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FACTORS AND TRENDS IN TRIP LENGTHS

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RESEARCH SPONSORED BY THE AMERICAN ASSOCIATION OF STATE HIGHWAY OFFICIALS IN COOPERATION WITH THE BUREAU OF PUBLIC ROADS

SUBJECT CLASSIFICATION:
TRAFFIC MEASUREMENTS
URBAN LAND USE
URBAN TRANSPORTATION SYSTEMS

HIGHWAY RESEARCH BOARD
DIVISION OF ENGINEERING NATIONAL RESEARCH COUNCIL
NATIONAL ACADEMY OF SCIENCES—NATIONAL ACADEMY OF ENGINEERING 1968
Systematic, well-designed research provides the most effective approach to the solution of many problems facing highway administrators and engineers. Often, highway problems are of local interest and can best be studied by highway departments individually or in cooperation with their state universities and others. However, the accelerating growth of highway transportation develops increasingly complex problems of wide interest to highway authorities. These problems are best studied through a coordinated program of cooperative research.

In recognition of these needs, the highway administrators of the American Association of State Highway Officials initiated in 1962 an objective national highway research program employing modern scientific techniques. This program is supported on a continuing basis by funds from participating member states of the Association and it receives the full cooperation and support of the Bureau of Public Roads, United States Department of Transportation.

The Highway Research Board of the National Academy of Sciences-National Research Council was requested by the Association to administer the research program because of the Board's recognized objectivity and understanding of modern research practices. The Board is uniquely suited for this purpose as: it maintains an extensive committee structure from which authorities on any highway transportation subject may be drawn; it possesses avenues of communications and cooperation with federal, state, and local governmental agencies, universities, and industry; its relationship to its parent organization, the National Academy of Sciences, a private, nonprofit institution, is an insurance of objectivity; it maintains a full-time research correlation staff of specialists in highway transportation matters to bring the findings of research directly to those who are in a position to use them.

The program is developed on the basis of research needs identified by chief administrators of the highway departments and by committees of AASHO. Each year, specific areas of research needs to be included in the program are proposed to the Academy and the Board by the American Association of State Highway Officials. Research projects to fulfill these needs are defined by the Board, and qualified research agencies are selected from those that have submitted proposals. Administration and surveillance of research contracts are responsibilities of the Academy and its Highway Research Board.

The needs for highway research are many, and the National Cooperative Highway Research Program can make significant contributions to the solution of highway transportation problems of mutual concern to many responsible groups. The program, however, is intended to complement rather than to substitute for or duplicate other highway research programs.

This report is one of a series of reports issued from a continuing research program conducted under a three-way agreement entered into in June 1962 by and among the National Academy of Sciences-National Research Council, the American Association of State Highway Officials, and the U. S. Bureau of Public Roads. Individual fiscal agreements are executed annually by the Academy-Research Council, the Bureau of Public Roads, and participating state highway departments, members of the American Association of State Highway Officials.

This report was prepared by the contracting research agency. It has been reviewed by the appropriate Advisory Panel for clarity, documentation, and fulfillment of the contract. It has been accepted by the Highway Research Board and published in the interest of an effectual dissemination of findings and their application in the formulation of policies, procedures, and practices in the subject problem area.

The opinions and conclusions expressed or implied in these reports are those of the research agencies that performed the research. They are not necessarily those of the Highway Research Board, the National Academy of Sciences, the Bureau of Public Roads, the American Association of State Highway Officials, nor of the individual states participating in the Program.

NCHRP Project 7-4 FY '64 & '65 Continued
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Knowledge of the characteristic trends in trip lengths is needed by transportation planners to determine future urban travel demands. The research presented in this report analyzes trip lengths of various cities with regard to the influence of trip purpose, size and spatial characteristics of the area, level of the transportation service, and some of the socio-economic factors involved. This report will be of particular interest to the transportation analyst who is estimating travel patterns for the expected urban growth in his area.

Our urban areas have experienced substantial growth in the past few decades, and the trends seem to indicate that the urban population will continue to increase at a relatively rapid rate. In order to adequately plan transportation facilities to serve these expanding metropolitan areas, information is needed to determine where and how people and goods will move in the years to come. Inasmuch as no one can actually see into the future, an impression of what to expect can be gained from studying the past and present travel experience of urban areas and relating this information with variables of physical growth.

Alan M. Voorhees and Associates studied trip lengths for work trips, shopping trips, social-recreational trips, nonhome-based trips, and truck trips from origin-and-destination data collected in several cities in the United States and Canada. Trip length information was compared to the city population and employment density pattern, the transportation network speed characteristics, the special distribution of income, and the historical and social patterns involved for each of the cities providing source data.

To analyze the data collected on this research project, a multiple regression technique was utilized. To further understand the results, a computer simulation was conducted to study cities of various urban forms. The results of these studies provide a set of guidelines for predicting trip lengths and evaluating traffic forecasts. The reported effect of various urban designs upon trip length, and hence travel demand, may assist the urban planner in providing the desired development for his city from a transportation viewpoint.

This research points out that improvements in forecasting procedures are needed. It suggests that a new trip distribution model should be developed which will take into consideration the spatial arrangement of trip opportunities, as well as travel time impedance. Preliminary work on such a model is presented.
## CONTENTS

1  SUMMARY

4  CHAPTER ONE  Introduction and Research Approach

5  CHAPTER TWO  Research Findings
   Trip Length Measurement and Characteristics
   Developing Hypotheses
   Testing the Hypotheses

22  CHAPTER THREE  Implications for Travel Forecasting
   Guidelines for Predicting Trip Length
   Improvements in Existing Forecasting Models
   A New Distribution Model

32  APPENDIX A  A Generalized User Cost Function and Its
   Application to Diversion Curve Theory

35  APPENDIX B  Use of the Gamma Distribution in Analysis of
   Trip Length

39  APPENDIX C  Simulation Studies

45  APPENDIX D  Predicting the Geographic Distribution of Family Income in an Urban Area

50  APPENDIX E  Procedure for Income Class Experiment in Washington, D.C., for Social-Recreation Trips

52  APPENDIX F  Stability of Friction Factors over Time for an Urban Area

54  APPENDIX G  Details of Network Coding Used in Historical Analyses

55  APPENDIX H  Performance of a Regional Gravity Model on a Subarea Basis

61  APPENDIX I  Fitting the Gamma Distribution to Gravity Model Travel Time Factors

62  APPENDIX J  Opportunity L-Values and the Opportunity Trip Distribution

63  APPENDIX K  Non-work Opportunity Trip Length Distributions

68  APPENDIX L  References and Bibliography
FIGURES

12 Figure 1. Average shopping center trip duration vs size of shopping center.
13 Figure 2. Work opportunity distributions across three cities, and approximate average trip durations.
13 Figure 3. Work opportunity distributions for Washington, D.C., in 1948 and 1955.
14 Figure 4. Average actual vs opportunity work trip durations.
15 Figure 5. Average trip duration vs work trip length index.
16 Figure 6. Average social-recreation trip duration vs trip length index.
16 Figure 7. Average nonhome-based trip duration vs trip length index.
17 Figure 8. Average work trip duration vs average distance of workers from CBD.
17 Figure 9. Standard deviation of trip duration vs mean trip duration.
18 Figure 10. Work trip variance vs population.
19 Figure 11. Comparison of prediction errors for income-stratified and unstratified gravity models, inter-district trips to CBD, Washington, D.C., 1955.
20 Figure 12. Social-recreation origin-destination/gravity model ratios vs income.
21 Figure 13. Twin Cities social orientation.
22 Figure 14. Change in urban density vs distance from CBD for: (Case 1) extension of present population and employment density patterns; (Case 2) filling in of unused land while maintaining the same density pattern; and (Case 3) concentration of additional population and employment in the downtown area and/or other sections of the metropolitan area.
26 Figure 15. Los Angeles 30-min peak-hour isochronal lines for 1957, 1960, 1962 and 1963.
27 Figure 16. Work trip duration variance vs average trip duration.
28 Figure 17. Work opportunity distributions for three selected zones in Washington, D.C., 1948.
29 Figure 18. Work travel time factors vs trip duration for three selected zones in Washington, D.C., 1948.
30 Figure 19. Work shape parameter vs mean opportunity trip duration for Washington, D.C., auto driver work trips, 1948.
30 Figure 20. Mean work travel time factors vs percent of opportunities passed, Washington, D.C.
30 Figure 21. Work travel time factors vs trip duration.
31 Figure 22. Comparison of actual and theoretical work trip duration distributions.
33 Figure A-1. Relationship between percent using freeway and distance difference.
34 Figure A-2. Relationship between percent using freeway and time difference.
34 Figure A-3. The normal distribution curve—a comparative cost measure.
35 Figure A-4. User cost curves.
36 Figure B-1. The gamma distribution.
38 Figure B-2. Comparison of predicted and observed auto driver work trips, Erie, Pa.
39 Figure B-3. Comparison of predicted and observed transit work trip durations, Washington, D.C., 1955.
40 Figure C-1. Trip time variance (theoretical city) vs work trip population and average network speed.
41 Figure C-2. Average opportunity distance vs distance from center (exponential city).
41 Figure C-3. Opportunity trip length distribution (theoretical city).
42 Figure C-4. Gamma distribution parameters for locations within theoretical city.
43 Figure C-5. Opportunity trip length vs city population.
44 Figure C-6. Relationships associated with the distance from a point (a) to a differential area (da).
47 Figure D-1. Cumulative log-normal plot, relative median family income, Washington, D.C.
48 Figure D-2. Relationships of estimated to observed relative median family incomes, Washington, D.C., 1960.
49 Figure D-3. Predicted minus observed residuals from relative median family income model, Washington, D.C.
51 Figure E-1. Comparison of observed and predicted distribution of social-recreation trips by income-difference group, Washington, D.C., 1955.
53 Figure F-1. Comparison of observed and predicted distribution of auto driver work trip durations, all income groups, Baltimore, Md., 1962.
53 Figure F-2. Comparison of observed and predicted distribution of work trip durations, all modes, Baltimore, Md., 1945.
54 Figure F-3. Comparison of observed and predicted distribution of transit work trip durations, Baltimore, Md., 1926.
57 Figure H-1. Locations of Connecticut areas selected for trip length distribution analyses.
58 Figure H-2. Comparison of actual and theoretical short-trip duration distributions, West Hartford, Conn.
58 Figure H-3. Comparison of actual and theoretical long-trip duration distributions, Groton, Waterford, and New London, Conn.
59 Figure H-4. Comparison of actual and theoretical nonhome-based trip duration distributions, Bridgeport, Conn.
59 Figure H-5. Comparison of actual and theoretical truck trip duration distributions, Bridgeport, Conn.
60 Figure H-6. Comparison of actual and theoretical work trip duration distributions, Groton, Waterford, and New London, Conn.
Figure I-1. Comparative fit of gamma and exponential Toronto data.

Figure J-1. Relationship of short L value to average opportunity duration, Pittsburgh, Pa.

Figure K-1. Home-based shop opportunity trip duration distributions for (a) five selected urban areas, and

Figure K-2. Home-based social-recreation opportunity trip duration distributions for (a) six selected urban
areas and (b) Washington, D.C., 1948 and 1955.

Figure K-3. Nonhome-based opportunity trip duration distributions for four selected urban areas.

Figure K-4. Truck opportunity trip duration distributions for three selected urban areas.

TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Nonhome-based Trip Ends</td>
</tr>
<tr>
<td>2.</td>
<td>Trip Length Characteristics and Major Influencing Factors</td>
</tr>
<tr>
<td>3.</td>
<td>Work Trip Duration Changes in Baltimore and Washington</td>
</tr>
<tr>
<td>4.</td>
<td>Nonhome-based Trips in CBD Areas</td>
</tr>
<tr>
<td>5.</td>
<td>Truck Trips by Land Use at Destination</td>
</tr>
<tr>
<td>6.</td>
<td>Experimental Design for Simulation Study</td>
</tr>
<tr>
<td>7.</td>
<td>Average Trip Times, Hypothetical City</td>
</tr>
<tr>
<td>8.</td>
<td>Percent of Mean Trip Length Variation, Explained by Variation in Population Size, Network Speed, and City Centralization</td>
</tr>
<tr>
<td>9.</td>
<td>Average Work Trip Duration by Income Class, Washington, D.C.</td>
</tr>
<tr>
<td>10.</td>
<td>Average Work Trip Duration by Income Class, Washington, D.C., Northwest Corridor</td>
</tr>
<tr>
<td>12.</td>
<td>Possible Changes in Trip Length, by Metropolitan Size</td>
</tr>
<tr>
<td>13.</td>
<td>Comparison of Parameter Estimates of the Cost Function</td>
</tr>
<tr>
<td>14.</td>
<td>Comparison of Familiar Statistical Distributions</td>
</tr>
<tr>
<td>15.</td>
<td>Table for Estimating Parameters of Gamma Distribution</td>
</tr>
<tr>
<td>16.</td>
<td>Sample Fit of Gamma Distribution Using Toronto Travel Time Factors</td>
</tr>
<tr>
<td>17.</td>
<td>Typical Results of Distribution Function Computations</td>
</tr>
<tr>
<td>18.</td>
<td>Results of Analysis of Geographic Distribution by Income</td>
</tr>
<tr>
<td>19.</td>
<td>Number of Trips by Income Class Differences, from O-D Study</td>
</tr>
<tr>
<td>20.</td>
<td>Number of Trips by Income Class Differences, from GM Study</td>
</tr>
<tr>
<td>21.</td>
<td>Ranking of Towns with Respect to Income and Opportunity Arrangement</td>
</tr>
<tr>
<td>22.</td>
<td>Connecticut (CIPP) Purpose Definitions and Average Auto Driver Trip Lengths</td>
</tr>
<tr>
<td>23.</td>
<td>Summary of Trip Length Characteristics</td>
</tr>
<tr>
<td>24.</td>
<td>Travel Time Factors</td>
</tr>
</tbody>
</table>
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FACTORS AND TRENDS IN TRIP LENGTHS

SUMMARY

The purpose of the research reported herein was to investigate the effect of various factors on trip length. This is significant in that (a) the average length of all trips makes up the total travel demand, and (b) different trip lengths indicate the need for different types and combinations of transportation facilities. Sound estimates of trip lengths are thus essential to transportation planning.

Trip lengths can be measured in a variety of ways—in terms of time, distance, cost, or a combination of these measurements. Trip length distribution can be expressed in terms of its average or variance. Due to data limitations, most of this research has dealt with average trip length either in time or in distance.

Trips can also be identified by trip purpose, mode, or the time of day when they occur. Following an investigation of the available data, it was decided that the following types of trips were the only practical breakdown for this study: (1) auto home-based work, (2) auto home-based shop, (3) auto home-based social-recreation, (4) auto nonhome-based, and (5) all truck.

The analyses of these various types of trips in a number of cities in the United States and Canada indicate that the three most important factors related to trip length are: (1) the size and physical structure of the urban area; (2) the transportation system; and (3) social and economic patterns.

In measuring the size and physical structure of an urban area, its population and spatial distribution of activities were found to be the most significant factors influencing work and nonwork trip lengths.

In most of the cities analyzed the average length of work, social-recreation and nonhome-based trips was found to be related to population. A regression analysis indicated that the average work trip length was proportional to changes in metropolitan population. The change in social-recreation trip length was proportional to the change in work trip raised to about the 0.7 power. For nonhome-based trip length the change was proportionate to changes in work trip length and changes occurring in the CBD areas.

The deviation of some cities from these relationships appears to be related to their unique structural characteristics. For example, New Orleans, one of the oldest and most compact of the cities studied, has an average work trip length, expressed either in miles or minutes, typical of that normally found in newer cities only one-tenth its size.

To compare the development pattern for different urban areas, a measurement called the “opportunity trip distribution” was devised. This was an estimate of the trip length characteristics that would occur, assuming that trips were not influenced as to their destination by either time or distance but only by the structural elements of the city. Changes in the opportunity distributions were accompanied by changes in trip lengths, particularly for work, social-recreation, and nonhome-based trips.

In analyzing shop trip lengths, it was apparent that these are not highly related to the general development pattern of the city. Shop trip lengths are responsive to changes in retailing location, which are in turn affected by retailing practice. Although retailing practices are continually changing, it does not appear at this time that future changes are likely to affect trip lengths substantially.
A strong relationship between average truck trip length and average shop trip length was detected in a regression analysis. On the other hand, truck trips are related to the spatial arrangement of residential and commercial land uses.

The transportation system and its operation were found to have a significant impact on the work trip length. For example, it was generally found that the cities with the fastest transportation systems had the longest trip length in miles, but not necessarily in minutes. Various simulation studies that were made suggested that the average work trip length increases in proportion to the square root of an increase in network speed, whereas trip time decreases at about the same rate.

Analysis of nonwork trips made in Los Angeles indicated that these trips are not as sensitive to changes in the speed of the transportation network as are work trips. This finding is based on all nonwork travel and may not be applicable to certain types of nonwork trips, such as social-recreation.

The socio-economic spatial structure of our cities has a significant effect on travel patterns, hence on trip length. The effects of socio-economic factors were found to be influential in the case of work and social-recreation trips. This has been frequently observed in urban transportation studies where "socio-economic adjustment factors" had to be used in calibrating the traffic models.

In the twin cities of Minneapolis and St. Paul, a strong reluctance to make trips between the two cities was observed. A subsequent attitude survey indicated that families usually were more familiar with, and more closely oriented to, the opportunities in their own community than they were with closer ones in an adjacent community. This same observation was also made in a study in the Wilkes Barre-Scranton (Pa.) area. However, in the Los Angeles area this tendency of community separation was not observed.

The spatial distribution of families in various income groups was found to be important in determining home-to-work linkages. In the simplest terms, workers of one income class can be oriented only to areas with similar salary levels.

For social-recreation travel, it was found that persons of a given socio-economic status tend to make social and recreation trips to an area with comparable status. A detailed statistical analysis in Washington, D.C., and Springfield, Ill., verified this hypothesis.

In an effort to provide a tool for predicting trip length or for evaluating a traffic forecast, the following set of guidelines was developed for work and nonwork trips:

**Work Trips**

1. **Size and physical structure:**
   
   (a) If an urban area grows by extending its present population and employment density patterns, the change in the average work trip length (time) will probably be proportional to the fourth root of the population change.
   
   (b) If an urban area grows largely by the filling in of unused land while maintaining its same density pattern, there will be no material change in work trip length (time).
   
   (c) If an urban area develops by concentrating additional population and employment in the downtown area and/or in other sections of the metropolitan area, the average work trip length (time) will probably decline. Various studies have shown that this decline could vary from 5 to 10 percent.
2. Network speed:
   (a) Changes in the average work trip distance (miles) will be directly proportional to the square root of changes in peak-hour network speed.
   (b) Changes in the average work trip duration (minutes) will be inversely proportional to the square root of changes in peak-hour network speed.

3. Socio-economic factors:
   (a) A more heterogeneous distribution of income in an urban area could reduce trip length by as much as 10 percent, but with present social attitudes this is not likely to happen.
   (b) Changes in existing historical and social patterns could change trip length by 5 percent, but it appears from the data investigated that these changes occur very slowly.

Nonwork Trips

1. Size and physical structure:
   (a) Nonhome-based trip lengths will change at about the same rate as work trips, whereas social-recreation trip lengths will change as the 0.7 power of the work trip length.
   (b) Shop trip length is not likely to change unless unanticipated changes occur in retailing practices.
   (c) Truck trip length is not likely to change unless commercial and residential land-use patterns change.

2. Network speed:
   (a) Shop and truck trip length does not appear at this time to be related to changes in average network speed.
   (b) Changes in the average social-recreation trip distance appear to be directly proportional to the cube root of changes in off-peak network speeds.
   (c) Changes in the average social-recreation trip duration (minutes) appear to be inversely proportional to the cube root of changes in off-peak network speeds.
   (d) Nonhome-based trip distance (miles) will be directly proportional to changes in network speeds provided there is no radical change in CBD travel.

3. Socio-economic factors:
   (a) More heterogeneous distribution of incomes within an urban area could increase social-recreation trip lengths as much as 10 percent, but would have little impact on other types of nonwork trips.
   (b) Changes in existing historical and social patterns could change nonwork trip lengths as much as 10 percent, but, as in the case of work trips, these changes are not likely to occur very rapidly.

A more complete picture of trip length characteristics is obtained when the dispersion around the mean (variance) is considered. The gamma distribution, a mathematical function considering both the mean and the variance, has been found to fit the form of the work trip distribution observed in most urban areas.

To use the gamma distribution as a tool in estimating future trip length distribution, it is necessary to establish the mean and variance of that distribution. The mean
can be estimated by using the guidelines just described. The change in variance can be approximated through the relationship between average trip length and the variance.

The results of this research indicate that additional refinements in forecasting procedures are necessary. Ways must be found of varying the L in the opportunity model and the $F$-factor in the gravity model as a function of the opportunity distribution of trips. Several approaches to this problem were investigated and are described in this report. But what seems to be really needed is a new distribution model which takes into consideration trip length and opportunity distribution simultaneously.

In our large metropolitan areas, stratification of the work trip by various income categories should help to advance the understanding of travel behavior and to improve land-use and traffic models. For social-recreation trips, stratification by income class would aid in reducing the bias now found in trip distribution models for this trip purpose category.

Because the level of transportation service affects trip distribution, the model employed in larger metropolitan areas should develop different trip distributions for both peak and off-peak periods, taking into consideration the level of transportation service during the particular period involved. To establish the level of transportation, both highway and transit networks should be employed.

These conclusions imply that there may be a need for certain changes in data collection. More information on the socio-economic characteristics of travelers—especially their incomes—at both the residence and the job end, would be useful and could be gathered in conjunction with the origin-destination surveys. In addition, transportation system inventories might include data on peak-hour characteristics for both the highway and transit networks.

CHAPTER ONE
INTRODUCTION AND RESEARCH APPROACH

The main purpose of the research reported herein was to obtain a better understanding of the nature of urban travel in order to improve transportation planning techniques. The question as to whether or not trip lengths were going to increase in the future had become quite a debatable issue. In fact, in several transportation studies it was assumed that trip lengths would not change. If this assumption proved to be wrong, the forecasts based on it were open to question. Therefore, this research project was aimed at evaluating this critical question.

Further, it was recognized that such a research program should also make it possible to design and plan transportation systems more effectively, inasmuch as trip length patterns naturally have an impact on the type of transportation facility that should be provided, and therefore would have an effect on the optimum transportation system.

A better understanding of urban travel would also help in planning new urban areas more effectively. Furthermore, this research might indicate ways by which transportation requirements could be minimized in the redevelopment of older urban complexes, as well as in the planning for new areas.

The present project was part of a number of such projects sponsored by the National Cooperative Highway Research Program. It was conducted in two phases, the first dealing with the work trip and the second dealing with the non-work trip. In carrying out the first phase, most of the fundamental factors about trip length were developed. Thus, the second phase was primarily limited to determining how these fundamentals varied from one type of trip to another.

The general approach outlined in the study design was divided into three steps, as follows:
1. Assembly of all information presently available on trip lengths, collected from the various transportation studies in the United States and Canada.

2. Development of a set of hypotheses for an analysis of these travel data.

3. Testing of these hypotheses by:
   (a) Comparing trip lengths found in various cities to their size, density, transportation system, and other appropriate characteristics.
   (b) Evaluating the results of the tests of hypotheses against historical data.

Upon completion of this work it was apparent that certain inconsistencies existed in the results. Due to some limitations in the data, they were not completely comparable. Therefore, it was decided to undertake certain simulation studies of a series of theoretical cities in an effort to see what could be derived regarding trip length from a theoretical point of view. In addition, some special analyses of traffic patterns were undertaken to explore additional hypotheses that were developed.

As a result of these various tests and a careful evaluation of the findings, it was possible to set up a series of guidelines for predicting trip length. These guidelines have been tested in numerous cases and have proved to be a valuable tool for evaluating traffic forecasts.

In addition, this research project has pointed out the need for improving forecasting procedures. Of utmost importance is a new distribution model which takes into consideration opportunity distribution and trip length. In the course of this study several techniques were developed that accomplished this objective. However, they must be improved and refined before they can really be called "operational."

CHAPTER TWO

RESEARCH FINDINGS

This chapter is concerned with the major findings of both phases of this project; namely, those dealing with work trip lengths and those dealing with nonwork trip lengths. It discusses some of the basic decisions that were made regarding the procedure for measuring trip length characteristics and the trip purposes that should be used. It then deals with the hypotheses that were developed, the various tests that were made, and the results of these tests. In many instances, mathematical calculations and detailed descriptions of the steps are included in the appendices.

TRIP LENGTH MEASUREMENT AND CHARACTERISTICS

Individual trip lengths may be measured in a variety of ways—time, distance, cost, or even effort. It would be desirable to establish which of these basic measurements, or what combination of them, is closest to the measurement used by travelers to make decisions about trip destinations. However, because a majority of transportation studies summarize most of these trip data in terms of either time or distance, there was little choice in the matter, although considerable research was conducted into the use of cost as a measure of trip length (see Appendix A).

Even though trip length in time and distance was derived from standard computer programs, there was considerable variation in the data because the approach to establishing network speed varied for each study. As it would have been a mammoth undertaking to correct for these variations, the data were used, taking into consideration these limitations. A further problem with these data existed in that varying techniques had to be used to determine intrazonal or terminal times. To make the data as compatible as possible, it was concluded that terminal times should be eliminated. Thus, there are certain limitations in the comparability of the data.

Individual trip lengths, regardless of what measurement is used, are generally combined in terms of overall statistics such as mean, variance, or the entire trip length distribution. The mean trip length reflects the average length of all individual trip lengths. The variance indicates the dispersion of different trip lengths around this mean. The gamma distribution was used to describe the trip length distribution. Details on the use of the gamma distribution in describing trip length distribution are presented in Appendix B.

Trip Categories

Individual trips can be classified in many ways, but the most important classification would be the purpose of the trip and the mode of travel. The mode can be broken down into auto driver, auto passenger, transit, truck, walk, etc. Most of the findings in this study are related to auto driver trips, because in the majority of transportation studies this was the most common way of reporting trips. However, some specific analyses were made of transit trips in the studies examined.

The purpose of a trip can be indicated as home, work, personal business, related business, goods, social, recreation, eat meal, serve passenger, medical, etc. After considering the data available, it was decided that the following grouping of mode and purpose should be studied:
1. Auto driver home-based work.
2. Auto driver home-based shop.
3. Auto driver home-based social recreation.
4. Auto driver nonhome-based.
5. All truck.

The following discussion describes these classifications in more detail and relates some of the problems encountered in working with these categories.

**Auto Driver Home-Based Work**

This category constitutes an important grouping, inasmuch as these trips occur during the peak period, which governs the design of transportation facilities. Generally, the trips represent about 25 percent of all the auto driver trips made in a typical day.

As defined by the Bureau of Public Roads Procedure Manual (96), a work trip includes:

... trips made to the location of a person’s place of employment, such as a factory, a shop, a store, or an office; and, also, to locations that must be visited in performing a normal day's work. The major occupation of a person is classed as “work” even though it be in the nature of a business. Trips made by a doctor in making his calls, and by a salesman calling on prospective customers are classed as trips to work. The purpose “work” would also apply to electricians, carpenters, plumbers, and others who are employed on construction projects and have no regular place of employment.

Some transportation studies, however, do not include as a home-based work trip doctor or salesman visits if they start at home, again causing some discrepancy in the data.

**Auto Driver Home-Based Shop**

As defined in the Bureau of Public Roads Procedure Manual (96), a shop trip includes:

... a trip made to do some shopping, regardless of the size of the purchase. Trips made to a store for the purpose of “just looking” are classed as shopping even though no purchase is made. Trips made for repairs to automobiles, radios, or other items, and for personal service such as haircuts, beauty treatments, cleaning and pressing clothes, etc., also should be recorded as shopping.

These trips represent about 20 percent of all the auto driver trips made in a typical day.

**Auto Driver Home-Based Social-Recreation**

As defined in the Bureau of Public Roads Procedure Manual (96), a social-recreation trip includes:

... cultural trips made to church, civic meetings, lectures, and concerts, as well as trips to attend parties or to visit friends. This item also includes trips made for golfing, fishing, movies, bowling, pleasure riding, etc.

These trips represent about 15 percent of all the auto driver trips made during a typical day.

**Auto Driver Nonhome-Based**

A nonhome-based trip is defined as any trip made which does not have its origin or destination at home. This was the most complex grouping of trips. This classification generally represents about 25 percent of all auto driver trips and includes many purposes. Table 1 gives the purpose category at either end of these trips for Worcester, Mass.; Springfield, Ill.; Virginia Peninsula; and Washington, D.C. The table shows that a large percentage of nonhome-based trips have work, personal business, and serve passenger at one end. No trip length data were available for these subgroupings of nonhome-based trips.

**Truck**

This generally refers to trips made by all types of trucks. However, in some instances light trucks (such as pick-up trucks) were excluded in the calculation of trip lengths for certain cities, presenting some problems in comparability of data. These trips are usually equivalent to about 20 percent of the auto driver trips made in a typical day.

It also should be noted that intrazonal truck trips were

---

**TABLE 1**

**Nonhome-Based Trip Ends**

<table>
<thead>
<tr>
<th>PURPOSE, FROM</th>
<th>WORCESTER</th>
<th>SPRINGFIELD</th>
<th>VA. PENINSULA</th>
<th>WASHINGTON</th>
</tr>
</thead>
<tbody>
<tr>
<td>OR TO NO. %</td>
<td>NO. %</td>
<td>NO. %</td>
<td>NO. %</td>
<td>NO. %</td>
</tr>
<tr>
<td>Work</td>
<td>61,653 26.8</td>
<td>42,112 31.6</td>
<td>49,423 27.7</td>
<td>112,447 41.1</td>
</tr>
<tr>
<td>Pers. business</td>
<td>44,947 19.5</td>
<td>10,003 7.5</td>
<td>73,174 41.0</td>
<td>26,612 9.7</td>
</tr>
<tr>
<td>Shop a</td>
<td>50,057 21.8</td>
<td>24,690 18.5</td>
<td>35,187 19.7</td>
<td>27,281 10.0</td>
</tr>
<tr>
<td>Serve passenger</td>
<td>46,622 20.2</td>
<td>31,222 23.5</td>
<td></td>
<td>45,553 16.6</td>
</tr>
<tr>
<td>Social-recreation</td>
<td>24,132 10.5</td>
<td>11,248 8.5</td>
<td>18,757 10.5</td>
<td>35,564 13.0</td>
</tr>
<tr>
<td>School</td>
<td>2,532 1.1</td>
<td>1,362 1.0</td>
<td>1,987 1.1</td>
<td>2,807 1.0</td>
</tr>
<tr>
<td>Other b</td>
<td>155 0.1</td>
<td>12,473 9.4</td>
<td></td>
<td>23,386 8.6</td>
</tr>
<tr>
<td>All</td>
<td>230,098 100.0</td>
<td>133,110 100.0</td>
<td>178,528 100.0</td>
<td>273,650 100.0</td>
</tr>
</tbody>
</table>

* Convenience and goods, apparel and furniture shop trips.
* Change travel mode, medical-dental, eat meal, etc.
not always reported in the summaries. Inasmuch as these trips often represent a high percentage of all truck trips, there is some additional bias in the data.

DEVELOPING HYPOTHESES

On the basis of the purposes that have already been discussed and the measurement of trip length in time and miles, Table 2 was developed for 34 cities for which data were obtainable. In all cases these data were obtained from studies that involved computer application or origin-destination data; but, as has already been indicated, the data were not completely uniform in terms of purpose and measurement of trip duration. This table, however, provided insight into some of the factors involved in various types of trips. It pointed to the fact that size and physical structure of the city have an impact on trip length, as it was apparent from the table that work trip lengths increase with population. It was also apparent from these observations that the transportation system has an impact on trip length. Generally, with an increase in network speed there was an increase in trip length and a slight decrease in trip duration.

In light of other research that has been done, it was quite apparent that socio-economic factors also affect trip length. This has been observed in Minneapolis-St. Paul, where it was found that these two cities act as two rather independent communities in their travel patterns. Therefore, the trip length is somewhat less than if they were to function as one entire community.

On the basis of these observations a series of hypotheses were developed which were the beginning point of this research effort. They are described as follows:

1. Work trip length should increase with population. Intuitively, it appears that the larger the city, the greater the number of opportunities to make longer trips.

2. An improvement in network speed should reduce the duration of a work trip, since a given number of opportunities could then be reached in less time. Such an improvement should also cause an increase in the distance traveled, because more opportunities could then be reached in a fixed amount of time.

3. Social-recreation and nonhome-based trip lengths should be related to population and average network speed for basically the same reasons given for work trips in (1) and (2).

4. Decreased average urban density should result in increased trip length because there should be fewer opportunities at a given distance from an origin.

5. Shop trip lengths should be related to retailing practices and not to city size. Retailing practices would involve the size and location of various shopping facilities, as well as the range and price of goods offered.

6. Truck trip lengths should be related to changes in the arrangement of residential, commercial, and manufacturing land uses. It was also hypothesized that truck trip lengths would be similar to shop trip lengths because they were, in effect, related to the same type of land uses.

7. Work, social-recreation, and nonhome-based trip lengths should be related to spatial arrangement of various associated activities.

8. As family income increases, work trip length increases. This hypothesis reflects the observation that high income groups usually have a longer work trip than lower income groups.

9. Social-recreation trip lengths should be related to the spatial distribution of various income classes. People of a given socio-economic status tend to make social and recreational trips to areas of comparable status.

TESTING THE HYPOTHESES

To test the first six hypotheses it was necessary to carry out two basic types of experiments. The first utilized the multiple regression technique. The second was a computer simulation study of cities of various urban forms.

To test hypothesis 7, a graphic analysis of the spatial arrangement of activities was made. Sufficient data were not available for any regression analyses.

To test hypotheses 8 and 9, an investigation was made of the effect of socio-economic structure on a number of cities.

Regression Analysis

The regression analysis used to test the first six hypotheses related to trip duration and length was in the form

\[ Y^a = p^b D^c S^d \] (1)

in which

\[ Y = \text{the average trip duration, } \bar{T} \text{, or distance, } \bar{L}; \]
\[ P = \text{the urban population; } \]
\[ D = \text{the average city density, in persons per square mile; } \]
\[ S = \text{the average network speed, in mph; and } \]
\[ a, b, c, d = \text{constants.} \]

This analysis was made for most of the basic trip categories and the variables which did not make a significant contribution were eliminated, resulting in the following equations (in each case the standard error of the regression coefficient is given in parentheses and the multiple correlation coefficient, \( R^2 \), is given):

Average auto driver trip duration, \( \bar{T} \), in minutes—

**Work**

\[ \log_{10}\bar{T} = -1.19 + 0.22 \log_{10}P + 0.10 \log_{10}D \] (2a)

or

\[ \bar{T} = 0.31 P^{0.22} D^{0.19} \]

\[ R^2 = 0.84 \] (2b)

\[ \log_{10}\bar{T} = -0.02 + 0.19 \log_{10}P \] (3a)

or

\[ \bar{T} = 0.98 P^{0.19} \]

\[ R^2 = 0.71 \] (3b)
<table>
<thead>
<tr>
<th>CITY</th>
<th>POPULATION (1,000's)</th>
<th>AVERAGE TRIP DURATION (MIN)</th>
<th>AVERAGE TRIP LENGTH (MI)</th>
<th>AVERAGE NETWORK SPEED (MPH)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>WORK</td>
<td>SHOP</td>
<td>SOC.-</td>
<td>NHB</td>
</tr>
<tr>
<td>1. Los Angeles, Calif.</td>
<td>6,489</td>
<td>16.0</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>2. Philadelphia, Pa.</td>
<td>3,635</td>
<td>20.1</td>
<td>7.2</td>
<td>11.6</td>
</tr>
<tr>
<td>3. Washington, D.C.</td>
<td>1,808</td>
<td>14.1</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>4. Pittsburgh, Pa.</td>
<td>1,804</td>
<td>12.6</td>
<td>10.9</td>
<td>13.4</td>
</tr>
<tr>
<td>5. Baltimore, Md.</td>
<td>1,419</td>
<td>16.7</td>
<td>6.6</td>
<td>11.5</td>
</tr>
<tr>
<td>6. Minneapolis-St. Paul</td>
<td>1,777</td>
<td>12.5</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>7. New Orleans, La.</td>
<td>845</td>
<td>9.1</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>8. New Orleans, La.</td>
<td>685</td>
<td>14.6</td>
<td>9.3</td>
<td>-</td>
</tr>
<tr>
<td>9. Fort Worth, Tex.</td>
<td>503</td>
<td>15.7</td>
<td>7.1</td>
<td>-</td>
</tr>
<tr>
<td>10. Lackawanna-Luzerne, Pa.</td>
<td>453</td>
<td>19.2</td>
<td>13.9</td>
<td>20.4</td>
</tr>
<tr>
<td>11. Broward County, Fla.</td>
<td>440</td>
<td>13.7</td>
<td>9.8</td>
<td>13.1</td>
</tr>
<tr>
<td>12. Ottawa-Hull, Ont.</td>
<td>406</td>
<td>12.6</td>
<td>7.4</td>
<td>-</td>
</tr>
<tr>
<td>13. Louisville, Ky.</td>
<td>210</td>
<td>11.0</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>14. Nashville, Tenn.</td>
<td>347</td>
<td>10.8</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>15. Milwaukee, Wis.</td>
<td>288</td>
<td>11.6</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>16. Virginia Peninsula</td>
<td>277</td>
<td>11.5</td>
<td>7.6</td>
<td>11.9</td>
</tr>
<tr>
<td>17. Nashville, Tenn.</td>
<td>258</td>
<td>9.4</td>
<td>8.3</td>
<td>10.9</td>
</tr>
<tr>
<td>18. Davenport, Iowa</td>
<td>227</td>
<td>7.7</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>19. Charlotte, N.C.</td>
<td>210</td>
<td>11.0</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>20. Cleveland, Tenn.</td>
<td>205</td>
<td>10.8</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>21. Erie, Pa.</td>
<td>177</td>
<td>9.4</td>
<td>6.6</td>
<td>8.8</td>
</tr>
<tr>
<td>22. Waterbury, Conn.</td>
<td>142</td>
<td>10.1</td>
<td>7.8</td>
<td>-</td>
</tr>
<tr>
<td>23. Springfield, Ill.</td>
<td>134</td>
<td>7.5</td>
<td>5.1</td>
<td>7.2</td>
</tr>
<tr>
<td>24. Pensacola, Fla.</td>
<td>128</td>
<td>8.7</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>25. Regina, Sask.</td>
<td>127</td>
<td>8.0</td>
<td>5.8</td>
<td>-</td>
</tr>
<tr>
<td>26. Greensboro, N.C.</td>
<td>123</td>
<td>8.9</td>
<td>9.8</td>
<td>-</td>
</tr>
<tr>
<td>27. Lexington, Ky.</td>
<td>112</td>
<td>9.1</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>28. Springfield, Mo.</td>
<td>110</td>
<td>8.4</td>
<td>6.0</td>
<td>8.0</td>
</tr>
<tr>
<td>29. Altoona, Pa.</td>
<td>103</td>
<td>11.1</td>
<td>6.8</td>
<td>-</td>
</tr>
<tr>
<td>30. St. Catharines, Ont.</td>
<td>999</td>
<td>13.6</td>
<td>8.4</td>
<td>10.4</td>
</tr>
<tr>
<td>31. Sioux Falls, S.Dak.</td>
<td>67</td>
<td>7.0</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>32. Tallahassee, Fla.</td>
<td>48</td>
<td>7.3</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>33. Hutchinson, Kans.</td>
<td>38</td>
<td>6.1</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>34. Beloit, Wis.</td>
<td>33</td>
<td>6.7</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

a These data were obtained from various sources and attempts were made to keep them as compatible as possible by removing terminal time effects wherever possible.

b Average convenience trip length.
c Average goods, apparel, and furniture shop trip length = 10.3 min.
d Average goods, apparel, and furniture shop trip length = 4.2 min.
e External trips are included in determination of these average trip lengths.
f Average goods, apparel, and furniture shop trip length = 14.4 min.
g Total person trips.
h Average of blue and white collar average work trip lengths.
i Average goods, apparel, and furniture shop trip length = 10.8 min.
Social-recreation
\[
\log_e \bar{t} = +0.78 + 0.12 \log_e P \\
(0.03)
\]

or
\[
\bar{t} = 2.18P^{0.12} \\
R^2 = 0.65
\]

Nonhome based
\[
\log_e \bar{t} = -0.46 + 0.20P \\
(0.03)
\]

or
\[
\bar{t} = 0.63P^{0.20} \\
R^2 = 0.67
\]

Average auto driver trip distance, \( \bar{L} \), in miles—

Work
\[
\log_e \bar{L} = -5.86 + 0.20 \log_e P + 1.49 \log_e S \\
(0.02) \\
(0.20)
\]

or
\[
\bar{L} = 0.003P^{0.20} S^{1.49} \\
R^2 = 0.85
\]

or
\[
\bar{L} = -0.77 + 0.19 \log_e P \\
(0.02)
\]

Social-recreation
\[
\log_e \bar{L} = -5.80 + 0.18 \log_e P + 1.40 \log_e S \\
(0.09) \\
(0.71)
\]

or
\[
\bar{L} = 0.003P^{0.18} S^{1.40} \\
R^2 = 0.61
\]

Nonhome based
\[
\log_e \bar{L} = -5.98 + 0.26 \log_e P + 1.25 \log_e S \\
(0.13) \\
(0.62)
\]

or
\[
\bar{L} = 0.003P^{0.26} S^{1.25} \\
R^2 = 0.57
\]

From these regression equations it can be seen that population and network speed explain much of the variation in work, social-recreation, and nonhome-based trip length and offer proof of hypotheses 1, 2 and 3. With respect to hypothesis 4, average urban density for the overall study area did not explain variation in trip length. Density patterns within a city, however, do have an impact, as is pointed out in the simulation study described in a subsequent section.

To check the regression results, certain historical analyses were made of data for the Washington and Baltimore areas (Table 3). According to these data, work trip duration increases as the 0.3 power of change in population, or somewhat faster than that revealed by the multiple regression analysis.

However, population changes alone may not always affect the average work trip duration. From 1958 to 1964 the work trip duration in Broward County, Fla., increased by about 4 percent over 10.5 min, even though population increased by 40 percent. This was probably due to the fact that growth did not extend throughout the urban area, as was the case in Washington and Baltimore. Instead, growth in population and employment occurred in the vacant land within the built-up area.

In addition to studying the relationship between trip length and city size and network speed, an analysis was made which showed that home-based social-recreation and nonhome-based trips could be related to work trip duration as follows:

\[
\bar{t}_{soc/rev} = 2.18\bar{t}_{work}^{0.60} \\
R^2 = 0.66
\]

Eq. 10 indicates that social-recreation trip duration does not increase as fast as does work trip duration. This is probably related to the fact that social-recreation trips are generally shorter trips and therefore do not increase as rapidly. Compared with the actual 9.4 percent increase in the social-recreation trip length in Washington, D.C., from 1948 to 1955, the equation's estimate of 8.6 percent is good. Comparisons for Broward County, Fla., are not as good. The actual change between 1958 and 1964 was 9.1 percent, but the predicted change was only 3 percent. This was probably due to the considerable change in recreational facilities in this area between 1958 and 1964, which must have generated longer trips.

According to Eq. 11, it was found that nonhome-based trips, which are an important part of total travel, grow in length at about the same rate as work trip duration. The actual change from 1948 to 1955 in Washington was 12 percent, and the change estimated by the equation was 12.8 percent. The equation estimates a 6.2 percent increase in NHB trip duration from 1958 to 1964 in Broward County, however, whereas the actual change was only 2.7 percent. The reason is that over a period of time this county has grown by filling in the unused land.

It also appears that changes occurring in nonhome-based trip lengths should be closely related to changes that are occurring in the downtown areas due to urban renewal and other downtown revitalization projects. Table 4 shows the
The importance of CBD nonhome-based trips in urban areas, with this type of trip accounting for between 13 and 20 percent of the total nonhome-based trips. Therefore, care should be exercised in checking nonhome-based trips by use of the previously described regression analysis. As downtown areas concentrate their activities in tight core areas, the nonhome-based trip length might decline. On the other hand, a wider dispersion or spread of activities in the downtown area would cause an increase in nonhome-based trip length and, hence, travel demand.

To test hypothesis 5, related to shop trip length, several regression and special analyses were made. The regression analysis did not reveal any relationships between trip length, population, speed, and density. It would appear that shop trip lengths are more responsive to changes in retailing practices (the size, location and variety of goods offered at shopping centers). For example, in the Virginia Peninsula study (85) shop trip durations were longer for larger shopping centers. The location of a shopping center, as well as its size, was found to have an effect on shop trip lengths, as indicated by the analyses of metrotowns in the Baltimore region (45).

In Washington, department store sales showed an absolute drop of $6.5 million between 1958 and 1963 (38). Moreover, the decline in average shop trip duration from 9.0 to 8.1 min between 1948 and 1955 seems to give further indication that retailers are moving closer to the people and, hence, varying the size and location of their stores. Items to be purchased on a shopping trip have an influence on the length of these trips, as was demonstrated by Berry, Barnum and Tennant (7). These authors showed that families tend to make longer shopping trips when they are purchasing very expensive items.

The correlation shown between average truck trip durations and average shop trip durations in Eq. 12 seems to reflect hypothesis 6—that the truck trip is similar to the shop trip.

The length of the truck trip is highly related to the arrangement of residential, commercial, and manufacturing land use, as indicated in Table 5. This table indicates that, except for the industrial area of Waterbury, Conn., residential and commercial land use contains the majority of the truck trip ends. The change of these land uses with respect to each other will have an impact on the truck trip length change. It is therefore important that the relationship observed between shop and truck trip lengths in the regression analysis should be considered in light of the changes occurring in the land use pattern. In Waterloo, Iowa, for example, a large growth in population was envisioned outside the present urbanized area by 1990 (a growth of approximately 40,000 people). This growth changed the truck trip lengths from 10.84 to 11.66 min (88), or 7 percent, even though the shop trip length remained approximately the same.

Analysis of historical data in Broward County showed that the average shop trip lengths changed from 9.0 to 10.0 min, an increase of 10 percent, between 1958 and 1964. The average truck trip length actually increased only 0.1 min (from 11.4 to 11.5 min), or 1 percent, during the same period. According to the regression equation, the percent change in truck trip lengths should have been 9.2, probably illustrating that other variables are involved (as indicated in the Waterloo analysis) which are not involved in shop trips.

### Table 4

<table>
<thead>
<tr>
<th>Study Area</th>
<th>Internal Nonhome-Based Trips</th>
<th>CBD</th>
<th>TOTAL</th>
<th>% CBD of TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calgary, Alta.</td>
<td>38,600</td>
<td>202,300</td>
<td>19.1</td>
<td></td>
</tr>
<tr>
<td>Erie, Pa.</td>
<td>6,200</td>
<td>45,000</td>
<td>13.8</td>
<td></td>
</tr>
<tr>
<td>Worcester, Mass.</td>
<td>16,600</td>
<td>125,700</td>
<td>13.2</td>
<td></td>
</tr>
<tr>
<td>Boston, Mass.</td>
<td>281,600</td>
<td>1,783,200</td>
<td>15.8</td>
<td></td>
</tr>
<tr>
<td>Springfield, Ill.</td>
<td>13,800</td>
<td>73,400</td>
<td>18.8</td>
<td></td>
</tr>
</tbody>
</table>

* Fig. 27, Highway nonhome-based trip ends, Procedure Manual for Estimating Internal and External Trip Ends (May 1966).
* Table A, Trip Data, Traffic Characteristics and Model Development (Oct. 1966).

The multiple regression analysis of trips for the various cities studied was naturally influenced by unique characteristics of the cities involved and, to some degree, by variations in the data. Thus, it was felt desirable to measure on a more theoretical basis the effect of population, network speed and urban density patterns on trip length.

The theoretical tests were accomplished by simulating a series of hypothetical cities on a digital computer and calculating the traffic patterns that would result. This was achieved by using the gravity model and standard assignment programs. Each of the cities was assumed to be square in shape, with zones of 4 square miles on a 2-mile major street grid. Cities with 14-, 20-, and 28-mile sides were used. The number of trips generated was based on 2,500 trip origins per square mile. It was also assumed that trip production and attraction for any zone were equal, but the distribution of these trips between zones would vary depending on the type of city to be simulated. These assumptions resulted in about 500,000, 1,000,000 and 2,000,000 trips for the various cities.

To measure this, three different patterns of density (decreasing exponentially from the center) were used in each city. The resulting central area densities were from 12,000 to 20,000 trips per square mile for the small city and from 15,400 to 27,600 in the largest city. This reflected characteristics observed by Clark (15) and others (see Ap-
TABLE 5
TRUCK TRIPS BY LAND USE AT DESTINATION

<table>
<thead>
<tr>
<th>STUDY AREA</th>
<th>COMM. &amp; OTHER</th>
<th>TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Altoona, Pa. a</td>
<td>36.9</td>
<td>4.2</td>
</tr>
<tr>
<td>Erie, Pa. b</td>
<td>58.0</td>
<td>14.0</td>
</tr>
<tr>
<td>Waterbury, Conn. c</td>
<td>25.7</td>
<td>21.0</td>
</tr>
<tr>
<td>Virginia Peninsula d</td>
<td>32.0</td>
<td>4.0</td>
</tr>
</tbody>
</table>

d Summary of Basic Tabulations (Apr. 1965).

TABLE 6
EXPERIMENTAL DESIGN FOR SIMULATION STUDY

<table>
<thead>
<tr>
<th>RELATIVE DENSITY AT CENTER</th>
<th>500,000</th>
<th>1,000,000</th>
<th>2,000,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low</td>
<td>25</td>
<td>25</td>
<td>25</td>
</tr>
<tr>
<td>Medium</td>
<td>30</td>
<td>25</td>
<td>25</td>
</tr>
<tr>
<td>High</td>
<td>35</td>
<td>30</td>
<td>25</td>
</tr>
<tr>
<td>Constant</td>
<td>–</td>
<td>25</td>
<td>25</td>
</tr>
<tr>
<td>Constant</td>
<td>–</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>Constant</td>
<td>–</td>
<td>35</td>
<td>35</td>
</tr>
</tbody>
</table>

The results of these studies are given in Table 7, which reveals that the average trip duration increases with population and decreases with increasing travel speed and with a greater concentration of density at the area's center. Increasing the exponent of the travel time factors decreases the average trip length. Increases in trip duration were found to be associated with large cities, smaller travel time exponents, and slower networks.

Analysis of Table 7 indicated that if the travel time factor is $1/r^2$ (approximately equal to the work travel time factor) and the density pattern is uniform, the change in trip duration was inversely proportional to the square root of the network speed change, whereas the change in trip distance was directly proportional to the square root of the change in network speed. It was also noted that the trip duration change was directly proportional to the population change raised to the 0.3 power.

A three-way analysis of variance was made to determine the amount of variation in trip length explained by population, network speed and urban density patterns. The effect

TABLE 7
AVERAGE TRIP TIMES, HYPOTHETICAL CITY

<table>
<thead>
<tr>
<th>AVG. SPEED (MPH)</th>
<th>AVG. TRIP DURATION (MIN)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$F = 1/r^2$</td>
</tr>
<tr>
<td>25</td>
<td>21.3</td>
</tr>
<tr>
<td>1,000,000</td>
<td>26.8</td>
</tr>
<tr>
<td>2,000,000</td>
<td>27.4</td>
</tr>
<tr>
<td>30</td>
<td>48.6</td>
</tr>
<tr>
<td>500,000</td>
<td>35.7</td>
</tr>
<tr>
<td>1,000,000</td>
<td>17.6</td>
</tr>
<tr>
<td>2,000,000</td>
<td>21.4</td>
</tr>
<tr>
<td>25</td>
<td>29.8</td>
</tr>
<tr>
<td>1,000,000</td>
<td>41.2</td>
</tr>
<tr>
<td>2,000,000</td>
<td>30.3</td>
</tr>
<tr>
<td>1,000,000</td>
<td>14.8</td>
</tr>
<tr>
<td>35</td>
<td>21.5</td>
</tr>
<tr>
<td>500,000</td>
<td>22.6</td>
</tr>
<tr>
<td>1,000,000</td>
<td>35.6</td>
</tr>
<tr>
<td>2,000,000</td>
<td>26.5</td>
</tr>
</tbody>
</table>
of population on average trip length was studied according to the third, fourth and fifth root relationships. It was also hypothesized that average trip length is directly related to average travel time. Table 8 summarizes this analysis for gravity model runs with travel time exponents of 0, 1, and 2.

The significant effect of population size is apparent. Where the rate of new highway construction approximates the rate of population and traffic increase (so that network speeds remain fairly stable over time), population changes will become the overriding factor.

Spatial Arrangement of Activities

To test hypothesis 7, an analysis was made of the relationships between trip length and the spatial arrangement of trip activities. A consideration of how to measure this spatial arrangement led to the concept of the opportunity distribution. (This is, in effect, the gravity model distribution with travel time factors equal to 1.0).

Figure 2 shows the opportunity distribution of work trips for several cities. The shapes of the curves explain a great deal about the natures of the cities involved. Erie, a rather small city geographically, has a very steep distribution pattern, whereas Seattle is spread out and so are the opportunities. The variation in work trip duration between cities, shown in Figure 2, certainly reflects the variation in opportunity distribution depicted.

Figure 3 illustrates what happens to the work opportunity distribution for a city over a period of time; in this case, Washington, D.C., between 1948 and 1955. The change in the mean was quite substantial and parallels the change in work trip length, which was discussed in the section on regression analysis.

Figures K-1, K-2, K-3 and K-4 show the opportunity distributions for home-based shop, home-based social-recreation, nonhome-based, and truck trips. These non-work trips

### Table 8

<table>
<thead>
<tr>
<th>VARIATION FACTOR</th>
<th>RULE TYPE</th>
<th>( F = 1/t^0 )</th>
<th>( F = 1/t^1 )</th>
<th>( F = 1/t^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population size</td>
<td>Cube root</td>
<td>57.4</td>
<td>56.8</td>
<td>58.7</td>
</tr>
<tr>
<td></td>
<td>Other</td>
<td>5.7</td>
<td>9.6</td>
<td>13.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>63.1</td>
<td>66.4</td>
<td>71.9</td>
</tr>
<tr>
<td></td>
<td>Fourth root</td>
<td>54.9</td>
<td>61.2</td>
<td>63.2</td>
</tr>
<tr>
<td></td>
<td>Other</td>
<td>8.2</td>
<td>5.2</td>
<td>8.7</td>
</tr>
<tr>
<td></td>
<td></td>
<td>63.1</td>
<td>66.4</td>
<td>71.9</td>
</tr>
<tr>
<td></td>
<td>Fifth root</td>
<td>52.3</td>
<td>58.8</td>
<td>66.3</td>
</tr>
<tr>
<td></td>
<td>Other</td>
<td>10.8</td>
<td>7.6</td>
<td>5.6</td>
</tr>
<tr>
<td></td>
<td></td>
<td>63.1</td>
<td>66.4</td>
<td>71.9</td>
</tr>
<tr>
<td>Network speed</td>
<td>Inverse</td>
<td>24.2</td>
<td>21.9</td>
<td>15.2</td>
</tr>
<tr>
<td></td>
<td>Other</td>
<td>1.3</td>
<td>2.2</td>
<td>3.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>25.5</td>
<td>24.1</td>
<td>20.7</td>
</tr>
<tr>
<td>City centralization</td>
<td></td>
<td>7.6</td>
<td>6.1</td>
<td>4.9</td>
</tr>
<tr>
<td>Total explained variance</td>
<td></td>
<td>96.2</td>
<td>96.6</td>
<td>97.5</td>
</tr>
<tr>
<td>Unexplained variance</td>
<td></td>
<td>3.8</td>
<td>3.4</td>
<td>2.5</td>
</tr>
</tbody>
</table>
Figure 2. Work opportunity distributions across three cities, and approximate average trip durations.

Figure 3. Work opportunity distributions for Washington D.C., in 1948 and 1955.
opportunity distributions give the same general patterns found for home-based work trips. However, the change in opportunity distribution does not always follow trip length change. This is particularly so for shopping trips, which, of course, reflect the findings of the multiple regression work.

**MEAN OPPORTUNITY TRIP LENGTH**

As part of the previously described simulation study, the relationship between the average opportunity and actual trip duration was investigated. The following relationships were developed:

\[
\text{for } F(t) = 1/t \quad \bar{t} = 1.33 \bar{O}^{0.85} \quad (14a)
\]

\[
\text{for } F(t) = 1/t^2 \quad \bar{t} = 2.41 \bar{O}^{0.50} \quad (14b)
\]

in which

\( F(t) \) = travel time factor at time \( t \);

\( \bar{t} \) = average trip duration; and

\( \bar{O} \) = average of the opportunity distribution.

The work trip duration and opportunity patterns found in six cities were then compared to the results of the simulation study, as shown in Figure 4. From the comparison of actual and simulated trip durations related to mean opportunity trip duration shown in the figure, it was concluded that the change in trip duration should be stated as follows:

\[
\bar{t}_2 = \bar{t}_1 \left( \frac{\bar{O}_2}{\bar{O}_1} \right)^{0.00} \quad (15)
\]

in which

\( \bar{t}_2 \) = average trip duration at time 2;

\( \bar{t}_1 \) = average trip duration at time 1;

\( \bar{O}_2 \) = mean of the opportunity distribution at time 2; and

\( \bar{O}_1 \) = mean of the opportunity distribution at time 1.

---

![Figure 4. Average actual vs opportunity work trip durations.](image-url)
In applying Eq. 15 to time series data from Washington between 1948 and 1955, it was found that it did not give quite as good results as did the gravity model using travel time factors equal to $1/t^2$. This may be due to the fact that any rule related to a change in the mean of the opportunity distribution will not be as accurate as one related to an entire change in the distribution.

**TRIP LENGTH INDEX**

In light of the Washington analysis an index was developed by the application of travel time factors of $1/t^2$ to the work opportunity distribution for seven cities. The resulting work trip length index versus trip duration for these cities is shown in Figure 5. Although the data are far from sufficient, the plots are somewhat linear through the population range of 800,000 or greater. It should also be noted that the 15 percent increase calculated for Washington, D.C., between 1948 and 1955 closely approximates the 14 percent increase that occurred.

A similar index was developed for home-based shop, home-based social-recreation, and nonhome-based trip durations. The index is basically the same except for the travel time factors of $1/t^4$, $1/t^3$, and $1/t^2$ for shop, social-recreation, and nonhome-based trip durations, respectively. The findings show no variation in shop duration. Social-recreation and nonhome-based trip durations, however, varied with changes in these trip length indices, as shown in Figures 6 and 7. But the lack of high consistency in the pattern again indicates that any general index is not too accurate.

**LOCATION OF WORKPLACES AND RESIDENCES**

The relationship between the actual work trip duration and work trip opportunities led to an attempt to relate empirically some spatial measures of the locations of workers' residences and workplaces. The average work trip distance was found to be almost a linear function of the average distance of workers from the central business district (Fig. 8), which is similar to the findings of Herla (37), who demonstrated that median trip length was a function of the radius of gyration of the city's population.

The standard deviation of the trip time distribution can also be explained as a function of the location of workers and jobs measured indirectly by average trip duration (Fig. 9).

To simplify the prediction of this variance (square of the standard deviation), this parameter was related to population. The results (Fig. 10) show that a linear relationship exists between the trip length variance and population for the smaller cities, as follows:

$$r^2 = 12.8P \quad 0 < P \leq 6 \quad (16)$$
in which $P$ is population in 100,000's. As population increases beyond 600,000 the relationship becomes nonlinear and approaches an asymptotic value of approximately 96.

From the relationships shown in Figures 8 and 9, the actual auto driver work trip length distribution can be determined by use of the gamma distribution. By substitution of the relationship shown in Figure 10, the gamma function can be used to estimate the distribution satisfactorily.

---

**Figure 6.** Average social-recreation trip duration vs trip length index.

**Figure 7.** Average nonhome-based trip duration vs trip length index.
Economic and Social Factors Influencing Trip Lengths Within Cities

The analyses which follow are designed to test hypotheses 8 and 9—the effect of income on work and social-recreation trip lengths, and impact of the social structure of a city, and length on a study area basis using multiple regression techniques.

Effects of Income

To investigate the difference in travel behavior of persons having different incomes, an effort was made to examine the way these trip makers view the cost of making trips of various durations. Data from Washington, D.C., were used to examine the effect of income on work and social-recreation trip durations. Data from Springfield, Ill., were used to see whether the findings in Washington for social-recreation trip durations would be applicable to a smaller urban area.

Work Trips.—In order to analyze the effect of income on work trips, an analysis was made of the differences in trip duration for workers of various income groups. This investigation began with the stratification of the Washington, D.C., O-D work trip matrix by income class. Income data were then obtained from the National Capital Planning Commission's 1955 Mass Transportation Survey.

The gravity model was run, using the same set of travel time factors stratified by income class. It should be noted that the socio-economic K-factors were not used. The results are given in Table 9.

The performance of the model using travel time factors derived from all income groups combined was good. The average trip duration and the trip frequency distributions were comparable to those developed from the origin-destination survey.

To check these results further, two tests were performed. First, inter-district volumes were checked between all districts and the downtown area. This was done to be sure that the model was actually producing a realistic trip pattern.

A comparison of the prediction errors for the inter-district movements having one end in the central business district is shown in Figure 11. Inasmuch as a regression line of 45° would indicate no improvement by stratification, the shallower slope of this line indicates the sizeable degree of improvement that was obtained. A summary measure of the amount of improvement was obtained by comparing the average percentage error for the two methods. For the unstratified model the average error was 18.5 percent, whereas for the stratified model the average error was 10 percent, a reduction of more than 45 percent.

An examination of work trips from the Northwest Cor-
Figure 10. Work trip variance vs population.

The model seemed to be performing in a satisfactory manner. The pattern indicated by Tables 9 and 10, that high income persons generally have longer work trip lengths, does not differ significantly from the findings of other researchers. However, the explanation for this does not seem to lie in the fact that high income groups have a greater propensity for travel, but instead is due to spatial distribution of residential areas and employment for different income groups.

To help support these findings a comparison was made of changes in the work trip duration and variance for different income classes in Washington, D.C., between 1948 and 1955, as given in Table 11. This analysis was undertaken to determine whether changes in the work trip length distribution are affected by the distribution of jobs and increased income.

Table 11 can be summarized as follows:

1. There is a tendency for higher income groups to make longer trips than lower income groups. In 1948 the highest income group spent almost 50 percent more time on the work trip than did the lowest income group. By 1955 this difference was only 25 percent.

2. The two lower income groups lengthened their average work trip length in the 7-year period, while the upper groups both decreased their average work trip length.

3. The changes in average work trip length are less than 10 percent and are not as significant as the changes in the variance or dispersion in the length of trip.

4. The higher income groups had a much larger variance in work trip duration in 1948 than did the lower ones, the difference in dispersion being as much as 110 percent. By 1955 the difference in dispersion had been reduced less than 30 percent. This change was accomplished by a doubling in the variance of trip length for the lowest income group, a 60 percent increase for the second group, and approximately 37 percent increase for the largest group.

These findings indicate that stratification of the work trip matrix by income could help to improve forecasting ability. An objection to the use of this finding, however, might be the fact that it would be difficult to predict family income in an urban area. Appendix D describes a procedure for making this prediction.

Social-Recreation Trips.—The analysis of income effects on social-recreation trips was designed to test the hypothesis that people of a given socio-economic status tend to make social and, to a somewhat lesser extent, recreational trips in areas which have comparable status. Data from the Washington, D.C., and Springfield, Ill., transportation studies were utilized in the testing of this hypothesis.

In Washington the approach was to compare actual (O-D) trips to a set of "idealized" trips; i.e., trips which do
not account for socio-economic conditions of the zones. A natural candidate for generating such trips is the gravity model, with the K-factors all set equal to unity—thus, spatial separation is all that would be considered. Income class was selected to measure the socio-economic status of a zone, and each of the 400 zones in Washington was assigned to an income class (1, 2, 3, or 4, from lowest to highest). The hypothesis was then reduced to the following: O-D trip volumes between zones of similar income classes are greater than gravity model trip volumes between those same zones.

The tests of this hypothesis, described in detail in Appendix E, were designed to determine whether the differences between the O-D and gravity model trip interchange volumes were due to chance or to income bias. It was found that differences in the trip interchanges were not due to chance but were due to differences in the income groups themselves.

In the Springfield, Ill., transportation study a special analysis was made of the reasons why certain socio-economic K-factors were needed for social-recreation trips. It was found that the gravity model tended to overestimate

---

**TABLE 9**

**AVERAGE WORK TRIP DURATION BY INCOME CLASS, WASHINGTON, D.C.**

<table>
<thead>
<tr>
<th>FAMILY INCOME ($)</th>
<th>AVG. TRIP DURATION (MIN)</th>
<th>1955 O-D</th>
<th>ERROR (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 - 4,499</td>
<td>17.5</td>
<td>18.1</td>
<td>-3.3</td>
</tr>
<tr>
<td>4,500 - 6,999</td>
<td>20.0</td>
<td>20.1</td>
<td>-0.5</td>
</tr>
<tr>
<td>7,000 - 9,999</td>
<td>20.2</td>
<td>19.4</td>
<td>+4.1</td>
</tr>
<tr>
<td>10,000 -</td>
<td>21.1</td>
<td>21.0</td>
<td>+0.5</td>
</tr>
<tr>
<td>All</td>
<td>20.3</td>
<td>20.0</td>
<td>+1.5</td>
</tr>
</tbody>
</table>

**TABLE 10**

**AVERAGE WORK TRIP DURATION BY INCOME CLASS, WASHINGTON, D.C., NORTHWEST CORRIDOR**

<table>
<thead>
<tr>
<th>FAMILY INCOME ($)</th>
<th>AVG. TRIP DURATION (MIN)</th>
<th>1955 O-D</th>
<th>ERROR (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 - 4,499</td>
<td>16.4</td>
<td>15.2</td>
<td>+7.9</td>
</tr>
<tr>
<td>4,500 - 6,999</td>
<td>25.1</td>
<td>24.6</td>
<td>+2.0</td>
</tr>
<tr>
<td>7,000 - 9,999</td>
<td>19.4</td>
<td>20.0</td>
<td>-3.0</td>
</tr>
<tr>
<td>10,000 -</td>
<td>21.2</td>
<td>21.3</td>
<td>-4.7</td>
</tr>
<tr>
<td>All</td>
<td>21.7</td>
<td>21.7</td>
<td>0.0</td>
</tr>
</tbody>
</table>

*Figure 11. Comparison of prediction errors for income-stratified and unstratified gravity models, inter-district trips to CBD, Washington, D.C., 1955.*
social-recreation trips from areas of low income to areas of high income and, conversely, underestimated trips from areas of low income to areas of low income.

The gravity model without $K$-factors does not account for the heavier social-recreation interaction between people of similar socio-economic status or the lack of interaction between different socio-economic classes. These findings are presented graphically in Figure 12.

It is necessary to keep in mind the question of the validity of representing the socio-economic condition of each zone by a class spanning a range of incomes, each of which in itself is only a representation of an aggregate. However, the findings of these two studies indicate the validity of the hypotheses related to income bias for social-recreation trips.

EFFECTS OF SOCIAL STRUCTURE

The clustering of trip patterns has been observed in some cities to be based on the social, rather than the economic, structure of the community. The phenomenon was found to

---

![Figure 12. Social-recreation origin-destination/gravity model ratios vs income.](image)
occur frequently in large metropolitan areas that enclose a number of separate communities. The effect of multiple city centers was found to be related to a social and informational orientation rather than to a merely governmental or physical separation (28, 87).

This clustering of trip patterns has been found to exist in cities such as Boston, Mass.; Cedar Rapids, Iowa; Minneapolis-St. Paul, Minn.; Lackawanna-Luzerne, Pa.; and Ottawa-Hull, Ont. In some of these cities there is a physical boundary associated with the social division. The fragmentation of the area probably began because of the travel friction introduced by the local geography. These patterns were then reinforced by separation of such information sources as newspapers and radio stations. These traditions of relative isolation may currently be carried on simply through custom and habit.

A detailed analysis to investigate this social structure influence was made in the Minneapolis-St. Paul area and in Lackawanna and Luzerne Counties in Pennsylvania.

In the Minneapolis-St. Paul area both political and physical boundaries separate the Twin Cities. In calibrating a gravity model for this area, it was found that socio-economic K-factors were required to bring travel across a

<table>
<thead>
<tr>
<th>INCOME LEVEL</th>
<th>TRIP DURATION (MIN)</th>
<th>1955</th>
<th>1948</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MEAN ( ± ) VARIANCE, MEAN ( ± ) VARIANCE,</td>
<td></td>
<td></td>
</tr>
<tr>
<td>LOW</td>
<td>13.2 ( ± ) 31.0</td>
<td>14.5</td>
<td>61.5</td>
</tr>
<tr>
<td>2</td>
<td>16.8 ( ± ) 51.2</td>
<td>17.0</td>
<td>83.0</td>
</tr>
<tr>
<td>3</td>
<td>18.5 ( ± ) 65.9</td>
<td>17.2</td>
<td>75.2</td>
</tr>
<tr>
<td>HIGH</td>
<td>19.2 ( ± ) 54.2</td>
<td>18.1</td>
<td>74.6</td>
</tr>
</tbody>
</table>

*The 3-min length is not included in order to account for the shift along the x-axis to the shortest reported trip length.*

**Figure 13. Twin Cities social orientation.**
boundary between the cities down to the level shown in the O-D survey. The necessary adjustments seemed to be unrelated to economic considerations or unusual clustering of trip attractions (28, 87). An attitude survey was conducted in the vicinity of the boundary. Residents were asked about their knowledge of various parts of the metropolitan area, including both the Minneapolis and St. Paul central business districts. It was found that the residents of certain zones were oriented toward Minneapolis, others toward St. Paul, and still others toward the Twin Cities in general. Some families had more knowledge of a shopping center in their own city than of another, nearer center across the boundary line (87). Figure 13 shows the close relationship between the boundary observed in the travel data and the attitudinal dividing line observed in the opinion survey. Failure to consider this social orientation resulted in an overestimate of average trip time. Only after the proper adjustments were made within the model did the predicted results agree with the O-D survey.

A similar pattern was observed in Lackawanna and Luzerne Counties, where the unadjusted gravity model estimate of total travel between the two counties did not duplicate the pattern found in the O-D survey.

From the survey it was found that approximately 33,000 trips a day were made between the two counties. The gravity model trip distribution overestimated trips between the two counties by 24 percent. Analysis of these trips by area and purpose indicated that the bias in the gravity model trip interchange was relatively uniform for all trip purposes and varied in relation to the length of the trip. As the trip length became longer, the ratio between the gravity model and survey interchange volumes decreased proportionally.

This condition reflects the self-containment of the two counties. Each county has its own central city, is served by its own newspaper, radio and television stations, and in general, the inhabitants of one county are not fully aware of the activities and opportunities in the other county.

Satellite cities within the counties with a high degree of self-containment interacted with surrounding areas at a substantially lower-than-average rate. Consequently, the gravity model predicted a greater-than-average number of inter-area trips. The need for socio-economic K-factors to compensate for this trend was found in areas which historically have developed as relatively isolated, independent, and balanced communities (e.g., they contained their own rather well-developed commercial facilities and, in many cases, a relatively high proportion of work opportunities relative to the population). This isolated development, in most cases, goes back to before the advent of the automobile and the extensive highway system. Even when the highway network in the area was improved and adjacent communities became more accessible, the people continued to work and trade within their own community. It is significant that invariably the areas requiring the "self-containment" adjustment are known by a community name. Residents of such areas tend to identify with their own area and not with the region as a whole. This bias will continue until those communities become more integrated with the metropolitan region.

It is reasonable to expect that similar situations exist within most cities where there are a number of distinct social groupings. Where the intra-group bounds are weak, or where there are not cohesive social groupings, it is likely that these patterns will not be visible in the travel data. These variations in travel patterns caused by social structure present a serious problem in travel forecasting because of the difficulties associated with predicting trends in social customs. In time, some of these customs might be expected to change. However, it does appear that these patterns do not change quickly, because they continue to exist in such metropolitan areas as Minneapolis-St. Paul.

CHAPTER THREE

IMPLICATIONS FOR TRAVEL FORECASTING

An important result of any research effort is the conclusions drawn from the testing of hypotheses. The purpose of this chapter is to discuss the findings of this study and their application to the field of traffic forecasting. The chapter presents guidelines for predicting trip lengths, improvements which can be made in existing forecasting models, and a new distribution model.

GUIDELINES FOR PREDICTING TRIP LENGTH

The various hypotheses developed and tested have improved the understanding of the factors influencing trip length. Some of the findings were contradictory; but after evaluating all of these tests it appears that certain guidelines can be developed which should prove useful in making estimates of what trip lengths are likely to be with changes in the size and physical structure of urban areas, changes in network speeds, and adjustments in socio-economic factors. An attempt has been made to summarize these guidelines and indicate the observations that led to these conclusions.

These guidelines, which present how size and physical structure, network speed and socio-economic factors influence trip length, are divided into those related to work trips and non-work trips.
**Work Trips**

**SIZE AND PHYSICAL STRUCTURE**

*Case 1.*—If an urban area grows by extending its present population and employment density patterns, the change in the average work trip length will probably be proportional to the fourth root of the population change (see Fig. 14).

The results of the multiple regression analyses in the various cities studied indicated that the trip length changed in proportion to the fifth root of the population change. Results in the simulation studies ranged from the third to the fifth root. However, historical studies in Baltimore and Washington, D.C., indicate that if a city grew by extending its present population and employment density patterns, its trip length increased in proportion to the third root. Thus, it would appear that a fourth root rule might be considered an average of the various studies that have been undertaken and, therefore, can be used for estimating purposes. The exact exponent will depend on the extent to which present density patterns are extending into the suburban areas.

*Case 2.*—If an urban area grows largely by the filling in of unused land, while maintaining its same density pattern, there will be no material change in work trip length (Fig. 14).

The historical data from Broward County, Fla., indicated that there was practically no change in trip length as it grew by the filling in of unused areas between various developments within the county. Simulation studies would indicate that if the density profile did not change there would be no change in trip length if the area was not enlarged. Thus, it is felt that this rule is appropriate for communities which are growing by the filling in of voids in land development.

*Case 3.*—If an urban area develops by concentrating additional population and employment in the downtown area and/or other sections of the metropolitan area, the average work trip length would probably decline (Fig. 14). These studies indicate that this decrease could vary from 5 to 10 percent.

Simulation studies indicated that with centralization there was a reduction in trip length. This had also been observed in various transportation studies tested, such as those in Hartford, Conn., and in the Baltimore-Washington region.

**NETWORK SPEED**

*Case 4.*—Changes in the average work trip distance (miles) will be directly proportional to the square root of changes in peak-hour network speed.

Although the multiple regression analysis indicated that work trip length seemed to increase at a faster rate than network speeds, it was felt that this was unreasonable in light of the findings of the simulation studies, which indicated that trip length increased in proportion to the square root of the change in network speed. The multiple regression analysis was measuring the impact of higher network speeds on the spread of the city, which by itself increased trip length. The simulation studies, therefore, probably give a better description of the impact of network speeds if the impact of the network speed on urban development is considered separately.

*Case 5.*—Change in the average work trip duration (minutes) will be inversely proportional to the square root of change in peak-hour network speed.

The multiple regression analysis indicated that network speeds had no impact on work trip duration. The simulation studies, however, showed that travel time was reduced with increased network speeds. For the same reasons as described in relation to trip length, it is felt that the findings of simulation studies are more appropriate.

**Socio-Economic Factors**

*Case 6.*—A more heterogeneous distribution of income in an urban area could reduce work trip length by as much as 10 percent, but with present social attitudes this is not likely to happen.

The analysis of work trips by different income groups in the Washington area indicated that the work trip length for any given income class was related to the spatial rela-
tionship between the residences of people in that income class and their jobs. With the present economic colonization of our cities, it is apparent that certain income classes are forced to travel farther to work than if there had been a more heterogeneous distribution of income throughout an urban area. The magnitude of this change was concluded from the analysis of 1955 work trips in the Washington area and from some of the studies undertaken by the Baltimore-Washington Interregional Study.

Case 7.—Changes in existing historical and social patterns could change work trip length by 5 percent, but it appears from the data investigated that these changes occur very slowly.

As has been indicated in Chapter Two, there are numerous situations where historical social factors had influenced trip length. In most cases these patterns had retarded trip length, because people tend to stay within their own community rather than go beyond its limits in the search for jobs. From various assignments that were developed in the St. Paul area, it was estimated that this influence, in effect, depressed trip lengths about 5 percent.

Non-Work Trips

SIZE AND PHYSICAL STRUCTURE

Case 8.—Nonhome-based trip lengths will change at about the same rate as work trips, whereas social-recreation trip lengths will change as the 0.7 power of the work trip duration.

These guidelines reflect the multiple regression analyses that were made, relating nonhome-based and social-recreation trip duration with work trip duration. They were also verified from historical data for Washington. The guideline on nonhome-based trip duration reflects the fact that as an area grows, travel is generally more complex, and longer nonhome-based trips are produced. With regard to social-recreation trips, the findings seem reasonable, inasmuch as the travel time factors have a higher exponent in the gravity model, and the simulation tests indicate that in such cases the length would be less.

Case 9.—Shop trip lengths are related to commercial practices (size and location of commercial areas relative to residential areas, as well as the variety of merchandise offered). Because of better retailing practice, shop trip length has been changing slowly, and it appears that little change can be expected in the future.

The multiple regression analysis of shop trips indicated that shop trip lengths are not related to any particular characteristics of urban areas, such as size or population. As indicated in Chapter Two, a review of shopping patterns in several areas, such as the Virginia Peninsula, found that these trip lengths are dictated largely by retailing practices. Therefore, any estimate of future shop trips will depend on retailing practices at that time. Recent trends in retailing indicate that although there is presently more development of larger centers, there is also some development of smaller outlets, such as the 7-11 Stores. Therefore, it would appear that at the present time no particular change in trip length is likely to occur unless these retailing practices change.

Case 10.—Truck trip length changes are related to variations that occur in the spatial arrangement of residential, commercial, and industrial land uses.

The multiple regression analysis indicated a strong relationship between shop and truck trip lengths. This would indicate that no change would occur in truck trip lengths because shop trip lengths have been remaining fairly stable. However, a traffic forecast in Waterloo, Iowa, indicated a change in truck trip lengths with a spread population in the outlying area, even though the shop trip length remained constant. Because truck trip lengths are also related to the locations of warehouses and businesses, care should be exercised in trying to relate truck trip length changes to those of shop trip lengths.

NETWORK SPEED

Case 11.—Average shop and truck trip lengths do not appear at this time to be related to changes in average network speed.

This guideline is based on the multiple regression analysis, which indicated that both truck and shop trip lengths are not sensitive to changes in the network speed. It also reflects the fact that shop and truck trips are generally short and the major portion of them are not involved with high-speed facilities.

Case 12.—The change in the average social-recreation trip distance (miles) appears to be proportional to the cube root of the change in the off-peak network speed.

This guideline reflects the multiple regression analysis, where it was found that social-recreation trip lengths changed as the 0.7 power of the work trip length change. As was pointed out previously, according to the simulation studies the work trip changed as the square root of the network speed. It was therefore felt that the social-recreation trip length would change at a slower rate than the work trip (approximately the 0.7 power). Inasmuch as these trips occur largely during the off-peak hours, the social-recreation trip length should be directly proportional to approximately the cube root of the change in network speed.

Case 13.—The change in the average social-recreation trip duration (minutes) appears to be inversely proportional to the cube root of the change in the off-peak network speeds.

This was based on the same logic as set forth in Case 12.

Case 14.—Nonhome-based trip lengths will change in the same manner as work trips.

The multiple regression analysis indicated that nonhome-based trip length increases with network speed. This is similar to findings for the work trip length, which were modified by results of the simulation study that indicated trip length changed as the square root of network speed. Because no simulation studies were made of nonhome-based trip length, it was felt that the general guideline, as indicated, should be used until additional evidence is developed.
SOCIO-ECONOMIC FACTORS

Case 15.—More heterogeneous distribution of incomes within an urban area could increase social-recreation average trip lengths as much as 5 percent, but would have little impact on other types of non-work trips. But with present social attitudes such a trend is not very likely.

This guideline primarily reflects the work of various special studies made on social-recreation travel, which indicated that the income colonization within cities has tended to force people to travel shorter distances than they would if incomes were more heterogeneously dispersed throughout the region. In other words, it is the opposite of work trips. Thus these factors tend to offset each other.

Case 16.—Changes in existing historical and social patterns could change non-work trip lengths as much as 5 percent, but, as in the case of work trips, these changes are not likely to occur very fast.

Again, this guideline reflects the special studies made in Minneapolis-St. Paul and other areas, which show that all non-work trip patterns are influenced by these historical factors. In other words, these same factors seem to have the same impact whether related to work or to non-work trips, therefore the general observations made for work trips probably apply for non-work trips.

Use of Guidelines

The guidelines described in the foregoing indicate clearly that the trip length patterns in any area will depend on many factors, some of which may even counteract each other. In predicting trip lengths for any community, the estimate should be based on the appropriate guidelines developed from anticipated changes in city size and physical structure, network speed, and socio-economic factors. For each area the answer is likely to be different, reflecting unique changes that will occur. But, for a “typical community,” it is quite clear that the main impact on trip length will be related to network speed, and socio-economic factors. For each area these same factors seem to have the same impact whether related to work or to non-work trips, whereas in smaller metropolitan areas the increase might be in the 15 percent range.

On such an assumption, it is possible to make some estimate of changes that are likely to occur in trip lengths. In larger metropolitan areas—over 2 million population—it appears that there will be some concentrations of population and employment in their downtown centers and in various suburban sections, but expansion in the outlying areas will continue. Therefore, the change in the work trip length would be between Case 1 and Case 3 in the guidelines, which would mean about a 10 percent increase in trip length with a 100 percent increase in population. This would also mean that the nonhome-based trip length would increase about the same amount, whereas the social-recreation trip length would increase only about 7 percent Case 8).

Experience in Los Angeles, where little increase in speed has occurred at the peak hour, has shown that it is very difficult to anticipate an increase in peak-hour speeds in large metropolitan areas (Fig. 15). However, these speeds might increase as much as 10 percent for the peak hour and 20 percent for off-peak hours. On the basis of Case 4, the work trip lengths might increase by 5 percent, and on the basis of Cases 12, 13, and 14, social-recreation and nonhome-based trip lengths might increase about 10 percent.

In metropolitan areas of less than 500,000 population, present growth patterns are likely to continue and trip lengths could increase in line with Case 1, which would mean that with a 100 percent increase in population, work trip length might increase about 20 percent, nonhome-based trip length would increase at the same rate, and social-recreation trip lengths would increase 14 percent (Cases 1 and 8). In many cities of this size present expenditures for highway facilities may have a substantial impact on peak-hour speeds. These speeds could possibly be 20 percent higher at the peak hour, as well as in off-peak periods, in which case the work and social-recreation trip lengths would increase about 7 percent, and nonhome-based trip lengths about 10 percent (Cases 4, 12, and 14).

These assumptions are summarized in Table 12 for the work, social-recreation, and nonhome-based trips. If each trip category is weighted by its length and its proportion of urban travel, and if they are added together, it would show that in larger metropolitan areas there would be only a modest increase in trip length—between 5 and 10 percent—whereas in smaller metropolitan areas the increase might be in the 15 percent range.

<table>
<thead>
<tr>
<th>TYPE OF TRIP</th>
<th>LARGER METROPOLITAN AREAS</th>
<th>SMALLER METROPOLITAN AREAS</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) CHANGES RELATED TO CITY STRUCTURE</td>
<td>Work 10%</td>
<td>20%</td>
</tr>
<tr>
<td></td>
<td>Social-recreation 7%</td>
<td>14%</td>
</tr>
<tr>
<td></td>
<td>Nonhome-based 10%</td>
<td>20%</td>
</tr>
<tr>
<td>(b) CHANGES RELATED TO NETWORK SPEED</td>
<td>Work 5%</td>
<td>10%</td>
</tr>
<tr>
<td></td>
<td>Social-recreation 7%</td>
<td>7%</td>
</tr>
<tr>
<td></td>
<td>Nonhome-based 10%</td>
<td>10%</td>
</tr>
<tr>
<td>(c) TOTAL CHANGE IN TRIP DURATION (MINUTES) a</td>
<td>Work 5%</td>
<td>10%</td>
</tr>
<tr>
<td></td>
<td>Social-recreation 0%</td>
<td>7%</td>
</tr>
<tr>
<td></td>
<td>Nonhome-based 0%</td>
<td>10%</td>
</tr>
<tr>
<td>(d) TOTAL CHANGE IN TRIP LENGTH (MILES) b</td>
<td>Work 15%</td>
<td>30%</td>
</tr>
<tr>
<td></td>
<td>Social-recreation 14%</td>
<td>21%</td>
</tr>
<tr>
<td></td>
<td>Nonhome-based 20%</td>
<td>30%</td>
</tr>
</tbody>
</table>

* The change in work trip duration (min) for a large metropolitan area, for example, would be (1.10)(1/1.05) = 1.05, or a 5% increase.
* The change in work trip length (miles) for a large metropolitan area, for example, would be (1.10)(1/1.05) = 1.15, or a 15% increase.
Figure 15. Los Angeles 30-min peak-hour isochronal lines for 1957, 1960, 1962 and 1963.
**Trip Length Distribution**

The guidelines were set up to establish trip lengths either in terms of average time or distance. However, a more complete picture of trip length is obtained when the trip length distribution is established. This can be done by using the gamma distribution. All that is required is to estimate trip variance and its mean.

Studies have shown that the work trip variance changes with work trip duration, as demonstrated in Figure 16. Thus, with estimates of the mean developed from the guidelines and the variance established by Figure 16, it is possible to construct the work trip distribution. The gamma distribution requires as parameters the values of the mean \( \bar{t} \), and the variance, \( \sigma_t^2 \), in determining the work trip distribution; that is,

\[
f(t) = K \frac{t^{(\bar{t}-\sigma_t^2)/\sigma_t} e^{-(t/\sigma_t)}}{\sigma_t^2} \quad (17)
\]

in which

- \( f(t) \) is the relative frequency of trips of duration \( t \);
- \( K \) is a constant;
- \( e \) is the base of natural logarithms;
- \( \bar{t} \) is mean trip length; and
- \( \sigma_t^2 \) is variance of \( t \).

![Figure 16. Work trip duration variance vs average trip duration.](image-url)
IMPROVEMENTS IN EXISTING FORECASTING MODELS

This research indicated significant changes that should be considered in the application of existing forecasting models for transportation and general planning purposes. These changes lie in five areas: (1) travel time; (2) trip classification; (3) land-use models; (4) \( L \) or travel time factors; and (5) land-use forecast.

Travel Time

It is quite clear from the analysis of the Los Angeles data that in larger metropolitan areas peak travel times should be used in the distribution of the work trip, and off-peak travel times should be used for other trips. In other words, the times that should be used are those which most closely reflect the conditions under which these trips are typically made.

The possibility of using travel costs instead of travel time was suggested by some studies as described in Appendix A. Before cost is used as a parameter, however, additional research should be conducted to determine if this method is superior to that of using peak or off-peak times, depending on trip purpose.

This research has clearly indicated that it is important to use a realistic travel time in the application of the gravity model. In forecasting traffic for larger metropolitan areas, this will often call for a capacity constraint, or some other type of technique which will adjust peak-hour travel times in accordance with forecasted traffic. However, even with such procedures, the guidelines previously described should be used to test the reasonableness of the travel forecast.

In connection with the opportunity model, the guidelines should also be used to determine if realistic values of \( L \) have been used in light of the trip length that is produced.

Trip Classification

Although there probably are many modifications that could be made in trip classification, the most significant one revealed in this research was that related to income. The results of income experiments on work trips in Washington, D.C., indicated that stratification of the work trip matrix by income would correct for the trip distribution bias. Income biases were also found for social-recreation travel. Thus, it would appear that in the larger metropolitan areas it is probably desirable to stratify both work and social-recreation trips by income.

Land-Use Model

The observations related to trip classification have a direct effect on the data requirements for travel forecasting. To implement these findings, it will be necessary to develop forecasts of the spatial distribution of trip origins and destinations by income class. Appendix D describes a model.
for the prediction of income in a residential area. Since this residential income model was developed, improvements have been made to it. However, a model to forecast salary structure in employment areas has not been developed, although it is needed not only for traffic forecasting but also for general planning purposes.

**L and Travel Time Factors**

It is quite clear from this research that different values of $L$ should be used for various opportunity distributions. Therefore, a set of values of $L$ employed in any forecast should be related to opportunity distribution. In the case of the gravity model, separate travel time factors should be used if there is a wide variation in opportunity distribution between zones. The impact of variations in the opportunity distribution between zones is much greater in the opportunity model than in the gravity model, and this must be recognized in its application.

**Land-Use Forecast**

It is quite clear from the research that has been done that inaccurate land-use forecasts quite often are the real reasons for many bad traffic estimates. Often, however, the guidelines related to trip length can be used to check the reasonableness of these land-use forecasts. For example, if a poor job has been done in distributing retailing activities, the application of the gravity model will show an unrealistic change in trip length. Although a poor land-use forecast will not have as much impact on the opportunity model, it should also be checked for reasonableness.

**A NEW DISTRIBUTION MODEL**

Based on the previously described findings as well as special studies of present distribution models (described in Appendices F, G, H, I, and J) it became apparent that a new distribution model could be developed—one which considered the spatial arrangement of trip opportunities and travel time impedance. Investigations of such a model were undertaken with data obtained in Washington, D.C., and Worcester, Mass.

Travel time ($F$) factors for 1948 were obtained for 11 zones in Washington, D.C., and were fitted using the gamma distribution. The travel time factors were then compared to corresponding opportunity distributions developed for each of these zones. Figure 17 shows 1948 opportunity distributions for three zones in Washington, D.C., which are representative of the mean and extreme ends of the observed opportunity distribution. Zone 048 is near the CBD, zone 255 is several miles from the downtown area, and zone 298 is in the suburban area.

Figure 18 shows the relationship of the travel time factors to trip duration for these three zones. A comparison of the two figures indicates an apparent relationship between mean opportunity lengths and the shape of the travel time factor curve. Figure 19 indicates the relationship between the shape parameter, $\frac{\tau^2}{\sigma^2}$ (see Appendix B for a more detailed description of this parameter), of the travel time factors and the mean opportunity length (min) for eight of the eleven zones analyzed. As the mean opportunity length of the opportunity distribution increased, the value of the shape parameter decreased. This relationship implies a greater weighting of nearby activities as opportunities arrange themselves at greater mean opportunity times from a particular zone. The observation seems to indicate that the travel time ($F$) factor in the gravity model (or the $L$ factor in the opportunity model) should be modified for variations in the opportunity distribution.

After the investigation in Washington, D.C., it became apparent that a measure of the spatial arrangement of activities could be used to develop varying travel time factors.

![Figure 18. Work travel time factors vs trip duration for three selected zones in Washington, D.C., 1948.](image-url)
An analysis undertaken in Worcester, Mass., developed travel time factors for various classes of trips arranged by the percent of opportunities passed. The relationship observed between these classes (percent of opportunities passed) and the mean travel time factor is shown in Figure 20.

This relationship indicates that the mean travel time factor decreased as the percent of opportunities passed increased. This is similar to the relationship observed between the shape parameter of the gamma distribution and the mean opportunity trip length observed for Washington, D.C., shown in Figure 19. The travel time factors stratified by representative opportunity classes are shown in Figure 21. These relationships indicate that the propensity to travel for work trips increases as the percent of opportunities passed decreases.

Figure 22 shows a plot of the actual versus the estimated work trip length distribution. The results are good. It should be pointed out, however, that this comparison was made with some inherent problems still in the model. For example, it showed an under-attraction of trips to the CBD and irregular travel time factors within each class of percent of opportunities passed.

Thus, it would appear that a new distribution model should be developed that recognizes the influence of both travel time and the spatial arrangement of opportunities.
Figure 22. Comparison of actual and theoretical work trip duration distributions.

Source: Worcester, Massachusetts
Internal Travel Only
Walk to Work Excluded
APPENDIX A

A GENERALIZED USER COST FUNCTION AND
ITS APPLICATION TO DIVERSION CURVE THEORY

Time is not the only expenditure considered in the urban travel decision. Some researchers have tried to demonstrate the importance of the level of service and psychological cost of making trips (36). Although the establishment of a means of measuring true psychological cost to the traveler is difficult, the consideration of the time and out-of-pocket cost of a trip should provide a more accurate representation of the travel environment. To accomplish this, it is necessary to determine the relative weights of monetary costs and time. That is, the proper combination of these two measures must be developed in order to define a realistic user cost function.

The objective of this part of the research was to investigate further the way in which trip-makers perceive separation. The measure of their perception is based on driver behavior, given a set of alternatives with different characteristics. The best available data of this type were accumulated in route selection or diversion studies. Research in developing diversion curves for assigning a portion of the zonal interchange volume to an expressway, as opposed to the best alternate route, indicates that the best results are most often obtained by using a combination of both comparative distance and comparative time (11, 12, 56). These data offer an opportunity to generalize on the relative weights that drivers give to time and distance; that is, to out-of-pocket costs associated with distance and the value of personal time. In this section, the expected shape of the user costs function is determined, and the parameters of the function are statistically estimated from behavioral data. The values of the parameters in the function represent the relative weights of the components of the cost function. (Division curve data for the Shirley Highway in Washington, D.C., the Dallas Central Expressway, the Gulf Freeway in Houston, and the Alvarado-Mission Valley Freeway in San Diego were used in this analysis.)

When choosing among alternatives, one ordinarily combines the gains and costs of the various alternatives according to his value system and picks that which is "best" for him. It is well recognized that different people, when faced with the same choice, do not always choose the same alternative. These differences are usually ascribed to two factors. The first is imperfect information concerning the true gains and costs of the alternatives. Second are basic differences in the value systems of people. In most instances, available data only describe variations in behavior and do not distinguish between the two sources of variation.

The hypothesis used in explaining the variations in behavior evidenced by diversion curves is that the variations stemming from individual values and information accuracy are normally distributed. For an algebraic difference, the comparative cost distribution will be normal, whereas for a cost ratio the comparative distribution will be approximately log-normal.

Support for this hypothesis is given by Figures A-1 and A-2, in which the normal probability plot of the percentage of people using the freeway is plotted against distance difference and time difference, respectively. Distance difference and time difference each represent a major portion of a user cost function. Both plot in a somewhat linear manner on probability paper, indicating that the comparative cost function could be normal.

It is to be expected that this distribution of route preference with respect to comparative costs would have a zero mean. Even though the distribution may not have a zero mean for distance difference or time difference when considered separately, it should when the total cost function is considered. This is because one would expect one-half the people to use the freeway and one-half to use the alternate when the comparative costs are equal, always remembering that costs are defined by user value systems.

The problem of estimating the proportion that will use each route then becomes one of determining the area under the curve on each side of the comparative cost measure.

Two illustrative cases are shown in Figure A-3 as $\Delta C_1$ and $\Delta C_2$, where $\Delta C$ refers to arterial cost minus freeway cost. In each case, the area under the curve to the left of the line corresponding to a given $\Delta C$ represents the proportion of trips taking the freeway, while the area to the right represents the proportion taking the arterial. Thus, for $\Delta C_1$, where freeway costs are lower than arterial costs, the area to the left of the vertical line indicates the larger proportion of persons who will use the freeway. The converse holds for the case $\Delta C_2$.

Estimating the area under the curve on either side of $\Delta C$ may be accomplished by integrating the equation for the normal curve to the number of standard deviations that is equivalent in position to $\Delta C$.

Similar reasoning can be extended to the multivariate case where more than two alternate routes are considered. This is accomplished by determining the value of $x$, corresponding to the percentage of trips using the freeway, and relating this to $\Delta C$ through regression.

A GENERALIZED COST FUNCTION

The two major components of user cost are out-of-pocket costs and the value of personal time. Out-of-pocket costs include such items as gasoline and oil consumption, tire wear, and maintenance. These costs are generally parabolic.
with respect to speed (31). The value of personal time in this analysis was assumed to be linear.

A generalized cost function can therefore be described as follows:

$$C = D[a_0 + a_1S + a_2S^2 + a_3(1/S)]$$  \hspace{1cm} (A-1)

in which

- \(C\) = cost;
- \(D\) = distance;
- \(S\) = speed; and
- \(a_i\) = the parameters to be estimated.

In this equation \(D(a_0 + a_1S + a_2S^2)\) represents out-of-pocket costs and \(D(a_3(1/S))\) represents the value of personal time. In addition, the estimates of the parameters indicate the relative importance of the different parts of the cost function. They can be compared for consistency to estimates obtained by the usual pricing procedures. A generalized cost function can be written for use of the freeway, and another for use of an alternate. From these, the difference between the two may be evaluated. This yields the following equation (used in the multiple regression analysis):

$$x = k + a_0(D_a - D_f) + a_1(D_aS_a - D_fS_f) + a_2(D_aS_a^2 - D_fS_f^2) + a_3(T_a - T_f)$$  \hspace{1cm} (A-2)

in which

- \(x\) = the number of standard deviations associated with a given probability of using the expressway;
- \(D, S, T\) = distance, speed, and time for the freeway and alternate route as denoted by the subscripts \(a\) and \(f\);
- \(a_i\) = the parameters of the equation to be estimated; and
- \(k\) = the constant term in the regression equation (which, according to our hypothesis, is set equal to zero).

The data used in this analysis were from the Shirley Highway in Virginia, the Gulf Freeway in Houston, the Central Expressway in Dallas, and the Alvarado-Mission Freeway in San Diego. Some zonal interchange volumes were not used because it was believed that the sample size represented by them was too small to be reliable. The remainder provided a total of 197 sample cases.

In testing the hypothesis, both theory and previous experience indicate the range of values the parameter estimates should take. Thus, \(k\), the constant term in the equation, is zero because, if the comparative costs of the two routes are equal, 50 percent of the people should take each route and the standard score associated with 50 percent usage is zero. Also, \(a_0\), \(a_1\), and \(a_2\) are the three parameters
in the parabolic equation relating per mile out-of-pocket costs to speed of travel. Thus, \( a_0 \) should be positive, \( a_1 \) should be negative and smaller in magnitude than \( a_0 \), and \( a_2 \) should be positive and smaller in magnitude than \( a_1 \). Both the sign and relative size of these parameters has been indicated in prior research (8). Finally, the \( a_0 \), which corresponds to an estimate of the value of personal time, should be positive.

In estimating the parameters of the equation, a departure from the usual type of multiple regression analysis was necessitated by the high degree of correlation among the independent variables. Briefly, the procedure followed was to obtain orthogonal estimates of each parameter assuming independence among the independent variables, and then multiply them by the variance-covariance matrix for those parameter estimates obtained through the usual multiple regression analysis. The variance-covariance matrix was scalar-multiplied by an iterative procedure to constrain the constant term in the equation, \( k \), to equal zero.

The final parameter estimates obtained by this procedure are given in Table A-1. The parameter estimates for all the different areas are statistically not significantly different from each other. The correlation coefficients, which all exceed 0.80, indicate that the data fit the theory very well.

The user costs estimated by this equation indicate the relative importance of the different components in the user cost equation. The final measurement is not in dollars and cents, but some arbitrary unit. However, a direct comparison may be made between the user cost equation derived here and those obtained by the more usual methods, such as were employed in Detroit and Chicago (18, 23), by multiplying each of the parameter estimates by a constant to place them in the same range as the Detroit and Chicago equations and then comparing the shapes of the curves. This is shown in Figure A-4, where each of the parameter estimates is multiplied by a constant to place them in the same range as the Detroit and Chicago equations.
TABLE A-1
COMPARISON OF PARAMETER ESTIMATES OF THE COST FUNCTION

<table>
<thead>
<tr>
<th>STUDY DATA USED</th>
<th>VALUE OF PARAMETER</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$a_1$</td>
</tr>
<tr>
<td>California</td>
<td>0.608</td>
</tr>
<tr>
<td>Texas</td>
<td>0.806</td>
</tr>
<tr>
<td>Washington, D.C.</td>
<td>0.656</td>
</tr>
<tr>
<td>All</td>
<td>0.726</td>
</tr>
</tbody>
</table>

estimates was multiplied by 3.2. As can be seen, all three curves have a similar shape. The implications of this study for trip length are that the users of the highway network are responsive to both time and distance as determinants of trip length. The conclusion would be that in the use of trip distribution models attention should be paid to peak-hour conditions when travel-time service is poor and that in areas with extensive freeway coverage consideration should be given to using cost as a parameter in the trip distribution procedure.

APPENDIX B
USE OF THE GAMMA DISTRIBUTION IN ANALYSIS OF TRIP LENGTH

The plotted relative frequencies of occurrence of several variables of interest in the study of trip length (i.e., trip length itself, travel-time factors, and the number of opportunities for trips) are strikingly similar. Each distribution is nonnegative and has a minimum observed value. The distributions are skewed, with a single mode near the origin, a point of inflection on the right shoulder, and a long tail. There are several mathematical functions which exhibit these characteristics. A function which has these characteristics and which is expressible in terms of few parameters is the so-called gamma distribution, a three-parameter distribution ($\alpha, \beta, m$). Figure B-1 shows the shape assumed for various values of the shape parameter $\alpha$ ($\beta = 1, m = 0$). The gamma distribution is closely related to familiar statistical functions, as can be seen from Table B-1, which indicates that the gamma, chi-square, cumulative Poisson, and Pearson type III distributions are all different forms of the same distribution.

The purpose in considering this distribution in detail is fivefold. First, the distribution provides an efficient way of describing the characteristics of behavior of trip length distributions. Second, the parameters of the fitted trip-associated distributions may be directly related to other physical characteristics of urban form and may be predicted in a correlation sense from knowledge of this form. Third, the parameter relationship may provide useful explanations of the importance of the determinants of trip length, location of opportunities, and propensity to travel. Fourth, the use of this distribution follows the classical sharpening of analysis which proceeds from crude qualitative measures to the use of measures of central tendency, thence to consideration of dispersion (standard deviation, variance, skewness), and finally to the detailed characteristics of the probability density function. Fifth, and most important, the gamma distribution may provide a basis for inductive reasoning, leading to a rational, improved, simple, and theory satisfying explanation for the significant characteristics of trip length.
Accordingly, some effort has been devoted to fitting the gamma distribution to a variety of trip-length-related variables recorded in transportation studies throughout the country. The task of fitting the distribution efficiently has been accomplished. Table B-1 gives the interrelationships among the gamma, chi-square, cumulative Poisson, and Pearson type III distributions.

**FITTING THE DISTRIBUTION**

The general gamma density function may be written

\[ f(t) = \frac{\beta^a}{\Gamma(a)} (t-m)^{a-1} e^{-\beta(t-m)} \]  

(B-5)

where \( f(t) \) is the relative density of occurrence of trips of length \( t \),

\[ \int_0^\infty f(t)dt = 1 \]  

(B-6)

\( a = \) the shape parameter;
\( \beta = \) the scale parameter;
\( m = \) the origin parameter;
\( e = \) the base of natural logarithms (2.71828); and

\[ \Gamma(a) = (a-1)! \text{ or } \int_0^\infty x^{a-1}e^{-x}dx \]  

(B-7)

It is often possible to transform the distribution when the origin parameter, \( m \), can be specified. Under these conditions characteristics of the first two moments of an observed distribution, the mean (\( \mu \)) and variance (\( \sigma^2 \)), can be used to determine values of \( a \) and \( \beta \), the shape and scale parameters, respectively. The relationships used in the method of moments are:

![Figure B-1. The gamma distribution.](image-url)
TABLE B-1

COMPARISON OF FAMILIAR STATISTICAL DISTRIBUTIONS

<table>
<thead>
<tr>
<th>DISTRIBUTION</th>
<th>EQUATION</th>
<th>NO.</th>
<th>REMARKS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chi-square</td>
<td>( P(x^2/n) = \frac{2^{n/2}}{\Gamma(n/2)} \int_0^{x^2} x^{n/2-1} e^{-x^2/2} dx )</td>
<td>B-1</td>
<td>If ( n ) is an even integer, gives cumulative Poisson (Eq. B-2).</td>
</tr>
<tr>
<td>Cumulative Poisson</td>
<td>( P(x^2/n) = 1 - \sum_{j=k}^{n-1} \frac{e^{-x^2/2} \left( x^2 / 2 \right)^j}{j!} )</td>
<td>B-2</td>
<td>If ( n = 2k ), gives incomplete gamma function (partial integral of gamma distribution, Eq. B-3).</td>
</tr>
<tr>
<td>Incomplete gamma function</td>
<td>( P(x^2/n) = \frac{2^a}{\Gamma(a)} \int_0^{x^2} x^{a-1} e^{-x^2} dx )</td>
<td>B-3</td>
<td></td>
</tr>
<tr>
<td>Pearson type III</td>
<td>( P(t</td>
<td>\gamma,\alpha) = y_0 \int_{\gamma/a}^{t} (1 + n/a)^{y_0} e^{-y_0} dy )</td>
<td>B-4</td>
</tr>
</tbody>
</table>

\[ a = \frac{\mu^2}{\sigma^2} \quad \text{(B-8)} \]

and

\[ \beta = \frac{\mu}{\sigma^2} \quad \text{(B-9)} \]

As the gamma distribution shape parameter, \( a \), decreases from large values (10 or more) to the values shown in Figure B-1, use of the method of moments to compute the parameters of the distribution becomes less efficient. Kendall (43) states that the maximum likelihood estimator (ML) approach is almost 4.6 times as efficient as the method of moments when \( a = 5 \), and even better for the values of \( a \) encountered in observed trip length distributions. The ML method described by Greenwood and Durand (30) has been used in this study.

The ML estimator of the gamma distribution parameters from a sample can be found by solving the simultaneous equations which result from differentiating the likelihood function, as follows:

\[
\frac{N\alpha}{3} = \sum_{i=1}^{n} (X_i - m) \quad \text{(B-10)}
\]

\[
N \log \frac{1}{\beta} = N \left( \frac{d}{da} \left( \log(\alpha) \right) \right) + \sum_{i=1}^{n} \log(X_i - m) \quad \text{(B-11)}
\]

\[
N = \frac{(a - 1)}{\beta} \sum_{i=1}^{n} \frac{1}{(X_i - m)} \quad \text{(B-12)}
\]

in which \( N \) = the number of observations in the sample; and \( X_i \) = the value of the \( i \)th observation.

TABLE B-2

TABLE FOR ESTIMATING PARAMETERS OF GAMMA DISTRIBUTION

<table>
<thead>
<tr>
<th>VALUE OF ( y )</th>
<th>VALUE OF ( \gamma_a )</th>
<th>VALUE OF ( y )</th>
<th>VALUE OF ( \gamma_a )</th>
<th>VALUE OF ( y )</th>
<th>VALUE OF ( \gamma_a )</th>
<th>VALUE OF ( y )</th>
<th>VALUE OF ( \gamma_a )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.10</td>
<td>0.5161</td>
<td>0.23</td>
<td>0.5352</td>
<td>0.36</td>
<td>0.5523</td>
<td>0.49</td>
<td>0.5677</td>
</tr>
<tr>
<td>0.11</td>
<td>0.5176</td>
<td>0.24</td>
<td>0.5366</td>
<td>0.37</td>
<td>0.5536</td>
<td>0.50</td>
<td>0.5689</td>
</tr>
<tr>
<td>0.12</td>
<td>0.5192</td>
<td>0.25</td>
<td>0.5380</td>
<td>0.38</td>
<td>0.5548</td>
<td>0.51</td>
<td>0.5700</td>
</tr>
<tr>
<td>0.13</td>
<td>0.5207</td>
<td>0.26</td>
<td>0.5393</td>
<td>0.39</td>
<td>0.5560</td>
<td>0.52</td>
<td>0.5711</td>
</tr>
<tr>
<td>0.14</td>
<td>0.5222</td>
<td>0.27</td>
<td>0.5407</td>
<td>0.40</td>
<td>0.5573</td>
<td>0.53</td>
<td>0.5722</td>
</tr>
<tr>
<td>0.15</td>
<td>0.5237</td>
<td>0.28</td>
<td>0.5420</td>
<td>0.41</td>
<td>0.5585</td>
<td>0.54</td>
<td>0.5733</td>
</tr>
<tr>
<td>0.16</td>
<td>0.5252</td>
<td>0.29</td>
<td>0.5433</td>
<td>0.42</td>
<td>0.5597</td>
<td>0.55</td>
<td>0.5743</td>
</tr>
<tr>
<td>0.17</td>
<td>0.5266</td>
<td>0.30</td>
<td>0.5447</td>
<td>0.43</td>
<td>0.5608</td>
<td>0.56</td>
<td>0.5754</td>
</tr>
<tr>
<td>0.18</td>
<td>0.5281</td>
<td>0.31</td>
<td>0.5460</td>
<td>0.44</td>
<td>0.5620</td>
<td>0.57</td>
<td>0.5765</td>
</tr>
<tr>
<td>0.19</td>
<td>0.5295</td>
<td>0.32</td>
<td>0.5473</td>
<td>0.45</td>
<td>0.5632</td>
<td>0.58</td>
<td>0.5775</td>
</tr>
<tr>
<td>0.20</td>
<td>0.5310</td>
<td>0.33</td>
<td>0.5486</td>
<td>0.46</td>
<td>0.5643</td>
<td>0.59</td>
<td>0.5786</td>
</tr>
<tr>
<td>0.21</td>
<td>0.5324</td>
<td>0.34</td>
<td>0.5498</td>
<td>0.47</td>
<td>0.5655</td>
<td>0.60</td>
<td>0.5796</td>
</tr>
<tr>
<td>0.22</td>
<td>0.5338</td>
<td>0.35</td>
<td>0.5511</td>
<td>0.48</td>
<td>0.5666</td>
<td>0.61</td>
<td>0.5806</td>
</tr>
</tbody>
</table>

* From (30).
### TABLE B-3
**SAMPLE FIT OF GAMMA DISTRIBUTION USING TORONTO TRAVEL TIME FACTORS**

<table>
<thead>
<tr>
<th>LENGTH OF TRIP (MI)</th>
<th>OBSERVED VALUE</th>
<th>PREDICTED VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3327</td>
<td>1609</td>
</tr>
<tr>
<td>2</td>
<td>1859</td>
<td>2082</td>
</tr>
<tr>
<td>3</td>
<td>2067</td>
<td>2242</td>
</tr>
<tr>
<td>4</td>
<td>1891</td>
<td>2339</td>
</tr>
<tr>
<td>5</td>
<td>1948</td>
<td>2146</td>
</tr>
<tr>
<td>6</td>
<td>1787</td>
<td>2004</td>
</tr>
<tr>
<td>7</td>
<td>1473</td>
<td>1838</td>
</tr>
<tr>
<td>8</td>
<td>1232</td>
<td>1664</td>
</tr>
<tr>
<td>9</td>
<td>1375</td>
<td>1491</td>
</tr>
<tr>
<td>10</td>
<td>1192</td>
<td>1326</td>
</tr>
<tr>
<td>11</td>
<td>844</td>
<td>1172</td>
</tr>
<tr>
<td>12</td>
<td>1385</td>
<td>1029</td>
</tr>
<tr>
<td>13</td>
<td>778</td>
<td>901</td>
</tr>
<tr>
<td>14</td>
<td>378</td>
<td>785</td>
</tr>
<tr>
<td>15</td>
<td>444</td>
<td>682</td>
</tr>
<tr>
<td>16</td>
<td>693</td>
<td>591</td>
</tr>
<tr>
<td>17</td>
<td>657</td>
<td>511</td>
</tr>
<tr>
<td>18</td>
<td>575</td>
<td>440</td>
</tr>
<tr>
<td>19</td>
<td>687</td>
<td>379</td>
</tr>
<tr>
<td>20</td>
<td>1078</td>
<td>326</td>
</tr>
<tr>
<td>21</td>
<td>1177</td>
<td>279</td>
</tr>
<tr>
<td>22</td>
<td>263</td>
<td>239</td>
</tr>
<tr>
<td>23</td>
<td>173</td>
<td>204</td>
</tr>
<tr>
<td>24</td>
<td>137</td>
<td>175</td>
</tr>
<tr>
<td>25</td>
<td>0</td>
<td>149</td>
</tr>
<tr>
<td>26</td>
<td>0</td>
<td>127</td>
</tr>
<tr>
<td>27</td>
<td>0</td>
<td>108</td>
</tr>
<tr>
<td>28</td>
<td>234</td>
<td>92</td>
</tr>
</tbody>
</table>

Simultaneous solution of these equations is cumbersome, and for \( m \) must be estimated from the smallest observations. If \( m \) is assumed to be known, simultaneous solution of Eqs. B-10 and B-11 is simplified. If the arithmetic and geometric means of the sample (\( \mu \) and \( G \), respectively) are available, substitution of \( \mu = a/\beta \) in Eq. B-11 gives

\[
\log \alpha - \frac{d}{da}(\log \Gamma(a)) = \log \mu - \log G \quad \text{(B-13)}
\]

Greenwood and Durand (30) have tabulated a well-behaved function of this relationship for interpolative purposes. Table B-2 presents part of this table.

The computational procedure is as follows:

1. Select the origin parameter, \( m \), and transform the variate to make \( f(0) = 0 \).
2. Compute the arithmetic, \( \mu \), and geometric, \( G \), means of the sample.
3. Compute \( y = \log \mu - \log G \).
4. Refer to the table (Table B-2) to find \( y_a \) and solve for \( a \) by \( a = y_a/y \).
5. Find \( \beta \) from \( \beta = a/\mu \).

### TABLE B-4
**TYPICAL RESULTS OF DISTRIBUTION FUNCTION COMPUTATIONS**

<table>
<thead>
<tr>
<th>CITY</th>
<th>( m )</th>
<th>( \bar{t} )</th>
<th>Variance</th>
<th>( \alpha )</th>
<th>( \beta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) AUTO DRIVER WORK TRIPS</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Seattle-Tacoma</td>
<td>2</td>
<td>21.15</td>
<td>172.72</td>
<td>2.91</td>
<td>0.14</td>
</tr>
<tr>
<td>Washington, 1955</td>
<td>0</td>
<td>14.33</td>
<td>80.41</td>
<td>2.54</td>
<td>0.18</td>
</tr>
<tr>
<td>Washington, 1948</td>
<td>0</td>
<td>12.60</td>
<td>40.20</td>
<td>3.79</td>
<td>0.30</td>
</tr>
<tr>
<td>Erie, Pa.</td>
<td>0</td>
<td>9.34</td>
<td>25.41</td>
<td>3.37</td>
<td>0.36</td>
</tr>
<tr>
<td>(b) TRANSIT WORK TRIPS</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Baltimore, 1926</td>
<td>12</td>
<td>32.40</td>
<td>284.75</td>
<td>3.94</td>
<td>0.11</td>
</tr>
<tr>
<td>Seattle-Tacoma</td>
<td>2</td>
<td>32.69</td>
<td>273.47</td>
<td>3.97</td>
<td>0.12</td>
</tr>
<tr>
<td>Washington, 1955</td>
<td>3</td>
<td>28.41</td>
<td>221.78</td>
<td>3.24</td>
<td>0.11</td>
</tr>
</tbody>
</table>

*Figure B-2. Comparison of predicted and observed auto driver work trips, Erie, Pa.*
A sample computation is given in Table B-3.

Figure B-2 shows the results of this process for auto driver work trips in Erie, Pa. As shown, the equation

\[ \% \text{ Trips at time } t = \frac{0.368 \times 10^2 t^{2.37} e^{-0.36t}}{(3.37)} \]  

(B-14)

fits the data quite well. Table B-4a gives the results for two other cities. The fits are satisfactory and the values of \( \alpha \) are all near 3.

Figure B-3 shows the fit obtained for transit work trips recorded in Washington, D.C., in 1955. Table B-4b gives the results obtained in three cities, with the values of the parameters of the distribution clustered above 3 for \( \alpha \) and near 0.10 for \( \beta \).

\[ \text{Percent of Total Trips} \]

\[ \text{Trip Time (Min)} \]

Figure B-3. Comparison of predicted and observed transit work trip durations, Washington, D.C., 1955.

APPENDIX C

SIMULATION STUDIES

VARIANCE OF HYPOTHETICAL CITIES

In addition to a study of the mean, a relationship was developed between the variance and population (measured in work trips), and network speed. It was observed that the variance of the opportunity trip distribution was related to the “actual” trip time variance obtained with a travel time exponent of one. Figure C 1 shows a power function which appears to represent quite well the relationships between travel time variance and city size and network speed, respectively.

THE EXPONENTIAL CITY

Students of urban activity have noted the relationship between density of activity and distance from the central
business district (19, 95). Clark (19) has studied this characteristic for several cities over time and has concluded that an extremely satisfactory mathematical approximation of the intensity of workplaces and population is as follows:

\[
f(r, \theta) = a e^{-b r} \quad 0 \leq r \leq 2\pi \quad (C-1)
\]

in which

- \( f(r, \theta) \) = the density of development at a distance, \( r \), usually in miles, from the central business district and at an angle \( \theta \), in radians, from an arbitrary base;
- \( e \) = the base of natural logarithms;
- \( a \) = a constant giving the central business district density; and
- \( b \) = a constant expressing the decline in density with distance from the central business district.

This form is called the "exponential city" in the remainder of this report. Certain mathematical characteristics of a city with an infinite radius are presented in the next section.

Figure C-2 shows the mean and standard deviation of the opportunity trip length distribution for an average distance of origins and destinations from the center of one distance unit. Figure C-3 shows the actual opportunity trip length distribution for the center city developed by the technique described in the next section. The gamma distribution provides an excellent fit to the data, both citywide and for individual locations within the city. Figure C-3 can be used to estimate the opportunity trip length distribution for an exponential city. The values of the mean and standard

---

Figure C-1. Trip time variance (theoretical city) vs work trip population and average network speed.
Figure C-3. Opportunity trip length distribution (theoretical city).

Gamma Distribution for Entire City

\[ f(t) = t^2 e^{-2t} \]

\[ a = 3 \]
\[ B = 2 \]
deviations presented in Figure C-2 can be related to the parameters of the gamma distribution, $\alpha$ and $\beta$, for any point within the city, as shown in Figure C-4. These results are consistent with Tomazinis' (79) findings on auto driver home-to-work trips.

Although the parameters $\alpha$ and $\beta$ do not necessarily assume a given set of values, and a set of workplaces and residences can be arranged in an urban area in many ways, there is substantial empirical evidence (although no generally accepted theoretical basis) that the exponential form gives a remarkably good representation of what takes place in urban areas throughout the world. In addition, Weiss (89) has shown that for more than 30 American cities in 1950 the relationship between the constant, $b$, and the total population, $P$, could be expressed by

$$b = 46.4 P^{-0.53} \quad (C-2)$$

As is shown in the next section, the average distance of population from the center of the city, $\mu$, is

$$\mu = 2/b \quad (C-3)$$

Clark (20) has recently reported that the $\alpha$ and $\beta$ parameters of population and jobs are the same. By the simulation described in the next section, the average opportunity trip distance for an exponential city with equal home and work parameters was empirically found to be

\[ f(d) = \frac{\beta \lambda^{\alpha-1} e^{-\beta d}}{\Gamma(\alpha)} \]

\[ f(d) = \frac{\beta \lambda^{\alpha-1} e^{-\beta d}}{\Gamma(\alpha)} \]

Figure C-4. Gamma distribution parameters for locations within theoretical city.
Substituting values for \( \mu \) and \( b \) from Eqs. C-2 and C-3 in Eq. C-4, the opportunity trip length is

\[
\bar{O} = 0.065P^{1/3}
\]  (C-5)

This relationship is plotted in Figure C-5, which shows that a doubling of population results in a 26 percent increase in average opportunity trip length.

It is the empirical form of American and Canadian cities that establishes the cube root of the population as an upper bound on the average opportunity trip length.

Mathematical Addendum for Probability
Trip Length Development

Given the exponential infinite city of Eq. C-1, in which parameter \( a \) represents the density at the city center and parameter \( b \) determines the degree of compactness of the city (large values of \( b \) are associated with compact cities), the density at a point \( r \) from the city center and at angle \( \theta \) can be expressed as

\[
f(r) = \frac{b^2}{2\pi} e^{-br}
\]  (C-6)

where \( a = \frac{b^2}{2\pi} \). The population of the city is 1.0. The density is assumed to be independent of \( \theta \). The cumulative fraction of population reached at a distance, \( s \), from the city center is

\[
F(s) = \int_0^s 2\pi f(r) dr = 1 - (1 + sb)e^{-sb}
\]  (C-7)

The average distance of opportunities from the city center is

\[
\mu = \int_0^\infty r 2\pi f(r) dr = \frac{2}{b}
\]  (C-8)

The density of total population in a ring at a distance, \( r \), from the center of the city is

\[
g(r) = 2\pi f(r) = b^2re^{-br}
\]  (C-9)

The maximum ring population density is located at a distance \( 1/b \) from the center of the city. The second moment

\[
\bar{O} = 0.065P^{1/3}
\]  (C-5)
of the population from the center is 6/b^2, and the variance of the distribution is 1/b^2.

**Mean Probability Trip Length**

Given a city with an exponential density of opportunities

\[
f(r, \theta) = \frac{\beta e^{-\beta r}}{2\pi} \quad 0 \leq r \leq \infty, 0 \leq \theta \leq 2\pi \quad (C-10)
\]

it is desired to obtain the average probability trip length, \(d(a)\), from a point a distance a from the origin to all other places in the city.

Figure C-6 shows the important relationships associated with the distance from a point (a) to a differential area (da). The average distance from a is obtained by integrating over the plane

\[
\bar{d}(a) = \int_0^\infty \int_0^{2\pi} d(a, r, \theta) f(r, \theta) r d\theta dr \quad (C-11)
\]

From the law of cosines,

\[
d(a, r, \theta) = a^2 + r^2 - 2ar \cos \theta \quad (C-12)
\]

Introducing the variable

\[
x = r/a \quad \text{and} \quad f(x, \theta) = \frac{\beta e^{-\beta x}}{2\pi} \quad (C-13)
\]

gives

\[
\bar{d}(a) = a^2 \int_0^\infty \int_0^{\pi/2} (x^2 + 1 - 2x \cos \theta)^{0.5} f(x, \theta) x d\theta dx \quad (C-14)
\]

But

\[
2 \cos^2 \gamma - 1 = \cos 2\gamma \quad (C-15)
\]

therefore,

\[
d(x, \theta/2) = [(x + 1)^2 - 4x \cos^2(\theta/2)]^{0.5} \quad (C-16)
\]

Introducing

\[
\phi = (\pi - \theta)/2 \quad (C-17)
\]

gives

\[
\bar{d}(a) = \frac{a^2 \beta^2}{\pi} \int_0^\infty \int_0^{\pi/2} [(x + 1)^2 - 4x \sin^2 \phi]^{0.5} e^{-\beta x} x d\phi dx \quad (C-18)
\]

Then, if

\[
K^2 = 4x/(x + 1)^2 \quad (C-19)
\]

\[
\bar{d}(a) = \frac{a^2 \beta^2}{\pi} \int_0^\infty \int_0^{\pi/2} [1 - K^2 \sin^2 \phi]^{0.5} (x^2 + x) e^{-\beta \phi} d\phi dx \quad (C-20)
\]

The integral in \(\phi\) is Legendre's complete elliptic integral of the second kind \(E(K, \pi/2)\), a function tabulated in common mathematical reference publications, and for which Hastings has developed a suitably accurate approximating polynomial in \(k\) and its natural logarithm.

Figure C-6 can be used with any distribution of productions to obtain the mean probability distribution for attractions at a given location. In the case of an exponential distribution of productions with the same average dispersion, the mean probability trip length is 1.47 times the average dispersion from the city center.

**NONEXPONENTIAL CITY FORMS**

Inasmuch as some urban plans (e.g., Baltimore's "Metro-towns") envision urban developments that are not of simple exponential form, the opportunity trip length for several alternative forms was investigated. Because developing the distribution was mathematically difficult, recourse was made to a simulation experiment in which 10,000 destinations were located at random in the desired density pattern and the distance characteristics for a number of points were obtained by using trigonometric relationships and a general purpose digital computer. The problem is formulated and further described in the preceding sections.

Four city forms were tested. Each was an exponential city wherein the parameter describing the slope was defined in terms of multiples of the average distance of the city population from the city center.

The first form was the exponential city in which the average radial population distance from the city center was 1.0 unit, and the average opportunity distance was 1.5 units. In the second case, the same city population was assumed to have a doubled population density at the city center, which resulted in the reduction of the average distance of the population from the city center to 0.71 units. For this city the average opportunity distance was 1.1 units, or 27 percent less than that of the simple exponential city. In the third case, the central city was assumed to have 80 percent of the population and workers. Each of four satellites was assumed to contain 5 percent of the population and be centered one distance unit away from the central city center. The peak density of the satellites was assumed to be 50 percent of that of the center city, and the mean satellite population distance from the satellite center was 3/2 unit. After correcting for distance so that the average distance from the center is 1 unit, the mean opportunity distance for the configuration would be 1.52 units, or only a little more than that (1.50) for a central city with the same average population radius. The fourth case consisted of five clustered exponential developments of equal size which when corrected for average distance from the center
resulted in an average opportunity distance of 1.39 units, or approximately 10 percent less than the basic exponential city.

As a limiting case, information was developed for a constant-density, circular city with an average distance from the center of one unit. The radius of such a city would be 1.5 units. Tanner (74) has shown that the mean opportunity in such a city would be 1.36 units, or nearly 10 percent less than that found in an exponential city. The practical range of average opportunity trip lengths in terms of the mean distance of the regional population from its center for more dispersed developments is from 1.36 to 1.52, a relatively small difference (approximately 10 percent).

---

APPENDIX D

PREDICTING THE GEOGRAPHIC DISTRIBUTION OF FAMILY INCOME IN AN URBAN AREA

Variations in family income were found to have no noticeable effect on travel behavior as such. The correlation between urban work trip length and family income is attributable to the spatial arrangement of residential areas with respect to employment areas. Persons with higher incomes in general find homes farther from the central city, with its high concentration of jobs, than do families with lower incomes. In addition, it was shown that the stratification of the work trip matrix by income groups results in a definite improvement of the trip distribution model.

Therefore, it was of interest to examine the distribution of family incomes within the urban area in order to determine whether or not these patterns of economic status are predictable. If so, these suggestions would be useful in the development of better trip distribution models. The goal of this investigation, then, was to predict the changes in family income for small areas, from time period $t$ to time period $t + 1$.

PHILOSOPHY OF THE RESEARCH

Many of the models developed to predict social and economic phenomena are the results of correlation or regression analyses performed on a large number of variables. The mathematical form of the final models, and the variables included in them, result from a search over many alternatives for the formulation having the greatest predictive ability.

This trend in model development has resulted from the imperfect knowledge which the researchers possess concerning the phenomena under study. Although such a “fishing” process is acceptable for developing some feel for the interrelationships involved in the real world, it is not a satisfying method for constructing an operational model. This research does attempt to formulate reasonable hypotheses concerning the spatial distribution of median family incomes. The hypotheses were tested with field data and the final model was selected on the basis of aptness of the formulation and significance of the variables, as well as predictive ability. In this way the explanation of the phenomena was developed first and then tested with regression analysis. A higher coefficient of determination could probably be achieved with more variables or more complex forms. Without a strong hypothesis about the causal relations involved, however, the ability of such a model to predict into the future can be doubted. A less accurate model whose relations hold over time is more satisfying than a model with no theoretical explanation which predicts 100 percent of the presently observed variation.

THE BASIC HYPOTHESIS

To formulate the model, considerable thought was given to establishing those factors which might cause a change of income in a small part of a metropolitan area. There have been clear regional and national trends in family incomes over the past years. Although the figures have always been increasing, the magnitude of the change is tied in with the structure of the local and national economies. Productivity, automation, and birth rates, as well as social and racial employment barriers, have affected these changes. A model which would predict such patterns would necessarily be large and complex. It would have to relate local and regional phenomena to activities on a wide scale.

Although such models are becoming more feasible, it would be inefficient to construct and use one for each metropolitan transportation study. Therefore, the responsibility for establishment of absolute income values will be left to the econometricians. This model will be based on relative median family incomes (the median family income for a traffic district divided by the average of the median incomes for the whole area). By performing this normalization operation, the model can be constructed to focus more closely on individual variations within the districts, because long-range economic variations will be eliminated.

The question to be answered during the formulation
process is: What factors affect the relative income within a given traffic district? The present relative income is a reasonable "best estimator" of future income, as long as no changes occur in the area. If no basic changes are known to have occurred, the best guess which can be made about relative income in the future is that it will remain unchanged. Therefore, all of the equations tested were in the form

\[ R_2^i = f(R^1_i, x_1^i, x_2^i) \quad (D-1) \]

in which

- \( R_2^i \) = relative income in district \( i \), time 2 (future);
- \( R_1^i \) = relative income in district \( i \), time 1 (present); and
- \( x_1^i, x_2^i \) = other variables.

Although relative income at the beginning of the projection period establishes a base value for future income, some indicator of change must be present in the model. Change could occur if many of the families residing within a district were able to improve their economic positions while the rest of the region was held at the present level. This seems unlikely to occur in one district and no others. In addition, it is unreasonable to suspect that people with changing (especially improving) economic positions will remain in the same location. A more reasonable cause for change might be a change in the people living in the district. Although it would be most accurate to predict the numbers and types of families who will depart from and arrive at the district, data for the development of such a model were not available. The next best measure was thought to be population change, in percentage points. If the population falls in a district, it is likely that conditions are deteriorating and that relative incomes are falling. If population increases, high income persons may be seeking better quality housing in an outlying district, or a low income district could be declining in relative income due to overcrowding and poor living conditions. In any case, population change indicates a departure from the conditions of stability; the data should indicate the effect on income of a given population change.

As described in the foregoing, a given population increase could result in different income changes in different types of residential areas. Some indicator of area type is necessary to separate districts with different living conditions. Although a number of social or physical indices might have been selected, net residential density was chosen because it was readily available. This variable was introduced at its anticipated level in the second, or design period. This was expected to reflect the conditions of housing development at the time income was being predicted.

There are, then, three independent variables in the model—one to indicate the base level of relative income, a second to represent instability or change, and the third to separate different types of residential areas. These variables are, respectively, relative income in the first time period, percentage change in district population, and net residential density in the design year. The model can be represented in the following form:

\[ R_2^i = f(R^1_i, \text{POP}_i, \text{DEN}_i) \quad (D-2) \]

in which

- \( \text{POP}_i \) = percentage change in population of district \( i \) from period one to period two;
- \( \text{DEN}_i \) = net residential density in district \( i \) at time period two.

THE DATA

This basic hypothesis was evolved with U.S. Bureau of the Census data collected in Washington, D.C., for the years 1950 and 1960. These were converted from a census tract basis to traffic districts of the 1955 Mass Transportation Study. The regression analyses were also performed under this research project. The model was intended to demonstrate that income stratification of work trip matrices was technically feasible for traffic projection.

Because this investigation was only a minor phase of the trip length project, only limited funds were available for its completion. Therefore, refinement of the data, investigation of more complicated variables, and further tests in other cities were not possible.

Traffic districts were used in the analysis because it was necessary to have the same district and study area boundaries in both time periods; relative incomes had to be relative to a common base. In addition, further analysis in the project required similar data for correlation with the travel characteristics collected in the 1955 transportation study.

FORMULATION OF THE HYPOTHESES

Past experience has shown that absolute incomes within a region tend to be distributed log-normally. An attempt was made to determine the distribution of relative incomes in Washington in 1950 by plotting the cumulative frequency distribution on probability paper. The line appeared to be quite straight on log-normal paper, although there was some departure of points from linearity in the high income region. Because values were available only in the form of medians for entire traffic districts, very high income families scattered throughout the metropolitan area were not adequately represented in the data; yet they are the cause of the log-normal distribution of incomes, with its long tail of high incomes. When estimates of the number of persons in the higher income groups were made and plotted, the line appeared to be very nearly straight, indicating the condition of log-normality (see Fig. D-1).

Assuming this to be the case, it becomes necessary to include the logarithm of relative income in the regression model, due to the requirement that variables possess a normal distribution. Because a preliminary scatter diagram indicated that \( R_2 \) was closely associated with the logarithm of \( R_1 \), the other two had the logarithms of \( R_2 \) and \( R_1 \) in the equations.

Two of the formulations used population changes as an additive term, because it was to reflect the deviation from \( R_1 \) which occurred due to a change in population. These two also included net residential density as an additive term. This, however, was felt to be unsatisfactory in an intuitive sense, inasmuch as density should influence the kind of
Effect population changes will have on an otherwise stable relative income. Therefore, two of the equations tested combined the last two independent variables by dividing population change by density. Thus, a high percentage population increase will be less effective in changing income if the families move into a densely settled district. Conversely, if the percentage population change is high (+), the new families will have a strong influence on relative income. The four equations tested were:

\[ R_s^2 = A + B \log (R_s^t) + C \text{POP}^t + D \text{DEN}_s^t \]  
\[ (D-3) \]

\[ \log (R_s^t) = A + B \log (R_s^t) + C \text{POP}^t + D \text{DEN}_s^t \]  
\[ (D-4) \]

\[ R_s^t = A + B \log (R_s^t) + C \text{POP}^t/\text{DEN}_s^t \]  
\[ (D-5) \]

\[ \log (R_s^t)q = A + B \log (R_s^t) + C \text{POP}^t/\text{DEN}_s^t \]  
\[ (D-6) \]

Data from each of the 67 traffic districts were regressed in these forms using a high-speed digital computer.

RESULTS OF REGRESSION ANALYSIS

Table D-1 shows the results of the regression. Eq. D-3 had the lowest coefficient of determination, \( r^2 \), as well as the highest standard error of estimate. In addition, density, which was to indicate the type of residential area, is significant with a probability of only 0.661. Therefore, this formulation was rejected.

Eq. D-4 shows density with a negative coefficient, as expected from the hypothesis. However, this variable is significant at only the 0.653 level, and population change is not significant at a desirable probability. This formulation was also rejected.

The remaining equations used the combined variable, population change divided by density. This proved satisfactory, inasmuch as this form is more logical according to the hypothesis for the use of density (that its impact is on the amount of change in income brought about by a change in population). All variables in these equations are highly significant according to the \( t \)-test and the standard...
TABLE D-1
RESULTS OF ANALYSIS OF GEOGRAPHIC DISTRIBUTION BY INCOME

<table>
<thead>
<tr>
<th>NO.</th>
<th>ITEM</th>
<th>EQUATION</th>
<th>STD. ERROR</th>
<th>LEVEL OF SIGNIF.</th>
</tr>
</thead>
<tbody>
<tr>
<td>D-3</td>
<td>Equation</td>
<td>( R_s^t = 1.018 + 0.6309 \log (R_i^t) + 0.0018 \text{POP}^t + 0.0004 \text{DEN}^t )</td>
<td>0.0699</td>
<td>1.000</td>
</tr>
<tr>
<td>D-4</td>
<td>Equation</td>
<td>( \log (R_s^t) = -0.0128 + 0.7242 \log (R_i^t) + 0.0010 \text{POP}^t - 0.0044 \text{DEN}^t )</td>
<td>0.0636</td>
<td>1.000</td>
</tr>
<tr>
<td>D-5</td>
<td>Equation</td>
<td>( R_s^t = 1.015 + 0.6132 \log (R_i^t) + 0.0087 \text{POP}^t / \text{DEN}^t )</td>
<td>0.0468</td>
<td>1.000</td>
</tr>
<tr>
<td>D-6</td>
<td>Equation</td>
<td>( \log (R_s^t) = -0.0257 + 0.7642 \log (R_i^t) + 0.0045 \text{POP}^t / \text{DEN}^t )</td>
<td>0.0464</td>
<td>1.000</td>
</tr>
</tbody>
</table>

* Level of significance of coefficient, by \( t \)-test.

errors of estimate are the lowest of the four. Eq. D-6 was selected because it predicted a slightly larger percent of the variation in \( R_s \) than did Eq. D-5.

To further check this relationship, a plot of predicted versus observed relative income in period two was made. Figure D-2 indicates that there are no obvious tendencies toward bias. This implies that the representation of the variables and the general formulation is acceptable. Because previous attempts showed that a spatial bias might result from concentrations of certain types of families, Figure D-3 was developed. It presents the residuals in each traffic district; again, no systematic bias was observed.
sions with regard to the stability of travel time factors in space and time. Whitmore (91) recently concluded that travel time factors are nearly constant for regions but vary from city to city. Lowry (30) reported substantial differences in travel time factors and that higher income Pittsburgh groups were less susceptible to distance separation than lower income groups. Braswell (10) reported a substantial variation in travel time factors among several Pittsburgh neighborhoods. These conclusions seem to indicate that travel time factors for an urban area are constant over time but vary by location within the urban area. The magnitude of the differences in the extremes of the travel time factors in the first 10 to 15 min should be analyzed with respect to the opportunity distribution, especially where the affected zones constitute a significant proportion of total trip generation.

Figure F-1. Comparison of observed and predicted distribution of auto driver work trip durations, all income groups, Baltimore, Md., 1962.

Figure F-2. Comparison of observed and predicted distribution of work trip durations, all modes, Baltimore, Md., 1945.
Figure D-3. Predicted minus observed residuals from relative median family income model, Washington, D.C.
CONCLUSIONS AND IMPLICATIONS

The analyses indicated that Eq. D-6 represents a viable model for the prediction of the spatial arrangement of relative family incomes. Its predictive ability is on a level with those of the other models used in the typical urban transportation planning process.

It is necessary to point out that the need for predicting district population changes exogenously is not limiting within the current planning process. All trip generation models require similar information, and reasonable models have been developed to provide it. This model, then, is no more demanding on the predictive ability of the planner than any other.

The fact that separate models are used to predict future population (i.e., residential location) and income does raise some serious questions. Why doesn't the location model tell about the income of the families? Certainly these two considerations are intimately related. Why isn't a complete econometric model available which will describe residential location and the spatial distribution of absolute incomes?

The answer to these questions is clear: the state of knowledge of these phenomena has not yet progressed to a sufficient degree, nor are the available data sufficient, to develop such a model.

The prediction of future income for traffic districts is basically a migration problem. It requires an understanding of the patterns of migration of various types of families and, more basically, the causes of these patterns. Knowledge of this nature cannot be based on simple description. It is not enough to know that high-income families tend to move to the suburbs. Specific family characteristics, such as income, job type, and number and age of children, must be related to site and location characteristics of the home. The analysis must be performed on traffic district distributions rather than on mean values. Such techniques as the cohort survival approach to population projection must be applied to families within an urban area. Modern computer technology has advanced enough to handle the cumbersome accounting problems that may evolve.

Models like the one presented here are of a stop-gap nature. Future research must be in the direction of understanding the behavior of the family unit and the stochastic nature of its response to environment stimuli. The end result must be an integrated theory of the family and its relation to the metropolis, rather than a fragmented set of subtheories. Current planning models are trying to simulate, in many discrete steps, decisions which families make as a part of their continuous behavioral processes.

APPENDIX E

PROCEDURE FOR INCOME CLASS EXPERIMENT IN WASHINGTON, D.C., FOR SOCIAL-RECREATION TRIPS

The procedure used for the income class experiment in Washington, D.C., for social-recreation trips can be briefly outlined as follows:

1. Washington was divided into 400 internal zones (from the 1955 Washington study), each of which was assigned a "median income," taken from 1950 census data. Four income classes were used, and some adjustment was made to account for the difference in zonal division between the 1955 Washington study and the 1950 census data. An approximate stratification of income is:
   1. 0–$3200
   2. $3200–$4400
   3. $4400–$5100
   4. $5100 and above

2. O-D trip tables were labeled "actual" and gamma model (GM) trip tables were labeled "expected." A sample was taken from each of these two sets of trip tables, considering the tables themselves to be populations.

3. Income class difference was found by subtracting the income class of the origin zone. This yielded class differences of 3, 2, 1, 0, –1, –2, and –3. The number of trips in each of these categories was tabulated for both the O-D and the GM (Tables E-1 and E-2).

4. A chi-square test was used to test the difference between these two discrete distributions.

Sampling methods accounted for the main difficulties encountered in the study. In the first place, the O-D survey, on which the GM is based, is an expanded sample, thus making the study sample a "sample of a sample." The representativeness of the latter is thereby subject to two sources of sampling error. The second sampling problem was due to the presence of a large number of zero trips between zones; i.e., the trip tables had a low density. This required that a tediously large sample be taken. It was decided that a random sample of three zones from each of the four income classes would be chosen as origin zones (twelve origin zones in all) with destinations at all 400 zones. Of the 160,000 (400 × 400) entries in the trip table...
matrix, a stratified sample of 4,800 (12 \times 400) was chosen. Of course, the same twelve origin zones were used for both the O-D and GM projections.

The number of trips was counted for each income class difference, as given in Tables E-1 and E-2. The distributions are shown graphically in Figure E-1.

The chi-square test was performed at the 5 percent level. This test is applicable if it is assumed that the two distributions are normal. The value of \( x^2 \) for this test is computed by

\[
x^2 = \sum_t \frac{(O_t - E_t)^2}{E_t}
\]  

(E-1)

in which \( O_t \) is the observed value (O-D); \( E_t \) is the expected value (GM); and the degrees of freedom = \((7 - 1) = 6\). From Tables E-1 and E-2, \( \chi^2 = (157 - 132)^2/132 + (419 - 864)^2/864 + (1226 - 2299)^2/2299 + (3632 - 3638)^2/3638 + (966 - 1484)^2/1484 + (867 - 1088)^2/1088 + (7377 - 5286)^2/5286 = 1,787 \). The value of 1,787 can be interpreted as follows:

The probability that a value of 12.86 be calculated for \( \chi^2 \) is 0.05 (hence, the test at the 5 percent level). Inasmuch as 1,787 is much greater than 12.86, it can be concluded that there is a very small probability (less than 0.05) that the difference between the observed and the expected values is due to random chance in the sampling.

\( (Washington, D.C., 1955) \)

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Figure E-1. Comparison of observed and predicted distribution of social-recreation trips by income-difference group, Washington, D.C., 1955.
APPENDIX F

STABILITY OF FRICTION FACTORS OVER TIME FOR AN URBAN AREA

Calibration is only the first step in the use of the gravity model, the ultimate goal being the projection of travel patterns for the purpose of designing transportation systems. In the past, most operational studies have assumed that the calibrated travel time factors by trip purpose remain constant over time for an urban area. An experiment was performed to investigate this assumption.

Travel data for the City of Baltimore, Md., for 1926, 1945 and 1962 were compared to the results of gravity models based on the network and trip generation characteristics of each time period. Throughout the analysis, the travel time factors developed in the 1962 study were applied (67).

The trip time distributions generated by the gravity models were compared to the corresponding actual trip time distributions. When allowances were made for major discrepancies in the data sets, it was apparent that the same propensity function could be applied to each. The criteria for evaluation were the same as those used for calibrating travel time factors, a comparison of the mean values of the gravity model with the actual distributions and a visual check for the duplication of the shape of the distribution (see Figs. F-1, F-2, and F-3).

The presence of transit trips in the 1945 data gave rise to fewer short trips in the actual distribution. (The gravity model overpredicts these short trips and underpredicts long trips.) Even so, the difference between the means of the two distributions was only 1.9 min, or 12.7 percent of the actual mean trip length. If the transit trips could have been removed from the data, a better fit would have been expected.

The gravity model only distributed trips up to 50 min long, because this was the upper value of the factors used in 1962. In 1926, trips of up to 120 min were recorded. The resulting gravity model distribution reproduced the shape of the actual distribution, but the means did not agree. The actual distribution, if factored to represent the percentage of all trips up to 50 min, produces a good fit in terms of mean values and overall shape. The difference in the means is 1.7 percent of the actual mean trip length.

On the basis of this work, it appears that gravity model travel time factors tend to remain relatively constant over time for a given urban area. The analysis of 1948 data from Washington, D.C., referred to in Chapter Three, indicated variations in travel time factors within the region that were probably caused by changes in mean opportunity lengths. Other investigators have reported similar conclu-
sions with regard to the stability of travel time factors in space and time. Whitmore (97) recently concluded that travel time factors are nearly constant for regions but vary from city to city. Lowry (50) reported substantial differences in travel time factors and that higher income Pittsburgh groups were less susceptible to distance separation than lower income groups. Braswell (10) reported a substantial variation in travel time factors among several Pittsburgh neighborhoods. These conclusions seem to indicate that travel time factors for an urban area are constant over time but vary by location within the urban area. The magnitude of the differences in the extremes of the travel time factors in the first 10 to 15 min should be analyzed with respect to the opportunity distribution, especially where the affected zones constitute a significant proportion of total trip generation.

Figure F-1. Comparison of observed and predicted distribution of auto driver work trip durations, all income groups, Baltimore, Md., 1962.

Figure F-2. Comparison of observed and predicted distribution of work trip durations, all modes, Baltimore, Md., 1945.
Data from earlier origin-destination studies were used to evaluate changes in trip length over time. Such data were available for Washington, D.C., for 1948 and 1955 and for Baltimore, Md., for 1926, 1945, and 1962. To utilize the available information to its best advantage, it was necessary to convert it for use with standard computer programs. In particular, a listing of appropriate travel times was needed. Acceptable times existed for Washington, 1955, and for Baltimore, 1962. Prior work by the Bureau of Public Roads indicated that the 1955 Washington network gave acceptable values for use with the 1948 data.

In the case of the earlier Baltimore studies, it was necessary to develop networks of the existing systems from which the travel time data could be obtained. Using maps from the respective studies, the corresponding networks were constructed and coded. An attempt was made to keep as close to reality as imperfect knowledge would permit.

**BALTIMORE 1926 TRANSIT SYSTEM**

The year 1926 was chosen for transit system analysis because streetcars represented the predominant mode of travel at that time in Baltimore. More than 70 percent of those employed in the CBD and more than 50 percent of those working in seven industrial districts used streetcars to get to work. Walk trips represented the second most prevalent method of getting to work. Under these conditions, it was felt that all trips could be treated as transit trips and that
the travel propensity function could be treated as similar to that for auto trips in 1945 and 1962. This method of analysis implies that the propensity function would remain constant over time, regardless of the primary mode of travel.

A basic transit network was constructed from a 1925 map of streetcar and bus lines in Baltimore. Trip volumes were given by a tabulation of the residential distribution of persons employed in the CBD (16 districts) and seven industrial districts. The home zones were developed from a 1-mile square grid pattern and tended to overlap the employment zones. Therefore, a zone was coded as either a generator or a receiver of trips, but not both. To simplify the task of coding a network from a crude map with limited information, a minimum number of transit lines was utilized. Every origin zone had access to at least one line of the network, and all destination zones were similarly connected to the system. Zone centroids were estimated from dot maps representing the distribution of residential population. In some cases, due to the coarse nature of the zone system, this procedure resulted in fairly long walks to the transit system. However, a check of zone-to-zone travel times gave a reasonable comparison with an isochronal map showing travel times from the CBD via the shortest route.

**Baltimore 1945 Highway System**

Inasmuch as auto trips represented the major mode of travel in 1945, a highway network was coded to the 1945 zones. Trip volumes were available from an O-D survey, but there was no way to segregate the trips by mode of travel. Therefore, all modes were assigned to the highway network. As with the 1926 network, limited information dictated that a minimum network be coded. A 1945 Baltimore street map, with the study cordon and traffic zones drawn in, was used. Because only the major streets in each zone were coded, the network is made up primarily of arterial streets. Speeds were estimated according to a systematic schedule which appeared reasonable.

**Washington 1948 Trip Data**

A 1948 O-D study supplied the trip volumes used to evaluate the trip length highway distribution in Washington. Because the basic system was not greatly changed, the 1955 highway network was used.

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**APPENDIX H**

**Performance of a Regional Gravity Model on a Subarea Basis**

The purpose of this analysis was to examine the performance of a statewide gravity model on a subarea basis. Figure H-1 is a map of Connecticut showing the four towns (subareas) examined. These towns were generally selected because of their differences in income and their surrounding trip opportunities. The ranking of these towns with respect to income and opportunity arrangement is given in Table H-1.

Available actual and theoretical trip length distributions synthesized with the gravity model were examined for bias. The trip length distributions analyzed were for work, long, short, nonhome-based, and truck purposes. The definitions for the auto purpose groupings, as well as their average trip lengths, are given in Table H-2. As can be seen from the table, the purpose groupings used in the statewide model were based on trip purpose and trip length.

Table H-3 summarizes the trip length characteristics for the towns analyzed. Representative trip length distributions for these grouped purposes are shown in Figures H-2 through H-6. This summary table and the representative trip length distributions show that the greatest biases occurred in the work and long trip length categories. Short trips, nonhome-based, and truck trips, on the other hand, had no discernible bias. The bias in work trip lengths is consistent with biases noted of work trip lengths by income and the spatial arrangement for trip opportunities found for Washington, D.C. The bias in long trips, which have social-recreation trips, indicates the income biases noted in Washington, D.C., and Springfield, Ill. Short trips, which contain shop trips, appear to have no consistent biases.

**Table H-1**

<table>
<thead>
<tr>
<th>Town</th>
<th>Income Class</th>
<th>Relation to Other State Trip Opportunities</th>
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</thead>
<tbody>
<tr>
<td>West Haven, Orange</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>Bridgeport</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>West Hartford</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>Groton, Waterford, New London</td>
<td>2</td>
<td>4</td>
</tr>
</tbody>
</table>
TABLE H-3  SUMMARY OF TRIP LENGTH CHARACTERISTICS

<table>
<thead>
<tr>
<th>TOWN</th>
<th>TOTAL NUMBER OF TRIPS</th>
<th>AVERAGE TRIP LENGTH (MIN)</th>
<th>AUTO WORK</th>
<th>AUTO LONG</th>
<th>AUTO SHORT</th>
<th>AUTO NONHOME BASED</th>
</tr>
</thead>
<tbody>
<tr>
<td>West Haven, Orange</td>
<td>62,840</td>
<td>13.8</td>
<td>12.3</td>
<td>13.5</td>
<td>13.7</td>
<td>9.9</td>
</tr>
<tr>
<td>West Hartford</td>
<td>132,920</td>
<td>10.1</td>
<td>10.1</td>
<td>11.6</td>
<td>10.6</td>
<td>9.9</td>
</tr>
<tr>
<td>Groton, Waterford, New London</td>
<td>12,920</td>
<td>7.6</td>
<td>7.4</td>
<td>6.3</td>
<td>5.5</td>
<td>6.3</td>
</tr>
<tr>
<td>TOWN</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AVERAGE TRIP LENGTH (MIN)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AUTO WORK RATIO, A/T</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AUTO LONG RATIO, A/T</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AUTO SHORT RATIO, A/T</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AUTO NONHOME BASED RATIO, A/T</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TRUCK</td>
<td></td>
<td></td>
<td></td>
<td></td>
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</table>

TABLE H-2  AVERAGE AUTO DRIVER TRIP LENGTHS

CONCEPTION (CPU) PURPOSE, DEFINITIONS AND GROUP

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<tr>
<th>PURPOSE</th>
<th>GROUP</th>
<th>AVG. ALTRO</th>
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APPARENT BIAS IN THEIR TRIP LENGTHS

It appears that trip lengths are not as strongly related to trip purposes as was found in other studies. This may be due to the inclusion of trips involving transit and non-home-based trips. The results suggest that the income and employment characteristics of the population in each town are similar. For non-home-based trips, however, the differences in trip lengths are significant. For non-home-based trips, the results are similar to those found in other studies. For home-based trips, the differences in trip lengths are not as significant. For home-based trips, the results are similar to those found in other studies.
Areas selected for work and non-work trip length distribution analysis (Actual vs. GM)

Figure H-1. Locations of Connecticut areas selected for trip length distribution analyses.
Figure H-2. Comparison of actual and theoretical short-trip duration distributions, West Hartford, Conn.

Figure H-3. Comparison of actual and theoretical long-trip duration distributions, Groton, Waterford, and New London, Conn.
Figure H-4. Comparison of actual and theoretical nonhome-based trip duration distributions, Bridgeport, Conn.

Figure H-5. Comparison of actual and theoretical truck trip duration distributions, Bridgeport, Conn.
Figure H-6. Comparison of actual and theoretical work trip duration distributions, Groton, Waterford, and New London, Conn.
APPENDIX I
FITTING THE GAMMA DISTRIBUTION TO GRAVITY MODEL TRAVEL TIME FACTORS

The travel time factors in the gravity model are selected so that the model accurately duplicates the frequency distribution of trip lengths. An attempt was made to find a functional form which could be satisfactorily fitted to the observed travel time factors developed for a number of cities. A meaningful functional representation of the travel time factors aids in interpreting extreme variations and understanding the fundamental determinants of this characteristic.

The statistical gamma distribution was tested to see if it could approximate the travel time factor curves used in the gravity model. This curve-fitting technique is shown in Figure I-1 for Toronto, where the fit is compared with the best of several obtained by Dieter (24). The general improvement in the fit is apparent.

Observed work trip travel time factors were obtained for several cities and fitted to the gamma distribution. Generally, the curve fits for auto driver and transit person trips were good. The moments of the distributions (the mean and the variance), and the parameters of the gamma distribution ($\alpha$ and $\beta$) are given in Table I-1.

There are substantial differences among the means and variances of the distributions. The alpha ($\alpha$) parameter of the gamma distribution lies between 1.1 and 1.5. Because of the nature of the distribution, the relative effect of alpha on the distribution decreases as the value of time increases. The beta ($\beta$) parameter plays an increasingly significant role and, for auto driver trips, varies relatively little from values near 0.2. The value for the limited sample of transit travel time factors is much lower (less than 0.1).

The findings of this analysis indicated that there were computer operational problems associated with trying to fit travel time factors with the gamma distribution. Therefore, from an operational standpoint the results of applying the gamma distribution to travel time factors were not satisfactory.

![Figure I-1. Comparative fit of gamma and exponential Toronto data.](image-url)

<table>
<thead>
<tr>
<th>CITY</th>
<th>MOMENTS OF OBSERVED DISTRIBUTION</th>
<th>GAMMA-DISTRIBUTION PARAMETERS</th>
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<tr>
<td></td>
<td>AVG. (MIN)</td>
<td>VAR. (MIN$^2$)</td>
</tr>
<tr>
<td>(a) TRANSIT WORK TRIPS</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Seattle-Tacoma, Wash.</td>
<td>22.0</td>
<td>427.7</td>
</tr>
<tr>
<td>Ottawa-Hull, Ont.</td>
<td>20.3</td>
<td>193.2</td>
</tr>
<tr>
<td>(b) AUTO DRIVER WORK TRIPS</td>
<td></td>
<td></td>
</tr>
<tr>
<td>St. Catharines, Ont.</td>
<td>5.5</td>
<td>36.9</td>
</tr>
<tr>
<td>Ottawa-Hull, Ont.</td>
<td>8.4</td>
<td>71.3</td>
</tr>
<tr>
<td>Greensboro, N.C.</td>
<td>8.1</td>
<td>81.8</td>
</tr>
<tr>
<td>Erie, Pa.</td>
<td>8.8</td>
<td>60.0</td>
</tr>
<tr>
<td>Waterbury, Conn.</td>
<td>6.2</td>
<td>64.7</td>
</tr>
<tr>
<td>Washington, D.C.</td>
<td>10.3</td>
<td>105.8</td>
</tr>
</tbody>
</table>
APPENDIX J

OPPORTUNITY L-VALUES AND
THE OPPORTUNITY TRIP DISTRIBUTION

It would be expected from the theory of the intervening opportunity model and from its methods of calibration that the $L$-value would correlate highly with the average of the opportunity trip length distribution. It has been shown in this research that there is a relationship between average actual trip length and average opportunity trip length. Schneider's (61) statement on the calibration of the opportunity model is that the $L$-value is directly related to the actual trip length. In the Chicago Area Transportation Study (18) the $L$-value was calibrated to a predetermined average trip length. In this research an attempt was made to relate the $L$-values to the opportunity trip length so that

Figure J-1. Relationship of short $L$ value to average opportunity duration, Pittsburgh, Pa.
future $L$'s might be projected from knowledge of the location of opportunities.

The research project reported here, however, was unable to investigate this area thoroughly for the following reasons:

1. A computer program for the opportunity model compatible with existing data was under development by the Bureau of Public Roads during the time of this research. The only operational program was not compatible with the BPR package programs. The magnitude of this research did not permit the investment of the considerable fraction of the available resources which would have been required to accomplish an effective investigation. This prevented the desired numerical investigation of the relationship between opportunity model $L$-values and average opportunity trip length on a controlled experimental basis.

2. Where the opportunity model had been calibrated to produce a desired trip length, data were insufficient to permit a comparison with the average opportunity trip length on an individual zone basis.

Data from the Pittsburgh Area Transportation Study (60) were examined in this analysis. The "long" $L$-values had been calibrated to provide accurate estimates of zonal interchange volumes to the central business district, and the "short" $L$-values were calibrated to provide accurate estimates of intrazonal traffic volumes. It is clear that neither of these methods of calibration is identical to that of determining those $L$-values which will satisfy a predetermined average trip length requirement.

Both the "long" $L$-values and the "short" $L$-values were correlated with the corresponding average opportunity trip length for individual zones. The correlation involving the "long" $L$-values was inconclusive ($r = 0.35$), primarily because so many zones had been assigned values at either an upper or a lower bound. Inasmuch as a wide range of trip lengths was represented in each group and the overlap between these trip lengths was also large, it was statistically impossible to achieve a high correlation.

For the "short" $L$-values, which are plotted against average probability trip length in Figure J-1, a correlation coefficient of 0.61 was obtained for the equation:

$$\text{short } L = (\bar{O})^{0.04}$$

in which $\bar{O}$ is the average opportunity trip length.

Because of the low correlation coefficient, even these results are inconclusive. They do, however, indicate that some relationship does exist.

---

**APPENDIX K**

**NON-WORK OPPORTUNITY TRIP LENGTH DISTRIBUTIONS**

See Figures K-1 through K-4 (pp. 64-67).
Figure K-1. Home-based shop opportunity trip duration distributions for (a) five selected urban areas, and (b) Washington, D.C., 1948 and 1955.
Figure K-2. Home-based social-recreation opportunity trip duration distributions for (a) six selected urban areas and (b) Washington, D.C., 1948 and 1955.


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