NATIONAL COOPERATIVE HIGHWAY RESEARCH PROGRAM
REPORT **51**

SENSING AND COMMUNICATION BETWEEN VEHICLES

HIGHWAY RESEARCH BOARD
NATIONAL RESEARCH COUNCIL
NATIONAL ACADEMY OF SCIENCES—NATIONAL ACADEMY OF ENGINEERING

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SENSING AND COMMUNICATION BETWEEN VEHICLES

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RESEARCH SPONSORED BY THE AMERICAN ASSOCIATION
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SUBJECT CLASSIFICATION
- HIGHWAY SAFETY
- TRAFFIC CONTROL AND OPERATIONS
- TRAFFIC FLOW
- TRAFFIC MEASUREMENTS

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NATIONAL COOPERATIVE HIGHWAY RESEARCH PROGRAM

Systematic, well-designed research provides the most effective approach to the solution of many problems facing highway administrators and engineers. Often, highway problems are of local interest and can best be studied by highway departments individually or in cooperation with their state universities and others. However, the accelerating growth of highway transportation develops increasingly complex problems of wide interest to highway authorities. These problems are best studied through a coordinated program of cooperative research.

In recognition of these needs, the highway administrators of the American Association of State Highway Officials initiated in 1962 an objective national highway research program employing modern scientific techniques. This program is supported on a continuing basis by funds from participating member states of the Association and it receives the full cooperation and support of the Bureau of Public Roads, United States Department of Transportation.

The Highway Research Board of the National Academy of Sciences-National Research Council was requested by the Association to administer the research program because of the Board's recognized objectivity and understanding of modern research practices. The Board is uniquely suited for this purpose as: it maintains an extensive committee structure from which authorities on any highway transportation subject may be drawn; it possesses avenues of communications and cooperation with federal, state, and local governmental agencies, universities, and industry; its relationship to its parent organization, the National Academy of Sciences, a private, nonprofit institution, is an insurance of objectivity; it maintains a full-time research correlation staff of specialists in highway transportation matters to bring the findings of research directly to those who are in a position to use them.

The program is developed on the basis of research needs identified by chief administrators of the highway departments and by committees of AASHO. Each year, specific areas of research needs to be included in the program are proposed to the Academy and the Board by the American Association of State Highway Officials. Research projects to fulfill these needs are defined by the Board, and qualified research agencies are selected from those that have submitted proposals. Administration and surveillance of research contracts are responsibilities of the Academy and its Highway Research Board.

The needs for highway research are many, and the National Cooperative Highway Research Program can make significant contributions to the solution of highway transportation problems of mutual concern to many responsible groups. The program, however, is intended to complement rather than to substitute for or duplicate other highway research programs.

This report is one of a series of reports issued from a continuing research program conducted under a three-way agreement entered into in June 1962 by and among the National Academy of Sciences-National Research Council, the American Association of State Highway Officials, and the U. S. Bureau of Public Roads. Individual fiscal agreements are executed annually by the Academy-Research Council, the Bureau of Public Roads, and participating state highway departments, members of the American Association of State Highway Officials.

This report was prepared by the contracting research agency. It has been reviewed by the appropriate Advisory Panel for clarity, documentation, and fulfillment of the contract. It has been accepted by the Highway Research Board and published in the interest of an effectual dissemination of findings and their application in the formulation of policies, procedures, and practices in the subject problem area.

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FOREWORD

By Staff

Highway Research Board

This final report will be of interest to traffic engineers, motor vehicle designers, and other public officials responsible for the design of safer vehicles and improved driver communication aids. The investigation presents an evaluation of four automobile taillight signal systems, an investigation of the motorist's use of turn signals, a study of longitudinal control systems for platoon movement, and a study of passing decisions on a multi-lane expressway. An infrared source-sensor system has been designed and tested to provide the driver with a visual display of relative velocity and spacing between vehicles. Aerial photogrammetric methods have also been devised to aid in the studying of traffic flow phenomena. It is believed that this research effort is one of the most comprehensive studies to date of modern inter-vehicular sensing and communication systems.

As the traffic demands on expressway-type facilities continuously increase the importance of correct driver decisions increases. It is believed that improvements in highway safety may be achieved through the development of improved inter-vehicular sensing and communication systems. It was with these thoughts in mind that this research was initiated. The research involved establishment of the operating requirements of a communication system designed to enable better communication between vehicles on expressway-type facilities. This development should reduce accidents, permit greater vehicle speed, and allow for more traffic volume.

The Ohio State University research team in this thorough and well-documented study has placed the emphasis on vehicle-borne, inexpensive, and reliable inter-vehicular communication systems based on the driver's natural ability and his needs in the high-speed, high density, car-following situation. Consideration has been given to the "pull-out-and-pass" decision on the freeway. Special problems associated with geometric influences on driver control behavior have also been studied.

Theoretical aspects of controlling platoon movement have been developed and part of the theory was verified by field studies. The principles of control systems have been developed based on sensing relative velocity between the leading and the trailing vehicle, and duplication of the acceleration pattern of the lead car by the trailing car.

The feasibility of a control system based on distance measurements between vehicles traveling in a platoon has been investigated and equations have been developed to determine the natural safe spacing of vehicles for different control systems. A prototype infrared source-sensor system was built and the system's effectiveness and reliability has been tested. The results were quite favorable.

Future research could involve further development and testing of the infrared source-sensor. Evaluation of this type of system during various types of weather

conditions seems desirable. Future research could also include the development of a system in which both the source and the sensor are located in one vehicle. The tri-light system of vehicle taillights offers advantages over the conventional system now in use and future research could involve a national full-scale test to determine statistically significant conclusions on a nationwide basis. It seems quite likely that conventional taillight systems can be improved. However, a major test is needed to further evaluate the tri-light system.

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ACKNOWLEDGMENTS

The research reported herein was performed as a cooperative effort by the Transportation Engineering Center and the Systems Research Group of the Engineering Experiment Station, Ohio State University, under the general coordination of Dr. Johannes F. Schwar, Professor of Civil Engineering and Director of the Transportation Engineering Research Center. Co-principal investigators for the project were Dr. Thomas F. Rockwell, Professor of Industrial Engineering, and Dr. Joseph Treiterer, Associate Professor of Civil Engineering, who acted, respectively, as supervisors of the Systems Research Group and the Transportation Research Group.

For the Systems Research Group studies, L. Tracewell and C. James designed and constructed effective, imaginative, and reliable hardware and visual displays; Dr. R. L. Ernst, Professor of Psychology, furnished consultation in statistical and psychological aspects; and Dr. A. Hanken (now at Georgia Institute of Technology) developed very useful piecewise linear multivariant regression routines. Other engineers and psychologists who actively participated included Norman Lobdell, G. V. Raju, K. Underwood, D. Kandel, L. J. Kerr, Sudhir H. Dalal, R. L. Samuelson, Jorge Matanzo, M. Rulon, P. Brown, and C. Wickens.

For the Transportation Engineering Group studies, the following investigations were carried out by the individuals noted:

W. C. Taylor, traffic accidents and traffic flow by dynamics;
Z. Nemeth, investigation of the use of turn signals;
J. Treiterer, theoretical aspects of traffic control;
R. M. Campbell, development of an infrared source-sensor system for traffic control;
J. Treiterer, N. E. Kaech, and H. Russell, longitudinal control systems for platoon movement; and
J. I. Taylor, traffic flow data and collection analysis.

Special acknowledgment is given to the Ohio Department of Highways for providing essential highway data.

SENSING AND COMMUNICATION BETWEEN VEHICLES

SUMMARY

This project involved establishment of the operating requirements of a communication system designed to enable better communication between vehicles on expressway-type facilities. Such development could reduce accidents, permit greater vehicle speed, and allow for more traffic volume. The requirements were assessed in the light of decreased time for making correct decisions. Consideration was also given toward compatibility with an automatic vehicle control system.

Evaluation and comparative examination of four intervehicular communication systems was completed. This involved both night and day study of car-following for no signal display, for the conventional brake light, for the tri-light system denoting brake and accelerator action, and for an acceleration information display of horizontal rows of green and red lights to indicate the magnitude of the vehicle's acceleration or deceleration. Studies of lane changing decisions were also made. In all of this research, testing was accomplished in the live traffic environment using instrumented vehicles.

Taxonomies of functional groupings of conceptual rear-end visual display components have been studied for the various signal systems previously tested. A prototype infrared sensing system has been developed and tested to indicate distance and relative velocities between vehicles. Field studies of traffic dynamics have been analyzed to determine the data which should be transferred by the sensing and communication system to increase traffic volume and improve safety and speed of traffic flow. Model development studies were made to quantitatively evaluate possible improvements which may be obtained through improved communication between vehicles.

CHAPTER ONE

INTRODUCTION

This report presents the results of the work performed by the Transportation Engineering Center and the Systems Research Group of The Ohio State University under NCHRP Project 3-3, entitled "Sensing and Communication Between Vehicles."

The work by the Systems Research Group was initially focused on the "need for intervehicular communication to decrease decision-making time on the highway, to reduce the probability of accidents, and to allow the movement of a greater volume of traffic." During the course of this research, the project team placed the greatest emphasis on vehicle-borne, inexpensive, and reliable intervehicular communication systems based on the driver's natural ability and his needs in the high-speed, high-density, car-following situation. In addition, consideration was given to the question of the "pull-out-and-pass" decision on the freeway, and the special problems associated with geometric influences on driver control behavior. In all of this research, testing was accomplished in the live traffic environment using instrumented vehicles.

The Transportation Engineering Center studies led to encouraging results, but cannot be considered exhaustive.

Nevertheless, sufficient territory was covered to obtain the following reassuring results:

1. The theoretical aspects of controlling platoon movement were developed and part of the theory was verified by field studies.

2. The basic principles of control systems, based on sensing relative velocity between the leading and the trailing vehicle, or duplication of the acceleration pattern of the lead car by the trailing car, have been developed. The feasibility of a control system based on distance measurements between vehicles traveling in a platoon was considered. Safety aspects were considered and equations were developed to determine the natural safe spacing of vehicles for different control systems.

3. A prototype of the Infrared Source Sensor System was built and the system's effectiveness and reliability were tested. Encouraging results were obtained for daytime as well as for nighttime driving.

SYSTEMS RESEARCH GROUP STUDIES

In view of the ever-increasing complexity of modern transportation systems and the particular problems relating to highway transportation, it is clear that improved methodology, hardware, and operating procedures must be developed to permit necessary gains in traffic flow, volume, and safety. Improvements including advances in the theory of roadway design, increased modernization of existing highways, and concurrent changes in automotive design practice are required if pace is to be kept with the growing numbers of vehicles that crowd the highways.

Advances such as these are of limited value unless there can be developed a relevant, verified theory integrating the driver, automobile, and highway to permit prediction of the consequences and interactions resulting from changes in any of the elements of the total system. Central to such a theory is an understanding of the role of the individual driver, who acts as the controller or regulator of the system. His performance depends critically on the information inputs he receives; consequently, intervehicular communications are seen to play a vital role in system effectiveness.

As a man-machine system operating in a complicated environment, the uniqueness of the driver-vehicle-highway complex lies in the wide range of skill and intelligence levels to be found in the human components of the system. System procedures and hardware components must be designed to accommodate human operators whose variance on any psychological or physiological scale can be very large. This feature immediately suggests that criteria under which alternative intervehicular communications systems might be judged should include such factors as simplicity and public acceptability in addition to actual performance measures such as longitudinal stability in car-following situations.

Empirical research which seeks to learn something of the role of communications in the driving task may be undertaken in a number of ways. This research placed test subjects in real on-the-road driving situations and measured the performance of the system of which they are a component. This approach does not allow the precise control over variables that might be available in a laboratory situation. Nevertheless, results obtained on the highway have the distinct advantage of being readily interpreted into meaningful conclusions. The philosophical problems and stark practical uncertainties which are ever-present when extrapolations are attempted from laboratory simulation results into real-world contexts are minimized when studies are conducted in a setting in which they are to be interpreted. Many potentially useful intervehicular communication systems can be developed from knowledge of the basic abilities and needs of the driver. What is needed is basic operating information about the performance of such intervehicular communication systems on the highway and in traffic. Toward this end, this research was directed.

Much of the research completed on this project has centered on evaluation of various types of intervehicular communication systems and the effects of displayed information on driver response and longitudinal stability in car-following situations. In this evaluation several experiments were conducted using two highly instrumented vehicles to test displays of specific forms of velocity, acceleration, deceleration, and pedal position information on a battery of car-following measures. All of the tests reported here were conducted on the highway in actual day and night traffic conditions. Therefore, the results of these tests are partially relieved of the effects of experimental bias and are more easily interpreted to real-life situations than laboratory studies.

In addition to the tests conducted on the various intervehicular displays, exploratory tests were also conducted to study the passing performance of drivers on multi-lane expressways to determine if it would be practical to install a system on the highway which would inform the driver about passing decisions. Exploratory tests were also made to determine the effects of roadway geometry on driver and vehicle preformance. Finally, an argument for a systems view to vehicle communication was presented.

The purpose of this summary is to present the highlights and the major conclusions of the various aforementioned experiments in condensed form.

Equipment Used

For the majority of tests experimental subjects drove a 1963 Chevrolet sedan (Fig. 1) fitted with a 50-channel oscillograph recorder (Fig. 2), which produced continuous traces of signal inputs on 400-ft rolls of 12-in. photographic paper. Sensors mounted throughout the vehicle permitted continuous recording of driver control movements (steering wheel, gas pedal, and brake pedal) together with vehicle dynamics such as velocity, horizontal and vertical accelerations, and their derivatives. In addition to these variables, headway (the distance between a lead car and the following instrumented vehicle) and relative velocity were measured by a thin wire and a take-up reel as shown in Figure 3. This information was also an input to the oscillograph recorder.

Figure 1. The instrumented passenger vehicle.

Taillight Signal Systems

DAYTIME STUDIES

Four intervehicular signal systems—Acceleration Information Display (AID), Tri-light, No Information Lights (NIL), and conventional—were compared. The display consisted of 5-in. square light units arranged as shown in Figure 4. Conventional brake lights were displayed by the

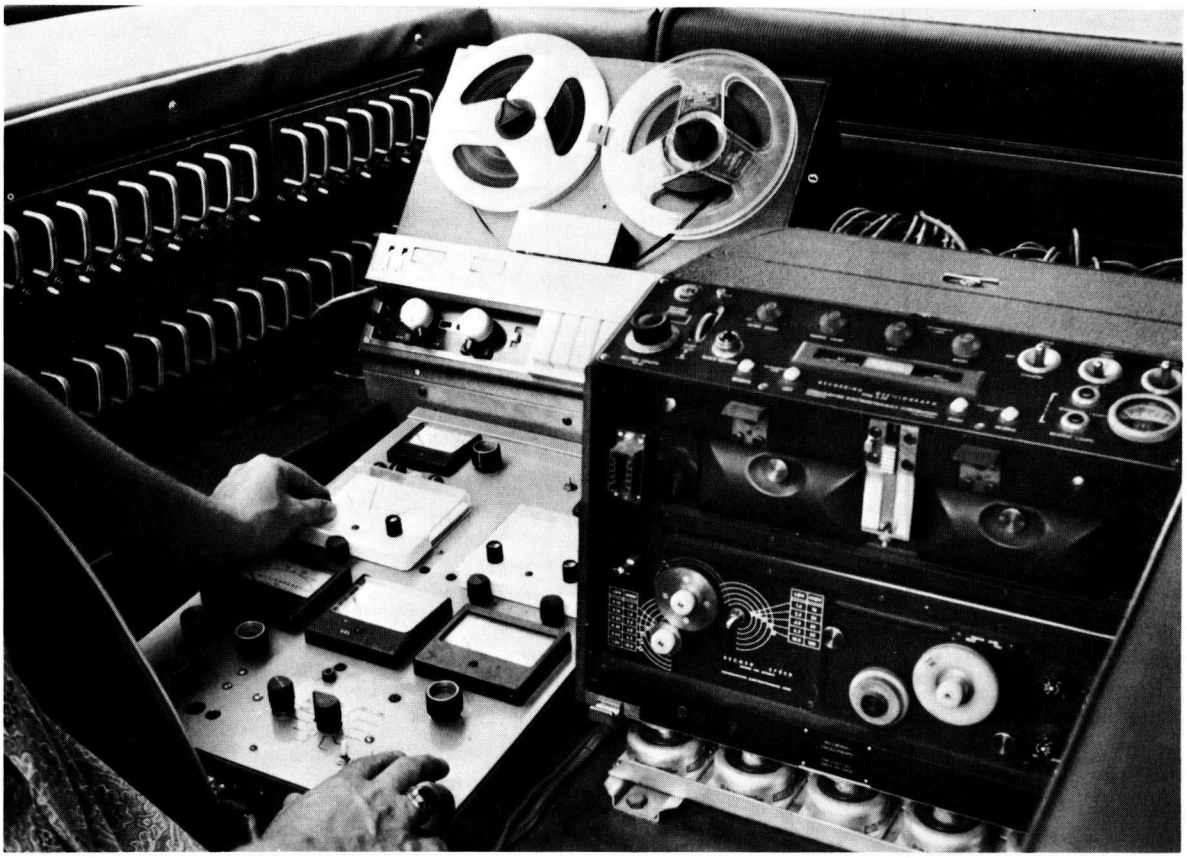

Figure 2. The experimenter's console and the C.E.C. oscillograph recorder.

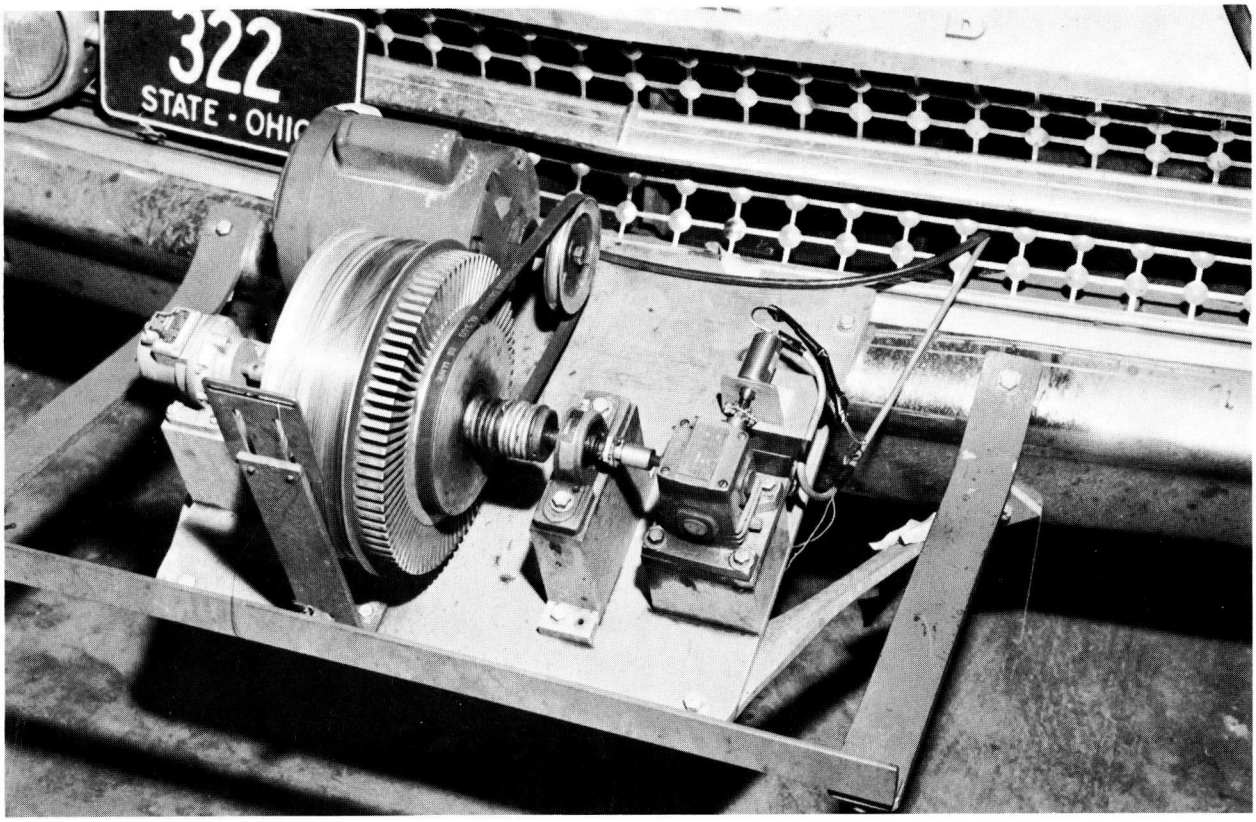

Figure 3. The "yo-yo" device for measuring relative velocity and headway between a lead car and the following (instrumented) vehicle.

outer red lights, and the Tri-light system used the vertical columns of red (brake pedal actuation), amber, and green (gas pedal actuation). The AID system employed the horizontal rows of red and green lights, actuated by vehicle acceleration, with the outer reds signaling slight deceleration, the outer two reds a greater deceleration, and so on until all red lights were illuminated for a violent braking maneuver. The green lights functioned in a similar manner for acceleration.

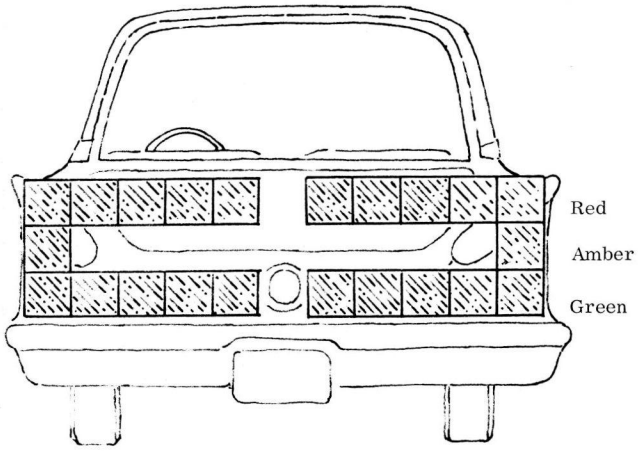

Figure 4. Arrangement of signal lights on lead vehicle.

The following vehicle (driven by the subject) was equipped with a take-up reel and recorder to permit continuous recording of headway and relative velocity, as well as vehicle velocity, pedal movements, and acceleration. Radio communication between vehicles and a headway meter visible to the experimenter in the following car were available for use.

The basic experimental procedure placed subjects in a car-following situation while a lead vehicle with the display performed a series of braking and coasting maneuvers. The eight different maneuvers used in the tests are described in Table 1.

The design involved four replications of each maneuver under each condition for each subject, which meant 160 total replications of each maneuver. Collision likelihood increases with testing frequency, and although decelerations did not exceed 0.5*g*, observers were stationed in both vehicles as an added precaution, particularly in the hard braking case.

Each maneuver was identified (together with any experimenter notations) on the recorder paper after film processing. Initial numerical information removed from the traces for each maneuver included the following:

1. Period of closure. Time interval between initiation of the maneuver and the first return to zero relative velocity.

TABLE 1

LIST OF MANEUVERS[a]

MANEUVER	DESCRIPTION
A	Coast to 50 mph, 150-psi brake to 35 mph (coast and mild brake)
B	Coast to 35 mph (simple coast)
C	Coast to 45 mph
D	150-psi brake to 45 mph
E	225-psi brake to 45 mph, coast to 35 mph (medium brake and coast)
F	225-psi brake to 35 mph (direct medium brake)
G	2-sec coast, 225-psi brake to 35 mph (medium brake with delay)
H	2 sec coast, 350-psi brake to 35 mph (hard brake with delay)

[a] All maneuvers began at 65 mph and the lead vehicle returned to that speed following the maneuver. All accelerations used the "co-pilot" at a 65-mph setting.

2. Gas pedal response. Time interval between initiation of the maneuver and the complete release of the following vehicle's gas pedal.

3. Gas pedal release interval. Time period that the subject's foot remains off the gas pedal.

4. Initial headway. Headway at initiation of maneuver.

5. Minimum headway. This should occur exactly at the end of the period of closure.

6. Minimum relative velocity or maximum closing velocity. A negative quantity, occurring during the period of closure.

7. Maximum relative velocity or maximum opening velocity. This will occur during the acceleration period following the period of closure.

8. Minimum velocity.

In the analysis, an evaluation of the signal systems was completed for each maneuver and various performance measures were derived from the foregoing with all subjects and replications pooled. Each subject served as his own control in system tests.

Analysis of the data indicated that both the AID and Tri-light systems resulted in decreased response times over the conventional system. Response times under conventional and NIL (where drivers were left to detect decelerations by such visual cues as cant and perceived relative velocity) were in excess of three times those obtained under the AID and Tri-light systems, as can be seen in Figure 5. The greatest gains were obtained with the Tri-light system. Large differences between performance under conventional and that under AID or Tri-light clearly indicated the benefits obtainable from signaling slight decelerations.

The data did not seem to indicate much difference between the various signal systems when the maneuver began with an immediate braking action, although AID and Tri-light improved performance in braking maneuvers that began with a coast.

As with the other two previously mentioned measures, AID and Tri-light systems resulted in improved performance as indicated by significant reduction of the average relative velocity maintained during a maneuver for maneuvers beginning with a coast.

Acceleration response times represent the time interval from the lead vehicle's acceleration until the following driver steps on the gas pedal. One would expect improved performance (reduced response time) when the lead vehicle signaled its accelerations. Both AID and Tri-light did, in fact, result in slightly lower acceleration response times than the conventional system.

Maximum relative velocity during the lead vehicle's acceleration forms a partial measure of the following driver's performance in interpreting and following the lead vehicle's acceleration. The AID system resulted in a significant improvement of performance with respect to this measure.

In summary, results of the more than 1,100 daytime highway trials performed indicate that car-following performance is significantly and substantially improved when conventional brake lights are replaced by the acceleration or gas pedal signals. When a vehicle begins coasting from 65 mph, for example, following-driver response times with the conventional systems were found to be four times greater than those obtained with the Tri-light system.

When performances under the two special systems (AID and Tri-light) were compared, it was found that the Tri-light system gave slightly better performance during lead vehicle deceleration and that both systems performed equally well during lead vehicle accelerations.

NIGHTTIME STUDIES

The experiment previously described was repeated at night, from 12:00 midnight to 4:00 AM. This experiment was conducted in essentially the same manner as the daytime study, with the NIL condition deleted for safety reasons.

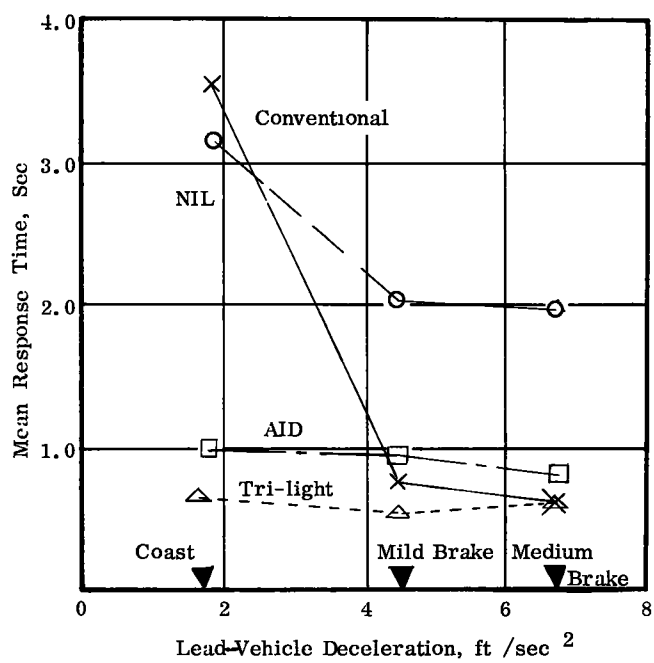

Figure 5. Mean response times for all subjects and replications.

The same eight-maneuver evaluation technique was used to compare the three signal systems. The findings of this study almost exactly paralleled those of the daytime study, with the AID and Tri-light systems yielding superior performance, especially with maneuvers beginning with a coast.

Velocity Display Signal System

Research was conducted to obtain some insight into the effects on car-following performance of a display mounted on the rear of the lead vehicle and indicating its velocity.

The physical display used in the taillight studies was modified to enable its use in this study. These modifications consisted of changing the lenses so that both the bottom and top rows of the display contained green lights, whereas the single lights in the middle row were changed from amber to red. After modification, the green lights were used to signal the lead car's velocity in 5-mph increments, and the red lights were used in the conventional display.

Figure 6 shows the configuration of these lights, together with the velocity for which each was activated. The appearance of the display at the beginning of each maneuver (65 mph) is shown in Figure 7. The speeds indicated here are those indicated on the speedometer. The eight-maneuver evaluation technique described earlier was used to compare this velocity display system with the conventional system.

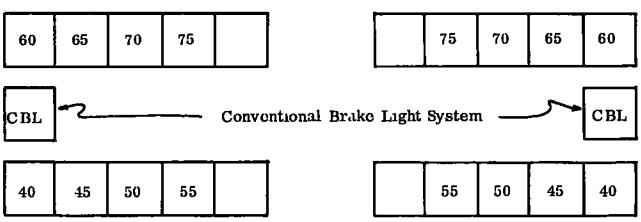

Figure 6. Velocity display system. Numerals represent observed speedometer readings. Lights remain on when velocity is equal to or greater than speeds indicated.

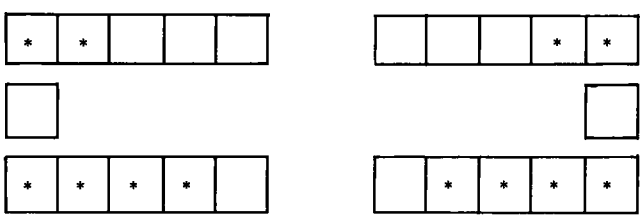

Figure 7. Appearance of velocity display at beginning of test maneuver (65 mph).

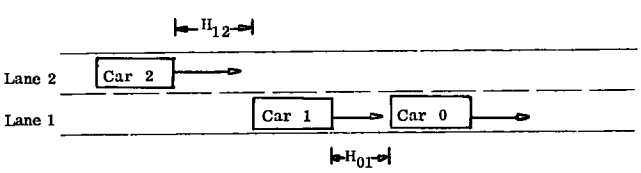

Figure 8. Pattern of traffic flow for expressing passing study.

Performance, as measured by response times, minimum relative velocity, maximum closing velocity, average relative velocity, percent reduction in headway occurring during a maneuver, and maximum opening velocities, was compared for the two systems. The first four measures, which serve to measure performance under the deceleration phase of the maneuver, all indicated that the velocity display system was superior for maneuvers whose deceleration consisted of only a coast. On maneuvers beginning with a braking action, the conventional system proved superior.

The seeming superiority of the velocity display in maneuvers consisting of coasts is probably due to the fact that this system gives positive information that the vehicle is slowing down, whereas the conventional system furnishes no information during a coast.

On the other hand, during a braking maneuver the conventional system furnishes evidence of deceleration as soon as the brake is applied, whereas the velocity display incorporates a lag until a 5-mph decrease is realized.

Passing on a Multi-Lane Expressway

The decision to enter a traffic lane at a given time is based on many factors, such as the following:

1. The willingness of the driver to accept the risk of collision (related to gap-acceptance studies).
2. The amount of information available to the driver concerning the headway between the vehicles involved.
3. The amount of information available to the driver concerning the relative velocity relationships between the vehicles involved.
4. The acceleration capabilities of the vehicle.
5. The geometry of the expressway.

The general objective of this study was to determine if the volume of traffic that can be carried by a highway can be increased by increasing the information available to the driver relevant to his passing decision.

This experiment was conducted on Interstate 71 south of Columbus, Ohio. Three vehicles were used to form a traffic pattern as shown in Figure 8. Car 0 was driven by an experimenter at a constant velocity, V_0, establishing the traffic flow in lane 1.

The subject was in car 1. He was instructed to follow car 0 at a constant headway, H_{01}. During this time he was denied use of his rearview mirrors. To begin a trial, the experimenter in car 2 adjusted his velocity to a predetermined value, thus establishing the flow in lane 2. At time t_0, when the headway between car 1 and car 2 reached a predetermined value, H_{12}, the experimenter accompanying the subject in car 1 simultaneously marked the recorder and allowed the subject use of his rearview mirrors. The mirrors were made operable by turning out the light behind the outside mirror, and remotely raising the flap covering the inside mirror.

At this time the subject made a decision to pass or not pass car 0. If he elected not to pass, he continued his task

of maintaining constant headway between himself and car 0. When the experimenter in car 2 reached car 0, he decelerated and prepared for the next trial. If the subject elected to pass, he initiated the maneuver by pulling into lane 2. A large amplitude in deflection of the gas pedal trace on the recorder showed the time, t_1, at which the maneuver was initiated.

Headways between car 1 and car 2, H_{12}, were set at twelve different levels during the experiment.

Two dependent variables were investigated—decision time and percentage of "go" responses. These were studied under all headways between cars 1 and 2. A "go" response was designated when the subject in car 1 pulled into the passing lane during a trial. Decision time was taken as the time difference between the start of the trial, t_0, and the point when the subject heavily depressed the gas pedal, t_1.

It is convenient to talk in terms of a threshold for "go" or "no-go" responses to the passing situation. The term implies that there exists some headway value above which the response will always be "go" and below which it will always be "no-go." It has been found [1, 2] that acceptance of a "gap" in entering a highway is not a step function denoting a definite threshold, but a trapezoidal function. This may be generalized to the necessary gap in traffic flow on the highway which a driver must accept in order to make a passing response.

For analysis, however, the threshold is defined in terms of the level at which 50 percent of the responses are "go" responses. Figures 9 and 10 show the percentage of "go" responses for each subject as a function of headway for both velocity conditions. As expected, there were large differences between subjects.

On the basis of the data and on the calculation of the minimum possible distance to make a pass, it was concluded that the subjects, in general, passed with resulting headway values, H_{12}, below that recommended by the National Safety Council (NSC) but above that determined as resulting in collision.

Increasing the safety and flow of traffic via increased information to the driver is potentially possible for two types of drivers. The first is the driver who passes only at values in excess of that needed for safety. He is the driver who may wait for a headway twice that necessary for a safe pass. When highway conditions are crowded, the likelihood of an excess headway is small, and the driver may

[1] CHANDLER, R. E., HERMAN, R., and MONTROLL, E. W., "Traffic Dynamics: Studies in Car Following." *Oper. Res.*, Vol. 6 (1958) pp. 165-184.
[2] HERMAN, R., *Theory of Traffic Flow*. Elsevier (1961).

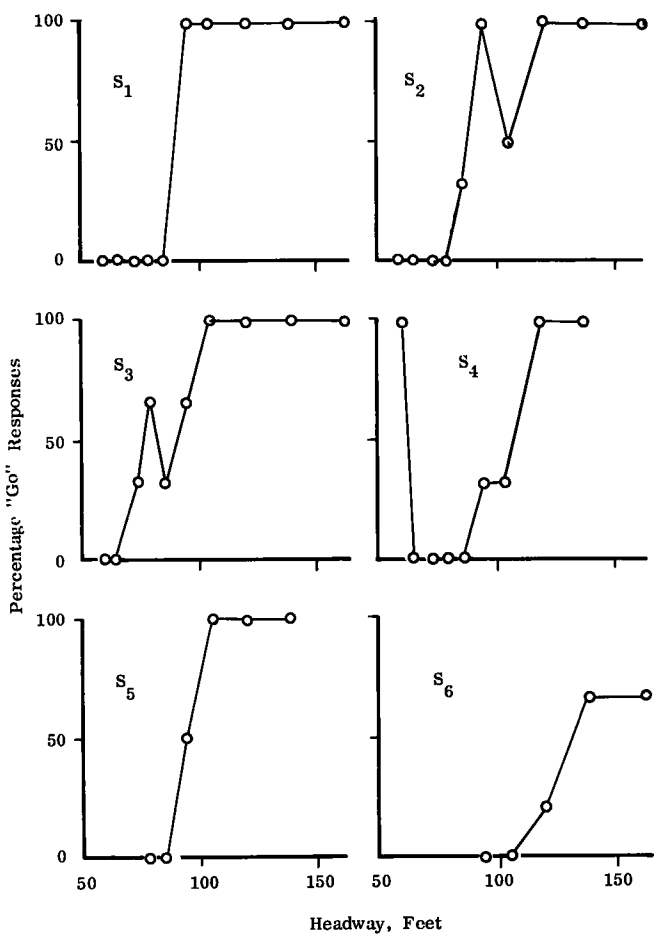

Figure 9. Percentage of "go" responses for each subject as a function of headway when $V_0 = 50$ mph and $V_2 = 60$ mph.

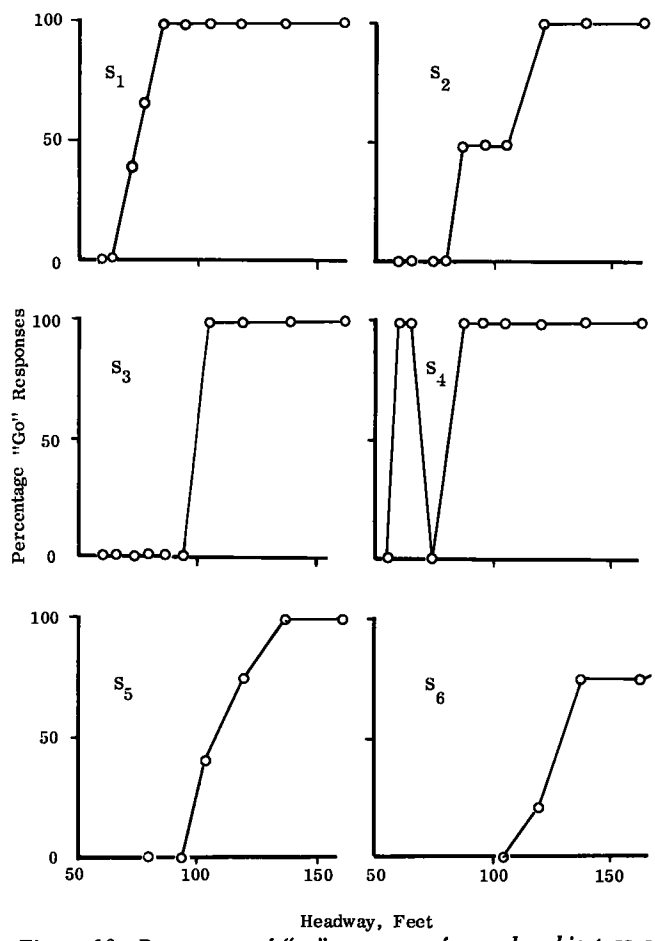

Figure 10. Percentage of "go" responses for each subject as a function of headway when $V_0 = 60$ mph and $V_2 = 70$ mph.

let numerous safe passing opportunities escape while waiting for a greater headway. This would have the effect of letting usable space on the highway go to waste.

The second type of driver is the one who passes at headways which are at a value low enough to cause the closing car to decelerate. In dense traffic this could have the effect of causing a wave of disturbance which would result in traffic stoppage and/or rear-end collisions in that lane.

As was pointed out, the subject drivers tended to pass at headways below the NSC-recommended car-following distance (although whether this is an adequate safety criterion is open to question). These drivers might profit by vehicle-borne or highway-based information on the proper time to execute a pass.

Driving Control Behavior and Roadway Geometry

A driver's response to information about other vehicles is influenced by the use which he and his vehicle can make of that information and by the attention which he must pay to controlling his own vehicle. This section describes some exploratory studies of single car-driver behavior and its relation to roadway geometry.

The purpose of this study was to explore driver control behavior on expressways as contrasted to driver-vehicle behavior reported in the previous studies. Here concern was not solely with vehicle velocity or acceleration, as such, but rather with the driver's input to the vehicle in terms of gas pedal and steering wheel movements over time as a function of road geometry, instructions (driving objectives), and vehicle dynamics, such as acceleration and velocity. The basic experiment is first described, wherein the unique data collection and analysis with the use of the computer enables simultaneous comparison through a computer plot of gas pedal movement, steering wheel movement, and velocity, acceleration, time, vertical curvature, and distance traveled (see Fig. 11). With these primary data, the following questions were briefly explored:

1. What are the natural characteristics of gas pedal movements in terms of amplitude and periodicities and how do subjects compare under the same driving conditions?
2. How does route profile (vertical curvature) affect gas pedal movement and velocity? To what extent might this clarify the accident problem on sags and crests on Interstate highways?
3. With time-based regression analysis, can the contribution of profile, acceleration, and velocity on gas pedal movement changes be found? To what extent do these variables lead or lag gas pedal movement changes?
4. How do steering wheel and gas pedal amplitudes and periodicities relate to driving objectives as imposed by several different driving instructions?

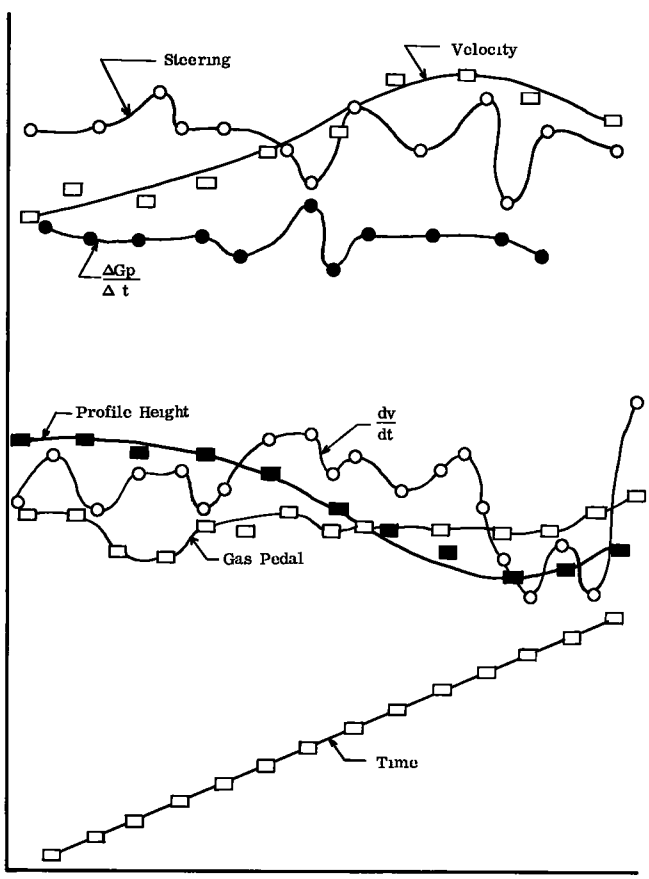

Figure 11. Sample computer output.

THE BASIC EXPERIMENT

All subjects drove an instrumented vehicle (1961 Ford station wagon) on Interstate Highway 71, a limited-access freeway. The test route consisted of a 50-mile trip north of Columbus, Ohio, a coffee break, and the 50-mile return. The subjects were told that they were participating in a fatigue study and that their normal driving behavior was desired.

Five variables were recorded—gas pedal position, steering wheel position, velocity, the vehicle's acceleration tangent to the road, and the vehicle's vertical acceleration. In addition, a recorder channel was reserved for marking on the data from the 81 points along the route which were previously marked by taped delineations. The experimenter tripped the marker when the vehicle was opposite each of these, thereby providing a means of correlating the recorded driving data with the road geometry.

The data from the film were punched on cards using a 1-sec sampling interval. These cards were processed to convert the time scale data for each subject to a common distance scale and to calculate additional measures. The resulting data (discrete traces of equi-distance spacing) were plotted to facilitate the search for patterns of behavior.

Response to vertical curvature fell into two basic categories. These are best described in terms of the ways in which drivers appeared to be controlling their vehicle. Two basic types of gas pedal trace were indicated—ragged traces and smooth traces. Inspection of the computer plots shows that the more ragged of the two gas pedal types produces very "flat" velocity curves; i.e., there is little velocity variation regardless of road geometry. This type of driver ap-

pears to make extensive use of the speedometer, inasmuch as responses occur after brief periods of velocity change. Hills are compensated for by increasing the amplitude of the medium gas pedal oscillations. The occasional velocity fluctuations which do occur seem to result from misestimates of the gas pedal amplitudes necessary to reverse the velocity trends. The regions of the highway which cause estimation errors most often are the inflections. Generally, these seem to create the impression that the vehicle is climbing after it has either stopped doing so or has begun to level out, so that the velocity climbs higher than anticipated and then must be reduced. Because velocity deviations for this type of driver seem to be errors of estimation of needed gas pedal adjustment, their magnitude varies from hill to hill (i.e., the magnitude of velocity change is random).

The second class of drivers consists of those who have a less rigid attitude toward velocity; i.e., they allow larger fluctuations in their speed. These drivers use a minimum of gas pedal movement for the task at hand, apparently attempting to predict the rate of pedal deflection necessary to keep velocity constant. They then use this predicted gas pedal movement as their control. Feedback about actual velocity is not used unless the velocity change is large (at least 5 mph). This class of subjects is also affected by inflections in the same way as the other class. In addition, however, this minimum gas pedal group often demonstrates velocity curves similar to those of Figure 12. It seems that these errors develop because (a) the estimation of the slope being traveled occurs as the vehicle starts up and down, and (b) the velocity is not checked often enough to detect the change.

In addition to the aforementioned results, this study also pointed out several characteristics of the relation between driving control behavior and roadway geometry. For example, it was found that subject gas traces appeared to share basic frequencies. When the subject's gas pedal performance was separated for straight sections of road and for hilly sections of road, a difference in the frequencies and amplitudes of the traces was evident. For hilly sections, the frequency of gas pedal movements decreased, but the amplitude of these movements increased.

This study also showed that deviations from a target velocity are most likely to occur on the upgrades and downgrades of sags and crests in the road. This fact may explain the findings of other studies that accident rates are highest at these points in the road.

Several regression analyses indicated that gas pedal movements bore a time-based relation to acceleration and velocity. Differences between subjects prevented quantification of the effects of time.

When several different types of tasks were imposed on the subject drivers, they were found to have effects on both steering movements and gas pedal movements, even though each task specified control of only one of these two variables.

Although the samples on which these particular results were obtained were too small to allow positive conclusions, the results certainly indicate that further efforts along these lines would be profitable.

Figure 12. Velocity curve of second driver class over a crest.

Systems Concepts in Intervehicular Communication

Throughout automotive history the design and development of automotive communication systems has proceeded on an evolutionary basis, wherein new and special-purpose components have been added and others removed with little consideration for the system as a whole. Some consequences of this form of development are evident in visual communication systems employed on today's automobile. Red running lights are used in the rear, although the color red usually denotes STOP. The two are frequently confused. Turn signals vary in color according to whether they appear on the front or the rear of a car; flashing amber (CAUTION) at the front, and flashing red (STOP) at the rear. One could continue with this list. This is the consequence of installing special-purpose components, which may perform very well individually but collectively lose effectiveness when viewed in a systems context. Highway signing reflects similar difficulties. Signing varies from state to state and the oft-used negative instruction merely tells the driver what he cannot do but not what he should do. Combined left- and right-hand exit patterns on freeways fail to recognize the inherent increase in response time where the number of possible choices is increased from two to three.

INTERVEHICULAR COMMUNICATION SYSTEMS

It is clear that consideration of intervehicular communications from a systems viewpoint requires some initial statements about system bounds. This is somewhat arbitrary, and the present choice is to consider the system as including those visual signaling devices installed on the vehicle for that purpose, together with routine and deliberate signals by the driver.

The functions that such a system might be expected to perform stem from the question, "Why communicate between cars?" One way of answering this question might lead to a description ascribing to intervehicular communication systems the following functions:

1. Reduce collision likelihood.
2. Facilitate and improve traffic flow.
3. Ease and simplify the driving task.
4. Reduce the need for other and more costly traffic control devices.

When the conventional system found on cars today is viewed in this context, several inadequacies are found compared to other possible systems. For example, a signal system found today would consist of the following:

1. Headlights, which besides helping the driver to see the road, perform the equally important task of frontal

identification; i.e., telling oncoming drivers that the vehicle is present.

2. Amber running lights, in front.
3. Amber front turning signals, integral with front running lights.
4. Red rear running lights.
5. Red rear turning signals, integral with rear running lights.
6. Red rear brake lights, integral with rear running lights.

These six components serve collectively to communicate three kinds of information to other vehicles; viz.,

1. The presence of the vehicle and its direction relative to the observer.
2. The driver's intent to turn, and his intended direction.
3. Whether or not the vehicle is braking.

Another signal system, Tri-light, includes the following as major signaling components:

1. Headlights.
2. Amber front turning signals.
3. Red rear brake lights.
4. Two rear amber lights, which are activated when neither the brake nor the gas pedal is depressed.
5. Two rear green lights, which are activated by a depressed gas pedal.
6. Amber rear turn signals, integral with the solid amber described in item 4.

This signal system communicates four kinds of information; viz.,

1. The presence of the vehicle and its direction relative to the observer.
2. The driver's intent to turn, and his intended direction.
3. Whether or not the vehicle is braking.
4. Whether or not the gas pedal is depressed.

In communicating these four kinds of information, the Tri-light system uses six components, whereas the conventional system requires six to communicate only three kinds of information.

Clearly, an assessment must be made on the number and level of information requirements for exterior signal systems before the choice of optimal signal systems can be made.

TRANSPORTATION ENGINEERING CENTER STUDIES

In investigations carried out by the Transportation Engineering Center, the distribution of accidents on rural Ohio freeways was found to be as given in Table 2. The first two types are caused by the lack of longitudinal control of vehicle movement. Together they constitute 36.4 percent of the accidents on rural Ohio freeways and it can be expected that any sensing and communication system designed to prevent these types of accident will make an important contribution to road safety on freeways.

No figures are available for urban expressways in Ohio. The distribution of accidents, however, indicates that the percentage of rear-end collisions and collisions with stopped or stopping vehicles is considerably higher on urban freeways. This observation is supported by data from the Ford and the John Lodge Expressways (Table 3). In this case an even higher contribution to road safety can be expected from the longitudinal control system, as almost 60 percent of the accidents involve rear-end collisions. An analysis of the relationship between headway distribution and accident rate (Appendix C) has shown that the accident rate increases with the percentage of vehicles traveling at closer distances to the lead car. The same study further indicated that the distance over which communication should be transmitted to improve traffic flow in platoon movements is between 50 and 160 ft, a rather short range, which was extended from a few feet to about 500 ft for traffic safety reasons in the first infrared source-sensor system (Chapter Seven) for longitudinal traffic control. Future research using more powerful sources, or an injection laser, could extend the range to about 1,000 ft.

In an investigation on the use of turn signals (Chapter Eight), the possibility was tested as to what extent drivers might make use of a new sensing and communication system. It was found that signaling frequencies increase with responsibility, the complexity of the driving maneuver, and the more complicated driving task with higher traffic densities. This indicates that drivers, probably subconsciously, feel the need for more communication and information with increasing traffic volumes.

Sensing and Communication Systems

From the studies mentioned in the foregoing, it became clear that any sensing and communication system should meet the following requirements:

1. It must be a longitudinal control system if the improvement of traffic safety on divided expressways, freeways, and Interstate highways is a primary objective.
2. It must be a longitudinal control system, which is aiding the driver or can operate automatically, if the velocity of vehicles is to be increased safely.
3. It must basically be a longitudinal control system with a range of at least 160 ft if increased traffic flow, and the prevention of traffic jams, is a major objective. Lateral control to facilitate lane switching must also be considered.

In the light of these conditions, an investigation was carried out on possible system concepts for communication and sensing between vehicles. The following media were considered:

1. Sound (exhaust noise, ultrasonic sound).
2. Radio.
3. Radar.
4. Light (infrared, visible, and ultraviolet).
5. Color-sensing devices.

The braking conditions in following the lead vehicle were investigated and the effects of highway geometry were studied.

From these rather extensive studies, infrared light has been singled out as a possible medium for sensing and communication, if reliability, simplicity, range, and cost are major considerations. It should be mentioned here that radar equipment has been tested on the road during these

TABLE 2

DISTRIBUTION OF ACCIDENT TYPES ON RURAL OHIO FREEWAYS

ACCIDENT TYPE	DISTR. (%)
1. Rear-end collision	25.5
2. Stopped or stopping vehicle	10.9
3. Sideswipe	6.6
4. Fixed object	32.5
5. Other	24.5

TABLE 3

DISTRIBUTION OF ACCIDENT TYPES ON FORD AND JOHN LODGE EXPRESSWAYS[a]

ACCIDENT TYPE	DISTRIBUTION (%)		
	FORD	JOHN LODGE	FORD AND LODGE
Rear-end collision	62	55	59
Sideswipe	27	35	30
Fixed object	9	9	9
Head-on	2	3	2

[a] MALO, A. F., and MIKA, H. S., "Accident Analysis of an Urban Expressway System." HRB Bull. 240, pp. 33-43 (1960).

investigations and good results have been obtained with a skilled operator. This type of equipment, however, is expensive and target identification imposes a serious problem.

Two versions of an infrared system have been designed and a prototype of the source-sensor system was built at a material cost of about $100. A description of the prototype is given in Chapter Seven.

The present version uses a pulsed infrared beam to transmit velocity data of the leading vehicle, which are evaluated together with velocity data of the trailing vehicle, and the differential velocity between the two vehicles is displayed. The distance between the two vehicles is determined by measuring the strength of the signal received by the following vehicle. This distance information is also displayed on the meter of the following vehicle, and a threshold value for safe spacing can be chosen by adjusting the amplification factor accordingly.

For the trailing vehicle, only the vehicle ahead in the same lane is of importance in the car-following situation. A system has therefore been developed to code traffic lanes with magnetic material similar to a magnetic tape. This coding strip will be sprayed, like road marking paint, as continuous strips along the center of each traffic lane, and a sensor on each vehicle will pick up the pattern controlling the transmitting code of the infrared sensing system. Thus, interference from adjoining traffic lanes will be eliminated, and the target (i.e., the leading car) can be clearly identified because of the signal code (frequency), the signal strength, and the shielding effect of vehicles traveling in a platoon.

It has been envisaged that the magnetic coding strip along the center of each traffic lane will finally be used for vehicle guidance in a fully automatic system.

The coding strip along the center of each traffic lane is the only requirement that must be added to the highway for the IR control system. The cost of applying a coding strip is minimal compared with the cost of sensors, guidance frequency cables, and other auxiliary equipment necessary for a longitudinal control system which is part of the highway structure. It further should be mentioned here that in addition to the comparatively high cost of the equipment contained in the highway structure, some receiving and servo equipment must be installed in each vehicle if the full benefit of such a system is to be realized. The argument that vehicles may be equipped any time when individual drivers are prepared to accept the control system in a step-by-step process after the highway has been furnished with the longitudinal control system without causing difficulties is not valid.

Although all vehicles will be detected, only those cars will benefit from the control system which are equipped with the necessary receiving and servo equipment, and even if 50 percent of the vehicles are equipped accordingly, no higher average speeds can be expected nor will the traffic volume be increased (Chapter Eleven). It can not be determined at this stage whether traffic safety will be improved or deteriorated under these conditions, but the full benefits will definitely not be realized as long as some vehicles are not an integral part of the control system. It can be expected that vehicles controlled by a driver aid system or an automatic system will not respond in the same way as purely driver-controlled vehicles, which then become sources of disturbance and hazards to traffic movement.

A theoretical approach was used to study the continuity of traffic flow and the propagation of disturbances (Chapter Ten and Appendix D). Results from this study indicate that traffic flow depends on two factors—the delay time in propagating changes, which will be extremely important for any driver aid or automatic system, and the ratio of the density of moving traffic to the density of traffic in a jam condition. The propagation and characteristics of kinematic waves and the formation of shock waves in traffic flow were explained (Chapter Nine) and some of the resulting consequences for automatic control systems were discussed. These studies finally lead to a study of longitudinal control systems for platoon movement (Chapter Eleven).

Figure 13 shows the most significant result of this study; i.e., the approximate region which will be accessible by using relative velocity between successive vehicles or duplication of the acceleration pattern of the lead vehicle by the trailing vehicle as control parameters. Both systems have characteristics which appear to be of great importance for longitudinal control systems. The most important one is that relative velocity control does provide attenuation in the propagation of disturbances but no attenuation of disturbances is indicated by the acceleration control system. The attenuation factor will actually determine the capacity of traffic flow within the range of these two systems. At 60 mph this range extends from about 2,600 to about 4,300 vph per traffic lane. Even the lower value of 2,600 vph

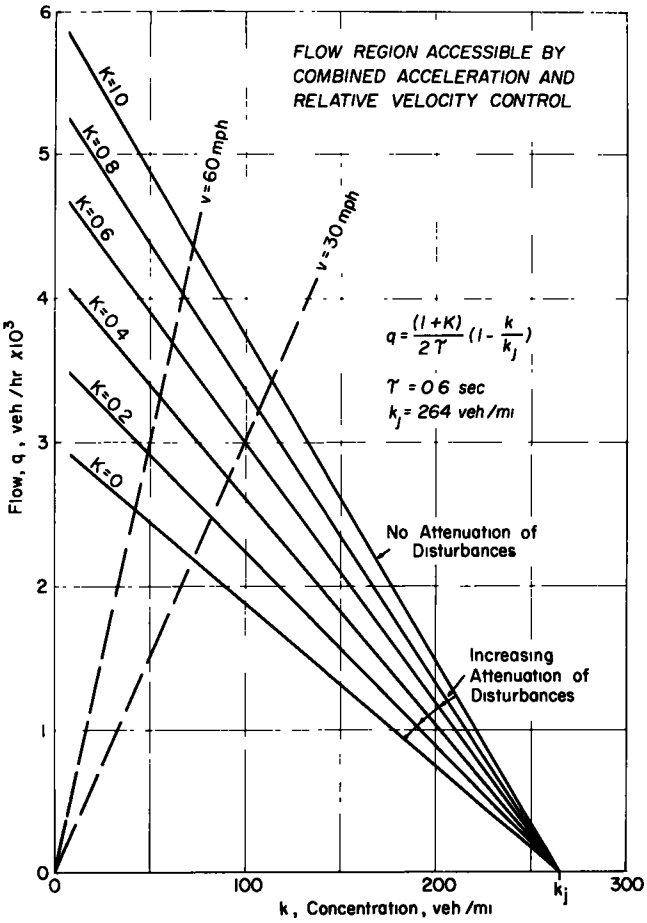

Figure 13. Flow region accessible by combined acceleration and relative velocity control.

per lane represents a potential increase in traffic flow, although volumes of this magnitude (2,400 vph) have been measured in uncontrolled traffic for short periods before a breakdown in traffic flow occurred. It must be realized, however, that the volume of 2,600 vph per lane will be possible continuously at a higher degree of safety if traffic is controlled by the relative velocity system.

Conclusions

The prototype of an IR sensing and communication system has been built to test the feasibility of such a system. Tests were mainly carried out on I 71 and about 900 miles were covered during these tests in daytime and nighttime driving. The system proved to be reliable over a distance of about 400 ft. Further development and tests will be necessary, preferably by a platoon of vehicles equipped with the IR system. So far the results have been encouraging, and it is hoped that minor difficulties with increased background noise could be solved with more powerful or coherent sources.

Theoretical studies indicate that intervehicular communication can be improved by using the IR source-sensor system. The vehicle spacing for platoon movement, controlled by sensing the relative velocity between successive vehicles, or by the acceleration pattern of the lead car, has been determined as the natural spacing. Traffic flow under these conditions will be marginally safe, which means that the maximum possible deceleration determined by the coefficient of friction of the road surface and the efficiency of the braking system can be applied to any vehicle in a platoon without resulting in rear-end collisions, if braking efficiency and the coefficient of friction are the same for all vehicles. In real-world conditions, there will be a variation in braking efficiency and in the coefficient of friction, which will necessitate some spacing in addition to the natural spacing to permit marginally safe traffic flow. No effort has been made to determine these values, and further research will be necessary.

The approximate range in traffic volume accessible by a combined acceleration and relative velocity control system has been determined for different velocities, and it has been found that a substantial increase in traffic volumes can be obtained.

CHAPTER TWO

TAILLIGHT SIGNAL SYSTEMS—DAYTIME STUDIES

Research on the project led to an experimental evaluation of four different intervehicular signal systems. These included the NIL (no light), the conventional, the Tri-light, and the AID or Acceleration Information Display systems. The physical attributes of these displays have been described in Chapter One.

It was felt that the best way to obtain reasonably objective comparisons among the Tri-light, AID, and conventional signal systems was to test each with the same subjects, vehicles, and maneuvers. A Curtis-Wright "co-pilot" was obtained to control the lead vehicle acceleration following a maneuver, thus permitting criteria regarding longitudinal stability during acceleration to be included in the analysis.

Use of NIL as a control condition stemmed clearly from the need to assess the effectiveness of the conventional brake light system.

The display consisted of 5-in. square light units arranged as shown in Figure 4. Conventional brake lights were displayed by the outer red lights, and the Tri-light system used the outer vertical columns of red, amber, and green. The AID system employed the horizontal rows of red and green lights, with the outer reds signaling slight deceleration, the outer two reds a greater deceleration, and so on until all red lights were illuminated for a violent braking maneuver. The green lights functioned in a similar manner for acceleration.

The following vehicle (driven by the subject) was equipped with a take-up reel and recorder to permit continuous recording of headway and relative velocity. Radio communication between vehicles and a headway meter visible to the experimenter in the following car were available for use.

MANEUVERS

The basic experimental procedure placed subjects in a car-following situation while a lead vehicle with the display performed a series of braking and coasting maneuvers. The eight different maneuvers used in the tests are described in Table 1.

The four deceleration rates employed in these maneuvers are described as follows:

1. Coasting was achieved by simply lifting the foot from the gas pedal with the automatic transmission selector in the DRIVE position. This yielded an average deceleration between 65 and 35 mph of 1.9 ft/sec^2. Coasting during the tests was accomplished by disengaging the "co-pilot."
2. Mild braking used a constant hydraulic system pressure of 150 psi and furnished an average deceleration of 4.4 ft/sec^2.
3. Medium braking was at 225 psi, yielding an average deceleration of 6.7 ft/sec^2.
4. Hard braking imposed a pressure reading of 350 psi to provide a deceleration of 13.4 ft/sec^2.

It should be noted that the medium brake is a very solid deceleration, and the hard brake approaches 0.5*g*. Decelerations slightly harder than this appeared to be within the capability of the vehicle, but were avoided to prevent excessive tire and mechanical wear during the experiment. Also, there was the ever-present danger of collision, which was definitely a factor because the study included trials in which no deliberate visual signals were given (NIL).

The design involved four replications of each maneuver under each condition for each subject, which meant 160 total replications of the hard brake maneuver. Collision likelihood increases with sample size, and although decelerations did not exceed 0.5*g* observers were stationed in both vehicles as an added precaution.

The acceleration curves for each of the eight maneuvers are shown in Figure 14. These curves are idealized, in that constant or average acceleration values are shown. In reality one would expect slightly nonlinear velocity changes,

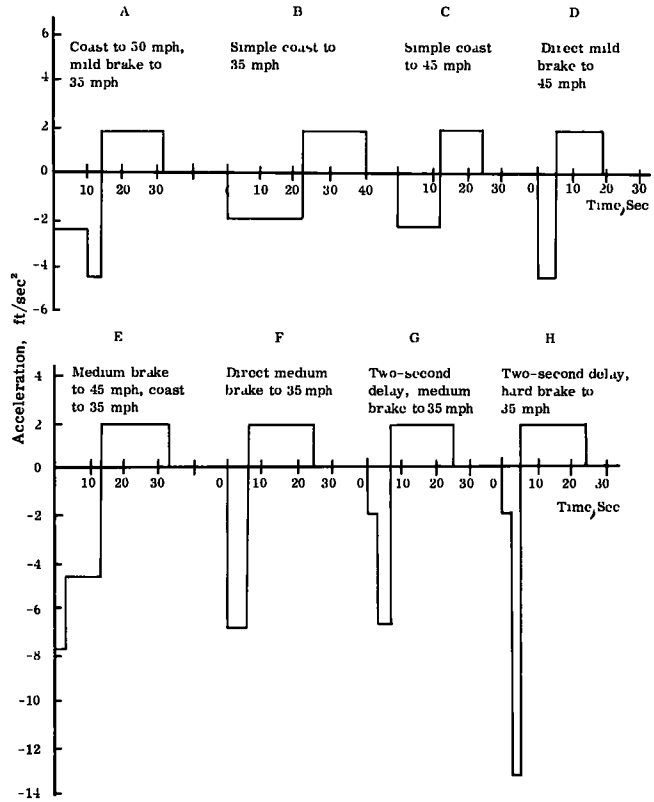

Figure 14. Idealized acceleration curves of leading vehicle for the various maneuvers.

particularly during braking. The vehicle's average acceleration under the "co-pilot" with two passengers was found to be 2.0 ft/sec^2.

SIGNAL SYSTEMS

The Acceleration Information Display (AID) was adjusted so that the accelerations required to illuminate lights on each side of the display were: 1 red pair, —1.25 ft/sec^2; 2 red pairs, —5.23 ft/sec^2; 3 red pairs, —7.86 ft/sec^2; 4 red pairs, —10.68 ft/sec^2; 5 red pairs, —13.20 ft/sec^2; 1 green pair, 1.68 ft/sec^2; 2 green pairs, 3.94 ft/sec^2; 3 green pairs, 5.78 ft/sec^2; 4 green pairs, 6.74 ft/sec^2. It is important to recognize that these are static measures; i.e., the vehicle was at rest during calibration. During the actual maneuvers longitudinal cant acted to lower the thresholds for the red lights, and the four deceleration rates used did in fact produce one to four illuminated red lights, respectively. Nonlinearities in velocity during the hard braking were possible because an initial five lights would change to four as the velocity decreased.

EXPERIMENTAL PROCEDURE

Each subject's run included four replications of each maneuver under each condition. To eliminate glare effects, replications were equally divided between northbound and southbound travel on Interstate Highway 71. The sequence

of maneuvers used in the experiment was randomized, as given in Table 4. The numerical values listed between maneuvers represent suggested random time intervals after the recovery speed of 65 mph was reached by the leading vehicle and before the initiation of the next maneuver. It was found, however, that traffic conditions usually dictated the time interval between maneuvers. As can be seen from Figure 14, the time required for the actual maneuver ranged from 19 to 40 sec.

Initiation of a maneuver did not occur unless all of the following conditions were satisfied:

1. Lead vehicle velocity 65 mph.[1]
2. Relative velocity not exceeding 3 mph.
3. Headway between 180 and 220 ft.

Actual driving time with maneuver presentations for one cycle of the test route was approximately 1 hr. To avoid subject fatigue, subjects participated in two 2-hr sessions on different days. The order of presentation of displays for each subject was the same: AID and NIL in the first session and Tri-light and conventional in the second session. This fixed order of presentation was dictated by a number of reasons. AID and Tri-light were first in each session to permit subject familiarization while driving to the test site. AID preceded Tri-light and conventional to avoid transfer of the meaning of one red light (a coast in AID, but possibly a hard brake in conventional and Tri-light). Also, NIL could not be the first system presented, in view of collision possibilities until the subjects became aware of the severity of the maneuvers.

[1] Observed speed; speedometer checks on the lead vehicle showed a true speed of 58.6 mph at indicated 65 mph, and a true speed of 36.6 mph at indicated 40 mph.

TABLE 4

RANDOMIZED MANEUVER SEQUENCES [a]

AID		NIL		CONV.		TRI-LIGHT	
NORTH	SOUTH	NORTH	SOUTH	NORTH	SOUTH	NORTH	SOUTH
H	B	H	A	A	B	D	C
16	65	98	24	23	94	70	49
C	F	C	D	H	D	E	F
39	37	44	70	3	32	19	58
F	G	F	G	B	E	H	C
26	43	17	81	96	18	42	17
B	D	E	B	D	D	E	A
67	88	51	17	25	86	16	41
E	B	C	E	E	B	A	D
12	14	87	85	78	19	63	52
D	E	H	G	A	G	H	F
31	59	12	05	46	11	67	83
H	D	F	H	E	C	B	D
46	82	17	82	44	9	55	42
G	E	D	F	H	E	G	G
39	31	36	62	55	14	2	64
F	G	B	A	D	F	D	E
75	91	75	56	63	8	4	81
B	F	A	H	B	G	C	B
98	16	21	66	17	52	59	36
A	C	B	B	G	H	A	H
1	86	23	51	37	38	30	51
G	A	E	C	C	C	C	B
13	12	46	11	13	91	8	79
C	H	G	E	G	A	F	H
24	10	16	31	36	6	6	83
D	A	D	D	F	F	G	E
58	59	50	34	96	21	43	72
A	C	A	F	C	A	F	G
92	26	61	94	18	79	97	33
E	H	G	C	F	H	B	A
82	62	90	73	1	13	52	95

[a] Numerical values are suggested random time intervals after recovery speed of 65 mph was reached by leading vehicle and before initiation of next maneuver.

SUBJECT INSTRUCTIONS

The subjects used in this experiment were given the following instructions prior to the actual data collection:

> We would like you to follow the vehicle in front of you at a distance of 200 feet. We will guide you to that distance. After you obtain the distance of 200 feet, we will let you follow the lead car for a while. After this, the lead car will begin a maneuver which will incorporate a deceleration and a subsequent acceleration. During this maneuver we would like you to drive in your normal manner; that is, we would like you to assume that you are following this man on the freeway and wish to avoid a collision, but do not wish to drop too far behind him.
>
> In various phases of the experiment the decelerations and accelerations will be signaled by different types of taillight signaling systems. These will be explained to you later.

Prior to each change in signal system the new system was explained to the subject, and he was shown the new signals.

DATA REDUCTION

The field testing was based on a complete replicate experiment with the following dimensions:

4 Signal systems
8 Maneuvers
9 Subjects (college males)
4 Replications

This means that a total of 1,152 trials were conducted and were used in the data analysis. Each maneuver was identified (along with any comments) on the recorder paper as soon as possible after development.

Initial numerical information removed from the traces for each maneuver included the following:

1. *Period of closure.*—Time interval between initiation of the maneuver and the first return to zero relative velocity.
2. *Gas pedal response time.*—Time interval between initiation of the maneuver and the complete release of the following vehicle's gas pedal.
3. *Gas pedal release interval.*—Time period that the subject's foot remains off the gas pedal.
4. *Initial headway.*—Headway at initiation of maneuver.
5. *Minimum headway.*—Should occur exactly at the end of the period of closure.
6. *Minimum relative velocity or maximum closing velocity.*—A negative quantity, occurring during the period of closure.
7. *Maximum relative velocity or maximum opening*

velocity.—Will occur during the acceleration period following the period of closure.

8. *Minimum velocity.*

These measures were recorded as the initial reduction of the data. An example of traces for a hard brake with delay maneuver under NIL are shown in Figure 15, together with dimensional locations of the eight measures listed in the foregoing. Inasmuch as relative velocity is considered as the derivative of headway, relative velocity during the period of closure is negative and that immediately following is positive.

Initial headways for all trials were between 180 and 220 ft. Statistical tests indicated that there were no significant differences in initial headways between any of the experimental tests. The signal system evaluations should not, therefore, be biased by variations in initial headways.

It is recognized that in an experiment of this kind drivers will be more alert than during normal freeway driving. This alertness was probably present for all trials and signal systems and, accordingly will not affect direct comparison of signal system performance. Absolute values (such as response times) will, however, reflect the experimental conditions and generally be lower than one would expect under normal driving conditions void of psychological set.

In the analysis, an evaluation of the signal systems was completed for each maneuver and performance measure with all subjects and replications pooled. Learning effects, measured by among-replication variations, were examined separately.

PERFORMANCE UNDER LEAD-VEHICLE DECELERATION

It is useful to divide system dynamics during a lead-vehicle maneuver into two phases. The first phase begins with the deceleration of the lead vehicle, and ends when zero relative velocity is regained. This phase is identified in Figure 15 as the period of closure. The second phase includes the remainder of the maneuver, terminating when the lead vehicle regains its initial speed of 65 mph.

The first phase, or period of closure, corresponds roughly to the lead-vehicle deceleration. Interest during this phase is directed toward three measures of car-following performance—response times, average relative velocity, and minimum relative velocity.

Response Time

The time required for a driver to respond to a lead-vehicle deceleration is defined as the interval from the initiation of the lead vehicle's deceleration to the following driver's release of the gas pedal. Although "release of the gas pedal" is not precisely defined, data from all maneuvers in the experiment showed a definite removal of the foot, and no problems were encountered in determining the time of gas pedal response. Mean response times (overall subjects and replications) for the four signal systems are given in Table 5, together with significance levels for the differences between mean response times.

Cursory inspection of Table 5 shows that although both the AID and Tri-light systems decreased response times over the conventional system, the greatest gains were obtained with the Tri-light display.

It is useful to note that the first three maneuvers all began with coasting by the leading vehicle. In this case conventional brake lights gave no signal, whereas an amber light was illuminated in the Tri-light and a single red light in the AID system. Neither NIL nor conventional brake lights signaled the coasting; the shorter response times with the

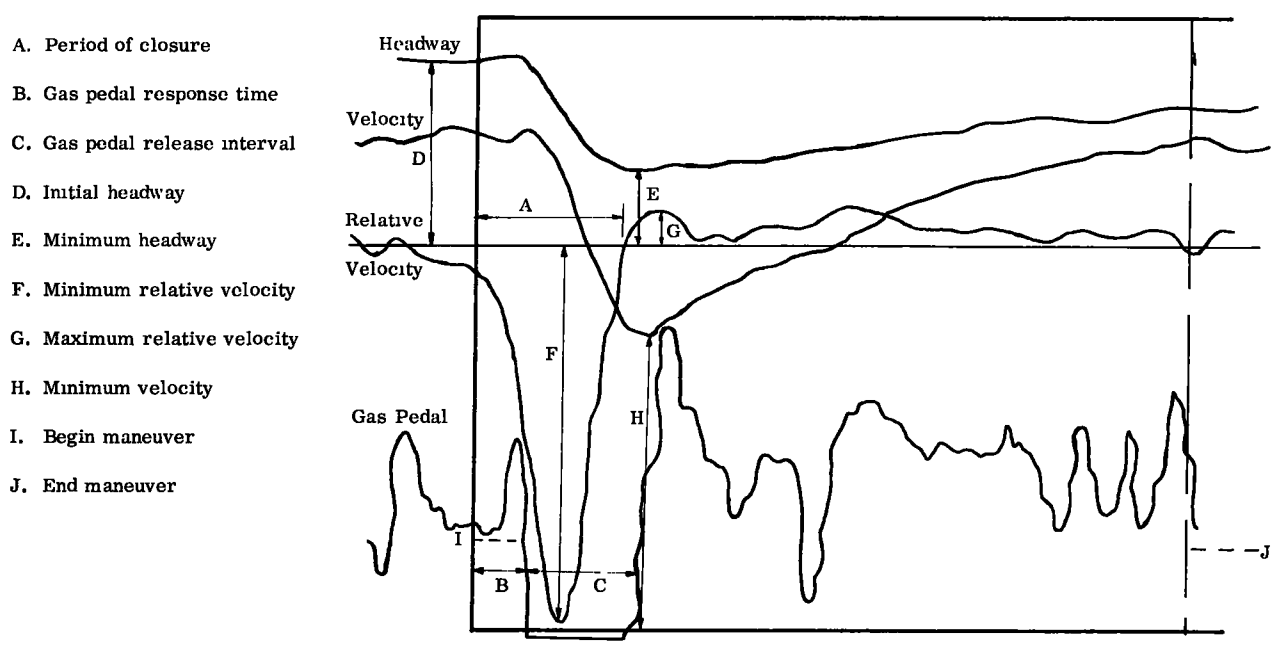

Figure 15. Traces of an actual hard-brake-with-delay maneuver under NIL.

TABLE 5

MEANS AND SIGNIFICANT DIFFERENCES OF RESPONSE TIMES

MANEUVER	MEAN RESPONSE TIME (SEC)				SIGNIFICANT DIFFERENCE			
	NIL	CONV.	AID	TRI-LIGHT	CONV. VS NIL	CONV. VS AID	CONV. VS TRI-LIGHT	AID VS TRI-LIGHT
Coast to 50 mph, mild brake to 35 mph	3.09	3.46	1.17	0.56	NS	AID 0.01	Tri 0.01	Tri 0.01
Coast to 35 mph	3.23	3.49	1.14	0.73	NS	AID 0.01	Tri 0.01	Tri 0.05
Coast to 45 mph	2.69	3.78	0.78	0.39	NIL 0.01	AID 0.01	Tri 0.01	Tri 0.01
Mild brake to 45 mph	1.52	0.31	0.95	0.52	Conv. 0.01	NS	NS	Tri 0.01
Medium brake to 45 mph, coast to 35 mph	1.52	0.75	0.86	0.78	Conv. 0.01	NS	NS	NS
Medium brake to 35 mph	1.44	0.61	0.79	0.57	Conv. 0.01	NS	NS	Tri 0.03
Coast 2 sec, medium brake to 35 mph	2.53	2.14	0.98	0.47	NS	AID 0.01	Tri 0.01	Tri 0.01
Coast 2 sec, hard brake to 35 mph	2.64	2.53	0.91	0.54	NS	AID 0.01	Tri 0.01	Tri 0.02

NIL system probably reflect the subject's increased alertness, knowing that no warning signals would be given for any level of lead-vehicle deceleration. The large difference between performance under conventional and that under AID or Tri-light clearly indicates the benefits obtainable from signaling slight decelerations. Response times under conventional and NIL (where drivers were left to detect decelerations by such visual cues as cant and perceived relative velocity) were in excess of three times those obtained under the AID and Tri-light systems.

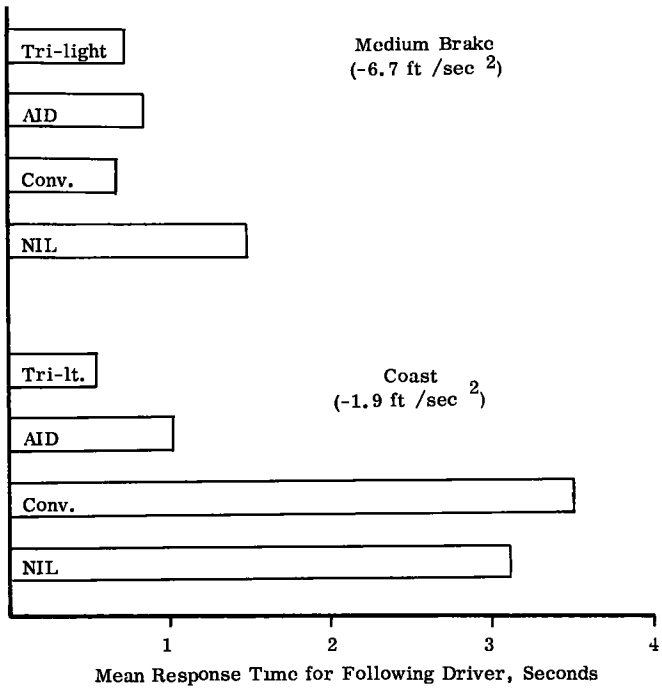

Figure 16. Mean response times for medium brake and for coast.

Figure 16 shows graphically the mean response times obtained for coasting by the lead vehicle, together with response times for a medium braking action by the lead vehicle. Here data from maneuvers 5 and 6 have been pooled, inasmuch as both began with a medium braking action. For these pooled data, NIL is significantly different from the other system, but there are no statistically significant differences among the AID, conventional, and Tri-light systems.

For the response times under coasting, the systems are all significantly different. The "superiority" of Tri-light to AID in this regard should be interpreted with moderation in view of possible interpretation of signals sent by the two systems. For coasting, the AID system's single light signals a low rate of deceleration (not too much cause for alarm), whereas the Tri-light's message that the foot has come off the gas pedal suggests possible subsequent hard braking.

Figure 5 shows response times for the four systems plotted against lead-vehicle deceleration. The curve for the NIL system is of particular interest here, because response times under NIL are largely dependent on a driver's ability to sense relative velocity. The three points for NIL in Figure 5 suggest that response times may become asymptotic to some constant value as lead-vehicle deceleration rates are increased.

Minimum Relative Velocity (Maximum Closing Velocity)

Minimum relative velocity is a negative quantity occurring during the maneuver's period of closure and representing the greatest rate of closure that occurs during the maneuver. As such, it becomes a useful measure of a driver's performance in attempting to maintain stable headway, because it represents the maximum rate of closure that occurs during the maneuver. Table 6, giving average minimum relative velocities (over subjects and replications) for the four signal systems and eight maneuvers, indicates the numerical reduction in maximum closure rates. It should be noted that the AID, Tri-light, and conventional systems

TABLE 6

MEANS AND SIGNIFICANT DIFFERENCES OF MINIMUM RELATIVE VELOCITIES

MANEUVER	MEAN MINIMUM RELATIVE VELOCITY (MPH)				SIGNIFICANT DIFFERENCE			
	NIL	CONV.	AID	TRI-LIGHT	CONV. VS NIL	CONV. VS AID	CONV. VS TRI-LIGHT	AID VS TRI-LIGHT
Coast to 50 mph, mild brake to 35 mph	— 7.34	— 7.35	— 7.06	— 5.54	NS	NS	Tri 0.01	Tri 0.01
Coast to 35 mph	— 5.62	— 6.40	— 4.24	— 3.94	NIL 0.05	AID 0.01	Tri 0.01	NS
Coast to 45 mph	— 5.56	— 6.06	— 3.34	— 3.38	NS	AID 0.01	Tri 0.01	NS
Mild brake to 45 mph	— 8.72	— 5.39	— 6.31	— 4.20	Conv. 0.01	NS	Tri 0.04	Tri 0.01
Medium brake to 45 mph, coast to 35 mph	—12.06	— 8.57	— 8.61	— 8.37	Conv. 0.01	NS	NS	NS
Medium brake to 35 mph	—12.47	— 9.00	— 9.00	— 8.57	Conv. 0.01	NS	NS	NS
Coast 2 sec, medium brake to 35 mph	—13.20	—10.00	— 9.12	— 7.53	Conv. 0.01	NS	Tri 0.01	Tri 0.01
Coast 2 sec, hard brake to 35 mph	—16.37	—13.89	—11.91	—11.29	Conv. 0.01	AID 0.01	Tri 0.01	Tri. 0.09

tend not to be significantly different for the maneuvers that begin with a braking action (viz., 4, 5, and 6) although the Tri-light continues to yield lower relative velocities. Once again, these results suggest the advantage obtainable from signaling small decelerations.

Average Relative Velocity During Closure

Another useful measure of longitudinal stability in car-following is average relative velocity during the period of closure.[2] This permits a measure of a driver's performance throughout the lead-vehicle's deceleration. Although not independent of minimum relative velocity, this average value represents over-all performance instead of extremes.

Average relative velocity during the period of closure is simply the area under the relative velocity curve during the period of closure, divided by the length of the period of closure; i.e.,

$$\overline{\mathrm{rv}} = \frac{1}{T} \int_{t=0}^{T} \mathrm{rv}\,(t)\,dt \qquad (1)$$

in which

$$\mathrm{rv}\,(t) < 0, 0 < t < T; \qquad (2a)$$

$$\mathrm{rv}\,(t) = 0, t = T \qquad (2b)$$

Actual computation of average relative velocity is simplified by using the headway trace. Because

$$\frac{dH\,(t)}{dt} = \mathrm{rv}(t) \qquad (3)$$

Eq. 1 becomes

$$\overline{\mathrm{rv}} = \frac{1}{T}\,(H(T) - H(0)) \qquad (4)$$

Accordingly, average relative velocity during the closure period may be computed by dividing the difference between initial and final headway during the period of closure by the period of closure.

Table 7 gives the means and significant differences obtained using this measure. Here again the AID and Tri-light systems tend to significantly reduce relative velocities when a maneuver involves initial coasting. The value of giving any sort of signal is apparent for the conventional system in maneuvers 3 and 4, where going from a coast to a mild brake (with brake light) actually produced a decrease in average relative velocity. Generally, this performance measure shows little comparative difference between AID and the Tri-light system.

PERFORMANCE UNDER LEAD-VEHICLE ACCELERATION

The previous sections discussed car-following performance during deceleration by the leading vehicle. This phase of car-following is particularly important in terms of collision likelihood, both between the two cars studied and later in a platoon.

Attention is now transferred to car-following performance during acceleration by the lead vehicle. This phase is possibly less directly important from a safety standpoint, but more important in terms of traffic volumes. Accordingly, we again assume car-following performance to be improved when relative velocity is reduced during a lead-vehicle's acceleration.

In accordance with the foregoing, two measures of car-following performance are considered. The first is acceleration response time, a measure of a driver's performance in detecting, interpreting, and taking action (depressed accelerator) in response to the lead-vehicle's acceleration. A second measure is the maximum relative velocity that occurs during the maneuver. This usually occurs just after the end of the period of closure, when the lead vehicle has started accelerating and the following driver may be still decelerating. Performance of the four signal systems in terms of these measures is discussed in the following.

[2] A commonly used measure of stability related to this is mean square relative velocity. However, there is no need in this analysis to resort to mean square measures, inasmuch as the relative velocity trace being averaged is not positive, and canceling cannot occur.

TABLE 7

MEANS AND SIGNIFICANT DIFFERENCES OF AVERAGE RELATIVE VELOCITIES

	MEAN AVERAGE RELATIVE VELOCITY (MPH)				SIGNIFICANT DIFFERENCE			
MANEUVER	NIL	CONV.	AID	TRI-LIGHT	CONV. VS NIL	CONV. VS AID	CONV. VS TRI-LIGHT	AID VS TRI-LIGHT
Coast to 50 mph, mild brake to 35 mph	—3.18	—3.35	—2.38	—1.88	NS	AID 0.01	Tri 0.01	Tri 0.09
Coast to 35 mph	—2.02	—2.72	—1.52	—1.92	NIL 0.01	AID 0.01	Tri 0.01	AID 0.06
Coast to 45 mph	—2.44	—2.78	—1.18	—1.38	NIL 0.07	AID 0.01	Tri 0.01	NS
Mild brake to 45 mph	—3.93	—2.26	—2.94	—2.14	Conv. 0.01	Conv. 0.03	NS	Tri 0.01
Medium brake to 45 mph, coast to 35 mph	—4.30	—3.42	—3.14	—3.29	Conv. 0.01	NS	NS	NS
Medium brake to 35 mph	—6.12	—4.36	—4.11	—4.30	Conv. 0.01	NS	NS	NS
Coast 2 sec, medium brake to 35 mph	—5.38	—4.57	—3.62	—3.14	Conv. 0.01	AID 0.01	Tri 0.01	NS
Coast 2 sec, hard brake to 35 mph	—6.02	—5.39	—4.20	—3.97	Conv. 0.05	AID 0.01	Tri 0.01	NS

Acceleration Response Time

Acceleration response time represents the time interval from the lead-vehicle's acceleration until the following driver steps on the gas pedal. The latter event is easily noted on the traces shown in Figure 15; it is a straightforward matter to record the instant the gas pedal was depressed in response to acceleration. Identifying the point in time at which the leading vehicle began accelerating is, however, a more difficult task.

It was pointed out earlier that the point of zero relative velocity (marking the end of the period of closure) will not generally correspond to the initiation of the lead-vehicle acceleration. Further, the lead-vehicle's velocity patterns do not directly appear on the recorder paper. To obtain the lead-vehicle's velocity pattern (and thereby its initial acceleration point) it is necessary to reconstruct the pattern by algebraically adding the following-vehicle's velocity function, $v_2(t)$, to the relative velocity, $[v_1(t) - v_2(t)]$. From the $v_1(t)$ trace, the initial point of the leading-vehicle's acceleration may be immediately located. Because lead-vehicle decelerations were controlled by a brake system pressure gauge, the location of the initial point of acceleration varied little among replications of a given maneuver and was, of course, independent of the signal system. For analysis, lead-vehicle velocity traces for 16 replications of each maneuver were reconstructed and the average time intervals from the beginning of the maneuver until lead-vehicle accelerations began were computed. For a given trial, then, this value was subtracted from the time interval until the gas pedal was depressed, yielding acceleration response time.

Table 8 gives means and significant differences of acceleration response times for the eight maneuvers. For this measure, the AID system tends to show a slight advantage over the Tri-light.

One would expect improved performance (reduced response time) when the lead vehicle signals its accelerations. For maneuvers ending in coasting, for example, one would

TABLE 8

MEANS AND SIGNIFICANT DIFFERENCES FOR ACCELERATION RESPONSE TIMES

	MEAN ACCELERATION RESPONSE TIME (SEC)				SIGNIFICANT DIFFERENCE			
MANEUVER	NIL	CONV.	AID	TRI-LIGHT	CONV. VS NIL	CONV. VS AID	CONV. VS TRI-LIGHT	AID VS TRI-LIGHT
Coast to 50 mph, mild brake to 35 mph	2.21	1.21	1.39	0.67	Conv. 0.03	NS	Tri 0.09	Tri 0.02
Coast to 35 mph	1.93	2.71	0.51	2.10	NS	AID 0.01	NS	AID 0.02
Coast to 45 mph	1.59	1.49	0.63	1.14	NS	AID 0.01	NS	NS
Mild brake to 45 mph	2.68	1.52	2.14	1.63	Conv. 0.01	Conv. 0.03	NS	Tri 0.05
Medium brake to 45 mph, coast to 35 mph	1.94	1.88	0.54	0.70	NS	AID 0.01	Tri 0.01	NS
Medium brake to 35 mph	2.59	2.25	2.37	2.84	NS	NS	NS	AID 0.09
Coast 2 sec, medium brake to 35 mph	2.49	2.25	2.08	2.51	NS	NS	NS	AID 0.08
Coast 2 sec, hard brake to 35 mph	2.98	3.41	2.26	2.58	NS	AID 0.01	Tri 0.01	NS

not expect significant differences between NIL and conventional brake lights, because neither signals acceleration (or, for that matter, the end of deceleration). Looking at maneuvers 2, 3, and 5, all of which end with coasting, no significant difference is found between NIL and brake lights. Performance under AID for these three "coast-before-acceleration" maneuvers represents a significant improvement over conventional brake lights. Performance gains under the Tri-light system for the same three maneuvers are not as great. This suggests that signaling the magnitude of acceleration might be preferable to simply displaying that the gas pedal has been depressed.

The results for the other maneuvers are difficult to interpret. Maneuvers 1 and 4 end in mild braking: here the Tri-light system appears numerically superior. The remaining maneuvers (6, 7, and 8) end in medium and hard braking and show advantages to the AID system. A more complete appraisal of performance during the acceleration period can be given in terms of maximum relative velocities.

Maximum Relative Velocity (Maximum Opening Velocity)

Maximum relative velocity during the lead-vehicle's acceleration forms a partial measure of the following-driver's performance in interpreting and following the lead-vehicle's acceleration. It should be pointed out that motivation for good following-driver performance (reducing relative velocity) during acceleration is less than during lead-vehicle deceleration. In the latter case, collision likelihoods help motivate the driver, whereas such encouragement does not directly appear during acceleration.

Table 9 gives means and significant differences of maximum relative velocities for the four systems tested. In maneuvers 2, 3, and 5, the acceleration is preceded by coasting, and for these maneuvers the data show no difference between NIL and conventional brake lights. This is to be expected, because under these conditions neither system signals acceleration or termination of deceleration. Benefits from the AID system for these maneuvers are marked, as shown in the table. For other maneuvers (ending in braking) results are mixed, but in general the AID system appears to be the best performer for this performance measure.

SIGNAL SYSTEM PERFORMANCE COMPARISONS

The evaluation of maneuver data for the NIL, conventional, AID, and Tri-light signal systems was based on five measures of car-following performance, as follows:

1. Time for gas pedal response to lead-vehicle deceleration.
2. Minimum relative velocity during lead-vehicle deceleration.
3. Average relative velocity during the period of closure.
4. Time for response to the lead-vehicle acceleration.
5. Maximum relative velocity during the lead-vehicle's acceleration.

It is convenient to recompile the results in terms of statistically significant performance differences between systems as described by these measures.

Conventional vs NIL

Table 10 gives an assessment of the effectiveness of conventional brake lights by comparing the NIL and conventional signal systems. If one system yielded a statistically significant improvement in performance, it is listed together with the corresponding significance level. Differences at and above the 10 percent level are considered not significant.

Initially, confining attention to performance during lead-vehicle deceleration (i.e., to the first three performance measures), NIL is seen as yielding slightly better performance during coasting (maneuvers 2 and 3). This is probably due to increased alertness for NIL, because neither system gives a signal for coasting. When the coast is shortened slightly and followed by a braking action (maneuver

TABLE 9

MEANS AND SIGNIFICANT DIFFERENCES FOR MAXIMUM RELATIVE VELOCITIES OR MAXIMUM OPENING VELOCITY

	MEAN MAXIMUM RELATIVE VELOCITY (MPH)				SIGNIFICANT DIFFERENCE			
MANEUVER	NIL	CONV.	AID	TRI-LIGHT	CONV. VS NIL	CONV. VS AID	CONV. VS TRI-LIGHT	AID VS TRI-LIGHT
Coast to 50 mph, mild brake to 35 mph	4.78	4.75	4.03	4.35	NS	AID 0.08	NS	NS
Coast to 35 mph	5.13	5.17	4.31	4.39	NS	AID 0.04	Tri 0.05	NS
Coast to 45 mph	5.22	5.04	3.75	3.48	NS	AID 0.01	Tri 0.01	NS
Mild brake to 45 mph	5.08	4.22	3.68	4.38	Conv. 0.06	NS	NS	NS
Medium brake to 45 mph, coast to 35 mph	5.47	5.53	4.47	4.83	NS	AID 0.01	NS	NS
Medium brake to 35 mph	5.20	4.25	4.33	4.86	Conv. 0.02	NS	NS	NS
Coast 2 sec, medium brake to 35 mph	4.46	4.83	3.63	4.40	NS	AID 0.01	NS	AID 0.09
Coast 2 sec, hard brake to 35 mph	4.56	5.41	4.35	4.13	NS	AID 0.06	Tri 0.01	NS

TABLE 10

STATISTICAL COMPARISON FOR CONVENTIONAL BRAKE LIGHT SYSTEM AND NIL

	MEASURES UNDER DECELERATION			MEASURES UNDER ACCELERATION	
MANEUVER	RESPONSE TIME	MIN. REL. VEL.	AVG. REL. VEL.	MAX. REL. VEL.	ACCEL. RESP. TIME
Coast to 50 mph, mild brake to 35 mph	NS	NS	NS	NS	Conv. 3%
Coast to 35 mph	NS	NIL 5%	NIL 1%	NS	NS
Coast to 45 mph	NIL 1%	NS	NIL 7%	NS	NS
Mild brake to 45 mph	Conv. 1%	Conv. 1%	Conv. 1%	Conv. 6%	Conv. 1%
Medium brake to 45 mph, coast to 35 mph	Conv. 1%	Conv. 1%	Conv. 1%	NS	NS
Medium brake to 35 mph	Conv. 1%	Conv. 1%	Conv. 1%	Conv. 2%	NS
Coast 2 sec, medium brake to 35 mph	NS	Conv. 1%	Conv. 1%	NS	NS
Coast 2 sec, hard brake to 35 mph	NS	Conv. 1%	Conv. 5%	NS	NS

1), no significant difference is seen in the systems. When the coasting is further reduced to 2 sec (maneuvers 7 and 8), conventional brake lights yielded significantly improved performance in terms of relative velocities. Initial braking by the lead vehicle (maneuvers 4, 5, and 6) results in superior performance by conventional brake lights for response time as well as relative velocities.

System comparison during the acceleration phase of the maneuver suggests a slight advantage to conventional brake lights.

Conventional vs AID

Table 11 gives a statistical summary of performance comparisons between conventional brake lights and AID. During lead-vehicle deceleration, the AID system yields superior performance for all maneuvers which began with a coast. For maneuvers beginning with braking, there is little difference in performance under the two systems. During lead-vehicle acceleration, the AID system generally produces gains in car-following performance.

Conventional vs Tri-light

A comparison of conventional brake lights with the Tri-light system is given in Table 12. The advantage of signaling gas pedal information during deceleration is evident for both the maneuver beginning with coasting and the short mild braking maneuver. The numerical magnitudes of these differences have been discussed earlier. The Tri-light system also shows a small advantage during the acceleration phase of the maneuver.

Tri-light vs AID

The foregoing sections have tended to indicate rather clearly the advantage given by both the AID system and the Tri-light system over conventional brake lights. It is of particular interest to compare performances under the two special systems.

The significant performance differences between the AID and the Tri-light system are given in Table 13. In terms of response to deceleration (braking or coasting), the Tri-light system appears to give better performance. For the relative velocity measures during deceleration, results tend to be indifferent, although the Tri-light system yielded better performance on those maneuvers (1, 7, and 8) in which a coasting action preceded braking.

For the acceleration phase of the maneuvers, no difference was found between the systems for maximum relative velocity. In terms of the time to respond to acceleration, however, the AID system shows a slight advantage. It might be concluded from this that the systems perform equally well in terms of relative velocities, with the Tri-light system yielding a significantly quicker response to deceleration.

It should be noted, of course, that the selection of maneuvers used may bias a Tri-light-AID comparison in favor of the Tri-light system. The intuitive appeal of the Tri-light system lies in the warning (amber for released gas pedal) that it gives following drivers for mild deceleration or prior to violent braking. The maneuvers used included a number of initial coasting actions of this sort.

The *a priori* value of acceleration signals, on the other hand, lies in the ability to signal actual vehicle dynamics as opposed to driver control action. This is particularly important over changes in highway vertical curvature. On an upgrade, for example, a slowing vehicle might have the gas pedal depressed, giving, in the case of the Tri-light system, misleading information. A similar situation can exist on downgrades, where a vehicle may be accelerating while the brake pedal is depressed. Here, both conventional and Tri-light systems give misleading information. One of the advantages of acceleration signals is that they

TABLE 11

STATISTICAL COMPARISON FOR CONVENTIONAL BRAKE LIGHT AND AID SYSTEMS

	MEASURES UNDER DECELERATION			MEASURES UNDER ACCELERATION	
MANEUVER	RESPONSE TIME	MIN. REL. VEL.	AVG. REL. VEL.	MAX. REL. VEL.	ACCEL. RESP. TIME
Coast to 50 mph, mild brake to 35 mph	AID 1%	NS	AID 1%	AID 8%	NS
Coast to 35 mph	AID 1%	AID 1%	AID 1%	AID 4%	AID 1%
Coast to 45 mph	AID 1%	AID 1%	AID 1%	AID 1%	AID 1%
Mild brake to 45 mph	NS	NS	Conv. 3%	NS	Conv.3%
Medium brake to 45 mph, coast to 35 mph	NS	NS	NS	AID 1%	AID 1%
Medium brake to 35 mph	NS	NS	NS	NS	NS
Coast 2 sec, medium brake to 35 mph	AID 1%	NS	AID 1%	AID 1%	NS
Coast 2 sec, hard brake to 35 mph	AID 1%	AID 1%	AID 1%	AID 6%	AID 1%

TABLE 12

STATISTICAL COMPARISON FOR CONVENTIONAL BRAKE LIGHT AND TRI-LIGHT SYSTEMS

	MEASURES UNDER DECELERATION			MEASURES UNDER ACCELERATION	
MANEUVER	RESPONSE TIME	MIN. REL. VEL.	AVG. REL. VEL.	MAX. REL. VEL.	ACCEL. RESP. TIME
Coast to 50 mph, mild brake to 35 mph	Tri 1%	Tri 1%	Tri 1%	NS	Tri 9%
Coast to 35 mph	Tri 1%	Tri 1%	Tri 1%	Tri 5%	NS
Coast to 45 mph	Tri 1%	Tri 1%	Tri 1%	Tri 1%	NS
Mild brake to 45 mph	NS	Tri 4%	NS	NS	NS
Medium brake to 45 mph, coast to 35 mph	NS	NS	NS	NS	Tri 1%
Medium brake to 35 mph	NS	NS	NS	NS	NS
Coast 2 sec, medium brake to 35 mph	Tri 1%	Tri 1%	Tri 1%	NS	NS
Coast 2 sec, hard brake to 35 mph	Tri 1%	Tri 1%	Tri 1%	Tri 1%	Tri 1%

correct the situation. It is probable that car-following studies on moderate to high vertical slopes would show a real advantage of the AID system over the Tri-light.

LEARNING EFFECTS

Subjects who participated in the experiment drove in two half-day testing sessions on separate days. In the first session the subject drove behind the AID system to the test site (25 min) and was then tested with AID and the NIL. The second day's session included trials under conventional brake lights and the Tri-light system, and on that day the latter system was activated during the drive to the site, permitting subject familiarization.

It was hypothesized that 25 min of driving under the new signal system might not provide adequate familiarization, in which case learning and performance improvement might continue during the actual experiment. To test for this effect, first and last replications for each system-maneuver combination were compared. Statistically significant learning effects (at the 1% level) were found, as follows:

1. Both the Tri-light and the AID systems showed reduction in response time during the experiment.
2. Performance under the AID system improved in terms of minimum relative velocity.
3. Minimum relative velocities under the NIL system became more negative during the experiment.

TABLE 13

STATISTICAL COMPARISON FOR TRI-LIGHT AND AID SYSTEMS

	MEASURES UNDER DECELERATION			MEASURES UNDER ACCELERATION	
MANEUVER	RESPONSE TIME	MIN. REL. VEL.	AVG. REL. VEL.	MAX. REL. VEL.	ACCEL. RESP. TIME
Coast to 50 mph, mild brake to 35 mph	Tri 1%	Tri 1%	Tri 9%	NS	Tri 2%
Coast to 35 mph	Tri 5%	NS	AID 6%	NS	AID 2%
Coast to 45 mph	Tri 1%	NS	NS	NS	NS
Mild brake to 45 mph	Tri 1%	Tri 1%	Tri 1%	NS	Tri 5%
Medium brake to 45 mph, coast to 35 mph	NS	NS	NS	NS	NS
Medium brake to 35 mph	Tri 3%	NS	NS	NS	AID 9%
Coast 2 sec, medium brake to 35 mph	Tri 1%	Tri 1%	NS	AID 9%	AID 8%
Coast 2 sec, hard brake to 35 mph	Tri 2%	Tri 9%	NS	NS	NS

This last effect is somewhat surprising and may reflect a subject's reversion from extreme initial caution when driving with the NIL condition.

Improved performance through learning occurred in response times for both AID and Tri-light, and earlier results of comparisons of the two systems were unaffected.

Performance gains in minimum relative velocity under the AID system may reflect a driver's learning to relate the number of red lights illuminated on the display to the lead-vehicle's deceleration. Because the AID system presents far more information than the other systems tested, this continued performance gain is not surprising. In order to compensate in part for this learning in the AID system, the comparative analysis between AID and Tri-light was repeated using only data for the last (fourth) replication. Table 14 gives the means and identifies significant differences. Results of the all-replication analysis from Table 13 are also included in Table 14 for comparison. Although the fourth replication data show a greater variance of sample means due to the reduction in sample size from 36 to 9, the improved performance under AID in reducing relative velocity is evident. More importantly, over all trials the Tri-light system gave significantly better performance for four maneuvers, whereas on the fourth replication the Tri-light was significantly different for only one maneuver out of the eight.

The foregoing suggests that as subjects gain experience with the AID system their performance in reducing relative velocity improves at a greater rate than with the Tri-light system. The data, however, show that the greatest perform-

TABLE 14

MEAN MINIMUM RELATIVE VELOCITIES FOR AID AND TRI-LIGHT, ALL REPLICATIONS AND FOURTH REPLICATION

	ALL REPLICATIONS [a]			FOURTH REPLICATION [b]		
MANEUVER	AID	TRI-LIGHT	SIG. DIFF.	AID	TRI-LIGHT	SIG. DIFF.
Coast to 50 mph, mild brake to 35 mph	− 7.06	− 5.54	Tri 1%	− 5.97	− 5.49	NS
Coast to 35 mph	− 4.24	− 3.94	NS	− 3.91	− 3.70	NS
Coast to 45 mph	− 3.34	− 3.38	NS	− 2.95	− 3.77	NS
Mild brake to 45 mph	− 6.31	− 4.20	Tri 1%	− 6.24	− 4.25	Tri 1%
Medium brake to 45 mph	− 8.61	− 8.37	NS	− 8.85	−10.15	NS
Medium brake to 35 mph	− 9.00	− 8.57	NS	− 8.09	− 8.37	NS
Coast 2 sec, medium brake to 35 mph	− 9.12	− 7.53	Tri 1%	− 8.44	− 7.75	NS
Coast 2 sec, hard brake to 35 mph	−11.91	−11.29	Tri 9%	−11.94	−12.14	NS

[a] $N = 36$. [b] $N = 9$.

ance gain came with the second replication, with little change in the third and fourth replication. Figure 17 shows the fractional performance gain in minimum relative velocity under AID as a function of replication. Extrapolation of this curve would suggest little change if later trials were performed.

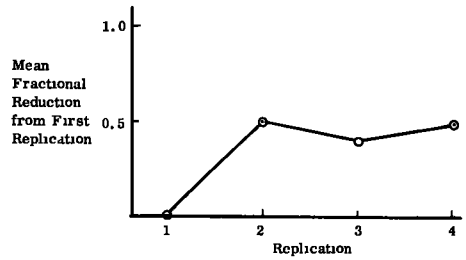

Figure 17. Fractional minimum relative velocity change under AID for 1 to 4 replications (subjects and maneuvers pooled).

COMMENTS ON CRITERIA USED IN EVALUATION

The same eight-maneuver evaluation technique used in the research presented in this chapter was also used in research discussed in Chapters Three and Four. It seems appropriate at this time to compare various performance measures which were previously defined and presented earlier in the analysis.

The performance measures can be broken into two groups: (1) criteria which measure performance during lead-car deceleration, and (2) criteria which measure performance under lead-vehicle acceleration. Those measures which fall in the first group include:

1. Response times.
2. Average relative velocities.
3. Minimum relative velocities or maximum closing velocity.
4. Percent reduction in headway occurring during a maneuver.

Included in the second group are the following measures:

1. Acceleration response time.
2. The maximum relative velocity or maximum opening velocity.

The response time that occurs during the deceleration phase of the maneuver must be considered the primary measure, both from a safety standpoint and by virtue of the fact that all of the other measures are at least partially related to this initial response time. The fact that all of the measures are related to response time would lead to the hypothesis that any signal system that furnishes information which results in reduced response times would prove to be the preferred system. This is supported by Table 15, which presents the response times under the coast-two-seconds-medium-brake-to-35-mph maneuver. The direction of increase is indicated by the response time column for the four signal systems. As can be seen, the general direction of increase for the other measures under deceleration is consistent with the direction of the response time measure.

Acceleration response time does not follow in exactly the same manner as the initial response time, although maximum opening velocity follows acceleration response time closely. This is probably due to the fact that the driver's decision to let the headway open up before accelerating is highly variable as opposed to his reluctance to let the headway close during initial lead-car deceleration.

It can be said that even though the individual performance measures are related, they provide a different outlook on the effects of the signal system. For example, it has been shown that minimum relative velocity or maximum closing velocity is closely related to response time, but the information about the maximum closing velocity is useful in itself.

SUMMARY

Four alternative rear-end automotive signal systems were tested and compared in terms of their performance in car-following situations. The experimental procedure allowed the display-equipped leading vehicle to go through a programmed sequence of braking, coasting, and acceleration maneuvers, and performance was measured by response times, headway reduction, and relative velocities. The four systems compared included conventional brake lights, a Tri-light system signaling gas pedal information, a display of acceleration magnitudes (AID), and a condition in which no information was displayed (NIL). Results of the more than 1,100 highway trials performed indicate that car-following performance is significantly and substantially improved when conventional brake lights are replaced by the acceleration or gas pedal signals. When a vehicle begins coasting from 65 mph, for example, following-driver response times with the conventional systems were found to be four times greater than those obtained with the Tri-light system.

When performance under two special systems (AID and Tri-light) were compared, it was found that the Tri-light system gave slightly better performance during lead-vehicle deceleration, especially when a coast preceded a braking maneuver, and that both systems performed equally well during lead-vehicle accelerations.

In general, this study has demonstrated that there do exist

TABLE 15

COMPARISON OF MEAN VALUES OF PERFORMANCE MEASURES UNDER THE COAST-TWO-SECONDS-MEDIUM-BRAKE-TO-35-MPH MANEUVER

	MEASURES UNDER DECELERATION			MEASURES UNDER ACCELERATION	
SIGNAL SYSTEM	RESP. TIME	MIN. REL. VEL.	AVG. REL. VEL.	MAX. REL. VEL.	ACCEL. RESP. TIME
Tri-light	0.47	— 7.53	—3.14	2.51	4.40
AID	0.98	— 9.12	—3.62	2.08	3.63
Conv.	2.14	—10.00	—4.57	2.25	4.83
NIL	2.53	—13.20	—5.38	2.49	4.46

signals of gas pedal information and acceleration magnitudes which furnish significant improvements over conventional brake lights in terms of car-following performance. These results must not be generalized beyond the conditions of this experiment on the subjects used. Young male drivers under such experimental conditions would probably not be representative of the driving population. However, because each driver served as his own control, the prediction is made that the same relative gain in performance from the Tri-light and AID systems would hold, although the magnitude of performance measures (e.g., response times) would be larger.

CHAPTER THREE

TAILLIGHT SIGNAL SYSTEMS—NIGHTTIME STUDIES

The three intervehicular signal systems tested in this portion of the experiment included the conventional system, the Tri-light system, and the AID system.

Descriptions of the three systems tested, together with details (e.g., instructions) of the eight-maneuver evaluation technique employed in the experimentation, are reported in Chapter Two. Exceptions to this procedure include the following:

1. Deletion of the NIL or "no light" condition (for safety reasons).
2. The addition of small running lights on the outside of the display (for safety reasons).
3. A change of time in experimentation from daytime to nighttime (experimentation was conducted from 12:00 midnight to 4:00 AM.)
4. Neglect of the "north-south" effect (which had originally been considered to compensate for the effects of glare on the taillight display).

DATA COLLECTED

This experiment tested the three signal systems under eight maneuvers. Each maneuver was repeated twice for the Tri-light and conventional signal systems and four times for the AID systems. Complete data were obtained for five subjects, which means that the total number of trials conducted was 320.

The data were collected in a manner similar to that employed in the daytime portion of the experiment. The same measures were obtained from the data as before, and similar methods of analysis were employed. The results of the analyses are presented in the following.

PERFORMANCE UNDER LEAD-VEHICLE DECELERATION

As was done in the analysis of the data collected during the daytime study, the major emphasis during lead-vehicle decelerations is placed on response times, average relative velocities, minimum relative velocities, and percent reduction in headway.

Response Time

Table 16 presents the levels of significance for the differences in the mean response times as calculated over all subjects and all trials. It reveals that both the AID and Tri-light systems resulted in significantly lower response times when compared to the conventional system.

As was found in the daytime study, the AID and Tri-light systems, in general, result in lower response times on the part of the subject driver. This is not too surprising because the AID and Tri-light systems signal coasting maneuvers and also signal the time between the release of the gas pedal and the depressing of the brake pedal.

It is also interesting to note that the few apparent significant differences that are found when the AID and Tri-light system are compared tend to favor the Tri-light system. This difference is possibly due to the fact that the Tri-light system requires less interpretation on the part of the subject.

Minimum Relative Velocity (Maximum Closing Velocity)

Table 17 gives the significant differences in taillight systems as reflected in the minimum relative velocity exhibited by the subject. Minimum relative velocity can be considered a measure of driver performance in the car-following situation. As such, driver performance is considered to be improved when the minimum relative velocity exhibited by the following drivers was decreased in magnitude. Table 17 shows significant differences in magnitude of minimum relative velocities.

The table indicates that there is a slight advantage in the AID and Tri-light systems when compared to the conventional system, if it is deemed desirable to minimize the magnitude of minimum relative velocities. Comparison of the Tri-light system with the AID system indicates that the Tri-light system is slightly better for those maneuvers in which a coast precedes mild deceleration.

Average Relative Velocity During Closure

Table 18 presents the significant differences that were noticed when the average relative velocity during the period

TABLE 16

SIGNIFICANT DIFFERENCES IN RESPONSE TIMES BETWEEN SIGNAL SYSTEMS TESTED

MANEUVER	MEAN RESPONSE TIME (SEC) CONV.	TRI-LIGHT	AID	SIGNIFICANT DIFFERENCE CONV. VS TRI-LIGHT	CONV. VS AID	AID VS TRI-LIGHT
Coast to 50 mph, mild brake to 35 mph	3.35	1.01	1.20	Tri 0.01	AID 0.01	NS
Coast to 35 mph	3.79	2.22	1.70	Tri 0.06	AID 0.01	NS
Coast to 45 mph	2.85	0.74	1.55	Tri 0.01	AID 0.02	Tri 0.01
Mild brake to 45 mph	0.80	1.72	1.31	Conv. 0.07	Conv. 0.01	NS
Medium brake to 45 mph, coast to 35 mph	1.53	0.91	1.13	NS	NS	NS
Medium brake to 35 mph	0.82	0.81	1.28	NS	Conv. 0.04	Tri 0.015
Coast 2 sec, medium brake to 35 mph	3.13	1.26	1.19	Tri 0.01	AID 0.01	NS
Coast 2 sec, hard brake to 35 mph	2.54	0.97	1.05	Tri 0.01	AID 0.01	NS

of closure was compared for the different maneuvers and signal systems. The measure of average relative velocity throughout the period of closure indicates that both the AID and the Tri-light systems result in lower average relative velocities during this period. These results confirm the daytime results. No definite differences can be seen when the AID and the Tri-light systems are compared.

Percent Reduction In Headway

Percent reduction in headway for a maneuver is defined as: (Initial headway-Minimum headway) / Initial headway. This measure, which is a function of average relative velocity during the period of closure and of response times, serves as a measure of driver performance during the deceleration phase of a maneuver. The performance indicated by this measure is important from both a safety and a traffic flow viewpoint. Table 19 gives the levels of significance of this measure when the three signal systems are compared.

This analysis indicates that both the AID and Tri-light systems help to reduce the percent reduction in headway that occurs in maneuvers of the type employed in the experiment. There does not appear to be any major difference between AID and Tri-light with respect to this particular measure.

PERFORMANCE UNDER LEAD-VEHICLE ACCELERATION

Chaper Two presents a discussion of the importance of the performance of a subject driver in car-following when the lead vehicle is accelerating following a deceleration.

TABLE 17

SIGNIFICANT DIFFERENCES OF MINIMUM RELATIVE VELOCITIES

MANEUVER	MEAN MIN. RELATIVE VEL. (MPH) CONV.	TRI-LIGHT	AID	SIGNIFICANT DIFFERENCE CONV. VS TRI-LIGHT	CONV. VS AID	AID VS TRI-LIGHT
Coast to 50 mph, mild brake to 35 mph	— 6.18	— 5.88	— 6.88	NS	Conv. 0.08	Tri 0.065
Coast to 35 mph	— 6.45	— 4.91	— 3.61	Tri 0.045	AID 0.01	AID 0.01
Coast to 45 mph	— 6.12	— 4.00	— 2.91	Tri 0.01	AID 0.01	AID 0.01
Mild brake to 45 mph	— 4.48	— 4.61	— 8.18	NS	Conv. 0.03	Tri 0.03
Medium brake to 45 mph, coast to 35 mph	— 7.82	— 7.61	— 8.12	NS	NS	NS
Medium brake to 35 mph	— 9.30	— 8.42	— 8.58	NS	NS	NS
Coast 2 sec, medium brake to 35 mph	— 9.45	—10.03	— 6.42	NS	AID 0.01	AID 0.01
Coast 2 sec, hard brake to 35 mph	—13.15	—11.36	—12.33	Tri 0.09	NS	NS

TABLE 18

SIGNIFICANT DIFFERENCES OF AVERAGE RELATIVE VELOCITIES DURING THE PERIOD OF CLOSURE

	MEAN AVG. RELATIVE VEL. (MPH)			SIGNIFICANT DIFFERENCE		
MANEUVER	CONV.	TRI-LIGHT	AID	CONV. VS TRI-LIGHT	CONV VS AID	AID VS TRI-LIGHT
Coast to 50 mph, mild brake to 35 mph	6.27	4.42	5.90	Tri 0.04	NS	Tri 0.15
Coast to 35 mph	5.26	5.73	3.51	NS	AID 0.01	AID 0.065
Coast to 45 mph	5.78	4.82	3.22	NS	AID 0.015	AID 0.085
Mild brake to 45 mph	3.79	5.02	6.06	NS	Conv. 0.02	NS
Medium brake to 45 mph, coast to 35 mph	9.51	6.40	6.95	Tri 0.02	AID 0.04	NS
Medium brake to 35 mph	9.19	8.31	7.95	NS	NS	NS
Coast 2 sec, medium brake to 35 mph	8.06	7.18	7.19	NS	NS	NS
Coast 2 sec, hard brake to 35 mph	9.55	7.31	7.82	Tri 0.04	AID 0.08	NS

As in the first study, the measures of particular interest in this phase of the analysis were acceleration response time and the maximum relative velocity that occurs during the maneuver.

Acceleration Response Time

Acceleration response time is defined for purposes of this analysis as the interval of time between the acceleration of the lead vehicle after an initial deceleration and the depression of the gas pedal by the following driver. Table 20 gives the significant differences of acceleration response times for the various maneuvers (Note: The means for the measure are in coded form.)

In general, it appears that the AID and Tri-light signal systems result in lower acceleration response times than the conventional system. The significance levels of the differences are especially high on maneuvers 3, 6, and 7 when the AID system is compared to the conventional system. The differences are not nearly as great when the Tri-light system is compared to the conventional system. This tends to support the conclusion derived from the daytime study (Chapter Two) that the signaling of acceleration might be preferable to simply deleting a light (as in conventional) to indicate acceleration.

This result is to be expected, as one would expect improved performance, exhibited by reduced response time, when the

TABLE 19

SIGNIFICANT DIFFERENCES OF PERCENT REDUCTION IN HEADWAY DURING PERIOD OF CLOSURE

	REDUCTION IN HEADWAY			SIGNIFICANT DIFFERENCE		
MANEUVER	CONV.	TRI-LIGHT	AID	CONV. VS TRI-LIGHT	CONV. VS AID	AID VS TRI-LIGHT
Coast to 50 mph, mild brake to 35 mph	0.45	0.25	0.33	NS	NS	Tri 0.10
Coast to 35 mph	0.44	0.34	0.29	NS	AID 0.05	NS
Coast to 45 mph	0.36	0.18	0.23	Tri 0.05	AID 0.10	NS
Mild brake to 45 mph	0.14	0.22	0.25	Conv. 0.075	Conv. 0.025	NS
Medium brake to 45 mph, coast to 35 mph	0.43	0.36	0.33	NS	NS	NS
Medium brake to 35 mph	0.39	0.35	0.32	NS	NS	NS
Coast 2 sec, medium brake to 35 mph	0.38	0.37	0.29	NS	AID 0.075	AID 0.10
Coast 2 sec, hard brake to 35 mph	0.42	0.35	0.31	NS	AID 0.05	NS

TABLE 20

SIGNIFICANT DIFFERENCES FOR ACCELERATION RESPONSE TIMES

MANLUVER	MEAN RESPONSE TIME, CODED			SIGNIFICANT DIFFERENCE		
	CONV.	TRI-LIGHT	AID	CONV. VS TRI-LIGHT	CONV. VS AID	AID VS TRI-LIGHT
Coast to 50 mph, mild brake to 35 mph	— 1.20	—17.45	3.30	Tri 0.04	NS	Tri 0.04
Coast to 35 mph	—25.30	—29.00	— 1.65	NS	Conv. 0.01	Tri 0.01
Coast to 45 mph	16.70	—17.45	4.60	Tri 0.01	AID 0.09	Tri 0.04
Mild brake to 45 mph	24.35	16.20	19.15	Tri 0.15	NS	NS
Medium brake to 45 mph, coast to 35 mph	—27.68	—25.40	—27.30	Conv. 0.02	NS	NS
Medium brake to 35 mph	23.00	17.00	15.40	NS	AID 0.04	NS
Coast 2 sec, medium brake to 35 mph	37.20	20.75	19.75	Tri 0.12	AID 0.05	NS
Coast 2 sec, hard brake to 35 mph	22.85	12.30	22.95	Tri 0.04	NS	Tri 0.04

lead vehicle signals its accelerations. All three of the systems used in this experiment signal acceleration, or at least the absence of large decelerations, by presenting or deleting lights from the display. The conventional system, which utilizes the absence of a brake light to signal possible accelerations or absence of braking, resulted in the longest acceleration response times. The Tri-light system, which indicated that the gas pedal was depressed, improved the response characteristics. The AID system, which furnished quantitative information about the degree of acceleration, resulted in response times slightly larger than the Tri-light system. This last somewhat contradictory result might be due to the fact that, because the acceleration used in the experiment was constant for all maneuvers, the subject only needed information about whether or not the vehicle was accelerating.

Maximum Relative Velocity (Maximum Opening Velocity)

As was mentioned earlier, relative velocity is a derivative of headway. The maximum relative velocity exhibited by a subject driver during a maneuver would occur during the lead-vehicle's acceleration. This maximum relative velocity serves as a partial measure of the following-driver's performance in interpreting and following the lead-vehicle's acceleration. Table 21 gives the significant differences of maximum relative velocities for the three systems tested.

As was found in the daytime study, in those maneuvers which had the acceleration preceded by a coast the Tri-light and AID systems resulted in lower maximum relative velocities than did the conventional system. Possibly due to the fact that both systems signaled the coast in a posi-

TABLE 21

SIGNIFICANT DIFFERENCES BETWEEN SIGNAL SYSTEMS AS REFLECTED IN MAXIMUM RELATIVE VELOCITIES

MANEUVER	MEAN MAX. RELATIVE VEL. (MPH)			SIGNIFICANT DIFFERENCE		
	CONV.	TRI-LIGHT	AID	CONV. VS TRI-LIGHT	CONV. VS AID	AID VS TRI-LIGHT
Coast to 50 mph, mild brake to 35 mph	6.27	6.49	6.15	NS	NS	NS
Coast to 35 mph	6.48	4.36	5.63	Tri 0.01	NS	Tri 0.10
Coast to 45 mph	6.85	4.36	5.03	Tri 0.01	AID 0.01	NS
Mild brake to 45 mph	5.57	9.76	5.48	NS	NS	NS
Medium brake to 45 mph, coast to 35 mph	7.09	4.85	5.09	Tri 0.01	AID 0.02	NS
Medium brake to 35 mph	5.64	5.18	5.39	NS	NS	NS
Coast 2 sec, medium brake to 35 mph	4.61	5.24	5.85	NS	NS	NS
Coast 2 sec, hard brake to 35 mph	6.69	5.27	5.39	NS	NS	NS

tive manner, a comparison of AID with Tri-light failed to show much of a difference.

SIGNAL SYSTEM PERFORMANCE COMPARISONS

The data previously presented are rearranged in the following to allow an over-all comparison to be made between individual systems.

Conventional vs AID

Table 22 gives a statistical summary of all the performance measures used in comparing the conventional and AID systems. This table indicates that for maneuvers beginning with a coast there appear to be definite advantages during the deceleration phases with the AID system. The con-

TABLE 22

STATISTICAL COMPARISON OF CONVENTIONAL SYSTEM AND AID SYSTEM

	MEASURES UNDER DECELERATION				MEASURES UNDER ACCELERATION	
MANEUVER	RESP. TIME	MIN. REL. VEL.	AVG. REL. VEL.	% RED. IN HEADWAY	ACCEL. RESP. TIME	MAX. REL. VEL.
Coast to 50 mph, mild brake to 35 mph	AID 0.01	Conv. 0.08	NS	NS	NS	NS
Coast to 35 mph	AID 0.01	AID 0.01	AID 0.01	AID 0.05	Conv. 0.01	NS
Coast to 45 mph	AID 0.02	AID 0.01	AID 0.015	AID 0.10	AID 0.09	AID 0.01
Mild brake to 45 mph	Conv. 0.01	Conv. 0.03	Conv. 0.02	Conv. 0.025	NS	NS
Medium brake to 45 mph, coast to 35 mph	NS	NS	AID 0.04	NS	NS	AID 0.02
Medium brake to 35 mph	Conv. 0.04	NS	NS	NS	AID 0.04	NS
Coast 2 sec, medium brake to 35 mph	AID 0.01	AID 0.01	NS	AID 0.075	AID 0.05	NS
Coast 2 sec, hard brake to 35 mph	AID 0.01	NS	AID 0.08	AID 0.05	NS	NS

TABLE 23

STATISTICAL COMPARISON OF CONVENTIONAL AND TRI-LIGHT SYSTEMS

	MEASURES UNDER DECELERATION				MEASURES UNDER ACCELERATION	
MANEUVER	RESP. TIME	MIN. REL. VEL.	AVG. REL. VEL.	% RED. IN HEADWAY	ACCEL. RESP. TIME	MAX. REL. VEL.
Coast to 50 mph, mild brake to 35 mph	Tri 0.01	NS	Tri 0.04	NS	Tri 0.04	NS
Coast to 35 mph	Tri 0.055	Tri 0.045	NS	NS	NS	Tri 0.01
Coast to 45 mph	Tri 0.01	Tri 0.01	NS	Tri 0.05	Tri 0.01	Tri 0.01
Mild brake to 45 mph	Conv. 0.07	NS	NS	Conv. 0.075	Tri 0.15	NS
Medium brake to 45 mph, coast to 35 mph	NS	NS	Tri 0.02	NS	Conv. 0.02	Tri 0.01
Medium brake to 35 mph	NS	NS	NS	NS	NS	NS
Coast 2 sec, medium brake to 35 mph	Tri 0.01	NS	NS	NS	Tri 0.12	NS
Coast 2 sec, hard brake to 35 mph	Tri 0.01	Tri 0.09	Tri 0.04	NS	Tri 0.04	NS

ventional system appears to offer advantages in maneuvers that began with braking, possibly because the AID system furnishes information that must be interpreted. During the acceleration phases of the maneuvers, there do appear to be some definite advantages with the AID system.

Conventional vs Tri-light

Table 23 presents a comparison of the conventional and Tri-light systems with respect to all of the measures discussed so far.

The results of this comparison seem to indicate that Tri-light offers advantages over conventional in deceleration aspects on all maneuvers, except possibly those beginning with a braking action. The Tri-light system also appears to have a beneficial effect on performance in acceleration portions of the maneuvers.

Tri-light vs AID

As in the daytime studies, both the Tri-light and AID systems show advantages over the conventional system. Table 24 gives a statistical comparison of the Tri-light and AID systems. It appears that the Tri-light offers a slight advantage over AID in improving response times, possibly due to the fact that in the latter deceleration and signaling are not instantaneous. AID appears to benefit the driver more than Tri-light with respect to the other three deceleration measures. The Tri-light system appears to offer a slight advantage in improving performance during acceleration phases of maneuvers.

This last seems rather surprising. It might be explained by the fact that each acceleration was preceded by a coast of some duration, even if for only a fraction of a second, and that the Tri-light system furnishes a more positive indication of coasts. This supposition is supported by the data obtained in the daytime study, which showed that Tri-light yielded better performance on those maneuvers in which a coasting action preceded braking.

LEARNING EFFECTS

The results of the daytime study indicated that learning might have an effect on performance, especially in the case of the AID system. Therefore, each subject in the nighttime study was given a period of familiarization with the AID system. During this period, each subject was required to perform under two replications of each maneuver. These data were subsequently deleted from the analysis that has been presented so far. They were, however, compared with the data used in the analysis to see if any evidence of learning could be found.

The only learning effect that was apparent in this study was that the performance under the AID system improved in terms of minimum relative velocity.

COMPARISON OF NIGHTTIME AND DAYTIME STUDIES

Slight differences in operating conditions, such as definition of beginning of a maneuver, do not allow for valid comparisons to be made between the mean values of the various measures. As has been indicated in the sections dealing with individual measures, the significant differences in the nighttime studies correspond generally with those obtained during the daytime studies.

TABLE 24

STATISTICAL COMPARISON OF THE AID AND TRI-LIGHT SYSTEMS

	MEASURES UNDER DECELERATION				MEASURES UNDER ACCELERATION	
MANEUVER	RESP. TIME	MIN. REL. VEL.	AVG. REL. VEL.	% RED. IN HEADWAY	ACCEL. RESP. TIME	MAX. REL. VEL.
Coast to 50 mph, mild brake to 35 mph	NS	Tri 0.065	Tri 0.15	Tri 0.10	Tri 0.04	NS
Coast to 35 mph	NS	AID 0.01	AID 0.065	NS	Tri 0.01	Tri 0.10
Coast to 45 mph	Tri 0.01	AID 0.01	AID 0.085	NS	Tri 0.04	NS
Mild brake to 45 mph	NS	Tri 0.03	NS	NS	NS	NS
Medium brake to 45 mph, coast to 35 mph	NS	NS	NS	NS	NS	NS
Medium brake to 35 mph	Tri 0.015	NS	NS	NS	NS	NS
Coast 2 sec, medium brake to 35 mph	NS	AID 0.01	NS	AID 0.10	NS	NS
Coast 2 sec, hard brake to 35 mph	NS	NS	NS	NS	Tri 0.04	NS

CHAPTER FOUR

VELOCITY DISPLAY SIGNAL SYSTEM

In addition to the four intervehicular signal systems described Chapters Two and Three, research was conducted to obtain some insights into the effects on car-following performance of a display mounted on the rear of the lead vehicle and displaying its velocity.

The actual physical display used in this experiment was a modification of the display used in the other four signal systems. These modifications consisted of changing the lenses so that both the bottom and top rows of the display contained green lights, and the single lights in the middle row were red. After the modifications, the green lights were used to signal the lead-car's velocity in 5-mph increments, and the red lights were used as the conventional display. A circuit was designed that enabled the operator of the lead vehicle to switch the conventional system off whenever the velocity information system was being utilized.

The velocity display was controlled so that the speed shown by the lights corresponded to the speed indicated on a Borg-Warner Police Special speedometer mounted in the lead car. Experimental trials conducted before the actual collection of data indicated that the lights were being activated within ± 1 mph of the actual velocity shown on the police speedometer.

Figure 6 shows the configuration of these lights, together with the velocity for which each of the lights was activated. The appearance of the display at the beginning of each maneuver is given in Figure 7.

DATA COLLECTION

Collection of data was conducted between the hours of 6 and 10 AM on I-71 south of Columbus. To avoid any bias that might possibly have been introduced by the effects of the glare, both systems were tested while traveling both south and north. Five young male subjects were exposed to four replications of eight maneuvers under both systems for a total of 64 trials.

The eight maneuvers employed in the study were the same as those used in the two previously mentioned studies. The method of data collection and the measures obtained from these data were also the same as those in the earlier studies. The analyses of the collected data are presented in the following.

PERFORMANCE UNDER LEAD-VEHICLE DECELERATION

For purposes of analysis, system dynamics are divided (as in earlier studies) into two phases. The first phase is defined as the period of closure; the second phase begins at the end point of the period of closure and terminates when the lead vehicle regains the 65-mph speed. Discussed measures occurring in the first phase include response time, average closing relative velocity, minimum relative velocity, and percent reduction in headway. Phase two includes maximum opening velocity and acceleration response time.

Response Time

Table 25 gives the mean response times (each entry represents the mean of 20 trials), with the significant results obtained by *t* tests applied to the data. Inspection shows that the velocity information system is superior for all maneuvers beginning with a coast. Conversely, the three maneuvers beginning with a braking action show quicker responses under the conventional system. The seeming superiority of the velocity display in the maneuvers beginning with a coast is probably due to the fact that the velocity display furnishes positive information that the vehicle is slowing down in the coast, whereas the conventional display furnishes no information during a coast.

On the other hand, during a braking maneuver the conventional system furnishes evidence of deceleration as soon as the brake is applied, but the velocity display incorporates a lag until a 5-mph decrease is realized.

Minimum Relative Velocity

Minimum relative velocity is a negative quantity representing the maximum closing velocity achieved during a maneuver. Mean minimum relative velocities (over all subjects and replications) are presented in Table 26.

Here, again, the results show improved performance under the velocity display system on coast maneuvers, "improvement" being defined as a decrease in the absolute value of the relative velocity when compared to the value obtained under conventional taillights. Also of note is the significantly better performance under the conventional system on all maneuvers beginning with a braking action. The reason for this was discussed previously.

Average Relative Velocity During Closure

Average relative velocity during the period of closure is used as another criterion of driver's performance during the lead-vehicle's deceleration phase. Table 27 gives the mean average relative velocities, together with significant differences.

Generally, it appears that the conventional system is slightly superior to the velocity display, because with the conventional system the average relative velocities are lower for all maneuvers beginning with a brake. The velocity information system is significantly better on only one maneuver, a coast to 45 mph.

TABLE 25

MEANS AND SIGNIFICANT DIFFERENCES OF RESPONSE TIMES

MANEUVER	MEAN RESPONSE TIME (SEC) CONV.	VEL. DISPLAY	SIGNIFICANT DIFFERENCE CONV. VS VEL. DISPLAY
Coast to 50 mph, mild brake to 35 mph	3.50	2.85	VD 0.001
Coast to 35 mph	5.32	2.31	VD 0.001
Coast to 45 mph	3.73	1.98	VD 0.001
Mild brake to 45 mph	1.27	1.86	C 0.050
Medium brake to 45 mph, coast to 35 mph	1.13	1.66	C 0.005
Medium brake to 35 mph	1.32	1.71	NS
Coast 2 sec, medium brake to 35 mph	2.56	1.91	NS
Coast 2 sec, hard brake to 35 mph	2.76	2.11	VD 0.050

TABLE 26

MEANS AND SIGNIFICANT DIFFERENCES OF MINIMUM RELATIVE VELOCITY

MANEUVER	MEAN MIN. RELATIVE VEL. (MPH) CONV.	VEL. DISPLAY	SIGNIFICANT DIFFERENCE CONV. VS VEL. DISPLAY
Coast to 50 mph, mild brake to 35 mph	8.62	—7.72	NS
Coast to 35 mph	6.42	—4.33	VD 0.005
Coast to 45 mph	5.87	—3.06	C 0.001
Mild brake to 45 mph	6.24	—8.88	C 0.001
Medium brake to 45 mph, coast to 35 mph	10.48	—12.11	C 0.050
Medium brake to 35 mph	10.99	—13.77	C 0.005
Coast 2 sec, medium brake to 35 mph	12.37	—13.20	NS
Coast 2 sec, hard brake to 35 mph	16.78	—17.68	NS

TABLE 27

MEANS AND SIGNIFICANT DIFFERENCES OF AVERAGE RELATIVE VELOCITY

MANEUVER	MEAN AVG. RELATIVE VEL. (MPH) CONV.	VEL. DISPLAY	SIGNIFICANT DIFFERENCE CONV. VS VEL. DISPLAY
Coast to 50 mph, mild brake to 35 mph	2.48	2.40	NS
Coast to 35 mph	1.76	1.88	NS
Coast to 45 mph	1.96	1.01	VD 0.001
Mild brake to 45 mph	1.83	2.66	C 0.001
Medium brake to 45 mph, coast to 35 mph	2.20	2.81	C 0.050
Medium brake to 35 mph	3.30	4.05	C 0.005
Coast 2 sec, medium brake to 35 mph	3.21	3.13	NS
Coast 2 sec, hard brake to 35 mph	3.39	3.73	C 0.150

TABLE 28

MEANS AND SIGNIFICANT DIFFERENCES OF PERCENT REDUCTION IN HEADWAY

MANEUVER	MEAN REDUCTION IN HEADWAY (%) CONV.	VEL. DISPLAY	SIGNIFICANT DIFFERENCE CONV. VS VEL. DISPLAY
Coast to 50 mph, mild brake to 35 mph	17.59	14.93	VD 0.100
Coast to 35 mph	18.20	10.85	VD 0.001
Coast to 45 mph	14.19	10.15	VD 0.001
Mild brake to 45 mph	8.58	11.03	C 0.100
Medium brake to 45 mph, coast to 35 mph	17.42	16.95	NS
Medium brake to 35 mph	13.90	16.66	C 0.050
Coast 2 sec, medium brake to 35 mph	14.99	15.40	NS
Coast 2 sec, hard brake to 35 mph	14.89	16.37	C 0.150

Percent Reduction In Headway

Another measure of driver performance during the deceleration phase of a maneuver is percent reduction on headway. As before, this is (Initial headway-Minimum headway)/Initial headway. Table 28 presents the mean of reductions in headway and the level of the significant differences. Inspection again reveals the pattern whereby the velocity information system appears superior for coasting maneuvers and inferior for braking maneuvers.

PERFORMANCE UNDER LEAD-VEHICLE ACCELERATION

A measure of performance in this second phase of driving is the maximum relative velocity, or maximum opening velocity, exhibited during a maneuver. The maximum relative velocity usually occurs soon after the end of the period of closure, as this is the time when the lead car is accelerating most rapidly. This measure is useful because it gives a partial indication of a driver's ability to ascertain the lead-vehicle's acceleration characteristics. Average maximum relative velocities are given in Table 29. The results indicate no significant differences between the two systems.

SUMMARY

Due to unavoidable differences in operating procedures, one is precluded from making valid comparisons between mean values obtained in this study and the means of other investigations. The results seem to be consistent in indicating significantly improved performance for the velocity infor-

TABLE 29
MEANS AND SIGNIFICANT DIFFERENCES OF MAXIMUM RELATIVE VELOCITY

MANEUVER	MEAN MAX. RELATIVE VEL. (MPH) CONV.	VEL. DISPLAY	SIGNIFICANT DIFFERENCE CONV. VS VEL. DISPLAY
Coast to 50 mph, mild brake to 35 mph	7.08	7.32	NS
Coast to 35 mph	8.43	7.08	VD 0.150
Coast to 45 mph	7.22	6.90	NS
Mild brake to 45 mph	5.95	5.59	NS
Medium brake to 45 mph, coast to 35 mph	7.76	7.26	NS
Medium brake to 35 mph	6.69	6.74	NS
Coast 2 sec, medium brake to 35 mph	8.54	7.37	NS
Coast 2 sec, hard brake to 35 mph	5.41	6.12	NS

mation system on maneuvers involving initial small negative accelerations. For large initial magnitudes of deceleration the conventional system generally appears significantly better. One could hypothesize that the foregoing results are the result of a delay in actuation of the velocity display. This delay, although very short, might be enough to cause the conventional system to appear significantly better on all maneuvers beginning with a braking action.

It would seem, therefore, that the type of display tested in this experimentation would not be suitable for use on automobiles, because it is inferior to the conventional system, in the braking situations, which from a safety standpoint are probably the most critical car-following situations.

This should not be taken as a condemnation of all velocity displays. Another display with possibly smaller speed change increments or some combination of conventional plus velocity displays might prove superior to the conventional in all types of maneuvers.

CHAPTER FIVE

PASSING ON A MULTI-LANE EXPRESSWAY

Research in the field of traffic flow on multi-lane expressways has often been approached from the standpoint of minimizing traffic disturbances; i.e., "acceleration noise." The entrance of a vehicle into a traffic lane may cause such a disturbance if its action requires braking or other dynamic changes by other vehicles in the lane. The entrance of a vehicle into a traffic lane may be due to several causes, including:

1. Entering a lane from an acceleration ramp.
2. Passing a slower vehicle.
3. Avoiding a collision.

The concern of the current study is with passing on multi-lane freeways. However, the results have general implications to the freeway entrance problem. The decision to enter a traffic lane at a given time is based on many factors, such as:

1. The willingness of the driver to accept the risk of collision (related to gap acceptance studies).
2. The amount of information available to the driver concerning the headway between the vehicles involved.
3. The amount of information available to the driver concerning the relative velocity relationships between the vehicles involved.
4. The acceleration capabilities of the vehicle.
5. The geometry of the expressway.

The ultimate objective of this type of research is to see if a highway's volume of traffic can be improved by increasing the information available to the driver relevant to his passing decision. This particular study seeks to describe the characteristics of a driver's decision to pass on expressways without any decision aids.

One method for increasing traffic volume is to increase density by reducing the headway between cars, if that headway is greater than that needed for safety and if such increased density does not create a precondition for undamped "shock waves," precipitated by a velocity perturbation in the stream. Another way of increasing highway volume is to reduce the amount of headway needed between a passing car and a closing car in the passing lane. This would have the effect of moving a car into the headway space in the passing lane, thereby raising the density in that lane and allowing the opportunity for another car to move in the outside lane. If velocities are maintained, volume will rise (depending on the position on a volume-density curve).

With these considerations in mind, an empirical study was undertaken to endeavor to measure a driver's performance in accepting and moving into a gap in an adjacent lane. It was felt that certain safe limits might exist on this maneuver, represented in terms of spacing, speeds, and the relative velocity with the closing vehicle in the adjacent

lane. If driving performance would be improved so as to reach these limits, a gain in volume might be obtained. The empirical study reported in the following was undertaken to determine how closely present performance approaches the limits of safety.

PROCEDURE

This experiment was conducted on Interstate 71 south of Columbus, Ohio. Three vehicles were used to form a traffic pattern as shown in Figure 8. Car 0 was driven by an experimenter at a constant velocity, V_0, establishing the traffic flow in lane 1.

The subject was in car 1. He was instructed to follow car 0 at a constant headway, H_{01}. The subject used a piece of plastic tape attached to the windshield to aid this task. During this time he was denied use of his rear mirrors. This was accomplished by shining a light from the rear of the outside mirror and covering the inside mirror with a remotely controlled removable flap. The speed of car 0, V_0, was set at 50 or 60 mph and established the stream velocity in lane 1.

To begin a trial, the experimenter in car 2 adjusted his velocity to a predetermined value (usually 10 mph greater than car 1), thus establishing the flow in lane 2. At time t_0, when the headway between car 1 and car 2 reached a predetermined value, the experimenter accompanying the subject in car 1 simultaneously marked the recorder and allowed the subject use of his rear-view mirrors. The mirrors were made operable by turning out the light behind the outside mirror and remotely raising the flap covering the inside mirror.

At this time the subject made a decision to pass or not pass car 0. If he elected not to pass, he continued his task of maintaining constant headway between himself and car 0. When the experimenter in car 2 reached car 0, he decelerated and prepared for the next trial. If the subject elected to pass, he initiated the maneuver by pulling into lane 2. A large amplitude in deflection of the gas pedal trace on the recorder showed the time, t_1, at which the maneuver was initiated.

Headway between car 1 and car 2 was set at 12 different levels during the experiment. The line of sight across a stationary wire located in the back window of car 1 and one of 12 bolts on a bar attached above the rear bumper determined the angle, hence the headway, during the trial, as shown in Figure 18. Accuracy was calculated at approximately ± 10 percent for a 1-ft vehicle lane centering error. When car 2 lined up with the wire and the bolt of car 1, the experimenter in car 1 started the trial.

Initially one subject was tested to determine practical limits for testing, then six subjects were tested under each of the 24 conditions given in Table 30. The relative velocity throughout the experiment was (−) 10 mph.

Two dependent variables were investigated—decision time, and percentage of "go" responses. These were studied under all headways of conditions 1 and 2. A "go" response was designated when the subject in car 1 pulled into the passing lane during a trial. Decision time was taken as the

TABLE 30

CONDITIONS TESTED

HEADWAY,[a] H_{12} (FT)	$V_0 = 50$ $V_2 = 60$	$V_0 = 60$ $V_2 = 70$
200	X	X
162	X	X
137	X	X
119	X	X
104	X	X
94	X	X
86	X	X
78	X	X
73	X	X
65	X	X
60	X	X
55	X	X

[a] When mirrors opened.

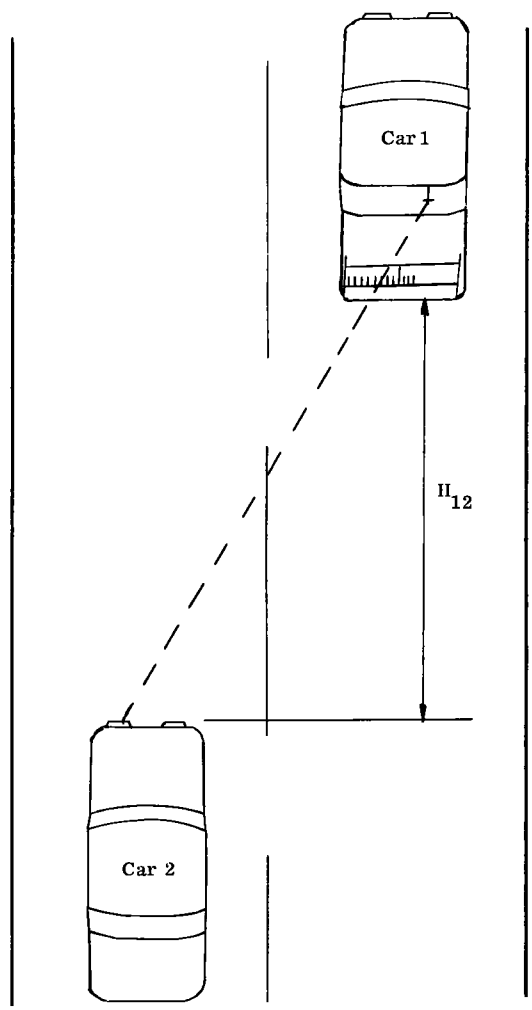

Figure 18. Line of sight used in determining various H_{12} headways.

time difference between the start of the trial, t_0, and the point when the subject heavily depressed the gas pedal.

RESULTS

Figures 19 and 20 show plots of decision times under the two velocity conditions. "t-tests" were done on the two velocity conditions and on the "go" and "no-go" response times. Differences were not statistically significant.

A one-way analysis of variance was computed to test the effects of headway on decision time for "go" responses. The results are given in Table 31. Pretests for the necessary conditions for ANOVA indicated that this statistical model was appropriate for this analysis.

From knowledge of human decision-making, decision times might be expected to be largest at or near the decision threshold ($\approx$ 100 ft). In an aggregate sense this does not appear to be the case, although a much larger sample size would be required to properly test this hypothesis.

It is convenient to talk in terms of a threshold for "go" or "no-go" responses to the passing situation. The term implies that there exists some headway (or time headway) value above which the response will usually be "go" and below which it will usually be "no-go." It has been found that acceptance of a gap in entering a highway is not a step function denoting a definite threshold, but a trapezoidal function. This may be generalized to the necessary gap in traffic flow on the highway which a driver must accept in order to make a passing response. For analysis, however, the threshold is discussed in terms of the level at which 50 percent of the responses are "go" responses. Figures 9 and 10 show the percentage of "go" responses for each subject as a function of headway for both velocity conditions.

Condition 1: Figure 9 shows that 4 of the 6 subjects showed consistent passing behavior. For example, subject 5 made "go" responses 100 percent of the time when the

TABLE 31

ANALYSIS OF VARIANCE ON THE EFFECTS OF HEADWAY ON "GO" RESPONSE DECISION TIME

CONDITION	F	F 0.10
$V_0 = 50$ mph, $V_2 = 60$ mph	0.617	1.78
$V_0 = 60$ mph, $V_2 = 70$ mph	1.358	1.90

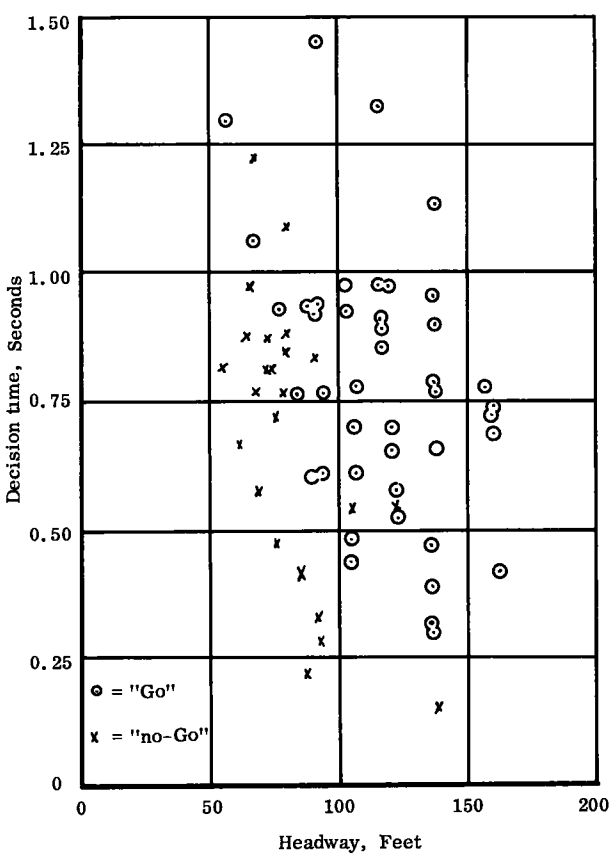

Figure 19. Decision time for responses when $V_0 = 50$, $V_2 = 60$ mph (all subjects).

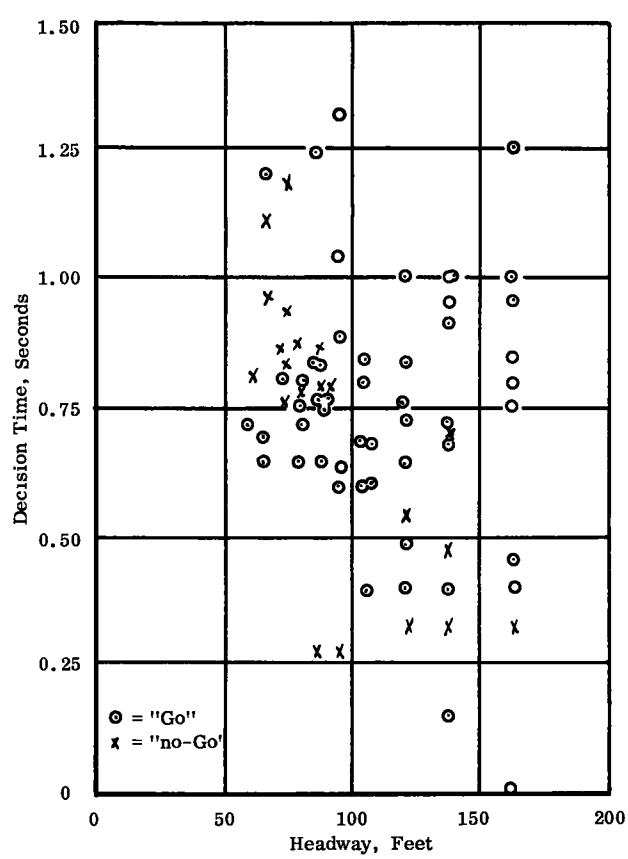

Figure 20. Decision time for responses when $V_0 = 60$, $V_2 = 70$ mph (all subjects).

headway was 104 ft or more. When it dropped to 94 ft he passed 50 percent of the time, and when it dropped to 86 ft he did not elect to pass at all. However, the threshold estimate is not as easy to determine for all subjects, as indicated by Figure 9.

A plot of the subjects' combined performance is shown in Figure 21 for both stream velocity conditions. This shows a threshold value of approximately 90 ft for these velocity conditions. Referring again to Figure 9, this threshold value is clearly supported by subjects 1 and 5 and conditionally by subject 3. Subject 2 appears to be responding with "go" decisions below the threshold for the group, and subjects 4 and 6 appear to respond above the threshold for the group.

Condition 2: Figure 10 shows that only one subject (4) has a non-monotonic curve. However, subject 4's inconsistency is the result of one "no-go" response on one trial at a headway of 73 ft, although responding with 100 percent "go" responses on all other trials. It should be noted that this subject was also given 6 trials with a 55-ft headway at the velocity condition, and responded with 0/6 "go" responses. This seems to indicate that the one trial at the 73-ft headway may have been merely a subject error, and that this subject's behavior is consistent with a threshold of around 60 ft at this velocity condition.

Figure 21 shows the group curve for the velocity condition and indicates a threshold value of approximately 100 ft. From Figure 10 it appears that subjects 2 and 3 support this threshold while subjects 1 and 4 respond with "go" responses below that threshold, and subjects 5 and 6 respond above it.

In comparing Figures 9 and 10, it is interesting to note that three subjects (2, 3, 5) had a higher threshold under the higher velocity condition, while two subjects (1, 4) had a lower threshold. Subject 6 appeared unaffected by the change in velocity condition. This may imply that different cues are relevant to individuals at different velocity conditions.

A probit analysis was performed on the data, and a regression line was recovered for each of the two velocity conditions. These are shown in Figure 22, with the derived sigmoid curves in Figure 23.

A comparison of Figures 21 and 23 is interesting. Figure 21 shows 100 ft and 90 ft as threshold values for the two velocity conditions. This is interesting because they are the reverse of those in Figure 23. The difference in the two figures may be explained by the higher percentage values at lower headways for the $V_0 = 60$, $V_2 = 70$ mph condition. In the probit analysis, these values may have shown their weight by pulling the curve for this condition to one side. Furthermore, the points representing 94 ft in Figure 21 seem to be slightly off the trend of the curve in both velocity conditions. If they were both disregarded and a line drawn connecting the surrounding points, the result would be a threshold value more in line with the probit analysis results. It must be concluded, therefore, that the threshold for the two conditions is approximately 90 to 100 ft.

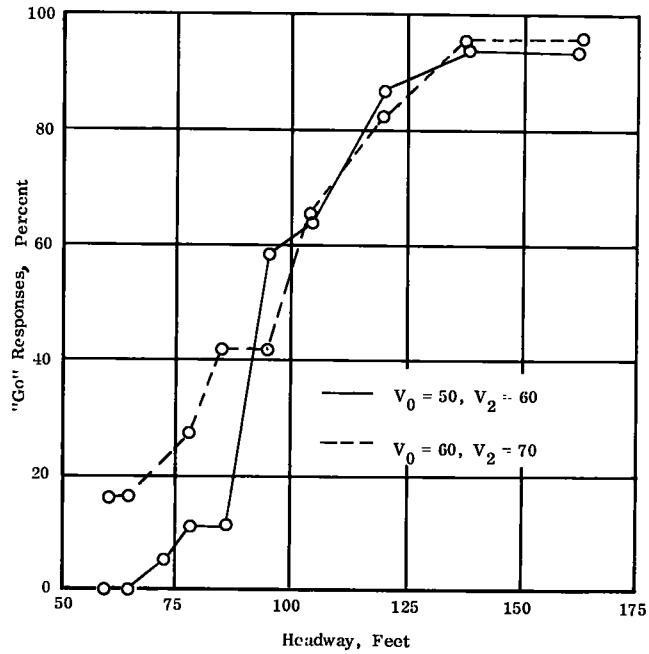

Figure 21. Percentage of "go" responses for subjects weighted equally for two velocity conditions.

Figures 24 and 25 show the minimum headway needed at the different velocities for car 1 to pass car 0 before touching car 2 and pulling away. That point is the closest point of the two cars and is the point of tangency on the graphs. These charts were derived by plotting V_1 and V_2 using known acceleration characteristics of car 1. Because the Kolmogorov-Smirnoff test on decision time distributions showed that a normal distribution could explain the data, the mean and standard deviations were calculated, and a decision time conservatively set to exceed 99 percent of this distribution was used as the delay before acceleration of car 1 took place. The acceleration curve, which describes the subject's passing response, was plotted following that decision time.

The point of tangency is the closest point between the acceleration curve and the velocity line of car 2. Beyond the point of tangency, the passing car's acceleration is pulling car 1 away from car 2.

These figures provide a graphical way to determine the necessary headway needed for a safe pass under some accepted standard of safety. For example, the National Safety Council (NSC) advises that cars follow at a minimum distance of one car length for each 10 mph. Under this rule, when $V_2 = 60$ mph (see Fig. 24) there should be a headway of approximately 120 ft between car 1 and car 2 at their nearest point. Inasmuch as the nearest point is the point of tangency, that point should be represented by a distance separation of 120 ft. The graph has shown that a 45-ft initial headway is needed to avoid just touching bumpers at the point of tangency, so the headway needed initially at this velocity is 45 + 120, or 165 ft. This assumes,

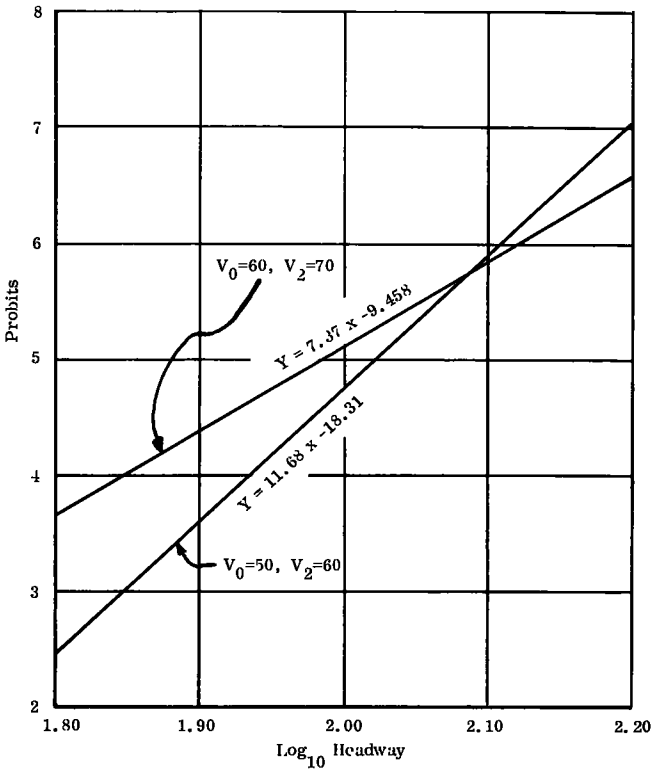

Figure 22. Probit analysis regression lines representing two conditions of velocity.

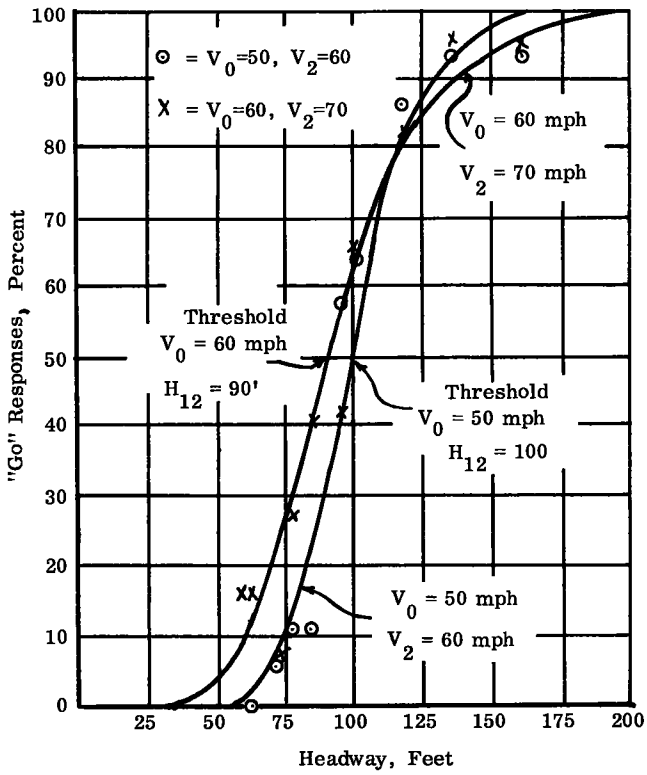

Figure 23. Sigmoid curves recovered from probit analysis regression lines for two velocity conditions.

of course, no required corrective action on the part of the driver in car 2.

Likewise, using Figure 25, if $V_2 = 70$ mph the minimum headway at the point of tangency should equal 140 ft, making the initial headway 140 + 45, or 185 ft, needed for safe passing by the NSC standard of safety.

Figure 9 shows that every subject, with the possible exception of subject 6, made "go" responses far below this criterion for safety. Similarly, Figure 10 shows every subject making "go" responses below the NSC safety standard. It might be mentioned that the 165 ft and 185 ft figures were computed for a standard car and represent a minimum headway desired by the NSC for normal car-following.

It is concluded that the subjects, in general, passed at headway values below that which would assure NSC clearance and above that determined as resulting in collision.

DISCUSSION

Increasing the volume flow of traffic via increased information to the driver is practical for two types of drivers. The first is the driver who passes only at values in excess of that needed for safety; the driver who may wait for a headway twice that necessary for a safe pass. When highway conditions are crowded, the likelihood of an excess headway is small, and the driver may let numerous safe passing opportunities escape while waiting for a greater headway. This would have the effect of letting usable space on the highway go to waste.

The second type is the driver who passes at headways low enough to cause the closing car to decelerate. In dense traffic this has the effect of causing a wave of disturbance, which could result in traffic stoppage and multiple rear-end collisions.

An illustration of the effects of passing may be shown by plotting the theoretical deceleration probability curve of the closing car (Fig. 26). The probability of deceleration (p. o. d.) represents the likelihood that the closing car on the inner lane will be forced to decelerate and is a function of the relative velocities and the headway between car 1 and car 2. If the headway is small, the p. o. d. is high. It can be seen from Figures 24 and 25 that any initial headway value smaller than 45 ft would result in collision, because 45 ft just avoids collision at the point of tangency. Therefore, any initial headway value below 45 ft should cause the p. o. d. to be 100 percent.

At different points on the curve, the p. o. d. may be said to take different forms. High p. o. d. may result in hard braking (region A, Fig. 26), whereas very low p. o. d. may result in no deceleration (region D, Fig. 26).

Theoretically, an efficient highway would find cars passing 100 percent of the time when the p. o. d. of the closing car is in the range of C and D in Figure 26. Drivers who

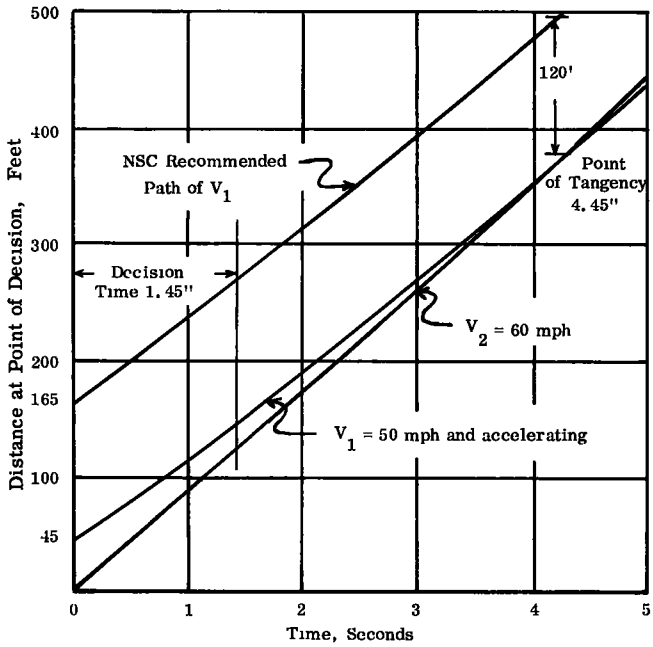

Figure 24. Velocity and acceleration characteristics when $V_0 = 50$, $V_2 = 60$ *mph.*

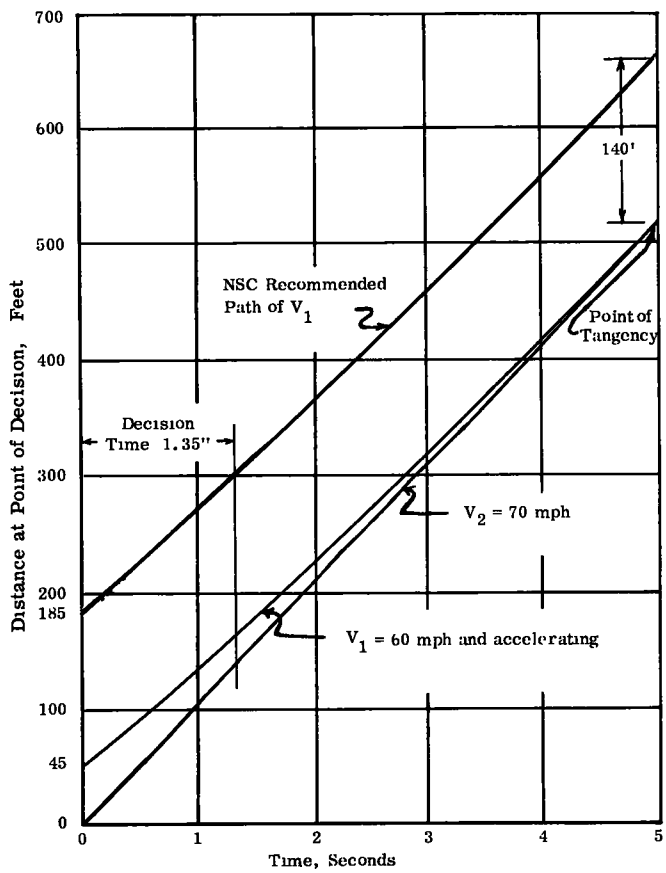

Figure 25. Velocity and acceleration characteristics when $V_0 = 60$, $V_2 = 70$ *mph.*

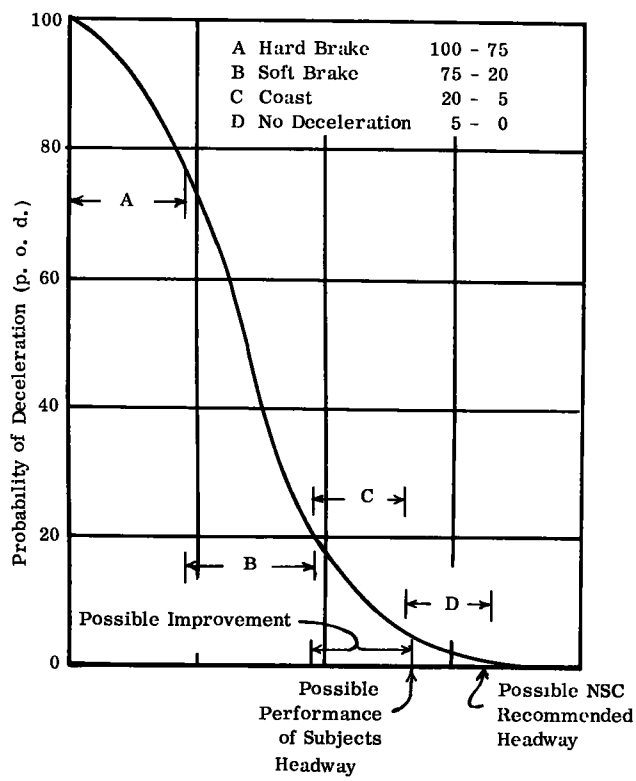

Figure 26. Theoretical curve of deceleration probability.

passed when the p. o. d. was in the area of A and B would cause traffic disturbances, while drivers who did not pass until beyond D would be wasting highway space.

It is assumed that the NSC would favor p. o. d. well within the D range. By placing the NSC desired performance criterion on the graph, it can be seen that the performance of the subjects may possibly fall within the C, D range and still leave some room for improvement. Obviously, this is conditional on the assumptions made about the shape of the curve and the absolute headway values along the abscissa. If more were known about the characteristics of the p. o. d. curve, it is possible that a more practical standard could be accepted: one which demonstrated clearly the advantages of increasing information available to drivers.

CHAPTER SIX

DRIVING CONTROL BEHAVIOR AND ROADWAY GEOMETRY

A driver's response to information about other vehicles is influenced by the use which he and his vehicle can make of that information and by the attention which he must pay to controlling his own vehicle. This chapter describes some exploratory studies of single car-driver behavior and its relation to roadway geometry, where the primary objective was to search for important variables and to classify behavior where possible. The basic data for each of these studies were collected from replications of a single experimental situation.

The purpose of this study was to explore driver control behavior on expressways as contrasted to driver-vehicle behavior reported in previous studies. In the current study concern was not solely with vehicle velocity or acceleration as such, but rather with the driver's input to the vehicle in terms of gas pedal and steering wheel movements over time as a function of road geometry, instructions (driving objectives), and vehicle dynamics, such as acceleration and velocity. The first step is to describe the basic experiment, wherein the unique data collection and analysis with the use of the computer enables simultaneous comparison, through a computer plot of gas pedal movement, steering wheel movement, velocity, acceleration, time, vertical curvature, and distance traveled. With these primary data, the following questions were briefly explored:

1. What are the natural characteristics of gas pedal movements in terms of amplitudes and periodicities and how do subjects compare under the same driving conditions?
2. How does route profile (vertical curvature) affect gas pedal and velocity? To what extent might this clarify the accident problem on sags versus crests on Interstate highways?
3. With time-based regression analysis, can we find the contribution of profile, acceleration, and velocity on gas pedal changes? To what extent do these variables lead or lag gas pedal movement changes?
4. How do steering wheel and gas pedal amplitudes and periodicities relate to driving objectives as imposed by several different driving instructions?

THE BASIC EXPERIMENT

All subjects drove an instrumented vehicle (1961 Ford station wagon) on Interstate Highway 71, a limited-access freeway. The test route consisted of a 50-mile trip north of Columbus, a coffee break, and a 50-mile return. The subjects were told that they were participating in a fatigue study and that their normal driving behavior was desired.

Five data variables were recorded: gas pedal position, steering wheel position, velocity, the vehicle's acceleration tangent to the road, and the vehicle's vertical acceleration. In addition, a recorder channel was reserved for marking on the data film the 81 points along the route which were previously marked by taped delineations. The experimenter tripped the marker when the vehicle was opposite each of these, thereby providing a means of correlating the recorded driving data with road geometric features.

The data from the recorder were punched on cards using a 1-sec sampling interval. These cards were processed to convert the time scale data for each subject to a common distance scale, and to calculate additional measures. The resulting data (discrete traces of equidistant spacing) were plotted to facilitate the search for patterns of behavior. An example of this output is provided in Figure 11.

THE CONTROL TRACES

The continuously recorded traces of the gas pedal and steering wheel contain more information about the driving process than can be extracted with a few statistical analyses of discrete trace samples. For this reason, the continuous traces that appeared promising after examining the data themselves were investigated.

Figure 27 is a set of gas pedal traces for several subjects, but all for the same section of road. Probably the most striking observation possible with these curves is the great difference between them.

The first possibility which suggests itself is that the more complex ones, such as number 7, have a basic structure similar to the smoother ones, such as number 1, with added components. It appears from inspection that trace 7 has three basic frequencies. To evaluate the contribution of each of these, tracings were first made of curve 7 with the small amplitudes eliminated and then with the medium amplitudes eliminated. These two smoothed curves are S1 and S2 in Figure 27. The second smoothing can be seen to resemble some of the other curves; indeed, all of the traces appear to have some of this lowest-frequency component.

The higher-frequency cycles are more difficult to make generalizations about. To determine how distinctly trace 7 could be classified into very small and medium cycles, distributions of the amplitudes and periods of its cycles were plotted. Separate plots were made for a level stretch of road and for a hilly section. It is clear from the distributions of periods shown in Figure 28 and in other plots not shown that the measures applied to trace 7 do not depict the existence of two distinct sets of cycles. The differences between level road and hills are not large and consist of a shift toward larger amplitudes and periods for the hilly section.

COMPARISON OF TRACES WITH ROUTE PROFILE

This section presents the results obtained from comparing the subjects' velocity, gas pedal, and acceleration as a function of profile. Computer plots were used in this comparison.

Response to vertical curvature fell into two basic categories. These are best described in terms of the ways in which drivers appeared to be controlling the vehicle. Two basic types of gas pedal trace were shown earlier: ragged traces and smooth traces. Inspection of the computer plots shows that the more ragged of the two gas pedal types produces very "flat" velocity curves; i.e., there is little velocity variation, regardless of road geometry. This type of driver appears to make extensive use of the speedometer, because responses occur after brief periods of velocity change. Hills are compensated for by increasing the amplitude of the medium gas pedal oscillations. The occasional velocity fluctuations which do occur seem to result from misestimates of the gas pedal amplitudes necessary to reverse the velocity trends. The regions of the highway which cause estimation errors most often are the inflections. Generally, these seem to create the impression that the vehicle is climbing after it has either stopped doing so or has begun to level out, so that the velocity climbs higher than anticipated and then must be reduced. A sketch of the velocity over a crest is shown in Figure 29, and the velocity over a sag is shown in Figure 30. Because velocity deviations for this type of driver seem to be errors of estimation of needed gas pedal adjustment, their magnitude varies from hill to hill; i.e., the magnitude of velocity change is random.

The second class of drivers consists of those who have a more lax attitude toward velocity control; i.e., they allow larger fluctuations in their speed. These drivers use a minimum of gas pedal movement for the task at hand. It appears that these drivers attempt to predict the rate of pedal deflection necessary to keep velocity constant. They then use this predicted gas pedal movement as their control. Feedback about actual velocity is not used unless the velocity change is large (at least 5 mph). This class of subjects is also affected by inflections in the same way as the other class. In addition, however, this minimum gas pedal change group often demonstrates velocity curves similar to that in Figure 31. It seems that these errors develop because (a) the estimation of the slope being traveled occurs as the vehicle starts up or down, and (b) the velocity is not checked often enough to detect the change.

A study by Mullins and Keese * showed that sections of highways composed of crests and sags had higher accident rates than sections of highway composed of flat surfaces. They found that the upgrades and downgrades of crests and sags had a higher accident rate per million vehicle-miles than did the crest peaks and the bottoms of the sags. The velocity curves obtained in the experiment reported here showed that discrepancies or errors from target velocity are most likely to occur on the slopes (upgrades and downgrades) of crests and sags. These discrepancies may account for the increase in accident rates because of the more hazardous situations they create. For example, drivers in queue with alternating positive and negative errors could create a situation in which dangerously short headways and associated high headway variances were exhibited.

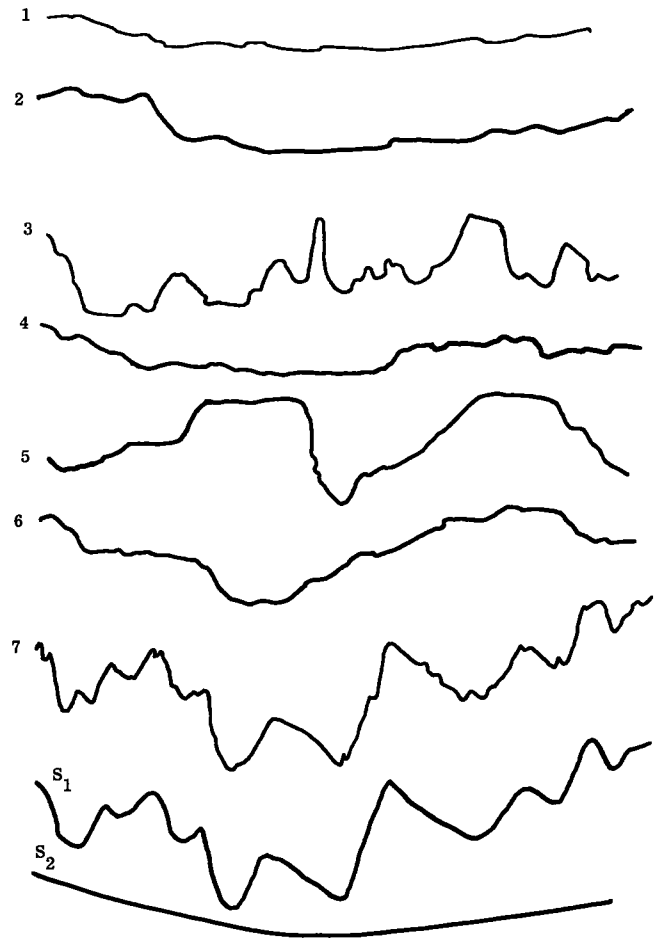

Figure 27. Examples of gas pedal traces over the same highway geometry.

NONLINEAR REGRESSIONS ANALYSIS OF GAS PEDAL MOVEMENT

The data contained on cards as the output of the road geometry program were analyzed with a piece-wise linear regression routine. Essentially, this multivariate routine fitted a set of linear surfaces to the data in the variable space. The basic equation of the regression plane obtained using the method of least squares is

$$V(t) = b_0 + b_1 X_1(t+\tau) + b_2 X_2(t+\tau) + b_3 X_3(t+\tau) \quad (5)$$

in which the b's represent the regression coefficients for the X's displaced by time τ.

The dependent variable, Y, used in the analysis was the change in gas pedal position, ΔGP, where Δ is the first

* Mullins, B. F. K., and Keese, C. J., "Freeway Traffic Accident Analysis and Safety Study." *Traffic Safety Res. Rev.*, Vol. 6, No. 4 (Dec. 1962).

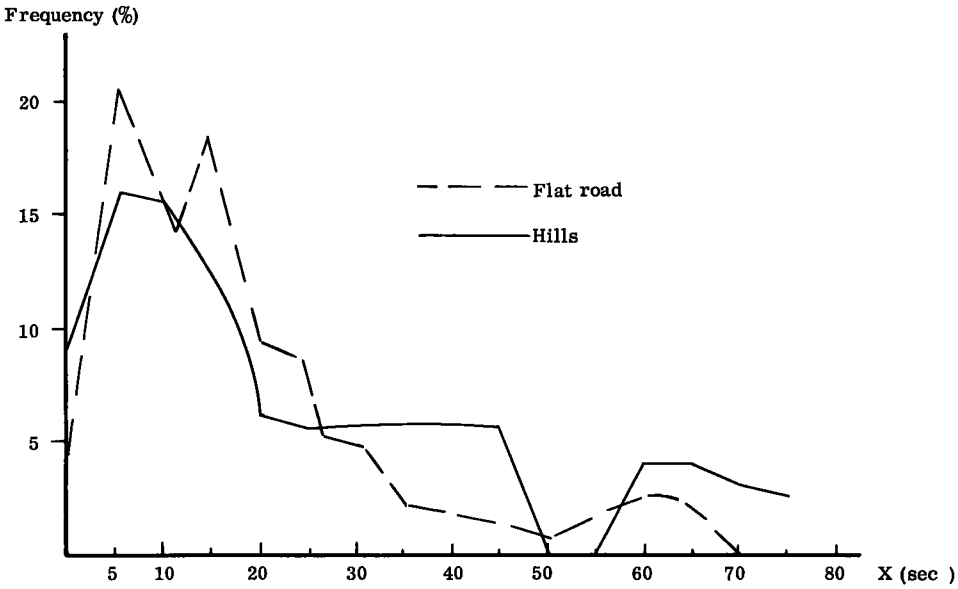

Figure 28. Frequency distribution of gas pedal periods for subject 7.

time difference, based on a 1-sec interval. The three independent variables used were velocity, V, acceleration, A, and approximate road slope, ΔP.

The analysis was performed for several time lags, as shown in Figure 32. The dependent variable was regressed against the independent variables for nine time relationships ranging from a 6-sec lag to a 10-sec lead. (Note: lag means ΔGP comes *after* in time.)

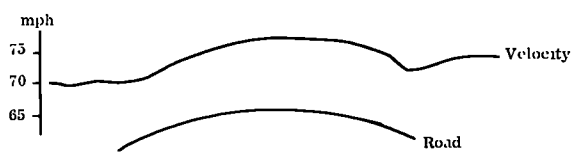

Figure 29. Velocity curve of first driver class over a crest.

Figure 30. Velocity curve of first driver class over a sag.

Figure 31. Velocity curve of second driver class over a crest.

The objective of this analysis was to obtain insight into the determinants and consequences of gas pedal behavior for a period of time before and after the actual gas pedal movement.

Three types of results were obtained from this analysis. First, for any one of the nine time intervals, τ, it was possible to determine the relative contribution of the independent variables to the variation of ΔGP. Second, each of the independent variables could be evaluated in terms of the magnitude of its contribution to ΔGP as a function of the nine time values (τ) used. Third, information was provided by the way in which the computer routine partitioned the variables. Figure 33 may help to clarify the nature of the third type of results. (For simplicity, only two variables are shown.) The figure shows a possible set of data fitted with two lines. The routine includes in its output the mean value of each variable in a fitted segment. In this case, the average of ΔGP and V for line 1 and for line 2 would be printed out. These values would indicate that a difference in the relationship between the variables occurs, depending on whether ΔGP is + or −, and, as expected, that + and − ΔGP tend to occur at different velocities.

It is clear that the regressions for those time points where ΔGP leads the independent variables (second type of result) are qualitatively different from those where it lags. The general situation is most easily summarized as in Table 32. The empty cells could be filled, but are left empty because the combinations they represent were not studied. Inspection of the data pertaining to the lower left cell showed a good fit of the data for a 10-sec lead

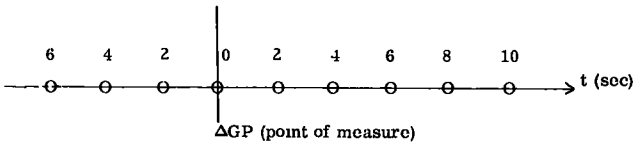

Figure 32. Time points used in regression analysis of gas pedal movement.

time X_3 (profile). At 70 mph this is about 1,000 ft, which appears reasonable for freeway driving.

There is a great deal of interrelationship between the different types of results. However, to simplify the presentation of results, each type is first considered separately. A separate analysis was run for each of four subjects; subject differences are mentioned where appropriate.

With respect to the coefficients for each time interval employed (data pertaining to the first type of results), the primary effect was the dominance of acceleration. The regression coefficient for A was often more than 20 times greater than those for V and P. The result is not surprising for lead times of 0, 2, or 4 sec, as these include the values of acceleration shortly after gas pedal deflection. The predominance of A for the other cases suggests that further study of this variable might profitably be undertaken.

The data pertaining to the third type of results are the most difficult to generalize. The example of Figure 33 oversimplifies the problem in two ways. First, there were four variables in the study instead of two as shown. Second, the routine often separated the data into five rather than two lines as shown. The most important general observation regarding this class of data is that the data were almost always split into separate groups for + ΔGP and − ΔGP. Within these groups, the pattern of regression coefficients was usually quite different. As with the dominance of acceleration in the preceding paragraph, this fact is not surprising for the lead time conditions, as it corresponds with the differences in vehicle dynamics, which are functions of ΔGP depression or release. The existence of substantial differences for the lag time conditions implies that there are different relationships leading to ΔGP depression than to ΔGP release. For example, one subject had opposite signs on his regression coefficient for A for the + and − ΔGP cases. The data for several subjects were fitted such that a set of data points comprising 5 to 10 percent of the total and having very large + ΔGP values was isolated. Even under lag time conditions, this set was often accompanied by larger than average coefficients for ΔP. There were numerous intra-subject differences for this third class of data.

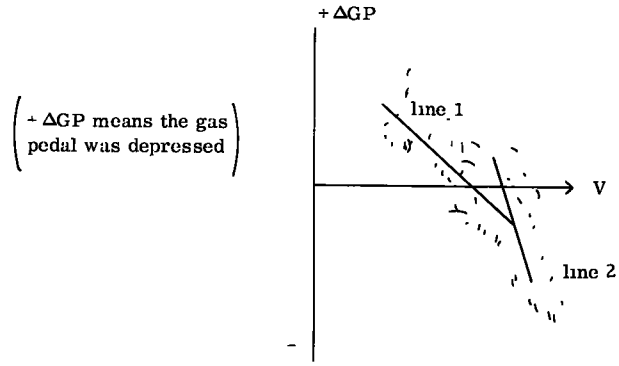

Figure 33. A possible fitting of data by two lines.

The variation in each regression coefficient over the times used (result type 2) must be interpreted in conjunction with the data of result type 3. Thus, the variations in the values of b_a, the regression coefficient for A, depend on the partitions of ΔGP. As an example, consider the variations of b_a for a particular subject, as shown in Figure 34. One line represents the values for the + ΔGP

TABLE 32

TYPES OF POSSIBLE OUTCOMES FROM THE REGRESSION ANALYSIS[a]

LAG \ LEAD	ΔGP	V	A	ΔP
ΔGP		Driver response to velocity	Driver response to acceleration	Possible driver response to slopes just traveled
V	Effects on vehicle velocity due to ΔGP			
A	Effects on acceleration due to ΔGP			
ΔP	Driver response to slopes ahead			

[a] Column entries lead line entries.

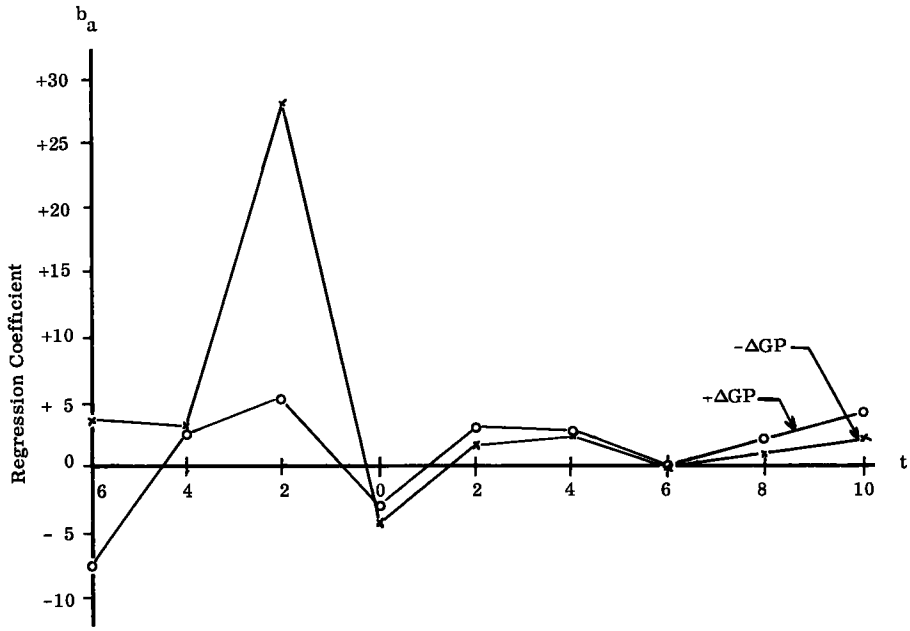

Figure 34. Values of b_a *for the nine time intervals (single subject).*

set, another line depicts b_a for $-\Delta$GP. Not all subjects produced the rather surprising positive b_a around the 2-sec lag.

Only a few of the subjects had relatively large b_v's for the time lag condition. When a significantly non-zero b_v did occur in the lag periods, it generally appeared for only one value of t. All subjects showed some positive b_v at some of the leading periods, but the time was not the same for all of them.

The ΔP correlations, b_p, were largest for lead times of 6, 8, and 10 sec. Some subjects displayed fairly large b_p's at lag times of 4 and 6 sec. The former would suggest that the driver responds to geometric changes far in advance of when they occur.

The primary consequence of this analysis is the fact that the use of such an approach seems to have value. There can be no question about the fact that knowledge of the time pattern of information acquisition and use is important to the understanding of driver behavior. Although the samples on which this particular analysis was based were too small to allow positive conclusions, the results certainly argue that further efforts would be profitable.

SOME DISTRIBUTIONS UNDER SPECIAL CONDITIONS

To learn more about the nature of the gas pedal and steering wheel tasks, it was decided to test a set of subjects under special conditions. The objectives in selecting the tasks were (1) to determine how well the drivers could manipulate the controls and the vehicle when specifically trying to do so, and (2) to determine the effects of increased complexity on controlling behavior.

The special conditions for accomplishing these objectives were created by running five subjects in the same manner as the previous 15, except that on the return (southbound) trip, these five were given a set of special tasks to perform, as follows:

1. Normal driving.
2. Hold steering wheel motionless.
3. Hold gas pedal motionless.
4. Hold a constant velocity.
5. Maintain a fixed lane position.
6. Follow a vehicle at a constant headway.

These tasks were performed one at a time. A movie camera was mounted on the vehicle roof and synchronized with the recorder to take one frame every 2 sec. One of the five subjects was a novice driver who had never driven on a freeway or an open highway, and who was about halfway through a driver training course.

For two of the subjects, the films were used to produce curves representing lane placement over the route. These were plotted above the steering wheel trace on the data film. The measures used consisted principally of amplitude and period distributions for steering wheel and gas pedal movements for the special conditions and for normal driving.

The results for the normal driving conditions are shown in Figure 35, which presents a set of distributions for three subjects and the four measures. The most obvious feature is the substantial similarity between subjects. The strong similarity between subjects for the normal driving condition is fortunate, for it makes evaluation of responses to the special task possible.

The gas pedal task affected the steering wheel distributions more than they did the gas pedal distributions. Steering wheel amplitudes became smaller and periodically were "flattened" (i.e., spread over a wider range of times

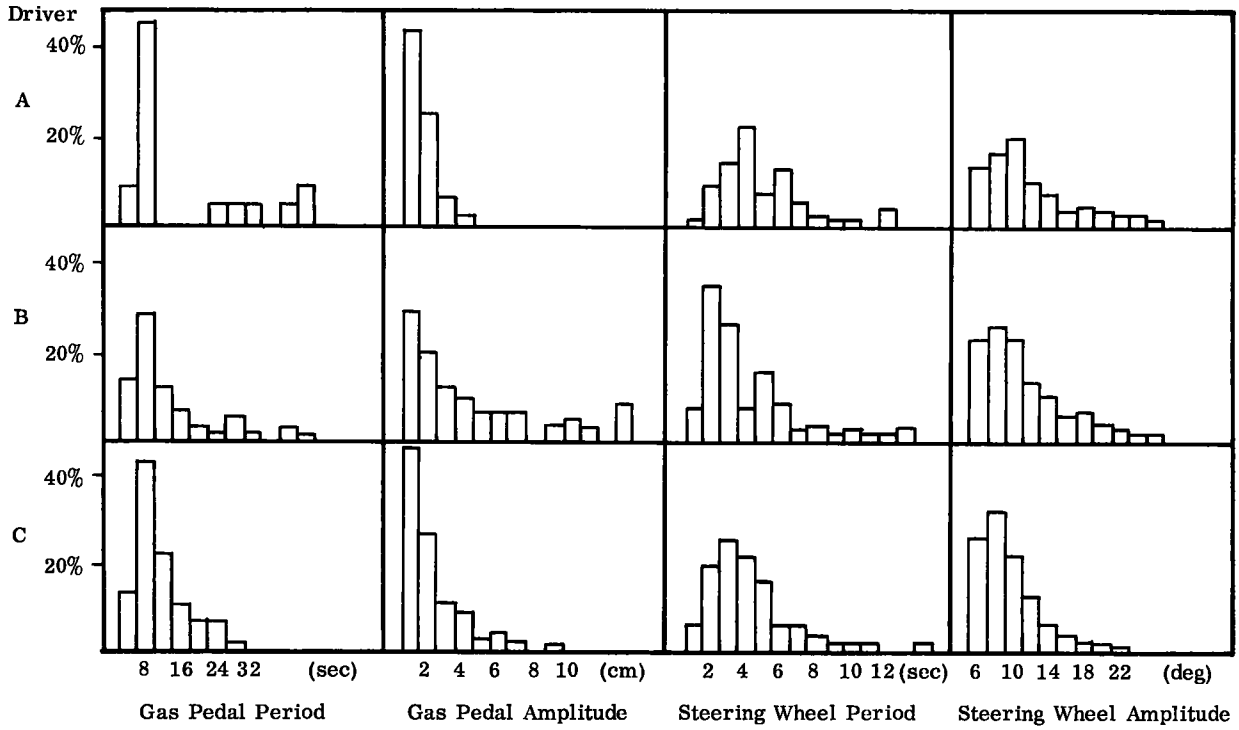

Figure 35. Frequency distributions for steering wheel and gas pedal movements under normal driving.

with the longer periods receiving proportionately more of the distribution). The constant throttle task had a sizeably larger effect on the steering wheel distributions than did the constant velocity task.

The gas pedal period distribution was flattened some. The gas pedal amplitudes were changed to the extent that a low-amplitude "spike" developed, whereas the distributions otherwise looked the same. All of the gas pedal effects were more severe for the constant throttle task.

The car-following task had negligible effect on steering wheel amplitudes. This was not true of steering wheel periods, however, as these became longer and more uniformly distributed for all subjects. The novice driver used more medium gas pedal periods than for normal driving, while the only change in the other distributions was the elimination of the longer periods. Although changes in gas pedal amplitude were not great, it was surprising to find that the distributions became more similar to each other than they were for the normal case (with respect to Fig. 35, subjects A and B became more like subject C).

In this series of tests all the subjects were free to adjust their behavior as they saw fit. Plots of lane position and the velocity curve were examined for relations between vehicle output and driving behavior.

The lane position plots showed little change as a function of task. During the steering wheel steady task there was a tendency to drift to one side or another for longer periods of time than with other conditions. It is well worth noting that performance in the lane position task was no better than at any other time. Indeed, the vehicle tended to deviate more sharply (although no farther) than during other tasks.

The velocity was well controlled during the constant velocity task. No unusual velocity patterns were evident for other tasks.

SUMMARY

The studies reported in this chapter point out several characteristics of the relation between driving control behavior and roadway geometry. For example, it was found that subject gas pedal traces appeared to share basic frequencies. When a subject's gas pedal performance was separated for level sections of road and for hilly sections of road, differences in the frequencies and amplitudes were evident. For hilly sections the frequency of gas pedal movements decreased but the amplitude of these movements increased.

This study also showed that deviations from a target velocity are most likely to occur on the upgrades and downgrades of sags and crests in the road. This fact may explain why other studies have shown that accident rates are higher at these points. Several regression analyses indicated that gas pedal movements bore a time-based relation to acceleration and velocity Differences between subjects prevented the quantification of the effects of time.

When several different types of tasks were imposed on the subject drivers, they were found to have effects on both steering wheel movements and gas pedal movements, even though each task specified control of only one of these two variables.

CHAPTER SEVEN

SYSTEMS CONCEPTS IN INTERVEHICULAR COMMUNICATION

Throughout automotive history the design and development of automotive communication systems has proceeded on an evolutionary basis wherein new and special-purpose components have been added and others removed with little consideration for the system as a whole. Some consequences of this form of development are evident in visual communication systems employed on today's automobile. Red running lights are used in the rear, although the color red usually denotes STOP. As a result, the two interpretations are frequently confused. Similarly, turn signals vary in color, depending on whether they appear on the front or the rear of a car, with flashing amber (CAUTION) at the front, and flashing red (STOP) at the rear. One could continue with the list. This is the consequence of installing special-purpose components, which may perform very well individually but collectively lose effectiveness when viewed in a systems context. Highway signing reflects similar difficulties. Signing varies from state to state and the oft-used negative instruction merely tells the driver what he cannot do but not what he should do. Combined left- and right-hand exit patterns on freeways fail to recognize the inherent increase in response time where the number of possible choices is increased from two to three.

GENERAL SYSTEMS CONCEPTS

Inasmuch as this chapter seeks to suggest ways in which system considerations may be applied to the design of future intervehicular communication systems, it is well to define some terms. A *system* may be considered as a set or collection of interconnected elements (man and machine, human and hardware) which are organized to accomplish stated functions under the influence of an external environment. *Systems research* seeks to describe the manner in which the elemental variables of the system are related to system effectiveness in performing these functions. *Systems design* involves selecting component elements so as to achieve desired performance levels for the system.

SYSTEMS DESIGN

Inasmuch as interest here is in the design of intervehicular communication systems, it is useful to examine several of the problems that must be considered during systems design. One way of describing the system design procedure might include the following questions:

1. What are the functions that the system will be expected to carry out? Functions should be stated as explicitly as possible, and the list should include those constraints to which adherence is desired.
2. What are the expected environments under which the functions are to be carried out? The environment within which a system operates encompasses all of those external factors which influence system performance but are not part of the system. Under this broad interpretation the environment may include economic, political, sociological, and psychological factors, as well as other relevant aspects of the physical surroundings.
3. What are the hardware elements (man and machine) which may be available for consideration as system components? These should include not only all currently available or technologically feasible items, but also those which may be expected to be available or feasible at the time the system is to be assembled. Accordingly, with due concern for development lead times and conservative forecasting, the hardware which the designer may include in his system considerations should span the range from items currently on the shelf to those which have a high probability of being available years hence.

Answers to these three questions provide the groundwork for systems design, for they should make possible concise statements about what is to be accomplished, the resources that may be used, and the conditions under which the system may be expected to perform.

As pointed out earlier, the design task is essentially one of specifying components to be assembled into the system. This activity requires the information specified by:

4. What is each component's individual performance in doing its specified task? Performance should be stated behaviorally rather than by physical dimension (e.g., a flashing turn signal should be described in terms of the same variables that would portray the effectiveness of the driver's arm signal).

It should be noted that this is not the same as asking how the component will perform as part of the system. Instead, the concern here is with behavior and performance that might be recorded in bench tests or certain specialized trials. Concise performance information on individual components will not always be available to systems designers, although evaluations of groups of components or subsystems are often adequate in filling information needs. The signal system test described earlier in this report represents information of this kind.

Given the information specified by the preceding four questions (which are not exhaustive) the designer must specify groupings of components as conceivable systems. The processes by which this is done are complex and have not been formalized without unacceptable losses to realism. The products of this activity, however, may be readily described as "candidate" systems. The phases of systems design in which we are most interested are those which involve the evaluation of these candidate systems.

TABLE 24

ACCIDENT ANALYSIS, CONNECTICUT TURNPIKE

LOCATION	TIME PERIOD [a]	DAY ACCIDENTS	DAY MILLION VEH.-MILES	DAY RATE	NIGHT ACCIDENTS	NIGHT MILLION VEH.-MILES	NIGHT RATE	DUSK ACCIDENTS	DUSK MILLION VEH.-MILES	DUSK RATE
(*a*) BEFORE LIGHTING INTENSITY REDUCED FROM 0.6 TO 0.2 FC (JAN. 1, 1961–JAN. 17, 1964)										
West of test section	1	138	118.90	1.161	109	53.51	2.037	44	25.12	1.752
	2	98	164.14	0.597	56	39.55	1.416	20	26.06	0.752
	3	90	168.08	0.535	35	30.47	1.149	12	21.80	0.550
	4	123	226.08	0.544	47	47.99	0.979	37	33.11	1.118
	5	107	180.84	0.592	110	81.24	1.354	70	34.90	2.006
	6	147	111.08	1.323	179	78.18	2.289	46	22.03	2.088
Test section	1	35	25.58	1.368	26	9.65	2.696	13	5.33	2.438
	2	31	34.41	0.901	18	7.02	2.563	5	4.89	1.023
	3	38	34.27	1.109	6	5.02	1.196	5	4.19	1.194
	4	31	46.57	0.666	2	7.92	0.252	4	5.98	0.669
	5	32	38.91	0.822	27	14.06	1.920	21	7.09	2.962
	6	52	24.84	2.094	27	13.16	2.052	23	5.14	4.471
East of test section	1	59	48.74	1.211	32	18.38	1.741	14	10.16	1.378
	2	59	65.33	0.903	11	13.33	0.825	4	9.28	0.431
	3	36	66.14	0.544	10	9.68	1.033	6	8.08	0.743
	4	67	91.44	0.733	11	15.54	0.708	1	11.72	0.085
	5	42	74.45	0.564	18	26.85	0.670	11	13.56	0.811
	6	38	47.18	0.805	35	25.00	1.400	12	9.77	1.228
(*b*) DURING TEST LIGHTING AT 0.2 FC (JAN. 18, 1964–NOV. 23, 1964)										
West of test section	1	58	46.25	1.254	53	20.75	2.254	22	9.74	2.259
	2	76	62.17	1.223	42	14.92	2.816	12	9.90	1.212
	3	46	65.05	0.707	23	11.97	1.951	8	8.84	0.948
	4	68	88.08	0.772	32	18.70	1.711	20	12.90	1.551
	5	56	69.40	0.807	54	30.94	1.745	11	13.39	0.822
	6	—	—	—	—	—	—	—	—	—
Test section	1	18	9.77	1.843	9	3.68	2.445	5	2.03	2.457
	2	18	12.83	1.403	7	2.61	2.685	2	1.82	1.098
	3	17	13.02	1.306	4	1.90	2.100	2	1.59	1.258
	4	15	17.89	0.838	2	3.04	0.657	6	2.30	2.614
	5	27	14.77	1.828	14	5.31	2.635	4	2.69	1.487
	6	—	—	—	—	—	—	—	—	—
East of test section	1	14	18.66	0.750	9	7.04	1.297	1	3.89	0.257
	2	10	24.56	0.407	17	4 99	3.407	1	3.49	0.287
	3	23	25.07	0.917	10	3.67	2.726	1	3.06	0.327
	4	25	35.19	0.710	8	5.98	1.337	6	4.51	1.330
	5	23	28.29	0.813	16	10.16	1.574	9	5.15	1.748
	6	—	—	—	—	—	—	—	—	—
(*c*) AFTER LIGHTING INTENSITY RESTORED FROM 0.2 TO 0.6 FC (NOV. 24, 1964–DEC. 31, 1965)										
West of test section	1	59	48.84	1.208	55	21.97	2.503	23	10.34	2.225
	2	33	67.67	0.488	30	16.24	1.848	9	10.74	0.838
	3	76	69.56	1.093	16	12.61	1.269	7	9.03	0.775
	4	63	95.24	0.661	33	20 25	1.630	12	13.96	0.860
	5	57	76.00	0.750	38	33.82	1.123	14	14.66	0.955
	6	91	77.44	1.175	113	54.49	2.074	28	15.34	1.826
Test section	1	9	10 26	0.877	13	3.88	3.353	7	2.15	3.260
	2	11	13.89	0.792	2	2.82	0 709	1	1.97	0.508
	3	10	13.82	0.724	5	2.02	2.473	4	1.69	2.371
	4	14	19.13	0.732	8	3.25	2.458	5	2.46	2.035
	5	17	15.83	1.074	5	5.69	0.879	3	2.88	1.041
	6	24	17.03	1.409	14	9.03	1.551	21	3.52	5.958
East of test section	1	13	20.19	0.644	13	7.63	1.704	6	4.22	1.420
	2	17	27.21	0.625	8	5.52	1.448	4	3.86	1.037
	3	18	27.45	0.656	6	4.02	1.494	1	3.35	0.298
	4	20	38.38	0.521	8	6.52	1.226	2	4.03	0.497
	5	17	31.39	0.541	8	11.27	0.710	2	5.71	0.350
	6	41	33.20	1.235	25	17.60	1.421	9	6.87	1.310

[a] Time period: 1 = Jan. 18 to Mar. 21; 2 = Mar. 22 to May 21; 3 = May 22 to July 10; 4 = July 11 to Sep. 11; 5 = Sep. 12 to Nov. 23; 6 = Nov. 24 to Jan. 17

TABLE 25

SUMMARY OF ACCIDENT RATES

PERIOD	LOCATION	LENGTH (MI)	ACCIDENT RATE (/M VEH-MI) DAY HOURS	NIGHT HOURS	DUSK HOURS	ALL HOURS
1/1/61–1/17/64	West	27	0.72	1.62	1.40	1.00
	Test	4	1.07	1.87	2.18	1.34
	East	16	0.77	1.08	0.77	0.83
	All	47	0.78	1.53	1.35	1.00
1/18/64–11/23/64	West	27	0 92	2.10	1.33	1.20
	Test	4	1.39	2.18	1.82	1.58
	East	16	0.72	1.88	0.90	0.94
	All	47	0.93	2.06	1.29	1.19
11/24/64–12/31/65	West	27	0.87	1.79	1.26	1.13
	Test	4	0.95	1.76	2.80	1.32
	East	16	0.71	1.29	0.86	0.81
	All	47	0.84	1.68	1.35	1.09

metry in the analysis, the sixth time period was excluded from all epochs, before, during and after.

Accordingly, a full-dress linear (Gaussian) model was constructed to represent the (square root of the) accident rate in each observed subclassification. Briefly, a parameter β was introduced to measure:

1. The general mean (β_1).
2. Each of the two independent comparisons between the overall accident rates of "east," "west" and "test" (β_2, β_3).
3. Each of four independent comparisons between the five "season" results as a whole (β_4, β_5, β_6, β_7).
4. Each of the two independent comparisons between the "before," "test" and "after" accident rates β_8, β_9).
5. Each of the two independent comparisons between the overall accident rates during "day," "dusk" and "night" (β_{10}, β_{11}).
6. Each of the four "interactions" between the two effects measured under item 4 and the two measured under item 5 (β_{12}, β_{13}, β_{14}, β_{15}). In particular this becomes a measure of the change in accident rate as related to nighttime lighting conditions.

This produced a model with 15 parameters, one of which was a measure of the difference between the before-and-after accident differential and the test differential, where this "differential" represents the difference between the accident rates during the hours of daylight, dusk, and night, respectively. Clearly this difference (if really present) is a measure of the possible increase in nighttime accidents caused by reducing the lighting during the test period. Because of the likelihood that this increase would be different (i.e., larger) during the winter months a further (16th) parameter was added to measure this difference for season 1 (winter) only.

The analysis of variance indicated the following:

1. The difference in accident rates (day, night, and dusk combined) is significant.
2. There is a significant difference in the accident rates during the "seasons" of the year at all locations.
3. The difference in accident rates by location (east, west and test) is significant.
4. The difference in accident rates "before" and "after" the period of lighting at 0.2 fc is not significant, but the accident rate during the 10 months at 0.2-fc lighting is significantly different from the other two "epochs." This applies to day, dusk and night.
5. Finally, and most important, although the night accident rate in the test section (2.18) at 0.2 fc is higher than the night accident rate in the test before and after (1.87 and 1.76) the difference is *not* statistically significant. It will be observed in Table 25 that all rates, by lighting condition and location, went up during the 10 months of testing.

In reviewing the results of the accident analysis it is well to recall that there were only 36 reported accidents at the test section during the 10-month test interval. This is a very small sample from which to detect significant differences in accident rates. The relatively accident-free characteristics of the Turnpike make it difficult to detect significant differences in accident rates over a short time and distance as used for this analysis.

date system using components found to be individually effective in these and other studies. One such system might be based on the Tri-light system, and would include the following as major signaling components:

1. Headlights.
2. Amber front turning signals.
3. Red rear brake lights.
4. Two rear amber lights, activated when both the brake pedal and the gas pedal are not depressed.
5. Two rear green lights, activated by a depressed gas pedal.
6. Amber rear turn signals, integral with the solid amber described in item 4.

The first three of these components are carry-overs from the current signal system. From the front of the vehicle, this candidate system differs from conventional ones in that front running lights are omitted. Table 33 noted that front running lights and headlights both perform the same task, that of frontal identification; it follows that information losses by deleting front running lights may not be significant. (In 1964 the Joint Vehicular Signal Systems Committee issued a resolution that front running lights be discontinued in new automobiles.)

At the rear of the vehicle, the Tri-light components furnish an active signal at all times. The rear turn signals are amber, in keeping with the "amber-means-caution" theme.

This candidate signal system communicates four kinds of information, as follows:

1. The presence of the vehicle and its direction relative to the observer.
2. The driver's intent to turn, and his intended direction.
3. Whether or not the vehicle is braking.
4. Whether or not the gas pedal is depressed.

In communicating these four kinds of information the candidate system uses six components, whereas the conventional system requires six to communicate three kinds of information.

Table 34 lists the possible combination of these components that would be seen by a nearby vehicle. Generally, it appears that the major ambiguities are not present, although some practical test would probably be required to determine if the direction of the vehicle (same direction or oncoming) would always be clear when a turn signal is activated during a coast without headlights.

Extensions of this procedure could consider other dimensions of signal components, such as color, shape, size, number, location, periodicity, and intensity. Other signal functions must also be considered; e.g., side running lights to permit detection of vehicles perpendicular to a direction of travel.

An attempt has been made to suggest several considerations that should be made in evaluating a candidate signal system. Systems design concepts mean that one should think of the total system or collection of components, rather than individual signaling devices. To be realistic, evaluations of performance must be made in terms of the system, and consequently avoid the dangers of sub-optimizations.

The foregoing discussion of intervehicular communication cannot be divorced from roadway communication to the driver via signs or reflectors, such as white lines and traffic control devices. In view of current highway communication systems, vehicle components must also be viewed in this context.

CHAPTER EIGHT

DEVELOPMENT OF AN INFRARED SOURCE-SENSOR SYSTEM

The goal of this portion of the project was to develop a means for monitoring the speed and distance of the preceding vehicle on a highway and to build and test a preliminary model of the equipment in a vehicle. This information was to be presented in some easily recognizable form in the following vehicle. No consideration was given to distinguishing between lanes in this preliminary model. The near infrared region of the spectrum was chosen for study because of its superior transmission in fog compared to the visible region and because ordinary glass optical systems could be used. Another consideration for choosing the near infrared region is that drivers in trailing vehicles would not be affected by the powerful light of the modulated source.

A system has been built and tested which will detect at a distance of 400 ft in clear weather. This system utilizes a pulsed infrared beam in the preceding vehicle, with the pulse frequency proportional to the preceding vehicle's speed. The relative speed of the following vehicle compared to that of the preceding vehicle is displayed on a meter, which reads miles per hour, in the following vehicle. The distance between the two vehicles is determined by measuring the signal strength at the following vehicle. It has been recognized that the signal strength for a given distance will vary with weather conditions, but it was chosen as a simple initial approach to the problem. This distance information is also displayed on a meter in the following vehicle.

DESCRIPTION OF SOURCE-SENSOR SYSTEM

The source for this system, which is placed at the rear of the preceding vehicle, is a pulsed infrared beam with the pulse frequency being a function of the vehicle speed. The light used was a 4½-in. sealed-beam lamp with a maximum rated initial candlepower of 35,000. The approximate beam, spread to 10 percent of maximum intensity, is ± 11° in the horizontal direction and ± 4½° in the vertical direction. The maximum rated initial candlepower is the intensity in the center of the beam at the source. The intensity at some distance, x, from the source is $35{,}000/x^2$, where x is given in feet. Thus, the intensity at 100 ft is 3.5 foot-candles. A Wratten No. 87C filter is used in front of the source. This filter eliminates the visible light, which could be a traffic hazard, and passes the near infrared light.

The pulsing of the source is provided by a rotating three-bladed disc in front of the lamp and filter and coupled to the output of a differential gear. One input to the differential is coupled to a rear wheel of the preceding vehicle through a flexible shaft. This provides an output signal frequency which is a function of the vehicle speed. The second differential input is coupled to a constant-speed motor. This serves to provide a pulsed beam when the preceding vehicle is stopped on the highway. A curve of source output frequency as a function of preceding vehicle speed is shown in Figure 36. Photographs of the source unit as it appears on the preceding vehicles are shown in Figure 37.

The sensor unit has been designed to detect the signal from the preceding vehicle and convert it to speed. This signal is then compared with the speed of the following vehicle. The following vehicle speed minus the preceding vehicle speed is displayed on a meter, calibrated in miles per hour, in the following vehicle. Once the following vehicle is close enough for the detector to lock onto the preceding vehicle (about 400 ft in clear weather for the units tested), the driver is always aware of whether he is traveling faster, slower, or at the same speed as the preceding vehicle. The intensity of the signal from the preceding vehicle is also measured and used as a measure of distance. It is realized that the signal intensity at a given distance will be a function of the weather conditions, but it was felt that this would be a simple method in which to approach this problem initially. This distance information is also displayed on a meter so that the driver of the following vehicle knows the distance between the preceding and following vehicles.

The actual distance between the vehicles is determined by pre-calibrating the distance measuring meter and circuit. The error in this method is estimated to be ± 12 percent, due to aberrations in the lens used with the detector. Dis-

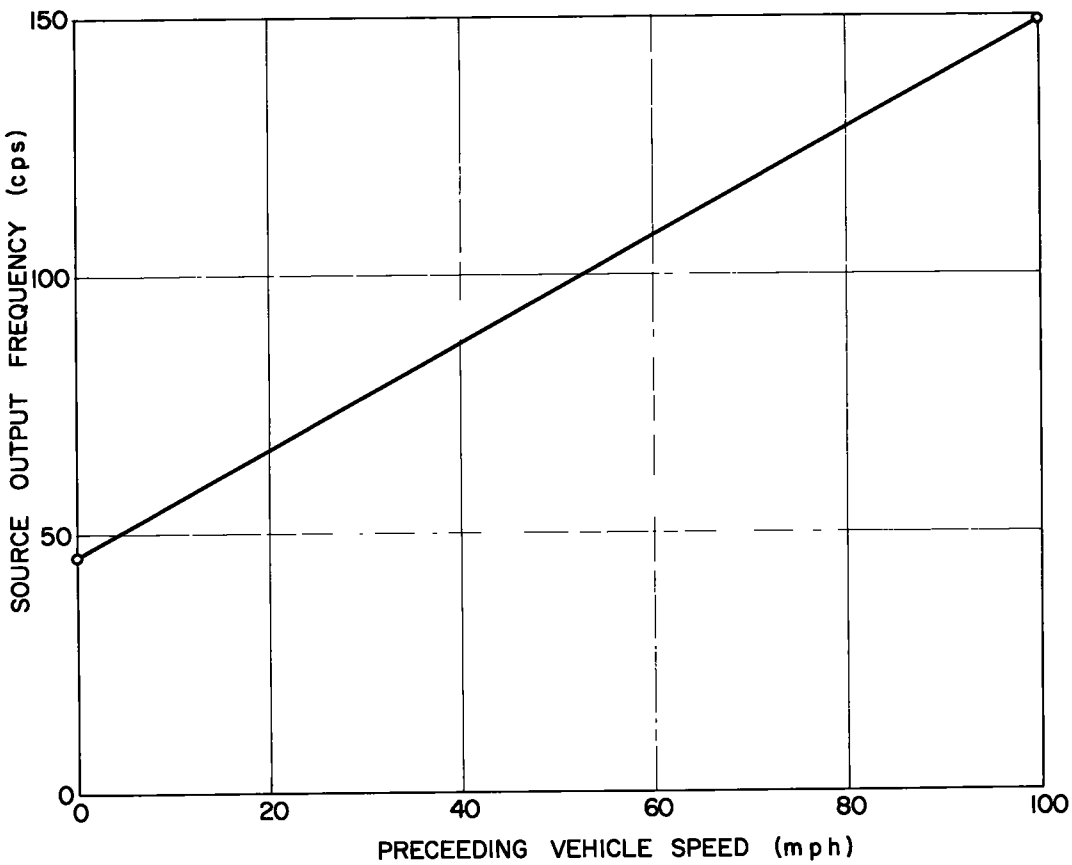

Figure 36. Source output frequency as a function of preceding vehicle speed.

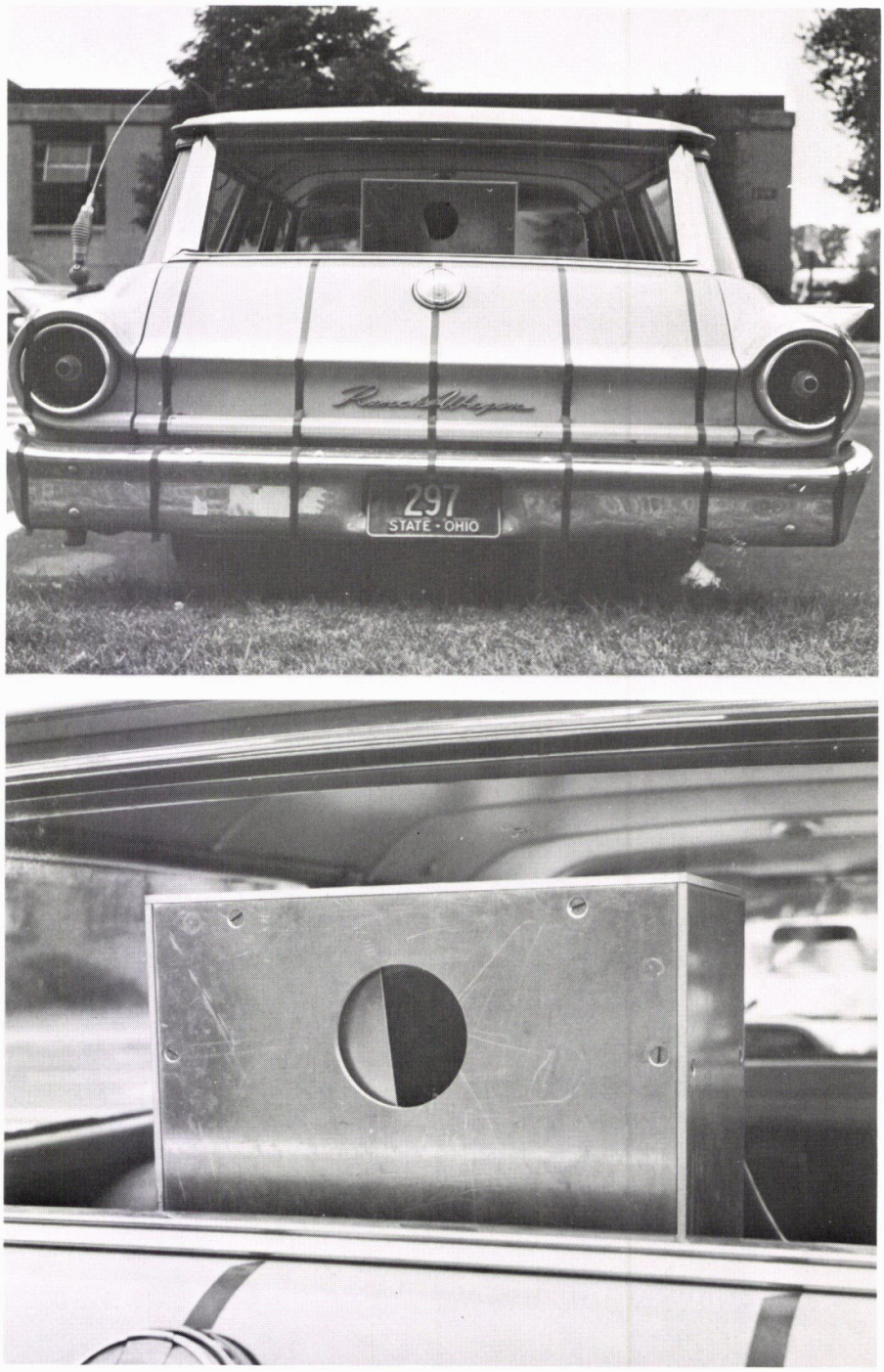

Figure 37. Source for source-sensor system in preceding vehicle.

tances have also been estimated by the driver of the following vehicle by using the reflector posts along the side of the highway. It is estimated that an accuracy of ± 50 ft can be obtained in this manner at a separation distance of 400 ft.

An attempt was made to use the calibrated telescope of a surveying transit to measure distance. Black vertical lines were placed on the rear of the preceding vehicle and a passenger in the following vehicle attempted to determine distance by aligning the lines on the rear of the preceding vehicle with the distance calibration lines in the telescope. At a distance of 400 ft, the error was ± 100 ft using this system. Because of its ease of use, the distance measuring circuit in the detector is being used at this time to determine the actual distances between the vehicles.

The circuit diagram for the sensor is shown in Figure

38. The detector shown is an RCA type SQ 2516 photojunction cell. This cell is sensitive in the region of 3,000 to 19,000 angstroms, with the peak at 15,000 angstroms. Its point of greatest sensitivity is in the near infrared, as desired. The output from the photojunction cell drives an emitter follower. The output from the follower drives the amplifiers for the frequency measuring circuit and the amplifier for the distance measuring circuit. A gain control is provided in the distance measuring circuit to calibrate the meter. A gain control is provided in the frequency measuring circuit and is usually set so that there is little or no noise signal output with zero signal input. Following the three standard amplifier stages in the frequency measuring section is a saturated amplifier. This amplifier must operate in saturation in order to maintain the magnitude of the output impulses constant. This output is fed to a capacitor charge and discharge type of frequency meter. By making the voltage to which the capacitor is charged independent of the frequency, the average current flowing out of the capacitor is proportional to the number of pulses per second if the time constants of the charge and discharge circuits are short compared to the pulse widths. In this case, the time constants are 62.5×10^{-6} sec and the shortest pulse widths are 300×10^{-6} sec, which satisfy the required conditions. A direct-current signal is also fed to the meter to balance out the zero speed signal from the preceding vehicle. A third signal, which comes from a tachometer generator driven from the following vehicle speedometer cable, is also applied to the meter. This provides the balance signal for producing the difference output. With these three input signals, the meter reads directly the difference in speed between the following vehicle and the preceding vehicle.

Figure 39 shows the lens mount of the sensor unit as it appears on the following vehicle. The detector itself is located in the rear of this mount. The meters and their associated circuitry are shown in Figure 40, with the relative speed meter on the left and the distance meter on the right. A typical measured calibration curve for the distance measuring circuit is shown in Figure 41.

MEASUREMENTS IN HIGHWAY VEHICLES

The source and sensor units described in the preceding section have been installed in two vehicles as shown in Figures 37, 39 and 40. Several hundred miles of road tests have been made in order to determine the best operating conditions for this equipment. Tests have been made in daylight under conditions of bright sunlight and cloudy overcast. Tests have also been made at night. All of these tests have been made in clear weather free from any degree of fog.

The best results to date have been obtained by using a 4⅜-in. diameter, 10-in. focal length, single-element lens

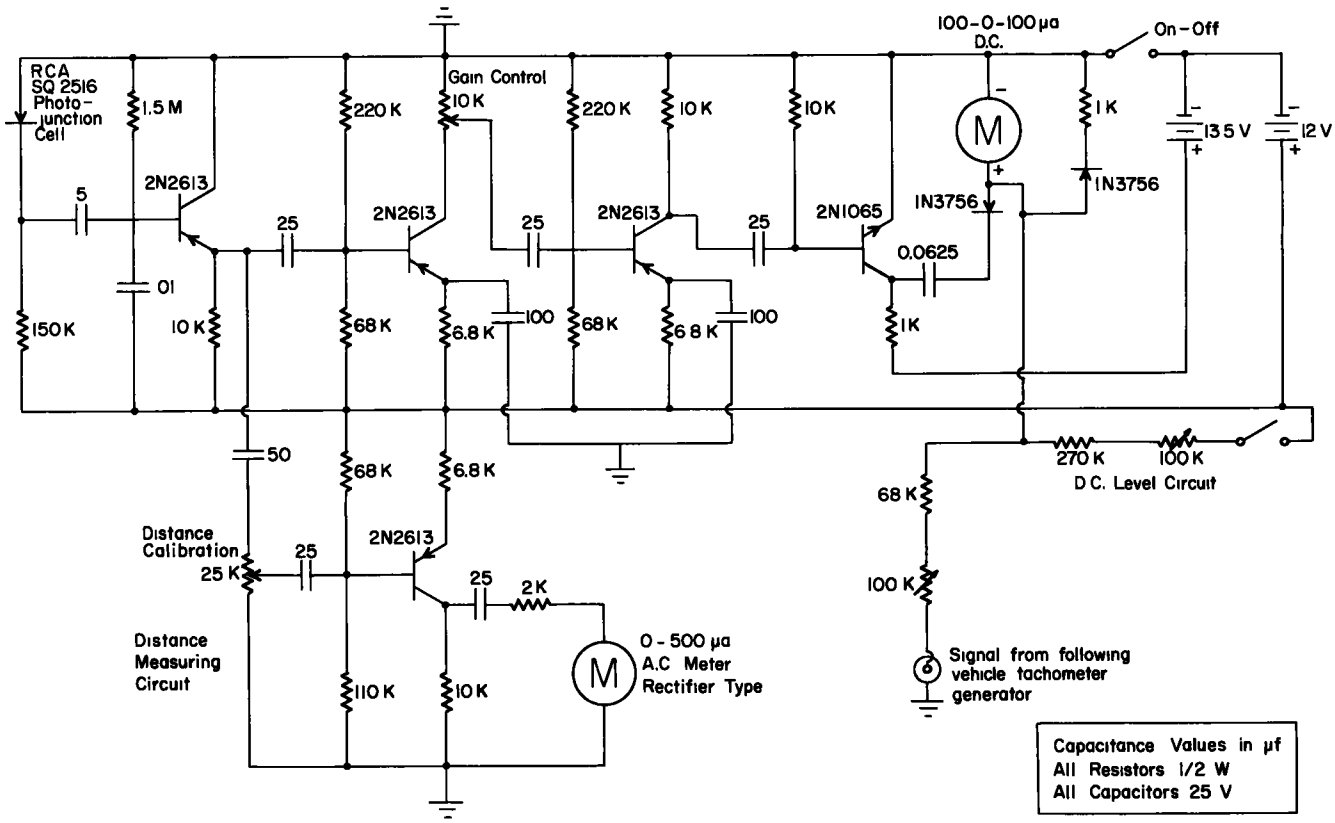

Figure 38. Circuitry for infrared speed and distance sensor.

Figure 39. Lens mount and detector mounted on bumper of following vehicle.

in conjunction with the S Q 2516 photojunction cell. This lens is located 7¼ in. in front of the detector and the whole unit is attached to the front bumper of the following vehicle 12 in. above the road surface. By locating the detector away from the lens focus it is not necessary to maintain exact alignment of the system. This allowance for some misalignment has made use of a tracking system for the detector unnecessary.

The best results with this system have been obtained at night or during periods of heavy clouds during the daytime. During these times it is possible to operate the following vehicle at a distance of 400 ft behind the preceding vehicle with no apparent noise appearing on the relative speed meter. For any distance up to 400 ft there are no fluctuations in this meter reading due to approaching cars, passing cars, or passing through underpasses. When the vehicles are operated under conditions of bright sunshine, the noise from all sources is less than ± 3 mph on the relative distance scale for distances up to 200 ft. For distances up to 400 ft, the maximum noise is ± 6 mph when traveling away from the sun. When traveling directly toward the sun the noise at this distance becomes ± 30 mph and higher. Even though there are conditions under which the noise can become large, it should be noted that the system operates best when it is most useful, such as at night.

A typical curve showing the distance meter reading as a function of distance between the vehicles has been shown in Figure 41. At constant distances up to 400 ft, the meter variation is ± 7 percent full scale at worst.

One attempt was made to use a radar unit to determine relative speeds of the two vehicles, but the results were poor. The unit used was an Electromatic Radar Speed Meter, Type S-5, operating at 10,515 megacycles. The reading on the meter was always the speed of the vehicle in which the radar unit was located. It was apparently receiving back from the highway surface a reflection which was always stronger than the reflected signal from the vehicle ahead.

SUMMARY AND POSSIBLE FUTURE DEVELOPMENT

A source-sensor, near infrared unit has been designed and tested which will give the driver in a following vehicle the

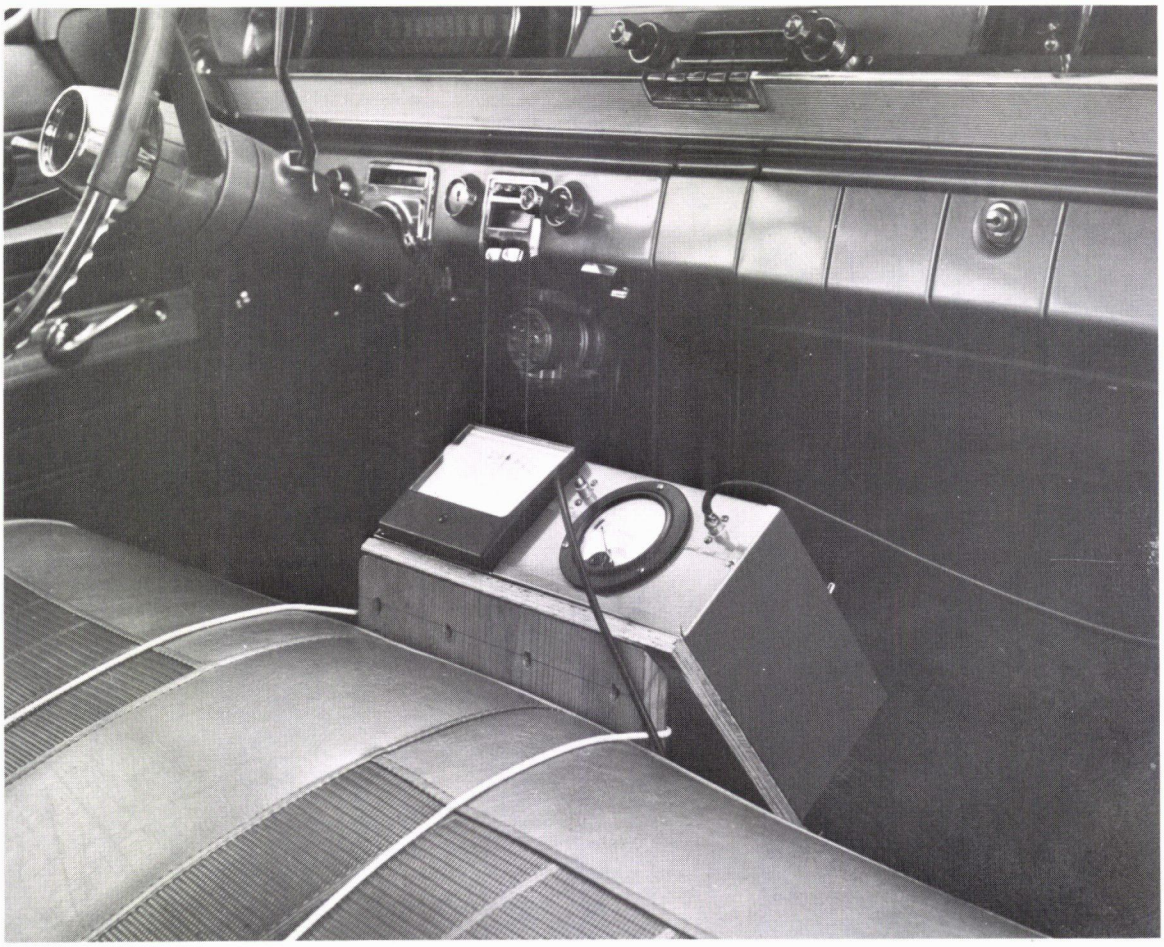

Figure 40. Meters and associated circuitry of sensor mounted in following vehicle.

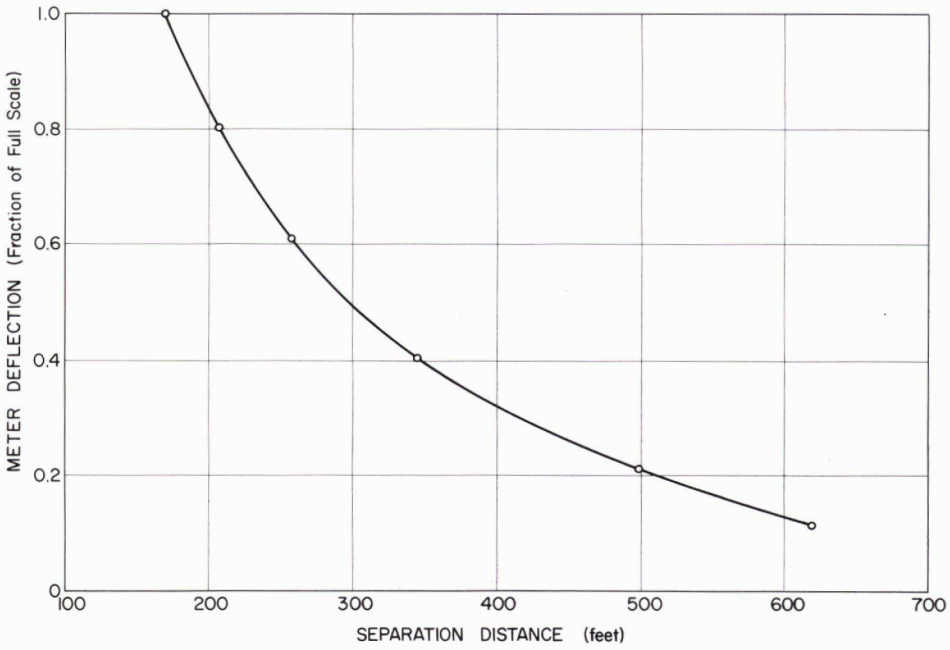

Figure 41. Typical measured calibration curve for distance measuring circuit.

speed of a preceding vehicle compared to that of the following vehicle, as well as the distance between the two vehicles. The detector in the following vehicle will operate at a distance of 400 ft behind the preceding vehicle in clear weather. Further work needs to be done with this system in regard to a smaller unit, particularly a smaller, more compact, and more powerful source. The present system has not been intended as a typical usable system, but rather as a system necessary to demonstrate the possibilities of this type of system.

It is quite evident that work needs to be done with regard to evaluation of this type of system in various types of weather other than ideal. It would certainly be desirable to test the operation of this type of system in fog, or in a snowstorm, or in a combination of both. Taylor and Yates * have shown that the transmission at a wavelength of 1.6μ over a 3.4-mile path in a snowstorm is 0.56×10^{-3} of that in clear weather. The transmission in light rain mixed with fog is 6.83×10^{-3} of that in clear weather over the same path at 1.6μ. There is a definite need for data of this type at the shorter distances encountered in highway traffic. It would probably also be necessary to test higher-intensity sources for these types of weather conditions.

It would also be desirable to develop a system in which the source and sensor are both located in the following vehicle. The information reaching the sensor would then be reflected back from the preceding vehicle. This system has the advantage of placing no reliability on equipment in the preceding vehicle.

As of yet no consideration has been given to distinguishing between lanes with this system. This problem should also be investigated and some system should be built and tested which considers lane coding.

* Taylor, J. H , and Yates, H. W. *Jour. Opt. Soc. Am.*, Vol. 47, p. 223 (1957).

CHAPTER NINE

INVESTIGATION OF USE OF TURN SIGNALS

Included in the research into sensing and communication between vehicles was a determination of the limitations of driver-actuated communications using turn signals as a control.

It was felt that the extent of turn-signal use under various conditions could indicate the extent to which drivers would make use of any proposed driver-actuated communication system. During the summer of 1963, an extensive field study was conducted, in the course of which more than 10,000 turning movements and lane changes were observed. The results of statistical evaluation of the data were used to test hypotheses on driver turn-signaling behavior.

Objective of Study

The objective of this study was to determine the limitations of turn signals as an intervehicular communication system. The method employed to achieve this objective was to (a) observe the frequency of turn signaling at selected test sites under a variety of conditions, (b) relate the observed frequencies to elements of the driver-vehicle-road complex, and (c) interpret the results in accordance with the objective of the study.

The application of this method is based on two initial hypotheses, as follows:

1. Different environmental conditions will evoke different frequencies of turn signaling. Factors chosen to represent environment were:
   (a) Presence of preceding vehicle.
   (b) Signaling behavior of preceding vehicle.
   (c) Presence of following vehicle.
   (d) Geometric design of test site.
   (e) Traffic control.
2. Frequency of turn signaling at a given location will be influenced by the following characteristics of the driver-vehicle unit:
   (a) Sex of driver.
   (b) Presence of passengers.
   (c) Direction of turn.
   (d) Age of vehicle.
   (e) Type of vehicle.

These two hypotheses constitute the basis for the selection of test sites and methods of field studies.

There are two basic conditions under which turn signals are used: (a) indication of intention to change direction of travel, as at intersections or at exit ramps; and (b) indication of intention to change lane, as during overtaking or shifting into turning lanes.

Observations of signaling under both conditions were included in the field studies.

TURN SIGNALING

The turn-signal system on recently manufactured automobiles in the United States consists of electric turn-signal lamps, which indicate, by flashing, the driver's intention to turn. Turn-signal lamps are mounted on both the front and the rear of the vehicle, the front-mounted lamps emitting white or amber light and the rear-mounted lamps

generally being an integral part of the red running and STOP lights.

The turn signal is for information only. It does not assure the turning vehicle the right-of-way. Under certain conditions, however, signaling will result in some kind of courteous response from drivers of other vehicles (for example, drivers will sometimes increase the gap for vehicles intending to merge).

It is readily observable at any intersection that not all drivers signal their intention to turn. Every driver will likewise know from his own driving experience that the same driver will not always signal under similar conditions. A novice driver is instructed to signal every turn, but through experience each driver will eventually develop personalized driving habits.

It is assumed that the tendency to expedite traffic movement, to reduce risk, and to follow traffic rules, together with the element of courtesy, all influence drivers in their turn-signaling behavior. Temporary influences, such as presence of passengers or driving a new car, will also affect turn signaling.

The low signaling frequencies observed at some locations indicate that the percentage of drivers who signal every turn and lane change is very low. Turn-signaling frequencies as low as 5 percent were observed at one location.

Legal Background

The majority of the test sites were located in the greater Columbus (Ohio) metropolitan area. Passing maneuvers were observed on the Ohio section of Interstate Route 71. The study of legal requirements concerning the use and specifications of turn signals, therefore, confined itself principally to sections of the Motor Vehicle Laws of Ohio and the Traffic Code of Columbus, Ohio.

The (Ohio) Uniform Vehicle Code was last revised in 1962. This revision included sections affecting turn signaling. For this reason a comparison was desirable with the older (1959) Motor Vehicle Laws of Ohio and the Traffic Code of Columbus. Sec. 11-604 of the Uniform Vehicle Code explicitly declares that "No person shall . . . turn any vehicle without giving an appropriate signal. . . ." This signal is to be given continuously during at least the last 100 ft before turning. The corresponding section of the Traffic Code of Columbus, Ohio (2131.11), is in full agreement with the Uniform Vehicle Code, requiring that "Drivers of vehicles . . . before turning . . . or changing their course shall give notice of intention by executing hand arm signals. . . ." Both sections require signaling of each turn, regardless of the presence or absence of vehicles in the immediate vicinity of the turning vehicle. The Motor Vehicle Laws of Ohio, however, required drivers to signal their intentions to turn only "in the event any traffic may be affected by such movement."

It was felt that a sample of drivers should be interviewed to provide an estimate as to what extent drivers are familiar with the traffic code regarding turn signaling. A sample of 200 drivers was interviewed. Each was asked if he knew whether the Traffic Code of Columbus required that (a) every turn be signaled, (b) turns be signaled when the situation makes it necessary, (c) signaling is optional. Drivers who claimed they did not know were asked to make a guess. The same question was asked concerning the Motor Vehicle Laws of Ohio. For both cases, only about 1 percent thought that signaling was optional. The responses are categorized as follows:

	KNEW (%)		GUESSED (%)	
LEGAL BASIS	RIGHT	WRONG	RIGHT	WRONG
Traffic Code of Columbus	24	3	55	18
Motor Vehicle Laws of Ohio	2	23	7	68

It is seen that 79 percent of the drivers interviewed knew or guessed that they must signal every turn according to the Traffic Code of Columbus. Only 2 percent knew and 7 percent guessed right that the Motor Vehicle Laws of Ohio only require signaling those turns which interfere with other traffic. In spite of the fact that only a very small group of drivers can be expected to know the corresponding traffic law, as indicated by the poll, it is shown later that a relationship between signaling frequencies and interference of traffic movement due to turns can be observed.

Statistical Analysis

The influence of a given parameter on turn signaling was evaluated by classifying turning vehicles according to the parameter investigated and comparing observed signaling frequencies. The statistical dependence of signaling frequency on the classification was evaluated by the application of 2-by-2 contingency tables. To check for consistency, several groups were formed and classified according to the same parameter. Following common practice, dependence was called "highly significant," "significant," or "not significant," as the significance level was found to be less than 0.01, less than 0.05, or greater than 0.05, respectively.

USE OF TURN SIGNALS AT INTERSECTIONS

Signaling frequencies were observed at intersections where drivers use turn signals to indicate their intention to change direction of travel. Eight test sites were chosen to represent intersections with various geometric layouts and various types of traffic control. The following is a brief description of these study sites.

STUDY SITE 1

Study site 1 is located at the intersection of North High Street and 15th Avenue. North High Street is a major arterial street in Columbus, Ohio, crossing the entire city in a north-south direction and dividing all intersecting streets into their respective east and west portions. As a typical intersecting street, 15th Avenue is designated East 15th Avenue on one side of N. High Street, and West 15th Avenue on the other side. Fifteenth Avenue constitutes a major entrance to the Ohio State University campus. Turning movements into and out of E. 15th Avenue are, there-

fore, quite heavy during most of the day. North High Street carries traffic on two lanes in each direction. Parking is permitted on both sides. There are bus stops on both the northwest and southeast corners. The posted speed limit is 35 mph. Pedestrian traffic crossing N. High Street is very heavy during most of the day. Fifteenth Avenue is a two-lane, two-way road carrying considerably less traffic than N. High Street. The proportion of the turning vehicles is much higher on 15th Avenue than it is on N. High Street.

Traffic volume on N. High Street varied between 806 and 1,023 vph during the field studies. On 15th Avenue, 172 vph represented the lowest, and 387 vph the highest hourly volume.

STUDY SITE 2

The traffic-signal-controlled T-intersection at North High Street and Oakland Park Avenue was selected as study site 2. North High Street is described in detail under study site 1. It differs in one respect from study site 1, however, in that a right-turning lane is added to the two northbound lanes approximately 100 ft in advance of the intersection. Parking is permitted on both sides of North High Street, except for the area adjacent to the right-turning lane.

North High Street carried approximately 1,100 vph at the time of data collection; Oakland Park Avenue carried less than 200 vph.

STUDY SITE 3

The intersection of Olentangy River Road and Lane Avenue was selected as study site 3. Only the southbound lanes of Olentangy River Road and eastbound left-turning lanes of Lane Avenue were studied at this test site. Both the southbound and eastbound approaches to this intersection have two lanes, which are separated from the opposing lanes by a median. At the intersection, the two approach lanes are supplemented by turning lanes.

Olentangy River Road has both right- and left-turning lanes, which extend for several hundred feet along the approach to the intersection. Lane Avenue has only one additional lane for the right turns, while one of the two main carriageway lanes is controlled as a "left turn only" lane by the traffic-actuated traffic light, which controls the entire intersection. The right-turn lanes are channeled by a raised traffic island and are controlled by YIELD signs. The left-turning movements are protected by a pre-phase green light, the length of which depends on the number of turning vehicles recorded by the traffic detector unit. At the time of the field studies, the length of the left-turning lane was always adequate to accomodate the queue of waiting vehicles. During data collection, traffic volume on Olentangy River Road amounted to approximately 1,700 vph, while Lane Avenue carried 850 vph. The posted speed limit is 45 mph on Olentangy River Road and 35 mph on Lane Avenue. Lane Avenue changes its geometric characteristics at this intersection from a four-lane divided highway west of Olentangy River Road to a two-lane two-way street east of it. There is no pedestrian traffic at the intersection.

At this study site both daylight and nighttime data were collected; 409 vehicles were observed twice. Signaling behavior was recorded as the vehicles changed into the turning lanes, and again as they turned at the intersection. At night, only direction of turn and signaling behavior were recorded.

STUDY SITE 4

The T-intersection of North Star Road and Kinnear Road was selected as study site 4. Traffic on Kinnear Road is controlled by a STOP sign. Both are two-lane two-way roadways. The posted speed is 45 mph on Kinnear Road and 35 mph on North Star. The sight distance is adequate in both directions for drivers stopped at the Kinnear Road approach. Traffic is very light at this intersection except during morning and evening peak hours. North Star and Kinnear Roads represent the western and southern boundaries of the Ohio State University campus. North Star is a residential street; Kinnear Road provides entrance to several industrial plants. This site was selected to provide comparison of traffic-signal-controlled and STOP-sign-controlled T-intersections.

The traffic volume at this intersection varied between 730 and 863 vph on North Star Road. The difference in hourly volumes was tested to see if it had any effect on signaling and was found to be not significant. The volume on Kinnear Road remained between 100 and 135 vph.

STUDY SITE 5

The intersection of 17th Avenue and the northbound exit ramp on Interstate 71 was selected as study site 5. Left turns from the exit ramp are controlled by a STOP sign and right turns by a YIELD sign. Seventeenth Avenue is a four-lane divided highway in the area of the intersection. Hourly volume on the exit ramp varied between 150 and 240 vph during data collection. Left-turning vehicles were queued in front of the STOP sign during the time of observation. There was no pedestrian traffic at this intersection. Posted speed limit on Interstate 71 is 70 mph.

STUDY SITE 6

This site is a traffic-signal-controlled T-intersection of a southbound exit ramp on Interstate 71 and West Broad Street. West Broad Street is an important east-west street that crosses the entire city and carries several numbered state highways through the metropolitan area. It is a four-lane road. The eastbound outside approach lane is provided with a continuous green signal indication. Pedestrian crossing is prohibited at this intersection. The hourly volume on the exit ramp exceeded 500 vehicles.

STUDY SITE 7

Study site 7 is a southbound exit ramp on Interstate 71 at West Broad Street. The intersection of the exit ramp and West Broad Street was discussed under study site 6. Interstate 71 is a four-lane divided expressway in the area of the study site. The hourly traffic volume on the two southbound lanes was approximately 1,650 at the time of the field studies. Nearly 30 percent of this volume entered the investigated exit ramp. The high turning volume is explained by the previously discussed importance of West Broad Street in the street system of Columbus. Synchronized observations at study site 6 and study site

7 made possible the comparison of turn-signaling behavior of the same group of drivers at two locations. Posted speed limit is 50 mph in the area of this study site.

STUDY SITE 8

At the two-level intersection of Interstate 71 and State Route 161, the northbound traffic on Interstate 71 was observed as it entered the deceleration lane preceding the exit ramp. More than 450 vph exited at this ramp during the observations. Interstate 71 is a four-lane divided expressway in this study area. Posted speed limit is 70 mph.

Data Collection

At each study site the site characteristics, time of observation, traffic volume, weather condition, and the following characteristics of each turning movement were recorded:

1. Direction of turn.
2. Sex of driver.
3. Age of vehicle.
4. Presence of passengers.
5. Type of vehicle.
6. Presence of preceding turning vehicle.
7. Signaling behavior of the preceding vehicle.
8. Presence of following vehicle.

The data collection required no special instrumentation. Clipboards equipped with tally counters, portable tape recorders, and field sheets designed for this purpose were used in this phase of the study. A code was worked out which enabled the field personnel to record any combination of the listed variables with one symbol. Tape recorders were used to record observations during heavy turning movements, but the increased decoding time did not make their use practical under normal conditions. Collected data were grouped by approach lanes. Generally, one person was responsible for one turning lane. Night traffic data were collected at one intersection. Care was taken to stay inconspicuous, even though it was unlikely that data would be affected by the noticeable presence of observers. More than 10,000 observations were made during the summer of 1963.

Variables Representing Environmental Conditions

PRECEDING TURNING VEHICLE

The assumption was made that signaling behavior of drivers is influenced by the presence of a preceding turning vehicle within 100 ft. As seen in Table 35, the presence of an immediately preceding turning vehicle corresponded to signaling frequencies that significantly differed in six out of eight cases from those observed in the absence of preceding vehicles. The summary shows that of the observations analyzed in this phase of the study, the vehicle immediately preceded by another turning vehicle signaled 45 percent of the turns as opposed to 64.5 percent in the absence of a preceding vehicle. In other words, the presence of a preceding vehicle appears to reduce signaling frequencies.

SIGNALING BEHAVIOR OF PRECEDING VEHICLE

It was assumed that the signaling behavior of a turning vehicle will influence an immediately following turning vehicle. Some 1,722 observations were made and the assumption was found to be correct. Signaling preceding vehicles were followed by signaling vehicles 59.3 percent of the time, as opposed to 47.6 percent in the case of no signaling preceding vehicle. The difference is statistically significant to the level of 0.01. It is seen in Table 35 that in six of eight cases the statistical dependence is not significant, but signaling preceding vehicles were always followed by higher signaling frequencies.

FOLLOWING VEHICLE

Table 36, relating signaling frequencies of observed vehicles to the presence or absence of a following vehicle within 100 ft, shows that the presence of a following vehicle had no significant influence on three out of four groups.

Table 37, relating signaling frequencies to turning vehicles classified by the presence or absence of both preceding and following vehicle in the immediate vicinity indicates that drivers traveling in the middle of a platoon signal less than drivers of single vehicles.

GEOMETRIC DESIGN OF INTERSECTIONS

Eight test sites were investigated for signaling frequencies in turning at intersections. Test sites 1, 2, 3, and 4 are intersections of city streets. The lowest signaling was observed at test site 3, where 53 percent of the turns were signaled. Turns at this intersection are controlled by turning phases and turning lanes. At sites 1, 2, and 4, where no special protection is provided for turning movements, signaling frequencies were found to be 69, 62, and 67 percent, respectively. There was no significant difference in the average signaling frequencies at these three locations.

At sites 5 and 6, where exit ramps of I-71 intersect city streets, signaling frequencies were 45 and 33 percent, respectively.

The lowest frequencies were observed at test sites 7 and 8. Test site 7 is an exit on I-71 and 29 percent of the turns were signaled. The same group of vehicles was observed at the end of the ramp at test site 5, where the signaling increased to 45 percent. At test site 8, drivers switching into the deceleration lane before entering exit ramp on I-71 were observed. Only 14 percent signaled their intention to change lane. Average observed signaling frequencies at all study sites are given in Table 38. To investigate more closely the relationship between signaling frequencies and geometric layout of test sites, observed signaling frequencies were grouped by direction of turn and by approach legs of intersections. Typical intersection layout was designed and all possible traffic lanes were evaluated in respect to their effect on turning movement. Fourteen different traffic movements were distinguished. Because each movement may or may not have the right-of-way in respect to the investigated turning

TABLE 35

FREQUENCY OF TURN SIGNALING

SEX OF DRIVER	DIR. OF TURN	OBS. VEH. BLING FOL-LOWED	OBS. VEH. BLING PRE-CEDED	PREC. VEH. SIG-NALLED	SIGNALING FREQUENCY (%) AND STATISTICAL SIGNIFICANCE					
Male	Right	Yes	Yes	Yes	55.5					
						NS	52.3			
				No	46.8					
								NS	52.0	
			No				51.7			
									NS	49.0
		No	Yes	Yes	42.7					
						NS	39.4			
				No	36.6					
								.01	47.7	
			No				56.1			
										.01
	Left	Yes	Yes	Yes	65.1					
						.01	58.0			
				No	46.7					
								.05	61.9	
			No				68.2			
									.01	66.0
		No	Yes	Yes	61.2					
						NS	60.7			
				No	59.8					
								.01	69.0	
			No				74.1			
										.01 [a]
Female	Right	Yes	Yes	Yes	68.2					
						NS	68.0			
				No	67.7					
								NS	66.7	
			No				65.1			
									NS	64.5
		No	Yes	Yes	65.5					
						NS	54.4			
				No	52.4					
								.05	63.0	
			No				68.5			
										.01
	Left	Yes	Yes	Yes	78.2					
						.05	70.8			
				No	58.0					
								.05	75.5	
			No				84.5			
									NS	77.0
		No	Yes	Yes	72.7					
						NS	66.3			
				No	59.5					
								.01	78.1	
			No				83.5			

[a] Dependence of signaling frequency on sex of driver.

movement, they represent 28 different factors. Giving a point value of 28 to the traffic lane with the most effect, and a point value of 1 to the one with the least effect on turning movements, a rating system was designed. The estimated effect of each traffic lane on turning movements was expressed by a point rate between 1 and 28. Applying this rating system, each test site was evaluated in respect to each turning movement, as shown in Table 39. Figure 42 plots the point ratings against the corresponding observed turn-signaling frequencies. A correlation can be observed in spite of the fact that no attempt was made to eliminate the effect of other factors. For example, it was shown in foregoing sections that single vehicles signal more than do cars traveling in the middle of a platoon. This might partially explain the low signaling frequencies at site 6, where during field observations cars were continuously queued on the STOP-sign-controlled exit ramp waiting to enter 17th Avenue.

TABLE 36

INFLUENCE OF FOLLOWING VEHICLE ON TURN SIGNALING

SEX OF DRIVER	DIR. OF TURN	SIGNALING FREQUENCY (%) WHEN FOLLOWED	NOT FOLLOWED	SIGNIFICANCE
M	R	52.0	47.1	NS
	L	61.0	69.0	0.01
F	R	66.7	63.0	NS
	L	75.5	78.1	NS

TABLE 37

INFLUENCE OF PLATOON INVOLVEMENT ON TURN SIGNALING

SEX OF DRIVER	DIR. OF TURN	SIGNALING FREQUENCY (%) SINGLE VEHICLE	CAR IN PLATOON	SIGNIFICANCE
M	R	56.1	52.3	0.05
	L	74.1	58.0	0.01
F	R	68.5	68.0	NS
	L	83.5	70.8	0.01

Variables Representing Driver-Vehicle Unit

SEX OF DRIVER

It was found that female drivers signaled more of their turns than did male drivers. The observed average signaling frequency of female drivers was 71 percent, as opposed to 57 percent by male drivers. Data grouped by test sites resulted in the following:

STUDY SITE	SIGNALING FREQUENCY (%) MALE	FEMALE	STATISTICAL DEPENDENCE
1	67.6	81.0	Highly sign.
2	56.6	72.6	Highly sign.
3	50.7	61.3	Significant
4	63.7	82.4	Highly sign.
5	44.4	52.4	Not sign.
6	28.4	52.5	Highly sign.
7[a]	—	—	—
8	12.0	25.5	Highly sign.

[a] No data collected at site 7.

TABLE 38

AVERAGE TURN-SIGNALING FREQUENCIES AT STUDY SITES

STUDY SITE NO.	DESCRIPTION	AVG. % SIGNALING
1	Signal-controlled, four-leg intersection; heavy turning volumes	69
2	Signal-controlled T-intersection; heavy traffic on through street	62
3	Traffic-actuated signal, turning phases, turning lanes, divider islands; four-leg intersection	53
4	T-intersection; STOP-sign control on side street	67
5	Signal-controlled T-intersection of exit ramp with arterial street	45
6	T-intersection of exit ramp with city street; STOP-sign control on exit ramp	33
7	Exit ramp on Interstate 71 in urban area	29
8	Lane change into deceleration lane at exit on Interstate 71	14

USE OF AUTOMOBILE

Drivers of trucks, taxicabs, and delivery cars, classified as commercial vehicles, were assumed to differ in signaling behavior from drivers of passenger vehicles. The data show that drivers of passenger vehicles signaled more at every test site included in this study (sites 1 to 6) than did drivers of commercial vehicles. Statistical analysis of the summarized data showed highly significant dependence.

PRESENCE OF PASSENGERS

The assumption that presence of passengers influences the driver's turn-signaling behavior was tested on data grouped by test sites. The following is a tabulation of the data and the results of the statistical analysis:

TEST SITE	SIGNALING FREQUENCY (%) DRIVER ALONE	DRIVER WITH PASSENGERS	STATISTICAL DEPENDENCE
1	56.8	74.7	Highly sign.
2	60.5	64.5	Not sign.
3	52.3	53.7	Not sign.
4	66.1	75.0	Significant
5	41.6	51.9	Highly sign.
6	24.0	90.0	Highly sign.
7[a]	—	—	—
8	13.9	14.0	Not sign.

[a] No data collected at site 7.

The presence of passengers always coincides with the higher signaling frequencies.

DIRECTION OF TURNS

Test sites 1 to 6 are included in this study. Left turns were signaled more at every location. Statistical dependence was found to be significant or highly significant at every location. The higher influence with other traffic associated with left turns is a possible explanation. This higher risk, however, was not present at sites 2, 3, and 5. At test sites 2 and 5, vehicles entering the T-intersection from the side street during the green phase did not interfere with any traffic movements.

The observed signaling frequencies were 57 percent vs 51 percent and 56 percent vs 42 percent, respectively, the higher percentage representing signaling for left turns. At site 3, left turns are controlled by turning lanes and

turning phases. Right turns are controlled by turning lanes and YIELD signs. Nevertheless, 65 percent of the left turns were signaled, compared with 30.5 percent signaling of right turns. The point rating system discussed in a foregoing section, which considers all possible traffic movements present, attempted to account for the difference in signaling frequencies for left and right turns.

AGE OF AUTOMOBILE

Late model automobiles (namely 1962 and 1963 models) were distinguished from older models during the field studies. The assumption was made that owners of new automobiles would be more careful drivers. It was expected that more careful driving would result in a higher signaling frequency. The data show that drivers of late model automobiles signaled more at every test site than did drivers of older automobiles. The statistical test on the summarized data indicated a highly significant statistical dependence. Because older cars are more likely to have technical defects than relatively new ones, it was felt to be necessary to collect some information on total percentage of cars with defective turn signals. During 1963, in Franklin County, Ohio, 10,282 trucks and 201,956 passenger cars were safety checked during the 1963 voluntary vehicle safety check. On trucks, 2.6 percent of the front and 2.8 percent of the rear turn indicators were found to be defective. On passenger vehicles only 0.8 percent and 0.6 percent were found to be defective on the front or rear, respectively. This is not significant to account for higher signaling frequencies by late model automobiles.

Conclusions

The objectives of this study phase were to investigate the use of turn signals at urban intersections and freeway exits, and relate variations in signaling frequencies to some characteristic factors of the environment and the driver-vehicle unit. It was hoped that some pattern in variation of signaling frequencies would be found, so as to enable drawing some conclusion about the driver who operates the turn indicators. The analysis of the data indicates the following relationships between frequencies of turn signaling and characteristics of environment and the driver-vehicle unit:

1. Presence of preceding turning vehicle reduces signaling frequencies.
2. Signaling by preceding vehicles increases signaling frequencies of the following vehicles.
3. Single vehicles signal more than vehicles traveling in the middle of platoons.
4. The number of interfering traffic movements, as well as the number of lanes, increases signaling frequencies.
5. Male drivers signal less than female drivers.
6. Drivers of commercial vehicles signal less than drivers of passenger vehicles.
7. Presence of passengers appeared to increase signaling frequencies.
8. Drivers of new vehicles signaled more than drivers of older vehicles.

The result of this study phase fulfilled the expectation, as the influence of a number of factors on driver's turn-signaling behavior was detected consistently in the observations. Some other factors did not prove to have any influence.

It appears that signaling is considered more important at intersections of city streets than at freeway exits. At sites 5 and 7, the same group of drivers was observed twice. First, as they entered the exit ramp at site 7 where

TABLE 39

OBSERVED SIGNALING FREQUENCIES AND POINT RATING OF TURNING MOVEMENTS

SITE	APPROACH	TURN DIRECTION	SIGNALING (%)	POINT RATING
1	N. High St.	L	88	88
		R	66	66
	15th Ave.	L	77	76
		R	57	66
2	N. High St., N	L	93	120
	N. High St., S	R	63	52
	Oakland Park Rd.	L	57	42
		R	51	36
3	Olentangy Rd.	L	54	62
		R	31	28
	Lane Ave.	L	69	86
4	N. Star Rd.	L	87	66
		R	70	50
	Kinnear Rd.	L	67	48
		R	62	40
5	Exit ramp	L	56	38
		R	42	24
6	Exit ramp	L	36	78
		R	28	32
7	I-71, N (Broad)	R	29	—
8	I-71, S (161)	R	14	—

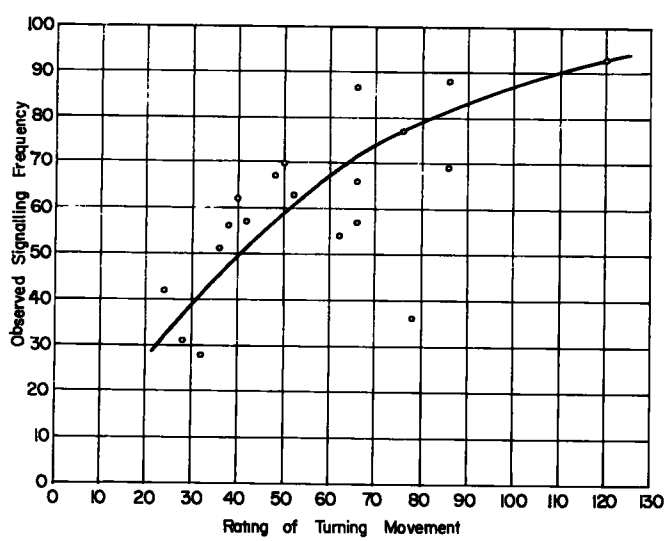

Figure 42. Correlation between observed signaling frequencies and point ratings of turning movements.

29 percent of them signaled. Second, they were observed as they entered Broad Street from the exit ramp during the green signal phase. There the signaling frequencies were found to be 45 percent, even though at a signalized T-intersection, such as site 5, there is no interference whatsoever between turning movements and through traffic.

It is also difficult to interpret why female drivers signal more than male drivers or why drivers of passenger vehicles signal more than drivers of commercial vehicles. One might say that increased confidence in one's driving practices may result in lower signaling frequencies.

USE OF TURN SIGNALS DURING OVERTAKING

In this study phase, signaling behavior of drivers on expressways was investigated. In the preceding section, signaling frequencies at exits (sites 7 and 8) were already discussed. It will be recalled that 29 percent of the turns into the exit ramp were signaled at site 7, where no deceleration lane is provided. At site 8 only 14 percent of the drivers signaled as they entered the deceleration lane connecting the exit ramp with Interstate 71.

In this phase of the study signaling practices were observed during overtaking maneuvers. The observations, made from test vehicles traveling along the highway, were recorded on a portable tape recorder by one or two observers traveling in one vehicle.

First Study

In the first study, observers traveling at a moderate speed on Interstate 71 recorded the signaling behavior of all overtaking vehicles. The following conditions were considered:

1. Sex of driver.
2. Presence of passenger.
3. Age of automobile.
4. Use of vehicle.
5. Direction of lane change (from left lane to right lane or from right lane to left lane).

In the nearly 2,000 observations made, it was found that 20.5 percent of all lane changes were signaled. Commercial vehicles signaled 51.2 percent compared with 15.9 percent signaling by passenger vehicles. This was the only significant difference in signaling frequencies. Statistical dependence of signaling on other factors was found to be not significant. The following is a tabulation of results:

FACTOR	SIGNALING FREQUENCY (%)	STATISTICAL TEST
Male drivers	15.7	Not sign.
Female drivers	16.9	Not sign.
Commercial vehicle	51.2	Highly sign.
Passenger automobile	15.9	Highly sign.
1963 and 1962 models	15.2	Not sign.
Older models	16.2	Not sign.
Driver alone	14.7	Not sign.
Driver with passengers	17.4	Not sign.

Second Study

It was found in the study of turn signaling at intersections that the complexity of a turning movement, measured by the number of possible conflicting traffic movements at the intersection, increased frequencies of turn signaling. In case of overtaking, the number of overtaken vehicles and the presence of vehicles following the overtaking vehicle could measure the complexity of overtaking maneuvers. Inasmuch as the data were collected on four-lane divided expressways, opposing traffic had no influence on overtaking vehicles.

Signaling behavior of overtaking vehicles was observed under four different conditions representing various degrees of risk involved in the maneuvers. These conditions were created by arranging one or two test (observer) vehicles in special positions in relation to each other. Altogether, four conditions were simulated, as described in the following and represented graphically in Figure 43. No commerical vehicles, sports cars, trailers, or station wagons were included in the observations.

CONDITION 1

In condition 1 one test vehicle was traveling in the outside lane at a velocity between 35 and 50 mph. Only those overtaking vehicles were observed which were not followed by other vehicles within 300 ft. This situation was considered as a basic overtaking maneuver that requires the minimum precaution from the driver of the overtaking vehicle. It was expected that the lowest signaling frequencies would be observed under this condition.

CONDITION 2

Two test vehicles were traveling at approximately 40 mph in the outside lane. They maintained a spacing of 300 to 400 ft until a single vehicle approached the rear test vehicle. The rear test vehicle then increased its speed and reduced the spacing to a minimum safe distance. The overtaking vehicle in this situation had to overtake two vehicles, the second of which expressed by the described maneuver some desire, or at least the ability, to travel at a higher velocity than the first vehicle. In this situation there would be a greater need for signaling of intention to overtake by the observed vehicle and, therefore, signaling frequencies were expected to be higher than for condition 1.

CONDITION 3

The simulation of this situation again required two test vehicles traveling at a 300- to 400-ft spacing as in condition 2. Those single vehicles were observed which had overtaken the rear vehicle and returned to the main lane. The rear vehicle then increased its velocity and closely followed the observed vehicle. Inasmuch as signaling of intention to change lanes is generally intended for following rather than preceding vehicles, signaling frequencies were expected to be higher in this situation than under conditions 1 and 2.

CONDITION 4

This situation is similar to condition 3, except that the overtaking rear test vehicle would follow the observed vehicle from the inside lane. To overtake the first vehicle, the observed vehicle had to cut in front of the rear test vehicle. This situation represented the highest need for signaling because the highest apparent risk was involved in this overtaking maneuver.

Discussion of Results

The nature of the data sought required low volumes on the test sites, which made data collection rather time consuming. Altogether, 800 observations were made. As shown by the following, the results fulfilled the expectations:

CONDITION	SIGNALED (NO.)	DID NOT SIGNAL (NO.)	% SIGNALED
1	25	175	12.5
2	31	169	15.5
3	66	134	33.0
4	87	113	43.5

As in the intersection study, drivers were found to be quite reasonable in the use of turn indicators if only the trend in signaling frequencies is considered.

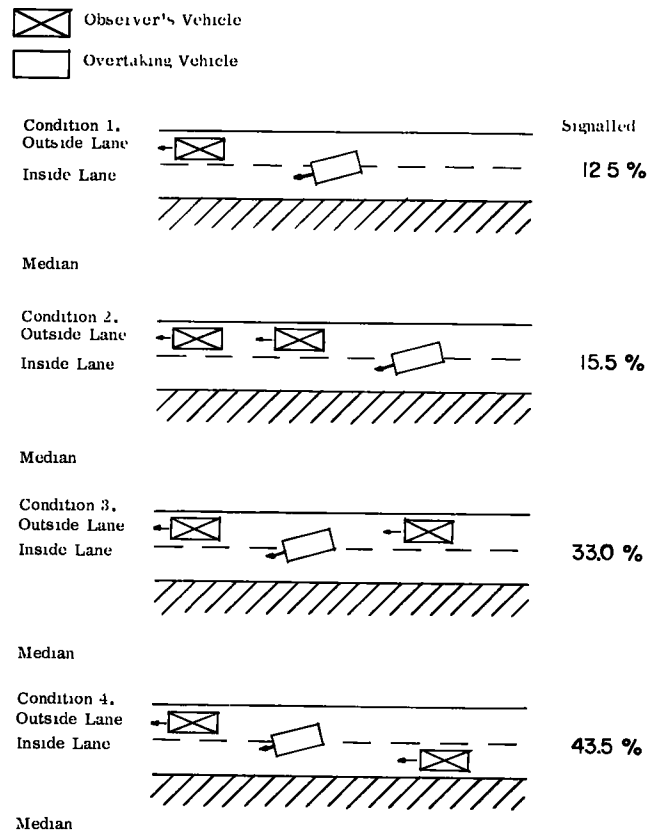

Figure 43. Observed signaling frequencies under four conditions of overtaking.

SUMMARY OF STUDY

Driving an automobile in today's fast moving traffic is a strenuous task. The driver continuously observes the environment, evaluates the observations, and makes decisions. Decisions are followed by actions. Delay in decision or action may result in accidents. However, delay can be reduced if drivers signal their intentions.

Of the intervehicular systems currently available, the turn indicators can be used to express intentions of two different movements; namely, change of lane and change of direction. Little is presently known about the extent to which such signals are used. It is assumed that drivers use turn signals to increase safety of turning movements and to expedite traffic flow.

Signaling frequencies were observed at intersections, on exit ramps, and during overtaking on expressways. More than 10,000 observations were made. Along with signaling behavior of the observed vehicle, several factors representing the environment and the special characteristics of the observed vehicle were recorded. The data were compiled and classified according to the investigated factor. Statistical dependence of turn-signaling frequencies on observed factors was tested. To check for consistency of results, not only summarized data, but also several groups of data, were investigated for the influence of some factor. For example, dependence of signaling frequencies on sex of drivers was found to be highly significant at each intersection.

It was found that factors which increased signaling frequencies can quite readily be considered to increase the responsibility of the driver. For example, single vehicles signaled more than drivers finding themselves in the middle of a platoon. When geometric layout and traffic control were investigated for their effect on signaling frequencies, it was found that a simple rating system based on possible conflict between turning vehicles and other traffic movement showed correlation between signaling and conflict. Figure 42 illustrates the point. Presence of passengers and driving a new automobile were also observed to increase signaling frequencies. Both can be assumed to increase the responsibility of the driver. Observation of signaling during overtaking under four conditions intended to simulate different degrees of risk, likewise indicated a relationship between conflict and signaling frequency. It would be more difficult to explain why drivers turning into the through street at a traffic-signal-controlled T-intersection would signal more than, for example, under condition 4 during overtaking. It is likewise perplexing why drivers entering an exit ramp on a high-speed expressway would only signal 29 percent, when the same drivers were observed to signal 45 percent at the end of the ramp, which terminated in a traffic-signal-controlled T-intersection, so no conflict could be expected by the driver of the turning vehicle.

On the results of this study, two general conclusions can be drawn, as follows:

1. Drivers are more apt to use turn indicators in city driving than in expressway driving.

2. Both in city driving and in expressway driving, signaling frequencies increase with the responsibility of the driver during the maneuver considered.

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CHAPTER TEN

TRAFFIC FLOW DATA COLLECTION AND ANALYSIS

Accurate traffic flow data for a group of vehicles as it progresses along the roadway may be obtained through the techniques developed in this study. A 70-mm camera, mounted in a helicopter, was used to obtain the photography. Average errors in the spacing and velocity determinations were 0.75 ft and 0.4 mph, respectively. The major bottleneck is the time and instrumentation required to extract the data from the photographs.

Analysis indicates fairly close agreement between observed shock-wave speed and the theoretical wave speed as developed by Lighthill and Whitham.* The sample is small, however.

The phenomena of traffic flow have been and are being investigated in the research studies of various disciplines, and numerous theories have been advanced to describe and predict the movement of vehicles along the roadway.

Two general approaches—macroscopic and microscopic—are taken by investigators of traffic flow. The macroscopic approach considers the principles governing the simultaneous movement of a large number of vehicles. The microscopic approach considers traffic movement on the basis of the behavior of the driver-vehicle unit.

At present there is no generally accepted or validated theory or model of traffic flow that deals with the propagation of disturbances in highway traffic, the amplification and attenuation of such disturbances, and their effect on traffic volumes and safety. The lack of knowledge concerning the interactions of vehicles as they progress along the roadway, which involve both the macroscopic and microscopic aspects of traffic flow, is at least partially due to the absence of satisfactory techniques for collecting the essential data. Better definition and understanding of the basic traffic flow parameters are required if theoretical investigations are to lead to improved traffic flow through modification of these parameters.

In order to study traffic flow characteristics, it is necessary to obtain spacing and velocity data, at relatively short time intervals, on a group of vehicles as it progresses along the roadway. Data collected in the past have been limited in the number of vehicles studied (car-following models), or the number of spacing and velocity determinations for each vehicle. If aerial photographs with standard overlap are taken from an airplane, a given vehicle is generally not imaged on more than two or three of the photographs, thereby permitting only two or three spacing determinations and one or two velocity determinations. Wires or pneumatic tubes on the roadway, or observers, can be used to collect data on many vehicles; but only a few spacing or velocity determinations on a given vehicle can be obtained practically, and data collection is limited to a preselected location.

The objectives of this study were (a) to develop photogrammetric techniques to obtain accurate spacing and velocity data on platoons of vehicle as they progress along the roadway, (b) to use these data to define traffic flow characteristics, and (c) to develop and/or validate theories of traffic flow.

DATA COLLECTION

A test car, identifiable from the air, was introduced into the traffic stream and followed by a helicopter, which maintained a position directly above the test car as it progressed along the highway.

Vertical photographs of the traffic and roadway for approximately ¼ mile ahead of and behind the test car are taken with a Maurer P-2, 70-mm reconnaissance camera at a known time interval. Essentially the same group of

* LIGHTHILL, M. J., and WHITHAM, G. B., "On Kinematic Waves II. A Theory of Traffic Flow on Long Crowded Roads." *HRB Spec. Rep. 79* (1964) pp. 7-35

vehicles is imaged on each photograph, as the driver of the test car is instructed to "float" with the traffic.

The data collection procedure is illustrated in Figure 44. A photograph is taken when the helicopter is at position H and the vehicles are at positions 1, 2, and 3. At the end of a preset time interval, another photograph is taken from position H′. The vehicles have moved to positions 1′, 2′, and 3′, respectively.

The photographic negatives were mounted on glass plates and the photocoordinates of each vehicle and several photo-identifiable natural ground control points were read at the OMI-Bendix AP/C Analytical Stereoplotter.

The photocoordinate data were transferred to punch cards for processing in an IBM 7094 computer. The vehicle photocoordinates were converted to ground coordinates through suitable transformations based on the ground control points. Then cumulative distance (termed D-distance in this study) along the roadway from an arbitrary point at the beginning of the test section to each vehicle on each photograph was computed from the vehicle ground coordinates.

Spacings were determined by subtracting the D-distance of the following vehicle from that of the lead vehicle for the same photograph. In Figure 44, the spacing between vehicles 1 and 2 is equal to $(D_2 - D_1)$ on the first photograph and $(D_2' - D_1')$ on the second photograph.

The velocity of a vehicle is determined by dividing the change in its D-distance on two successive photographs by the known time interval between exposures. In Figure 44, the velocity of vehicle 1 is $(D_1' - D_1)/\Delta t$ and the velocity of vehicle 2 is $(D_2' - D_2)/\Delta t$ where Δt is the time interval between the two exposures.

Seven photographic flights were made using either infrared Ektachrome or Plus-X panchromatic film. Photographs of the traffic surrounding a test car were taken at 1-sec intervals. The photographic scale approximated 1:12,000 in each flight. Each photograph covered a little less than ½ mile of roadway.

A sequence of 101 photographs from one flight was selected for detailed analysis. This sequence provided a good example of platoon interactions, the traffic characteristics changing from free-flowing to congested and then back to free-flowing within the 100-sec time period.

Photo-identifiable ground control points, such as lamp post bases and manhole covers, were selected at an average spacing of 186 ft. A total of 41 points was required to control the 7,500 ft of roadway included in the study section. A third-order ground survey was made to determine the ground coordinates of these points.

The photocoordinates of the center of the photograph, each ground control point, and the front-center of each vehicle imaged on each of the 101 photographs were determined. An average of 38 vehicles and eight to ten ground control points appeared on each photograph.

The cumulative distance traveled along the roadway by each vehicle in each photograph was computed. The spacings between vehicles on the same photograph and the velocity of each vehicle whenever it appeared on consecutive photographs were computed from the cumulative distances. Approximately 3,700 spacings and velocities were computed.

TIME-DISTANCE DIAGRAMS AND TRAFFIC FLOW DATA

The D-distances of each vehicle were plotted as ordinates against time as an abscissa to obtain vehicle trajectories on a time-distance diagram. The time-distance diagrams obtained from the data previously described are shown as Figures 45 and 46 for the inside (median) and outside (curb) lanes, respectively. These diagrams graphically illustrate variations in volume, density, spacings, and velocities as the group of vehicles progresses along the roadway.

The traffic flow data obtained are in error due to inaccuracies in the data collection and reduction equipment and techniques, and due to approximations made in the com-

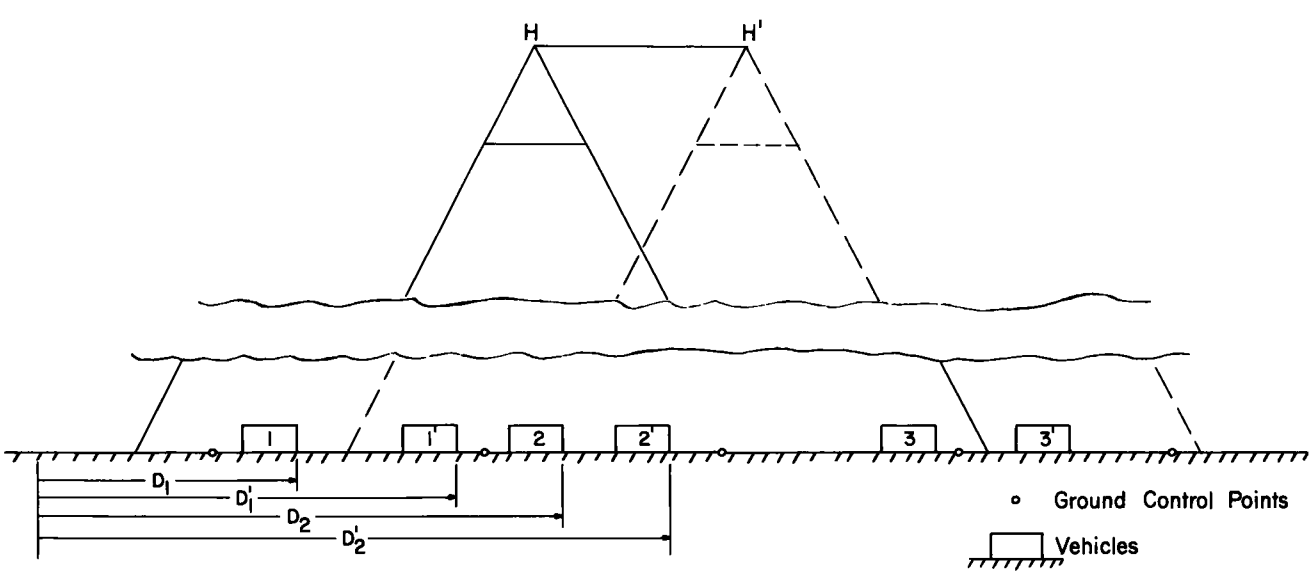

Figure 44 Procedure for collecting traffic flow data by aerial photography.

putation procedures. An error analysis indicated the average errors in the spacing and velocity determinations were 0.75 ft and 0.4 mph, respectively. The error analysis included consideration of the effect of the focal plane shutter, tilt of the camera axis, vertical alignment of the roadway, vehicle height above the roadway, accuracy of the photocoordinate determinations, lens and film distortions, and accuracy of the time interval between exposures.

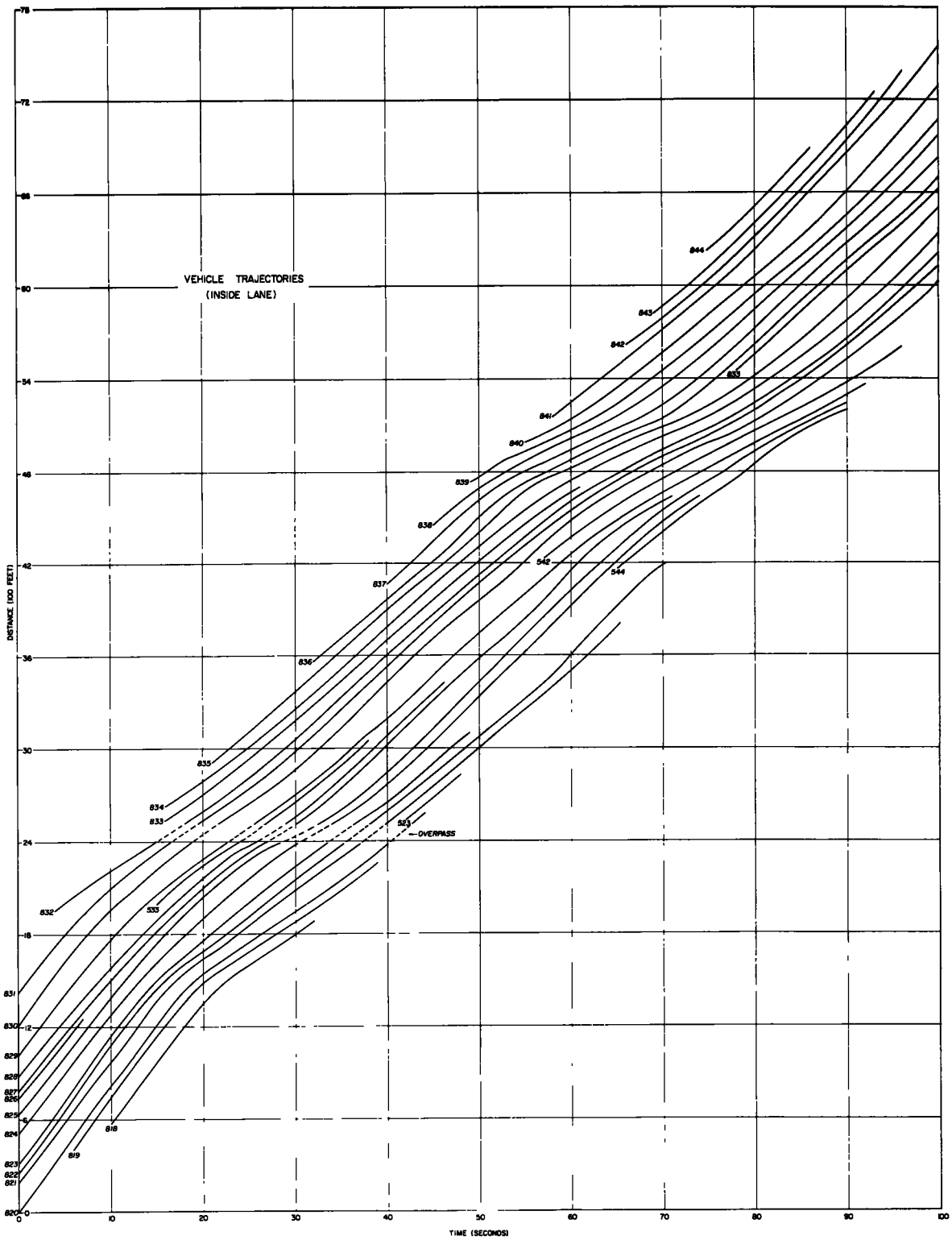

Figure 45. Vehicle trajectories for inside (median) lane.

The photogrammetric techniques formulated and tested in this study can provide a type of data not heretofore available to those studying traffic flow phenomena—accurate traffic flow data continuous in both time and space. Accurate vehicle trajectories, with corresponding spacing and velocity data, were obtained. The major bottleneck was, and still is, the reduction of the data available on the photographs to usable form. This factor alone may well

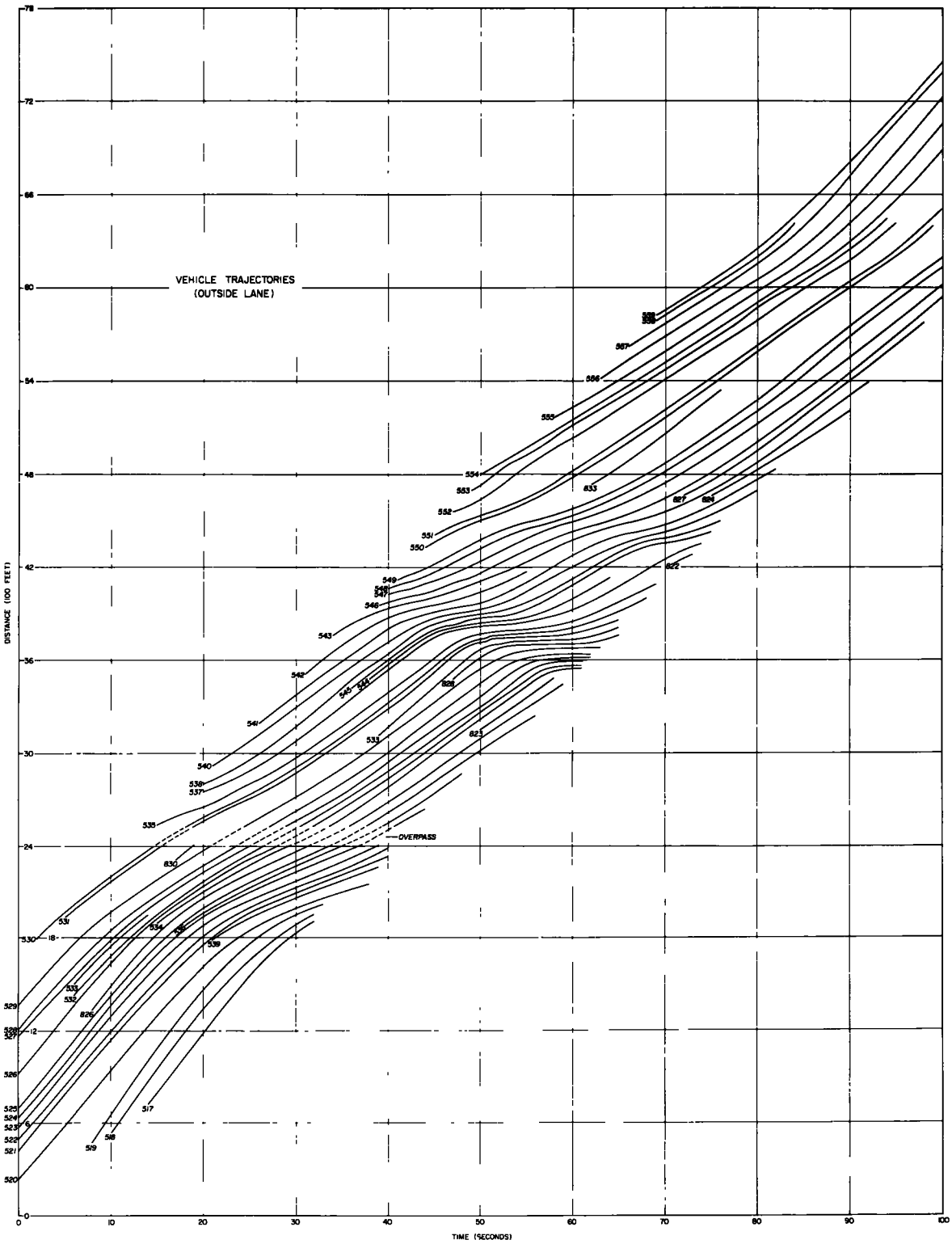

Figure 46. Vehicle trajectories for outside (curb) lane.

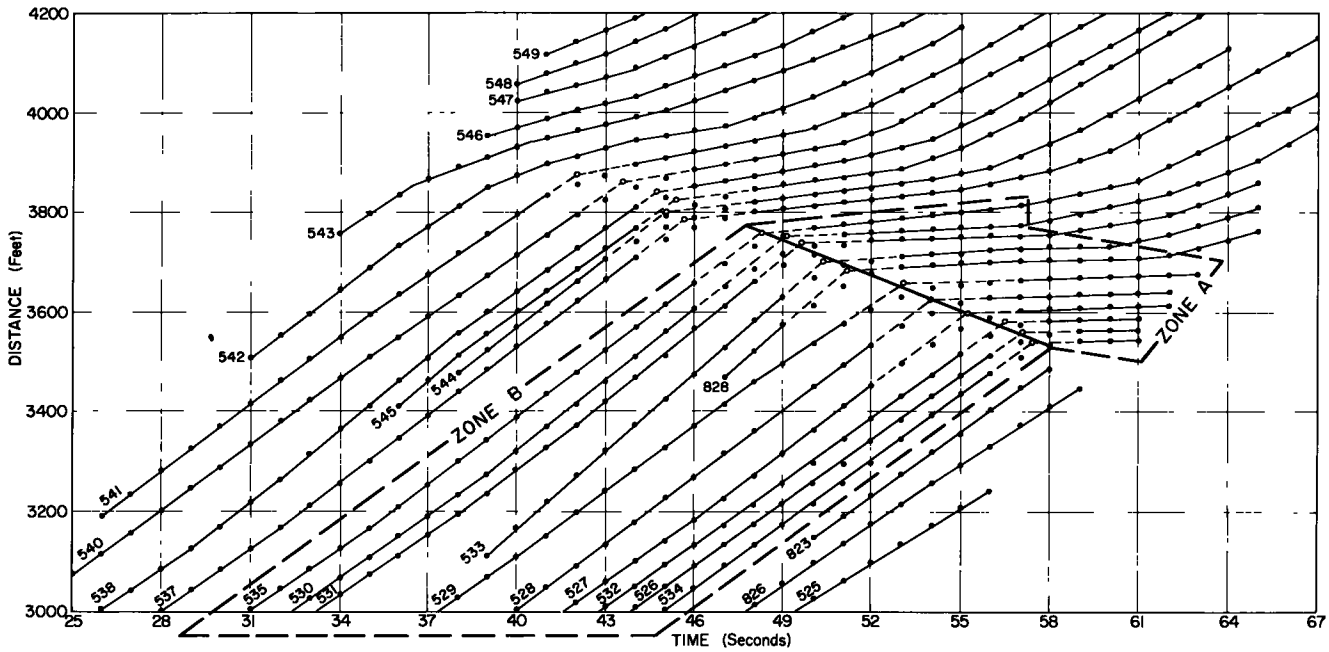

Figure 47. Enlarged portion of vehicle trajectories (from Figure 46) for outside (curb) lane.

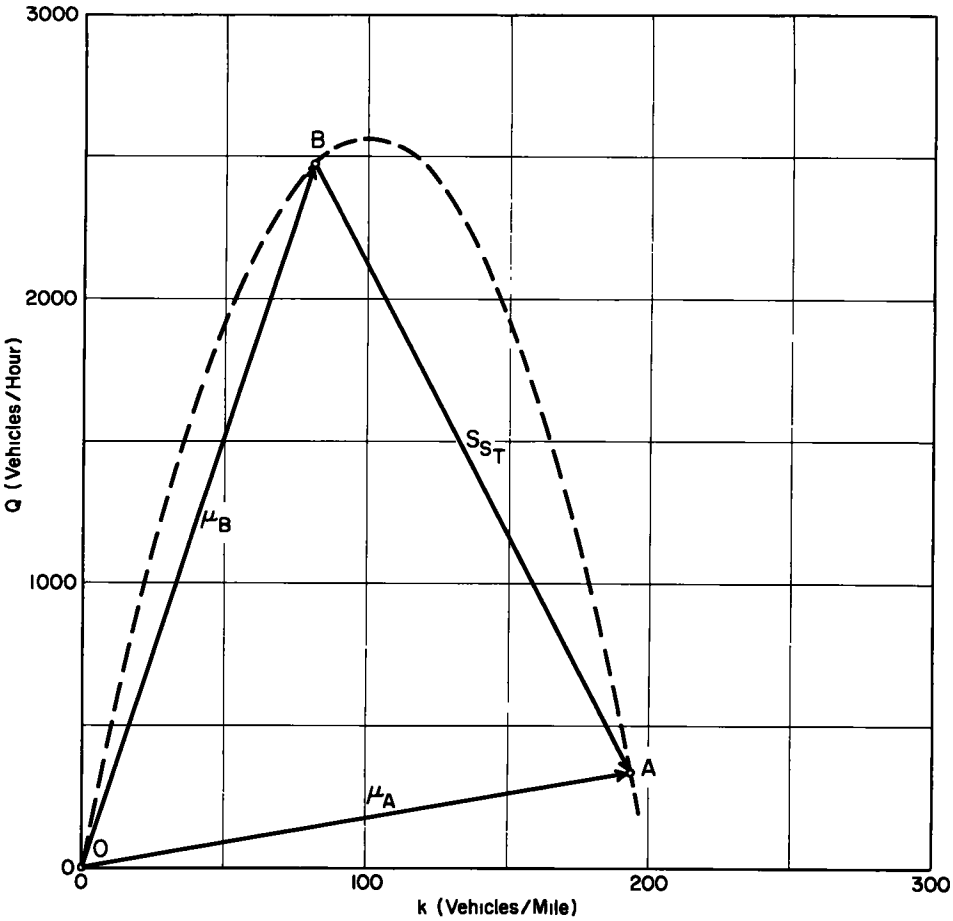

Figure 48. Flow conditions projected from zones A and B of Figure 47.

determine the economic feasibility of the aerial survey techniques for traffic studies.

It is impractical, if not impossible, to obtain the same data, with comparable accuracy, through ground-based measurement techniques.

ANALYSIS

A preliminary investigation of the "wave" phenomenon discussed by Lighthill and Whitham * was conducted.

Kinematic waves are likely to occur on any stretch of road when the traffic is more dense in front, and less dense behind. As the less dense (and higher speed) traffic "catches up" with the more dense (slower moving) traffic, a kinematic wave is generated. When vehicles enter the disturbance, their speed is reduced suddenly.

Figure 47 shows an enlarged portion of the vehicle trajectories for the outside (curb) lane of traffic as shown in Figure 46. These trajectories were derived from photographs of traffic flow on Interstate Highway 71 in Columbus, Ohio, during the afternoon peak flow. They illustrate the rapid deceleration accompanying a kinematic wave, as previously mentioned. Unfortunately, the helicopter was moving at a faster speed than the vehicles, and the disturbance was not recorded after a time of 58 sec.

The space-mean-speed, μ_B, of the eleven vehicles in zone B (behind the disturbance) is 30.5 mph. The density, k_B, computed from the mean spacing, is 81.0 veh per mile. The flow, Q_B $(= k_B \mu_B)$, is 2,470 veh per hour. This value is relatively high for traffic flow, but the sample is small (11 vehicles) and the data were collected on a 12-ft lane of Interstate Highway 71 in an urban area.

The space-mean-speed in zone A (ahead of the wave), μ_A, is 1.57 mph; the density, k_A, is 194 veh per mile; and the flow, Q_B, is 304 veh per hour.

These two flow conditions, before and after the wave, are indicated as points B and A, respectively, in Figure 48. The mean vehicle paths would be parallel to the radius vectors OB and OA. A hypothetical flow-concentration curve has been sketched in for illustrative purposes.

The speed of a shock wave, according to Lighthill and Whitham, is equal to $\Delta Q/\Delta k$, the slope of the chord joining the two points on the flow-concentration curve which represent conditions behind and ahead of a shock wave (points B and A, Fig. 48). The theoretical shock velocity is ${}^{S}S_T = (Q_A - Q_B)/(k_A - k_B) \times (304 - 2470)/(194 - 81) = -19.2$ mph.

The straight-line (constant velocity) portions of the vehicle trajectories in zones A and B were extended until they met (Fig. 47). These intersections are marked by small circles. The slope of the straight line fitted to these circles was determined to be −16.0 mph, and represents the observed shock velocity.

The theoretical and observed values are of the same magnitude, but this preliminary study only indicates the type of analysis which could be carried out with more data.

* Op. cit.

CHAPTER ELEVEN

THEORETICAL ASPECTS OF TRAFFIC CONTROL

A new problem which has arisen from the increasing number of motor vehicles on the roads and streets is how to control traffic so that an optimum number of vehicles can be transported continuously. To solve this problem, the stability of traffic flow appears to be even more important than the absolute maximum in traffic capacity, if traffic flow is liable to break down under the absolute maximum condition. Studies of the volume-density relationships (q-k curve) indicate that traffic flow is highly unstable at maximum flow conditions, and small disturbances can cause a breakdown, which often leads to intolerable delays during peak-hour traffic.

Although such breakdowns were observed at many freeways, it is extremely difficult to determine what influences actually caused the breakdown. Volumes up to 2,400 veh per lane per hour were observed for short periods before a platoon of efficiently moving vehicles was reduced to a traffic jam, and it often takes a long time before the pile-up again assumes the state of flowing traffic. Shock waves were named as the cause of these pileups by some of the control engineers for traffic surveillance systems; however, the term "shock waves" is used rather vaguely.

In this investigation, a theoretical approach was used to study the continuity of traffic flow and to develop a theory based on the flow-density relationship, because these two factors can be measured for traffic control purposes. The necessary equipment has been developed and has been installed at the Holland Tunnel in New York, and at the Eisenhower Expressway in Chicago, with good success.

RELATIONSHIP BETWEEN TRAFFIC FLOW AND CONCENTRATION

Lighthill and Whitham's (*1*) hypothesis that the flow, q, in vehicles per hour is a function of the concentration, k, in vehicles per mile, which determines the space mean speed

for a specific condition, has been supported by the investigations using aerial survey methods (see Chapter Ten). Using

$$q = v\,k \tag{6}$$

in which

$q =$ flow of traffic, in vehicles per time unit;
$v =$ space mean speed; and
$k =$ concentration of traffic, in vehicles per length of road

and letting $\phi\ (x,\ t)$ be the cumulative number of vehicles which have passed a point x on the road by the time t, the flow of traffic can be expressed as the time rate at which vehicles are passing a fixed point, x, or

$$q = \frac{\partial \phi}{\partial t} \tag{7}$$

and the concentration of traffic is

$$k = \frac{\partial \phi}{\partial x} \tag{8}$$

Then

$$d\phi = \frac{\partial \phi}{\partial x}\,dx + \frac{\partial \phi}{\partial t}\,dt \tag{9}$$

If a stable continuous flow of traffic is to be maintained, ϕ is a constant, $d\phi/dx = 0$, and the velocity, $v = dx/dt$, is a constant along ϕ.
The equation of continuity can then be written as

$$\frac{\partial \phi}{\partial x} + \frac{\partial k}{\partial t} = 0 \tag{10}$$

Using Eq. 6, or

$$q = c\,k \tag{11}$$

in which

$$c = \frac{\partial q}{\partial k} \tag{12}$$

changes along the route can be defined by partial differentiation as

$$\frac{\partial q}{\partial x} = c\frac{\partial k}{\partial x} \tag{13}$$

and changes in density with time as

$$\frac{\partial k}{\partial t} = \frac{1}{c}\frac{\partial q}{\partial t} \tag{14}$$

The continuity equation may then be written as

$$c\,\frac{\partial k}{\partial x} + \frac{\partial k}{\partial t} = 0 \tag{15}$$

or

$$c\,\frac{\partial q}{\partial x} + \frac{\partial q}{\partial t} = 0 \tag{16}$$

Each of these equations has a solution of $\partial k/\partial t = c'$ as a characteristic (*2*) along which k and q are constant. It follows then that the speed ($v = q/k$) is also constant along a characteristic; i.e., in a kinematic wave as defined by Lighthill and Whitham (*1*). Kinematic waves caused by small disturbances travel at smaller speeds than traffic relative to the road, and appear to propagate opposite to the direction of traffic flow. A vehicle going through kinematic waves (i.e., flow characteristics which carry constant velocities) is temporarily slowed down, and will then accelerate to the previous speed of traffic flow. Because it has been shown previously that the continuity equations (Eqs. 15 and 16) have solutions for $\partial k/\partial t = c'$ as a characteristic along which both k and q are constant, $c\ (k)$ is also a constant. The differential equation

$$c = dx/dt \tag{17}$$

becomes, by integration,

$$x = c\,t + x_0 \tag{18}$$

with x_0 representing the point where the characteristic intersects with the x axis, in the x-t plane (Fig. 49). The flow velocity, v, then has the constant value $v_0\ (x_0)$ along the characteristic through x; i.e.,

$$v\ (x,\ t) = x - c\,t \tag{19}$$

Considering the changes in velocity, dv/dt, when a vehicle proceeds on its path, $v = dx/dt$, along a characteristic, the following conditions are valid:

$$v = v_0\ (x_0) \tag{20}$$

and

$$\frac{dv}{dt} = v_0'(x_0)\,\frac{dx_0}{dt} \tag{21}$$

$$x = c\,t + x_0 \tag{22}$$

$$x_0 = x - c\,x_0\,t \tag{23}$$

$$\frac{dx_0}{dt} = \frac{dx}{dt} - t\,c'\,x_0\frac{dx_0}{dt} - c\,x_0 \tag{24}$$

$$\frac{dx_0}{dt} = \frac{dx/dt - c\,x_0}{1 + c'\,x_0\,t} \tag{25}$$

However, $v = dx/dt$ along the trajectory of a car in the time-space diagram and

$$\frac{dv}{dt} = v_0'\,x_0\,\frac{v - c\,x_0}{1 + c'\,x_0\,t} \tag{26}$$

Because the continuity equations (Eqs. 15 and 16) do have solutions as characteristics along which k and q, and consequently v, are constants, any acceleration to which the vehicle may be subjected while going through a characteristic tends to approach zero. If $c'\ (x_0) > 0$, then upon this condition the characteristics will form a fan. Should $c'\ (x_0) \leq 0$, the expression will become negative, and the characteristic will be stationary or move in a direction opposite to the flow of traffic. It can then be expected that the characteristic will intersect with other characteristics, causing a discontinuity or shock wave in the traffic flow; that is (from Eq. 22),

$$x_0 = x - c\,t \tag{27}$$

Although x_0 represents the point where the characteristic intersects with the x axis, it is also a constant along the characteristic, and c represents the speed of the kinematic wave propagating constant flow, density, and velocity. Discontinuities are generated when two continuities propagated by kinematic waves meet. The speed of propagation can be determined as follows:

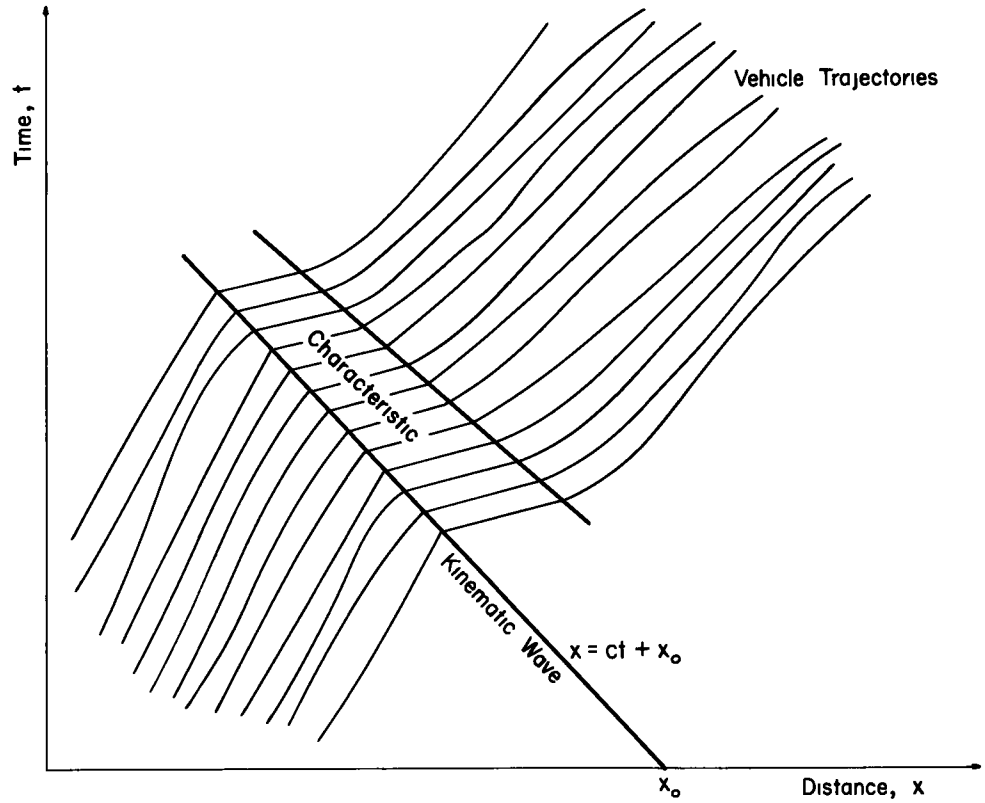

Figure 49. Theoretical conditions for a kinematic wave and the characteristic.

Along the vehicle paths $x = x(t)$ in a traffic lane carrying a platoon of cars, disturbances are propagated and

$$\frac{d\phi}{dt} = \frac{\partial\phi}{\partial x}\frac{dx}{dt} + \frac{\partial\phi}{\partial t} \tag{28}$$

Substituting Eqs. 7 and 8 in Eq. 28 gives

$$\frac{d\phi}{dt} = -k\frac{dx}{dt} + q \tag{29}$$

It has been shown that along a characteristic

$$\frac{dx}{dt} = \frac{dq}{dt} = c \tag{30}$$

and

$$\frac{d\phi}{dt} = q - k\,c \tag{31}$$

From the limited data which have been obtained by photogrammetric techniques Eq. 31 appears to be valid. Fairly continuous conditions with regard to q, k, and c were recorded for time intervals of about 35 sec. Unfortunately, the characteristic was lost after that period when following the test vehicle in the adjoining traffic lane with the helicopter. More research is needed to further verify this theory, as no attempt has been made to follow a kinematic wave.

Following a discontinuity (shock wave) traveling at a speed

$$u = dx/dt \tag{32}$$

the number of vehicles entering the shock wave must be equal to the number leaving the shock wave, or

$$\frac{d\phi_1}{dt} = \frac{d\phi_2}{dt} \tag{33}$$

in which ϕ_1 represents the flow before the shock wave and ϕ_2 the flow behind it. Thus, when entering,

$$d\phi_1/dt = q_1 - k_1\,u \tag{34}$$

and when leaving,

$$\frac{d\phi_2}{dt} = q_2 - k_2\,u \tag{35}$$

and along the path of the shock wave

$$q_1 - k_1\,u = q_2 - k_2\,u \tag{36}$$

and

$$u = \frac{q_2 - q_1}{k_2 - k_1} \tag{37}$$

It might be concluded from Eq. 37 that the velocity of the shock wave is zero and therefore some explanation should be given here. It has been shown that conditions in a kinematic wave are constant and continuous along the characteristic. This, however, does not imply that they are the same for all kinematic waves and—as shock waves are generated when two kinematic waves intercept each other—the respective traffic flows, q_1, q_2, and concentration, k_1, k_2, are expected to be different. This does not con-

tradict the assumption that the flow differential relation (Eq. 33) holds for the short duration of a shock wave. In fact, if $q_1 = q_2$ and $k_1 = k_2$, the speed of propagation will be the same for both characteristics, the kinematic waves cannot intercept each other, and no discontinuity will be generated.

From observations at the John Lodge Freeway in Detroit, and at other freeways, it appears that kinematic waves (i.e., the propagation of disturbances accompanied by a decrease in speed and traffic flow) are frequently responsible for the buildup of traffic jams. The term "shock wave" is sometimes used in publications for kinematic waves as originally described by Lighthill and Whitham (*1*).

The influence of a kinematic wave can be shown by the following example: Figure 50, showing vehicle trajectories obtained through the evaluation of aerial photographs at time intervals of 1 sec, indicates the generation of a kinematic wave between zones A and B. The straight-line (constant) speed portions were extended until they met. The intersections are marked by small circles. It can be seen that the disturbance travels at an almost constant speed of 14.8 mph. The small variations from a straight line can be expected to be caused to some extent by measuring errors.

Traffic conditions in zone A show uniform velocities and a very high flow of 2,350 vph. It is interesting to note that lane changes (vehicles 545, 544, 433, 828, and 823) have no noticeable influence, even at this high traffic flow. The kinematic wave, however, slows traffic down from an original speed of about 30.5 mph to between 2.5 and 7.9 mph. Recovery time for traffic to go through this range of low speed is between 7 sec (at 7.9 mph) and more than 9 sec (at 2.5 mph). However, traffic which has been running close to or at the peak capacity does not recover fully. After going through the disturbance, flow is reduced to about 1,600 vph.

Constant inflow at the rate of about 2,300 vph will widen the characteristic zone of seriously reduced traffic flow, forming a jam condition for a length of road which is spreading at the speed of the kinematic wave. It is interesting to note that traffic recovers to a flow of approximately 1,650 vph only. This capacity has been observed to be maintainable continuously under normal conditions with fairly uniform traffic composition. It was also found that the peak capacity of 2,300 vph can only be maintained for a short period of time at present traffic conditions, with full driver control, and no supporting driver aid system.

AUTOMATIC CONTROL SYSTEMS

It was found that the velocity of propagation for disturbances (kinematic waves) is as given by Eq. 12.

It also can be assumed that any changes in speed will be propagated more readily if vehicles travel with small gaps between the leading and trailing vehicles. In this situation, drivers have to react faster than at wide spacings of vehicles. As there is no rigid connection between vehicles, however, all changes in velocity will be propagated with a delay time, τ, which is dependent on the concentration of vehicles. That is,

$$\tau = \frac{1}{\mu k} \qquad \text{or } \mu = \frac{1}{\tau k} \tag{38}$$

where μ is a constant velocity.

Traffic flows at a speed, v, and as it will be the driver of the trailing car who has to react to speed changes of the lead car, disturbances will be propagated opposite to the direction of traffic flow at the velocity μ. Thus,

$$c = v - \mu \tag{39}$$

or

$$\frac{dq}{dk} = v - \mu \tag{40}$$

or, substituting Eq. 6,

$$\frac{dq}{dk} = \frac{q}{k} - \mu \tag{41}$$

or

$$k \frac{d}{dk}\left(\frac{q}{k}\right) = -\mu \tag{42}$$

Integrating gives

$$\frac{q}{k} = -\mu \int_{k_j}^{k} \frac{dk}{k} \tag{43}$$

and

$$\frac{q}{k} = -\mu \ln \frac{k}{k_j} + c \tag{44}$$

Because $q = 0$ for jam condition, $k = k_j$ and

$$q = -\mu k \ln \frac{k}{k_j} \tag{45}$$

Substituting Eq. 38 gives

$$q = \frac{-1}{\tau} \ln \frac{k}{k_j} = \frac{1}{\tau} \ln \frac{k_j}{k} \tag{46}$$

This relationship reveals that traffic flow is a function of a coupling factor; i.e., the speed at which disturbances are propagated. The factor, τ, is not uniform in a driver-controlled system and also depends on the spacing of vehicles. The expression "ln (k/k_j)" indicates that the relationship of traffic density to the density in a traffic jam condition, determined by traffic composition and by traffic control, also influences the magnitude of traffic flow. It further explains the different shape of the q—k curve found in experiments with closely controlled traffic in facilities like the Lincoln and Holland Tunnels in New York where rather high traffic densities were experienced.

The most important result, however, appears to be that if the delay time, τ, can be kept constant, as it would be in an automatic longitudinal control system, traffic capacity will be determined by the ratio of the chosen density and the packing density (i.e., the jam condition plus a delay factor depending on the sensitivity of the control system).

Disturbances (kinematic waves) will be propagated in such a system but no shock waves will be generated because the characteristics will travel along parallel paths and will not intercept each other, if vehicles are spaced uniformly.

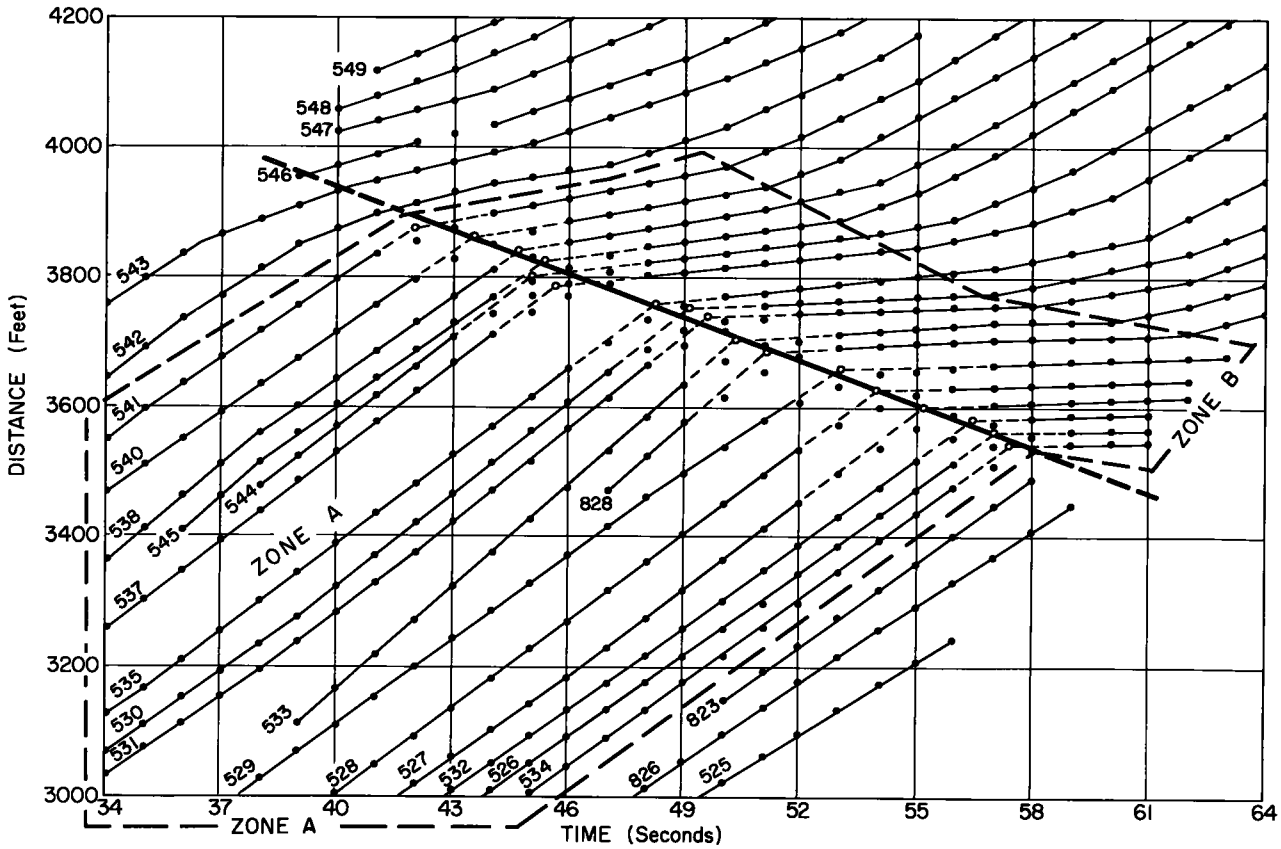

Figure 50. Influence of a kinematic wave.

CONCLUSIONS

It has been demonstrated that traffic flow depends on two factors—the delay time, τ, in propagating changes, and the ratio of traffic density of moving traffic to the density of traffic in jam conditions. Velocity and traffic composition are hidden factors in

$$q = \frac{1}{\tau} \ln \frac{k_j}{k} \tag{46}$$

because speed is a function of volume and density, and the number of vehicles per unit length of traffic lane is related to traffic composition. The propagation and characteristics of disturbances (kinematic waves) and the formation of shock waves in traffic flow have been explained and some of the resulting consequences for automatic control systems have been discussed.

The validity of the theoretical aspects of traffic flow and control, however, should be verified by field data. Methods for the measurement of data on traffic flow, kinematic, and shock waves have been developed. However, more research is needed in this area, although the limited data available appear to support the theoretical considerations.

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CHAPTER TWELVE

LONGITUDINAL CONTROL SYSTEMS FOR PLATOON MOVEMENT

Sensing and communication between vehicles can be applied to two different areas, as follows:

1. Improving road safety, and reducing traffic accidents.
2. Increasing traffic capacity of existing facilities and increasing the safe journey speed.

Both aspects have been considered, because it has been shown (Appendix C) that the greatest rewards from a sensing and communication system can be expected on freeways during peak-hour traffic under urban or semi-urban conditions. This applies to accidents (on freeways the accident rate is lower than on other roads, but due to the high traffic volumes the total number of accidents is high) as well as to the increase of capacity and speed.

The most frightening accident on freeways is the mass pileup of rearend collisions caused by weather conditions or by kinematic wave or shock wave at high traffic densities.

The many concepts of useful communicated information were reduced to a longitudinal control system in light of the problems encountered in freeway traffic. Possible means of communication were investigated, and it was found that an infrared source-sensor appears to be a feasible system for longitudinal control if simplicity, cost, reliability, and automatic identification of the target (i.e., the leading car in the car-following situation) are considered to be the most important factors.

Basically, two systems can be envisaged—a vehicle-contained sensing and communication system and a system which is essentially part of the structure of the freeway. Both systems must include facilities for the vehicle as well as for the highway. The infrared (IR) system, which is essentially a vehicle-contained system, relies on a minimum of equipment added to the highway structure for guidance and lane coding. A continuous, roadway-contained, sensing and communication system must transfer information to the moving vehicles. This also requires some receiving and servo equipment in the vehicle. Thus, some special vehicle-contained equipment will be necessary under all circumstances.

The study reported in this chapter covers the basic principles of any longitudinal control system and is valid for the vehicle-contained and for the highway-contained sensing and communication systems.

Two different control functions appear to be most useful to achieve longitudinal control in traffic flow. These are:

1. Duplication of the acceleration pattern of the lead vehicle by the trailing vehicle.
2. Differential speed between the lead vehicle and the trailing vehicle.

Both control functions are compatible with the IR system. A system based merely on the spacing of vehicles does not appear to be very promising, for reasons discussed later.

SOME BASIC CONSIDERATIONS OF TRAFFIC FLOW

Assume that an acceleration is applied to the trailing vehicle after a time delay, τ, to adjust its velocity to the velocity of the leading vehicle. N vehicles are traveling in a platoon, so that the $(n+1)$th vehicle is following the nth vehicle. Also, let $V_n(t)$ be the speed of the nth vehicle at time t, and $\dot{V}_n(t)$ its longitudinal acceleration. Then $\dot{V}_n(t+\tau)$ represents the acceleration of the nth car at time $t+\tau$. The factor τ means that an attempt will be made by the trailing vehicle after some delay to adjust its speed to the speed of the leading car. If the leading vehicle is decelerating the trailing vehicle will travel with the original speed for the time τ before any deceleration will be introduced. During this delay time the gap between the vehicles will be reduced, and it is assumed that both vehicles will then decelerate at the same rate. The remaining gap size between the vehicles represents a safety factor. Absolute safety can be attained if the gap is big enough to allow the trailing vehicle to come to a safe stop when the leading vehicle hits a fixed object in the road. Marginal safety can be attained when the following car has time to react and by braking fully, can come to a safe stop behind the lead vehicle. In an emergency both vehicles will skid to a safe stop with a minimum gap left between the vehicles. It is assumed that maximum deceleration will be the same for both vehicles and the accident situation, which may call for violent decelerations beyond the capability of a motor vehicle, is not considered. This, in fact, is the condition with regard to the spacing of vehicles which is frequently found in dense traffic on urban freeways.

For both conditions (i.e., the safe traffic flow and the marginally safe traffic flow) traffic flow can be determined (*9*). For safe flow, the separation between vehicles consists of the distance covered, $v\tau$, during the reaction time, τ, the actual braking distance, $v^2/(2\,\mu\,g)$, and the distance between the centers of the stopped vehicles. μ is the coefficient of friction and the gravitational constant, g, is 32 ft per sec per sec. Let t be the combined time, consisting of making a decision and reacting. The safe spacing is then

$$s = \frac{v^2}{2\mu g} + v\,t + c \tag{47}$$

the flow is

$$q = \frac{v}{s} = v(v^2/2\mu g + vt + c)^{-1} \tag{48}$$

and

$$\frac{dq}{dv} = 0 = \left(\frac{v^2}{2\mu g} + vt + c\right)^{1} - v\left(\frac{v^2}{2\mu g} + vt + c\right)^{-2}\left(\frac{2v}{2\mu g} + t\right) \tag{49}$$

from which

$$v^2 = 2\mu g c \tag{50}$$

and

$$v = \sqrt{2\mu g c} \tag{51}$$

For $\mu = 0.6$ (dry pavement) and $c = 20$ ft, and for maximum traffic flow, $v = 27.6$ ft per sec. $= 18.8$ mph.

The reaction time, τ, has been determined to be in the range of 1 to 2 sec. Gazis, Herman, and Rothery (*2*) report an average reaction time of 1.55 sec. Ohio State University observers informally report a mean reaction time of 1.4 sec. For the calculations in this report an average reaction time of $\tau = 1.5$ sec was chosen. Describing the reaction time will yield a higher maximum flow.

For marginally safe flow the spacing is given by

$$s = vt + c \tag{52}$$

and the flow by

$$q = v(v\,t + c)^{-1} \tag{53}$$

The most important difference in the characteristics of safe and marginally safe flow is that an optimum speed can be determined for the safe flow whereas marginally safe flow has no optimum speed. Eq. 53 shows that traffic flow increases with v and some value must be introduced for a maximum speed to arrive at a realistic value at low densities. Figure 51 shows the function of traffic flow against traffic density for the idealized conditions of safe and marginally safe traffic flow using an average reaction time of 1.5 sec. Also shown is the observed mean flow-concentration curve for the Eisenhower Expressway in Chicago. No reason can be given as to why traffic is moving slower at higher densities than would have been necessary for safe flow with the rather long reaction time assumed.

The individual driver is probably more interested in a reduction of the trip time than in the magnitude of total traffic flow. Figure 52 shows traffic flow as trip time; it appears to be very significant that the trip time can be reduced by almost 57 percent if a 10 percent reduction of the traffic flow can be accepted. The obtainable traffic volume, however, is rather low, as the assumed conditions meet the requirements for safe flow spacing.

It has been said before that reducing the reaction time will bring some increase in safe traffic flow. Figure 53 shows traffic flow as a function of reaction time; it can be seen that in adopting the standards for "safe flow" (i.e., sufficient space between vehicles to allow for a safe stop of the trailing vehicle even if the lead vehicle should hit a fixed object on the road) rather low traffic volumes can be obtained even with an unrealistically short reaction time. This flow would be reduced by another 10 percent to get more acceptable trip times. The effect of different reaction times from 0.5 to 1.5 sec is shown by three flow concentration curves in Figure 54. A desirable speed of 50 mph would reduce these values by about 15 percent if the very safe flow conditions are to be maintained. It is interesting to note that traffic conditions in the New York tunnels, where maximum flow combined with highest safety standards must be maintained, are similar to the conditions described before. The average speed in these tunnels is about 20 mph at high traffic volumes.

Marginally safe flow is extremely sensitive to a reduction time, especially at higher speeds (Fig. 55). Grime (*1*) measured a mean value of about 0.7 sec for an alert, ready driver to depress the brake pedal after an expected

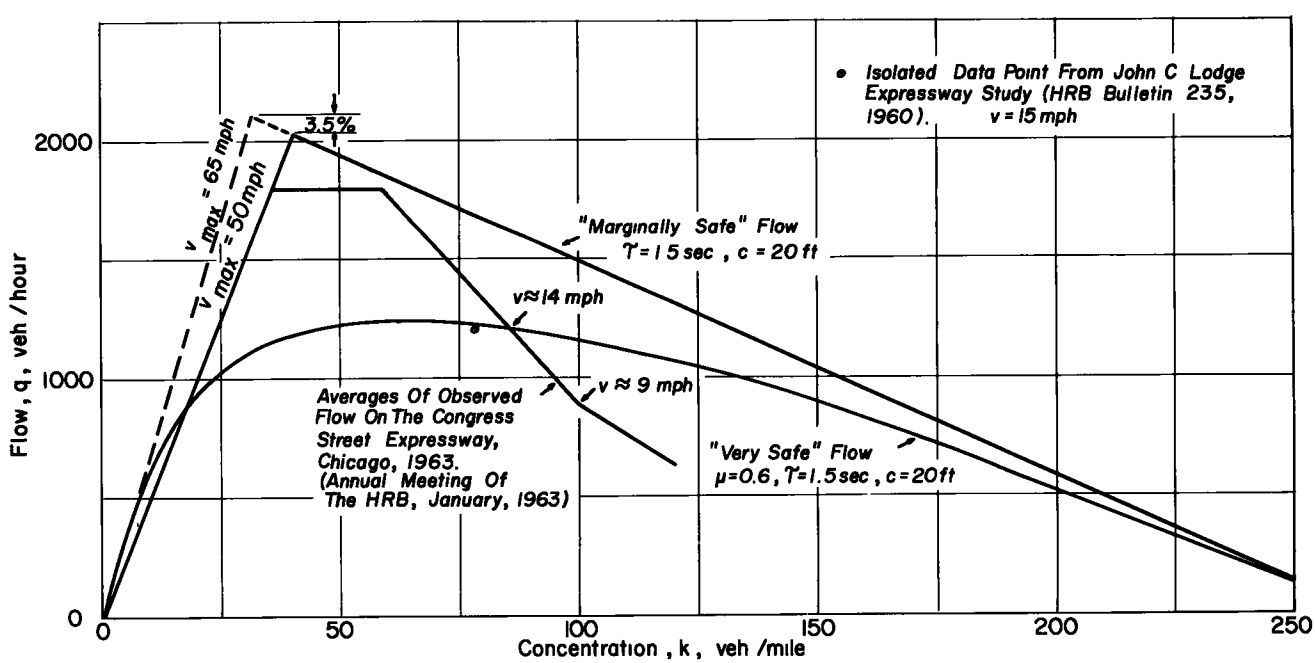

Figure 51. Vehicle density for "marginally safe" and "very safe" traffic flow.

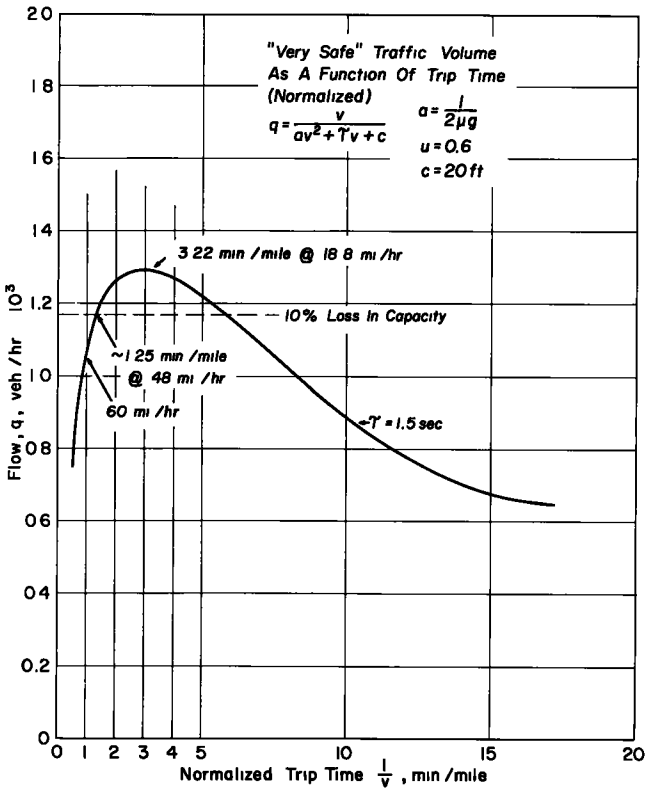

Figure 52. Traffic flow vs trip time.

stimulus under an information load corresponding to city driving. Thus, for the average driver a range in τ from 0.7 to 1.5 sec appears to be a reasonable assumption. Figure 55 shows that this range can produce a marginally safe traffic flow from about 1,900 to 3,400 vph per traffic lane. Sensing and communication between vehicles will most likely produce a uniform and considerably reduced reaction time, τ, depending on the sensitivity of the system. Although little gain in traffic volume and velocity can be expected from a system relying on the safe flow concept, considerable gain in volume and velocity can be expected from a marginally safe system. The increase in traffic flow in such a system is a function of τ and the velocity, v. Figure 56 displays these functions for $\tau = 1.5$, 1.0, and 0.5 sec. It can be seen that the gain in volume is rather small for velocities over 50 mph at reaction times of 1.0 and 1.5 sec. A reaction time of $\tau = 0.5$ sec, however, still provides substantial gain at velocities over 70 mph.

CONTROL SYSTEMS

It has been said previously that basically two different control systems are possible, as follows:

1. Acceleration control; i.e., the following car duplicates the acceleration pattern of the lead car after a delay (reaction time, τ).
2. Differential speed control; i.e., a system which controls the velocity of the trailing car so that the speed difference between the leading and the trailing cars becomes zero.

Acceleration Control

It has been shown that the gain produced by a sensing and communication system for safe flow condition is rather small. Therefore, the acceleration control system will only be discussed for marginally safe flow.

Figure 57 shows the time-distance diagram for two vehicles traveling at a marginally safe spacing. Consider the two cars of zero length separated in steady flow by a distance vt. The vertical separation of the two paths is then t, a constant, and the horizontal separation is vt, which changes with the velocity.

If the lead car undergoes an acceleration, the following

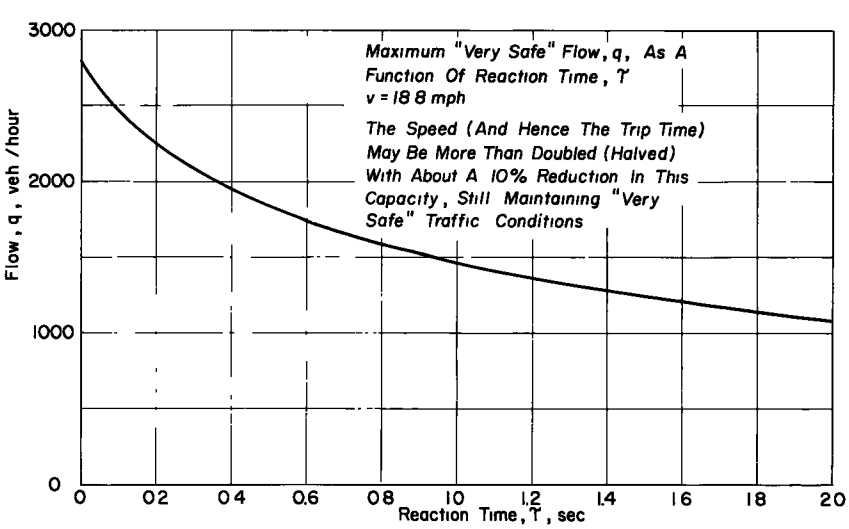

Figure 53. "Very safe" traffic flow vs reaction time.

car duplicates the acceleration a time, τ, later. If the initial spacing is vt, it duplicates the lead car's acceleration at the same point on the road. The vertical spacing, τ, however, will not change. Adding a constant spacing, s_j, the spacing in a traffic jam condition to account for the length of the vehicles, yields that an initially marginally safe spacing

$$s = v\,t + s_j \tag{54}$$

remains marginally safe as long as the lead car does not reverse its velocity.

Consider vehicles in a traffic lane starting to move from a jam condition with the spacing s_j and $v_n(t) = 0$ for $0 \le t \le n\tau$ with $v_0(0) = 0$. The movement is controlled by the acceleration control system as described before and

$$\dot{v}_n(t+\tau) = \dot{v}_{n-1}(t) \tag{55}$$

Then

$$\int_0^t \dot{v}_n(p+\tau)dp = \int_0^t \dot{v}_{n-1}(p)dp \tag{56}$$

where p is a dummy variable of integration. The right-hand side of this equation is $v_{n-1}(t) - v_{n-1}(0)$. To integrate the left-hand side, let $u(p) = v_n(p+\tau)$ so that $\dot{u}(p) = \dot{v}_n(p+\tau)$ and

$$\int_0^t \dot{v}_n(p+\tau)dp = \int_0^t \dot{u}(p)dp = u(t) - u(0) = v_n(t+\tau) - v_n(\tau) \tag{57}$$

The integral applying the foregoing boundary conditions is then

$$v_n(t+\tau) = v_{n-1}(t) \quad , t \ge 0 \tag{58a}$$

which may be written

$$v_n(t) = v_{n-1}(t-\tau) \quad , t \ge 0 \tag{58b}$$

Integrating from the jammed state, the spacing between the $(n-1)$th and the nth car, $s_n(t)$, is the integral of their relative velocity plus the initial jammed spacing, s_j, or

$$s_n(t) = \int_0^t [v_{n-1}(p) - v_n(p)]\,dp + s_j \tag{59}$$

Using

$$v_n(t) = v_{n-1}(t-\tau) \quad , t \ge \tau \tag{60}$$

$$s_n(t) = \int_0^\tau v_{n-1}(p)dp + \int_\tau^t v_{n-1}(p)dp - \int_\tau^t v_{n-1}(p-\tau)dp \tag{61}$$

and under $q = p - \tau$

$$\int_\tau^t v_{n-1}(p-\tau)dp = \int_0^{t-\tau} v_{n-1}(q)dq = \int_0^\tau v_{n-1}(q)dq + \int_\tau^{t-\tau} v_{n-1}(q)dq \tag{62}$$

so that

$$s_n(t) = \int_{t-\tau}^t v_{n-1}(p)dp + s_j \quad , t \ge \tau \tag{63}$$

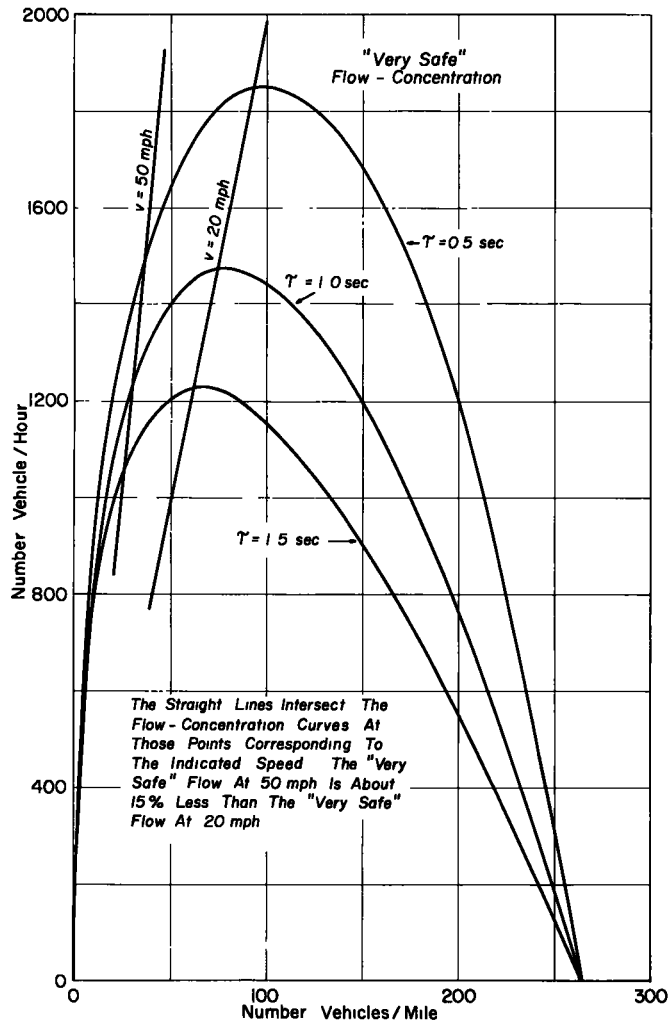

Figure 54. Effect of reaction time on traffic flow at "very safe" levels.

At any time for which $v_{n-1}(t)$ has been constant for the preceeding τ sec,

$$s_n(t) = \tau\, v_{n-1}(t) + s_j \tag{64}$$

and for the steady state

$$s_n(\infty) = \tau\, v(\infty) + s_j \tag{65}$$

From these calculations it appears that the spacing compatible with acceleration control is the same spacing as derived from the marginally safe flow. This may be considered as the natural spacing for optimum traffic flow.

Differential Velocity Control

Assume that the velocity of the lead car is continuously available to the trailing vehicle and that the differential velocity, $v_{n-1} - v_n$, can also be determined continuously. The response of the trailing car is characterized by a lag time of the sensing and communication system, and the reaction time of the driver, if it is used as a driver aid system.

The system responds to the relative velocity between cars traveling in a platoon in accelerating or decelerating trailing vehicles at a rate proportional to such relative velocities between vehicles. Then

$$\dot{v}_n(t+\tau) = \lambda\,[v_{n-1}(t) - v_n(t)] \qquad (66)$$

in which λ is the constant of proportionality. Stability of the flow is an important consideration for longitudinal control systems. Appendix D gives the considerations and the mathematical treatment of the stability problem, which, for an essentially sinusoidal variation in the speed of the leading vehicle, leads to the conclusion that the amplitude of the variation will be amplified as it passes from car to car down the traffic stream unless

$$2\,\lambda\,\tau \leq 1. \qquad (67)$$

Figure 58 shows the maximum "marginally safe" flow for velocities of 20, 40, and 60 mph with differential velocity control when the stability condition (Eq. 67) is satisfied. It can be seen that only reaction times of less than about 0.8 sec will produce results which are of interest for the increase of present traffic capacities.

The natural spacing of vehicles can be determined in a similar way as this has been done for the acceleration controlled system. Assume a platoon of vehicles in a traffic lane beginning to accelerate from a jam condition with the spacing, s_j, to a velocity according to the governing equations of relative velocity control. The spacing, s_n, between the nth and the $(n+1)$th car is then

$$s_n(t) = s_j + \frac{1}{\lambda}\,v(t+\tau) \qquad (68)$$

If λ is chosen to just satisfy the stability condition $2\lambda\tau = 1$, the minimum spacing satisfying this condition is

$$s_n(t) = s_j + 2\tau\,v_n(t+\tau) \qquad (69)$$

or, in a general form,

$$s = s_j + 2\tau v \qquad (70)$$

in which τ is the average response (reaction) time.

Defining the steady flow density as $k = 1/s$, the volume is then $q = v\,k$. Figure 59 shows the stable natural flow as a function of velocity for different reaction times. It can be seen that only a small increase can be obtained at speeds over 40 mph and reaction times longer than 0.7 sec. Shorter response (reaction) times can lead to a considerable increase, and with $\tau = 0.4$ a traffic volume of 3,500 vph per lane has been calculated for a speed of 60 mph. Figure 60 shows the range in traffic volumes with different reaction times and the resulting traffic density in vehicles per mile per traffic lane.

Comparing relative velocity control with acceleration control it can be seen that the natural spacing for relative velocity control is twice that for acceleration control, and the resulting traffic volume is half that of the volume obtained at equal speeds with acceleration control.

Distances between vehicles calculated as the natural spacing have been termed as marginally safe, although the spacing reduces to the jam spacing before the speed of the following vehicle is reduced to zero. For practical applications, an additional safety factor might be introduced with the k_j spacing, and the volume will then be somewhat less than the "natural" flow shown in the diagrams.

PROPAGATION OF DISTURBANCES

It has been pointed out previously that improvement in traffic capacity from a relative velocity control system begins with reaction times smaller than about 0.7 sec.

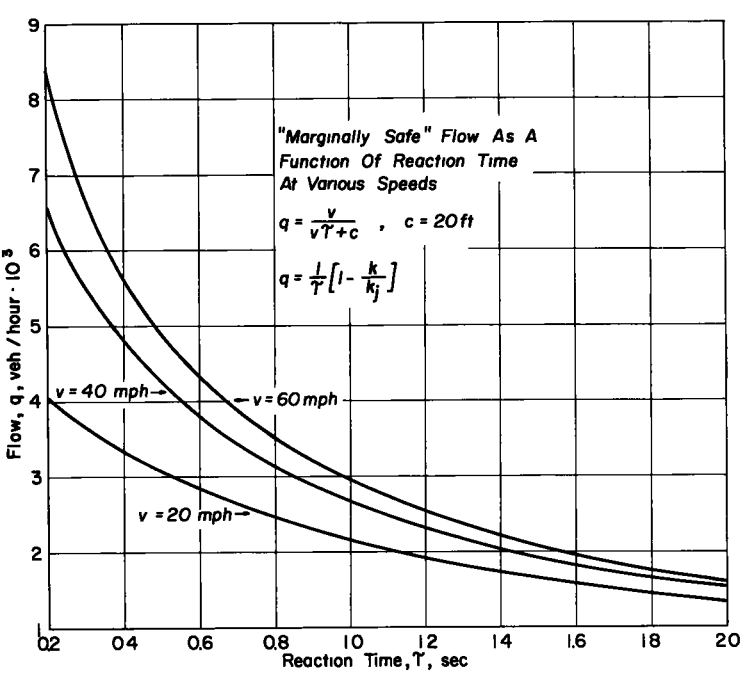

Figure 55. Effect of reaction time on traffic flow at "marginally safe" levels.

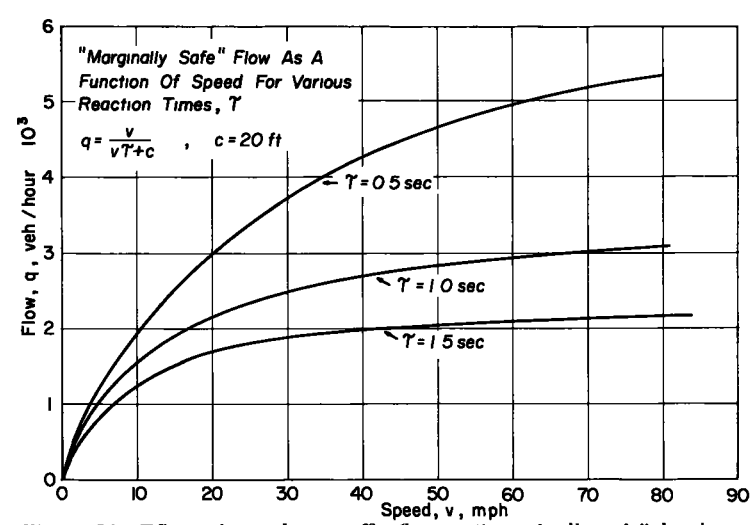

Figure 56. Effect of speed on traffic flow at "marginally safe" levels.

This suggests that automatic relative velocity control should be the aim of the whole development. For such a system, however, the propagation of disturbances will be of great importance. This problem has been considered in more detail in Appendix D, and only an indication of the method and the results is presented here.

Considering a most sensitive relative velocity control system with no delay in reacting to speed changes of the lead vehicle, traffic flow will be governed by

$$\dot{v}_n(t) = \lambda\,[v_{n-1}(t) - v_n(t)] \qquad (71a)$$

which may be written as

$$\dot{v}_1(t) = \lambda\,[v_0(t) - v_1(t)] \qquad (71b)$$

or

$$\dot{v}_1(t) + \lambda\, v_1(t) = \lambda\, v_0(t) \qquad (71c)$$

in which $v_0(t)$ is the forcing function and $v_1(t)$ is the system response. Multiplying through by an integrating factor, $e^{\lambda t}$, gives

$$\frac{d}{dt}\,[e^{\lambda t}\, v_1(t)] = \lambda\, e^{\lambda t}\, v_0(t) \qquad (72)$$

which, with $v_1(t_0) = a_1$, readily integrates to

$$e^{\lambda t}\, v_1(t) - e^{\lambda t_0}\, a_1 = \lambda \int_{t_0}^{t} e^{\lambda p}\, v_0(p)\,dp \qquad (73)$$

or

$$v_1(t) = a_1\, e^{-\lambda(t-t_0)} + \lambda \int_{t_0}^{t} e^{\lambda(p-t)}\, v_0(p)\,dp \qquad (74)$$

In a platoon of cars this value of $v_1(t)$ then becomes the forcing function for the second car; i.e.,

$$\dot{v}_2(t) = \lambda\,[v_1(t) - v_2(t)] \qquad (75)$$

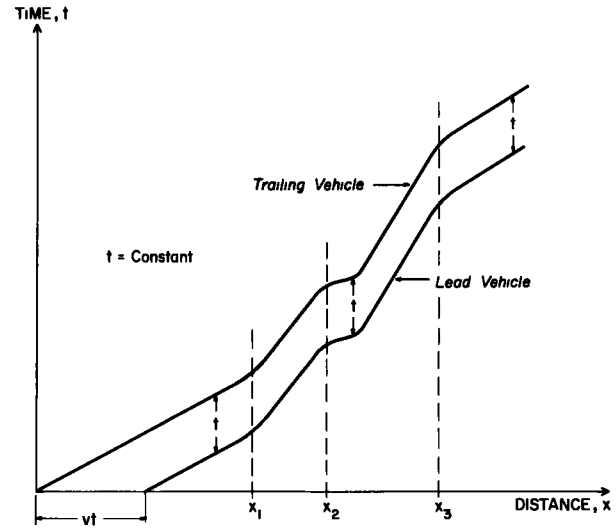

Figure 57. Time-distance diagram for two vehicles traveling at a "marginally safe" spacing.

which is integrated in the same way for $t \geq t_0$ with $v_2(t_0) = a_2$. Repeating this process n times yields the velocity for the nth car, or

$$\dot{v}_n(t) = \lambda\,[v_{n-1}(t) - v_n(t)] \qquad (76)$$

A simpler solution can be obtained if

$$v_1(t) = v_0(1 - \lambda^{-1}\, e^{\lambda}\, \lambda t e^{-\lambda t}) \qquad (77)$$

corresponding to the first car braking from the steady speed, v_0, momentarily coming to rest, and then acceler-

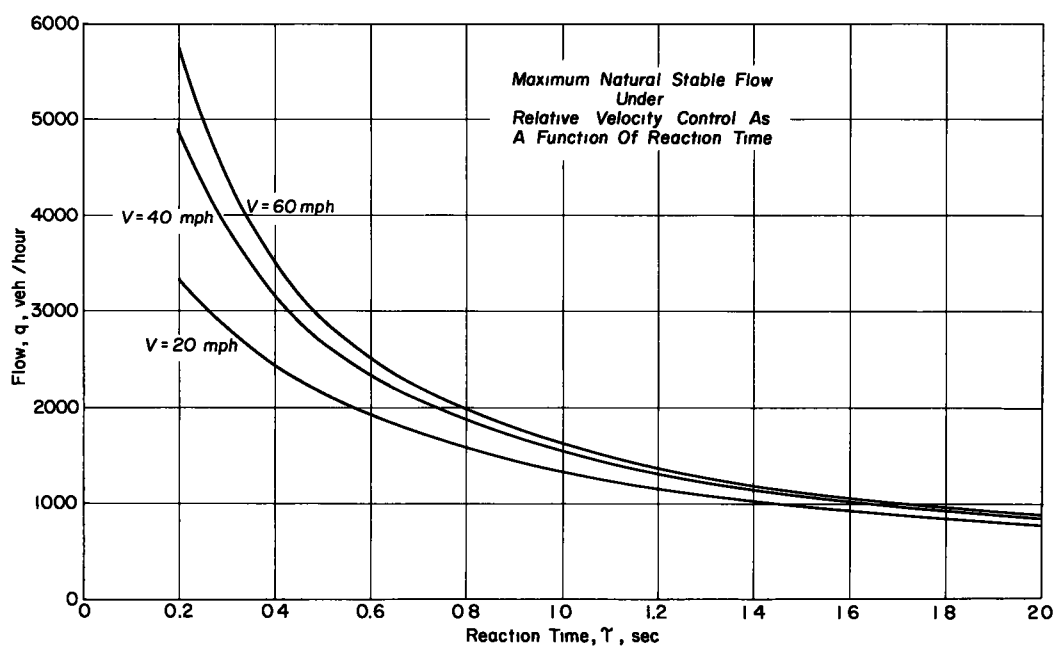

Figure 58. Effect of reaction time on traffic flow at the naturally stable level.

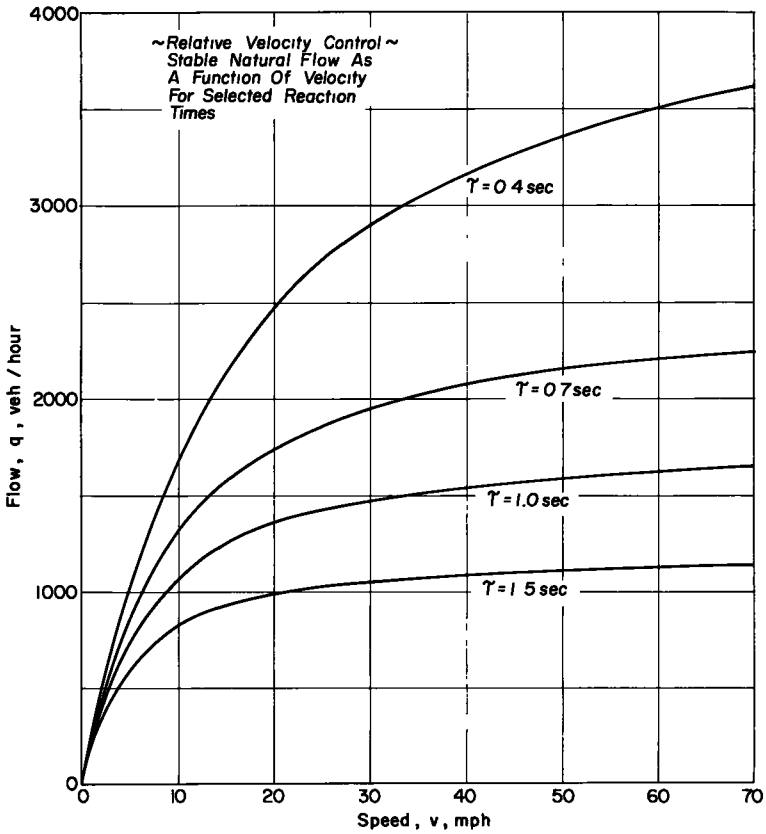

Figure 59. Effect of speed on traffic flow at the naturally stable level.

ating slowly to the original speed, v_0. If the following cars are all traveling at speed v_0 originally,

$$v_n(t) = v_0\left(1 - \lambda^{-1} e^{\lambda} \frac{(\lambda t)^n}{n!} e^{\lambda t}\right) \tag{78}$$

is the velocity program of the following cars under relative velocity control with no reaction delay. This result is obtained by the iteration process of the general solution,

$$\frac{(\lambda t)^n}{n!} e^{-\lambda t} = \frac{\lambda x}{x!} e^{-\lambda} \tag{79}$$

This is a Poisson distribution function, and the resulting distortions in the velocity of vehicles traveling in a platoon may therefore be called a Poisson wave. Figure 61 shows a sequence of graphs illustrating how this wave propagates. It can be seen that only the first vehicle comes to a full stop as required by the foregoing condition for Eq. 78. The second vehicle reduces its speed to 20 percent, the third to about 30 percent, and the fourth to about 40 percent of the original speed, and after 10 sec the steady-flow condition will be reassumed by the first five vehicles. Disturbances in a relative velocity controlled system do attenuate if the stability condition $2\lambda\tau \leq 1$ is maintained.

Other velocity controlled systems may be simulated by using the appropriate reaction (response) time.

By its very nature, the acceleration control system does not provide any attenuation of disturbances. It can be seen from Figure 57 that disturbances do not travel along the road in such a system, but can be expected to remain stationary in relation to the road. More research will be necessary on the subject. It appears, however, from the results obtained so far that high traffic volumes can be obtained from quickly responding, non-attenuating, acceleration control systems, whereas the slowly responding, attenuating, relative velocity control system will yield much lower possible traffic volumes. It hardly must be emphasized that a self-dampening system is desirable because amplified disturbances might well lead to the demanding of acceleration values beyond the capability of a motor vehicle. Even a non-attenuating system without any amplification can produce rather uncomfortable riding characteristics.

It appears that the answer lies somewhere between the two control systems. Considering a control system under which the following car's acceleration after a delay, τ, is given by a linear combination of the lead car's velocity and acceleration,

$$\dot{v}_n(t+\tau) = K\,\dot{v}_{n-1}(t) + \lambda\,[v_{n-1}(t) - v_n(t)] \tag{80}$$

with K and λ constant.

The evaluation and transformation of this equation (see Appendix D) leads to the approximate definition of the flow region accessible by combined acceleration and

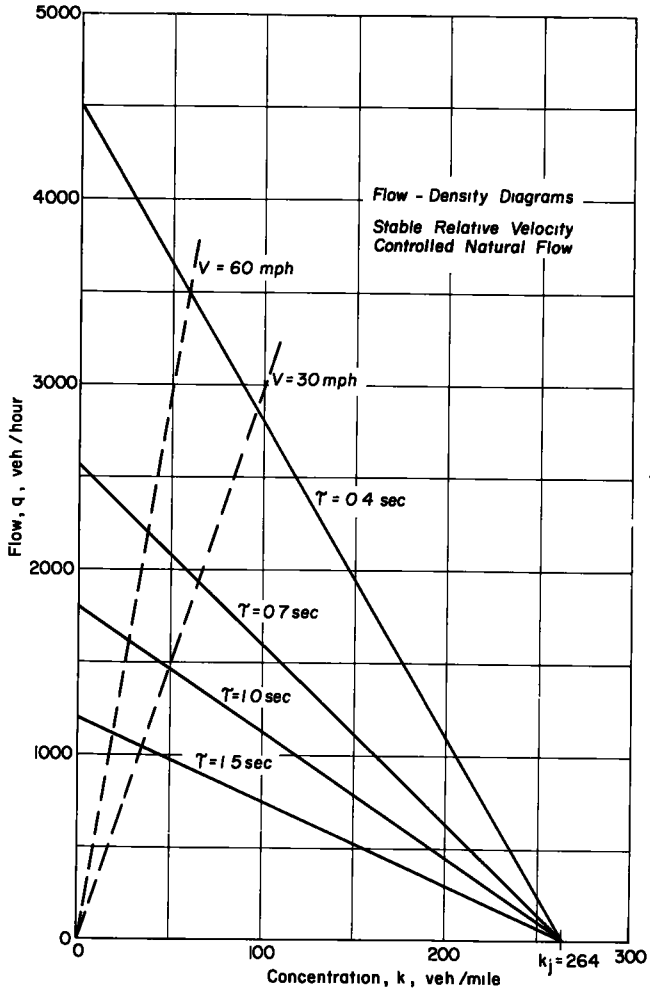

Figure 60. Flow-density diagram for stable relative velocity controlled natural flow.

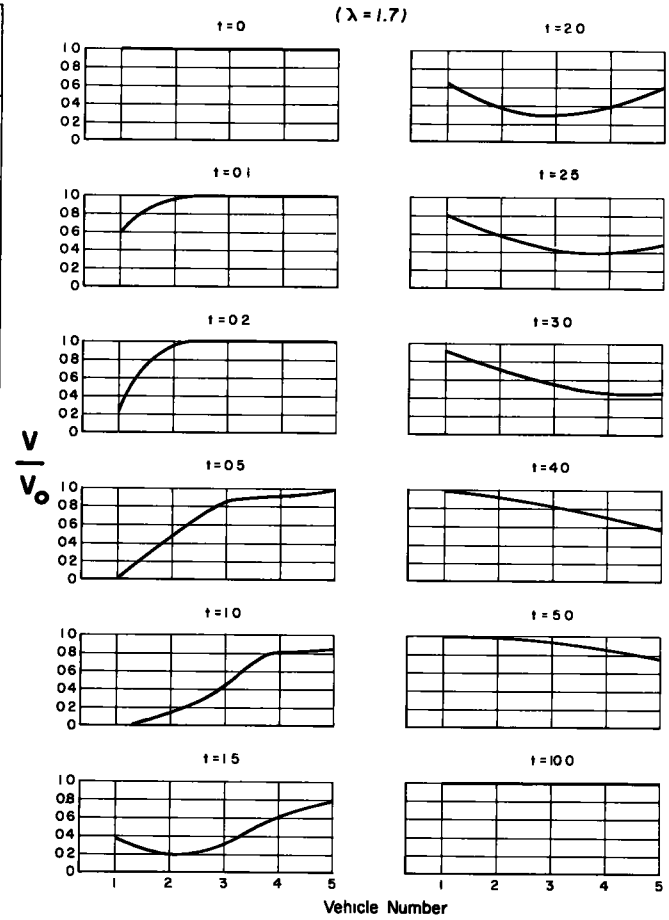

Figure 61. Time sequence illustrating propagation of a single Poisson wave.

relative velocity control. Figure 62 shows that area for a uniform reaction (response) time of $\tau = 0.6$ sec. The proportion of acceleration control is indicated by the factor K; $0 \leq K \leq 1$. It can be seen that the range is from more than 2,000 vph per lane at 30 mph and full relative velocity control to more than 4,300 vph per lane at 60 mph and full acceleration control. The speed range from 30 to 60 mph has been chosen because it appears as a feasible range with the present design standards of urban freeways.

HEADWAY CONTROL

The prototype which has been built making use of the IR system employs headway measurement in addition to relative speed control, and it appears desirable to discuss the problem of headway control more in detail. Headway control of the IR system was designed as an additional safety factor to aid differential speed control. Its purpose is twofold, as follows:

1. Headway control is employed to compensate for any drift in the electronic system. If the trailing car assumed a slightly higher speed than the leading car, caused by some slight error, the trailing car would finally reduce the headway to zero if both vehicles are traveling at a constant speed. To maintain a collision-free flow, however, a minimum spacing (i.e., the natural spacing) must be observed. Furthermore, the natural spacing reduces to the jam spacing before the speed of the following car vanishes. Under this condition, no space would be left theoretically between the bumpers of vehicles traveling in a platoon.

2. The second consideration takes care of the possibility that any automatic system will most probably be introduced first as a driver aid system. Some possibility must be provided to set the headway in accordance with his driving attitude.

These two aspects are taken care of by the headway control provided in the prototype of the IR system, although this control parameter is only a secondary function of the system.

It is of some interest to consider a system which is based on the desired spacing between successive vehicles.

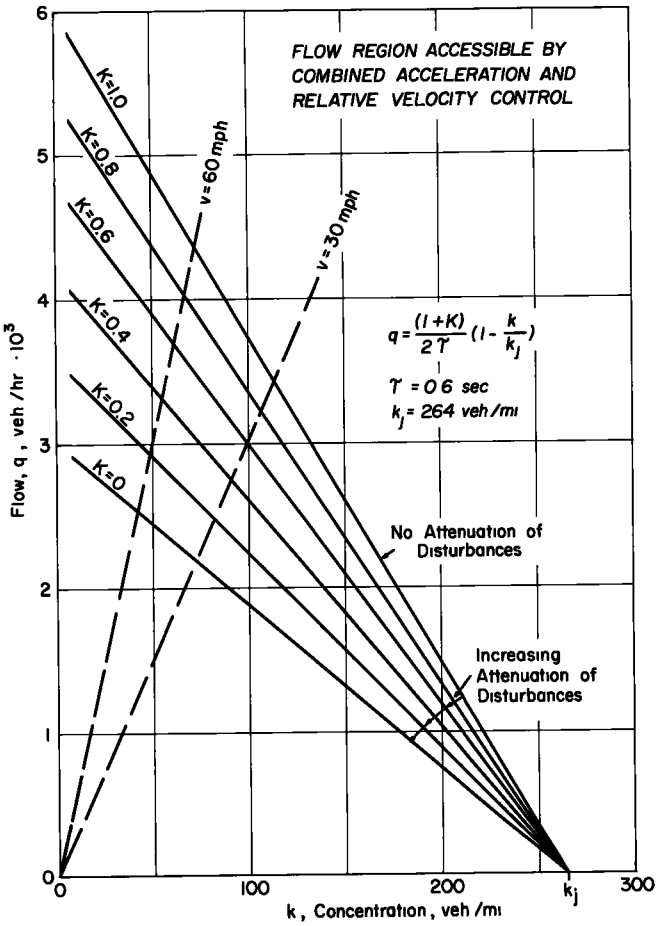

Figure 62. Area of flow-concentration diagram available for a uniform reaction time of 0.6 sec.

Then the spacing is proportional to the speed of the following car, and

$$v_n(t) = \mu\,[x_{n-1}(t) - x_n(t) - c] \qquad (81)$$

in which c is a factor depending on the length of the car. Appendix D describes the theory of such a system, and only the consequences in controlling vehicle movement by headways are discussed here.

If spacing is in error of only one car length in a headway controlled system, the acceleration demand for the following car will increase to almost 60 ft per sec per sec. Such accelerations are clearly beyond the capability of present motor vehicles. Weakening the response leads to a reduction of the dampening characteristics, and will lead to overshooting and oscillatory movement. It appears that such a control system will produce, to say the least, a rather unpleasant ride and appears to be unacceptable for road traffic.

CONCLUSIONS

The theoretical investigation of control systems has shown that considerable improvement in traffic flow and traffic safety can be obtained with the help of a longitudinal control and guidance system. The range of possible improvements in traffic flow appears to be determined by relative velocity control at the lower end and duplication of the acceleration pattern of the leading car at the upper end. Other considerations are the stability of the control system and the attenuation of disturbances propagated along a platoon of cars. It has been found that a relative velocity controlled system will be stable if $2\lambda\tau \leq 1$, and that disturbances will attenuate in such a system. An acceleration controlled system can be expected to be stable insofar as shock waves will not be formed, and disturbances will not be amplified if certain conditions in the spacing of vehicles and in the response of the system are met. Nevertheless, every disturbance will be propagated along the platoon of moving vehicles.

It appears that the solution to the problem of longitudinal control will be between the two extremes of relative velocity control and acceleration control. More research, and especially field tests, will be necessary before a final answer on the required attenuation of disturbances can be given.

For traffic safety, two possibilities have been studied—the "safe traffic flow," which will provide a safe stopping distance for the trailing car even if the leading vehicle should suddenly hit a fixed object in its traffic lane; and the "marginally safe traffic flow," which assumes that all vehicles traveling in a platoon can come to a stop and no rear-end collisions will occur, if the maximum possible deceleration is applied to any one of the vehicles in the platoon.

Using "natural spacing" as required by the control conditions, it appears that rear-end collision can at least be reduced, if not completely avoided, for normal traffic conditions with the acceleration and the relative velocity control system. More research on the subject will, however, be necessary to determine the minimum spacing which must be added to the natural spacing for optimum safety. This will reduce the maximum traffic flow somewhat for both systems, but the reduction is expected to be small.

Mean reaction times around 0.5 sec are not uncommonly reported. Should further studies indicate that this mean reaction time may be achieved with a suitable driver aid system for longitudinal control, these calculations indicate that a traffic capacity of 3,000 to about 4,000 vph per lane may be achieved, satisfying the necessary stability conditions and a reasonable attenuation ($K = 0.8$) of disturbances in the flow.

More research will be necessary to determine the conditions which will provide sufficient stability. It also appears desirable to test the IR system in platoon movements of about 10 vehicles. So far, all road tests have been carried out with only two vehicles.

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APPENDIX A

VEHICLE INSTRUMENTATION

Research efforts reported herein were accomplished for the most part on Interstate expressways. Extensive use was made of two highly instrumented vehicles. This discussion, however, emphasizes one of the research vehicles, an instrumented Chevrolet (Figs. 1 and 2), because of its representative characteristics and because it played the major role in this research. The following discussion considers each of the principal systems and characteristics of this vehicle, and discusses comparable systems in both vehicles when they differ substantially.

EQUIPMENT POWER

The nature of the recorder used in the first experimental vehicle necessitated several hundred watts of 110-v, 60-cycle power. This was obtained through use of a motor-generator set mounted in the rear of the vehicle. A standard electric fuel pump with a pressure regulator provided the necessary gasoline from the vehicle's fuel supply, and a flexible tube and muffler vented the exhaust through the rear window.

The Chevrolet (second vehicle), however, was equipped with both 12-v and 24-v dc alternator systems driven directly from the vehicle's engine, each capable of supplying 100 amp at or near engine idle. Lead-acid storage batteries served as ballast in these dc systems. All requirements for 100-v ac power in the Chevrolet were met with solid-state inverters.

RECORDING EQUIPMENT

All major recording requirements were met with oscillograph recorders; a 6-channel Honeywell Visicorder was employed in the first vehicle, and a 50-channel Consolidated Electrodynamics Corporation recorder (Fig. 2) was used in the second. Supplemental voice recording and single-channel, low-frequency dc recordings were performed with a Bell 4-track stereo tape deck.

The use of oscillograph recorders has several major advantages: (1) a visual record is obtained, particularly important when doing exploratory research; (2) each channel can be monitored as it is recorded; (3) no additional equipment is necessary to work with the recorded data; and (4) a permanent record is obtained at minimum cost.

INSTRUMENT POWER

The majority of the transducers employed in these two vehicles required 10-v dc power, which was obtained from the vehicle power supply regulated to the desired accuracy by solid-state voltage regulators.

TRANSDUCERS

The type of transducer employed in this equipment was entirely a function of the signal being monitored. All control positions, such as steering, throttle, and brake, were monitored with conventional potentiometers in a

balanced bridge circuit suitable for that function. Other transducers included a dc tach-generator coupled to the speedometer cable to provide a voltage proportional to speed, accelerometers and jerk meters mounted in the appropriate planes of interest, and other specialized transducers as needed for specific applications.

Figure 3 shows the "yo-yo" device used for measuring the headway and relative velocity of the instrumented vehicle with respect to the leading vehicle. In operation, the electric motor applies a frictional torque to a drum which has 600 ft of 0.005-in. silver alloy wire wound on it. The free end of this wire is attached to a leading vehicle. A potentiometer is gear-driven from this drum, such that its resistance changes in proportion to the amount of wire removed from the drum, while a dc tach-generator provides a voltage proportional to the rate at which wire is withdrawn from the drum. These changes are, in turn, proportional to headway and relative velocity.

WIRING LAYOUT

Cables lead from all transducers, power supplies, and sensors to a central junction point in the rear (trunk) of the vehicle. From this point, instrument power and the transducer signals are distributed to the appropriate circuit board in the 40-board rack located at the rear of the vehicle's passenger compartment. Here, one board contains the signal processing equipment for one of the channels being recorded. Leads then connect from this module board rack to the adjacent recorder. All auxiliary equipment is connected to and gets its power through the central junction point in the rear of the vehicle.

APPENDIX B

DIFFERENTIAL MODELS RELATING TO INTERVEHICULAR COMMUNICATIONS

A number of analytic models or mathematical representations of vehicle flow and physical phenomena have been suggested as having possible relevance to studies of intervehicular communication. This appendix reviews several of these models and discusses their applicability to communication problems. Emphasis is given to differential follower models, fluid flow analogies, and kinematic wave representations.

DIFFERENTIAL FOLLOWER MODELS

Attention is directed initially to the single-lane dynamics of pairs of vehicles as shown in Figure B-1, in which H represents time or distance headway. Let $X_n(t)$, $X'_n(t)$, and$X''_n(t)$ denote the position, velocity, and acceleration of car n at time t. Then if T is reaction time, $H'_{n+1}(t+T)$ is the relative velocity of car $n+1$ at time $t+T$.

General differential followers are written in at least three forms:

$$X''_n = f(H_n, X'_n, H'_n, H''_n) \qquad \text{(B-1}a\text{)}$$

$$X'_n = f(H_n, H'_n, H''_n, X''_n, X''_{n-1}) \qquad \text{(B-1}b\text{)}$$

$$H_n = f(X'_n, X''_n, X'_{n-1}, X''_{n-1}) \qquad \text{(B-1}c\text{)}$$

Some authors (e.g., *1*) have included other factors, such as response to brakelights, in their general models, but there is no known attempt to include these sorts of factors in analytic solutions or in experiments.

As can be seen, the forms listed are nearly equivalent. For example, Kometani and Sasaki (*2*) worked with

$$X_n(t-T) - X_{n+1}(t-T) = A\,X'^2_n(t-T) + B'\,X'^2_{n+1}(t-T) + B\,X_{n+1}(t) + b_0 \qquad \text{(B-2)}$$

in which b_0 is a constant accounting for car length and minimum tolerable motionless headway, whereas Helly (*1*) used

$$X''_n(t+T) = C_1\,X'_{n-1}(t) - X'_n(t) + C_2\,X_{n-1}(t) - X_n(t) - D \qquad \text{(B-3)}$$

in which D is desired moving headway. The two expressions differ only in the functional forms by which X, X', X'' are represented. The first form (Eq. B-2) employs a quadratic velocity term, whereas Eq. B-3 includes acceleration in the term D.

These differential followers are simply time-dependent differential equations relating position, velocity, and acceleration of one car to (usually) its predecessor. Their justification in every case rests with the degree to which they conform to real data. Thus, the different forms which have been used are simply attempts to pick the functional combination which best fits automotive behavior.

Pipes (*3*), and Chandler, Herman, and Montroll (*4*) were among the first to use differential followers. They used models of the form

$$X'_n = f(H_n) \qquad \text{(B-4}a\text{)}$$

and

$$X''_n = f(H_n, H'_n) \qquad \text{(B-4}b\text{)}$$

respectively. The Chandler et al. experiments determined the constants multiplying H_n and H'_n. Their results indi-

cated a very low correlation between X''_n and H_n. Thus, being able to neglect either X''_n or H_n, they dropped H_n, and obtained a perfect differential of the form

$$X''_n(t) = A\,H'_n(t-T) \qquad \text{(B-5}a\text{)}$$

which conveniently integrates to

$$X'_n = A'\,H_n(t-T) \qquad \text{(B-5}b\text{)}$$

a linear relationship. Kometani and Sasaki independently selected a similar form, obtained as a result of considering the distance car n would travel in time T, plus the distance it travels in stopping from a given V, and subtracting the distance which a slower leading vehicle moves over the same time.

Linear relationships of X'_n and H_n do not yield stability or other proper behavior in mass flow. They suffice only for small variations in velocity and are unrealistic in that no upper (or lower) limits are placed on X'_n. In order to correct such weaknesses, Kometani and Sasaki developed the expression given by Eq. B-2, which is of the general form

$$H_n = f(H'^2_n, X'_n) \qquad \text{(B-6)}$$

Experiments using movie recordings were run under urban conditions (only two cars in the area, however), during which the behavior of a follower was forced by lead-vehicle velocity fluctuations. Coefficients for Eq. B-2 were obtained for two speed ranges; these values are given in Table B-1.

The factors of interest are the low dependence on squared velocities, and the shift in "reaction time" between the two speed classes. These experiments were accompanied by a Fourier analysis of the speed fluctuation of the following car when the lead car undergoes sinusoidal variations in velocity. They assumed the motion of any vehicle, n, is represented by

$$X_n(t) = V_0 + Z(t) \qquad \text{(B-7)}$$

The input (lead vehicle) is

$$Z(t) = S \sin wt \qquad \text{(B-7}a\text{)}$$

and the motion of the followers is a series of the form

$$Y(t)\ (= \text{velocity}) = A_1 \sin(wt - \phi_1) + A_2 \sin(wt - \phi_2) + \qquad \text{(B-8)}$$

A consequence of this analysis was that spacing will increase or decrease according to the linear form $\frac{1}{2}[\alpha S^2 + \beta \sum_{n-1}^{\infty} A_n^2]$, where S and A are obtained from frequency analysis.

Differentiation of the quadratic follower yields

$$H'_n(t) = 2a\,H'_n(t-T) + B\,X''_n(t-T) \qquad \text{(B-9}a\text{)}$$

or, more generally,

$$\Delta H'_n(T) = C\,X''_n(t) \qquad \text{(B-9}b\text{)}$$

which suggests that velocity changes over an interval of length T (reaction time) are proportional to following vehicle acceleration at the beginning of the interval. This certainly seems reasonable when T is small with respect to vehicle dynamic changes. It also provides a simple means of verifying the differential follower. From Table B-1 the empirical value of T, which is 0.5 sec, can be imposed on

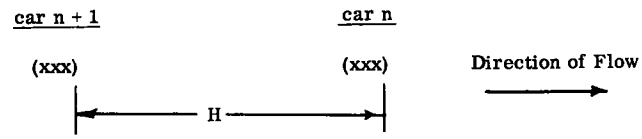

Figure B-1. Geometry of two vehicles following in single lane.

the time axis of the Ohio State University test vehicle output. It appears from inspection that the velocity curve could be closely approximated by linear pieces of length 0.5 sec. In this respect the Kometani and Sasaki formulation (Eq. B-2) appears to be empirically testable.

The form $V''_n(t) = f(H'(t-T))$, mentioned earlier, has been investigated by Herman and Potts (*5*), who employed linear, two-piece linear, and inverse headway forms. Experiments by Chandler, Herman, and Montroll (*4*) confirmed the inverse rule as being the best of the three. The tests were conducted at General Motors and in the New York tubes. In each experiment a least-squares fit of A (the determined value of the constant in Eq. B-5a) as a linear function of $H_n(t-T)$ calculated as the mean spacing was completed. Although the fits were not exceptionally good, it did not appear that nonlinear functions would yield improvement. The slope, A_0 of the least-squares function was employed as the coefficient in the inverse rule; i.e.,

$$X''_n(t) = A_0\left[\frac{H'_n(t-T)}{H_n(t-T)}\right] \qquad \text{(B-10)}$$

The value of A_0 varied somewhat with each set of runs. It seems to increase with the average velocity of the runs, which suggested that improvement might be obtained if it was a function of X'_n. The value of A_0 can be considered, at best, a statistical property. By the nature of its derivation it is a macro quantity (i.e., gross flow measure) more than a micro (i.e., based on single-car properties). In the plot presented for the Lincoln Tunnel tests, the derived A_0 is 20.3 mph, but calculation shows that for certain single runs in that series the constant, A, took values of 8 mph and of 41 mph. There thus appears to be nothing binding about A_0; i.e., it is a statistical quantity. Some slight

TABLE B-1

COEFFICIENTS FOR KOMETANI AND SASAKI DIFFERENTIAL FOLLOWER EQUATION

COEFF.	VALUE FOR VEL. $\leq$ 30 MPH	25 MPH $\leq$ VEL. $\leq$ 50 MPH
A	$-B'$	$-B'$
B'	0.00028 sec²/m	0.0085 sec²/m
B	0.58 sec	0.78 sec
T	0.5 sec	0.75 sec
b	4.13 m	—

additional insight regarding A_0 can be had by considering the alternative form of the inverse H follower. Integrating the X''_n form yields

$$X'_n(t) = V = A_0 \ln(H(t)/L) \qquad \text{(B-11)}$$

in which L was chosen as car length because H is measured from front to front of cars. Thus, for the Lincoln Tunnel case, $H = 2.7L$, or about 30 to 35 ft, results in a velocity of 20.3 mph on the average.

The velocity form of the relation is of further interest in that the authors comment favorably on the close relation of their A_0 values with the velocities reported and derived for maximum flow in the New York tubes. The flow relation derived by Greenberg (*6*) is

$$q = C\,K \ln(K_j/K) \qquad \text{(B-12)}$$

in which C is similar to A_0, K_j is jamming concentration, and K is the independent variable (in this case concentration). Comparison of Eq. B-11 with Eq. B-12 makes it reasonable to state that $q = \text{flow} = Vd$, where d is the density of cars. It seems extremely reasonable to define jamming concentration, K_j, as that concentration where cars are bumper to bumper; i.e., where $H = L$. Density equals headway divided into some unit of length. Thus,

$$K_j = C/L \qquad \text{(B-13}a\text{)}$$

and

$$K = C/H \qquad \text{(B-13}b\text{)}$$

Setting

$$V\,d = d\,A_0 \ln(H/L) = d\,A_0 \ln\left(\frac{C/K}{C/K_j}\right) = A_0\,d \ln(K_j/K) \qquad \text{(B-14)}$$

yields

$$q = A_0\,K \ln(K_j/K) \qquad \text{(B-15)}$$

since, by definition, $d = K$.

That various investigators have found their functions to fit better when the coefficients correlating H and X'' are very small is not at all surprising. This is especially the case in view of the fact that the other relationship usually correlated is H' vs X''. By several lines of argument, there must be a high correlation between H and X', which implies a high correlation between H and H'.

It is clear that, for car-following, the distance average velocity must be the same for all vehicles involved. The more precise the given headway relationship desired, the more the followers must match velocities. Discrepancies in velocity in car-following are linearly related to changes in H, with the constant multiplier being time. A similar relation exists between H' and X'', but there need not be such a relation between ends of the chain (between X'' and H).

One way to evaluate X', H relationships is to consider the constraints which act on X''. Consider the car-following option: If $X'_2 > X'_1$, the restrictions are vehicle deceleration capability and collision avoidance. The latter constraint can be handled by assuming that X'_1 is not changing. The constraint then becomes

$$X''_2\,t = H' \qquad \text{(B-16}a\text{)}$$

where

$$t\,H' < H \qquad \text{(B-16}b\text{)}$$

If it is assumed that under the conditions H' is twice the average value,

$$1/t > H'/2H \qquad \text{(B-17)}$$

which yields

$$X''_2 > \frac{(H')^2}{2H} \qquad \text{(B-18)}$$

A plot of this restriction for several values of H' and H is shown in Figure B-2. Note that such a constraint comes into play only when the driver chooses to use the minimum X'' for a given (H, H') pair. Any values of X''_2 greater than these minima can be used at any point of the space above (to either the left, or above, or both) the minimal point. The similarity of this X'' constraint to certain car-following models is clear.

The next consideration must be what it is that a driver does when he attempts to maintain H' or H'' at some value. Assume that a non-zero H'' exists, and that he wants to keep a zero H'. Given this situation, there are at least three ways in which the driver can obtain H' and H'' information. He can use vehicle cues, such as exhaust smoke or car tilt. He can use lead-car environment, such as rate of passing roadside objects. Finally, he can use his estimations of H and its derivatives. (The important additional source of information, expected lead-car behavior, has been temporarily neglected.) Any information the follower obtains about H'' from purely H data must come from his extracting time derivitives of H. In terms of task load, however, it is noted that if the driver is to obtain actual H'' data, he must by the continuity theorems sample H at a very high rate —in fact, nearly continuously. Even then, he is likely to develop gross errors if there are any errors in his headway time data. In order to match accelerations, the driver must watch the lead car continuously. Such a process is unlikely, and becomes even more so when one considers the rapid changes that occur in X''. We are thus led to expect that if the driver uses H data for knowledge of X''_1 or H'' he probably does so in a gross fashion. That is, he literally compares one headway with another taken a bit later and makes an estimate of the resulting H'. Taking a third point and comparing the two intervals can give him an estimate of H''. Note that the latter estimate is unnecessary unless very precise H is required, or unless the lead car is stopping rapidly. (In the fast-stop case, there are so many cues and changes of such large magnitude that H' estimates are hardly necessary.) The control process as it is now described is at all times a function of threshold of detection, which is in turn a function of X', X'', H, H', and H''. With the foregoing considerations as a guide, it should be possible in some cases to write relationships and constraint equations relating to sampling rate, situation dynamics, and reaction times.

MODELS BASED ON FLUID FLOW

Models based on fluid mechanics and kinematics are useful when treating instability of traffic flow and go a long way toward isolating critical variables in traffic flow which do not involve the human factor.

The best known fluid analogy is that by Greenberg (*6*), in which the flow density relationship previously mentioned (Eq. B-12) was developed. Imagine a length of pipe whose diameter changes at various rates with respect to the distance, *S*, from one end. To say the flow through this pipe is treated as one-dimensional is to say that differences in forward velocity across the diameter are neglected; i.e., at any point on any diameter the velocity vector parallel to the axis of the pipe is the same as any other. It should be noted that the possibility of a changing pipe cross-section has not been eliminated. Traffic flow models all use one-dimensional flow, and in particular are strictly concerned with laminar flow. (Indeed, there is but a single layer of particles.) If the fluid is compressible, use may be made of thermodynamic descriptions such as isothermal, adiabatic, or isentropic, all of which are conceivable by analogy in traffic flow.

The basic equations governing pure fluid flow include a continuity equation, a momentum equation, an energy equation, an equation of state, a thermodynamic relation such as isentropic behavior, and others relating to specific situations. For steady flow, the continuity equation takes the form

$$K_1 V_1 A_1 = K_2 V_2 A_2 \qquad \text{(B-19)}$$

whereas for unsteady flow one must include provision for changes in density (at a point) with time, resulting in

$$\frac{d}{dX}(KV) + \frac{dK}{dt} = 0 \qquad \text{(B-20)}$$

Both the energy and the momentum equations are derived from Newton's second law, and are thus sometimes lumped under the title "motion equation" when the differential form is used. This form can be obtained by considering an element of fluid in the pipe, and equating forces on it with its acceleration. One obtains an expression of the form

$$\frac{1}{G}\frac{dp}{ds} + \cos\theta + \frac{A_s}{g} = 0 \qquad \text{(B-21)}$$

The cos θ term is concerned with gravitational effects for streamlines at an angle of 90-θ with horizontal, and is dropped for applications to traffic under an assumed level flow. The symbol G is usually written as gamma, the specific mass, and when Eq. B-21 is multiplied through by the gravitational force, g, gives

$$\frac{1}{K}\frac{dp}{ds} + A_s = 0 \qquad \text{(B-22)}$$

where K is density. Inasmuch as A_s is the particle acceleration along a streamline,

$$V' = \frac{-1}{K}\frac{dp}{ds} \qquad \text{(B-23)}$$

This result (Eq. B-23) involves assumptions of steady, frictionless flow, the latter of which presents no difficulties to traffic models.

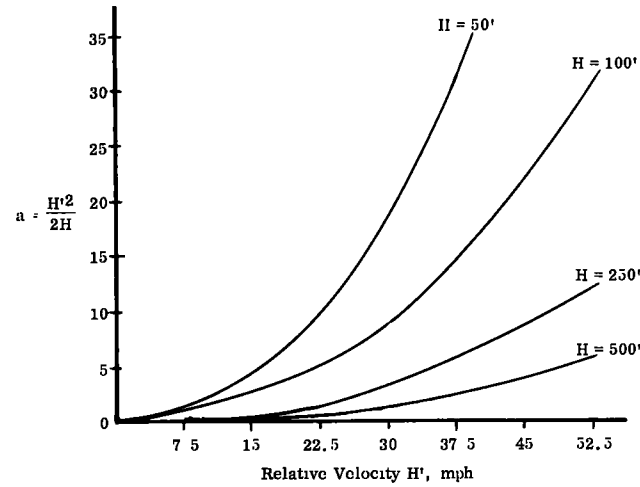

Figure B-2. Behavior of acceleration constraint (Eq. B-18) as a function of H *and* H'.

The remaining unexplained form is pressure, p. Pressure imposes problems for analogy, and must be handled by introducing a momentum equation for a particle of fluid,

$$\Sigma F = K Q (V_{out} - V_{in}) \qquad \text{(B-24)}$$

in which

F = force on particle (in this case, horizontal);
K = density;
Q = volume flow of fluid through a small rectangle;
V_{out} = fluid velocity out (downstream end) of rectangle; and
V_{in} = fluid velocity in (upstream end) of rectangle.

It is desired to use Eq. B-24 to derive necessary conditions for small discontinuities of flow. By considering flow on both sides of a discontinuity of magnitude dK, dV, the continuity equation (Eq. B-19) yields

$$K V A = (K + dK)(V + dV) A \qquad \text{(B-25)}$$

This reduces to

$$K\,dV + V\,dK = 0 \qquad \text{(B-26)}$$

and when applied properly to the momentum equation (it is assumed that the only component of F is pressure in the pipe) produces an expression for the velocity necessary for formation of a shock wave; viz.,

$$V^2 = dp/dK \qquad \text{(B-27)}$$

One can say that this is the velocity at which a small disturbance will travel through the fluid. Let this velocity be C, the speed of sound. Then

$$dp = C^2\,dK \qquad \text{(B-28)}$$

yields

$$V' = -1/K\,dp/ds = -(C^2/K)(dK/ds) \qquad \text{(B-29)}$$

which Greenberg calls "the equation of motion of a one-dimensional fluid." He calls C a "parameter that is de-

termined from the state of the fluid." For example, using the assumptions of an ideal gas, one gets $C = g\,K\,R\,T$, where the only non-constant is T, temperature.

Implicitly, Greenberg solves the problem of unsteady flow by saying that

$$V = V\,(s, t) \qquad \text{(B-30)}$$

and putting everything into the differential form,

$$\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial s} + C^2\frac{\partial K}{\partial s} = 0 \qquad \text{(B-31)}$$

Once this is done, it is possible to introduce the continuity equation again, and to obtain the relations

$$(K\,dV/dK)^2 = C^2 \qquad \text{(B-32}a\text{)}$$

$$du/dK = -\,C/K \qquad \text{(B-32}b\text{)}$$

and

$$V = C\ln\,(K_j/K) \qquad \text{(B-32}c\text{)}$$

It is imperative to note that this derivation included an assumption that V is a function only of density K. This is applied to the partial derivatives after account of the nonsteady equation of motion has been taken, and is thus not contradictory. Of course, it mandates

$$V = f\,(K) \qquad \text{(B-33)}$$

$$V' = f'(K) = f_1(K, K') = f_2(H, H') \qquad \text{(B-34)}$$

Because nothing is introduced in the derivation to require an additional relation affecting V, the only unanswered question is the function, f_2. This is what the fluid model supplies. It is now clear that the constant, A_0, used in differential followers is analogous—in fact, equal—to C^2.

It can be seen that the fluid model and the inverse H model are based essentially on the same assumptions. One added ingredient in the fluid model is the understanding afforded of C. A relevant comment here is that C depends on various variables of the system, these in turn being related to C depending on the assumptions made about the total system behavior. Recall the ideal gas relationship, whcih yielded $C = (gGRT)^{\frac{1}{2}}$, where C is a function of the variable temperature. Temperature, in kinematic thermodynamics, is related to the energy level of a group of particles, which in turn is related to their squared mean velocities. Without more careful analysis, there is, at least, the implication that for traffic flow A_0 should be made a function of mean traffic velocity.

KINEMATIC WAVES

A unique treatment of traffic dynamics has been made by Lighthill and Whitham (*7*) using kinematic waves. It is of interest at this point primarily for the light it sheds on the work previously considered.

A kinematic wave results from the assumption that flow can be described with a relation between density, distance, and flow; i.e., with a continuity equation alone. With the functional relationships in part yet undefined,

$$\frac{\partial K}{\partial t} + \frac{\partial q}{\partial x} = 0 \qquad \text{(B-35)}$$

One fact which follows from the form of this equation is that just one wave velocity exists for any set of conditions. This is in contrast to most descriptions of flow, which involve other factors in addition to continuity. The more complex descriptions usually result in quadratic forms, which in turn result in more than one wave velocity for a given set of conditions.

What is a wave? Admittedly, previous discussion has been directed to "ordinary flow" rather than waves. The theory of kinematic waves does not deal with ordinary flow, but considers pulses of flow. Imagine a long stream of cars passing through a section of highway, and suppose that all conditions are steady with the exception of the input flow. (Perhaps the input is controlled by a light, and the flow varies with the road which is being emptied into the section under study.) Then at any time and point (actually, subsegment) on the section, there will be a particular flow. By the assumptions mentioned, particular values of flow will "pulse" along the length of the segment. A kinematic wave is essentially a flow pulse. The set of waves is the set of differing flow values that travel along the study segment. With respect to the unitary wave velocity, the pulses travel in one direction only; for other models one can get pulses traveling in two. The direction of the waves is the same as that of the traffic flow, but the velocity is seldom as great.

By assuming the possibility of writing a relation between q and K with x held constant, one can write

$$C = \frac{\partial q}{\partial K} \quad \text{(x a constant)} \qquad \text{(B-36}a\text{)}$$

which yields

$$\frac{\partial q}{\partial t} + C\frac{\partial q}{\partial x} = 0 \qquad \text{(B-36}b\text{)}$$

after multiplying the continuity equation through by C. This says that if q is to be constant one must have

$$V = \frac{\partial x}{\partial t} = -\,C \qquad \text{(B-37)}$$

and thus flow is constant on waves with velocity C. Because fluid velocity is given by $V = q/K$, C may be expressed as

$$C = d/dK\,(VK) = V + K\,dV/dK \qquad \text{(B-38)}$$

if q is constant. In this form, C is a function of q. This form makes clear that the velocity of the waves will be greater or less than that of the stream velocity as dV/dK is positive or negative. In traffic flow, velocity is considered a decreasing function of K. It follows that one could easily make a few predictions about what would happen if ways are found to shift the value of dV/dK.

Kinematic waves, it will be noticed, are actually pulses of flow, and should not be confused with shock waves such as those treated in fluid mechanics. The kinematic theory does, however, provide a means of considering its own form of shock wave, one which is entirely analogous to fluid mechanic shock. By application of the same basic rules (continuity) to the conditions involving one wave's overtaking another, one can find an expres-

sion for kinematic shock wave velocity. Essentially, for the flow to transition from one wave to another of different flow, we must equate shock velocity, U, to $\Delta q/\Delta K$. Thus, shock velocity is $(q_2 - q_1)/(K_2 - K_1)$ when two waves are involved.

If a flow-density curve is available, techniques are available to determine flow velocity, wave velocity, and shock velocity when something is known about the environment. Several of these are described in the references.

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APPENDIX C

RELATIONSHIP BETWEEN HEADWAY DISTRIBUTION AND ACCIDENTS

DEFINITION OF STUDY BASE

One of the problems encountered in establishing a reference base relates to the variability of the probability of accidents and of traffic flow dynamics. This prompted the classification of the base into a number of environmental and geometric sections. For instance, in discussing the probability of accidents, it is more meaningful to state the probability of an accident occurring on a vertical curve, than to merely state the probability of an accident occurring on an undefined mile of freeway.

The objectives of the study were to "reduce the probability of accidents, increase the average speed of vehicles, and allow the movement of a greater volume of traffic." Each of these three objectives, however, is not equally applicable on every section of highway. For example, there is currently no need for a communication system to allow movement of a greater volume of traffic on rural Interstate highways. The capacity of these highways is more than adequate for the present volumes. Thus, a further classification into specific situations where one or more of the objectives of the project are applicable becomes necessary.

If density is used as the independent variable and velocity and volume are plotted as dependent variables, a graph of the general form shown in Figure C-1 results. Three different relationships are found as the density increases from zero to a very compact situation. Each of these three density ranges has been separated for detailed study in this phase of the project. The reason for this separation is that as the relationships between the variables change, the problem associated with them also changes. The three separate density ranges will be referred to as zones A, B, and C.

The characteristics of zone A are those of a demand situation which is sufficiently low to permit each driver to operate his vehicle in an independent manner. This results in the average velocity remaining constant throughout this zone. Inasmuch as the volume of flow is equal to the average velocity times the density, the volume increases linearly with density.

Zone B represents the density range in which the average velocity is decreased due to vehicle-to-vehicle interactions. The increase in density, however, is large enough to offset the decrease in velocity, thus the volume through this range remains approximately constant. In this density range, the drivers do not operate in an independent manner; their operation is affected by other vehicles.

Zone C represents the density range in which the state of congestion has progressed to the stage where the decrease in velocity is greater than the increase in density. As a result, the volume of flow decreases. In the density range, as in zone B, the driver does not operate in an independent manner, but is affected by other vehicles.

These three zones may be characteristic of three different sections of highway separated in space, or the same section separated in time. Every section of urban freeway is likely to pass through all three zones at some time during the day, whereas certain rural sections of the

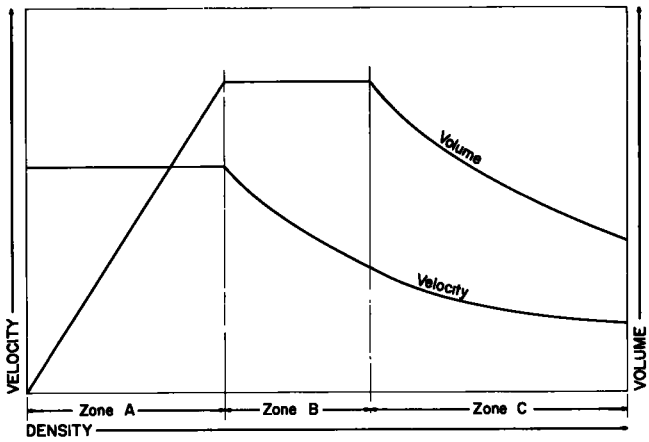

Figure C-1. Generalized form of density-volume-velocity relationships.

Interstate highway will never, at least for some time to come, operate in any zone except zone A.

Two independent methods have been developed for classifying the entire system of freeways and expressways into subsections with similar characteristic base values for either the probability of accidents or the relationships of traffic dynamics. The purpose of these classifications is to break the problems, as described in the introduction, into problem areas that can be more rigidly defined and attacked. The relative importance of each of the three objectives can now be specified for each such subsection. This was not possible when looking at the freeways in their entirety.

The relative importance of the three objectives is significantly different for different speeds of traffic. The economic importance of the objectives at various speed levels is shown in Figure C-2, which shows that as speed increases the time cost decreases at a decreasing rate. The cost of accidents also decreases, not as rapidly as time costs, and reaches a fairly constant value at a speed of about 45 mph (*1*).

Because the objectives of this study are stated in comparative terms, it is more meaningful to look at differential cost rather than at absolute values, between two points on the graph. It is quite clear, for instance, that when the speed increases beyond 60 mph, operating costs increase more rapidly than time cost decreases. Thus, if speed is increased beyond 60 mph, without changing vehicle design, the net result would be increased cost. For this reason, the costs associated with a velocity of 60 mph were considered as base time costs. A true base cost for accident costs could not be established as readily, because the frequency of accidents may be reduced at any velocity level. At this point, therefore, the base cost of accidents was assumed to be zero; in other words, it would be possible to eliminate all accidents with an intervehicular communication system. This assumption is the limiting case, but is useful in the classification of freeways into sections for study purposes.

The time cost per mile for a speed of 60 mph is given as $0.0195 by Haikalis and Hyman. Thus, the minimum cost attainable for accidents and time costs is $0.0195 per vehicle-mile. It is not so much the base cost which is relevant to this analysis, but the manner in which these costs vary with the speed. For example, at 60 mph the total cost above the base cost is due to accident costs. However, if the speed is reduced to 40 mph, the increase in accident costs is only $0.0020 per mile while the increase in time costs is $0.0098 per mile. At this speed the time costs represent about 75 percent of the increase costs beyond the base of $0.0195 per vehicle-mile. Figure C-2 shows that as the speed is decreased even further, the percentage of the costs attributable to time costs decreases.

With this perspective view on vehicle operating costs it is possible to look at the subsections listed previously and determine which of the three objectives of this study has the promise of the greatest return. This provides a lead toward the economically most feasible type of communication system.

Based on the foregoing information two separate study areas were delineated for further study: The sections with low demand, with the goal toward reducing accidents; the sections with high demand, primarily with the goal toward increasing flow, and only secondarily toward reducing accidents. Because the communications systems which will accomplish these two goals are likely to differ, it was important to separate them for detailed study.

ESTABLISHMENT OF ACCIDENT BASE

The initial step in the establishment of a set of base data to which accident rates could be compared was to find a common means of evaluating all accidents. The term used for this purpose was the direct economic cost.

Values on the cost of accidents were found in HRB Bull. 263 (*2*), from which the values given in Table C-1 were extracted. This table gives the relationship between cost and severity of accidents. The last column is an estimate of present-day (1963) accident cost as a means of determining where the need for communication systems might exist.

Another factor evaluated was the type of accident. This analysis was necessary because of the difference in the types of accident occurring in each study section. These costs are given in Table C-2.

Tables C-1 and C-2 make possible the comparison of different sections of highway, once the frequency of accidents by type and severity is known. Various physical factors had to be considered in this study. Pertinent data for those elements which significantly affected the frequency of accidents are given in the following tables.

Traffic volume was found to have a significant effect on the frequency of accidents on urban freeways, but not in those on rural freeways. The relationship found on urban freeways was given by Moskowitz (*3*) as

$$\text{Acc. rate/mil. veh-mi} = 0.1698(\text{ADT})^{0.1786} \qquad \text{(C-1)}$$

The relationship between traffic volume and the accident rate was investigated for rural freeways in Ohio.

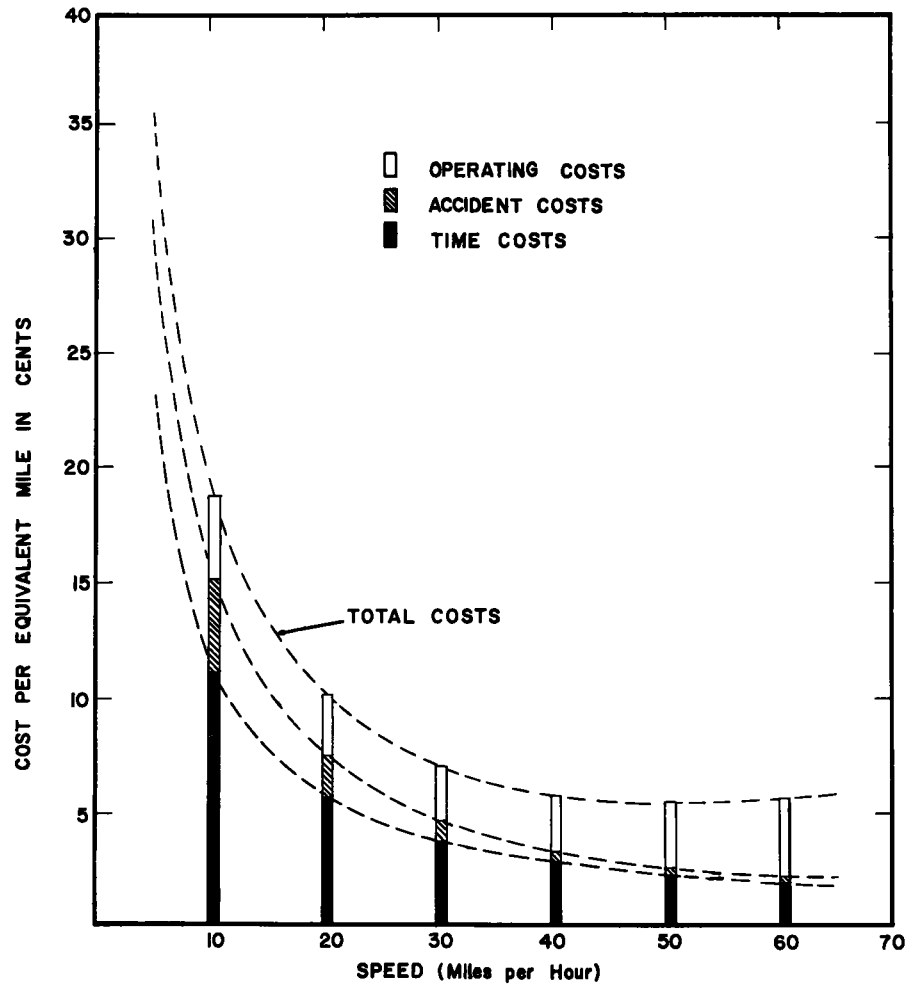

Figure C-2. Relationship of cost parameters to speed.

This analysis included data from all completed sections of the Interstate System in Ohio and the Ohio Turnpike. Unexpectedly, regression analysis of these data indicated that volume did not significantly affect the accident rate.

Nevertheless, high volumes do affect the accident rate and low volumes do not, making it apparent that the accident rate in urban areas is somewhat higher than it

TABLE C-1

DIRECT ECONOMIC COST OF ACCIDENTS

ACCIDENT CLASS	COST PER ACCIDENT ($) MASS., 1953	UTAH, 1955	ILL., 1958 [a]	EST., 1963
Non-injury	203	299	212	250
Injury	862	1,277	1,600	1,800
Fatal	5,212	3,690	9,500	10,000
All, avg.	382	491	412	450

[a] Based on an estimated 1.84 involvement per accident

TABLE C-2

DIRECT ECONOMIC COST OF ACCIDENTS BY ACCIDENT TYPE

ACCIDENT TYPE	COST PER ACCIDENT ($) MASS., 1953	UTAH, 1955	EST., 1963
Collision with:			
Fixed object	414	366	475
Other object	105	158	175
Other vehicles:			
Angle	327	666	400
Rear-end	482	573	550
Head-on	709	1642	800
Sideswipe	276	483	350
Non-collision	608	583	700
All, avg.	382	491 [a]	450

[a] This value is high due to the fact that 36.4 percent of all collisions in Utah are rear-end collisions, which are relatively expensive, whereas only 20.5 percent of Massachusetts collision accidents are of the rear-end type.

is in rural areas. A literature search indicated that this was true, as indicated by Table C-3, from which a value of 1.00 accidents per million vehicle-miles was selected as the base value to be used on rural freeways.

The distribution of accident frequencies is significantly different from one rural freeway to another. The average cost of accidents used in this analysis was adjusted to account for this difference. Table C-4 lists the distribution of accident frequencies as determined in the analysis of accidents on rural freeways in Ohio.

Using the 1963 estimated cost of accidents given in Table C-1 and averaging the distributions of Table C-4, the average cost of an accident is found to be $987 on rural freeways.

This value is more than twice the value found for streets and rural highways, other than freeways, which indicates that accidents tend to be more severe on this type of highway. This finding confirmed the validity of the decision to study communication systems that had as the primary objective prevention of accidents on rural freeways.

The distribution of accident types was studied to determine whether this would have a significant effect on accident costs. Table C-5 gives the percentage of accidents on rural freeways, by type. These percentages, when combined with the cost of accidents by type (Table C-2) do not significantly affect the average cost of an accident.

The geometry of the highway appeared to have little effect on accident type. In the case of the upgrade portion of a vertical curve, the percentage of rear-end accidents increased from 25.5 to 32.8 and the fixed object accidents from 32.5 to 39.3. In the case of horizontal curvature, the percentage of fixed object accidents increased from 32.5 to 49.3, whereas the percentage of rear-end accidents decreased from 35.5 to 20.9. These percentages would not change the average accident cost beyond the limitations of accuracy used in this study. They will, however, play a more important role in determining where, when, and what information must be communicated.

The accident relationships which exist on urban freeways were not studied in the same detail as on rural freeways because of the different primary objective of the communication system in each of the two areas.

The variations in accidents with certain geometric factors were discussed previously. A more detailed discussion of these relationships is included here.

The percentage of all accidents, by type, that involve a collision with another vehicle was found to be 43 on rural freeways in Ohio. Collision with a fixed object and non-collision-type accidents accounted for the other 57 percent. These percentages vary with the geometry of the highway. Table C-6 gives the percentages of collision accidents under a number of geometric conditions. The effect of these variations on the cost of accidents can be calculated by combining these values with those given in Table C-2 and the accident rates found for different geometric situations. The results of this analysis are shown in Figure C-3. These values indicate the theoretical

TABLE C-3

ACCIDENT RATES FOR SELECTED FREEWAYS

FREEWAY LOCATION	REF.	ACCIDENTS (NO./MIL. VEH.-MI.)
Urban:		
Texas	(4)	2.28
Detroit	(5)	2.81
Rural:		
Ohio		1.29
Ohio Tpk., 1962		0.73
June-Nov. 1963		0.66
California	(3)	1.00

TABLE C-4

SEVERITY OF ACCIDENTS ON OHIO RURAL FREEWAYS

ACCIDENT SEVERITY	DISTRIBUTION (%) OHIO FREEWAYS, 1961, 1962	OHIO TURNPIKE, 1961, 1962, 1963
Fatal	1.7	1.6
Injury	39.5	35.0
Property damage	59.8	63.4

TABLE C-5

DISTRIBUTION OF ACCIDENT TYPES ON RURAL OHIO FREEWAYS

ACCIDENT TYPE	DISTRIBUTION (%)
Rear-end	25.5
Sideswipe	6.6
Fixed object	32.5
Stopped or stopping vehicle	10.9
Other	24.5

TABLE C-6

DISTRIBUTION OF COLLISION ACCIDENTS ON OHIO EXPRESSWAYS, BY GEOMETRY

ACCIDENT TYPE	ACCIDENT DISTRIBUTION (%) TOTAL	UP GRADE	DOWN GRADE	HORIZ. CURVE
Collisions with other veh.	43.0	50.5	35.5	39.0
Other	57.0	49.5	64.5	61.0

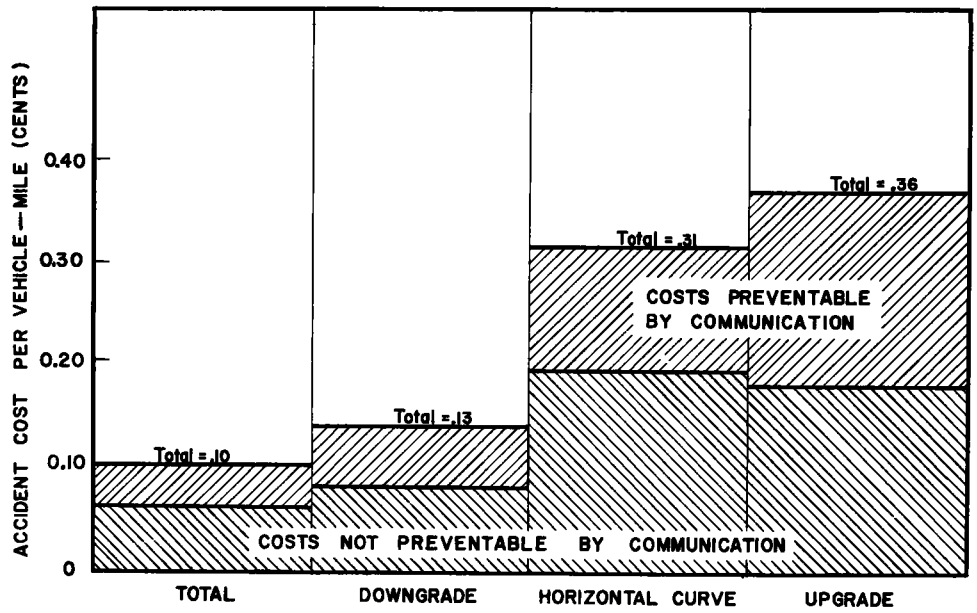

Figure C-3. Accident costs preventable and not preventable by increased intervehicular communication at certain geometric features.

maximum limit of accident benefits that can be derived from increased intervehicular communication at these specified locations.

The upgrade sections of vertical curves have the highest rate of preventable accident costs. The savings in this situation could be as high as \$0.0018 per vehicle-mile compared to the average savings of \$0.00043 per vehicle-mile for the entire highway downgrades. It must be remembered, however, that there are more miles of tangent, approximately level highways than there are vertical and horizontal curves, and that nearly 65 percent of the accidents on rural freeways occur on tangent, level sections.

Thus, although increased communications may show the greatest return per unit length on vertical curves, the largest absolute return will come from the straight, level sections of freeways.

HEADWAY DISTRIBUTION AND ACCIDENT RATES

The variation in accident rates, reflected by accident costs per million vehicle-miles, has been established. A study was conducted to determine the variations in flow patterns associated with locations of different accident rates. It was felt that some relationship between variations in flow and variations in accident rates could be established. This would indicate the type of information that must be communicated to the drivers in order to reduce the probability of accidents.

The two components of traffic flow that are considered to be most significant in simulation studies of the longitudinal control type are relative velocity and headway, or, more specifically, a combination of these. If the relative velocity is high and the headway is short, a dangerous situation exists. Thus, if a combination of these two characteristics is found to be more prevalent in a high-accident area than in a low-accident area, and a relationship between these two events can be found, this relationship may provide the key to one method of reducing the occurrence of accidents.

The theoretical relationship between relative velocity, spacing, and accidents can be derived from the equations of translational motion. That is, the combinations of relative velocity and spacing at which the following car cannot avoid a rear-end collision in case of emergency braking (except by lateral movement) can be calculated, based on certain assumed parametric values. Then, by varying these values, their effect on this relationship can be determined.

The equations used to develop this theoretical envelope of relationships in which accidents are unavoidable by the driver of the second vehicle if the lead vehicle assumes maximum possible deceleration are

$$(V_{1,2})^2 = (V_{1,1})^2 + 2\,a_1\,S_1 \qquad \text{(C-1)}$$

$$(V_{2,2})^2 = (V_{2,1})^2 + 2\,a_2\,S_2 \qquad \text{(C-2)}$$

$$S_1 + d = S_2 \qquad \text{(C-3)}$$

$$S = V_{2,1}\,(t_r) \qquad \text{(C-4)}$$

in which

$V_{1,2}$ = velocity of lead vehicle at time 2;
$V_{1,1}$ = velocity of lead vehicle at time 1;
$V_{2,2}$ = velocity of following vehicle at time 2;
$V_{2,1}$ = velocity of following vehicle at time 1;
a_1 = acceleration of lead vehicle;
a_2 = acceleration of following vehicle;

S_1 = distance moved by lead vehicle from time 1 to time 2;
S_2 = distance moved by following vehicle from time 1 to time 2;
S = distance following vehicle moved during reaction time, t_r; and
d = initial spacing between vehicles.

Solving these equations for d, when the final velocity of each vehicle is 0, gives

$$d = V_{2,1}\, t_r + \frac{(V_{2,1})^2}{2a_2} - \frac{(V_{1,1})^2}{2a_1}. \qquad \text{(C-5)}$$

Inasmuch as $V_{2,1} - V_{1,1}$ is the relative velocity of the two vehicles at time 1, or previous to any action by the lead vehicle, this value can be plotted versus d. Then the locus of such points defines the envelope within which an accident is preventable without lateral movement. The position of this safety envelope is sensitive to changes in any of the parameters $V_{1,1}$, $V_{1,2}$, t_r, and a_1.

The safety envelopes formed by values of 16 ft/sec/sec for a_1 and a_2, 60 mph for $V_{1,1}$, and reaction times of 0.50, 0.75, 1.00, 1.25 and 2.00 sec were constructed from the equations given. Data on relative velocity and headway distributions on various segments of rural highways in Ohio were analyzed to determine whether a relationship exists between the percentage of vehicles outside this envelope and the accident rate.

The data obtained from the field observations were plotted on a relative velocity-spacing graph containing the various safety envelopes and the percentage of the vehicles falling outside each envelope was determined. These percentages were plotted against the accident rates at that location and a least-squares analysis was conducted to determine the equation of the line of best fit through the data, and the coefficient of correlation. The results of this analysis are given in Table C-7.

Figures C-4, C-5, C-6, C-7, and C-8 show the plots and the regression line determined for reaction times of 0.50, 0.75, 1.00, 1.25, and 2.00 sec, respectively. The correlation coefficient is nearly the same for all reaction times selected. This analysis indicated that some relationship exists between the percentage of drivers outside this definition of safety and the accident rates, but it failed to show a strong relationship.

A similar study was reported by Crowther and Shumate (*6*). The results of that study indicate a good relationship between short headways and accident rates. The authors conclude that any program which is successful in reducing the number of vehicles following at short headways is likely also to reduce the accident rate.

Although the same conclusion cannot be drawn from this analysis, the data secured for the Crowther-Shumate analysis were not exactly the same as the data used in the analysis conducted in connection with this research project.

TABLE C-7

LEAST-SQUARES EQUATION AND COEFFICIENT OF CORRELATION FOR HEADWAY DISTRIBUTION (REACTION TIME) VS ACCIDENT RATE

REACTION TIME (SEC)	MINIMUM HEADWAY (FT)	EQUATION [a]	COEFF. OF CORRELATION
0.50	44	$A = 1.52 + 0.91\,P$	0.64
0.75	66	$A = 1.45 + 0.18\,P$	0.62
1.00	88	$A = 1.37 + 0.09\,P$	0.59
1.25	110	$A = 1.29 + 0.07\,P$	0.63
2.00	150	$A = 1.18 + 0.06\,P$	0.60

[a] A = accident rate per million vehicle-miles.
P = percentage of vehicles outside the safety envelope.

TRAFFIC FLOW DYNAMICS

A literature search was conducted to determine the relationships between volume, density, and speed that exist on the freeways today. These relationships, when defined, will serve as a basis for comparing relationships obtained through the use of intercommunications systems.

Three different freeways with similar characteristics were compared. Freeways in Detroit (*7*), Chicago (*8*), and Atlanta (*9*), for which data were available, were used to draw volume-density curves (Fig. C-9). Certain similarities were found, and some differences. Three zones, common to all three freeways, could be differentiated on the density scale. Zone A exists up to a density of approximately 30 veh per mile, zone B from 30 to about 60 veh per mile, and zone C at densities greater than 60 veh per mile. The peak volume occurred at a density of 50 to 55 veh per mile in all three cases.

The major difference was in the peak volumes that occurred. The Detroit freeway peaked at over 2,000 vph per lane, the Chicago freeway at 1,800, and the Atlanta freeway at 1,600. This difference may be caused by differences in geometry at the study locations, such as the nearness to access or exit points, bottleneck, or differences in gradients. For purposes of this study, the differences are not as important as the similarities. The fact that the volume increases linearly with the density up to a peak volume at a density of 50 veh per mile, and then falls off rapidly in all these cases is the relationship of importance.

Volume-density curves obtained in tunnel flow (*10*) were plotted on the same graph as those for the multilane freeways just discussed. The data lead to the conclusion that the number of lanes influences these relationships significantly. The volume-density relationships exhibited in the Lincoln Tunnel and the Holland Tunnel are less steep in the low-density range and reach a peak volume at a considerably higher density than on the three-lane freeways. The tunnels maintain a higher volume at high densities than do the freeways.

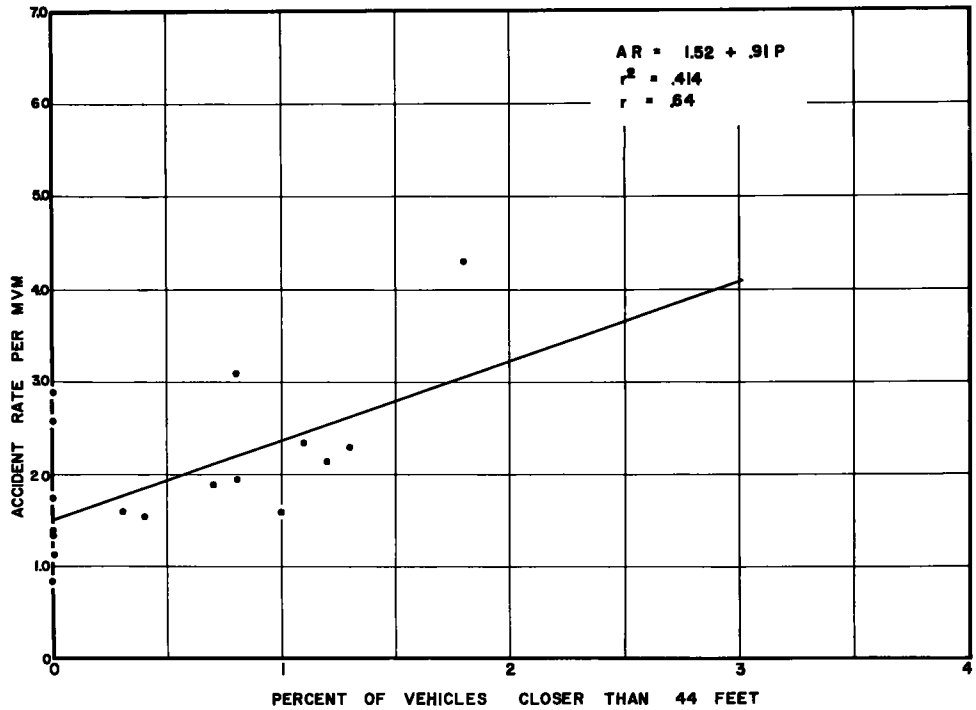

Figure C-4. Relationship between headway distribution and accident rate for reaction time of 0.50 sec (44 ft).

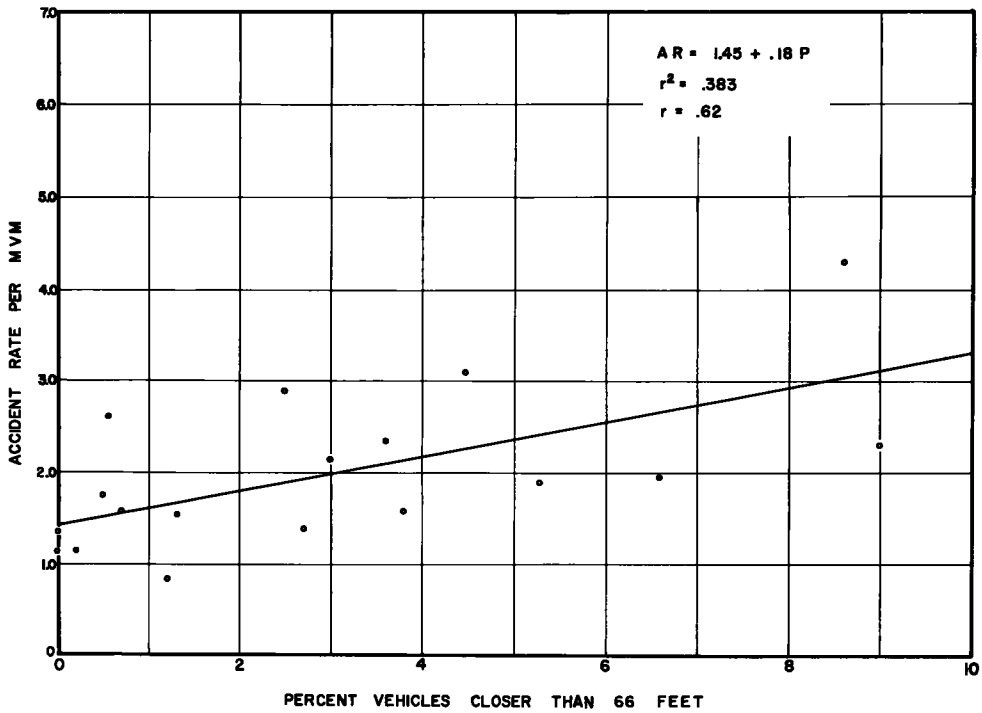

Figure C-5. Relationship between headway distribution and accident rate for reaction time of 0.75 sec. (66 ft).

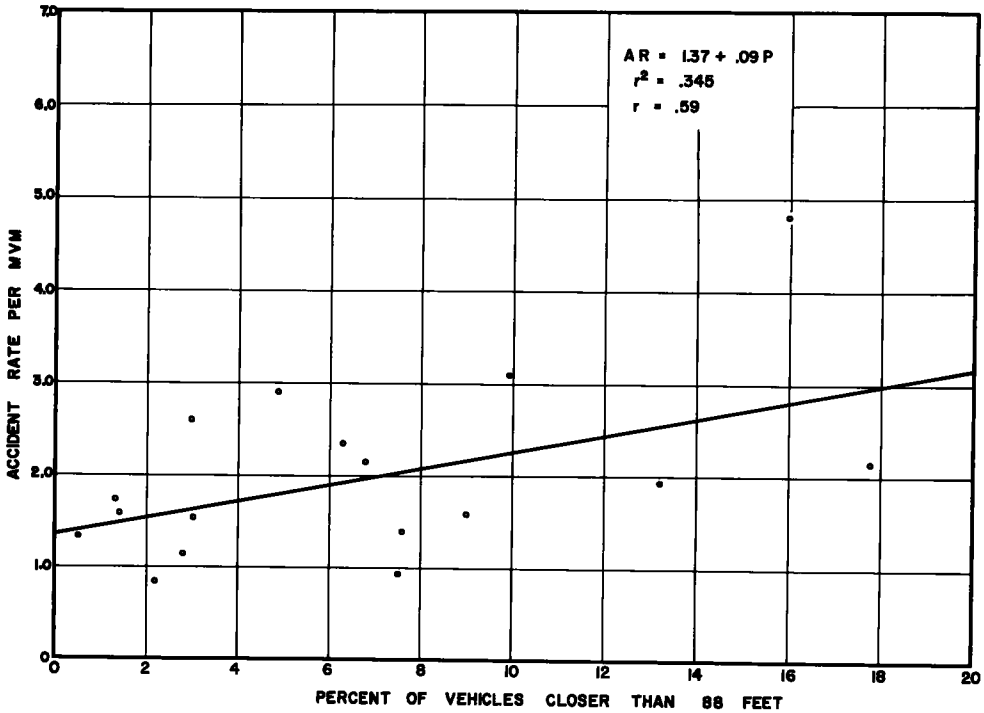

Figure C-6. Relationship between headway distribution and accident rate for reaction time of 1.00 sec (88 ft).

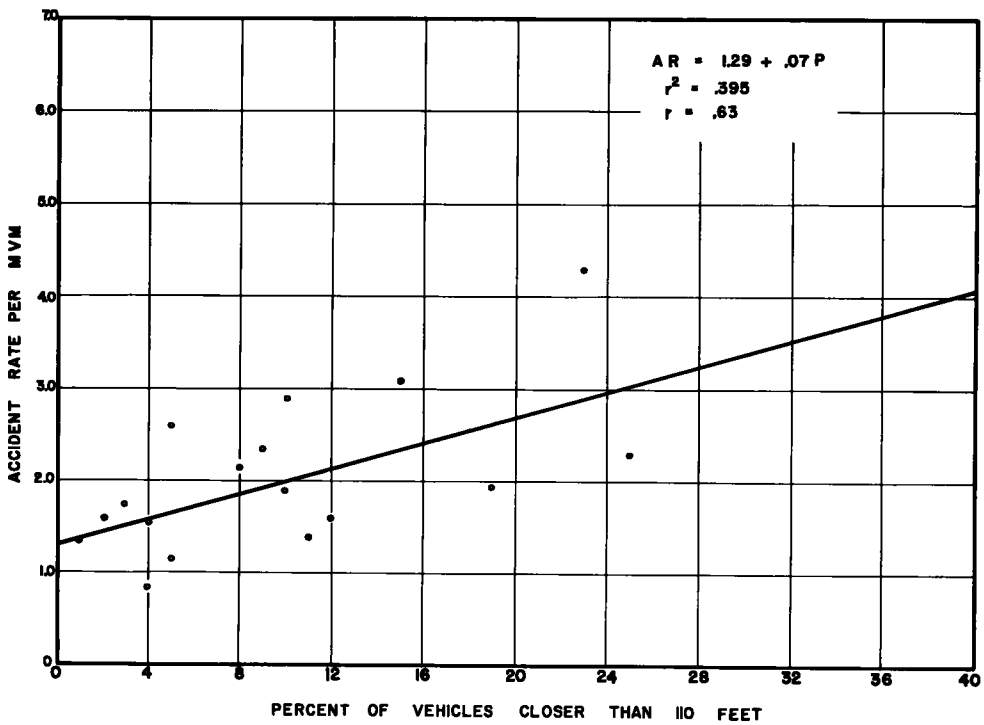

Figure C-7. Relationship between headway distribution and accident rate for reaction time of 1.25 sec (110 ft).

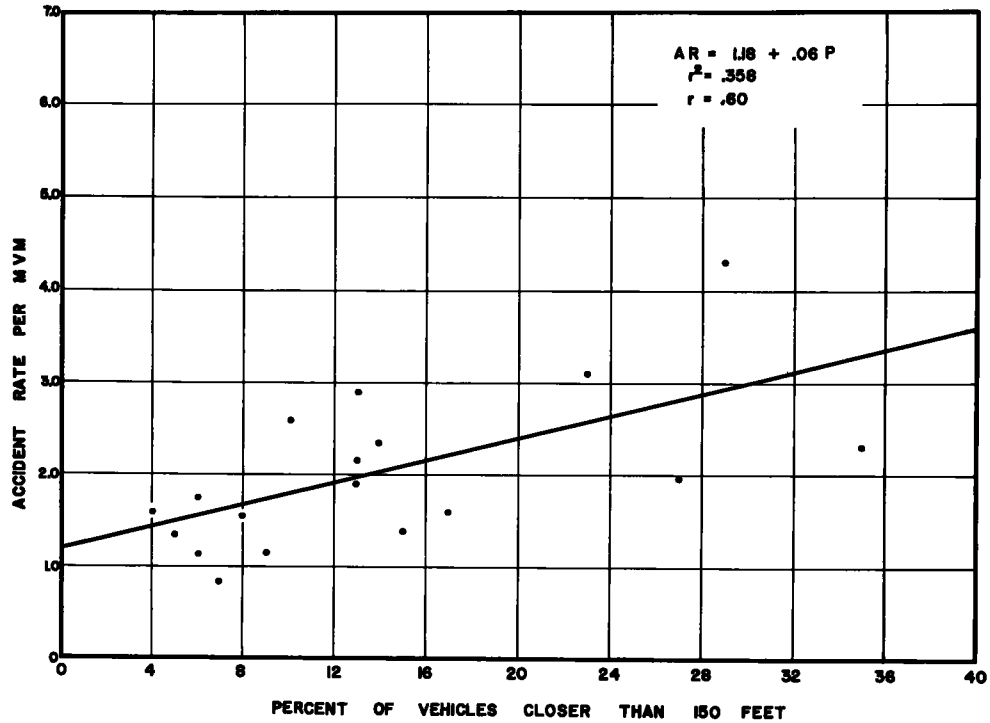

Figure C-8. Relationship between headway distribution and accident rate for reaction time of 2.00 sec (150 ft).

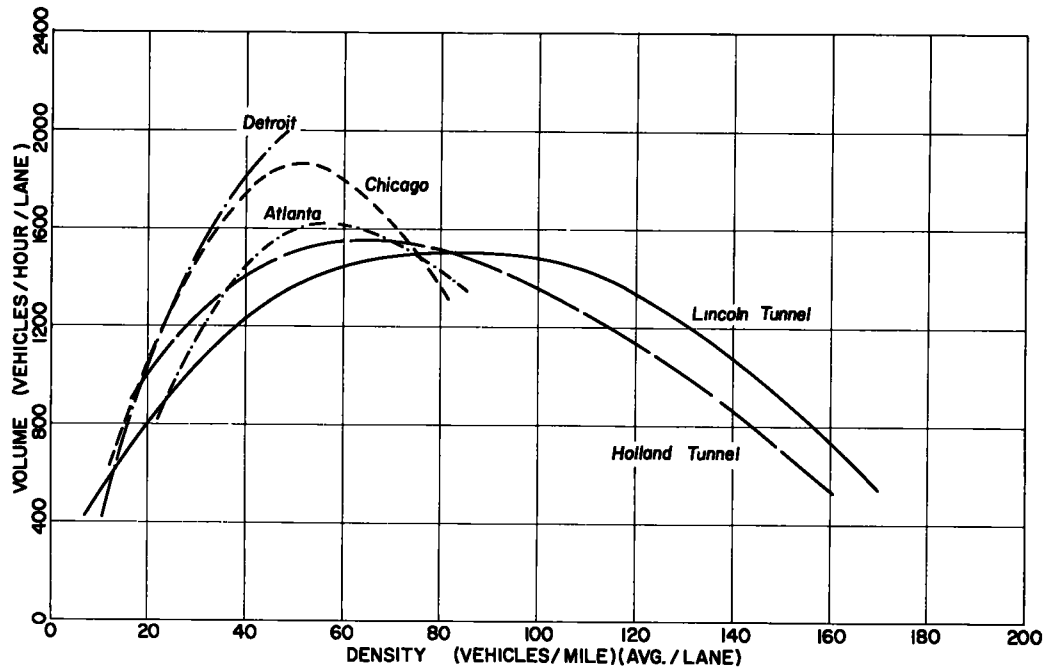

Figure C-9. Volume-density curves for selected freeways.

Higher volumes on freeways mean that for the same vehicle spacing, drivers will maintain a higher velocity on multi-lane facilities than in the tunnels. The reason for this behavior may well offer a clue as to what type of information needs to be communicated to change the volume-density relationship in a desirable way.

The basic difference between flow in a tunnel and flow on a multi-lane facility is the option to change lanes, when these are available on the multi-lane freeways. In fact, as the density on multi-lane freeways increases to the point where lane changing is no longer possible, the volume falls off to the volumes found in tunnels at the same density. At density of about 80 veh per mile, or an average spacing of approximately 40 ft, the volume-density relationships are similar for the two types of facilities.

The significance of this difference to this project is extremely pertinent. It indicates an area in which increased communication may yield higher volumes and higher speeds on freeways. If the driver behaves differently with the communication presently available to him, informing him of conditions in adjoining lanes of a freeway, an increase in the amount of information on these conditions might increase this behavior even further.

In summary, there is a difference between traffic flow characteristics for one-lane flow and multi-lane flow. The reason for this difference, at least in part, appears to be a knowledge of traffic conditions existing in adjoining lanes. If this is true, the difference may be increased by communication of information on these traffic conditions.

TRAFFIC FLOW

There is no simple method for establishing a theoretical maximum limit to traffic volumes or densities which can be obtained by increased intervehicular communication. Part of the reason is that the effect of communicable information on flow is not known. Certain observations have been made; however, no acceptable theory of traffic flow has yet been developed.

One of the objectives of the research being conducted was to determine the effect of different types of communication on certain driver parameters. If, in addition to this information, the effect of driver parameters on traffic flow can be established, the link between driver reaction to increased information and the resulting effect on traffic flow will be completed.

The car-following model described in Chapter Eleven is one means by which the effect of parameter variations on flow dynamics is evaluated.

Testing and evaluating of this effect could be simplified if a subclass of driving behavior were to be separated out for more detailed study. Because this study deals with density, it was felt that this subclass should be related to density.

Traffic density can most easily be measured and expressed as a function of gaps between successive vehicles. This at the same time provides a convenient measure of increased density, because reducing these gaps leads to dense traffic conditions. Therefore, the approach of this study was to investigate the effect of improved communication on g_i, where g_i is the gap between the ith and the $(i+1)$th car.

The distribution of gaps has been measured by others many times and it has been found that a Pearson type III distribution is a good theoretical distribution for approximating the actual distribution. The equation for this distribution is

$$f(x) = \frac{K\, e^{\lambda t}\, x^{K-1}}{(K-1)!} \qquad \text{(C-6)}$$

in which x is the gap length in time and λ and K are parameters describing the extent of randomness.

The parameters K and λ are related to each other by

$$K/\lambda = x \qquad \text{(C-7)}$$

A set of values for K and λ that describe the observed gap distribution at the volumes under consideration (approximately 2,200 vph) are $K=4$ and $\lambda=2.45$ (*11*).

The effect of gap reduction, $\sum_{i=0}^{h} g_i$, cannot be studied unless some method is available to describe to what extent gap length, g_i, can be changed. Unfortunately, no such data are available; however, an approximation of this relationship can be developed from logical reasoning and introspection. First, there is some minimum spacing beyond which no reduction in spacing will be possible by improved communication. Second, the probability of inducing a driver to decrease the gap in front of him increases as the size of the gap increases. Third, the absolute value of the change in gap size will increase as the gap size increases. Fourth, there is some value in spacing at which the driver is driving independently from the lead vehicle and therefore cannot be induced to close the gap by the methods under study.

A family of curves relating the probability of a given change to the distance changed over a bounded range will satisfy these conditions. If it is assumed that the probability of inducing a change follows a Poisson distribution, this family of curves will be Poisson curves.

Assumed values for the limiting boundaries are 33 ft * and 400 ft and the relationship to be investigated in this study says that the value of Δg in this distribution is

$$\Delta g = (g - 33)/2 \qquad \text{(C-8)}$$

in which g is any gap length in general. The equation of the Poisson curve thus becomes

$$P(x) = \frac{e^{-\Delta g}}{x!}\, \Delta g^{x} \qquad \text{(C-9)}$$

in which $P(x)$ is the probability that exactly x randomly arranged vehicles will be observed in a unit length of road. Then the expected value of any curve in this family is the mean value, or Δg.

* If a reaction time of 2.5 sec (as recommended by the AASHO "Blue Book") and a maximum deceleration of 0.359 is used, a minimum distance of approximately 33 ft is obtained

Therefore, the total gap reduction, $\sum_{i=0}^{h} g_i$, can be calculated by multiplying the percentage of a certain size gap times the volume times the mean value of the gap reduction summed over all intervals.

For purposes of identifying a subclass of vehicles for detailed study, a curve of gap reduction versus gap size was constructed (Fig. C-10). A cumulative curve of gap reduction versus gap size (Fig. C-11) was also constructed to determine the optimum (in terms of reduced gaps) range through which the information should be communicated to the driver.

Based on the assumptions used, Figures C-4 through C-8, and the more linear part of Figure C-11, indicate that the optimum subclass of drivers to whom communication should be conveyed are those with a gap size from 50 ft to 160 ft. This is quite important in that it sets boundaries in the limits, both maximum and minimum, in which information need be communicated. Because there are special problems related to communicating information at both extremes of distance, an analysis of this type can be used to indicate to what extent these problems need to be considered.

The results of this example indicate an increase of about 29 percent in the density by communicating only to the drivers whose gap is not between 50 ft and 160 ft.

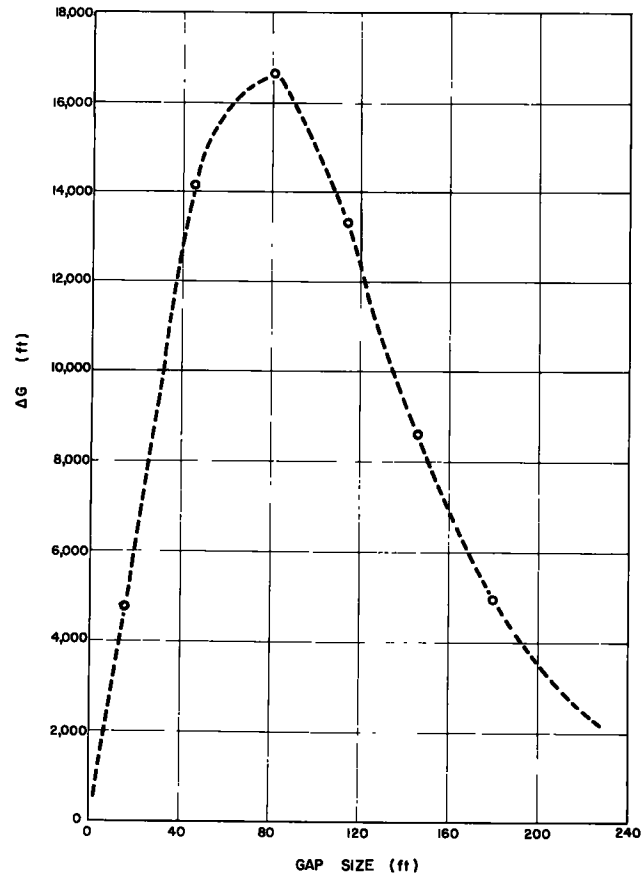

Figure C-10. Relationship between gap reduction and gap size; ΔG = possible gap reduction, Δg, times the number of vehicles with the gap size indicated.

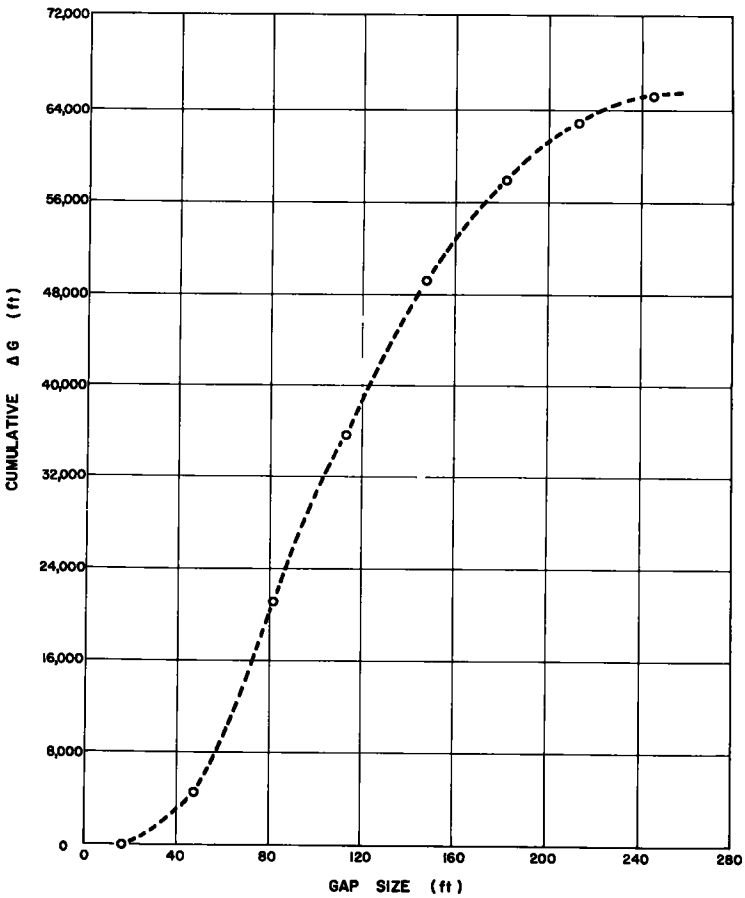

Figure C-11. Relationship between cumulative gap reduction and gap size.

The conclusion of this study is that there is expected to be some range of gap sizes that will yield the largest return of benefit in terms of increased density. The identification of this range can be calculated, based on assumptions relative to the effect of increased information to the driver.

The problem associated with the communication of information over very close and very far distances can be limited to the range of distances that will provide the largest benefit per unit of investment. The example analysis indicated that this range need not be very large for a sizable increase in the density of flow.

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APPENDIX D

THEORY OF INTERVEHICULAR CONTROL SYSTEMS

RELATIVE VELOCITY CONTROL

The second "pure" control system studied was the relative velocity follower, for which the acceleration of the following car is proportional to the time rate at which the gap between it and the car immediately ahead is increasing, this acceleration being applied to the following car after a delay, τ. Retaining the notation used previously,

$$\dot{V}_n(t+\tau) = \lambda\,[V_{n-1}(t) - V_n(t)] \qquad (66)$$

which represents the dynamical relation governing changes in the state of the flow for this class of model control systems.

The stability of the flow is an important consideration for these control systems. It is clear from actual considerations that $V_n(t)$ must be absolutely integrable (i.e., the car must eventually run out of gas). By the Riemann-Lesbeque lemma, $V_n(t)$ and $\dot{V}_n(t)$ have Fourier transforms, or

$$\overline{V}_n(\omega) = \int_{-\infty}^{\infty} e^{-i\omega t}\, V_n(t)\, dt \qquad \text{(D-1)}$$

and

$$\overline{\dot{V}}_n(\omega) = \int_{-\infty}^{\infty} e^{-i\omega t}\, \dot{V}_n(t)\, dt = i\omega \int_{-\infty}^{\infty} e^{-i\omega t}\, V_n(t)\, dt = i\omega\, \overline{V}_n(\omega) \qquad \text{(D-2)}$$

The transform of the governing differential equation is obtained by multiplying through by $e^{-i\omega t}$ and integrating over the whole line; that is,

$$\int_{-\infty}^{\infty} e^{-i\omega t}\, \dot{V}_n(t+\tau)\, dt = \lambda \int_{-\infty}^{\infty} e^{-i\omega t}\, V_{n-1}(t)\, dt - \lambda \int_{-\infty}^{\infty} e^{-i\omega t}\, V_n(t)\, dt = \lambda\,[\overline{V}_{n-1}(\omega) - \overline{V}_n(\omega)] \qquad \text{(D-3)}$$

The left side of this equation may be written

$$e^{i\omega\tau} \int_{-\infty}^{\infty} e^{-i\omega(t+\tau)}\, \dot{V}_n(t+\tau)\, dt = e^{i\omega\tau} \int_{-\infty}^{\infty} e^{-i\omega p}\, \dot{V}_n(p)\, dp \qquad \text{(D-4)}$$

in which $p = t + \tau$, so that

$$\int_{-\infty}^{\infty} e^{-i\omega t}\, \dot{V}_n(t+\tau)\, dt = i\omega e^{i\omega\tau}\, \overline{V}_n(\omega) \qquad \text{(D-5)}$$

Therefore, the differential equation becomes

$$i\omega e^{i\omega\tau}\, \overline{V}_n(\omega) = \lambda\,[\overline{V}_{n-1}(\omega) - \overline{V}_n(\omega)] \qquad \text{(D-6)}$$

Solving for $V_n(\omega)$ yields

$$\overline{V}_n(\omega) = [1 + (\omega)\,\lambda^{-1}\, e^{i\omega\tau}]^{-1}\, \overline{V}_{n-1}(\omega) \qquad \text{(D-7)}$$

and performing this operation once again

$$\overline{V}_n(\omega) = [1 + i\omega\lambda^{-1}\, e^{i\omega\tau}]^{-2}\, \overline{V}_{n-2}(\omega) \qquad \text{(D-8)}$$

so that n-fold iteration gives

$$\overline{V}_n(\omega) = [1 + i\omega\lambda^{-1}\, e^{i\omega\tau}]^{-n}\, \overline{V}_0(\omega) \qquad \text{(D-9)}$$

If the inversion is defined as

$$V_n(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{i\omega t}\, \overline{V}_n(\omega)\, d\omega \qquad \text{(D-10)}$$

the inversion of the foregoing equation will be

$$V_n(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{i\omega t}\, V_n(\omega)\, d\omega = \frac{1}{2\pi} \int_{-\infty}^{\infty} [1 + i\omega\lambda^{-1}\, e^{i\omega\tau}]^{-n}\, V_0(\omega)\, d\omega \qquad \text{(D-11)}$$

If, now, $V_n(t)$ is replaced by $U - v_n(t)$, where U is a constant "steady-state" speed, the form of the system of

the governing differential-difference equation will be unaltered; i.e.

$$V(t) = -\dot{v}(t) \tag{D-12}$$

so that Eq. 66 becomes

$$-\dot{v}_n(t+\tau) = \lambda\,[-v_{n-1}(t) + v_n(t)] \tag{D-13}$$

or

$$\dot{v}_n(t+\tau) = \lambda\,[v_{n-1}(t) - v_n(t)] \tag{D-14}$$

which is the same differential equation. The foregoing consideration will then hold for these v_n's as well, and

$$\bar{v}_n(\omega) = [1 + i\omega\lambda^{-1}\,e^{i\omega\tau}]^{-n}\,\bar{v}_0(\omega) \tag{D-15}$$

The v_n's are again absolutely integrable, but may closely approximate a pure sinusoidal $e^{i\omega t}$, so that a "resonance" can occur, if, for any ω,

$$1 + i\omega\lambda^{-1}\,e^{i\omega\tau} = 0 \tag{D-16}$$

Hence, the stability of the flow, in the sense that disturbances are not amplified, requires that

$$|1 + i\omega\lambda^{-1}\,e^{i\omega\tau}| \geq 1, \text{ for all } \omega, \tag{D-17}$$

be a necessary condition.

To see the implications of this condition, it is only necessary to remark from the theory of complex variables that

$$z = x + iy = [x^2 + y^2]^{-\frac{1}{2}}\,e^{i\theta} \tag{D-18}$$

in which $\tan\theta = x/y$. So we may write

$$\begin{aligned} 1 + i\omega\lambda^{-1}\,e^{i\omega\tau} &= 1 - \omega\lambda^{-1}\sin\omega\tau + i\omega\lambda^{-1}\cos\omega\tau \\ &= [(1 - \omega\lambda^{-1}\sin\omega\tau)^2 + (\omega\lambda^{-1}\cos\omega\tau)]\,e^{iv} \end{aligned} \tag{D-19}$$

in which

$$\tan v = (\omega\lambda^{-1}\cos\omega\tau)(1 - \omega\lambda^{-1}\sin\omega\tau)^{-1} \tag{D-20}$$

Thus, the amplitude factor will satisfy the inequality

$$|1 + i\omega\lambda^{-1}\,e^{i\omega\tau}| \geq 1 \tag{D-21}$$

whenever

$$(1 - \omega\lambda^{-1}\sin\omega\tau)^2 + (\omega\lambda^{-1}\cos\omega\tau)^2 \geq 1 \tag{D-22}$$

and, expanding,

$$\omega^2\lambda^{-2}\,(\sin^2\omega\tau + \cos^2\omega\tau) - 2\omega\lambda^{-1}\sin\omega\tau \geq 0 \tag{D-23}$$

so

$$\omega^2\lambda^{-2} \geq 2\omega\lambda^{-1}\sin\omega\tau \tag{D-24}$$

or

$$|\omega| \geq 2\lambda|\sin\omega\tau| \tag{D-25}$$

for all ω, whenever $2\lambda\tau \leq 1$, the equality holding only at $\omega = 0$.

Once it is clear that only the neighborhood of $\omega = 0$ is of interest, $\sin\omega\tau$ may be expanded about zero, retaining only the first terms and the foregoing result

$$\left|\frac{\omega}{2\lambda}\right| \geq |\sin\omega\tau| \approx \omega\tau \tag{D-26}$$

becomes

$$\frac{\omega}{2\lambda} \geq \omega\tau \quad 1 \geq 2\lambda\tau \tag{D-27}$$

Although the response of the system to a unit input function may be calculated, the pressure of time has prevented carrying this through in sufficient detail for discussion here.

To calculate the natural spacing between cars controlled by relative velocity, it is supposed initially that the vehicles are parked together with no space between them (an approximate join spacing will be added later). For a reaction time, τ, then,

$$\dot{v}_n(t) = 0 \quad 0 \leq t \leq n\tau,\ n = 0, 1, 2, \ldots \tag{D-28}$$

$$\dot{v}_n(t) = \lambda\,[v_{n-1}(t-\tau) - v_n(t-\tau)], \quad n\tau < t, \tag{D-29}$$

together with the initial condition $v_0(0) = 0$, the spacing between the nth and $(n-1)$th car is

$$s_n(t) = \int_0^t [v_{n-1}(p) - v_n(p)]\,dp \tag{D-30}$$

which leads to

$$*S_n(t) = \begin{cases} 0, & 0 \leq t \leq (n-1)\tau, \quad n = 1,2,3,\cdots \\ \displaystyle\int_{(n-1)\tau}^{t} v_{n-1}(p)\,dp, & (n-1)\tau \leq t \leq n\tau \\ \displaystyle\int_{(n-1)\tau}^{n\tau} v_{n-1}(p)\,dp + \int_{n\tau}^{t} [\dot{v}_{n-1}(p) - v_n(p)]\,dp, & t \geq n\tau \end{cases} \tag{D-31}$$

Consider $\displaystyle\int_{n\tau}^{t} [v_{n-1}(p) - v_n(p)]\,dp$ for $t \geq n\tau$

Let $p = q - \tau$ so that $q = (n+1)\tau$ when $p = n\tau$ and $q = t + \tau$ when $p = t$. Then

$$\int_{n\tau}^{t} [v_{n-1}(p) - v_n(p)]\,dp = \int_{(n-1)\tau}^{t+\tau} [v_{n-1}(q-\tau) - v_n(q-\tau)]\,dq \tag{D-32}$$

But it has been shown previously that

$$\frac{1}{\lambda}\dot{v}_n(q) = v_{n-1}(q-\tau) - v_n(q-\tau) \tag{D-33}$$

so the right side becomes

$$\frac{1}{\lambda}\int_{(n+1)\tau}^{t+\tau} \dot{v}_n(q)\,dq = \frac{1}{\lambda}[v_n(t+\tau) - v_n((n+1)\tau)] \tag{D-34}$$

To compute $v_n\,[(n+1)\tau]$, it is noted that $v_n\,(t) = 0$ for $0 \leq t \leq n\tau$ and $\dot{v}_n(t)$ for $0 \leq t \leq n\tau$ so that for $n\tau \leq t \leq (n+1)\tau$, $\dot{v}_n = \lambda v_{n-1}\,(t+\tau)$. Utilizing these conditions,

$$v_n(t) = \lambda\int_{n\tau}^{t} v_{n-1}(p-\tau)\,dp \tag{D-35}$$

therefore,

$$v_n[(n+1)\tau] = \lambda\int_{n\tau}^{(n+1)\tau} v_{n-1}(p+\tau)\,dp \tag{D-36}$$

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* Since $v_n(t) = 0;\ 0 \leq t \leq n\tau$,

$v_{n-1}(p) - v_n(p) = 0;\ \ 0 \leq t \leq (n-1)\tau$

but from $(n-1)\tau$ to $n\tau$ the $(n-1)$th vehicle accelerates while the nth vehicle does not react for a time τ; i.e., $n\tau$.

and letting $s = p - \tau$ so that $s = n\tau$ when $p = (n+1)\tau$ and $s = (n-1)\tau$ when $p = n\tau$,

$$v_n[(n+1)\tau] = \lambda \int_{(n-1)\tau}^{n\tau} v_{n-1}(s)ds \qquad \text{(D-37)}$$

Now, returning to

$$S_n(t) = \int_{(n-1)\tau}^{n\tau} v_{n-1}(p)dp + \int_{n\tau}^{t} [v_{n-1}(p) - v_n(p)]dp, \quad n\tau \le t \qquad \text{(D-38)}$$

and utilizing the results immediately above,

$$\begin{aligned} S_n(t) &= \int_{(n-1)\tau}^{n\tau} v_{n-1}(p)dp + \frac{1}{\lambda}\{v_n(t+\tau) - v_n[(n+1)\tau]\} \\ &= \int_{(n-1)\tau}^{n\tau} v_{n-1}(p)dp + \lambda^{-1} v_n(t+\tau) - \int_{(n-1)\tau}^{n\tau} v_{n-1}(s)ds = \lambda^{-1} v_n(t+\tau) \end{aligned} \qquad \text{(D-39)}$$

Therefore, with complete generality

$$S_n(t) = \lambda^{-1} v_n(t+\tau) \qquad \text{(D-40)}$$

is the natural spacing for point cars. Adding in the spacing in a traffic jam, the natural controlled spacing is

$$S_n(t) = S_j + \lambda^{-1} v(t+\tau) \qquad \text{(D-41)}$$

With this result for the spacing and the stability condition $2\lambda\tau = 1$, it is possible to calculate the maximum stable natural flow under relative velocity control. It is clear that higher steady-state flow will occur for larger values of λ. The largest λ yielding stable flow is $\lambda = (2\tau)^{-1}$. With this value of λ, the natural steady-state spacing is

$$S = S_j + 2\tau V \qquad \text{(D-42)}$$

and the corresponding flow is

$$q = V(S_j + 2\tau V)^{-1} \qquad \text{(D-43)}$$

Setting the steady state mean density

$$k = (S_j + 2\tau V)^{-1} \qquad \text{(D-44)}$$

and letting $S_j = k_j^{-1}$,

$$k = (k_j^{-1} + 2\tau V)^{-1} \qquad \text{(D-45)}$$

and

$$kk_j^{-1} + k2\tau V = 1 \qquad \text{(D-46a)}$$

or

$$2\tau V = (k^{-1} - k_j^{-1}) \qquad \text{(D-46b)}$$

$$V = (2\tau)^{-1}(k^{-1} - k_j^{-1}) \qquad \text{(D-46c)}$$

Then, since $q = V k$

$$q = (2\tau)^{-1} k(k^{-1} - k_j^{-1}) \qquad \text{(D-47)}$$

$$q = (2\tau)^{-1}(1 - kk_j^{-1}) \qquad \text{(D-48)}$$

Thus, steady natural flow under relative velocity control is exactly one-half the steady natural (marginally safe) flow under acceleration control, at the same values for mean density and mean reaction time. However, under acceleration control disturbances do not attenuate, but are propogated undiminished down the lane of cars. In traffic following stable relative velocity control, each sinusoidal component except the zero frequency is diminished in amplitude as the disturbance passes from car to car down the lane (the higher frequencies being more rapidly attenuated).

Because the natural spacing is a linear function of reaction time, the average natural spacing is the spacing corresponding to the mean reaction time of the drivers (if each driver drives according to his own "constant" reaction time).

The natural spacing for point cars (Eq. D-40) vanishes prior to $v_n(t)$; that is, the natural flow, under relative velocity control, is not collision free. On the contrary, with natural spacing, collisions are inevitable. Because the maximum $v_n(t)$, such that $v_n(t+\tau) = 0$, is $ug\tau$, is would appear that adding a spacing ½ $ug\tau^2$ to the natural spacing would give the following car room to stop. Such reasoning, however, might be specious and the entire subject of the minimum collision-free spacing under relative velocity control requires further study.

RELATIVE VELOCITY CONTROL WITH ZERO REACTION TIME

The traffic flow in this instance is governed by

$$\dot{v}_n(t) = \lambda[v_{n-1}(t) - v_n(t)] \qquad \text{(D-49)}$$

The acceleration of the first vehicle is then

$$\dot{v}_1(t) = \lambda[v_0(t) - v_1(t)] \qquad \text{(D-50)}$$

which may be written as

$$\dot{v}_1(t) + \lambda v_1(t) = \lambda v_0(t) \qquad \text{(D-51)}$$

This in turn integrates to

$$\int_{t_0}^{t} \frac{d}{dp}[e^{\lambda p} v(p)]dp = \lambda \int_{t_0}^{t} e^{\lambda p} v_0(p)dp \qquad \text{(D-52)}$$

which yields

$$e^{\lambda t} v_1(t) - e^{\lambda t_0} v(t_0) = \lambda \int_{t_0}^{t} e^{\lambda p} v_0(p)dp \qquad \text{(D-53)}$$

But if $v_1(t_0) = a_1$, $v_1(t)$ may be expressed as

$$v_1(t) = a_1 e^{-\lambda(t-t_0)} + \lambda \int_{t_0}^{t} e^{\lambda(p-t)} v_0(p)dp \qquad \text{(D-54)}$$

The governing equation for the second car is

$$\dot{v}_2(t) = \lambda[v_1(t) - v_2(t)] \qquad \text{(D-55)}$$

where $v_1(t)$ may be looked upon as the forcing function for the second car.

This equation may then be integrated to yield, as before,

$$v_2(t) = a_2 e^{-\lambda(t-t_0)} + \lambda \int_{t_0}^{t} e^{\lambda(q-t)} v_1(q)dq \qquad \text{(D-56)}$$

in which a_2 is defined in the same manner as a_1; i.e., $v_2(t_0) = a_2$ and, in general, $v_n(t_0) = a_n$. Substituting the first result (Eq. D-54), with t replaced by q, into Eq. D-56,

$$v_2(t) = a_2 e^{-\lambda(t-t_0)} + \lambda a_1 \int_{t_0}^{t} e^{-\lambda(t-t_0)} e^{\lambda(q-t)} dq$$
$$+ \lambda^2 \int_{t_0}^{t} dq \int_{t_0}^{q} e^{\lambda(p-t)} v_0(p) dp \qquad \text{(D-57)}$$

and integrating gives

$$v_2(t) = a_2 e^{-\lambda(t-t_0)} + a_1 \lambda e^{-\lambda(t-t_0)} (t - t_0)$$
$$+ \lambda^2 \int_{t_0}^{t} dq \int_{t_0}^{q} e^{\lambda(p-t)} v_0(p) dp \qquad \text{(D-58)}$$

It is now possible to write an expression for the velocity of the nth vehicle, or

$$v_n(t) = e^{-\lambda(t-t)} \sum_{m=0}^{n-1} \frac{[\lambda(t-t_0)]^m}{m!} a_{n-m}$$
$$+ \lambda^n \int_{t_0}^{t} \underset{(n)}{} e^{\lambda(p-t)} v_0(p) dp \qquad \text{(D-59)}$$

in which

$$\int_{t_0}^{t} \underset{(n)}{} e^{\lambda(p-t)} v_0(p) dp$$
$$= \int_{t_0}^{t} ds \int_{t_0}^{s} dr \int_{t_0}^{r} dx \cdots \int_{t_0}^{q} e^{\lambda(p-t)} v_0(p) dp \qquad \text{(D-60)}$$

and $a_{n-m} = {}_{n-m}(t_0)$.

The identity

$$\frac{d^n}{dt^n} \int_{t_0}^{t} \frac{(t-u)^{n-1}}{(n-1)!} f(u) du = f(t) \qquad \text{(D-61)}$$

enables integration of the second term in Eq. D-60.

Verification of this identity is quite elementary; that is,

$$\int_{t_0}^{t} \frac{d^n}{dt^n} \left[\frac{(t-u)^{n-1}}{(n-1)!} \right] f(u) du$$
$$= \int_{t_0}^{t} \frac{(n-1)!(t-u)^0}{(n-1)!} f(u) du$$
$$= \int_{t_0}^{t} f(u) du = f(t) \qquad \text{(D-62)}$$

because $f(t_0) = 0$.

Integrating the identity n times from t_0 yields

$$\int_{t_0}^{t} \underset{(n)}{} \frac{d^n}{dt^n} \left\{ \int_{t_0}^{t} \frac{(t-u)^{n-1}}{(n-1)!} f(u) du \right\} dt = \int_{t_0}^{t} \underset{(n)}{} f(q) dq \qquad \text{(D-63)}$$

But the left side is just an n-fold perfect differential, so it follows that

$$\int_{t_0}^{t} \underset{(n)}{} \frac{d^n}{dt^n} \left\{ \int_{t_0}^{t} \frac{(t-u)^{n-1}}{(n-1)!} f(u) du \right\} dt$$
$$= \int_{t_0}^{t} \frac{(t-u)^{n-1}}{(n-1)!} f(u) du \qquad \text{(D-64)}$$

therefore,

$$\int_{t_0}^{t} \frac{(t-u)^{n-1}}{(n-1)!} f(u) du = \int_{t_0}^{t} \underset{(n)}{} f(u) du \qquad \text{(D-65)}$$

Letting $u = t + t_0 - p$, then $du = -dp$ so $p = t$ when $u = t_0$ and $p = t_0$ when $u = t$,

$$\int_{t_0}^{t} \frac{(t-u)^{n-1}}{(n-1)!} f(u) du$$
$$= -\int_{t}^{t_0} \frac{(p-t_0)^{n-1}}{(n-1)!} f(t + t_0 - p) dp$$
$$= \int_{t_0}^{t} \frac{(p-t_0)^{n-1}}{(n-1)!} f(t + t_0 - p) dp \qquad \text{(D-66)}$$

The original equation is then

$$f(p) dp = \frac{(p-t_0)^{n-1}}{(n-1)!} f(t + t_0 - p) dp \qquad \text{(D-67)}$$

and if $f(t) = e^{\lambda t} v_0(t)$, this becomes

$$\int_{t_0}^{t} \underset{(n)}{} e^{\lambda p} v_0(p) dp$$
$$= \int_{t_0}^{t} \frac{(p-t_0)^{n-1}}{(n-1)!} e^{\lambda(t+t_0-p)} v_0(t + t_0 - p) dp \qquad \text{(D-68)}$$

and multiplying through by $\lambda^n e^{-\lambda t}$ yields

$$\lambda^n \int_{t_0}^{t} \underset{(n)}{} e^{\lambda(p-t)} v_0(p) dp$$
$$= \lambda^n \int_{t_0}^{t} \frac{(p-t_0)^{n-1}}{(n-1)!} e^{\lambda(p-t_0)} v_0(t + t_0 - p) dp$$
$$= \frac{\lambda^n e^{\lambda t_0}}{(n-1)!} \int_{t_0}^{t} (p-t_0)^{n-1} e^{-\lambda p} v_0(t + t_0 - p) dp \qquad \text{(D-69)}$$

This allows writing the velocity of the nth vehicle as

$$v_n(t) = e^{-\lambda(t-t_0)} \sum_{m=0}^{n-1} \frac{[\lambda(t-t_0)]^m}{m!} a_{n-m}$$
$$+ \frac{\lambda^n e^{\lambda t_0}}{(n-1)!} \int_{t_0}^{t} (s-t_0)^{n-1} e^{\lambda s} v_0(t - t_0 - s) ds \qquad \text{(D-70)}$$

a result which specializes to the solution of Pipes (*1*) for the special cases he treats and may be compared with the result of Newell (*2*).

Particularily simple solutions to

$$\dot{v}_n(t) = \lambda [v_{n-1}(t) - v_n(t)] \qquad \text{(D-71)}$$

result if

$$v_1(t) = V_0(1 - \lambda^{-1} e^{\lambda} \lambda t e^{-\lambda t}) \qquad \text{(D-72)}$$

which corresponds to the first car braking from the steady-speed V_0, momentarily coming to rest, and then slowly accelerating back toward V_0. It is assumed that $v_n(0) = v_0$ for $n = 2, 3, \ldots$.

The equation for the second car is

$$\dot{v}_2(t) = \lambda [v_1(t) - v_2(t)] \qquad \text{(D-73)}$$

or

$$\dot{v}_2(t) + \lambda v_2(t) = \lambda v_1(t) \qquad \text{(D-74)}$$

Multiplying by the integrating factor $e^{\lambda t}$ and integrating yields, as before,

$$e^{\lambda t} v_2(t) = v_2(0) + \lambda \int_{0}^{t} e^{\lambda t} v_1(t) dt \qquad \text{(D-75)}$$

But $v_2(0) = V_0$ and $v_1(t)$ is defined by Eq. D-72, so

$$e^{\lambda t} v_2(t) = V_0 + \lambda V_0 \int_0^t e^{\lambda t} (1 - \lambda^{-1} e^{\lambda} \lambda t e^{-\lambda t}) dt \quad \text{(D-76)}$$

and

$$e^{\lambda t} v_2(t) = V_0 + \lambda V_0 \left[\frac{1}{\lambda} e^{\lambda t} - \frac{1}{\lambda} e^{\lambda} \lambda \frac{t^2}{2} \right]_0^t \quad \text{(D-77)}$$

Therefore, with simplification,

$$v_2(t) = V_0 - V_0 \lambda^{-1} e^{\lambda} \frac{(\lambda t)^2}{2!} e^{-\lambda t} \quad \text{(D-78a)}$$

or

$$v_2(t) = V_0 \left(1 - \lambda^{-1} e^{\lambda} \frac{(\lambda t)^2}{2!} e^{-\lambda t} \right) \quad \text{D-78b)}$$

The velocity of the nth vehicle may now be written by inspection

$$v_n(t) = V_0 \left(1 - \lambda^{-1} e^{\lambda} \frac{(\lambda t)^n}{n!} e^{-\lambda t} \right) \quad \text{(D-79)}$$

A spacing quite analogous to the "very safe" spacing under acceleration control may be calculated for this control system; i.e.,

$$\dot{v}_n(t) = \lambda [v_{n-1}(t) - v_n(t)] \quad \text{(D-80)}$$

We consider a steady stream of traffic moving at the speed V_0 and require that the nth car have sufficient space to come to a stop without collision, if the $(n-1)$th car comes to rest instantaneously $(v_{n-1} = 0)$. The braking of the following car is then governed by

$$\dot{v}_n(t) + \lambda v_n(t) = 0 \quad \text{(D-81)}$$

and, integrating,

$$\int_0^t \frac{dt}{d} [e^{\lambda t} v_n(t)] dt = 0 \quad \text{(D-82)}$$

yields

$$e^{\lambda t} v_n(t) - v_n(0) = 0 \quad \text{(D-83)}$$

But

$$v_n(0) = V_0 \quad \text{(D-84)}$$

so

$$v_n(t) = e^{-\lambda t} V_0 \quad \text{(D-85)}$$

Integrating Eq. D-85 gives

$$\int_0^t v_n(t) dt = V_0 \int_0^t e^{-\lambda p} dp \quad \text{(D-86)}$$

in which

$$v_n(t) = \frac{dt}{d} [x_n(t)] \quad \text{(D-87)}$$

Therefore,

$$\int_0^t \frac{d}{dt} [x_n(t)] dt = V_0 \int_0^t e^{-\lambda p} dp \quad \text{(D-88)}$$

which gives

$$x_n(t) - x_n(0) = \frac{V_0}{-\lambda} (e^{-\lambda t} - 1) \quad \text{(D-89)}$$

But the initial displacement $x_n(t = 0) = 0$, so

$$x_n(t) = \frac{V_0}{\lambda} (1 - e^{-\lambda t}) \quad \text{(D-90)}$$

After a long time (i.e., $e^{-\lambda t} \approx 0$) the displacement $x_n(t)$ approaches the very safe spacing under relative velocity control, s, and

$$s = \lambda^{-1} V_0 \quad \text{(D-91)}$$

adding in the spacing in a traffic jam, s_j,

$$s = s_j + \lambda^{-1} V_0 \quad \text{(D-92)}$$

COMBINED ACCELERATION AND RELATIVE VELOCITY CONTROL

This section considers a control system under which the following car's acceleration, after a delay, τ, is expressed as a linear combination of the lead car's relative velocity and acceleration, or

$$\dot{V}_n(t + \tau) = K \dot{V}_{n-1}(t) + \lambda [V_{n-1}(t) - V_n(t)] \quad \text{(D-93)}$$

in which K and λ are constants.

Taking the Fourier transform of Eq. D-93,

$$i\omega\tau e^{i\omega\tau} \overline{V}_n(\omega) = Ki\omega \overline{V}_{n-1}(\omega) + \lambda [V_{n-1}(\omega) - V_n(\omega)] \quad \text{(D-94)}$$

or

$$\overline{V}_n(\omega)(\lambda + i\omega e^{i\omega\tau}) = (\lambda + i\omega K) \overline{V}_{n-1}(\omega) \quad \text{(D-95)}$$

so $\overline{V}_n(\omega)$ may now be expressed as

$$\overline{V}_n(\omega) = \left[\frac{\lambda + i\omega K}{\lambda + i\omega e^{i\omega\tau}} \right]^1 \overline{V}_{n-1}(\omega) \quad \text{(D-96)}$$

As in previous sections, n-fold iteration will yield

$$\overline{V}_n(\omega) = \left[\frac{\lambda + i\omega K}{\lambda + i\omega e^{i\omega\tau}} \right]^n \overline{V}_0(\omega) \quad \text{(D-97)}$$

Now, if there is to be stability in this case,

$$\left| \frac{\lambda + i\omega K}{\lambda + i\omega e^{i\omega\tau}} \right| \leq 1 \quad \text{(D-98)}$$

which may be written

$$\left(\frac{\lambda + i\omega K}{\lambda + i\omega e^{i\omega\tau}} \right) \left(\frac{\lambda - i\omega K}{\lambda - i\omega e^{-i\omega\tau}} \right) \leq 1 \quad \text{(D-99)}$$

and

$$\frac{\lambda^2 + \omega^2 K^2}{\lambda^2 + \omega^2 - 2\omega\lambda \sin \omega\tau} \leq 1 \quad \text{(D-100)}$$

which gives

$$\lambda^2 + \omega^2 K^2 \leq \lambda^2 + \omega^2 - 2\omega\lambda \sin \omega\tau \quad \text{(D-101)}$$

and

$$2\omega\lambda \sin \omega\tau \leq \omega^2 (1 - K^2) \quad \text{(D-102)}$$

The crucial range here is in the neighborhood of zero, i.e., $\omega\tau \approx 0$, so that $\sin \omega\tau \approx 0$, so that $\sin \omega\tau \approx \omega\tau$ and, therefore, $2\,\omega\,\lambda \sin \omega\tau \approx 2\omega\lambda(\omega\tau)$. Under these conditions, the inequality becomes $2\lambda\tau \leq 1 - K^2$, as a necessary condition for stability. But $k = 0$ implies the condition of Section 2, whereas $\lambda = 0$ implies the situation discussed in Section 1, if all vehicles are to have the same steady-state velocity.

The spacing between vehicles, $s(t)$, is given by

$$s(t) = s_0 + \int_0^t [V_{n-1}(p) - V_n(p)]dp \qquad \text{(D-103)}$$

or, utilizing Eq. 93,

$$V_n(t+\tau) = KV_{n-1}(t) + \lambda[V_{n-1}(t) - V_n(t)] \qquad \text{(D-104)}$$

Therefore,

$$[\dot{V}_{n-1}(p) - V_n(p)] = \lambda^{-1}[\dot{V}_n(p+\tau) - K\dot{V}_{n-1}(p)] \qquad \text{(D-105)}$$

and $s\ (t)$ becomes

$$s(t) = s_0 + \lambda^{-1} \int_0^t [\dot{V}_n(p+\tau) - K\dot{V}_{n-1}(p)]\, dp \qquad \text{(D-106)}$$

Now, using the conditions $V_n(t) = 0$, $0 \le t \le \tau$ for $n = 1$, 2, and $V_0\ (0) = 0$; letting $s = p + \tau$, so that $ds = dp$ and $s = \tau$ when $p = 0$ and $S = t + \tau$ when $p = t$,

$$\begin{aligned} s(t) &= s_0 + \lambda^{-1} \int_0^{t+\tau} \frac{dV_n(s)}{ds}\, ds - \lambda^{-1} K \int_0^t \frac{dV_{n-1}(p)}{dp}\, dp \\ &= s_0 + \lambda^{-1}[V_n(t+\tau) - V_n(\tau)] \\ &\quad - \lambda^{-1}\mathrm{K}[V_{n-1}(t) - V_{n-1}(0)] \end{aligned} \qquad \text{(D-107)}$$

But, from the foregoing, $V_n\ (\tau) = 0$ and $V_{n-1}\ (0) = 0$, so $s\ (t)$ may be expressed as

$$s(t) = s_0 + \lambda^{-1}[V_n(t+\tau) - KV_{n-1}(t)] \qquad \text{(D-108)}$$

If $\lambda = 0$ and $K = 1$, $s(t)$ will be an indeterminate expression and one must return to the natural spacing given under "Relative Velocity Control."

The asymptotic spacing is found when the flow is steady; i.e.,

$$V_n(t+\tau) = V_{n-1}(\mathrm{t}) \qquad \text{(D-109)}$$

or

$$s = s_j + \lambda^{-1}(1-K)V \qquad \text{(D-110)}$$

The steady natural flow satisfying the stability condition, $\lambda^{-1} = 2\tau(1-K^2)^{-1}$, found earlier, allows writing

$$s = s_j + 2\tau(1-K^2)^{-1}(1-K)V \qquad \text{(D-111)}$$

or

$$s = s + 2\tau(1+K)^{-1}\, V \qquad \text{(D-112)}$$

The average steady spacing is the average of the steady spacings as before, allowing τ to be thought of as a mean reaction time. The average density may be defined as the reciprocal of the average spacing,

$$k = 1/s \qquad \text{(D-113)}$$

or

$$k = [s_j + 2\tau(1+K)^{-1}V]^{-1} \qquad \text{(D-114)}$$

The natural flow

$$q = kV = V[s_j + 2\tau(1+K)^{-1}V]^{-1} \qquad \text{(D-115)}$$

may also be written as a function of the density, by solving for V in Eq. D114, or

$$V = (k^{-1} - k_j^{-1})\left(\frac{1+K}{2\tau}\right) \qquad \text{(D-116)}$$

in which $k_j^{-1} = s_j$. Substituting this expression for V in $q = VK$ gives

$$q = k(k^{-1} - k_j^{-1})\left(\frac{1+K}{2\tau}\right) \qquad \text{(D-117}a\text{)}$$

or

$$q = (1 - k/k_j)\left(\frac{1+K}{2\tau}\right) \qquad \text{(D-117}b\text{)}$$

It is to be noted here that the limiting stable natural flow under combined acceleration and relative velocity control is intermediate between that of pure acceleration control

$$q = \frac{1}{\tau}\,(1 - k/k_j) \qquad \text{(D-118)}$$

and that of pure relative velocity control

$$q = \frac{1}{2\tau}\,(1 - k/k_j) \qquad \text{(D-119)}$$

As in the case of pure relative velocity control, the maximum collision free flow is not yet known, but it is thought to be slightly smaller than the natural flow for small τ. It is possible to show that collisions do not occur for zero reaction time; that is, for natural flow governed by

$$\dot{V}_n(t) = K\dot{V}_{n-1}(t) + \lambda[V_{n-1}(t) - V_n(t)] \qquad \text{(D-120)}$$

when, for stability, $0 \le k \le 1$.

In the case of point cars in natural flow, the spacing is

$$s_n(t) = \lambda^{-1}[V_n(t) - KV_{n-1}(t)] \qquad \text{(D-121)}$$

from which it is clear that a negative value of $s_n\ (t)$ would correspond to a collision, so $s_n\ (t) \ge 0$.

To show this, the contrary is assumed; that is, for some t, $s_n\ (t) < 0$ or $V_n - KV_{n-1} < 0$. Then for all cars moving forward, $V_n - V_{n-1} < 0$, because $0 \le K \le 1$, and therefore $\lambda(V_{n-1} - V_n) > 0$. Furthermore, if by definition $g\ (t) = V_n\ (t) - KV_{n-1}(t)$, and since $g(t)$ is not always negative (i.e., it is initially zero), it must "go" negative. From the continuity of $\dot{g}\ (t)$, there is a right-hand neighborhood of the zero where it first starts to "go" negative, such that $\dot{g}\ (t) < 0$ for t in that neighborhood. However, from the governing differential equation,

$$\dot{V}_n(t) - K\dot{V}_{n-1}(t) = \lambda[V_{n-1}(t) - V_n(t)] \qquad \text{(D-122)}$$

or

$$\dot{g}(t) = \lambda[V_{n-1}(t) - V_n(t)] \qquad \text{(D-123)}$$

it has previously been shown that the quantity on the right side is always positive, therefore $\dot{g}\ (t) > 0$. This contradiction implies that the initial assumption was incorrect; i.e., collisions cannot occur. This proof also gives some indication of the difficulty to be expected when calculating the maximum collision-free flow with non-zero reaction time.

HEADWAY CONTROL

Under the headway control system, the headway (or spacing control) itself is the control input, the acceleration of the following car being taken as proportional to the deviation of the time headway from the desired headway. This kind of control may be taken as a natural extension of the rule-of-thumb put forth in the California Vehicle Code, that

the driver try to maintain a spacing of one car length for every 10 mph in speed. According to this rule, the desired spacing between successive vehicles is simply proportional to the following car's speed; that is,

$$v_n(t) = \mu[x_{n-1}(t) - x_n(t) - c] \qquad \text{(D-124)}$$

with due allowance for the length of a car, c. The difference between the true speed and the desired speed may then be taken as an error signal controlling the acceleration of the following vehicle. From the previous section,

$$\dot{v}_n(t) = K\, v_n'(t) - Kv_n(t) \qquad \text{(D-125)}$$

where the desired speed is that given by Eq. D-124. Therefore,

$$\dot{v}_n(t) = K\{\mu[x_{n-1}(t) - x_n(t) - c] - v_n(t)\} \qquad \text{(D-126)}$$

The application of a reaction time is omitted here because of the unsatisfactory performance of this kind of control, even as an automatic system. Further, a second-order system of differential-difference equations is by no means elementary. This equation may be written, with $v_n(t) \equiv \dot{x}_n(t)$, and $\dot{v}_n(t) \equiv \ddot{x}_n(t)$, as

$$\ddot{x}_n + K\dot{x}_n + K\mu x_n = K\mu(x_{n-1} - c) \qquad \text{(D-127)}$$

which may be integrated in a variety of ways. To simplify the notation somewhat, write $f = x_{n-1} - c$ and $x \equiv x_n$, or

$$\ddot{x} + K\dot{x} + K\mu x = K\mu f \qquad \text{(D-128)}$$

Let $y_1 = x$, $y_2 = \dot{x}$ so the vector

$$\underset{\sim}{y} = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \qquad \text{(D-129)}$$

and the above equation becomes

$$\underset{\sim}{y} = A\underset{\sim}{y} + \underset{\sim}{F} \qquad \text{(D-130)}$$

in which

$$A = \begin{pmatrix} 0 & 1 \\ -K\mu & -K \end{pmatrix} \qquad \text{(D-131)}$$

and

$$\underset{\sim}{F} = \begin{pmatrix} 0 \\ K\mu f \end{pmatrix} \qquad \text{(D-132)}$$

Eq. 132 may be verified by performing the indicated matrix multiplication; i.e.,

$$\dot{\underset{\sim}{y}} = \begin{pmatrix} \dot{x} \\ \ddot{x} \end{pmatrix} \qquad \text{(D-133)}$$

$$\begin{pmatrix} \dot{x} \\ \ddot{x} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -K\mu & -K \end{pmatrix}\begin{pmatrix} x \\ \dot{x} \end{pmatrix} + \begin{pmatrix} 0 \\ K\mu f \end{pmatrix} \qquad \text{(D-134}a\text{)}$$

or

$$\begin{pmatrix} \dot{x} \\ \ddot{x} \end{pmatrix} = \begin{pmatrix} \dot{x} \\ -K\mu x - K\dot{x} \end{pmatrix} + \begin{pmatrix} 0 \\ K\mu f \end{pmatrix} \qquad \text{(D-134}b\text{)}$$

which yields two equations—

$$\dot{x} = \dot{x} + 0 \qquad \text{(D-135)}$$

which is just an identity, and

$$\ddot{x} = -K\dot{x} - K\mu x + K\mu f(t) \qquad \text{(D-136}a\text{)}$$

or

$$\ddot{x} + K\dot{x} + K\mu x = K\mu f(t) \qquad \text{(D-136}b\text{)}$$

and this is the original equation (Eq. D-128). Multiplying

$$\dot{\underset{\sim}{y}} = A\underset{\sim}{y} + \underset{\sim}{F}(t) \qquad \text{(D-137)}$$

by the integrating factor e^{-At} gives

$$e^{-At}\dot{\underset{\sim}{y}} - e^{-At}A\underset{\sim}{y} = e^{-At}\underset{\sim}{F}(t) \qquad \text{(D-138}a\text{)}$$

or

$$\frac{d}{dt}[e^{-At}\underset{\sim}{y}] = e^{-At}\underset{\sim}{F}(t) \qquad \text{(D-138}b\text{)}$$

and integrating gives

$$\int_0^t \frac{d}{dt}[e^{-At}\underset{\sim}{y}]\,dt = \int_0^t e^{-Ap}\underset{\sim}{F}(p)\,dp \qquad \text{(D-139)}$$

$$e^{-At}\underset{\sim}{y}(t) - \underset{\sim}{y}(0) = \int_0^t e^{-Ap}\underset{\sim}{F}(p)\,dp \qquad \text{(D-140)}$$

Letting $\underset{\sim}{y}(0) = \underset{\sim}{y}_0$, we may write

$$\underset{\sim}{y}(t) = e^{At}\underset{\sim}{y}_0 + \int_0^t e^{A(p-t)}\underset{\sim}{F}(p)\,dp \qquad \text{(D-141)}$$

Now we have only to evaluate e^{At}. Then

$$e^{At} = T\,e^{\lambda t}\,T^{-1} \qquad \text{(D-142)}$$

in which

$$T^{-1}AT = \lambda \qquad \text{(D-143)}$$

and λ is a diagonal matrix. Multiplying from the left by T,

$$TT^{-1}AT = T\lambda \qquad \text{(D-144)}$$

But from elementary matrix theory, $TT^{-1} = 1$. Therefore,

$$AT = T\lambda \qquad \text{(D-145)}$$

in which

$$T = \begin{pmatrix} 1 & 1 \\ \lambda_1 & \lambda_2 \end{pmatrix} \qquad \text{(D-146)}$$

$$\lambda = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \qquad \text{(D-147)}$$

$$A = \begin{pmatrix} 0 & 1 \\ -K\mu & -K \end{pmatrix} \qquad \text{(D-148)}$$

so

$$\begin{pmatrix} 0 & 1 \\ -K\mu & -K \end{pmatrix}\begin{pmatrix} 1 & 1 \\ \lambda_1 & \lambda_2 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ \lambda_1 & \lambda_2 \end{pmatrix}\begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \qquad \text{(D-149)}$$

$$\begin{pmatrix} \lambda_1 & \lambda_2 \\ -K\mu - K\lambda_1 & -K\mu - K\lambda_2 \end{pmatrix}\begin{pmatrix} \lambda_1 & \lambda_2 \\ \lambda_1^2 & \lambda_2^2 \end{pmatrix} \qquad \text{(D-150)}$$

which yields the two identities, $\lambda_1 = \lambda_1$ and $\lambda_2 = \lambda_2$, and the two characteristic equations

$$\lambda_1^2 + K\lambda_1 + K\mu = 0 \qquad \text{(D-151}a\text{)}$$

and

$$\lambda_2^2 + K\lambda_2 + K\mu = 0 \qquad \text{(D-151}b\text{)}$$

with the solutions

$$\lambda_1 = \frac{-K - \sqrt{K^2 - 4K\mu}}{2} \qquad \text{(D-152)}$$

and

$$\lambda_2 = \frac{-K + \sqrt{K^2 - 4K\mu}}{2} \qquad \text{(D-153)}$$

Inasmuch as

$$T^{-1} = \frac{1}{\lambda_2 - \lambda_1} \cdot \begin{pmatrix} \lambda_2 & -1 \\ -\lambda_1 & 1 \end{pmatrix} \tag{D-154}$$

Eq. D-142 becomes

$$\begin{aligned} e^{At} &= T \begin{pmatrix} e^{\lambda_1 t} & 0 \\ 0 & e^{\lambda_2 t} \end{pmatrix} \begin{pmatrix} \lambda_2 & -1 \\ -\lambda_1 & 1 \end{pmatrix} \frac{1}{\lambda_2 - \lambda_1} \\ &= \frac{1}{\lambda_2 - \lambda_1} T \begin{pmatrix} \lambda_2 e^{\lambda_1 t} & -e^{\lambda_1 t} \\ -\lambda_1 e^{\lambda_2 t} & e^{\lambda_2 t} \end{pmatrix} \\ &= \frac{1}{\lambda_2 - \lambda_1} \begin{pmatrix} 1 & 1 \\ \lambda_1 & \lambda_2 \end{pmatrix} \begin{pmatrix} \lambda_2 e^{\lambda_1 t} & -e^{\lambda_1 t} \\ -\lambda_1 e^{\lambda_2 t} & e^{\lambda_2 t} \end{pmatrix} \\ &= \frac{1}{\lambda_2 - \lambda_1} \begin{pmatrix} \lambda_2 e^{\lambda_1 t} - \lambda_1 e^{\lambda_2 t} & e^{\lambda_2 t} - e^{\lambda_1 t} \\ \lambda_1 \lambda_2 (e^{\lambda_1 t} - e^{\lambda_2 t}) & \lambda_2 e^{\lambda_2 t} - \lambda e^{\lambda_1 t} \end{pmatrix} \end{aligned} \tag{D-155}$$

so

$$\begin{aligned} e^{At} \underset{\sim}{F}(p) &= e^{At} \begin{pmatrix} 0 \\ K\mu f \end{pmatrix} \\ &= \frac{1}{\lambda_2 - \lambda_1} \begin{pmatrix} (e^{\lambda_2 t} - e^{\lambda_1 t}) K\mu f \\ (\lambda_2 e^{\lambda_2 t} - \lambda_1 e^{\lambda_1 t}) K\mu f \end{pmatrix} \end{aligned} \tag{D-156}$$

which, with t replaced by $t - p$, becomes

$$e^{A(t-p)} \underset{\sim}{F}(p) = \frac{1}{\lambda_2 - \lambda_1} \begin{pmatrix} (e^{\lambda_2 (t-p)} - \lambda_1 (t-p)) K\mu f \\ (\lambda_2 e^{\lambda_2 (t-p)} - \lambda_1 e^{\lambda_1 (t-p)}) K\mu f \end{pmatrix} \tag{D-157}$$

Having

$$\underset{\sim}{y}(t) = e^{At} \underset{\sim}{y}_0 + \int_0^t e^{A(t-p)} \underset{\sim}{F}\, dp \tag{D-158}$$

and taking the (1, 1) element,

$$\begin{aligned} y_1 = x(t) = (\lambda_2 e^{\lambda_1 t} - \lambda_1 e^{\lambda_2 t}) \frac{\underset{\sim}{x}(0)}{\lambda_2 - \lambda_1} \\ + (e^{\lambda_2 t} - e^{\lambda_1 t}) \frac{\dot{\underset{\sim}{x}}(0)}{\lambda_2 - \lambda_1} \\ + \frac{K\mu}{\lambda_2 - \lambda_1} \int_0^t (e^{\lambda_2 (t-p)} - e^{\lambda_1 (t-p)}) f(p)\, dp \end{aligned} \tag{D-159}$$

and the (1, 2) element

$$\begin{aligned} y = \dot{x}(t) = (\lambda_1 \lambda_2 e^{\lambda_1 t} - \lambda_2 \lambda_1 e^{\lambda_2 t}) \frac{\underset{\sim}{x}(0)}{\lambda_2 - \lambda_1} \\ + \lambda_1 \lambda_2 (e^{\lambda_1 t} - e^{\lambda_2 t}) \frac{\dot{\underset{\sim}{x}}(0)}{\lambda_2 - \lambda_1} \\ + \frac{K\mu}{\lambda_2 - \lambda_1} \int (\lambda_2 e^{\lambda_2 (t-p)} - \lambda_1 e^{\lambda_1 (t-p)}) f(p)\, dp \end{aligned} \tag{D-160}$$

When $K < 4\mu$ one gets imaginary values for λ_1 and λ_2. If $h = K/2$ and $2i\nu = (K_2 - 4K\mu)$,

$$\lambda_1 = -h - i\nu \tag{D-161}$$

$$\lambda_2 = -h + i\nu \tag{D-162}$$

$$\lambda_2 - \lambda_1 = 2i\nu \tag{D-163}$$

$$\lambda_2 \lambda_1 = h^2 + \nu = K\mu \tag{D-164}$$

and

$$\begin{aligned} (\lambda_2 e^{\lambda_1 t} - \lambda_1 e^{\lambda_2 t}) = e^{-ht} \{ (-h + i\nu)(\cos \nu t - i \sin \nu t) \\ - (h + i\nu)(\cos \nu t + i \sin \nu t) \} = e^{-ht}\, 2i\nu \cos \nu t \end{aligned} \tag{D-165}$$

Likewise,

$$e^{\lambda_2 t} - e^{\lambda_1 t} = e^{-ht}\, 2i \sin \nu t \tag{D-166}$$

so $x(t)$ becomes

$$\begin{aligned} x(t) = e^{-ht} \cos \nu t\, (\dot{x}(0)) + e^{-ht} \nu^{-1} \sin \nu t\, (\dot{x}(0)) \\ + \nu^{-1} (h^2 + \nu^2) \int_0^t e^{-h(t-p)} \sin \nu (t - p)\, f(p)\, dp \end{aligned} \tag{D-167}$$

a damped oscillatory solution. Weakening K corresponds to reducing h and thus reducing the damping present. It is felt that a system which leads to overshoot and oscillation on paper will almost certainly be unacceptable on the road.

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