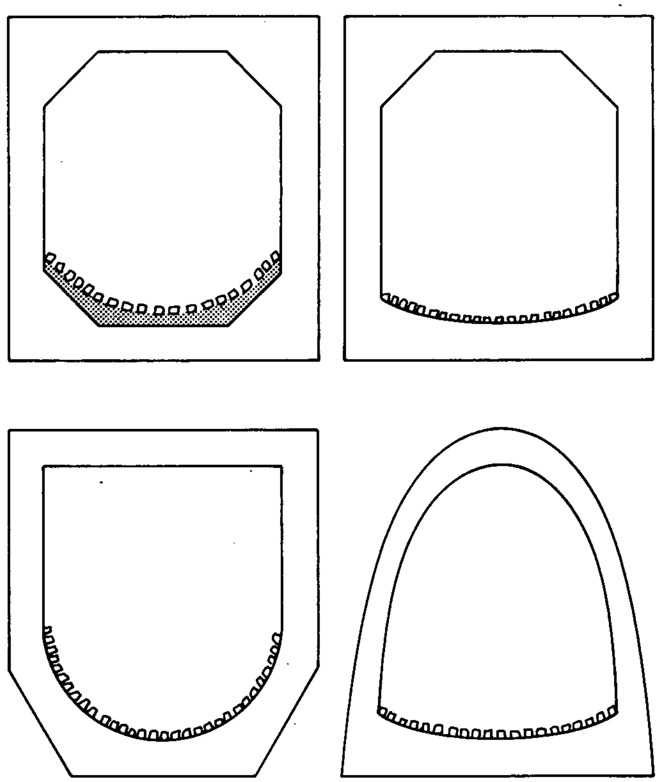


Figure K-2. Typical cross sections of monolithic concrete culverts.

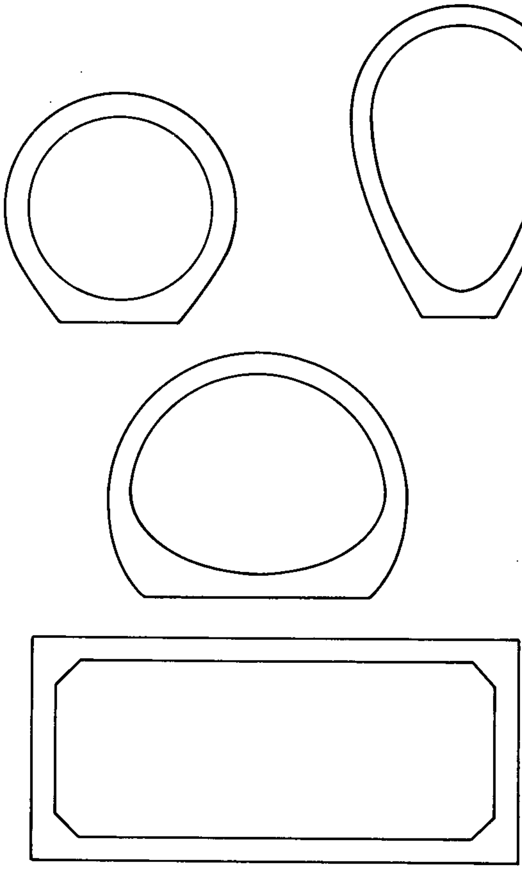


Figure K-3. Typical prefabricated concrete sections.

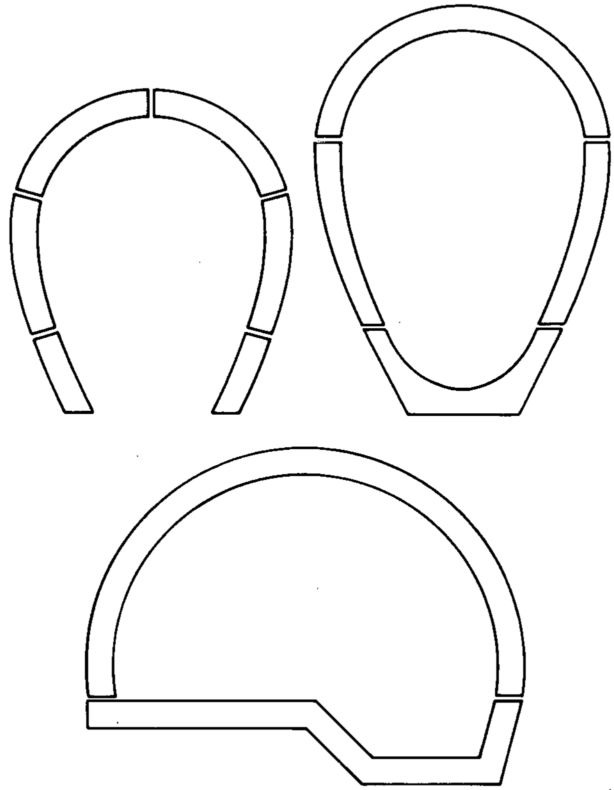


Figure K-4. Typical prefabricated concrete elements.

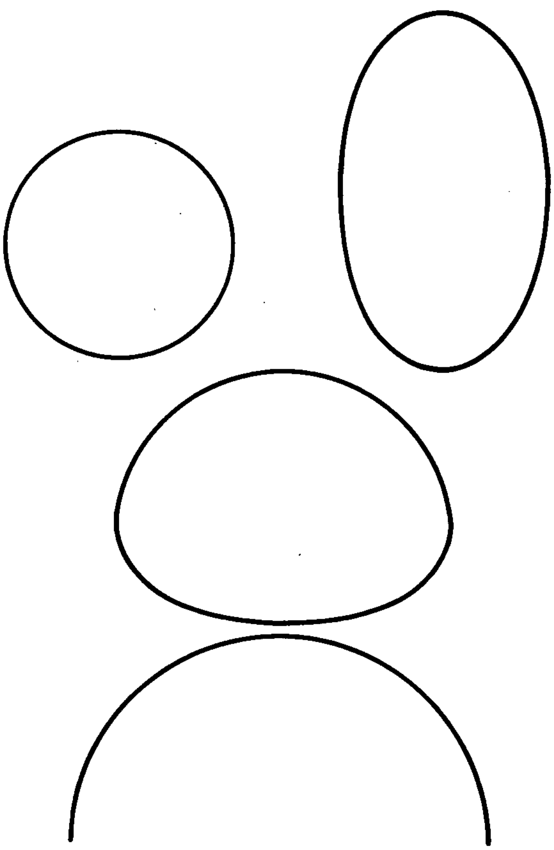


Figure K-5. Typical prefabricated steel sections.

construction; its advantages—adaptability to varying flow rates and relatively high structural strength without reinforcement—are not sufficiently important to offset the difficulties encountered in manufacture and construction. The same is true of channel cross sections. Elliptic and mouth cross sections are common for corrugated steel cul-

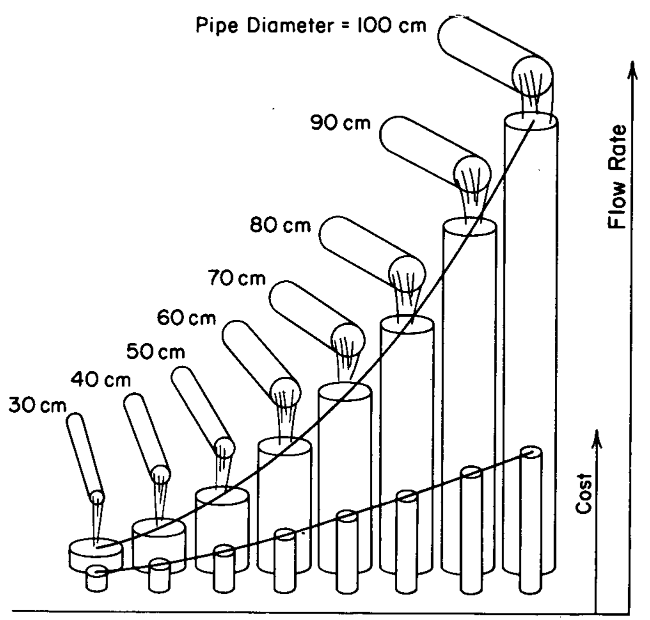


Figure K-6. Culvert capacity and cost as a function of diameter.

verts, but they are not used with other materials. Rectangular cross sections are common when the required size of the culvert is greater than the maximum size of precast circular pipe sections. Precast pipes of rectangular cross section are not common because they are less economical than precast pipes of circular cross section. As an intermediate solution, precast rectangular blocks with a circular cavity are often used in the Soviet Union (*106*). The dimensions of various precast culvert elements used in the Soviet Union are given in Table K-1.

The different materials used for culverts are as follows:

1. Ceramic pipes, except for wooden ones, are probably the oldest pipes used by man. The basic raw materials are clays, consisting of rock crystal (SiO_2) and bauxite ($A_2 0_3$) with a lime content of not more than 1 percent. After drying, the mixture is burnt and the result is a lime-free ceramic material that has a solid, corrosion-free, and impermeable texture. The hardness is characterized by a Mohs number of 8 to 9, which is higher than that of natural sediments (the hardness of quartz sand is 7 to 8); hence, ceramic pipes are highly abrasion-resistant. However, because ceramic pipes are very expensive, they are seldom used in culvert construction. The diameter of ceramic pipes is specified in national standards; for instance, the standard diameters and the permissible concentrated loads on ceramic pipes produced in Germany in lengths of 1.50 m or less are given in DIN 1230. Typical examples are given in Table K-2.

2. Concrete is the most common material used in culvert construction. Different methods (based on the principles of compaction, vibration, pressing, rotation, vacuum, and their combination) have been developed to produce solid precast concrete pipe sections. Probably centrifugal pipes yield the best results and they are, therefore, used extensively in many countries, including Germany, France, Denmark, and Holland.

   a. Concrete pipes without reinforcement are produced according to various national standards (DIN 4032 in Germany, BS 556 in Britain, GOST 6483-53 in the Soviet Union, etc.). Precast pipes of circular cross section are used almost exclusively in culvert construction except in the Soviet Union, where rectangular cross sections are also common. In several countries—Germany, France, Italy, and England—a special type of ceramic-concrete pipe (BK pipe) is produced; this is a normal ceramic pipe strengthened by a concrete covering.

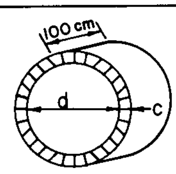
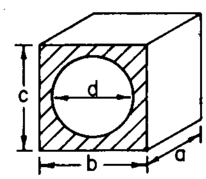
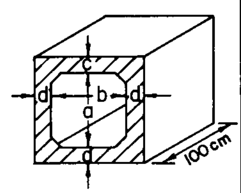
   b. Reinforced concrete and prestressed concrete pipes are most common in European culvert construction. Their quality requirements are specified in national standards or technical guidelines. The reinforcement consists of longitudinal bars with a diameter greater than 6 mm and spiral reinforcement. If the wall thickness is less than 70 mm, single spirals are used; otherwise, double spirals are employed. Diameters vary between 50 and 240 cm. The length of the pipe section is usually less than 5 m, although centrifugal pipes are often manufactured in lengths of 8 m. The quality of the concrete must be 300 or higher (B280 in Hungary); that is, the 28-day minimum working stress must not be less than 300 kg/cm^2 or 4,200 lb/in^2. The corresponding classes of concrete, according to British standards (*107*), are Z¾ and A¾. The transverse or radial reinforcement is calculated according to the second stress condition (cracks may develop in the tensile zone),* and the permissible stresses are chosen accordingly. As is discussed later, the Soviet design code differs from this generally accepted principle.

   c. Reinforced concrete and prestressed concrete pressure pipes are similar to the pipes discussed previously, except that they are reinforced for internal pressures. Because internal pressure does not have to be considered in highway culvert construction, pressure pipes are seldom used for culverts.

   d. Reinforced concrete pipes with steel linings are completely watertight and offer considerable resistance to the abrasive effects of bed-load transported by the stream. Because no special requirement exists on watertightness in culvert construction, and because the steel lining increases the cost considerably, the use of such pipes for culverts is rare; however, a few

TABLE K-1

DIMENSIONS OF PRECAST CONCRETE CULVERT ELEMENTS USED IN THE SOVIET UNION

Element	Dimensions (cm) a	b	c	d	Volume (cubic meters)	Weight (metric tons)
	–	–	10	100	0.35	0.9
	–	–	14	150	0.72	1.8
	–	–	16	150	0.85	2.1
	–	–	16	200	1.10	2.7
	–	–	20	200	1.40	3.5
	–	–	18	250	1.50	3.75
	–	–	22	250	1.88	4.70
	99	136	132	100	0.99	2.5
	50	198	182	150	0.95	2.4
	50	256	240	200	1.50	3.75
	50	310	298	250	2.20	5.5
	100	125	10	11	0.56	1.4
	100	125	10	14	0.63	1.6
	100	160	10	11	0.63	1.6
	125	150	10	13	0.72	1.8
	125	150	11	17	0.88	2.2
	125	190	10	13	0.80	2.0
	150	200	11	16	1.07	2.7
	150	200	13	20	1.31	3.3
	150	270	13	16	1.34	3.4
	200	250	13	17	1.55	3.9
	200	330	16	17	1.97	5.0
	200	250	17	23	2.05	5.2

* The analysis of reinforced concrete sections in most European countries is performed in accordance with three different stress conditions, as follows: (1) first condition—uncracked section, (2) second condition—cracked section, and (3) third condition—post-yielding (ultimate strength).

TABLE K-2

PERMISSIBLE CONCENTRATED LOADS ON CERAMIC PIPES

PIPE DIAMETER (CM)	PERMISSIBLE CONCENTRATED LINE LOAD (KG/M)	
	NORMAL	STRENGTHENED
30	2,400	4,000
40	2,600	5,000
50	3,000	5,000
60	3,000	5,000
70	3,000	6,000
80	3,000	6,000
90	3,000	6,000
100	3,000	6,000
120	3,000	6,000

highway culverts in Germany are built of this type of pipe.

e. Cast-in-place concrete culverts are usually built only if local conditions do not allow the construction of precast culverts. Because the structure is usually monolithic, it is more sensitive to settlement than culverts built of precast concrete pipes. Different cross-sectional shapes can be formed by use of slip forms. A special type of formwork, that was developed in Holland, consists of an inflated rubber hose as the internal form; rubber hoses are available in diameters 3 m and less. The economy of culverts built by this method is comparable to that of a precast concrete pipe culvert. Nevertheless, regardless of the new types of forms that help to shorten the time of construction, culverts of precast pipe sections are preferred to cast-in-place culverts. The requirement for the quality of concrete to be used in cast-in-place culverts is not as high as for precast pipes; usually B200 (200 kg/cm^2) concrete is acceptable.

3. Steel pipes are highly elastic, a quality that makes them adjustable to nonuniform conditions, and they can resist large loads; however, they are very sensitive to corrosion, and therefore are not usually used in culvert construction unless protective coatings are applied. Although steel culverts are seldom used in the eastern European countries because of the serious shortage of steel, they are often used in many western countries. Corrugated steel plates are bent to the required shape and a protective coating is applied to resist corrosion. The pipe cross section is usually circular or elliptic with a maximum diameter of 6.5 m, or frogmouth-shaped with a maximum span of 8 m. The thickness of the corrugated steel plates varies from 1.6 to 7.0 mm, depending on the loading conditions, and the plates are manufactured in different lengths (from 9.93 to 2.54 m) and widths (from 0.85 to 1.83 m). Two different types of corrugation are used; Type I has a pitch of 68 mm and a depth of 13 mm, and Type II has a pitch of 150 mm with a depth of 50 mm.

It might be of interest to mention that one of the earliest applications of corrugated steel plates for culvert construction was in Russia in 1875 and, before World War I, more than 5,000 culverts were built of corrugated steel. In 1915, however, serious failures occurred along the Orenburg-Tashkent railroad and authorities decided to abandon completely the use of corrugated steel in culvert construction. Since then corrugated steel plates have been used to repair old brick or stone culverts, but no new corrugated steel culverts have been built in Russia.

4. Plastic pipes manufactured in Europe consist mostly of polyvinyl-chloride (PVC) or polyethylene (PE). Because of the limited strength and the relatively high cost of plastic pipes, their use as culverts is not promising in the immediate future. In Germany, however, a new kind of plastic pipe, the so-called "Wickelrohr," will soon be available in diameters up to 1.6 m (*104*), and there is hope that this plastic pipe will be economically and structurally competitive with currently used concrete pipes.

In the Soviet Union, attempts have been made to introduce plastic materials into culvert construction as "plastic concrete" (*108*), which is a mixture of sand (80 to 90 percent), furfurol-acetone monomer (6 to 20 percent), hardening agent (benzo-sulfuric acid, 2.5 to 3.5 percent), and furfurol (0.2 to 1.5 percent). Experimental culverts, as shown in Figure K-7, with circular cross sections 1 m in diameter were built, and simultaneous laboratory and field experiments were conducted to compare the plastic concrete culvert with an unreinforced concrete culvert. According to the results of these experiments, the plastic concrete culverts were superior in every respect to the concrete culverts. Because the flexibility of a plastic concrete pipe is much higher than that of a concrete pipe, it adjusts more readily to nonuniform soil conditions. The only disadvantage of the plastic concrete appeared to be its sensitivity to water. If the mixture is allowed to absorb water, the hardening process is retarded considerably, or the plastic concrete does not harden at all; this, of course, seriously limits its use as a culvert material until this difficulty can be eliminated. The application of similar plastic material is being tested in West Germany (*104*), but detailed data are not presently available.

STRUCTURAL DESIGN OF CULVERTS

Determination of Loads

Culvert design in most European countries is based on the Marston-Spangler theory. In Germany the earth pressure on a culvert is calculated according to the standard DIN 4033, which specifies the three classes of conduits shown in Figure K-8. For an embankment condition,

$$P = K_e \gamma H B_c \qquad \text{(K-2)}$$

and for a ditch or semi-ditch condition,

$$P = K_d \gamma H B_d \qquad \text{(K-3)}$$

in which P is the total dead load per unit length; and K_e and K_d are earth pressure coefficients to be determined from charts. The value of K_e depends on a factor termed the settlement ratio, r_{sd}, which characterizes the type of bedding and the type of culvert. If a rigid culvert is supported on rock or very hard soil, the value for r_{sd} is 1; for ordinary

Note: All dimensions are in centimeters

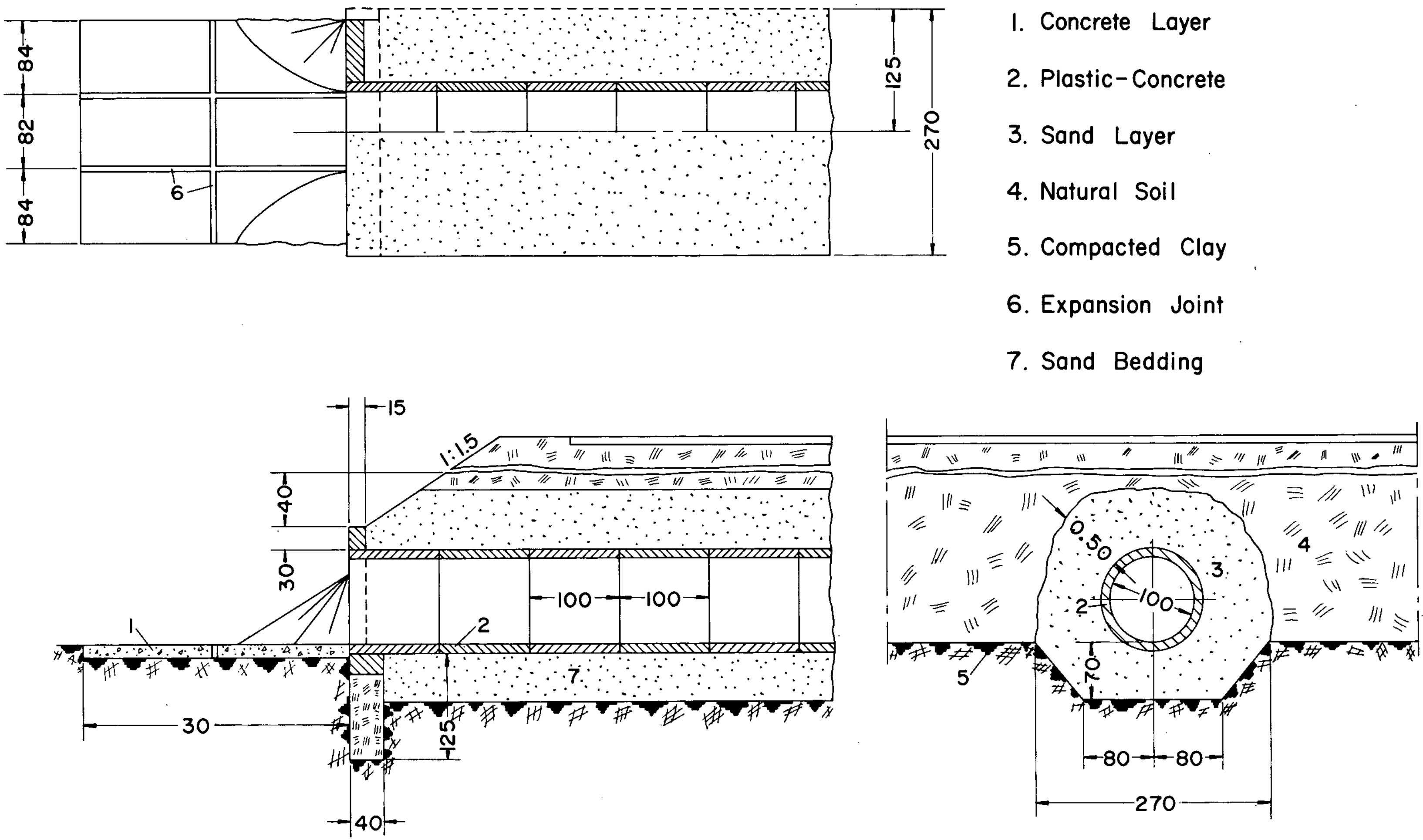


Figure K-7. Experimental plastic concrete culvert.

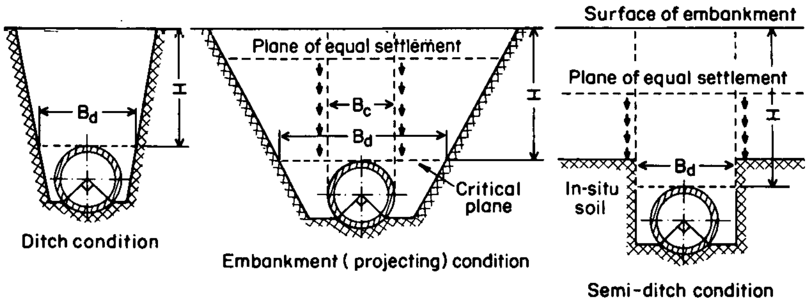


Figure K-8. Classes of conduits according to DIN 4033.

soil bedding, r_{sd} ranges from 0.8 to 0.5, whereas its value is between 0.5 and 0 for a foundation that yields with respect to the adjacent natural ground. Horizontal earth pressure, if considered at all, is calculated by the Rankine theory; however, it is usually neglected. In addition to the earth loads described previously, live loads are determined according to DIN 1072 by use of the chart shown in Figure K-9.

Recently, Wetzorke (*109*) carried out a series of experiments in connection with the fracture resistance of ditch conduits, and made several recommendations regarding the method of load determination; his recommended method is, to some extent, different from that discussed previously. The vertical earth load is calculated by

$$P = K_d' \gamma H B_d \qquad \text{(K-4)}$$

in which K_d' is taken from either Figure K-10 or Figure K-11. Although Wetzorke recommends that an uncompacted backfill be assumed in the calculations and that Figure K-10 be used, the coefficient K_d' may be obtained from Figure K-11 if thorough compaction is provided.

Because in most cases the pipe is more rigid than the surrounding soil, a load concentration occurs above the crown of the pipe, and this load concentration is inversely proportional to the degree of compaction achieved for the soil between the wall of the trench and the pipe. If the soil in this zone is not compacted, the total load calculated by Eq. K-4 will act on the pipe; but, if the soil is well compacted and the pipe is rigid, the design load, P', can be reduced according to the relation

$$P' = P \frac{d + B_d}{2B_d} \qquad \text{(K-5)}$$

in which d is the pipe diameter; and B_d is the width of the trench at the top of the culvert. Should the culvert be flexible, a larger allowable load reduction is given by

$$P' = P \frac{d}{B_d} \qquad \text{(K-6)}$$

The distinction between rigid and flexible pipes is determined by Klein's criterion (*110*):

$$\frac{E}{E_s}\left(\frac{t}{r_m}\right)^3 = k \qquad \text{(K-7)}$$

in which E and E_s are the moduli of elasticity of the pipe material and the soil, respectively; t is the wall thickness of the pipe; r_m is the average radius of the pipe; and k is a dimensionless number. If k is less than 1, the pipe is termed as flexible. Steel, plastic, aluminum, and asbestos-cement pipes are classified as flexible according to this criterion, whereas concrete and reinforced concrete pipes are rigid.

As proposed by Wetzorke (*109*), the live load, Q, is calculated from

$$Q = I_f q d \qquad \text{(K-8)}$$

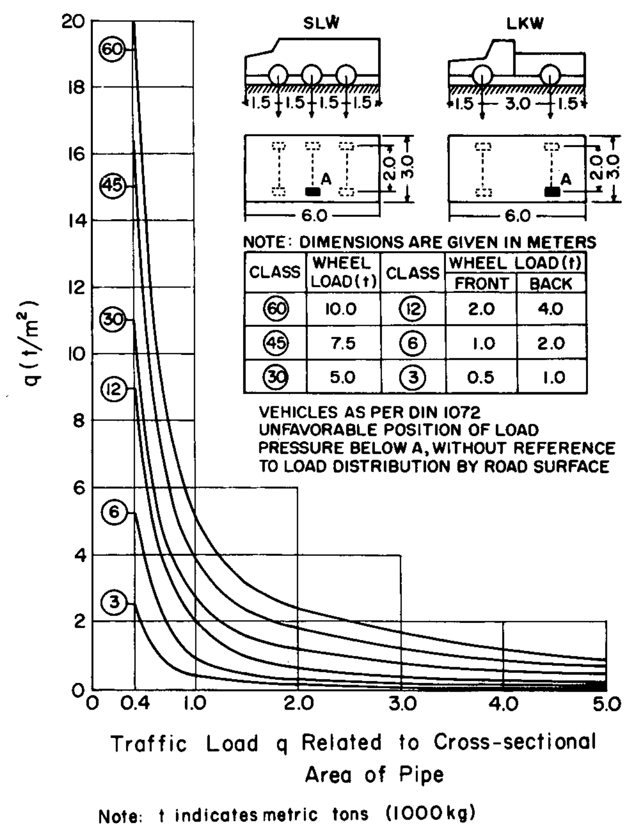
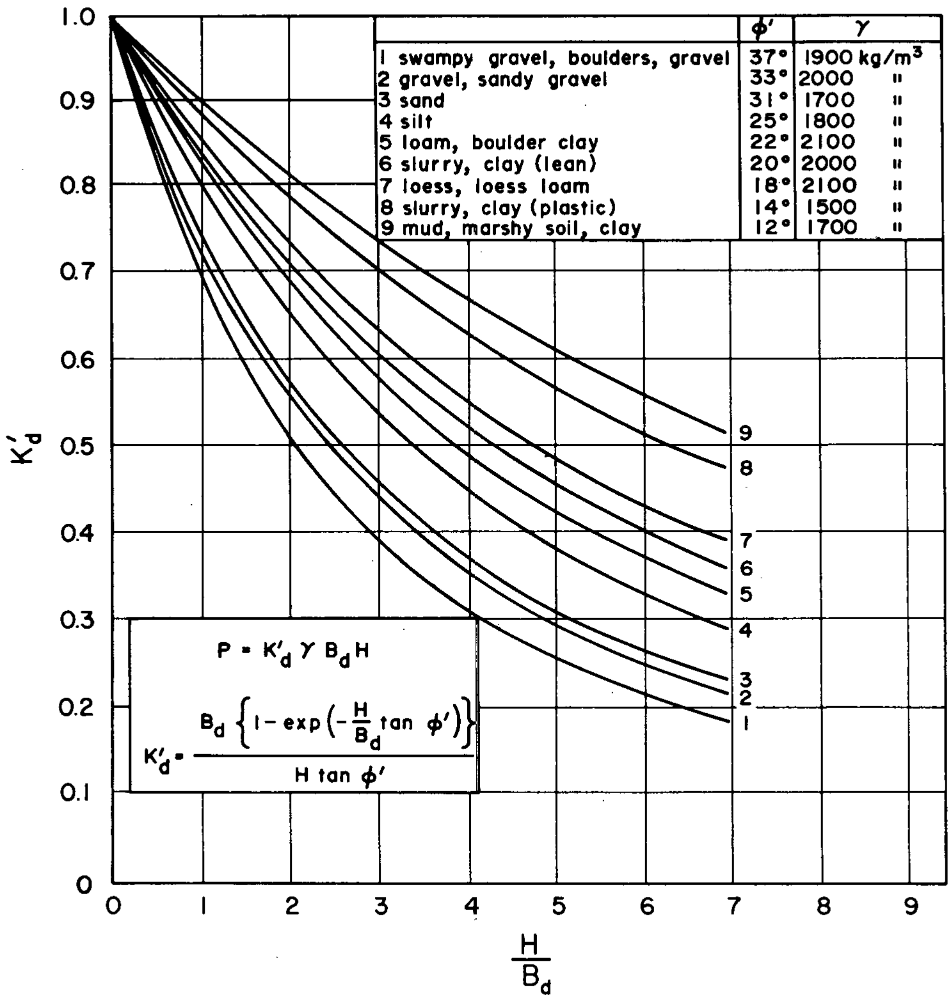


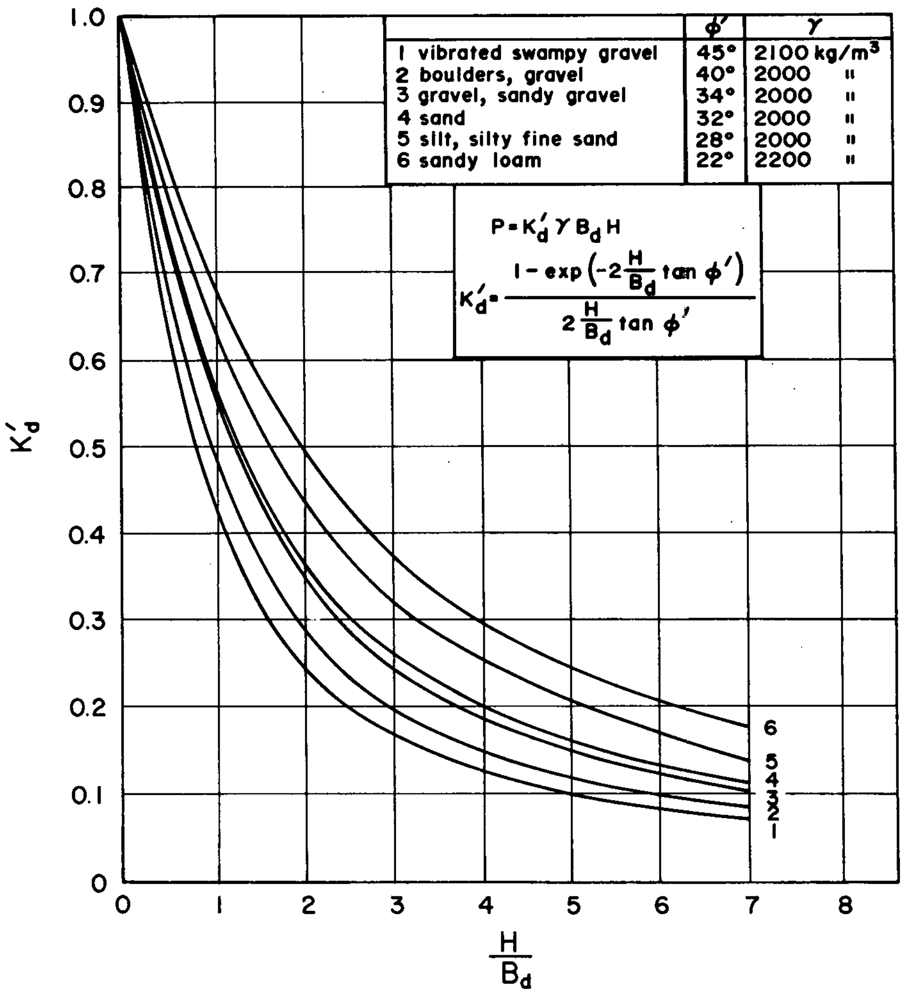
Figure K-9. Loads on pipes due to traffic.



B = Trench Width H = Depth of Cover Above Crown

If the fill differs from the original soil less favorable friction values shall be chosen

Figure K-10. Soil loads on pipes in trenches without compacted backfill.



B_d = Trench Width H = Depth of Cover Above Crown

If the fill differs from the original soil less favorable friction values shall be chosen

Figure K-11. Soil loads on pipes in trenches with compacted backfill.

TABLE K-3

VALUES OF IMPACT FACTOR FOR DIFFERENT PAVEMENT TYPES

TYPE OF PAVEMENT	IMPACT FACTOR FLEXIBLE PIPE	RIGID PIPE
Concrete or asphalt	1.2	1.5
Pebble	1.4	1.7
Gravel or stone	1.7	1.7
Pavement in poor condition	2.2	2.5

in which q is the unit live load taken from Figure K-9; and I_f is a concentration or impact factor to be determined from Table K-3. If the trench width is taken according to DIN 18306 (internal trench width equal to d plus 0.7 m), the earth loads for concrete pipes can be taken directly from Figures K-12 to K-20. Figures K-21 to K-24 show the sum of the soil and traffic loads for most normal situations.

The required minimum load-bearing capacity (crushing load), $P_{\min}$, of the pipe is calculated from the relationship

$$P_{\min} = \frac{F_e P' + F_w Q}{K} \tag{K-9}$$

in which F_e and F_w are safety factors for the soil and traffic loads, respectively; and K is a bedding factor to be taken from Figure K-25. The safety factor for the earth pressure, F_e, is 1.5 for favorable bedding conditions, but F_e is taken equal to 1.8 if the bedding conditions are unfavorable or groundwater is present. For hard road surfaces with a suitable foundation, F_w is taken equal to 1.5; for light road surfaces without any great load distributing effect, F_w equals 2.0 for H less than 1 m and 1.5 for H equal to or greater than 1 m. Different safety factors are applied in special cases.

A somewhat different approach is taken in the Soviet Union, and the standard soil pressures (dead load) acting on the pipe are calculated by

$$p_v = C_v \gamma_s H \tag{K-10a}$$

and

$$p_h = C_h \gamma_s H \tag{K-10b}$$

in which C_v and C_h are load coefficients; H is the height of the embankment measured from the crown of the culvert to the top of the pavement; and γ_s is the standard value for the unit weight of soil. The value of the coefficient C_v is determined from

$$C_v = 1 + A_c \tan^2\left(45° - \frac{\phi_s}{2}\right) \tan \phi_s \tag{K-11a}$$

in which

$$A_c = \frac{mh}{H}\left(2 - \frac{mB_c h}{H^2}\right) \tag{K-11b}$$

in which h is the distance between the foundation plane and the crown of the culvert; m is a coefficient to be determined from Table K-4 according to the type of soil; and ϕ_s is the standard value for the angle of internal friction of the soil. If mh/H is equal to or greater than H/B_c, A equals H/B_c. Values for ϕ_s and γ_s are determined by tests if the culvert is individually designed, but if standard drawings are adopted, values of 35° and 1,800 kg/m³, respectively, are chosen. If the culvert is built on a highway where high-quality compaction is provided with a degree of compaction equal or greater than 95 percent of Standard Proctor, the calculated value of C_v may be decreased by 30 percent. In this case, however, the degree of compaction must be checked in a soils laboratory located at the site of the construction and the results must be documented.

The temporary (live) load is determined as follows. The vertical pressure in metric tons per square meter is given either by

$$q_v = \frac{19}{H + 3} \tag{K-12}$$

if H is greater than 1 m, where H is expressed in meters, or by considering a uniform vertical pressure distribution resulting from a wedge over the culvert making angles of 30° with the vertical; the horizontal pressure may be calculated from the relation

$$q_h = q_v \tan^2\left(45° - \frac{\phi_s}{2}\right) \tag{K-13}$$

but horizontal earth pressures are neglected according to the Russian design code; as previously mentioned, steel culverts generally are not used.

The dead load calculated by Eq. K-10a must be multiplied by a safety factor, F, usually chosen as 1.2. Another safety factor is introduced in the calculations by using a value of ϕ given by $\phi = \phi_s \pm 5°$. Depending on whether the greater or smaller value of ϕ corresponds to the more dangerous situation, the 5° is added to or subtracted from ϕ_s. As far as the live load is concerned, no special safety factor is used, but the value of angle of internal friction is decreased by 5°.

According to an interesting method developed by Yaroshenko (*111*), underground conduits should be regarded as structures working together with the surrounding mass of soil, and each element of these conduits should be designed in such a way that the forces acting on the structure are minimized. To achieve this, the sections should be flexible, the headwalls should be free from the pressures of the embankment, the bedding should be flexible rather than rigid, and the deformations and joint separations should remain within tolerable limits. As a result of the earth

TABLE K-4

COEFFICIENT DEPENDING ON SOIL TYPE

TYPE OF SOIL	m
Very hard soil	15
Compacted soil (dense and medium dense sand, sandy clay, and hard clay)	10
Loose soil (loose sand, and clay with low plasticity)	5

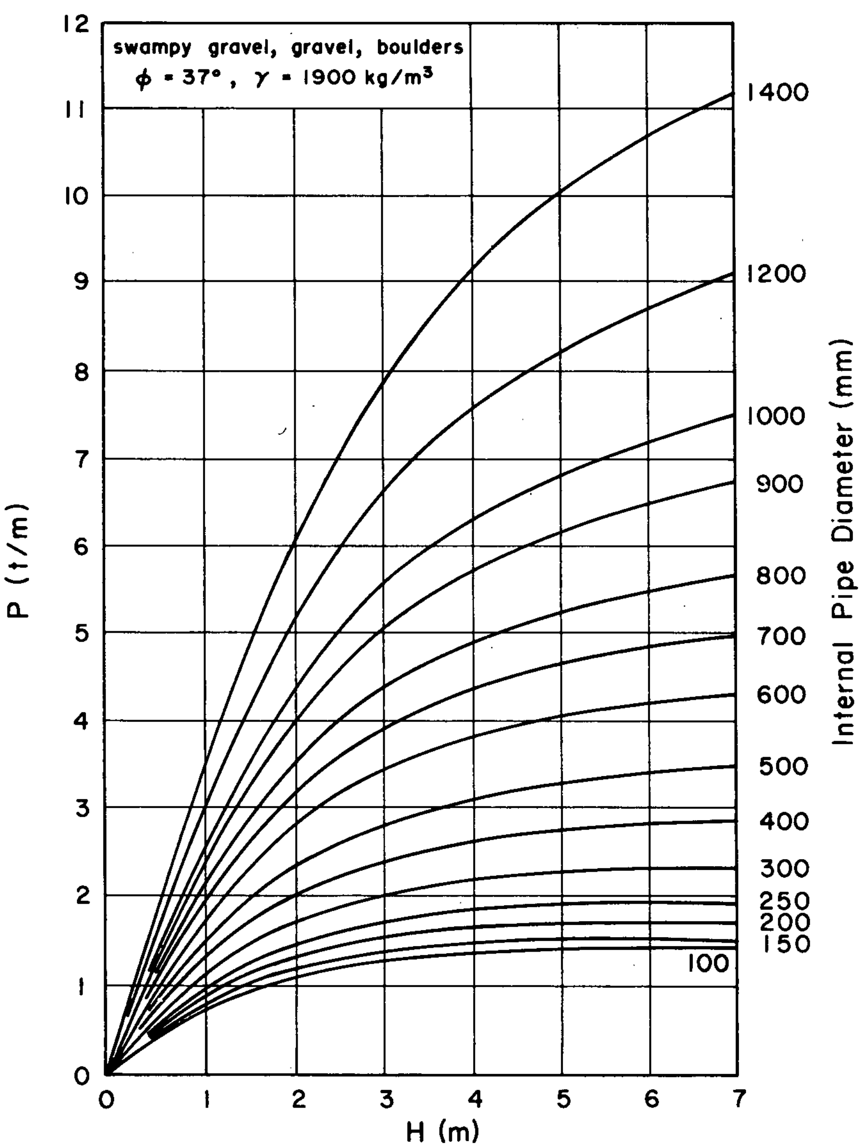


Figure K-12. Soil loads on concrete pipes in gravel soils.

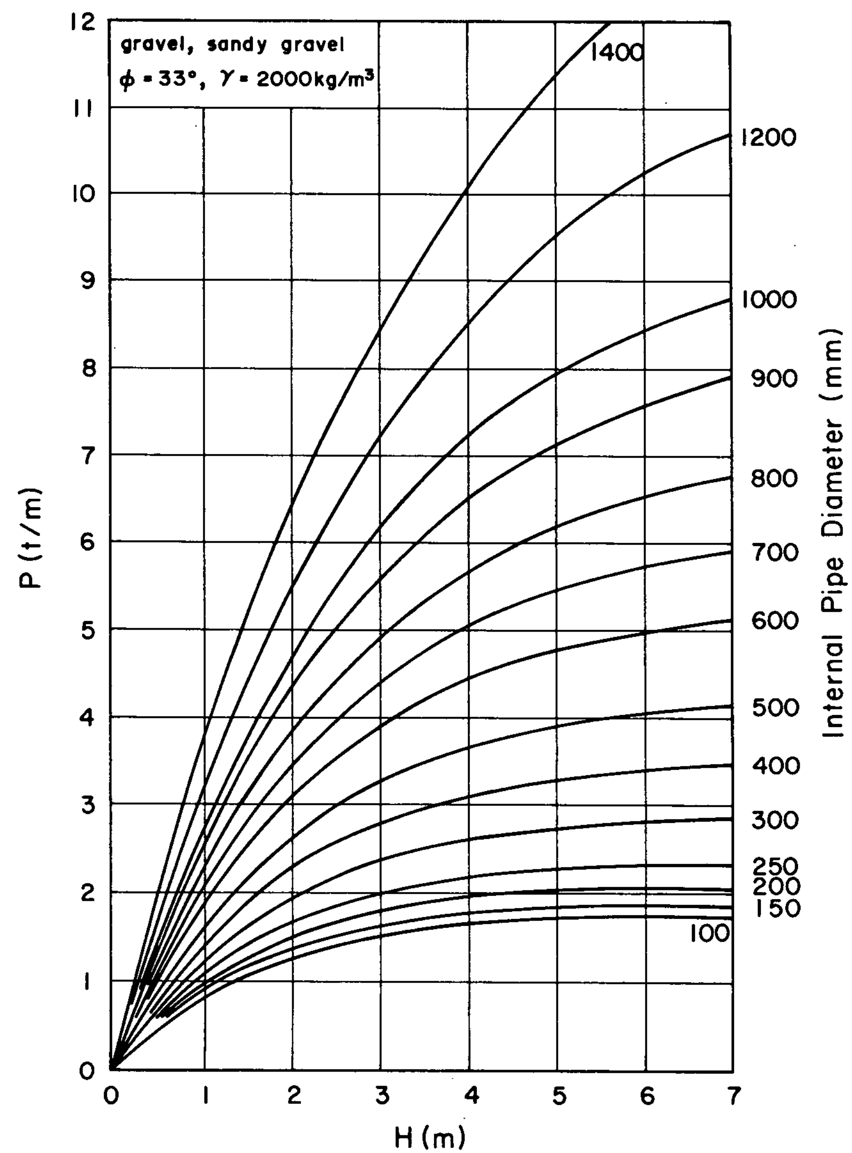


Figure K-13. Soil loads on concrete pipes in sandy gravel soils.

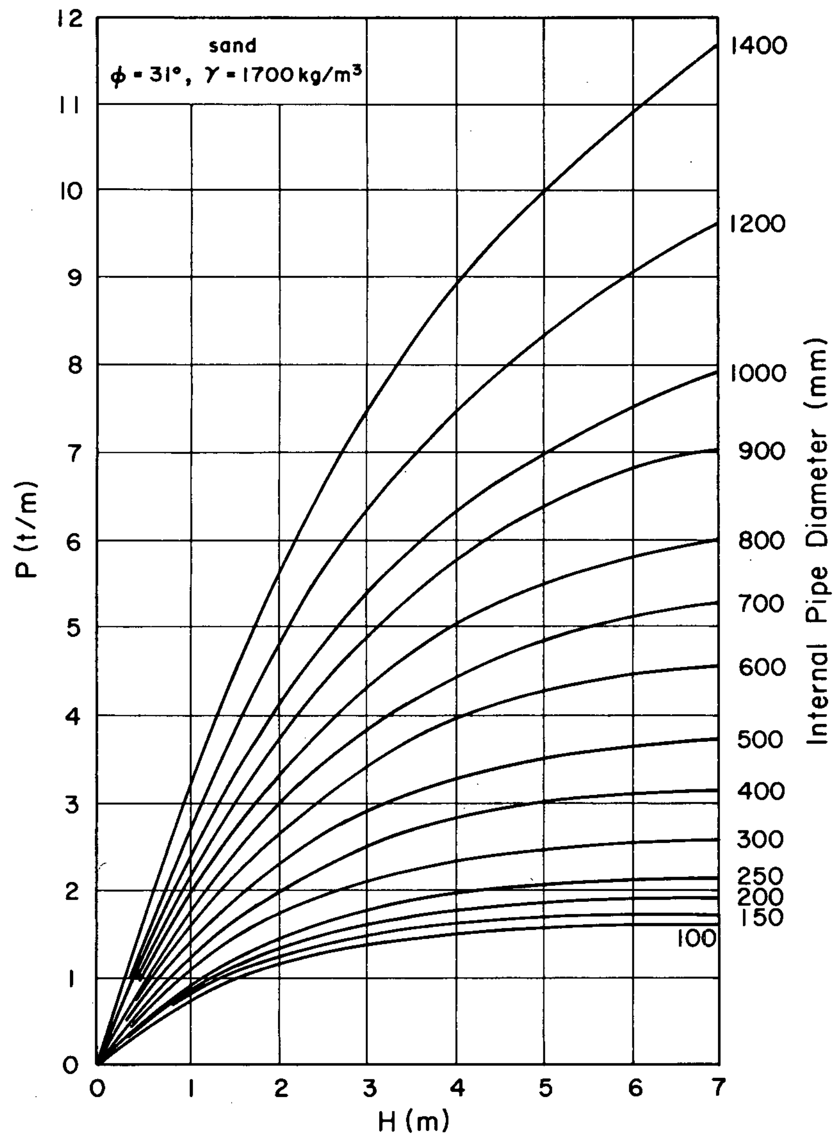


Figure K-14. Soil loads on concrete pipes in sand.

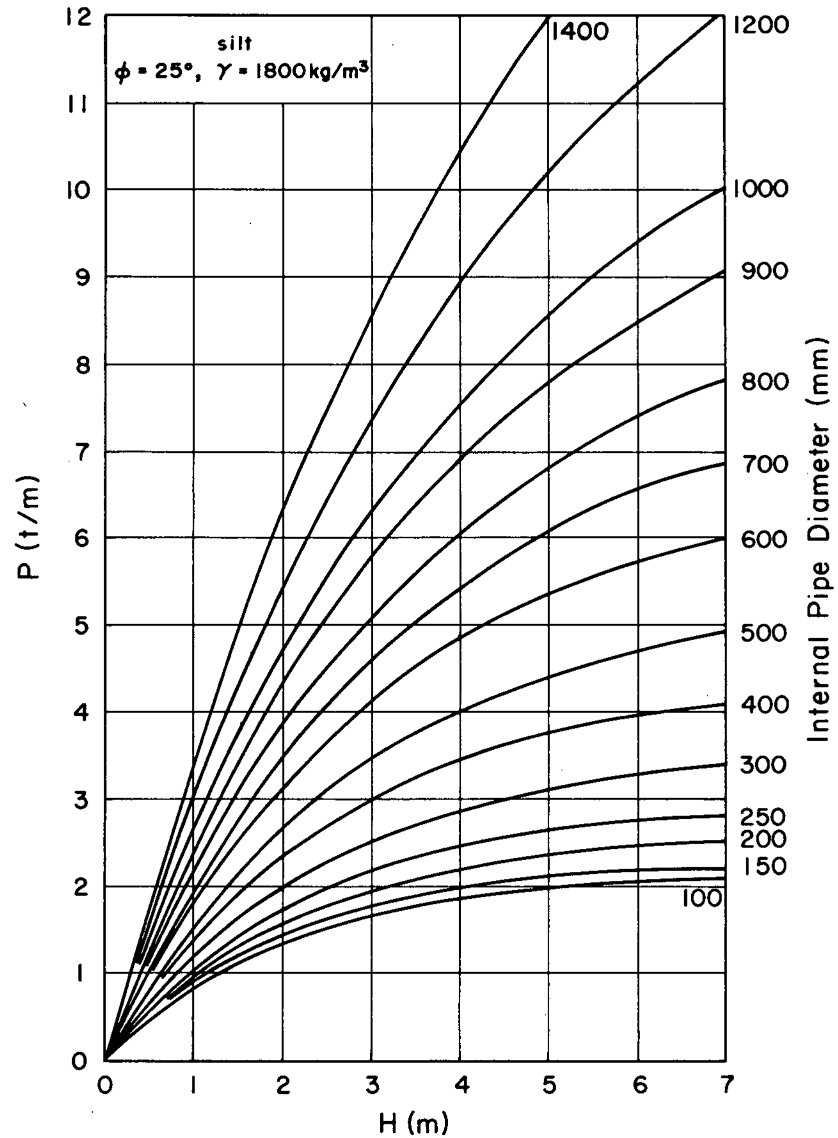


Figure K-15. Soil loads on concrete pipes in silt.

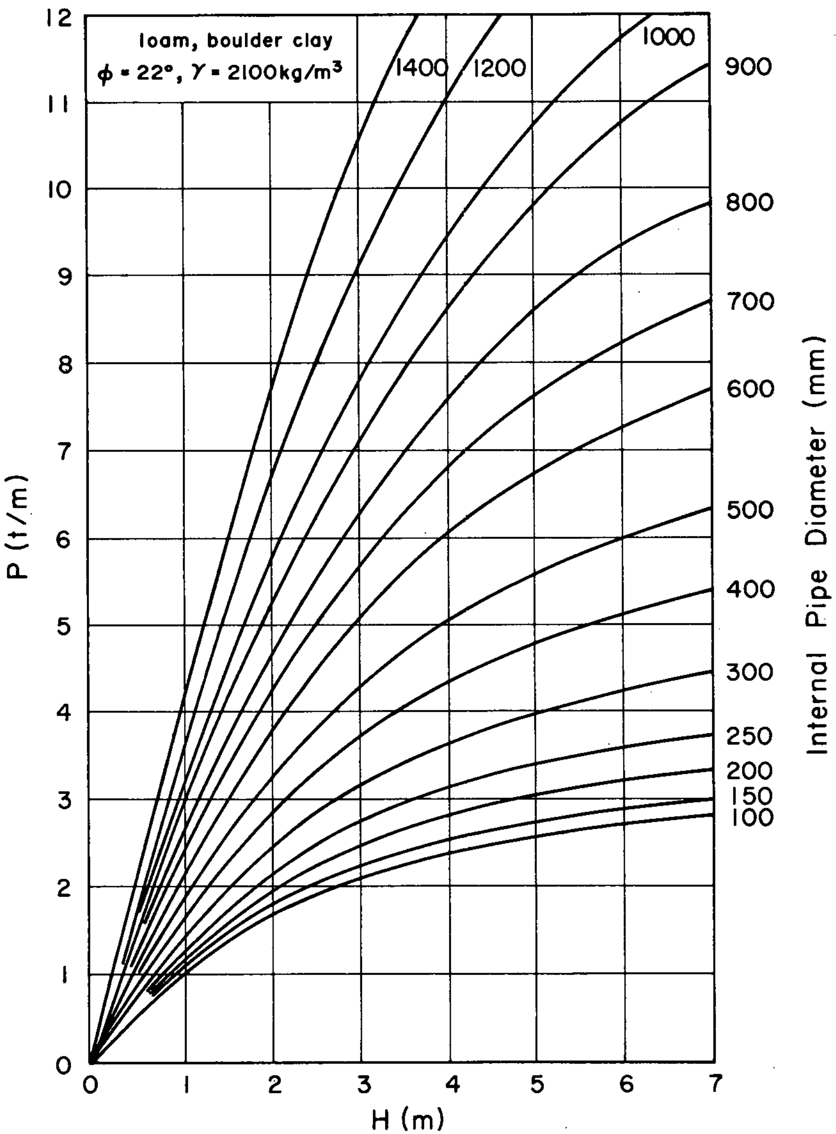


Figure K-16. Soil loads on concrete pipes in loamy clays.

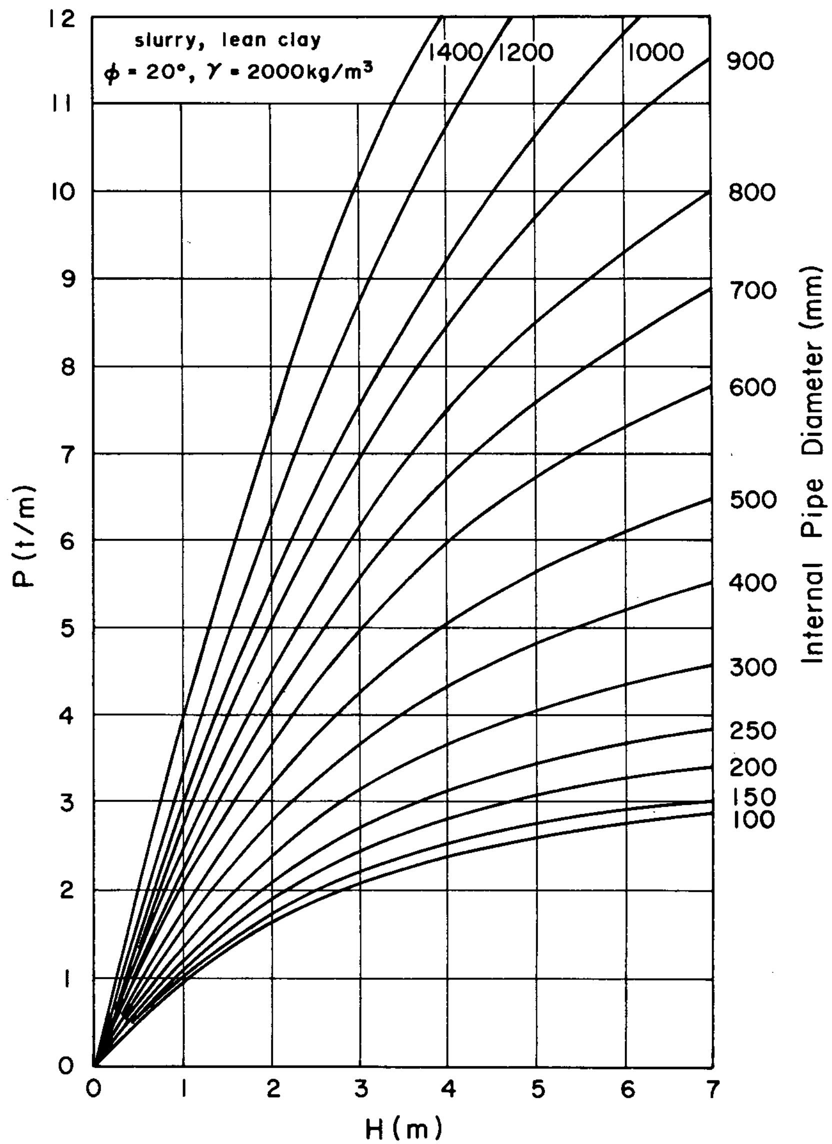


Figure K-17. Soil loads on concrete pipes in lean clays.

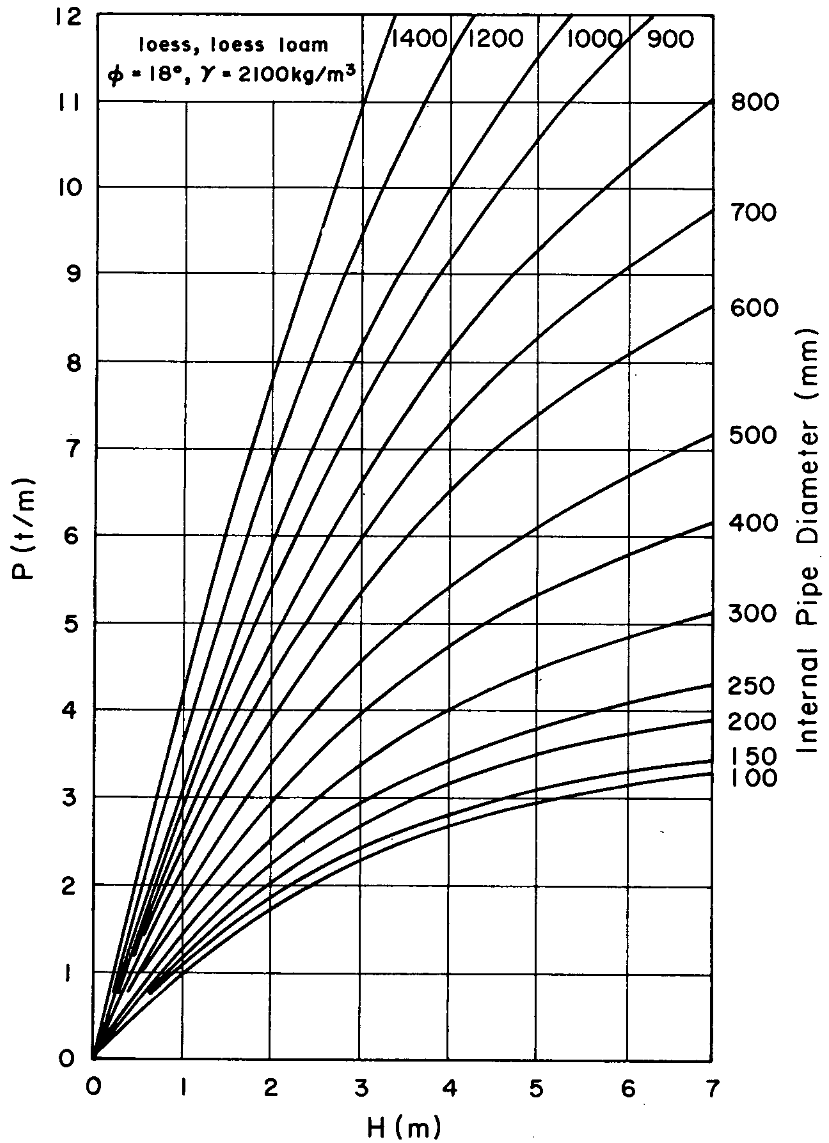


Figure K-18. Soil loads on concrete pipes in loess.

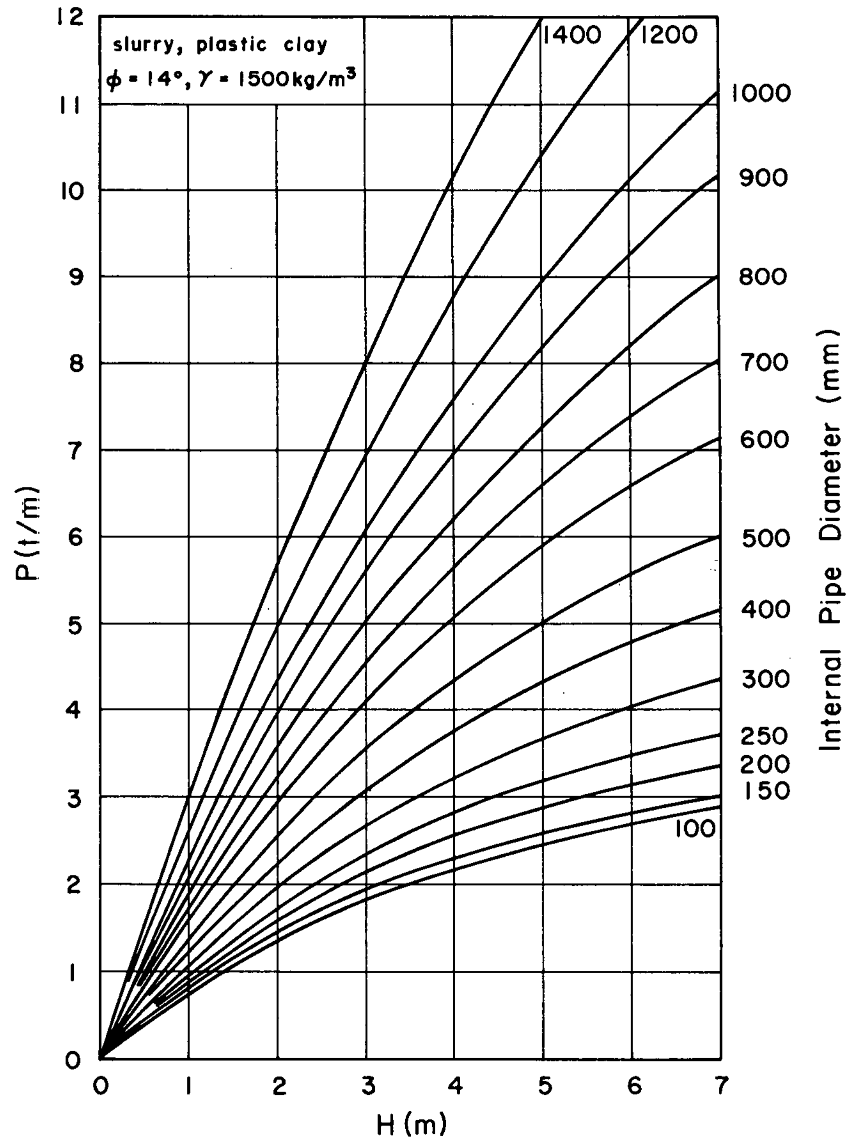


Figure K-19. Soil loads on concrete pipes in plastic clays.

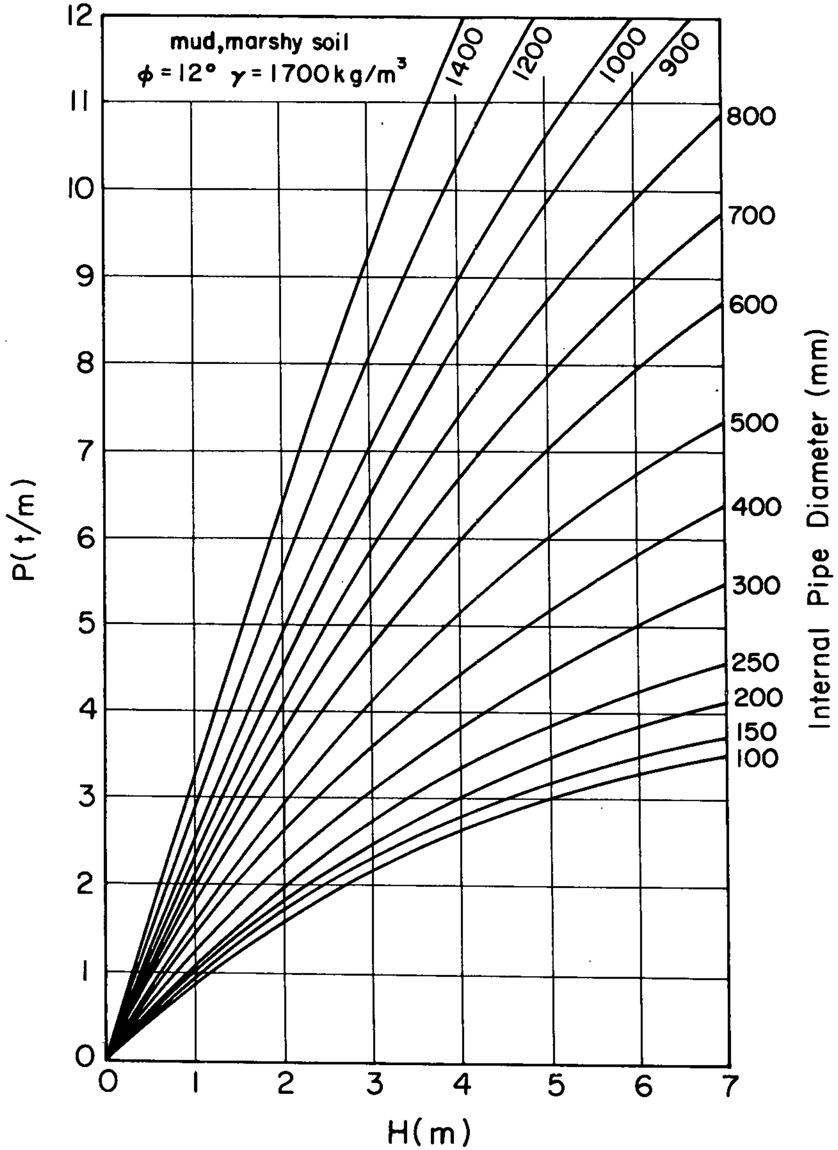


Figure K-20. Soil loads on concrete pipes in marshy soils.

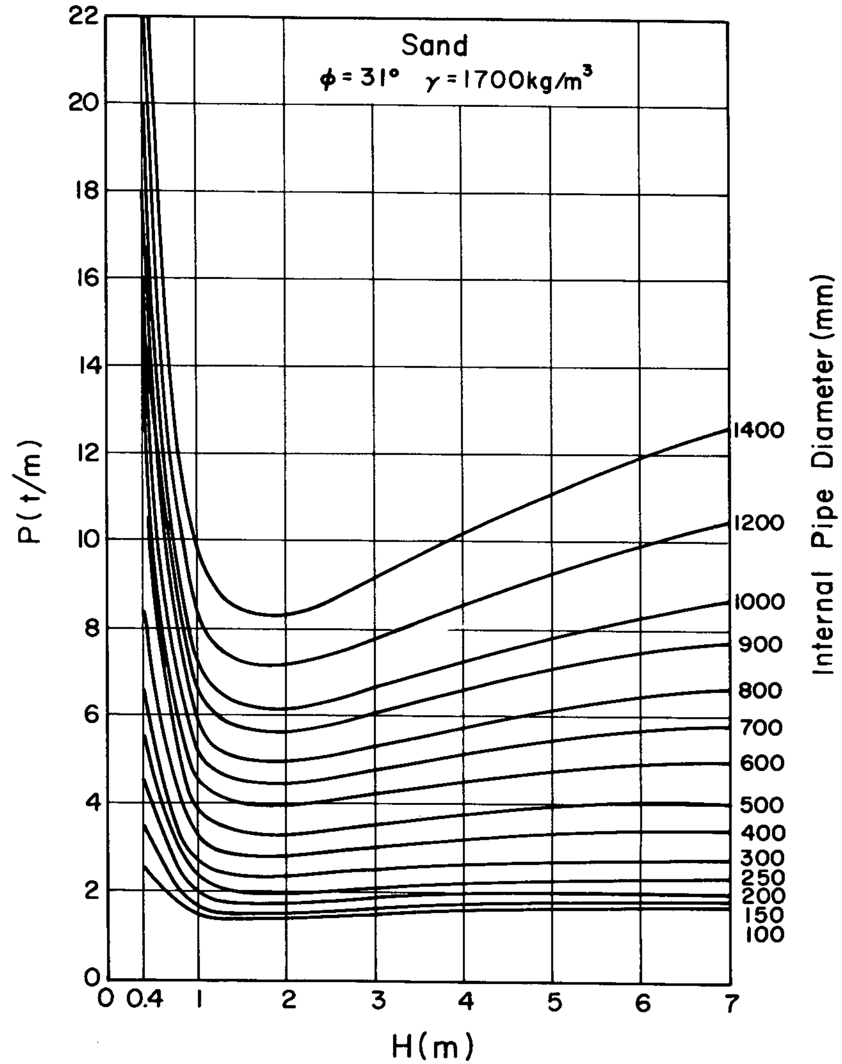


Figure K-21. Soil and traffic loads on concrete pipes in sand.

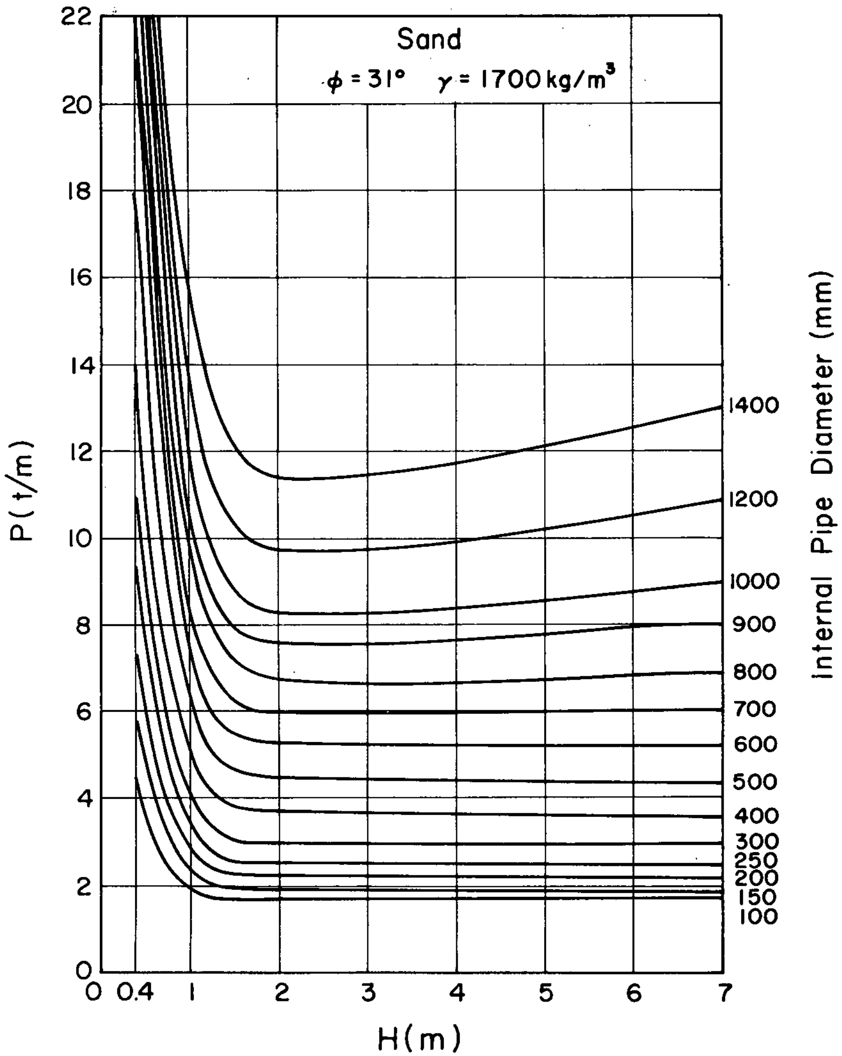


Figure K-22. Soil and traffic loads on concrete pipes in sand.

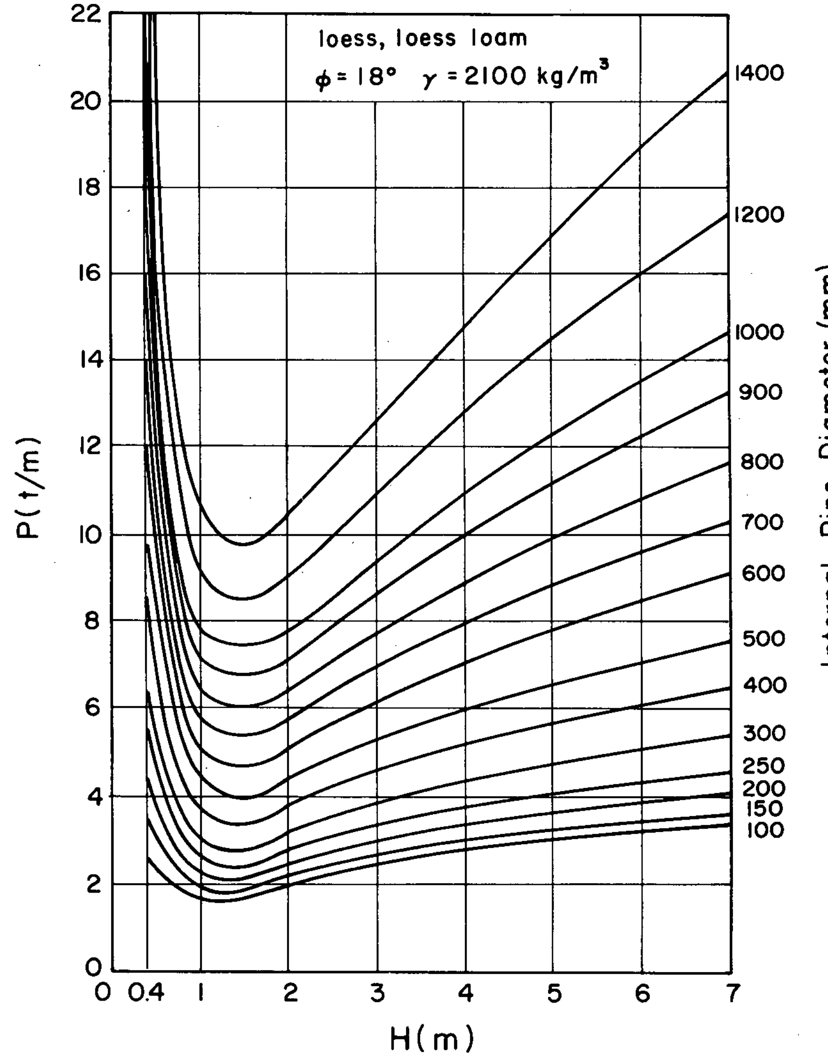


Figure K-23. Soil and traffic loads on concrete pipes in loess.

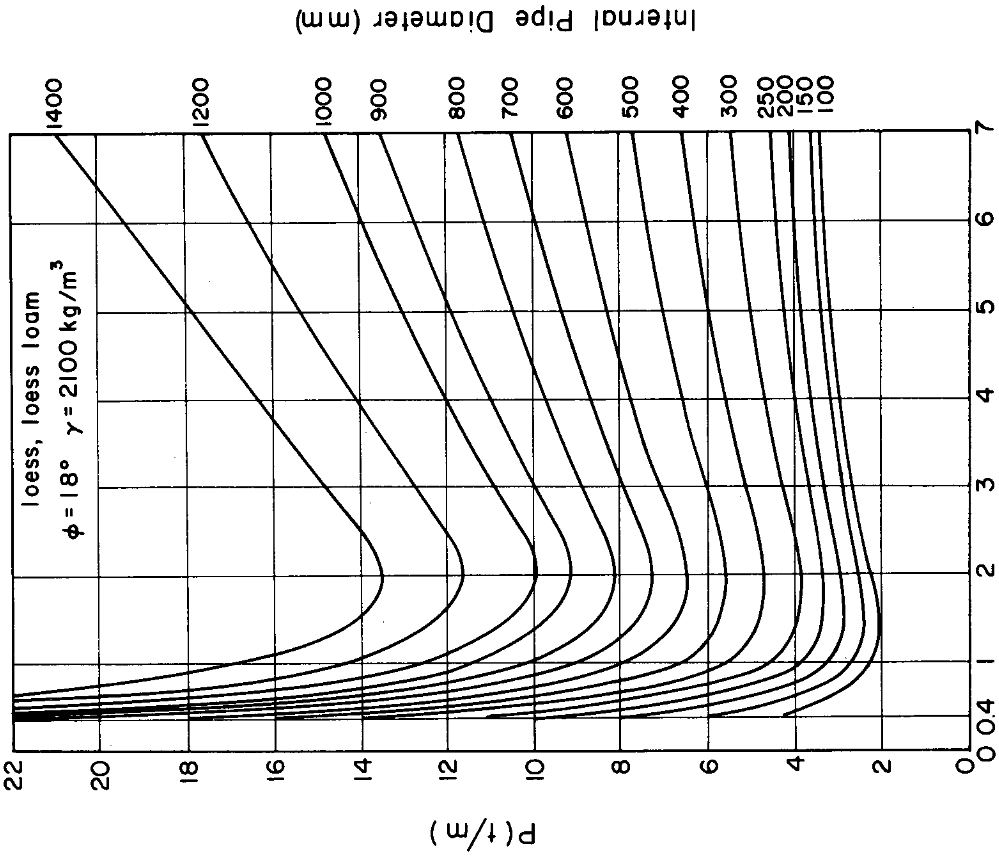


Figure K-24. Soil and traffic loads on concrete pipes in loess.

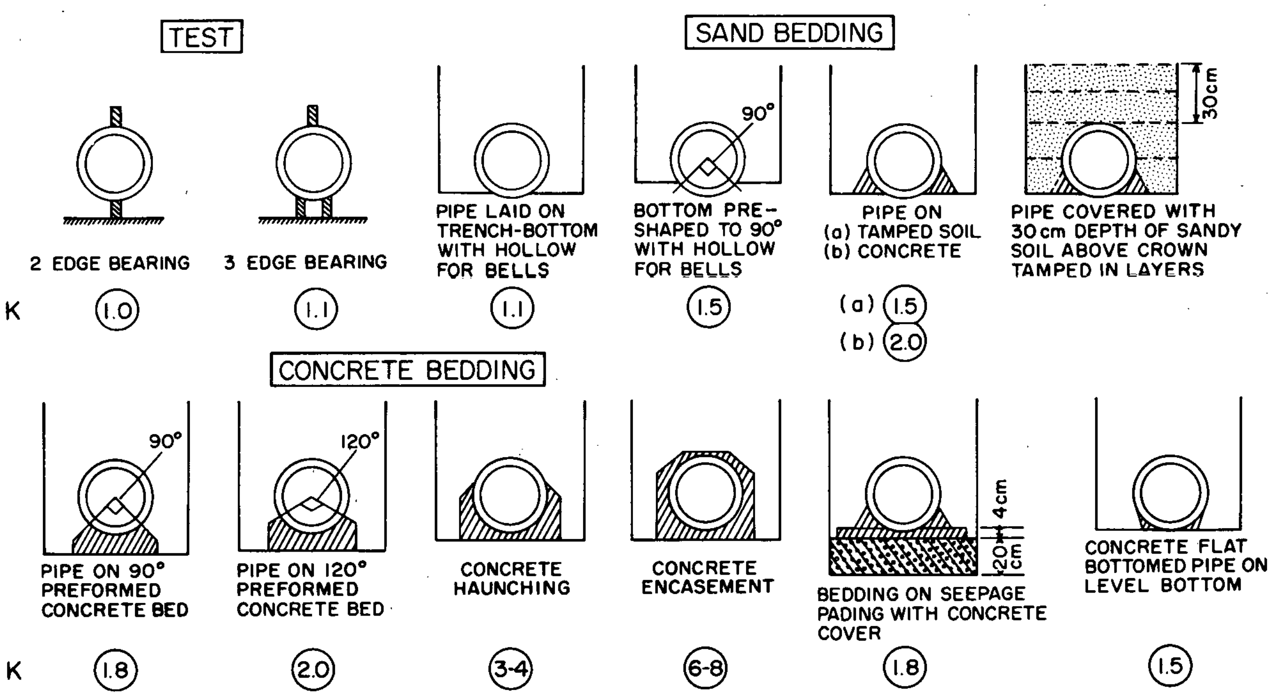


Figure K-25. Bedding factors for various types of bedding.

pressure on the culvert, the crown settles an amount Δh due partly to the flexural deflection of the section and partly to the compression of the supporting soil. The earth column above the culvert attempts to settle an equal amount but the relative movement of the earth column is opposed by the frictional forces between it and the adjacent earth masses. Thus, part of the weight of the earth column above the culvert will be transmitted to the adjacent soil, and the pressure on the culvert will be less than γH. It is also possible that the movement of the culvert is less than the settlement of the adjacent soil; the result in this case is a relative upward movement Δh of the culvert crown, and exactly the opposite effect will occur (Fig. K-26). As a result of the relative downward movement of the adjacent soil mass, additional loads will be transmitted by friction to the earth column above the structure, and the resulting average vertical pressure on the culvert crown will be higher than the geostatic one.

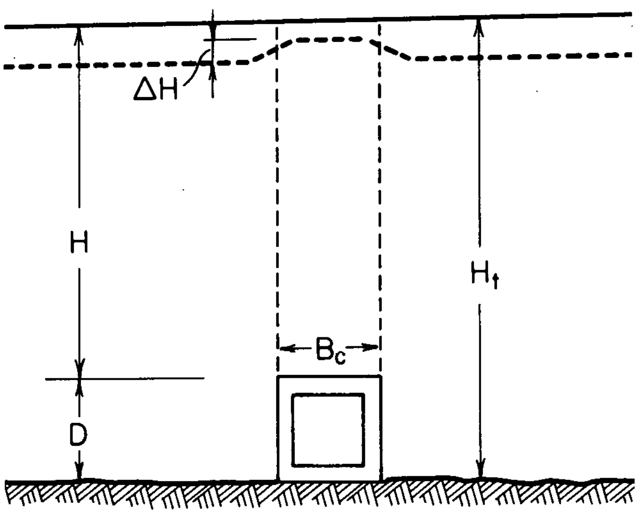


Figure K-26. Schematic diagram of rigid box culvert.

The assumed frictional forces would be distributed along the full height H above the culvert only if the earth column above the culvert were incompressible. However, the embankment is not incompressible, and frictional forces will develop only along that part of the height where differential movements take place. This height, termed the height of equal settlement, will be equal to the height of the earth column that will undergo a total settlement of Δh. If this height is called H_e, there will be a zone above it, the depth of which is equal to H_c, within which no relative movement occurs, and hence there is no redistribution of the stresses due to friction. Therefore, if culverts are rigid or are placed on rigid foundations, the settlement of the adjacent soil masses is likely to be larger than the vertical movement of the culvert crown, and consequently the vertical earth pressure on the culvert will increase above the geostatic one. On the other hand, for flexible structures and bases, the earth pressure will be less than the geostatic pressure. Experiments conducted in the Soviet Union have indicated that these pressures will reach their final values in about four or five months.

The magnitude of the pressure increase or decrease on the conduit is determined by the following reasoning. First, it is assumed that the frictional forces are directly proportional to the active earth pressure. Accordingly, the frictional force at a depth z is equal to $\gamma z K_a \tan \phi$, in which K_a is the coefficient of active earth pressure, and ϕ is the angle of internal friction. With the foregoing assumption and reference to Figure K-27, the earth pressure on a rigid culvert can be obtained in the following manner:

1. If H_c is 0 (Fig. K-27a),

$$P = C_t' \gamma B_d^2 \qquad \text{(K-14)}$$

in which

$$C_t' = \frac{H}{B_d}\left[1 + \frac{H}{B_d} K_a \tan \phi\right] \qquad \text{(K-15)}$$

2. If H_c is greater than 0 (Fig. K-27b),

$$P = C_t'' \gamma B_d^2 \qquad \text{(K-16)}$$

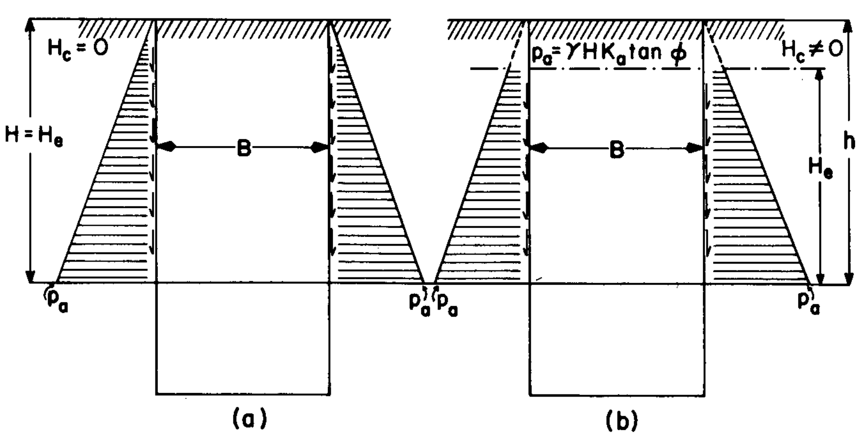


Figure K-27. Forces acting on soil column above pipe.

in which

$$C_t'' = \frac{H}{B_d}\left[1 + K_a \tan\phi \left(\frac{H}{B_d} - \frac{H_c^2}{B_d H}\right)\right] \quad \text{(K-17)}$$

The earth pressures acting on flexible culverts are determined from the following:

1. If H_c equals 0,

$$P = C_u'\gamma B_d^2 \quad \text{(K-18)}$$

in which

$$C_u' = \frac{H}{B_d}\left[1 - \frac{H}{B_d} K_a \tan\phi\right] \quad \text{(K-19)}$$

2. If H_c is greater than 0,

$$P = C_u''\gamma B_d^2 \quad \text{(K-20)}$$

in which

$$C_u'' = \frac{H}{B_d}\left[1 - K_a \tan\phi \left(\frac{H}{B_d} - \frac{H_c^2}{HB_d}\right)\right] \quad \text{(K-21)}$$

In both these cases H_c has to be determined separately from

$$\frac{H_c}{H} = 1 - \frac{\beta}{H^3} \quad \text{(K-22)}$$

in which, for the case of circular sections, β is obtained from

$$\beta = \frac{3d^2Hsp_r}{K_a \tan\phi} \quad \text{(K-23)}$$

in which p_r is defined as the ratio of the culvert height above the undisturbed soil surface to its outer diameter, as shown in Figure K-28, and s is chosen from the following empirical values:

Rigid structure on rock foundation	1.0
Rigid structure on dense soil	0.7
Rigid structure on elastic soil	0.3
Flexible structure on any type of soil	0

A graphical solution for sp_r as a function of H_e/d was worked out by Yaroshenko and is shown in Figure K-29; in this solution, ϕ was assumed equal to 30° ($\tan\phi K_a = 0.192$).

An assumption similar to that of Yaroshenko was made by Pruska (*112*) when considering the settlement difference, as shown in Figure K-30, as a measure of the additional load acting on the crown of the culvert. Using the equations of elasticity for the stress distributions in a semi-infinite continuum, he concluded that the total maximum vertical load acting on the crown of the culvert is given by

$$P = \gamma HB_d + \frac{1}{2}\left[\frac{\gamma\pi B_d(2Hd + d^2)}{2H \cot^{-1}\frac{2H}{B_d} + B_d \ln \frac{B_d^2 + 4H^2}{B_d^2}}\right] \quad \text{(K-24)}$$

This expression applies for all cases where the thickness of the backfill is at least four times the diameter of the culvert. On the basis of practical observations, Guerin (*113*) has suggested that the maximum additional load may not exceed 50 percent of the geostatic value.

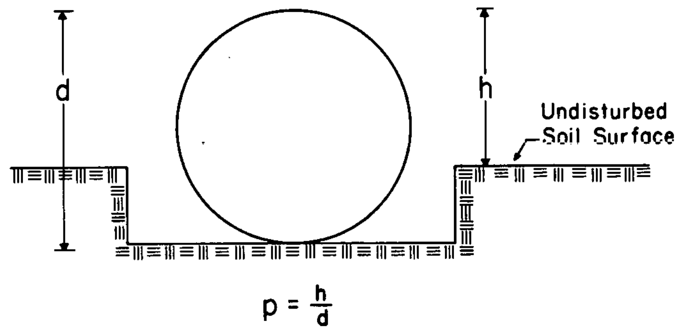


Figure K-28. Positive projecting conduit.

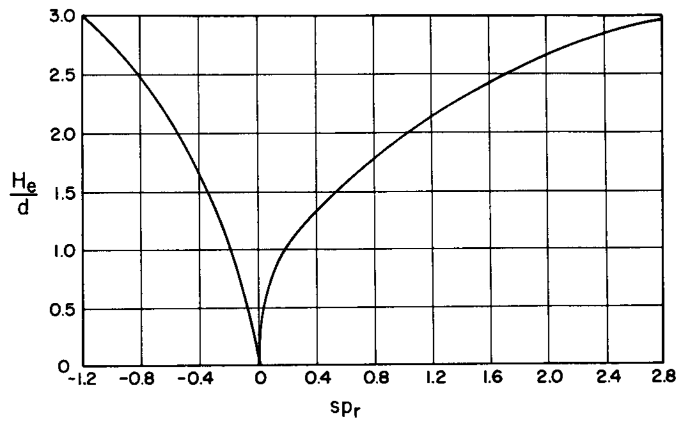


Figure K-29. Graphical solution for the plane of equal settlement.

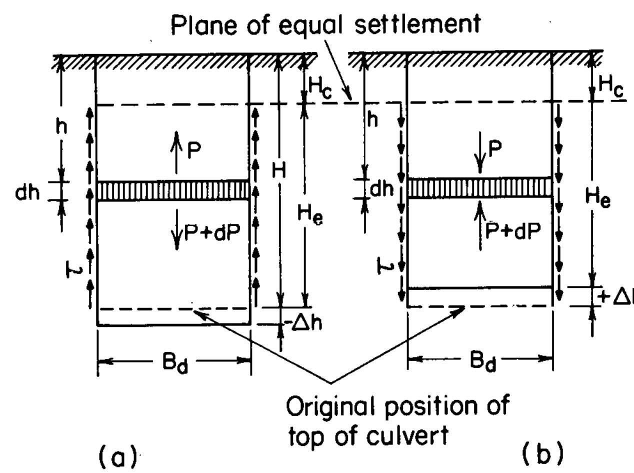


Figure K-30. Effect of relative settlement on culvert load.

Effect of Bedding

Because the structural behavior of a culvert is greatly affected by bedding conditions, these are often specified in standards. According to DIN 4033 (German standard), six different types of bedding are suggested, as shown in

Figure K-31, but concrete beddings are generally preferred. Five years ago, a survey in Hungary led to the conclusion that 60 percent of the pipe culverts laid directly on natural soil or on nonconcrete bedding suffered failure to some extent. Even in cases where the pipes were bedded on compacted soil, no definite improvement in structural performance was noticed; this may be attributed to the difficulty in achieving uniform compaction in the field in order to prevent irregular settlements. Concrete bedding, however, has proven to be very effective; this has also been emphasized by Wetzorke (*109*). If the pipe is laid on a concrete base, the angle of support is usually taken to be 90°, as shown in DIN 4033. For pipe culverts of large diameter, definite angles of support are often prescribed. Various formulas, such as that of Marquardt (*114*), for the bedding of pipes on preformed beds with various angles of support are based on the assumption that the supporting forces are distributed uniformly or according to the cosine law. The experiments by Wetzorke showed that considerable settlements of culverts may occur due to a fluctuating groundwater table. To avoid this problem, he suggests that the drain area be covered with concrete and the lower part be bedded in concrete.

The Soviet experiments mentioned earlier indicated that the normal and tangential stresses acting on the conduit depend on the flexibility of the conduit, the physical properties of the soil and, to a large extent, the method and degree of compaction of the fill. If, for instance, the pipe is laid directly on the ground without shaping a bed to the form of the underside of the pipe, the stress distribution on the pipe will be nonuniform and unfavorable. The load capacity of a pipe increases with the width over which the base pressures are distributed and the uniformity of these pressures. The Hungarian Building Code MSZ 15300 shows in its Appendix how the load capacity of a circular or egg-shaped pipe section increases with improved bedding conditions as compared with its knife-edge bearing test strength (e.g., for 180° embedment, the load capacity could increase to 2.5 times its lowest value).

The ratio of the horizontal and vertical pressures is normally considered to range between 0.2 and 0.5, depending on the method of construction, the properties of the soil, and the degree of compaction of the backfill; however, this ratio could be as high as 1.0 for flexible pipes, indicating that the distribution of normal stresses along the pipe perimeter is almost uniform. This uniform stress distribution, favorable from the structural point of view, will usually develop after the in-service deformations of the flexible pipe have occurred. Hence, for flexible pipes, it is not the final loading condition that is the most critical, but the one immediately after construction, when the deformations and the subsequent horizontal pressures have not developed completely.

Design of Culvert Cross Section

The structural design of culvert sections is usually based on the assumption that the culvert is an annular pipe section acted on by a specified load distribution. The corresponding stress distribution is then calculated, and a design criterion is formulated. The design criterion is usually speci-

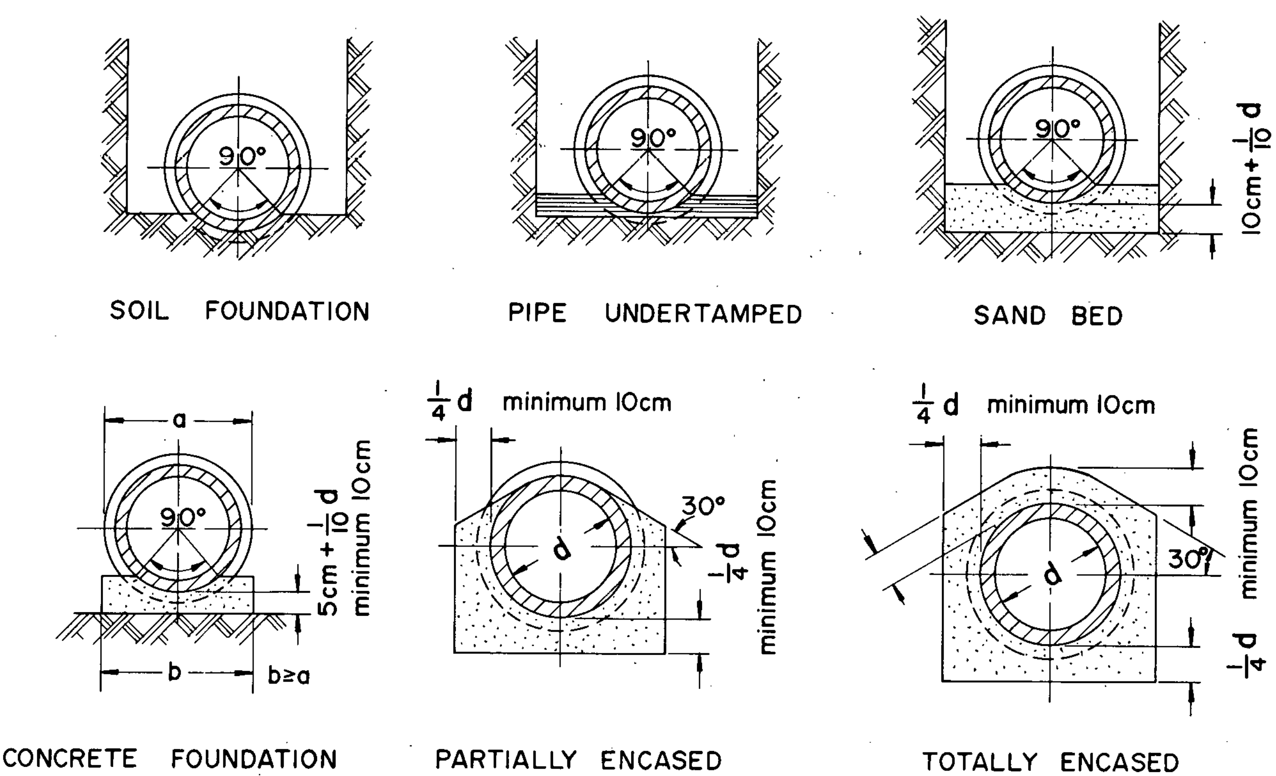


Figure K-31. Bedding conditions according to DIN 4033.

fied by a flexural stress value that must not be exceeded. The safety factor is normally included in the loading assumptions.

In the Soviet Union the structural design of highway culverts is based on principles of "limiting tolerances"; that is, situations are analyzed to determine when specified tolerances or working conditions cease to be satisfied under the design load (*115*).

Three limiting situations are usually considered:

1. The first limit is associated with the conditions that the culvert must not fail by exceeding the load-bearing capacity (strength, stability, and fatigue) of the pipe or by the development of substantial plastic deformations.
2. The second limit specifies that excessive over-all deformations (displacement, settlement), which endanger the normal operation of the structure, must not occur.
3. The third limit requires that the structure must resist cracking in order to avoid difficulties in normal operation.

Regardless of whether the culvert is built of steel, reinforced concrete, or prestressed reinforced concrete, the calculations are performed for all three limiting situations. For plain concrete culverts, however, only the first limiting situation is considered by designing the culvert for ultimate strength. The adequacy of the soil foundation for culverts is usually analyzed for the first and second limiting situations.

After values for the dead load, p, and the live load, q, have been determined, rigid circular culvert sections are analyzed for bending moment, M, without taking into account normal and transverse forces, by

$$M = n r_m^2 (p + q)(1 - K) \qquad \text{(K-25)}$$

in which r_m is the average radius of the pipe; n is a coefficient to be determined by the foundation conditions ($n \geqq 0.2$); and K is equal to $\tan^2\left(45° - \frac{\phi}{2}\right)$.

Because no information regarding the values of n can be found in the Soviet literature or in Soviet standard SN 200-62, special investigations were carried out by the "Lengiprotransport" (Institute for Highway Engineering in Leningrad, *116*). Two different types of bedding—rigid and soil—with variable bedding angles were studied. In the first step it was assumed that the distributions of the radial and tangential components of earth pressure obey the following laws:

Radial components:

$$p_r = (p + q)(1 - K)\sin^2(\theta + 90°) + K(p + q) \qquad \text{(K-26}a\text{)}$$

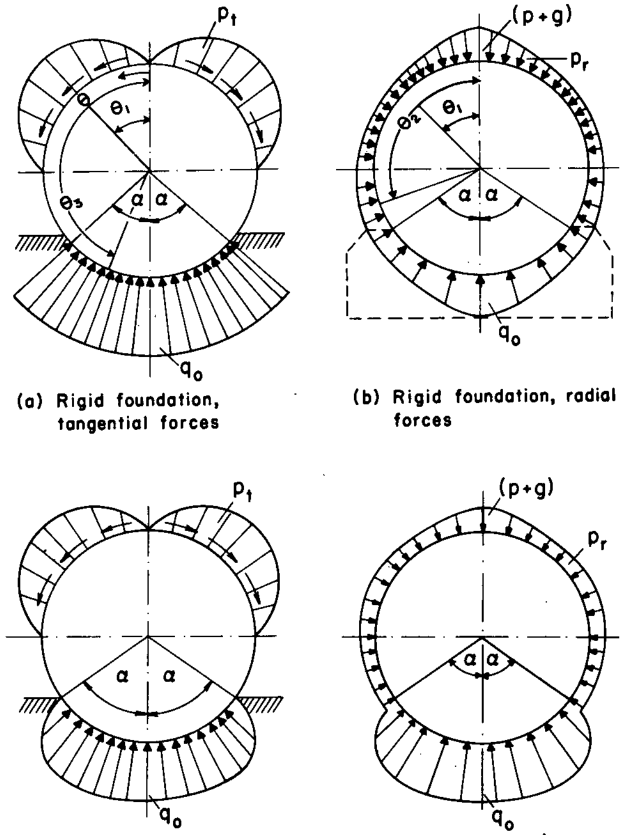
in which $p_r = p + q$ if $\theta = 0$ where θ is defined in Figure K-32a.

Tangential components:

$$p_t = \tfrac{1}{2}(p + q)(1 - K)\sin^2\theta \qquad \text{(K-26}b\text{)}$$

in which the maximum value of p_t occurs at $\theta = 45°$ and is given by

$$(p_t)_{\max} = \tfrac{1}{2}(p + q)(1 - K) \qquad \text{(K-26}c\text{)}$$



(a) Rigid foundation, tangential forces
(b) Rigid foundation, radial forces
(c) Soil foundation, tangential forces
(d) Soil foundation, radial forces

Figure K-32. Assumed pressure distributions on buried pipe.

In the next step, based on the investigations by Felme (*116*), it is assumed that the radial pressure, p_r, produced by the reaction of a rigid foundation can be determined by

$$p_r = -\eta \frac{2\cos\theta}{r_m(\sin 2\alpha + 2\alpha)} \qquad \text{(K-27}a\text{)}$$

if the culvert is placed on a rigid foundation, and by

$$p_r = \eta \frac{3(\cos^2\theta + \cos\alpha\,\cos\theta)}{r_m(3\sin\alpha + \sin^3\alpha - 3\alpha\,\cos\alpha)} \qquad \text{(K-27}b\text{)}$$

if the culvert is placed on a soil foundation, in which η is a constant to be determined by the equilibrium condition in the vertical direction. If, for instance, the pipe is supported by a rigid foundation and acted on by tangential forces, as shown in Figure K-32a, the value of η is determined as follows. The resultant of the vertical components of the tangential forces is

$$R_v' = 2\int_{\theta=0}^{\theta=\frac{\pi}{2}} \tfrac{1}{2}(p + q)(1 - K)\sin 2\theta\,\cos\theta\, r_m d\theta = \frac{2(p + q)(1 - K)\, r_m}{3} \qquad \text{(K-28}a\text{)}$$

and it must be equal to the resultant of the vertical components of the radial forces acting along the rigid foundation

$$R_v'' = 2\eta \int_{\theta=0}^{\theta=\alpha} \frac{2\cos\theta}{r_m(\sin 2\alpha + 2\alpha)} r_m \cos\theta \, d\theta = \eta \quad \text{(K-28}b\text{)}$$

thus,

$$\eta = \frac{2(p+q)(1-K)r_m}{3} \quad \text{(K-29)}$$

The moment distribution for the assumed radial and tangential loads is determined separately, and their sum is taken as the nominal value of the bending moment. The calculations are conveniently performed by use of either the force equilibrium method or the elastic load method. The following equations are obtained for the different sections of the culvert:

$$M_1 = M_{01} + X_1 + X_2 r_m \cos\theta_1; \text{ for } 0 \leqq \theta \leqq \frac{\pi}{2} \quad \text{(K-30}a\text{)}$$

$$M_2 = M_{02} + X_1 + X_2 r_m \cos\theta_2; \text{ for } \frac{\pi}{2} \leqq \theta_2 \leqq (\pi - \alpha) \quad \text{(K-30}b\text{)}$$

$$M_3 = M_{03} + X_1 + X_2 r_m \cos\theta_3; \text{ for } (\pi - \alpha) \leqq \theta_3 \leqq \pi \quad \text{(K-30}c\text{)}$$

in which X_1 is a unit moment and X_2 is a unit horizontal force; M_{01}, M_{02}, and M_{03} are the moments acting at sections $\theta_1 = 0$, $\theta_2 = \frac{\pi}{2}$, and $\theta_3 = (\pi - \alpha)$, respectively. The expressions for X_1 and X_2 are

$$X_1 = -\frac{1}{\pi} r_m^2 (p_t)_{\max} \left[\frac{3}{4}\pi - \frac{2}{3} - \frac{A}{2}\left(\alpha - 2\sin\alpha + \frac{1}{2}\sin 2\alpha\right)\right] \quad \text{(K-31}a\text{)}$$

and

$$X_2 = \frac{2}{\pi} r_m (p_t)_{\max} \left[\frac{\pi}{3} + \frac{1}{9} + \frac{A}{8}\left(\frac{1}{2}\sin 2\alpha - \alpha\cos 2\alpha\right)\right] \quad \text{(K-31}b\text{)}$$

in which

$$A = \frac{8}{3(\sin 2\alpha + 2\alpha)} \quad \text{(K-32)}$$

The maximum bending moment can be determined by taking different values for α. A similar analysis may be carried out for radial loading, and the values of the maximum bending moment can be obtained as a function of the bedding angle, α. The sum of the two results gives the nominal value of the bending moment, which yields the value of n in Eq. K-25. The analysis for the soil foundation follows the same procedure. The results obtained from such calculations are plotted in Figure K-33. In addition, values of n were determined to give the effect of a concentrated load, and these values, also shown in Figure K-33, can be used with the equation

$$M_{\max} = nPr_m \quad \text{(K-33)}$$

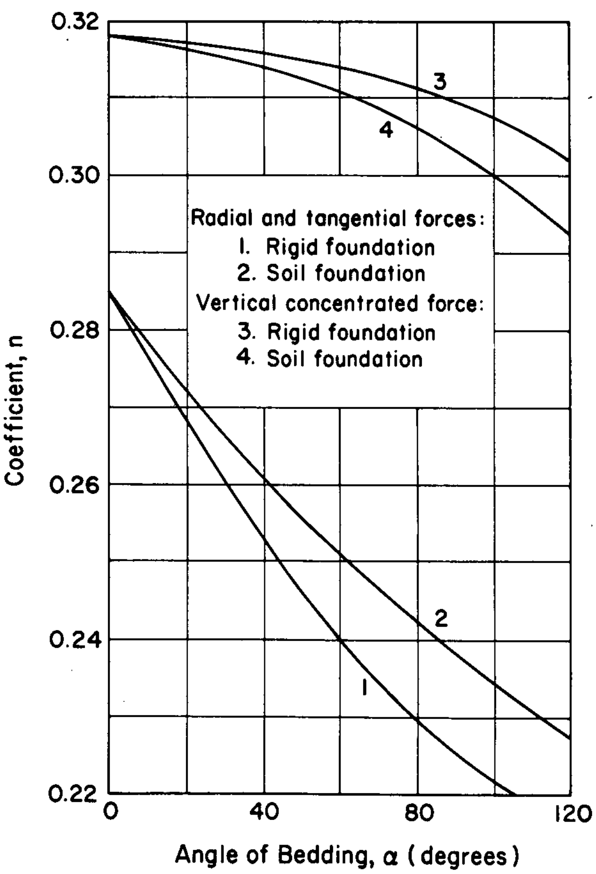


Figure K-33. Coefficient n *as a function of the bedding angle.*

The validity of the relationship that expresses the distribution of the soil reactions used in deriving the formula for the bending moments of a cylindrical pipe section was checked by static tests carried out by the National Research Institute for Structures (Russia). A circular reinforced concrete pipe section with a diameter of 1 m, according to Standard Project Number 7194, was placed on a compacted sand layer with a bedding angle of 110° and tested by application of a vertical, concentrated load. The theoretical assumptions were checked by comparing the nominal bending moments calculated by Eq. K-30 with the moments determined from the dimensions of the cracks which developed in the concrete at the same load. The ratio of the nominal moment to its actual value varied from 1.28 to 2.96, and the errors were on the safe side. It may be of interest to note that the maximum bending moment obtained by this method is considerably greater than that given by other methods (for instance, that of Harosy, *117*) which propose an n coefficient of less than 0.2. This deviation is accounted for by the fact that the tangential force components are included in the calculations.

The foregoing considerations do not apply to flexible culverts. The horizontal pressures must not be neglected, and the loading condition that occurs immediately after the completion of construction will determine the structural behavior of the culvert. According to Yaroshenko, it is erroneous to assume that the bending moment in the wall of a corrugated steel pipe is negligibly small because of the almost uniform distribution of normal stresses. The considerable deformations (10 percent of the diameter) that develop in virtually all normal corrugated steel culverts can be explained only by the existence of bending moments.

The actual loading conditions can be described as follows. At first the culvert is acted on by a vertical pressure equal to or possibly greater than the geostatic pressure, and the horizontal soil support is not large at this point. The coefficient, κ, given by the ratio of the horizontal earth pressure to the vertical pressure is near the value of $\tan^2\left(45^\circ - \frac{\theta}{2}\right)$, and the nonuniform distribution of earth pressures along the perimeter of the pipe results in the development of considerable bending moments. The stresses acting at the points of maximum bending moment (the four end points of the vertical and horizontal diameters) reach the state of plastic flow; that is, four plastic hinges develop and the deformations increase sharply. This situation is shown in Figure K-34, which shows the percentage decrease in the vertical diameters as a function of time. These deformations are accompanied by an increase in the horizontal soil reaction, which eventually tends toward a uniform pressure distribution around the perimeter of the pipe.

The relative magnitude of the horizontal earth pressure depends on the stiffness of the pipe and the mechanical properties of the soil, and it can be determined approximately by using the following reasoning of Yaroshenko. If it is assumed that the horizontal earth resistance increases linearly with an increase in the horizontal pipe diameter, one may write

$$p_h = K_a(p+q) + G\Delta d_h \tag{K-34}$$

in which G is a coefficient of subgrade reaction; and Δd_h is the horizontal diameter change given by

$$\Delta d_h = \frac{0.18r^4(p+q)(1-\kappa)}{EI} \tag{K-35}$$

Substitution of Eq. K-35 into Eq. K-34 gives

$$p_h = K_a(p+q) + \frac{0.18Gr^4(p+q)(1-\kappa)}{EI} \tag{K-36}$$

Because

$$\kappa = \frac{p_h}{p+q} \tag{K-37a}$$

one obtains

$$\kappa = \frac{K_aEI + 0.18Gr^4}{EI + 0.18Gr^4} \tag{K-37b}$$

A typical resulting pressure distribution is shown in Figure K-35, and values for G and K_a are given in Table K-5.

Another interesting analysis was carried out by Harosy (*117*), who assumed that the load q_o varies according to the relationship

$$q_o = (p+q)(1-\kappa)\frac{4\theta^2}{\pi^2} \tag{K-38}$$

For moment, M, and axial compression, T, at the crown,

$$M = -0.144r^2(p+q)(1-\kappa) \tag{K-39}$$

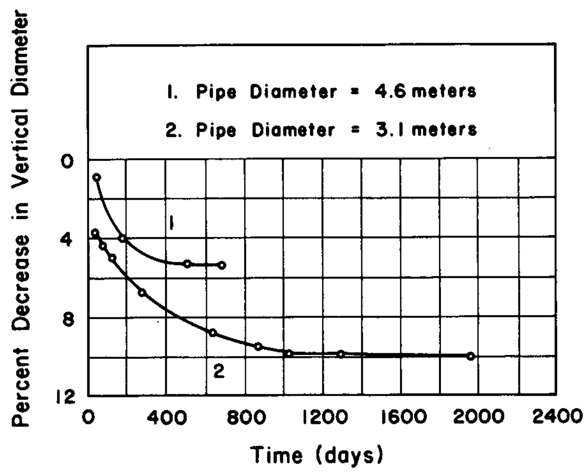


Figure K-34. Deflection of flexible pipes as a function of time.

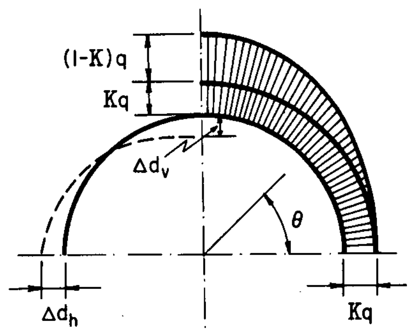


Figure K-35. Assumed pressure distribution on buried pipe.

which is similar to Eq. K-25, and

$$T_{\pi/2} = r(p+q)\left[1 - \frac{8}{\pi^2}(1-\kappa)\right] \tag{K-40}$$

TABLE K-5

COEFFICIENTS TO DETERMINE DEFORMATION AT PLASTIC HINGE IN CULVERT

SOIL TYPE	K_a	G (KG/CM³)
Sand or sandy clay:		
Compact	0.35	2-5
Dense	0.25	5-8
Clayey sand:		
Plastic	0.70	2-5
Stiff	0.50	5-8
Clay:		
Plastic	0.75	2-5
Hard	0.70	5-8

The elongation of the horizontal diameter and the shortening of the vertical diameter are given by

$$\Delta d_h = -\Delta d_v = 0.18 \frac{r^4(p+q)(1-\kappa)}{EI} \qquad \text{(K-41)}$$

from which the approximate value of the bending stresses, calculated by use of

$$M_{\pi/2} = 0.15r^2(p+q)(1-\kappa) \qquad \text{(K-42)}$$

would be

$$\sigma_t' = \frac{0.15r^2(p+q)(1-\kappa)}{Z} \qquad \text{(K-43)}$$

in which Z denotes the section modulus of the pipe; the compressive stresses from the axial load, as calculated from Eq. K-44, must be added to the value determined previously.

$$\sigma_t'' = \frac{T}{A} = \frac{r(p+q)}{A}\left[1 - \frac{8}{\pi^2}(1-\kappa)\right] \qquad \text{(K-44)}$$

For the pressure distribution diagram shown in Figure K-36, the moments and forces in any section to the left of the point of discontinuity may be obtained from the relations

$$M_{\mathrm{I}} = \left[\left(\frac{2\theta_o}{\pi} - 1\right) + \cos\theta_o \cos\theta\right](p+q)(1-\kappa)r^2 \qquad \text{(K-45}a\text{)}$$

$$N_{\mathrm{I}} = (1 - \cos\theta_o\cos\theta)(p+q)(1-\kappa)r + \kappa qr \qquad \text{(K-45}b\text{)}$$

$$T_{\mathrm{I}} = \cos\theta_o \sin\theta(p+q)(1-\kappa)r \qquad \text{(K-45}c\text{)}$$

and to the right of the point of discontinuity from the relations

$$M_{\mathrm{II}} = \left[\frac{2\theta_o}{\pi} - \sin\theta_o\sin\theta\right](p+q)(1-\kappa)r^2 \qquad \text{(K-46}a\text{)}$$

$$N_{\mathrm{II}} = \sin\theta_o \sin\theta(p+q)(1-\kappa)r + \kappa qr \qquad \text{(K-46}b\text{)}$$

$$T_{\mathrm{II}} = \sin\theta_o\cos\theta(p+q)(1-\kappa)r \qquad \text{(K-46}c\text{)}$$

On the basis of theoretical considerations as described previously, several hinged culverts were built in Russia; hinges were provided at the crown, the invert, and the springlines by reducing the cross-sectional area at these locations. From 1949 to 1951 experiments were carried out by Yaroshenko to compare the performance of hinged culverts with that of regular rigid culverts. These experiments indicated that deformations occurred more rapidly in hinged culverts than in continuous rigid culverts to a certain point beyond which a further increase in the load led to a cessation of the deformations. Rigid culverts, however, showed continuous deformation; accordingly, the ultimate load for the hinged culverts was considered double that for rigid culverts of the same dimensions and cross section. The reason for this is that, from the very beginning, the lateral support for the hinged culverts was larger than that for the rigid ones, and the value of κ increased from the initial value of 0.18 to almost 1. On the other hand, the value of κ was nearly equal to zero for the rigid culverts and increased only later when, as a result of the overstressing, plastic hinges developed at the crown, the invert, and the springlines.

From this it follows that, whenever the deflection of rigid culverts is small or before plastic hinges develop, the passive earth resistance is small, and the pipe does not obtain sufficient lateral support to counteract deformation due to vertical loads, resulting in considerable bending moments, whereas in the flexible culverts plastic hinges act from the very beginning and secure adequate lateral support to reduce the moments in the cross section. When one is designing hinged culverts, not only the stresses but also the deflections of the structure must be checked.

The stresses in the reinforcement at the hinges should be at the yield point, and the concrete section should be proportioned to safely carry the axial loads. The design may be performed according to the following steps:

1. At the location of the plastic hinges, the cross-sectional area should be large enough to transfer the normal forces and, in addition, the section modulus should be large enough to provide a safety factor of 1.5 against bending stresses; that is, 1.5 M must be less than $Z\sigma_y$, in which σ_y is the yield stress of steel and Z is the section modulus of the reinforcing steel only.

2. It should be checked whether the reinforcement is sufficient to resist the moments developed in the hinges;

$$M_{\lim} = Z\sigma_y \qquad \text{(K-47)}$$

3. The amount of vertical deformation should be checked by

$$\Delta d_v = \frac{(p+q)(\kappa - K_a)}{G} \qquad \text{(K-48}a\text{)}$$

for which the values of K_a and G are given in Table K-5 and κ is taken equal to its maximum value, 0.9. The radial deformation, as a result of the plastic hinge, will be

$$\Delta d_r = \frac{2M_{\lim}a}{E_h I_h} + 1.2\frac{M_{\lim}r^2}{E_r I_r} \qquad \text{(K-48}b\text{)}$$

in which a is the length of the hinge; $E_r I_r$ is the reduced stiffness of the pipe cross section; and $E_h I_h$ is the longitudinal stiffness of the pipe at the plane of the hinges. This deformation should not exceed 1/100 of the pipe diameter.

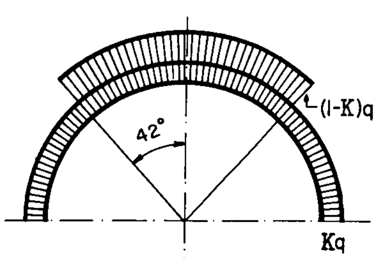


Figure K-36. Assumed pressure distribution on buried pipe (according to Harosy).

Longitudinal Design of Culverts

Culverts are structures that are likely to undergo differential settlements as a result of the variable loads acting on them. To avoid the development of fissures and fractures, it is customary to construct long culverts from individual short units, either precast or cast-in-place, with strong, watertight joints. As a result of differential settlements, the culvert sags in the center, thereby causing compressive stresses at the top and tensile stresses at the bottom.

In most foreign countries Tschebotarioff's suggestions (*113*) are used to design culverts to withstand differential settlements. As shown in Figure K-37, it is assumed that the radius of curvature of the longitudinal axis is R; then the resulting moment, M_d, according to Navier's hypothesis is

$$M_d = -\frac{EI_d}{R} \qquad \text{(K-49)}$$

and the associated bending stress is

$$|\sigma|_{\max} = \frac{M_d r}{I_d} = \frac{Er}{R} \qquad \text{(K-50)}$$

If R is approximately equal to $L^2/8\Delta$, in which Δ is the maximum deflection at the center, then

$$|\sigma|_{\max} = \frac{8\Delta Er}{L^2} \qquad \text{(K-51)}$$

Assuming that the effective lever arm is ⅔ of the diameter,

$$M_d = -\frac{EI_d}{R} = -\frac{4}{3} rX \qquad \text{(K-52)}$$

from whence

$$X = \frac{6\Delta I_d E}{rL^2} \qquad \text{(K-53)}$$

is the tensile force on which the longitudinal design of the section should be based.

Rendulic (*113*) expresses the longitudinal tensile force as the sliding resistance caused by the weight of the overburden acting as a normal force:

$$X_{\max} = \tfrac{1}{2}\gamma H^2 B_c \tan^2(45° - \phi/2) \qquad \text{(K-54)}$$

in which H is the height of the embankment above the pipe; B_c is the width of the culvert; γ is the unit weight of the backfill material; and ϕ is the angle of internal friction. In the Soviet Union, the empirical values given in Table K-6 are used to determine the longitudinal force acting on small-diameter culverts. If H is known, the longitudinal reinforcement or the connections between the precast units can be designed.

A simple but accurate method for the longitudinal design of culverts has been described by Siko (*118*) who considers the culvert as a beam on an elastic support. From the relationship established by Winkler for beams on elastic foundations, the beam deflection, y, is given by

$$y = \frac{p}{C_w} \qquad \text{(K-55)}$$

in which C_w is Winkler's coefficient; also, from strength of materials

$$M = -EI_d \frac{d^2y}{dx^2} \qquad \text{(K-56)}$$

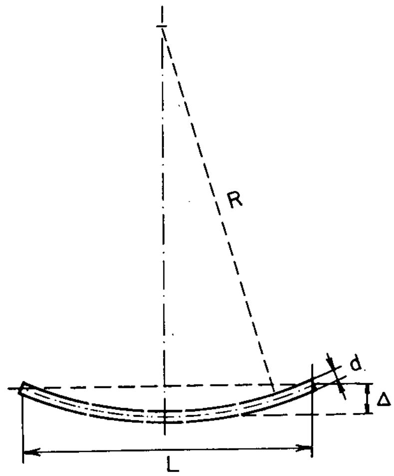


Figure K-37. Simplified method for the longitudinal design of culverts (after Tschebotarioff).

Instead of solving exactly the more complicated differential equation, an approximate method, as indicated in Figure K-38, could be used. Combining Eqs. K-55 and K-56 gives

$$M = -\frac{EI_d}{C_w}\frac{d^2p}{dx^2} \qquad \text{(K-57)}$$

Using finite difference approximations, the first derivatives can be written as

$$\left.\frac{\Delta p}{\Delta x}\right|_{x_i} = \frac{p_i - p_{i-1}}{\Delta x} \qquad \text{(K-58}a\text{)}$$

and

$$\left.\frac{\Delta p}{\Delta x}\right|_{x_{i+1}} = \frac{p_{i+1} - p_i}{\Delta x} \qquad \text{(K-58}b\text{)}$$

from which the second derivative can be expressed as

$$\frac{\Delta^2 p}{\Delta x^2}\, x_i = \frac{\left.\frac{\Delta p}{\Delta x}\right|_{x_{i+1}} - \left.\frac{\Delta p}{\Delta x}\right|_{x_i}}{\Delta x} = \frac{p_{i+1} - 2p_i + p_{i-1}}{\Delta x^2} \qquad \text{(K-59)}$$

Substituting these values into the equation for M_d, one gets

$$M_d = \frac{EI_d \Delta^2 p}{C_w \Delta x^2} = \frac{EI_d}{C_w \Delta x^2}(p_{i+1} - 2p_i + p_{i-1}) \qquad \text{(K-60)}$$

TABLE K-6

LONGITUDINAL FORCES ON SMALL-DIAMETER CULVERTS

PIPE DIAMETER (M)	LONGITUDINAL TENSILE FORCE (METRIC TONS), BY HEIGHT OF EMBANKMENT (M)				
	3	5	10	15	20
1.00	0.3	1.0	4.0	8.0	13.0
1.25	0.5	1.5	6.0	12.0	20.0
1.50	0.7	2.0	8.0	17.0	30.0

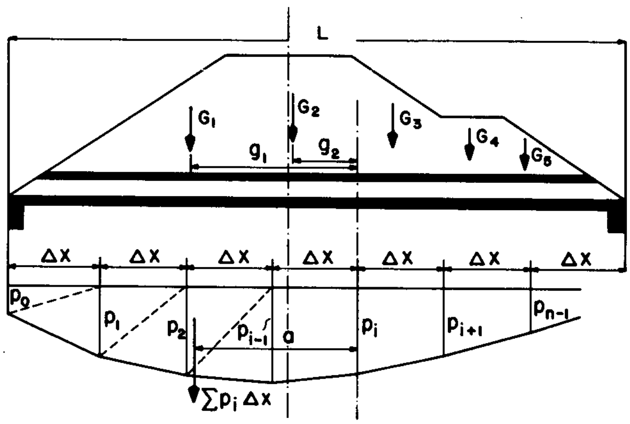


Figure K-38. Longitudinal design of culverts (after Siko).

If the total length, L, of the culvert is divided into n segments, each Δx long, contact pressures of unknown magnitude will be generated in the soil at each dividing point, as well as at the ends of the tube. However, for each section

$$M_i = \frac{EI_d}{C_w \Delta x^2}(p_{i+1} - 2p_i + p_{i-1}) = M_i^k \qquad \text{(K-61)}$$

in which M_i^k is the sum of the moments of the external forces and the unknown contact pressures p_i about section i. For equilibrium the moment calculated from the difference equation and the moments created by all external forces must be equal at each station; thus, one has

$$M_i = \frac{EI_d}{C_w \Delta x^2}(p_{i+1} - 2p_i + p_{i-1}) = \sum_{i=1}^{n} G_i g_i - p_o \frac{\Delta x}{2}$$
$$[i\Delta x - (2/3)\Delta x] - p_1 \frac{\Delta x}{2}\left[i\Delta x - \frac{\Delta x}{3}\right] - p_1 \frac{\Delta x}{2}$$
$$[(i-1)\Delta x - (2/3)\Delta x] - \ldots - p_{i-1}\Delta x(2/3)\Delta x$$
$$-\frac{p_i \Delta x^2}{2}\frac{\Delta x}{3} \qquad \text{(K-62)}$$

A similar equation can be written for any intermediate dividing point, thus yielding $n-1$ equations for the solution of $n+1$ unknowns. The additional two equations are obtained from statics; namely, the sum of vertical forces

$$\Sigma p \Delta x - \Sigma G = 0 \qquad \text{(K-63}a\text{)}$$

and the moment of these forces about any convenient point

$$(\Sigma p \Delta x)g - \Sigma G g = 0 \qquad \text{(K-63}b\text{)}$$

In the case of symmetry, the number of unknowns, as well as the number of equations, is reduced. When one has thus obtained the pressure ordinates, p_i, a catenary polygon with the forces G and p can be constructed, and the design moments can be determined graphically. Alternatively, the values of p_{i+1} and p_{i-1} can be substituted into the equation

$$M_i = -\frac{EI_d}{C_w \Delta x^2}(p_{i+1} - 2p_i + p_{i-1}) \qquad \text{(K-64)}$$

to obtain the ordinates. The accuracy of this method can be improved by increasing the number of divisions.

The Semi-Graphical Design of Egg-Shaped Culverts (*113*)

If the culvert cross section is egg-shaped rather than circular, the analytical solution to the problem would be too cumbersome, and therefore it is more practical to design the statically indeterminate structure by purely graphical or semi-graphical methods. The load should be determined in the manner discussed earlier and accurately plotted over the section. Next, the section is divided into elements with lengths Δs, and the elastic weights $\Delta s/EI$ are computed. The center of gravity of the elastic weights is determined by drawing a horizontal and vertical force polygon, considering the elastic weights as forces. When the elastic center has been established, the section is reduced to a statically determinate structure by dividing it at the crown. After a unit moment, M_1, and a unit horizontal force, x_1, have been applied, the unit displacement factors, a_{11} and a_{22}, are determined. To expedite the computation, the Δs lengths and the y distances are scaled from the drawing. Then, a_{11} is equal to $\Sigma \Delta s/EI$ and a_{01} represents the displacement of the elastic center as a result of the external moments; i.e., taking the sum of the products $(\Delta s/EI)\ px$, as indicated in Figure K-39. The quotient a_{01}/a_{11}, or $\Sigma \Delta s/EI\ px\Delta x$ divided by $\Sigma \Delta s/EI$ expressions will give the internal moment, M, as a result of the distributed load, p. The displacement factor, a_{22}, is determined similarly, taken as the sum of the moments of the elastic weights $\Delta s/EI$ about the horizontal axis through the elastic center. The X_1 force is obtained from the quotient a_{02}/a_{22}. As a result of symmetry, the vertical internal force X_2 equals zero. Thus, when calculating the internal forces one has to consider only the moments

$$M = M_o - M_1 - X_1 y \qquad \text{(K-65}a\text{)}$$

and the axial forces

$$X = X_o - X_1 \cos a \qquad \text{(K-65}b\text{)}$$

DURABILITY CONSIDERATIONS

Durability of Concrete

Measures to enhance the durability of unreinforced and reinforced concrete culverts are usually specified in national standards or design codes. In Germany, for instance, such directives are given in DIN 4030 (concrete in aggressive water and soil). It is generally accepted that special measures must be taken if the sulfate content of the water in the soil exceeds 0.1 percent. Some of the commonly used methods are (1) use of a sulfate-resisting cement (high-alumina cement), (2) admixing sulfate-resisting ingredients, such as sodium silicate in the amount of 5 to 10 percent, (3) decreasing the porosity of the concrete by special methods, and (4) applying a protective coating of plaster, bituminous epoxy-tar, or plastic. It has been mentioned previously that unreinforced or reinforced concrete centrifugal pipes are the most common type used in culvert construction; one of the reasons for this is that a very high density is obtained in the concrete by the centrifugal method. Experiments indicate (*119, 120*) that use of the centrifugal method can increase density of concrete by

10 to 15 percent with respect to that of concrete compacted mechanically.

Protective coatings of multi-layered plaster are still commonly used in many countries (such as Hungary and Poland); however, it is almost completely unknown in western European countries. As one example, the following multi-layer plaster protective coating is standardized in Hungary:

1. A base coat consisting of about 0.25 in. of mortar (1:4 mix with a water-cement ratio of 0.55 and $D_{max} =$ 5 mm) is applied with a rounded trowel to the prepared concrete surface and left unscreeded and rough. After setting has begun, the limits of each layer are trimmed along a sloping straight line; beyond those limits the excess material is scraped off and the surface is cleaned.

2. In about 24 hr (or when the base coat can still be scratched with the fingernail), a rough layer about 0.25 in. thick (1:3 mix with a water-cement ratio of 0.45 and $D_{max} = 2.5$ mm) is applied over the base coat; 2 hours after placing (or at initial set), it is finished smooth with a wooden trowel.

3. After 20 to 22 hr, a bleeding layer approximately 3⁄16 in. thick (1:2 mix with a water-cement ratio of 0.35 and $D_{max} = 0.8$ to 1.5 mm) is applied and finished smooth with a wooden trowel.

4. A cement paste finish (1 part cement to 1 part water) about 1⁄16 in. thick is applied after 48 hr, finished smooth with a wooden trowel, and refinished with a small wooden trowel after 8 hours and again after another 3 hours. Finally, another cement paste layer about 1⁄64 to 1⁄32 in. thick is added and finished with a steel trowel after 1 hr.

The finish may consist of a single layer if the cement paste is applied with a round brush directly over the bleeding layer when it is still soft and just about ready to set; it is then finished after 1 hr with a steel trowel. If exposed to fresh air frequently, the cement paste layer may be replaced by a cement slurry (1:1 mix) finish, 5⁄32 in. thick, and finished with a rough wooden trowel and then with a steel trowel. The setting times between subsequent layers can be reduced by up to 50 percent under favorable conditions. More than three layers of plastering are hardly ever required; in excessively thick plasterwork the different thermal and shrinkage coefficients of the individual layers consisting of different mixes produce undesirable effects. Curing is most important; regular spraying is continued for 14 days. The completed plaster coating is considered acceptable if the amount of water passing through it does not exceed 0.5 fl oz per square foot per day. In addition to the foregoing preparation methods, the following rules are observed:

1. The cement pastes shall have high tensile strength and low shrinkage characteristics.

2. The temperature of both cement and water shall be about 75° to 80° F at the time of placement.

3. The grain size distribution of the sand and aggregate shall be continuous according to the following specifications:

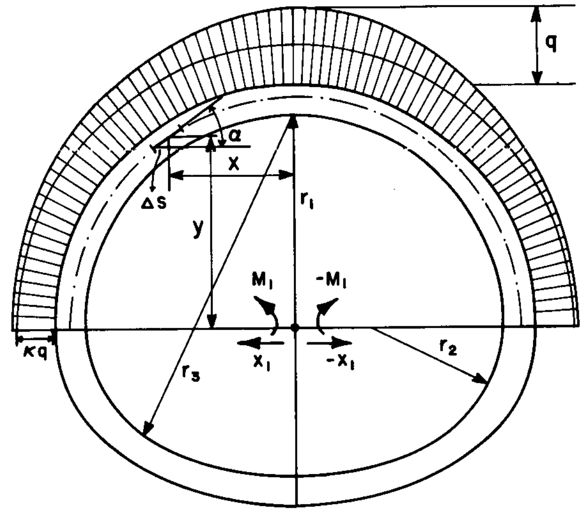


Figure K-39. Assumed load distribution on an egg-shaped culvert.

a. Coarse aggregate ($D_{max} = 5$ mm)

0.0 to 0.2 mm	10-15%
0.2 to 1.0 mm	25-30%
1.0 to 2.5 mm	25-30%
2.5 to 5.0 mm	40-25%

b. Fine aggregate ($D_{max} = 2.5$ mm)

0.0 to 0.2 mm	10-15%
0.2 to 1.0 mm	25-30%
1.0 to 2.5 mm	65-55%

4. Impermeable plaster coatings can be applied only to structures that are already load-bearing.

5. To minimize expansion and shrinkage, finishing shall be accomplished in a space protected from the sun, practically free from drafts, and with steady temperature and humidity conditions.

6. To avoid cracking, the difference in temperature between concrete wall and plaster shall not exceed 35° F.

7. The ideal temperature for applying the plaster coating is between 50° and 70° F.

8. Plastering on newly constructed concrete walls shall not be commenced when the temperature is below 40° F; also, for temperatures in the range of 70° F and greater, 50° to 70° F, and 40° to 50° F, plastering shall not begin within 8, 12, and 16 days, respectively, after stripping.

9. The concrete surface shall be thoroughly cleaned of all dirt, dust, oil, soot, etc., including any calcium efflorescence, and shall be roughened prior to the application of any plaster coat. The dust shall be cleaned with water under pressure prior to plastering, so that the wall will not absorb any water from the plaster.

10. Honeycombed areas shall be chiselled out and replaced with concrete having a cement content approximately 200 to 250 lb/yd³ higher than that in the wall; small areas can be replaced with mortar (1:3 mix).

Some of the most common mistakes and errors in plastering work are the following:

1. The surface is not cleaned properly.
2. A cement slurry base is used, thus reducing the adhesion between the plaster and the concrete.
3. Dry cement is applied to the wall surface, and the properly mixed plaster has a water surplus to transfer to the cement; haircracks will develop.
4. The surface is over-rubbed with a steel trowel; the fine cement particles will work themselves to the surface, and haircracks will develop as a result of shrinkage.
5. Curing is inadequate.
6. The work is done by unskilled laborers.

Bituminous bonded-type protective coatings are usually applied in the manufacturing plant rather than in the field. Not only are the working conditions much more favorable in the plant, but also the surface preparation (cleaning, finishing) and drying, all of which are essential for sound hot-placed protective coatings, can be completed much more thoroughly and safely. These factors are most important because hot bituminous coatings will adhere only to dry surfaces. Before the first layer is applied, the surface must be dried with infrared lamps, gasoline lamps, or flamethrowers. Alternatively, instead of the use of the drying process, cold bitumen emulsions can be applied successfully; the hot bitumen will readily bond to materials of this type.

The use of PVC sheets for coating culverts appears to be promising; it is common practice to use four layers of 120-gauge sheets. The ability of thermoplastic materials (Opanol, Dynogen, Isofol) to stretch to many times their original length is most helpful in overcoming overstresses and stress concentrations caused by differential settlements and lack of local support. Isofol, for example, has a tensile strength of 190 to 250 kg/cm^2 (2,700 to 3,500 psi) and an ultimate elongation of 280 to 340 percent before rupture. PVC-type protective coatings have been used with good results in Czechoslovakia (*121*). A number of admixtures (softeners, stabilizers, lubricants, fillers) have been added to PVC powder for economic reasons and to improve its impermeability and resistance to heat and corrosion. The sheets were 1 to 2 mm (about 1/16 in.) thick and have performed well against corrosion and attacks by chemicals; they are sensitive, however, to gasoline, kerosene, oil, acetone, and ether. Also, it is recommended that prolonged exposure to the sun be avoided.

Because of the lack of bond, the structure practically has to be wrapped in the PVC sheets. Provided that this is possible, the finishing, draining, and drying of the surfaces can be avoided, and this is a tremendous advantage when it comes to field applications. As a result of their ductility, the sheets can span over surface wrinkles without being damaged. Another advantage of the PVC sheet over bituminous coatings is that there is no danger of deterioration or loss of ductility due to aging. The PVC sheets are usually glued (hot or cold) to the culvert wall; this can be done with hot asphalt, various epoxy resins, and other special bonding agents. Prior to the application of such coatings, the surface shall be cleaned thoroughly so that the soft material is not damaged by dust or other rough particles. The sheets can be spliced either hot or cold; cold splicing is similar to rubber patching, and the bonding agent, as a matter of fact, is rubber cement. Hot splicing can be done either by ironing or with high-frequency dielectric heating (melting together the abutting or overlapping sheets at 180° to 200° C or 350° to 400° F) without using any foreign material. Experience indicates that few durability problems concerning concrete and reinforced concrete culverts arise if the appropriate protective measures corresponding to the activity of the water and soil are taken. The life of a culvert is considered to be about 90 years.

Destruction of concrete may result from frost action, which in cement paste has been described as follows. When water begins to freeze in a capillary cavity, the volume increase that accompanies the freezing of the water requires that (1) the cavity dilate by an amount equal to 9 percent of the volume of water frozen, (2) the excess water be forced out through the boundaries of the specimen, or (3) some combination of the preceding two effects. During this process hydraulic pressure is generated, and the magnitude of that pressure depends on (1) the distance to an "escape boundary," (2) the permeability of the intervening material, and (3) the rate at which ice is formed. Experience and research have indicated that disruptive pressures will be developed in a saturated specimen of cement paste unless every capillary cavity in the paste is not farther than 0.003 or 0.004 in. from the nearest escape boundary. Such closely spaced boundaries are provided by the correct use of a suitable air-entraining agent.

The necessity for using entrained air in concrete does not arise from a lack of space to accommodate the increase in water volume caused by freezing. All concretes contain at least ½ percent of air space, and this is more than enough to accommodate the increase in water volume produced by freezing of mature paste. The quantity of air required, ranging upward from 3 percent of the volume of concrete, is that which supplies a sufficient number of bubbles per unit volume of paste to reduce the distances between the bubble boundaries to an adequately low value, or, in other words, the quantity necessary to provide a sufficient number of bubbles to divide the paste into very thin layers.

Curve A in Figure K-40 shows the effect of cooling a saturated specimen of paste through one cycle of freezing and thawing (*122*). Because the paste was mixed in a vacuum, the specimen contained no entrained air. The graph shows only departures from normal thermal changes, the departures having been caused by the freezing of water in the paste. During cooling to about −24° C, this specimen had a strain of $1{,}600 \times 10^{-6}$; when it was warmed back to the original temperature, it showed about 500×10^{-6} residual strain. This effect is inevitable when freezing and thawing a specimen of saturated paste containing no air bubbles. The same kind of curves are obtained from tests on concrete without entrained air. Curve B of the same figure shows the effects of entrained air bubbles in a specimen containing 16 percent entrained air. With this high concentration of air bubbles, freezing and thawing produce no appreciable dilation during the cycle and no residual dilation at the end of the cycle. This same behavior is normally observed in air-entrained concrete.

Durability of concrete depends to a large extent on the selection and control of the aggregates. Those aggregates with high open-pore porosity are likely to produce concrete having poor resistance to frost, because they absorb water easily. When natural aggregates, such as flint gravels, most igneous rocks, and limestone, are used, trouble from wetting and drying cycles is not likely to be serious. As long as the aggregate itself is resistant to frost (that is, has a low water absorption), the shape and grading of the aggregate and the cement-aggregate ratio have little effect. For the same water-cement ratio, however, leaner mixes are more frost-resistant. Because low water content and low water-cement ratio increase the resistance of concrete to frost, the water-cement ratio for exposed concrete should not exceed 0.45, and vibratory densification is preferred.

Resistance to abrasion is of importance for concrete laid in culverts. To achieve such resistance, the mix is usually designed for high compressive strength, rich mixes having greater resistance than lean mixes. Mixes of 1:45 by weight appear to give best results. A low water-cement ratio is very important, and it has even more effect on resistance to abrasion than it has on the compressive strength. The use of tough aggregate, such as crushed igneous rock, gives good results; flint aggregates are too brittle and are generally ineffective. Good densification of the concrete improves its resistance to abrasion.

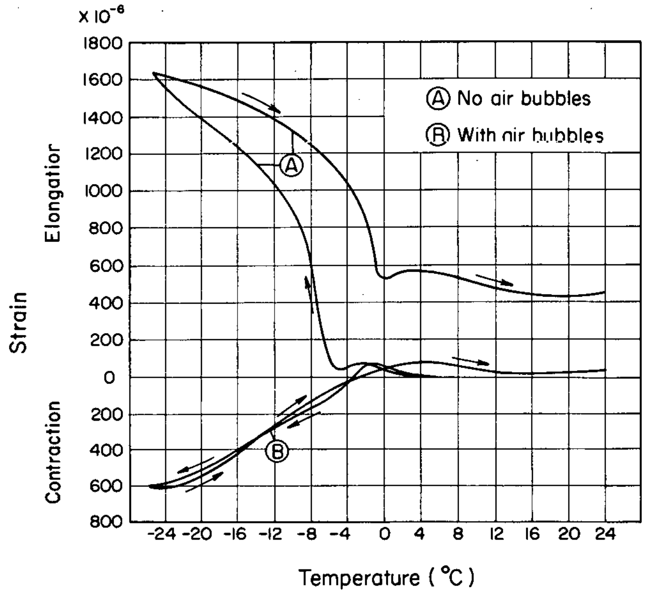


Figure K-40. Mechanical behavior of cement paste.

Durability of Steel

Protection of underground steel structures against corrosion is one of the major problems in the over-all fight for the prolongation of service life. The seriousness of this problem is clearly indicated by the fact that special committees have been established in all industrialized countries in the world to study the corrosion processes and to develop protective methods. In some countries (the Soviet Union, Germany, England, Hungary, etc.) the destruction of steel structures as a result of corrosion has reached alarming proportions, and intensive research activity was started about 10 years ago. These investigations have led to the formulation of a generally accepted theory (*123*) that is summarized in the following paragraphs.

Wet soil is a heterogeneous, porous, colloidal system; it may be considered an electrolyte, and corrosion may be analyzed as primarily an electrochemical process. Basically, three different processes are involved in the destruction of underground steel structures.

1. The anodic process for iron in moist soils is characteristic of that in liquid electrolytes. Only in very dry and highly air-penetrable soils is the anodic process similar to the anodic behavior of iron under atmospheric conditions; that is, when it is considerably retarded as a result of insufficient moisture for hydration of the ions. In prolonged activity there can be observed a gradual suppression of the anodic process as a result of secondary reactions producing insoluble protective corrosion products.

2. The basic cathodic depolarization process in soil corrosion is the ionization of oxygen. Only in very acid soils is it possible to have a parallel process of cathodic depolarization due to discharge and liberation of hydrogen. Transport of oxygen in the soil to the corroding surface can be accomplished by diffusion, convection, and dynamic (directed) mechanisms. In the absence of significant fluctuations in pressure and temperature, the basic means for oxygen transport in soils with a low porosity or consisting of small grains is by diffusion. The rate of diffusion of oxygen is determined by the thickness of the soil layer, its structure, and its moisture content; this rate decreases sharply with an increase in moisture or with an increase in the content of colloidal particles. The rate of oxygen diffusion can diminish tens of thousands of times with an increase in either the moisture content or the amount of clay and colloidal components in sandy soil. For this reason very favorable conditions can develop for corrosion macrocouples of differential aeration. Because of this, the character of corrosion, and particularly corrosion control, can change drastically, depending on soil conditions. Corrosion due to the activity of microcouples operates primarily by cathodic control in most soils (with the exception of very porous and dry soils).

3. Ohmic resistivity of soils can fluctuate within wide limits. With the exception of very dry soils, it is not a principal controlling factor for corrosion processes caused by the activity of microcouples. Only in corrosion caused primarily by long macrocouples can the ohmic resistivity become a basic factor in corrosion.

Table K-7 gives the values of anodic and cathodic controls, as calculated from anodic and cathodic polarizations (P_A and P_C) (*124*), for soil corrosion. It follows from these data that the majority of soils have a predominantly cathodic control; however, very dry soils (Nos. 7 and 8) exhibit cases of predominant anodic control.

TABLE K-7

ANODIC AND CATHODIC POLARIZATION AND CONTROL FOR CORROSION OF STEEL IN VARIOUS SOILS

NO.	SOIL CHARACTERISTIC	ANODIC POLARIZATION, P_A (V/A)	PORTION OF ANODIC CONTROL, $\frac{100 P_A}{P_A+P_C}$ (%)	CATHODIC POLARIZATION, P_C (V/A)	PORTION OF CATHODIC CONTROL, $\frac{100 P_C}{P_A+P_C}$ (%)	RATE OF STEEL CORROSION (G-YR/M^2)	SPECIFIC RESISTANCE OF THE SOIL (OHM/CM)
1	Very wet sand-clay base	10	4	220	96	252	900
2	Salty wet sand	3.3	6	50	94	1,572	1,800
3	Gray clay, with 7% to 10% moisture	50	9	500	91	120	40,800
4	Very wet clayey-sandy soil with pebbles	40	12	300	88	345	180,000
5	Very wet gray clay	36	13	235	87	252	720
6	Slime from a sewer ditch	40	19	170	81	270	30,000
7	Dry sand-clay base	125	53	110	47	92.5	240,000
8	Dry loam with lime	500	56	380	44	26	3,900

The corrosivity of a soil depends on many factors, such as specific resistance, moisture content, acidity, pH, salt content, and air permeability. However, a definite simple relationship between soil corrosivity and its physiochemical properties has not been established. The basic reason for this is that the interaction of micro- and macrocorrosion couples has been overlooked. Small objects (for example, specimens) corrode principally as a result of microcouple activity, and a relatively uniform corrosion damage is produced. This type of corrosion is characterized by the magnitude of cathodic and anodic polarization, and it does not depend on the electrical resistivity of the soil. With an increase in the size of the underground structure, a larger portion of corrosion deterioration is due to macrocouple activity, which is, in turn, directly related to cathodic polarization of the soil. Still more extensive underground structures, such as pipelines, suffer to a large degree from the activity of macrocouples created by nonuniform oxygen penetration in the soil of adjacent sections; this is characterized by local pitting. The basic criteria of soil corrosivity for such structures is the change in the rate of oxygen penetration and in the specific resistance of the soil along the pipeline route.

Anaerobic corrosion of iron in the soil can be considered as a variant of the electrochemical process of corrosion, in which the biological factors promote electrochemical corrosion by their action on the electrode processes. Anaerobic bacteria accelerate corrosion only in cases where the entire structure is under anaerobic conditions. Otherwise, acceleration of corrosion in the nonaerated soil is to a large degree influenced by the activity of differential aeration macro-corrosion couples.

A theoretical examination of the mechanism of underground corrosion leads to the following deductions:

1. Corrosion due to the activity of macrocouples, primarily a result of differential aeration of various sections of the soil, is highly localized (pit formation) and is more dangerous than corrosion by microcouples which, as a rule, is of a more uniform character.

2. Anodic macrocouples (i.e., dangerous local corrosion on sections of long structures) will form as a result of decreased oxygen penetration.

3. Insulating coatings on underground pipelines must be particularly perfect on the anodic sections, because the corrosion process would otherwise concentrate on the bare spots and cause deeper corrosion penetration in these local areas than if there were no coating at all.

4. In addition to the application and improvement of the principal types of protection against underground corrosion (insulating coatings, cathodic protection, and elimination of stray currents), increasing the homogeneity of the soil (fill) immediately adjacent to steel structures will reduce the probability of local couple formation and weaken the activity of those already present.

5. Insulation of sections of a long structure passing from one soil to another with sharply differing oxygen penetration will decrease the probability of forming long differential aeration macrocouples and thus reduce the intensity of local corrosion; this measure, however, will have little effect if stray currents along the pipe are of sufficient voltage to jump the insulated junctions.

6. Corrosion experiments conducted on separate small specimens of steel in a given soil cannot provide a true evaluation of the intensity of corrosion on extensive underground installations passing through that particular soil.

7. The corrosivity characteristics of various soils studied during the examination of old pipelines cannot be used in the evaluation of analogous soils along other routes where these soils may follow a different order.

8. In relation to large buried objects, it is correct to speak not of the corrosivity of the soil but the corrosivity of a given section of the route. The determination of soil corrosivity with small objects can be made on the basis of anodic and cathodic polarization characteristics under the given conditions. Determination of the corrosion activity of a given section of soil along a long structure can be

made on the basis of evaluating the change in oxygen penetration or its proportional cathodic polarization along the route and of the mean ohmic resistivity of the given section.

Steel and corrugated steel culverts are normally protected by galvanization or by a bituminous coating. Although much research work to find more reliable methods of protection is currently going on in most European countries, no definite results have been reported so far. Bituminous coating, if properly applied, provides the necessary protection; however, dynamic loads may cause cracks to develop in the coating, and corrosion is initiated at these cracks. Somewhat related to this situation, it was noted in Hungary that the durability of a steel culvert is related to the frequency of its dynamic loading; pipes exposed to frequent dynamic loads corrode much faster than those experiencing relatively few dynamic loads. It is generally accepted that the life of a properly coated steel or corrugated steel culvert is about 50 years, whereas an uncoated culvert or one exposed to frequent dynamic loads may last for only 10 or 15 years. Abrasion of a culvert is not serious compared to corrosion, but it ruins the protective coating which, in turn, results in excessive corrosion.

INSTALLATION

The shaping of the construction pit is directly related to the method of culvert design because its dimensions affect the magnitude and distribution of loads acting on the culvert. National standards or design codes specify the working space and trench widths; the various standards give values only slightly different from one another. As an example, the German standard DIN 18300 gives the following dimensions for the width of the construction pit or trench:

Unsheeted pit with a slope greater than 60° . . $d + 70$ cm
Unsheeted pit with a slope less than 60° . . . . . $d + 40$ cm
Sheeted pit . . . . $d + 70$ cm

If the outer diameter, d_o, is less than or equal to 60 cm, d_o must be taken equal to 60 cm for depths of 1.75 m and less, whereas a minimum value of 80 cm must be used for d_o if the depth of the trench is greater than 1.75 m.

The slope angle depends on the surface loads, if any, and the soil characteristics. A protective strip 60 cm wide must be kept free along the edge of the trench. If the soil is sand or gravel with little or no cohesion and a maximum grain size of 60 mm or less, the slope must be 45° or less. Slope angles up to 60° may be used if the grain size of the cohesionless sand or gravel exceeds 60 mm, or the soil is cohesive or has to be quarried by pickax. Vertical slopes may be cut without sheeting only in solid stone or rock. Safety measures are specified in standards—for instance, DIN 18306 and DIN 18303; these standards give examples of the type of support or sheeting to be used for different conditions.

Special attention is devoted to dewatering the construction pit. Experience has indicated that culvert failures can often be attributed to the uneven loosening of the soil foundation caused by the use of inappropriate dewatering methods. When dewatering problems are encountered, contractors try to solve them by collecting the seepage in sumps and ditches (open pumping); this practice is often used, even when the situation requires a different solution. It is a common mistake to believe that the only purpose of dewatering is to remove water from the soil. If the soil is fine-grained, for instance, the actual amount of water removed from the soil is unimportant; in such a situation, a well-point dewatering system is often used to stabilize the soil (i.e., to avoid harmful loosening of the soil caused by excessive hydraulic gradients). If open pumping is used instead of a well-point system, or if the installation of the well-point system is not perfect, soil loosening occurs, and this may result in uneven settlement and a consequent failure.

As with bedding conditions, the structural performance of a pipe culvert is significantly influenced by the method of backfilling. A definite distinction is made between embedding the culvert and refilling the pit. Embedding is the first part of the backfilling operation, and it generally includes filling the space between the pipe and the sides of the trench up to 30 cm above the crown of the pipe. It is universally accepted that only approved material containing no large stones can be used. Different standards and specifications allow the embedding work to be placed in layers not exceeding 15 to 20 cm in loose depth; according to the English specification, the maximum loose depth is 9 in. The compaction of this material must be very carefully controlled, because this soil adjacent to the pipe greatly affects the pressure distribution on the pipe, as shown by experiments (*109*). If the soil in this zone is well compacted, the pipe will carry less vertical load. Proper compaction is most important in the case of corrugated steel culverts. Although the second part of the backfilling operation is less sensitive, proper precautions associated with each particular item of compaction equipment must be taken. DIN 4033 specifies that a minimum soil cover above the crown of the pipe must be reached before a certain type of compaction equipment may be used.

APPENDIX L

CULVERT DESIGN IN JAPAN *

Pipe culverts can be classified into reinforced concrete pipe (including centrifugal reinforced concrete pipe, specified in JIS—Japanese Industrial Standard—A 5303), prestressed concrete pipe, and corrugated metal pipe. Although centrifugal reinforced concrete pipe and corrugated metal pipes are used widely in Japan, prestressed concrete pipes are becoming prevalent in recent years because of their strength. Reinforced concrete pipes, including centrifugal reinforced concrete pipes, are not applicable in cases where the soil cover is small and wheel loads are large or where the soil cover is large; in these cases, reinforced concrete box culverts, arch culverts, or corrugated metal pipe culverts are used. On the other hand, prestressed concrete pipes are more suitable under high fills because their strength against external pressure is high.

CENTRIFUGAL REINFORCED CONCRETE PIPE

The qualities and the details of centrifugal reinforced concrete pipes are specified in JIS A 5303. The pipe diameter can be determined graphically from the anticipated amount of flow and the grade of pipe. With a knowledge of the pipe diameter, soil cover, and class of conduit, as shown in Figure L-1, the bedding angle can be determined from Figure L-2. For cases where the soil cover is not indicated in Figure L-2, prestressed concrete pipe or corrugated metal pipe, rather than centrifugal reinforced concrete pipe, should be used.

PRESTRESSED CONCRETE PIPE

The method of determining the pipe diameter is the same as that for centrifugal reinforced concrete pipe. Because the qualities and details of prestressed concrete pipes are not yet specified in the Japanese Industrial Standards, a temporary classification has been provided by the Japan Road Association; this classification is given in Table L-1 for three strengths of pipe.

DESIGN CONDITIONS FOR CONCRETE PIPES

The design conditions for concrete pipes are as follows:

1. Backfill and embankment soil: The backfill and embankment soil is assumed to be sand or sandy soil having a unit weight of 1,800 kg/m^3 and an angle of internal friction equal to 30°.

2. Live load: The live load for design purposes is the so-called T-20 load that is shown in Figure L-3; this same load is used for design of steel bridges. The live load acting on the culvert is considered independent of the soil cover and is calculated by

$$W_t = \frac{3.78b}{H + 0.1} \qquad \text{(L-1)}$$

in which W_t is the live load acting on the pipe in kg/m; H is the soil cover above the top of the pipe in meters; and

* The information for this appendix was obtained through the courtesy of Tadashi Kondo, Subchief of Design Section, Tokyo-Nagoya Expressway Department, Japan Public Highway Corporation. The report by Mr. Kondo has been edited, and notation and terminology have been altered to maintain consistency throughout the report. Every effort has been made to present faithfully the information furnished by Mr. Kondo; the researchers accept responsibility for any inadvertent errors.

TABLE L-1

CLASSIFICATION OF PRESTRESSED CONCRETE PIPE

INTERNAL PIPE DIAMETER (CM)	CORE THICKNESS (MINIMUM VALUE) (MM)	LENGTH (STANDARD VALUE) (MM)	EXTERNAL LOAD STRENGTH (KG/M)		
			TYPE 1	TYPE 2	TYPE 3
50	40	4,000	12,900	10,900	9,300
60	42	4,000	13,100	11,000	9,300
70	45	4,000	13,200	11,000	9,300
80	50	4,000	13,300	11,000	9,300
90	55	4,000	13,500	11,100	9,300
100	60	4,000	13,700	11,200	9,400
110	65	4,000	14,000	11,400	9,400
120	70	4,000	14,400	11,700	9,600
135	75	4,000	15,200	12,000	9,800
150	85	4,000	16,100	12,700	10,100
165	95	4,000	17,100	13,500	10,500
180	100	4,000	18,200	14,400	10,900
200	110	4,000	19,400	15,100	11,400
220	120	3,600, 2,400, or 2,360	19,700	15,600	11,900
240	120	3,600, 2,400, or 2,360	19,800	16,000	12,300

b is the ditch width at the top of the pipe or the external pipe diameter in meters. The impact factor is taken as 0.3 (or 1.3, depending upon interpretation).

3. Earth load: Earth loads on the culvert are determined according to the classical Marston-Spangler procedure.

4. Other loads: Loads caused by horizontal earth pressure, earthquakes, and temperature changes are assumed to be negligible.

Based on the pipe diameter, soil cover, and class of conduit, the bedding and type of pipe can be determined from Figure L-4. The indicated curves have been obtained from the Marston-Spangler formula.

The classes of conduits, shown in Figure L-1, are divided into three types. The conduit is termed projecting when the top of the pipe is higher than the natural ground and fill is constructed over them. A semi-ditch condition occurs when the top of the pipe in a trench is lower than the height of the natural ground and fill is constructed above natural ground; this condition is applicable when the projection ratio, p_r', is equal to or greater than 0.5. The ditch condition is found when the pipe is installed in a trench with no fill above the natural ground.

Bedding conditions, shown in Figure L-5, are divided into three categories. Type A provides for a concrete bed under the lower 120° of the pipe; Type B provides for a deep sand bed under the lower 120° of the pipe; and Type C provides for a shallow sand bed or formed sand bed under the lower 90° of the pipe.

The following suggestions are offered regarding the use of Figure L-4:

1. Type A bedding conditions should be avoided if possible.

2. For high fills the sequence of operations should be as follows: (1) construct the embankment without the pipe to a height above the proposed top of the pipe, and (2) excavate a trench in the compacted embankment and install the pipe as a semi-ditch conduit, thereby lowering the earth load on the pipe.

3. Because the earth loads on conduits under semi-ditch and ditch conditions are about the same when the soil cover is less than about 5 m, the semi-ditch condition can be used instead of the ditch condition.

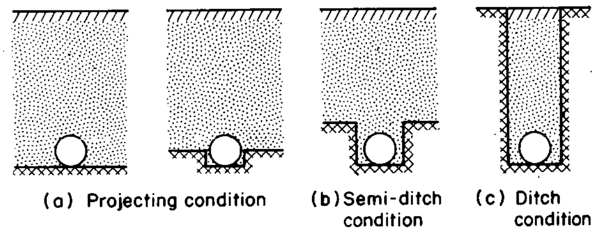


Figure L-1. Classes of conduits.

CORRUGATED METAL PIPE

As indicated in Table L-2, corrugated metal pipe is classified according to the shape of the corrugation, cross-sectional shape, and joint structure. Two types of corrugation are used; one has corrugations with a pitch of about 68 mm and a depth of about 13 mm, whereas the second has about a 150-mm pitch with a 50-mm depth. The cross-sectional shapes are shown in Figure L-6. Plate thicknesses are based on soil cover and pipe diameter, and are determined from Tables L-3 to L-7. The top of the pipe must be below the pavement structure; the minimum soil cover is normally 60 cm, although 30 cm is permitted for roads with light traffic and for temporary haul roads.

Bedding materials must be sand, sand and gravel, or a sandy soil with low compressibility, high density, and a

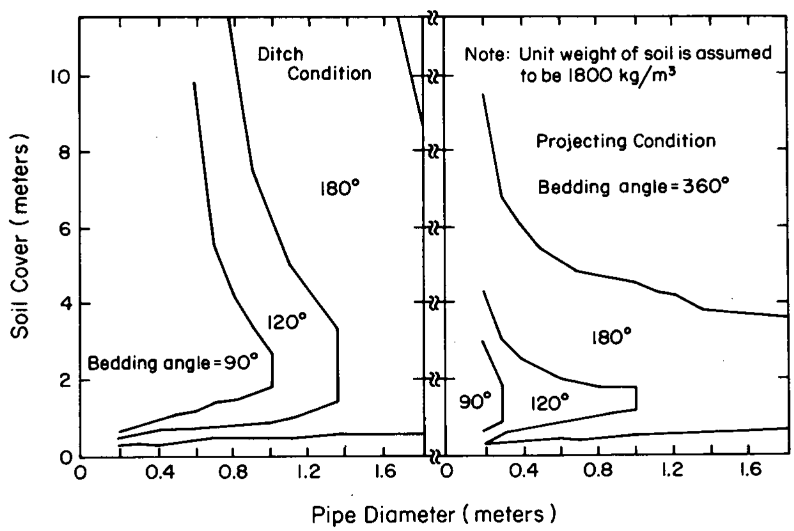


Figure L-2. Determination of bedding angle for centrifugal reinforced concrete pipe.

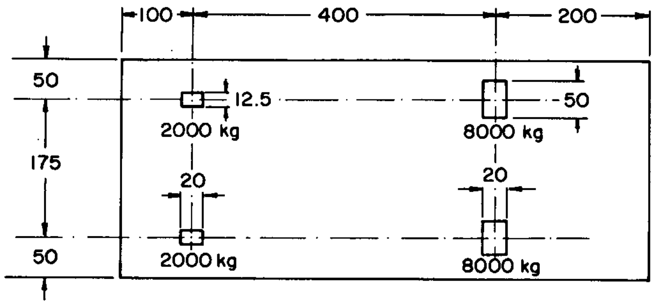


Figure L-3. T-20 load for design of steel bridges.

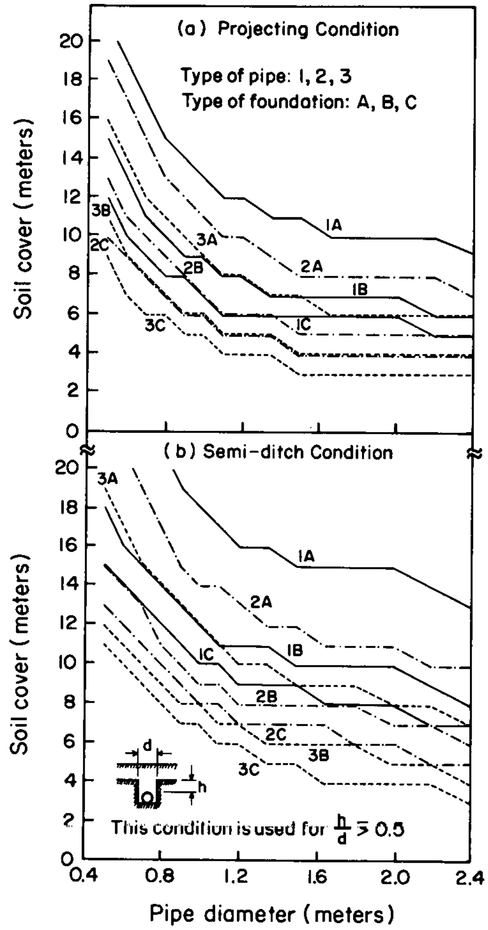


Figure L-4. Determination of pipe and bedding for prestressed concrete pipe.

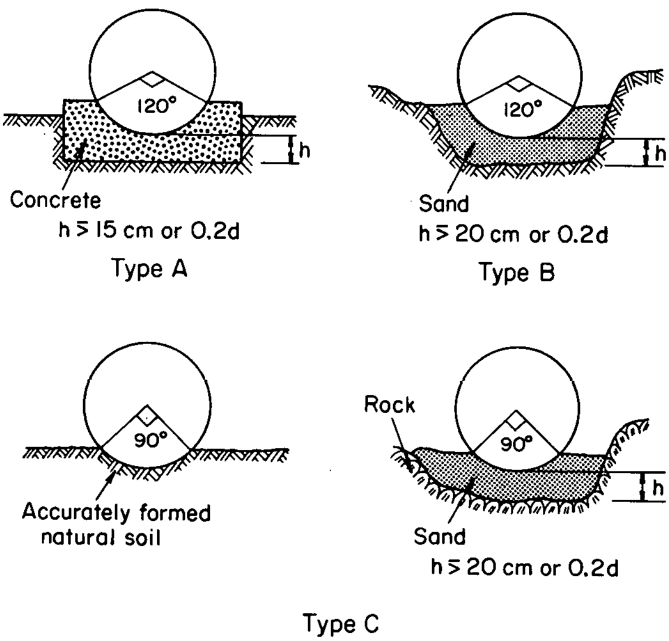


Figure L-5. Bedding conditions for concrete pipes.

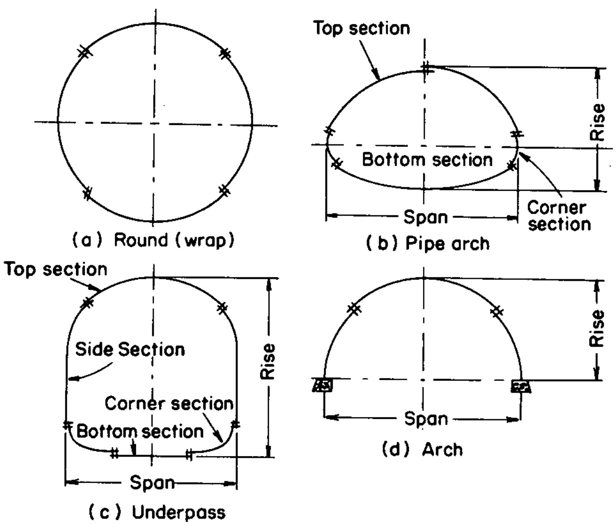


Figure L-6. Sectional shapes of corrugated metal pipe culverts.

maximum particle size of 10 cm. The bedding conditions for corrugated metal pipes are divided into four types. When the lower one-fourth of a pipe rests on a good sandy soil, gravel, or sand and gravel (Fig. L-7a) it is termed a stiff foundation. A so-called normal foundation is shown in Figure L-7b. For a rock foundation (Fig. L-7c) the

TABLE L-2

CORRUGATED METAL PIPE CULVERTS

SHAPE OF CORRUGATION	CROSS-SECTIONAL SHAPE	JOINT STRUCTURE TYPE	JOINT STRUCTURE PROCEDURE
I	Round	Flange	Bolt
		Wrap	Bolt
		Wrap	Rivet
		Wrap	Weld
		Butt	Weld
II	Round Pipe arch Underpass Arch	Wrap	Bolt

TABLE L-3

PLATE THICKNESSES FOR TYPE I ROUND CULVERTS

PIPE DIAMETER (CM)	PLATE THICKNESS (MM), BY SOIL COVER (M)												
	MIN. TO 3.0	3.1 TO 4.5	4.6 TO 6.0	6.1 TO 7.5	7.6 TO 9.0	9.1 TO 10.5	10.6 TO 12.0	12.1 TO 13.5	13.6 TO 15.0	15.1 TO 18.0	18.1 TO 21.0	21.1 TO 24.0	24.1 TO 30.0
40	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6	2.0	2.0	2.7	2.7
45	1.6	1.6	1.6	1.6	1.6	1.6	1.6	2.0	2.0	2.0	2.7	2.7	2.7
50	1.6	1.6	1.6	1.6	1.6	1.6	2.0	2.0	2.0	2.7	2.7	2.7	3.2
60	1.6	1.6	2.0	2.0	2.0	2.0	2.0	2.0	2.7	2.7	2.7	3.2	3.2
75	2.0	2.0	2.0	2.0	2.0	2.7	2.7	2.7	3.2	3.2	3.2	4.0	4.0
90	2.0	2.0	2.7	2.7	2.7	3.2	3.2	3.2	4.0	4.0	4.0	4.0	4.0
110	2.7	2.7	2.7	2.7	3.2	4.0	4.0	4.0	4.0	4.0	4.0		
120	2.7	2.7	2.7	3.2	4.0								
140	2.7	2.7	3.2	4.0	4.0								
150	3.2	3.2	4.0	4.0									
170	3.2	3.2	4.0										
180	3.2	4.0											

Note: Type II culverts are desirable when the pipe diameter is equal to or greater than 150 cm.

TABLE L-4

PLATE THICKNESSES FOR TYPE II ROUND CULVERTS

PIPE DIAMETER (CM)	PLATE THICKNESS (MM), BY SOIL COVER (M)											
	MIN. TO 1.7	1.8 TO 3.0	3.1 TO 4.5	4.6 TO 6.0	6.1 TO 7.5	7.6 TO 9.0	9.1 TO 10.5	10.6 TO 12.0	12.1 TO 13.5	13.6 TO 15.0	15.1 TO 16.5	16.6 TO 18.0
150	2.7	2.7	2.7	2.7	2.7	3.2	3.2	3.2	3.2	3.2	4.0	4.0
200	2.7	2.7	2.7	3.2	3.2	3.2	4.0	4.0	4.0	4.0	4.5	4.5
250	3.2	2.7	3.2	3.2	4.0	4.0	4.0	4.5	4.5	5.3	5.3	5.3
300	4.0	3.2	4.0	4.0	4.5	5.3	5.3	5.3	6.0	6.0	6.0	7.0
400	4.5	4.0	4.5	5.3	5.3	6.0	6.0	7.0	7.0			
450	5.3	4.5	5.3	6.0	6.0	7.0						

TABLE L-5

PLATE THICKNESSES FOR PIPE-ARCH CULVERTS

SPAN (CM)	PLATE THICKNESS (MM), BY SOIL COVER (M)														
	MIN. TO 0.4	0.5 TO 0.7	0.8 TO 1.0	1.1 TO 1.3	1.4 TO 1.6	1.7 TO 1.9	2.0 TO 2.2	2.3 TO 2.5	2.6 TO 2.8	2.9 TO 3.1	3.2 TO 3.4	3.5 TO 3.7	3.8 TO 4.0	4.1 TO 4.3	4.4 TO 4.6
200	2.7	2.7	2.7	2.7	2.7	2.7	2.7	2.7	2.7	2.7	2.7	2.7	2.7	2.7	3.2
250	3.2	3.2	3.2	2.7	2.7	2.7	2.7	2.7	2.7	2.7	2.7	3.2	3.2	3.2	3.2
300	4.0	4.0	3.2	3.2	3.2	3.2	3.2	3.2	3.2	3.2	3.2	4.0	4.0	4.0	4.5
350	4.5	4.0	4.0	4.0	3.2	3.2	3.2	3.2	4.0	4.0	4.0	4.0	4.5	4.5	5.3
400	5.3	5.3	4.5	4.0	4.0	4.0	4.0	4.0	4.0	4.5	5.3	5.3	5.3	6.0	7.0
450	6.0	5.3	5.3	4.5	4.5	4.0	4.5	4.5	4.5	5.3	5.3	6.0	6.0	7.0	7.0
500	7.0	6.0	5.3	5.3	5.3	5.3	6.0	6.0	7.0	7.0					

TABLE L-6

PLATE THICKNESSES FOR UNDERPASS TYPE OF CULVERT

SPAN (CM)	RISE (CM)	PLATE THICKNESS (MM), BY SOIL COVER (M) MIN. TO 1.5	1.6 TO 3.0	3.1 TO 4.5	4.6 TO 6.0	6.1 TO 7.5	7.6 TO 9.0	9.1 TO 10.5	10.6 TO 12.0	12.1 TO 13.5	13.6 TO 15.0
170	170 to 200	2.7	2.7	2.7	3.2	3.2	3.2	4.0	4.0	4.0	4.0
to		to	to	to	to	to	to	to	to	to	to
200	200 to 250	3.2	2.7	3.2	3.2	4.0	4.0	4.0	4.0	4.5	4.5

TABLE L-7

PLATE THICKNESSES FOR ARCH CULVERTS

SPAN (CM)	PLATE THICKNESS (MM), BY SOIL COVER (M) MIN. TO 0.7	0.8 TO 1.0	1.1 TO 1.3	1.4 TO 1.6	1.7 TO 1.9	2.0 TO 2.2	2.3 TO 2.5	2.6 TO 2.8	2.9 TO 3.1	3.2 TO 3.4	3.5 TO 3.7	3.8 TO 4.0	4.1 TO 4.3	4.4 TO 4.6	4.7 TO 4.9	5.0 TO 5.2	5.3 TO 5.5	5.6 TO 5.8	5.9 TO 6.2
150	2.7	2.7	2.7	2.7	2.7	2.7	2.7	2.7	2.7	3.2	4.0	4.5	5.3	5.3	6.0	6.0	6.0	7.0	7.0
300	3.2	3.2	2.7	2.7	2.7	2.7	2.7	3.2	3.2	4.0	4.0	4.5	5.3	6.0	6.0	7.0	7.0		
350	4.0	4.0	3.2	3.2	2.7	3.2	3.2	4.0	4.0	4.5	5.3	6.0	6.0	7.0	7.0				
400	4.5	4.0	4.0	3.2	3.2	3.2	4.0	4.0	4.5	5.3	6.0	6.0	7.0	7.0					
450	6.0	5.3	4.5	4.0	4.0	4.0	4.5	5.3	6.0	6.0	7.0	7.0							
500	7.0	6.0	6.0	5.3	4.5	5.3	6.0	6.0	7.0	7.0									
550	7.0	7.0	6.0	6.0	5.3	6.0	6.0	7.0	7.0										
600			7.0	7.0	6.0	7.0	7.0												
650					7.0	7.0	7.0												
700						7.0													

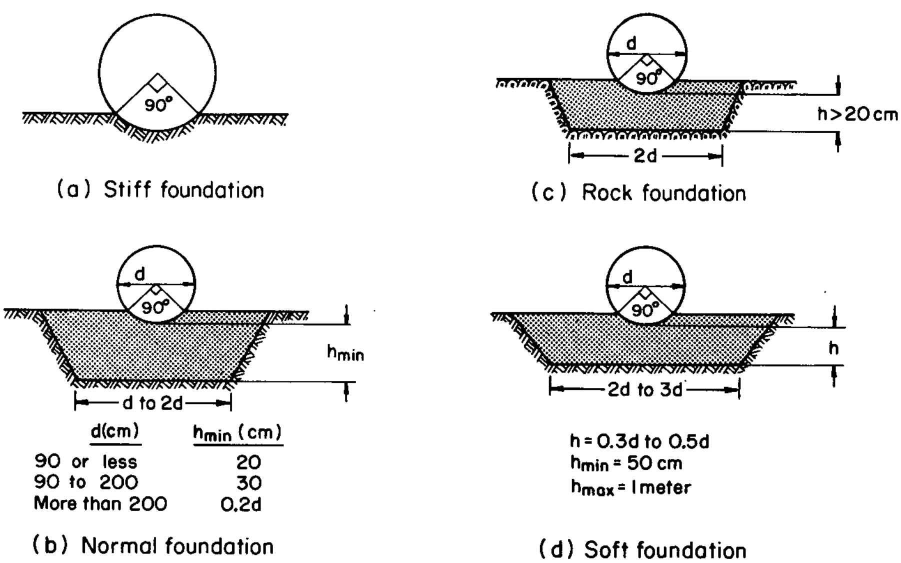
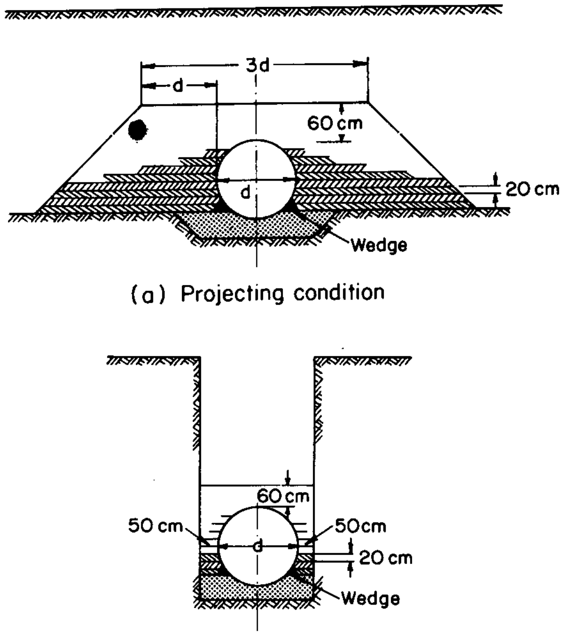


Figure L-7. Bedding conditions for corrugated metal pipes.

rock is excavated to a minimum depth of 20 cm below the bottom of the pipe for a width of twice the pipe diameter, and the pipe is bedded in a sandy soil; the thickness of the bed is a minimum of 20 cm when the fill height is 5 m, with an increase of 4 cm per meter for each additional meter of fill height. In the case of a soft foundation (Fig. L-7d) the minimum thickness of the bed is 50 cm for a width of 2 to 3 times the pipe diameter; the bed thickness is usually 0.3 to 0.5 times the pipe diameter with a maximum of 1 m.

Backfilling around corrugated metal pipe is accomplished with good, well-compacted material in about 20-cm lifts in such a way that the difference in height between the fill on either side of the pipe is as small as possible, as shown in Figure L-8. The wedges shown in Figure L-8 are compacted with compacting rods. The pipe is covered by as much as 60 cm of the same material as used for backfill.



(a) Projecting condition

(b) Ditch condition

Figure L-8. Backfill around corrugated metal pipe.

APPENDIX M

CANADIAN CORRUGATED METAL STRUCTURES

Owing largely to the efforts of Dr. G. G. Meyerhof of Nova Scotia Technical College, significant progress in the area of shallow corrugated metal structures has been made in Canada in recent years. Apart from only minor modifications, the loading and deformation computations follow the established methods of Spangler (*29*) and White (*9*). However, new formulae have been proposed for failure criteria. The most significant practical application of these efforts to date has been the successful installation of an elliptical arch plate culvert with a 40-ft span.

As just indicated, in Canadian design practice culvert loads are determined according to either the Marston-Spangler load distribution or the uniform load distribution of White. Live loading is included for shallow cover heights, but it is neglected for cases where the cover is more than 10 ft above the culvert. For determination of deflection, a simplification of the Iowa formula is used. If the flexural rigidity, EI, is neglected, the bedding factor, K, taken as 0.081, the deflection lag factor, D_ℓ, taken as 1, and the culvert load, W_c, taken as $2rp_v$ (in which r is the radius of the culvert and p_v is the vertical free field stress at the level of the top of the culvert), the deflection, Δx, based on the Iowa formula

$$\Delta x = \frac{D_\ell K W_c r^4}{EI + 0.061\, e r^4} \qquad \text{(M-1)}$$

becomes

$$\Delta x = 2.7\frac{p_v}{e} \qquad \text{(M-2)}$$

in which e is the coefficient of subgrade reaction. To account for the decreased soil support in cases where the cover height is small, the following modified formula is used:

$$\Delta x = \frac{2.7\, p_v}{\left[1 - \left(\dfrac{r}{r+H}\right)^2\right] e} \qquad \text{(M-3)}$$

in which H is the average cover height. As is normally the case for practice in the United States, a maximum deflection of 5 percent of the culvert diameter is recommended.

For small-diameter corrugated metal culverts where the backfill is carefully controlled, the yield strength of the material is usually considered to be entirely adequate as a failure criterion. For larger structures, especially those with relatively shallow cover heights, it is considered desirable to analyze the structure for stability against buckling. Meyerhof and Baikie (*69*) determined that the buckling stress, f_b, of steel sheets bearing against a uniform, compact soil is:

$$f_b = \frac{2}{A}\sqrt{\frac{eEI}{1-\nu_c^2}} \tag{M-4}$$

in which A is the cross-sectional area of the conduit wall per unit length; and ν_c is Poisson's ratio for the conduit material. The critical buckling stress, f_c, is taken as

$$f_c = \frac{f_y}{1 + f_y/f_b} \tag{M-5}$$

in which f_y is the yield strength of the culvert material. The buckling stress, f_b, according to Meyerhof and Fisher (*45*), is practically independent of the conduit radius, provided the ratio r/L exceeds 2, in which L, the relative stiffness of the culvert and the soil, is given by

$$L = \sqrt[4]{\frac{EI}{(1-\nu_c^2)\,e}} \tag{M-6}$$

Hence, the critical buckling stress for cases where the cover height is greater than the conduit diameter becomes

$$f_c = \frac{f_y}{1 + \left(f_y \frac{A}{2}\right)\sqrt{(1-\nu_c^2)/eEI}} \tag{M-7}$$

When the cover height is less than the conduit diameter, Eq. M-7 is modified to read:

$$f_c = \frac{f_y}{1 + \left(f_y \frac{A}{2}\right)\sqrt{(1-\nu_c^2)\,2r/eEIH}} \tag{M-8}$$

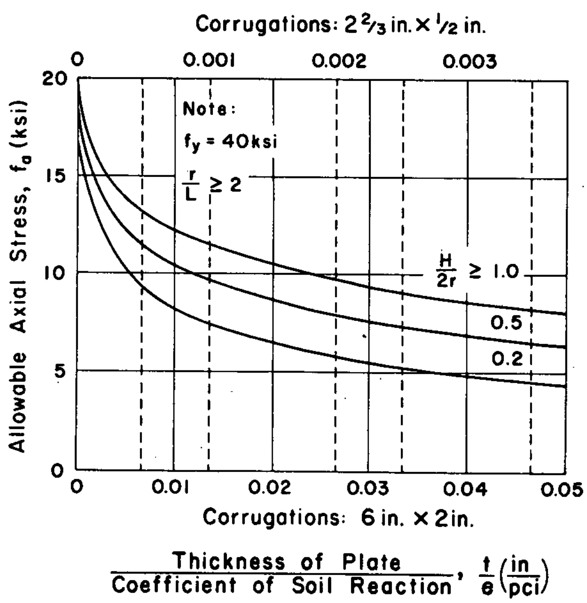


Figure M-1. Allowable stress for uniformly loaded underground steel structures.

Laboratory model tests on flexible pipes, uniformly loaded under low cover height conditions, have indicated that, for cover heights less than about one-quarter of the pipe diameter, the buckling strength is very sensitive to changes in cover height; failure under these conditions in the model tests generally occurred by sudden collapse of the upper portion of the pipe. Allowable stresses for uniformly loaded steel plate structures with various heights of cover are shown in Figure M-1.

Because a large proportion of the loading for cases of low cover height may be due to live loading, allowance must be made for load concentrations. Model tests have indicated that, for low cover conditions, reasonable agreement between observed and theoretical buckling strengths can be obtained by reducing the coefficient of soil reaction, e, to about one-quarter to one-half of its normal value. Further modification of the foregoing procedure is required for load eccentricity; Meyerhof (*21*) reported that, for unfavorable conditions of external friction, both theory and model tests indicate a buckling strength for eccentric loading about one-half that for concentric loading.

Canadian design practice generally involves the use of a safety factor of 2 for corrugated metal conduits. However, careful attention is given to determining realistic physical properties for both the structure and the fill. Conservative values of the coefficient of soil reaction, e, are used, and consideration is given to saturation of the fill. Meyerhof (*21*) has suggested the following relationships for the determination of e:

For dry and moist sands. . . . $e = K_s H/1.5r$ (M-9)

For submerged sands . . . . . . $e = K_s H/3r$ (M-10)

For clays . . . . $e = K_c/1.5r$ (M-11)

in which K_s and K_c are constants of soil reaction for sands and clays, respectively. Their values may be obtained from triaxial tests or, alternatively, estimated on the basis of the soil density or consistency, as indicated in Table M-1. Design based on these methods presumes careful construction control to ensure implementation of the intended design. Where the foundation soil consists of either soft soil or rock, a bedding of compacted granular material is required to provide relatively uniform conditions around

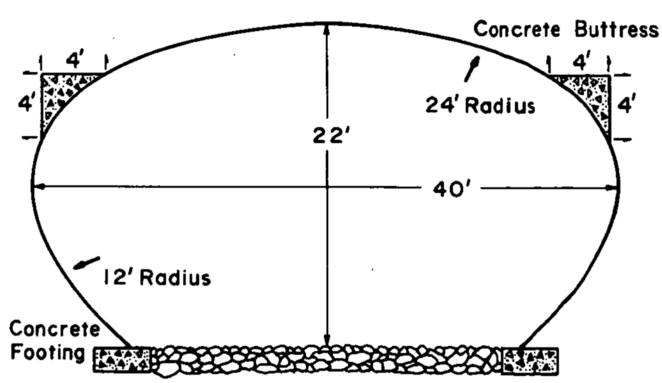


Figure M-2. Culvert section used on the Ontario County crossing.

TABLE M-1

VALUE OF CONSTANT OF SOIL REACTION FOR SANDS AND CLAYS

RELATIVE DENSITY OF SAND	PERCENTAGE OF STANDARD PROCTOR DENSITY	CONSTANT, K_s (PCI)	RELATIVE CONSISTENCY OF CLAY	CONSTANT, K_c (PSI)
Loose	Under 90	Under 4	Stiff	Under 1,000
Compact (medium)	90-100	4-12	Very stiff	1,000-2,000
Dense	100+	12+	Hard	2,000+

Note: The values for clay correspond to 95 to 100 percent of standard Proctor density.

the conduit; dense granular material is also preferable for the backfill. Compaction of the backfill is particularly important under the haunches and for a distance of about one diameter around the structure. Although it is desirable, granular material is not essential and, in general, fill materials normally used for well-constructed embankments are satisfactory.

One especially significant application of the Canadian research achievements in flexible culverts is the successful installation of a 40-ft-span arch plate culvert on the Ontario County crossing of Armstrong Creek near Whitby; a diagram of the culvert cross section is shown in Figure M-2. Special consideration has been given in the design to buckling stability. Model tests indicated the existence of critical areas at the 10 o'clock and 2 o'clock positions on the culvert wall. It was found that, when the fill height is low, the passive resistance acting over these areas may be insufficient and serious deformations may occur, resulting in catastrophic snap-through buckling. Cast-in-place concrete "ears," as shown in Figure M-2, were used to reduce the likelihood of this type of failure.

The successful completion of a culvert of this size has introduced the possibility of a vast new field of application for similar corrugated metal structures. The Canadian designers consider spans of 60 to 80 ft feasible. Such spans would permit the use of corrugated metal structures in place of conventional bridges for both drainage structures and overpasses; cost savings have been conservatively estimated at about 25 to 50 percent.

In general, Canadian research and practice has a great deal to offer the flexible culvert designer, particularly where low cover heights and large spans are concerned. Under these conditions, buckling stability may become critical. Largely on the basis of extensive model testing, a design method suitable for considering these factors has been established, and a number of impressive structures have been constructed.

APPENDIX N

BIBLIOGRAPHY

An extensive 80-page bibliography containing approximately 900 items on the analysis, design, and installation of pipe culverts and covering the period 1900 to 1968 was compiled by the researchers and submitted with the project report. The bibliography is not published, but may be obtained on a loan basis by writing to: Program Director NCHRP, Highway Research Board, 2101 Constitution Avenue, N.W., Washington, D.C. 20418.

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