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**NATIONAL COOPERATIVE HIGHWAY RESEARCH PROGRAM  
 REPORT**

69

*please review chapters 4 & 5 with our  
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# EVALUATION OF CONSTRUCTION CONTROL PROCEDURES

## AGGREGATE GRADATION VARIATIONS AND EFFECTS

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NATIONAL COOPERATIVE HIGHWAY RESEARCH PROGRAM  
REPORT

**69**

**EVALUATION OF  
CONSTRUCTION CONTROL PROCEDURES  
AGGREGATE GRADATION  
VARIATIONS AND EFFECTS**

**S. B. HUDSON AND H. F. WALLER  
MATERIALS RESEARCH & DEVELOPMENT  
RALEIGH, NORTH CAROLINA**

RESEARCH SPONSORED BY THE AMERICAN ASSOCIATION  
OF STATE HIGHWAY OFFICIALS IN COOPERATION  
WITH THE BUREAU OF PUBLIC ROADS

SUBJECT CLASSIFICATION:  
BITUMINOUS MATERIALS AND MIXES  
CEMENT AND CONCRETE  
CONSTRUCTION  
MINERAL AGGREGATES

**HIGHWAY RESEARCH BOARD  
DIVISION OF ENGINEERING      NATIONAL RESEARCH COUNCIL  
NATIONAL ACADEMY OF SCIENCES—NATIONAL ACADEMY OF ENGINEERING      1969**

## NATIONAL COOPERATIVE HIGHWAY RESEARCH PROGRAM

Systematic, well-designed research provides the most effective approach to the solution of many problems facing highway administrators and engineers. Often, highway problems are of local interest and can best be studied by highway departments individually or in cooperation with their state universities and others. However, the accelerating growth of highway transportation develops increasingly complex problems of wide interest to highway authorities. These problems are best studied through a coordinated program of cooperative research.

In recognition of these needs, the highway administrators of the American Association of State Highway Officials initiated in 1962 an objective national highway research program employing modern scientific techniques. This program is supported on a continuing basis by funds from participating member states of the Association and it receives the full cooperation and support of the Bureau of Public Roads, United States Department of Transportation.

The Highway Research Board of the National Academy of Sciences-National Research Council was requested by the Association to administer the research program because of the Board's recognized objectivity and understanding of modern research practices. The Board is uniquely suited for this purpose as: it maintains an extensive committee structure from which authorities on any highway transportation subject may be drawn; it possesses avenues of communications and cooperation with federal, state, and local governmental agencies, universities, and industry; its relationship to its parent organization, the National Academy of Sciences, a private, nonprofit institution, is an insurance of objectivity; it maintains a full-time research correlation staff of specialists in highway transportation matters to bring the findings of research directly to those who are in a position to use them.

The program is developed on the basis of research needs identified by chief administrators of the highway departments and by committees of AASHO. Each year, specific areas of research needs to be included in the program are proposed to the Academy and the Board by the American Association of State Highway Officials. Research projects to fulfill these needs are defined by the Board, and qualified research agencies are selected from those that have submitted proposals. Administration and surveillance of research contracts are responsibilities of the Academy and its Highway Research Board.

The needs for highway research are many, and the National Cooperative Highway Research Program can make significant contributions to the solution of highway transportation problems of mutual concern to many responsible groups. The program, however, is intended to complement rather than to substitute for or duplicate other highway research programs.

This report is one of a series of reports issued from a continuing research program conducted under a three-way agreement entered into in June 1962 by and among the National Academy of Sciences-National Research Council, the American Association of State Highway Officials, and the U. S. Bureau of Public Roads. Individual fiscal agreements are executed annually by the Academy-Research Council, the Bureau of Public Roads, and participating state highway departments, members of the American Association of State Highway Officials.

This report was prepared by the contracting research agency. It has been reviewed by the appropriate Advisory Panel for clarity, documentation, and fulfillment of the contract. It has been accepted by the Highway Research Board and published in the interest of an effectual dissemination of findings and their application in the formulation of policies, procedures, and practices in the subject problem area.

The opinions and conclusions expressed or implied in these reports are those of the research agencies that performed the research. They are not necessarily those of the Highway Research Board, the National Academy of Sciences, the Bureau of Public Roads, the American Association of State Highway Officials, nor of the individual states participating in the Program.

NCHRP Project 10-2A FY '65

NAS-NRC Publication 1744

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## FOREWORD

*By Staff*

*Highway Research Board*

This report describes four individual studies concerned with the gradation of aggregates used in highway construction. The research dealt with aggregate sampling and test procedures, gradation variations, and some of their effects and was conducted as the final phase of the research previously reported in *NCHRP Report 34*, "Evaluation of Construction Control Procedures—Interim Report." Practical recommendations are included for improved aggregate sampling techniques, for determining inherent variance of aggregate gradations, and for specifications changes. Highway materials and testing engineers, specification writers, and construction engineers will find the report of particular interest.

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A major problem associated with the acceptance of aggregates for use in highway applications has been that the specifications have not always been statistically sound. That is, they have not taken into consideration the inherent variability of test procedures, materials, and processes. Also, they have not allowed for the statistical possibility that some small percentage of economically produced materials will not conform to specified limiting values. Thus, when the results from currently used acceptance tests approach specification limits, the degree of confidence that can be placed in the results is seriously challenged. This is extremely critical from an engineering standpoint because aggregates constitute more than 90 percent of the materials used in roadway and bridge construction, and their quality and gradation are primary factors in successful performance of these structures. Therefore, there is a real need for a statistical method for setting realistic maximum and/or minimum numerical limits for acceptance of materials produced under controlled conditions.

*NCHRP Report 34* dealt with an analysis of the sources of variation that cause apparent or actual departure of aggregate gradations from those specified. The study by Miller-Warden Associates Division of Materials Research and Development, Inc., reported herein, was concerned with four specific items of the over-all problem, as follows:

1. Effect of variations in gradation of coarse aggregate on characteristics of portland cement concrete. In the initial study (*NCHRP Report 34*) it was shown that under current construction controls there are variations in the actual gradation of the coarse aggregate used in concrete. A statistically designed experiment was used during the study reported herein to determine the effect of these variations on the workability and compressive strength of the resultant concrete.

2. Variation in gradation of aggregates in bituminous hot-mix plants. The scope of the initial work was confined to coarse aggregate in portland cement con-

crete. This was extended during the second phase to include an investigation of variations in gradation of both fine and coarse aggregates and asphalt contents at several points in the production process of bituminous hot-mix plants.

3. Effect of increment size on sampling accuracy. An experiment was conducted to establish the practical relationship between maximum particle size and minimum increment size for determining the gradation of a LOT of aggregate. Several LOTS of known gradation were prepared and sampled with sampling scoops of predetermined capacity during the course of the study.

4. Mathematical study of the pattern of variations in gradation of aggregates. During the course of previous studies it appeared that variations in the gradation of aggregates followed an inherent pattern. All of the data from this project and NCHRP Project 10-3 were analyzed to determine the possibilities for selecting a single sieve for use as a quick check on compliance with gradation specifications.

§ The present trend in some highway departments toward a statistical approach to assurance of quality in highway construction evidences the growing recognition of the inherent variability of test procedures, materials and processes. The findings of this study should be enlightening within the broad context of quality control and should contribute to a better base for applying sound engineering judgment to the establishment of practical and realistic tolerance limits. If, for example, coarse aggregate with a good performance record is actually outside of current gradation specifications, tools are now available to support experience-based judgment in realistically modifying sampling techniques, acceptance procedures, and specification limits. Specific recommendations are made for revisions of specification limits; however, these apply only to those situations where statistically sound sampling plans and acceptance procedures are employed to define actual variations within known confidence limits.

Related published reports emanating from NCHRP projects are as follows:

*NCHRP Report 5*, "Effects of Different Methods of Stockpiling Aggregates—Interim Report."

*NCHRP Report 17*, "Development of Guidelines for Practical and Realistic Construction Specifications."

*NCHRP Report 34*, "Evaluation of Construction Control Procedures—Interim Report"

*NCHRP Report 46*, "Effects of Different Methods of Stockpiling and Handling Aggregates."

The combined effect of the investigations in the problem area, within NCHRP and by other agencies, has been the development of a better understanding of statistical concepts as they apply to an over-all acceptance program for highway aggregates. This should contribute to the development of practical approaches for improving the control of aggregates in highway construction.

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# EVALUATION OF CONSTRUCTION CONTROL PROCEDURES

## AGGREGATE GRADATION VARIATIONS AND EFFECTS

### SUMMARY

The work reported herein consists of four related studies which supplement the initial phase of the research reported in *NCHRP Report 34*, "Evaluation of Construction Control Procedures, Interim Report." The major findings of the work which have immediate practical application are summarized in the following paragraphs.

The findings of the concrete study indicate that savings in manpower and testing time can be effected by reducing the sampling frequency of coarse aggregates for concrete. Variations in gradation of coarse aggregates over the extreme range actually found to exist at operating two-bin concrete plants caused a change of about 3 in. in the slump and about 330 psi in the 28-day strength. This range of values is about the same as is caused by a change of 2% in the moisture content of the fine aggregate. The most significant finding was that compressive strength remained substantially constant, regardless of large variations in the gradation of the coarse aggregate, provided the slump was maintained at a uniform level. Rescreening or resizing coarse aggregates is contra-indicated, and it is recommended that restrictive gradation specifications be broadened to accommodate gradation variations actually found to exist.

From the study of operating hot-mix bituminous paving plants, it was found that indicated variations in the gradation of binned aggregates were largely due to within-batch variation. Results of tests on duplicate samples of the same aggregate going into the same batch showed widely different gradations. This means that mathematically combined gradations based on the results of tests on single bin samples are not necessarily representative of the actual gradation in the completed mix. It is recommended that compliance with job-mix formula gradation requirements be based on quality history charts rather than on the results of individual tests.

The study of the effect of the size of increment taken while sampling coarse aggregates indicates that the use of usual sampling tools does not seriously bias the results. This finding means that to determine the actual gradation of a batch of coarse aggregate, a better estimate is obtained by accumulating a test portion by a large number of small increments, such as scoops, than by a few larger increments, such as shovelfuls.

The mathematical study has confirmed that there is a typical pattern to variations of aggregate gradation. Immediate application of this finding is that gradation specification should have the greatest tolerance on those sieves that pass 50 to 70% of the aggregate. The required size of test portions depends on both the maximum particle size and the gradation of the aggregate.



## INTRODUCTION

The purpose of this report is to present the results of a continued study of sources of variation affecting construction control procedures. Previous work as described in *NCHRP Report 34 (6)* included unbiased random sampling of coarse aggregates from various points at eight operating plants located in six states. In connection with this work an equation and a nomograph were developed for determining the minimum size of total sample required to determine aggregate gradation with a chosen degree of accuracy.

In general, results of this work indicated that the actual variation in gradation of coarse aggregate for concrete, at point of use, exceeded the limits of some current specifications and that sampling methods had a large effect on the apparent variation in gradation. A limited amount of work was devoted to a mathematical study of the parameters that determine the pattern of over-all variation of aggregate gradations. The conclusion of the initial project left unsolved the question as to the proper approach to gradation control; that is, whether the indicated actual variations in gradation of aggregate for concrete were acceptable or whether stricter physical controls should be specified.

To answer some of the questions raised by the previous work, this continuation project includes four studies related to variations in aggregate gradations. One study is concerned with the effects of variations in gradation of coarse aggregate, comparable to those found in the previous work, on the strength and workability of portland cement concrete. A second study roughly parallels the previous work and consists principally of the sampling of aggregates at hot-mix asphalt plants, before and after separation by screening, to evaluate the efficacy of the screen controls in maintaining uniformity of the aggregate gradation in the paving mixture.

A major accomplishment of the previous work was the determination of the effect of test portion size on the accuracy of the gradation test used for construction control. However, the effect of increment size was not investigated. Accordingly, a third study in this continuing work is devoted to an investigation of possible bias of gradation test results due to the size of the increments; that is, the smallest portions of aggregate used to make up the sample to be tested for an acceptance decision.

The results of the aggregate samplings at concrete plants showed some indication of a pattern of variation in gradation that is independent of the size of the over-all variation. Test results for those sieves passing 50 to 70% of the aggregate showed the largest variations. Because this pattern would directly affect the specification tolerances defining allowable variations in gradation, a fourth study reported herein includes further mathematical work to verify this finding.

The methods of attaining the objectives of the various phases, and the results of the research conducted are presented in detail in the following chapters and appendices. However, for convenience, the statistical methods common to analysis of the data from all phases are discussed here.

### DATA ANALYSIS

The data accumulated in this experimental program have been analyzed by various statistical techniques. It is assumed that the reader is familiar with the meaning of the commonly used terms and methods of calculation; therefore, only the statistical parameters that are pertinent to the presentation and interpretation of the data are defined.

The analysis is based on the concept of a normal distribution defined by a true mean ( $\bar{X}'$ ) and the true standard deviation ( $\sigma'$ ). Estimates of these values are obtained from the data and expressed as the estimated mean ( $\bar{X}$ ) and the estimated standard deviation ( $\sigma$ ), which is a means of expressing variation from the average as a numerical value. By determining the standard deviation of gradation test results, for example, an estimate of the percentage of results that will be contained within any given limits can be calculated. For convenience, the variance ( $\sigma^2$ ), which is the square of the standard deviation, is used instead of  $\sigma$  in some parts of this report because variances can be added and subtracted directly, whereas standard deviations cannot be combined in this manner.

In research work previously accomplished by this agency, it has been shown that variations in aggregate gradation among individual test results are due to a combination of several different components of variance.

A brief explanation of these variance components is given in the following. Their relationship is shown diagrammatically in Figure 1.

#### *Inherent Variance*

The inherent variance ( $\sigma_a^2$ ) is a measure of the variation due to the random distribution of particles within an aggregate mass. It cannot be eliminated or reduced by process control. It is extremely important, because it provides a minimum limiting value which must be considered when statistical methods are employed to determine the size of a sample or test portion for a predetermined degree of accuracy.

#### *Testing Error*

The variation due to testing error is the within-test-portion variance ( $\sigma_t^2$ ) due to the lack of repeatability of the test

procedure. This is not an error in the sense of someone using the wrong technique, but an error due to the random variations associated with any test procedure. Testing error for the gradation test was determined by having the same operator, using the same equipment, repeat the test on randomly selected test portions, and determining the difference between the two tests made on the same test portion.

#### Experimental Error

The sum of the variances due to inherent variation and testing error ( $\sigma_a^2 + \sigma_t^2$ ) is called experimental error and is measured by the variance ( $\sigma_e^2$ ). This value is important because it is this combined variance that affects repeatability and reproducibility of an aggregate gradation test on *duplicate* test portions. When a precision statement is to be written, this experimental error must be used as the basis.

#### Sampling Error

The sampling error, measured by the variance ( $\sigma_s^2$ ), is a result of the combined effects of all within-batch variations, such as differences in gradation between different parts of a batch and not due to inherent variance or testing error.

#### Within-Batch Variance

The within-batch variance ( $\sigma_b^2$ ) is the sum of the sampling and experimental errors and is found by taking two test portions or increments from suitably separated points within the same batch, performing a gradation test on each increment, and determining the difference between the two. Because, in most cases, the nonuniformity represented by this variance will be greatly reduced by subsequent mixing, this is a false variation and does not necessarily affect the quality of construction.

#### Batch-to-Batch Variance

The batch-to-batch, or within-lot variance ( $\sigma_l^2$ ) is an actual variation and of real significance, because it can cause differences in the performance of different batches. The size of this variance depends almost entirely on the combined effects of the methods of handling, transporting, and proportioning aggregates, and the resulting degree of segregation.

#### Over-All Variance

The over-all variance among individual test portions taken from a lot is symbolized by  $\sigma_o^2$ , which is made up of  $\sigma_a^2 + \sigma_t^2 + \sigma_b^2 + \sigma_l^2$ . Obviously, this is the largest and most important variance of all, because it contains the combined effects of the other variances. This variance is the most significant consideration in the writing of realistic specifications, in the establishment of optimum sampling plans, and in determining the number of test portions required to obtain a desired degree of accuracy.

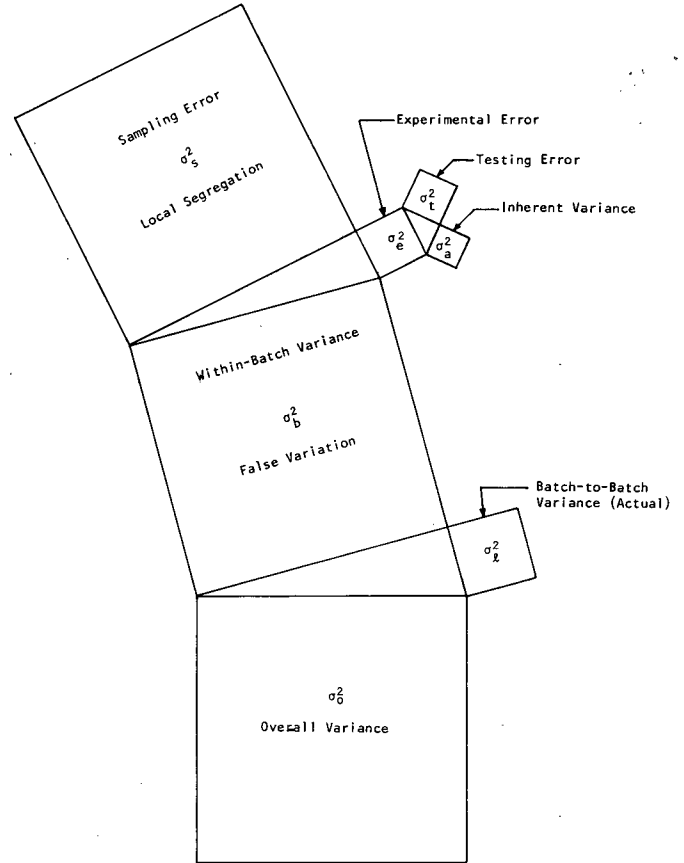


Figure 1. Sources of variations of gradation, hot-bin study.

#### Summary of Variances

The variances previously discussed and the equations for their computation are summarized in Table 1.

#### GRADATION MODULUS $\bar{A}$

To measure and assess the effects of changes in gradation by the use of statistical methods, it is desirable to describe the gradation by a single number rather than a multiplicity of percentages. The fineness modulus (FM), originated by Abrams, is such a parameter and is useful when dealing with aggregates for portland cement concrete. However, the FM was intentionally designed to exclude the influence of the percentage of the material passing the No. 100 sieve. This, therefore, makes the FM unsuitable for use in dealing with aggregates for bituminous concrete or when other aggregate mixtures contain a significant quantity of minus No. 100 material.

Recent studies have resulted in the concept of  $\bar{A}$ , which is simply  $1/100$  of the sum of the percentages passing the ten standard sieves, starting with the  $1\frac{1}{2}$ -in. and including the No. 200 sieve. An investigation of theoretical concepts, confirmed by limited experimental investigation, indicates that  $\bar{A}$  is a fundamental constant, related to the relative surface area effects of the aggregate gradation in any mix-

TABLE 1  
SUMMARY OF VARIANCES

VARIANCE	DESIG- NATION	CAUSE	HOW ESTIMATED	EQUATION
Inherent (within-test portion)	$\sigma_a^2$	Inherent	Computed	$\sigma_a^2 = \frac{P(100-P)\bar{g}^*}{454W}$
Testing error (between tests)	$\sigma_i^2$	Testing error	By experiment	$\sigma_i^2 = \frac{\Sigma(X_1 - X_2)}{2n}$
Sampling error (among increments)	$\sigma_s^2$	Sampling error	By difference	$\sigma_s^2 = \sigma_b^2 - (\sigma_a^2 + \sigma_i^2)$
Within-batch	$\sigma_b^2$	Multiple (sum of $\sigma_a^2, \sigma_i^2, \sigma_s^2$ )	By experiment	$\sigma_b^2 = \frac{\Sigma(X_A - X_B)^2}{2n}$
Batch-to-batch (within-lot)	$\sigma_i^2$	Segregation	By difference	$\sigma_i^2 = \sigma_o^2 - \sigma_b^2$
Over-all	$\sigma_o^2$	Result of com- bined causes	By experiment	$\sigma_o^2 = \frac{\Sigma X^2 - \frac{(\Sigma X)^2}{n}}{n-1}$

\* See Chapter Five for explanation and use of equation.

ture of particle sizes (see Appendix A). For example, with asphaltic concrete aggregates in the usual range of  $\bar{A}$  from 4.00 to 6.00, a change of 0.50 in the value of  $\bar{A}$  would change the asphalt demand by about 1% by volume, which is enough to affect the performance of the mixture. Thus,  $\bar{A}$  appears to be a measure of relative coarseness of aggregate

mixtures, sufficiently sensitive to reflect significant variations in aggregate gradation. The computer program included calculation of  $\bar{A}$  for gradations obtained from each location.

Details of alternate methods of computing  $\bar{A}$  are given in Appendix A.

## CHAPTER TWO

# STUDY OF EFFECT OF VARIATIONS IN GRADATION OF COARSE AGGREGATE ON CHARACTERISTICS OF PORTLAND CEMENT CONCRETE

## INTRODUCTION AND RESEARCH APPROACH

Previous work as described in *NCHRP Report 34* indicated that actual variations in the gradation of coarse aggregates (1 in. maximum size) at the weigh hopper of operating concrete plants was greater than the limits contained in some current specifications. A review of state highway gradation requirements for this size coarse aggregate for portland cement concrete indicates that three states have limits within the range of 30 to 60% passing the 1/2-in. sieve, and 13 have limits of 25 to 60%. The limits of percent passing the 1/2-in. sieve specified by the other states are not given or are not applicable. Although standard sieves were used in the previous investigations, interpolation of the results indicated that a range of about 26 to 65% passing the 1/2-in. sieve would be necessary if 95% of the

test results on random samples were to meet this gradation requirement consistently.

These findings raise the question as to the effect of variations in coarse aggregate on the strength and workability of the resultant concrete. If these variations do not seriously impair the quality of the concrete, it may be advisable to widen current limits that are unduly restrictive so that the gradation specifications will be fully enforceable. On the other hand, if observed variations in gradation do adversely affect the quality of the concrete to a significant degree, changes in methods of handling coarse aggregate may be indicated at some plants.

These changes might take the form of maintaining separate stockpiles of different sized aggregate or re-screening aggregate into separate sizes, and proportioning these sizes of coarse aggregate into the concrete batches from

separate bins. Although means for effecting these controls are available, such requirements, in cases where they are not fully justified, will be reflected in the cost of the concrete.

Two commonly accepted criteria for quality and uniformity of structural concrete are strength (usually 28-day compressive strength) and workability as measured by the slump test or compaction factor.

The effects of variations in gradation of coarse aggregates, as from a theoretical gradation curve, have long been a controversial subject among researchers. More than 35 years ago, one of the conclusions from research conducted by the Bureau of Public Roads (1) was that, within wide limits, variations in gradation of coarse aggregate had no consistent effect on the compressive or tensile strength of the concrete.

Recently expressed opinion (2) has been to the effect that re-screening of coarse aggregates would improve uniformity of strength; other opinion (3) is that relatively wide ranges of variation in gradation of coarse aggregate larger than  $\frac{3}{8}$ -in. have no appreciable effect on workability. A study (4) made with special apparatus has indicated that gradation has little effect on workability with rich mixtures but does affect mixtures having a low cement content.

The purpose of the work reported herein was to clarify these areas of uncertainty and to quantify the effect of measured changes in actual gradation of coarse aggregate on the strength and workability of concrete, of the type used in highway structures, by means of a designed experiment.

The general plan was to produce small batches of air-entrained concrete from three gradations of two types of coarse aggregate (gravel and crushed stone), keeping the percentage by weight of cement and coarse and fine aggregate constant, as would be the case in actual practice. Batches were produced with two slumps (1 to 2 in. and 2 to 4 in.) and consequently had different water-cement ratios. Batches were tested for workability, or placeability, by the slump test and by use of the compaction factor apparatus (Fig. 2). Four standard test cylinders were made from each batch for 7- and 28-day compressive strength tests. Two cylinders were tested at each age, because statistical calculations indicated that there would be little advantage in testing three cylinders at each age.

The various combinations of aggregate type, gradation, and workability (slump) were identified by batch number. The order of making batches for each repetition was then determined by use of a table of random numbers. Batch identification and sequence are given in Table B-5.

The gradations of the aggregates used are given in Table B-1. The three gradations of coarse aggregate are shown in Figure 3.

The mixtures using coarse aggregate having gradation B were designed by the method of ACI 613, using 6.0 bags of cement per cubic yard. The same proportion of coarse to fine aggregates was used in the batches made from gradations A and C.

The batches made from gradations A and C were mixed in a laboratory mixer with the same quantity of water as

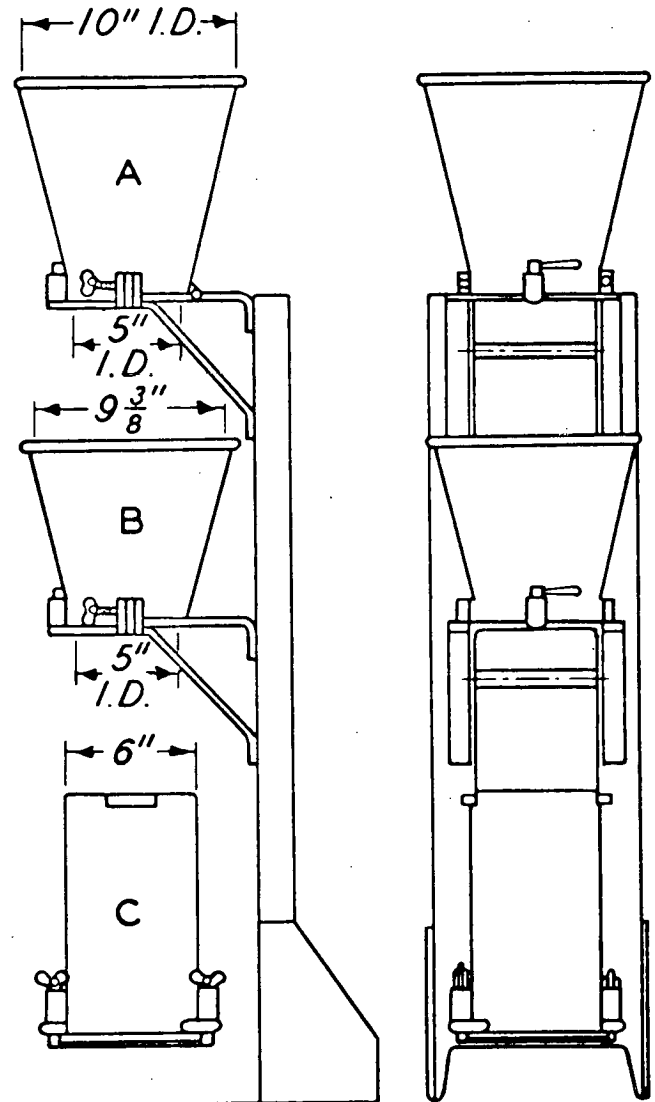


Figure 2. Compaction factor apparatus.

that required to obtain the slump of mixtures made from gradation B of the same type of aggregate.

The compaction factor was determined for each batch and four standard 6 × 12-in. test cylinders were molded from the remaining concrete. To minimize cylinder-to-cylinder variation, the cylinders were made by the group method; that is, all of the four cylinders were filled one-third full of concrete, this layer was rodded, then all cylinders were filled two-thirds full, and so on.

To reduce the effects of day-to-day variations, the entire experiment (2 aggregates × 3 gradations × 2 slumps) was repeated on five different days. Thus, the total number of batches was 2 aggregates × 3 gradations × 2 slumps × 5 repetitions = 60 batches. Standard procedures were followed for making, curing, capping, and testing the cylinders, one-half of which were tested at 7 days and one-half at 28 days.

In addition to the test for compressive strength, each

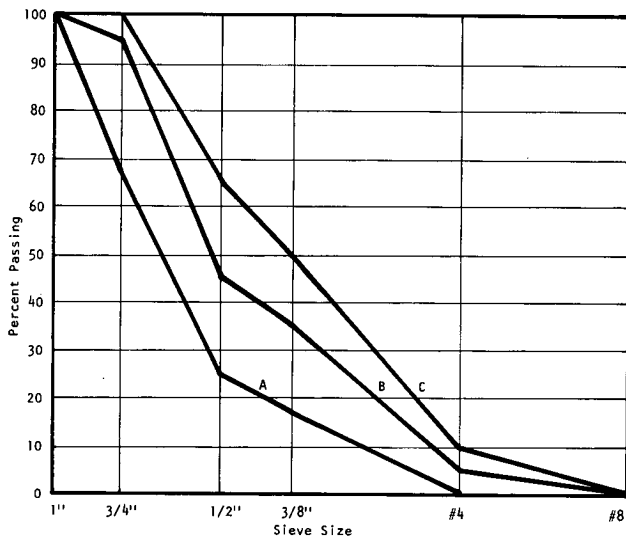


Figure 3. Gradations of coarse aggregate used in designed experiment.

batch of concrete was tested for slump, air content, weight per cubic foot (air meter base and cylinder weight), loose weight as determined by the compaction factor apparatus, and compacted weight determined by rodding and vibrating the concrete in the compaction factor apparatus mold.

Aggregates were obtained from stockpiles, transported to the laboratory and spread to air dry for one to two weeks. Coarse aggregate was separated into sizes on the rotary sieve, and plus 1 in. and minus No. 8 were discarded. Sized aggregates were held in 50-gal drums until needed. Batches were prepared by weighing specified percentages on a platform scale. The proportion of fine and coarse aggregate and cement corresponded to commercial (Philadelphia Building Code) mixes using the same aggregates in the Philadelphia area. Twenty-two bags of cement were obtained from one lot, and held in sealed bags until weighed out the day previous to use. Fine aggregate for one batch was contained in one bag, the coarse aggregate in two bags. Cement for one batch was contained in a 5-gal pail. The water-cement ratio was determined by trial on batches 5 and 11 for 4-in. slump. Adjustment of water for 1-in. slump was computed from the *PCA Handbook*. The Darex air-entraining agent was obtained from laboratory supply and was added to the mixing water in the proportion of 2.8 cc per batch, or about 1½ oz per sack of cement. Batch size was about 1½ cu ft.

The work was done by a group of experienced technicians at the laboratories of the Ambric Testing and Engineering Associates, Philadelphia (CCRL approved). The making of the batches and of initial tests was done under the direct supervision of the Principal Investigator, who also witnessed the 28-day tests.

Details of materials and equipment, and proportions used, are presented in Tables B-1 to B-4.

Batches were mixed in a 3½-cu-ft Gilson S-T mixer at approximately 25 rpm. Before the first batch the mixer

was conditioned by washing it clean and "buttering" it with a mixture of 1 part cement to 3 parts sand with sufficient water added to obtain a plastic consistency. The mixer was then brought to a scraped-clean condition before batching began. The sand and cement were added first and mixed ½ min. The coarse aggregate was added and mixing continued another ½ min. The total water was then added and the entire batch mixed for an additional 3 min. The mixer discharge opening was covered with wet burlap and the batch was allowed to stabilize for 3 min. The batch was then remixed for 2 min, discharged, and the mixer scraped clean.

The mixed batch was discharged into a mortar box, remixed slightly with a mortar hoe, and tested for slump. The material used for the slump test was returned to the mortar box, remixed, and the air meter base and cylinder molds were filled. The cylinders were made by filling each of the four molds one-third full, rodding this layer, placing another layer in each mold, and so on. Each cylinder was identified by date and batch number. The filled air meter base was weighed on a platform scale to the nearest 0.01 lb and the air content tested. This material was then discarded. The remaining material in the mortar box was used for the compaction test. The upper cone of the apparatus was filled, and the contents released into the lower cone. This material was then allowed to drop into the 6 × 12-in. container. This container was struck off, weighed, and the contents returned to the mortar box. The contents of the mortar box were remixed and the material was used to refill the 6 × 12-in. compaction container which, at this point, was placed on a vibrating steel plate. The container was filled in three layers, each of which was rodded during vibration in the same way as for a test cylinder. The container was struck off, the contents weighed, and all concrete discarded. All equipment except the mixer was then washed clean. Total time for each batch was approximately 30 min.

Test cylinders were covered immediately with plastic "cylinder socks" and allowed to remain undisturbed overnight. They were then moved to the stripping area, but were not stripped or the socks removed until after about 48 hr. After stripping they were placed in a standard moist room until removal for capping and testing.

The cylinders were removed from the moist room, weighed, and capped in preparation for the compressive strength test. Capping material was U. S. Gypsum Hydrocal White, a high-strength gypsum cement. Caps were formed against oiled marble slabs.

Cylinders were tested on a 350,000-lb capacity Forney testing machine, calibrated and certified on November 11, 1963.

## FINDINGS

The 60 measurements of each characteristic are given in Tables B-6 to B-13, inclusive. The means of the five repetitions of each measurement are summarized in Table 2. The relationships among these means were established by multiple regression analysis; the resulting regression equa-

TABLE 2  
SUMMARY OF VARIABLES, CONCRETE STUDY

BATCH NO.	GRADATION	W/C		$\bar{A}$	TYPE	$S_{e7}$ (PSI)		$S_{e28}$ (PSI)		SLUMP (IN.)	LOOSE WEIGHT (PCF)	COM-PACTED WEIGHT (PCF)	UNIT WEIGHT (PCF)	CYLINDER WEIGHT (PCF)	AIR (%)
		$X_1$	$X_2$			$X_3$	$Y_1$	$Y_2$	$Y_3$		$Y_4$	$Y_5$	$Y_6$	$Y_7$	$Y_8$
1	Stiff	SA	5.36	4.34	1	2,720	3,400	2.70	136.32	144.85	145.70	143.21	4.08		
2		SB	5.36	4.55	1	2,710	3,290	1.85	133.09	147.00	146.56	144.42	3.82		
3		SC	5.36	4.70	1	3,015	3,770	1.05	129.96	146.17	145.76	143.81	4.22		
4		GA	4.92	4.17	2	2,970	3,800	2.40	133.13	145.55	145.20	143.01	3.66		
5		GB	4.92	4.40	2	3,210	3,910	1.15	126.63	145.75	145.34	143.11	3.60		
6		GC	4.92	4.55	2	3,080	3,850	0.65	128.49	143.99	143.88	142.08	4.08		
7	Plastic	SA	5.72	4.34	1	2,315	2,950	5.90	142.72	146.31	145.40	141.14	4.38		
8		SB	5.72	4.55	1	2,440	3,015	4.50	139.71	145.04	144.06	140.47	4.52		
9		SC	5.72	4.70	1	2,720	3,570	2.95	136.32	143.82	144.70	142.33	3.96		
10		GA	5.28	4.17	2	2,635	3,370	5.30	138.51	144.35	143.78	141.21	4.12		
11		GB	5.28	4.40	2	2,820	3,610	3.10	134.29	143.95	143.34	140.98	4.04		
12		GC	5.28	4.55	2	3,040	3,715	2.10	130.90	144.71	143.50	141.49	3.94		
		$\Sigma$	63.84	53.42	18	33,675	42,250	33.65	1,610.07	1,741.49	1,737.22	1,707.26	48.42		
		$\bar{X}$	5.3200	4.4517	1.50	2806.25	3520.83	2.8042	134.17	145.12	144.77	142.27	4.0350		

tions are summarized in Table 3. The values predicted by the use of these equations are shown by the dashed lines in Figures 4, 5, 6, and 7.

#### Effect of Gradation on 7-Day Compressive Strength

The results of the 7-day tests on the different gradations are given in Table 4 and are represented by the solid lines in Figure 4.

The general trend was for the 7-day compressive strengths to be higher as the coarse aggregate was changed

from a coarse to a fine gradation. The largest effect was produced in the plastic crushed stone mix, where the fine gradation had about 15% greater strength than the coarse gradation.

The effect of gradation was highly significant statistically. The regression equation is

$$S_{e7} = 3,372 - 803 W/C + 806 \bar{A} + 79 T \quad (1)$$

which accounts for about 90% of the total variation. The equation indicates that the effect of a change of gradation

TABLE 3  
SUMMARY OF REGRESSION EQUATIONS,<sup>a</sup> CONCRETE STUDY

CHARACTERISTIC	REGRESSION	EQ. NO.	$r^2$
7-Day compr. strength (psi)	$= 3372 - 803 W/C + 806 \bar{A} + 79 T$	1	0.90
28-Day compr. strength (psi)	$= 3746 - 829 W/C + 889 \bar{A} + 151 T$	2	0.81
Slump (in.)	$= -4.7 + 6.5 W/C - 6.5 \bar{A} + 1.1 T$	3	0.96
Uncompacted weight (lb/cu ft)	$= 124.4 + 16.1 W/C - 17.1 \bar{A} + 0.05 T$	4	0.95
Compacted weight (lb/cu ft)	$= 167.3 - 2.4 W/C - 1.4 \bar{A} - 2.1 T$	5	0.40
Unit weight (lb/cu ft)	$= 174.7 - 3.5 W/C - 1.5 \bar{A} - 3.0 T$	6	0.81
Cylinder weight (lb/cu ft)	$= 173.1 - 5.6 W/C + 0.7 \bar{A} - 2.9 T$	7	0.77
Entrained air (%)	$= 0.46 + 0.69 W/C - 0.04 \bar{A} + 0.04 T$	8	0.49

<sup>a</sup> W/C = water-cement ratio, in gallons per bag;

$\bar{A}$  = gradation modulus of all solids, including both cement and aggregates, in the concrete mixture (meaning and application given in Appendix A);

$T$  = a dummy variable used to show the effect of aggregate type; i.e., whether gravel or crushed stone;

$r^2$  = the coefficient of determination. This coefficient ( $r^2$ ) multiplied by 100 is the percentage of the total variation accounted for by the regression equation. The remaining variation is due to unknown and random causes.

**TABLE 4**  
**MEASURED 7-DAY COMPRESSIVE STRENGTHS**

MIX TYPE	AGGREG. GRADATION	7-DAY COMPRESSIVE STRENGTH <sup>a</sup> (PSI)	
		CRUSHED STONE AGGREG.	GRAVEL AGGREG.
Stiff	Coarse	2720	2970
	Normal	2710	3210
	Fine	3015	3080
Plastic	Coarse	2315	2635
	Normal	2440	2820
	Fine	2720	3040

<sup>a</sup> Average of five tests of two cylinders each.

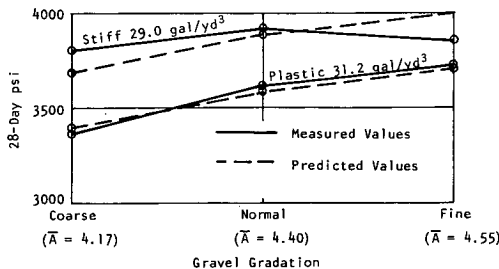
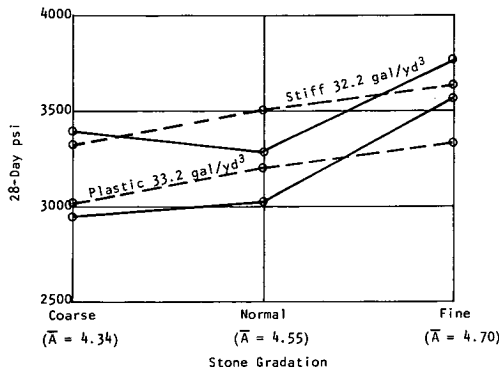
toward increased fineness, as measured in  $\bar{A}$  units, had an almost equal and opposite effect to an increase in the water-cement ratio, as measured in units of gallons per bag of cement.

As shown in Figure 4, the values predicted from this equation agree well with the measured values.

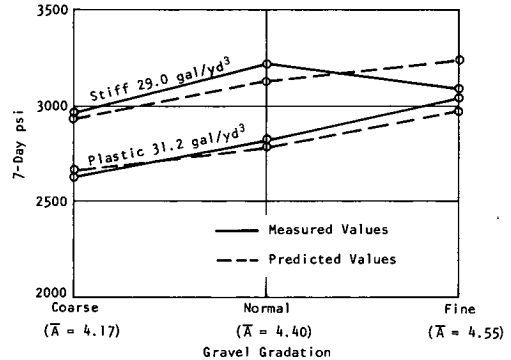
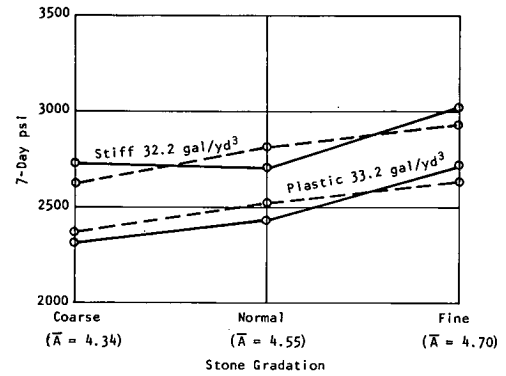
**Effect of Gradation on 28-Day Compressive Strength**

The results of the 28-day tests are given in Table 5 and shown in Figure 5.

As in the case of the 7-day compressive strengths, the



*Figure 5. Effect of gradation on 28-day compressive strength.*



*Figure 4. Effect of gradation on 7-day compressive strength.*

general tendency was for the 28-day compressive strengths to increase as the coarse aggregate gradation changed from coarse to fine. The greatest difference was in the case of the crushed stone mix of plastic consistency, where the increase in compressive strength, from coarse to fine gradation, was 620 psi, or about 21%.

The effect of gradation was statistically significant. The regression equation is

$$S_{c28} = 3,746 - 829 W/C + 889 \bar{A} + 151 T \quad (2)$$

**TABLE 5**  
**MEASURED 28-DAY COMPRESSIVE STRENGTHS**

MIX TYPE	AGGREG. GRADATION	28-DAY COMPRESSIVE STRENGTH <sup>a</sup> (PSI)	
		CRUSHED STONE AGGREG.	GRAVEL AGGREG.
Stiff	Coarse	3400	3800
	Normal	3290	3910
	Fine	3770	3850
Plastic	Coarse	2950	3370
	Normal	3015	3610
	Fine	3570	3715

<sup>a</sup> Average of five tests of two cylinders each.

which accounts for about 81% of the total variation. As in the case of the 7-day compressive strengths, this equation indicates that an increase in the fineness of the coarse aggregate has a nearly equal and opposite effect to an increase in the water-cement ratio.

It should be noted that the differences in the level of the 7- and 28-day compressive strengths are due to mixture composition and should not be related to aggregate type. The composition of the mixture is given in Table B-3.

#### Effect of Gradation on Slump

As shown in Figure 6 and given in Table 6, the slump decreased as the coarse aggregate gradation became finer. The largest observed difference was in the case of the gravel mix of plastic consistency, where the slump decreased about 3 in. as the gradation was changed from coarse to fine. This decrease is equivalent to a change of about 2% in the moisture content of the fine aggregate. The gradation had a highly statistically significant effect on the slump. The regression equation is

$$\text{Slump} = 6.5 W/C - 6.5 \bar{A} + 1.1 T - 4.7 \quad (3)$$

which accounts for about 96% of the total variation. This equation indicates that the effects of changes in water-cement ratio, as measured in gallons per bag, and in gradation as measured in  $\bar{A}$  units, are equal and opposite.

#### Effect of Gradation on Uncompacted Weight

The effect of the different gradations on the uncompacted weight of the concrete is shown in Figure 7 and given in Table 7.

It was intended to measure the effect of gradation on workability by the compaction ratio; that is, the ratio of the loose weight of concrete dropped into a mold to the weight of the same concrete compacted in the same mold (5). However, a study of the data indicated that changes in the loose weight were the best index of changes in workability. As can be seen by comparison of Figures 6 and 7, the effect of changes in gradation on the loose weight were almost identical to the effect on slump. The

gradation had a statistically significant effect on the uncompacted weight of the concrete. The regression equation is

$$\text{Uncompacted weight} = 124.43 + 16.13 W/C - 17.11 \bar{A} + 0.05 T \quad (4)$$

which accounts for about 95% of the total variation. The effects of changes of water-cement ratio and gradation are nearly equal and opposite and have the same sign as for the slump test.

#### Effect of Gradation on Other Measurements

As given in Table 8, the  $F$ -ratios computed from the regression mean squares indicate that the experimental variations in gradation (as measured in  $\bar{A}$  units) had no statistically significant effect on compacted weight, unit weight, cylinder weight, or the percentage of entrained air of the concrete.

#### Combined Effects

The combined effects of changes in gradation and water-cement ratio on strength and workability are indicated by

$$S_{e28} = 3,746 - 829 W/C + 889 \bar{A} + 151 T \quad (2)$$

$$\text{Slump} = 6.5 W/C - 6.5 \bar{A} + 1.1 T - 4.7 \quad (3)$$

Substituting the average value for type of aggregate (1.5) these equations become:

$$S_{e28} = 3,970 - 829 W/C + 889 \bar{A} \quad (9)$$

$$\text{Slump} = 6.5 W/C - 6.5 \bar{A} - 3.0 \quad (10)$$

TABLE 6  
CONSISTENCY MEASURED BY SLUMP

MIX TYPE	AGGREG. GRADATION	SLUMP <sup>a</sup> (IN.)	
		CRUSHED STONE AGGREG.	GRAVEL AGGREG.
Stiff	Coarse	2.70	2.40
	Normal	1.85	1.15
	Fine	1.05	0.65
Plastic	Coarse	5.90	5.30
	Normal	4.50	3.10
	Fine	2.95	2.10

<sup>a</sup> Average of five tests.

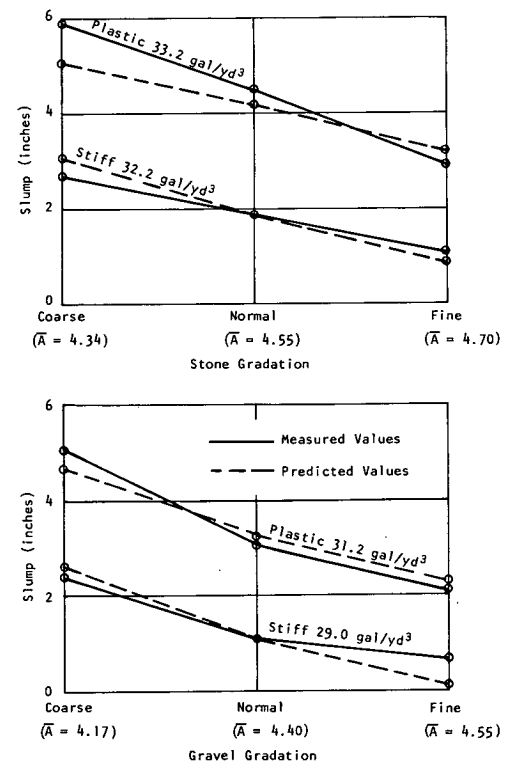


Figure 6. Effect of gradation on slump.



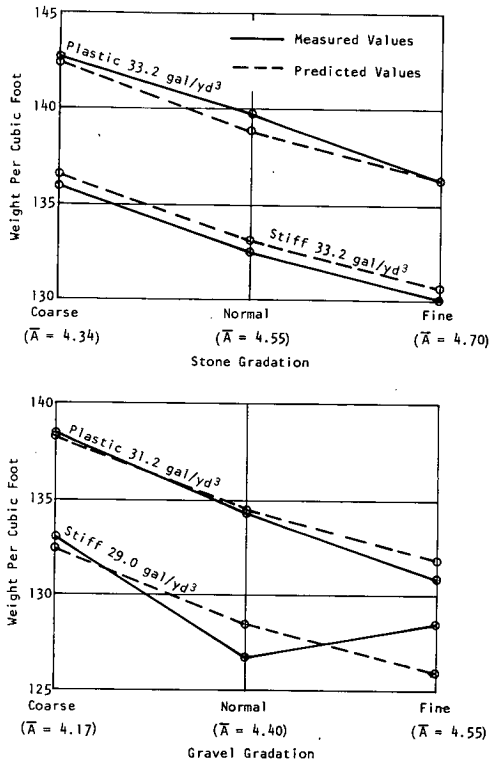


Figure 7. Effect of gradation on uncompact weight.

The average slump of the mixtures of stiff consistency was about  $1\frac{1}{2}$  in. The range of the gradation modulus ( $\bar{A}$ ) was from about 4.2 to 4.7. Substituting these values of  $\bar{A}$  in Eq. 10 results in corresponding W/C values of 4.9 and 5.4. If these pairs of extreme values are substituted in Eq. 9, the predicted 28-day compressive strengths are  $S_{c28} = 3,970 - 829(4.9) + 889(4.2) = 3,640$  and  $S_{c28} = 3,970 - 829(5.4) + 889(4.7) = 3,670$ .

TABLE 8  
SUMMARY OF TESTS OF SIGNIFICANCE

CHARACTERISTIC	F-RATIO <sup>a</sup> DUE TO		
	W/C	$\bar{A}$   W/C	TYPE   W/C + $\bar{A}$
7-Day compr. strength	53	18	< 1
28-Day compr. strength	27	7	< 1
Slump	77	99	9
Loose weight	97	58	< 1
Compacted weight	< 1	< 1	5.36
Unit weight	< 1	< 1	36
Cylinder weight	4	4	19
Percent air	8	< 1	< 1

<sup>a</sup> With 1 and 8 degrees of freedom,  $F = 5.32$  at the 5% level and  $F = 11.26$  at the 1% level.

TABLE 7

WORKABILITY MEASURED BY UNCOMPACTED WEIGHT

MIX TYPE	AGGREG. GRADATION	UNCOMPACTED WEIGHT <sup>a</sup> (PCF)	
		CRUSHED STONE AGGREG.	GRAVEL AGGREG.
Stiff	Coarse	136.3	133.1
	Normal	133.1	126.6
	Fine	130.0	128.5
Plastic	Coarse	142.7	138.5
	Normal	139.7	134.3
	Fine	136.3	130.9

<sup>a</sup> Average of five tests.

Similarly, the average slump of the concrete of plastic consistency was about 4 in. The range of W/C ratio computed from Eq. 10 is 5.2 to 5.8 for the corresponding range of  $\bar{A}$  of from about 4.2 to 4.7. Substituting the extreme values in Eq. 9 gives predicted 28-day strengths of  $S_{c28} = 3,970 - 829(5.2) + 889(4.2) = 3,390$  and  $S_{c28} = 3,970 - 829(5.8) + 889(4.7) = 3,340$ .

As shown in Figure 8, these results indicate that, although changes in gradation do affect both strength and slump, the strength remains substantially constant if the slump is maintained constant by slight adjustments in the mixing water, as is usually the case under actual job conditions.

The three-dimensional drawing (Fig. 8) illustrates the effects indicated by Eqs. 2 and 3. For example, the rear edge of the plane of relationship shows that if the gradation modulus ( $\bar{A}$ ) has a constant value of 4.7, the 28-day compressive strength increases from about 3,420 psi to about 4,090 psi as the water-cement (W/C) ratio is decreased from 5.7 to 4.9 gal per sack of cement. The front edge of the plane shows that when  $\bar{A} = 4.1$  the compressive strength increases from about 2,890 to 3,550 psi over the same range of W/C ratio values. The left edge of the plane shows that the 28-day compressive strength decreases from about 3,420 to 2,890 psi as the increase in the number of large particles in the coarse aggregate corresponds to a change in the value of  $\bar{A}$  from 4.7 to 4.1 when the W/C ratio is held constant at 5.7 gal per sack. Similarly, the right-hand edge of the plane shows that the strength decreases from about 4,090 to 3,550 psi for the same change in gradation when the W/C ratio is 4.9 gal per sack. The effects on 28-day compressive strength of combinations of W/C ratio and  $\bar{A}$ , intermediate to those previously stated, are represented by the surface of the plane, and values for particular combinations can be found by the use of Eq. 2.

When the values of  $\bar{A}$  and W/C ratio resulting in a constant slump, computed by use of Eq. 3, are plotted on the plane of relationship, nearly horizontal lines result.

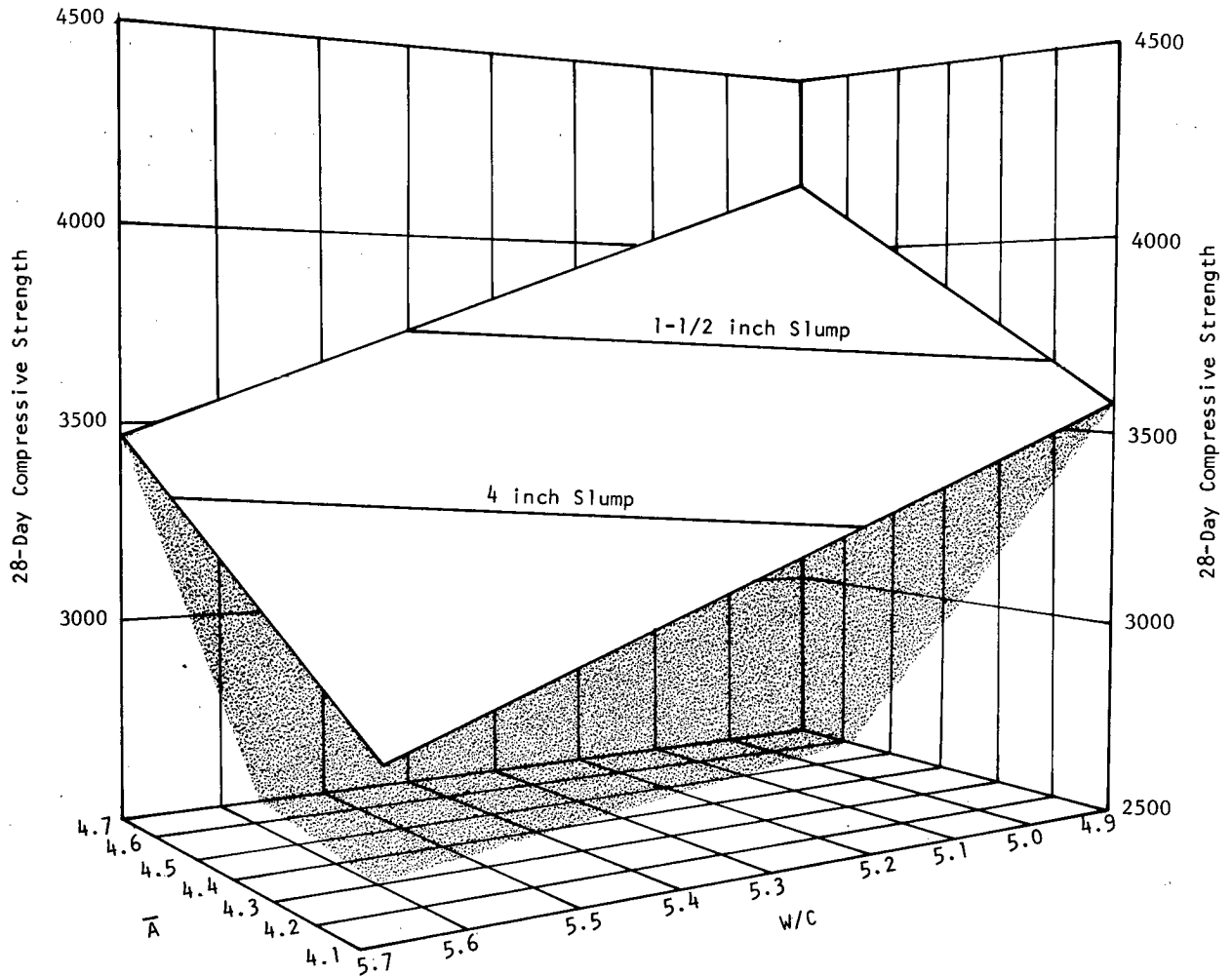


Figure 8. Relationships among gradation modulus ( $\bar{A}$ ), water/cement ratio, and 28-day compressive strength of concrete.

This indicates that over the range of W/C ratio and  $\bar{A}$  investigated in this study, the 28-day compressive strength is not significantly affected by changes in gradation provided the slump is held constant by adjusting the W/C ratio, which in practice is accomplished by small changes in the quantity of mixing water.

### CONCLUSIONS, APPLICATIONS, AND SUGGESTED RESEARCH

#### Conclusions and Applications

Analyses of the data obtained under the conditions of the designed experiment described in this report indicate that:

1. Changes in the gradation of coarse aggregate as commonly used in structural concrete, over the range included in the research study, do not significantly affect the 7- or 28-day compressive strength of the resulting portland cement concrete, providing the slump is held constant by the practice of making small adjustments in the quantity of mixing water.

2. Unit weight, cylinder weight, and the percentage of entrained air are not significantly affected by variations in the coarse aggregate gradation.

3. Specification limits for gradation of coarse aggregate for use in structures should be broadened, if necessary, to accommodate the actual variations in gradation indicated by unbiased random sampling at the point of proportioning into the batch. These limits should apply only when a statistically defensible acceptance sampling plan is employed to define the actual variations of gradation at the same sampling point.

4. Economies in inspection and testing can be effected by choice of realistic specification limits for the gradation of coarse aggregate for use in concrete for structures, provided the sampling plan is designed with regard to the minor criticality of this characteristic.

5. Unnecessary increases in cost can be avoided if requirements for mechanical controls of gradation of coarse aggregate for concrete (re-screening) are not included in specifications.

6. The uncompacted weight test for workability of

concrete is highly correlated with the slump test. Because it is less influenced by the operator, it may have value as a laboratory method of measuring relative workability of concrete.

#### Suggested Research

Based on the findings of this study it is recommended that parallel research be conducted to determine the actual effects of variations in gradation of coarse aggregate used

in the construction of concrete pavements. Such research should be conducted using gradations containing particles of up to 2½-in. maximum size, and should include observations of the workability and finishing characteristics of the concrete mixtures as well as laboratory tests. The findings should have direct application to the drafting of specification requirements for methods of handling, stockpiling, and proportioning coarse aggregates for use in concrete pavement construction.

### CHAPTER THREE

## HOT-BIN GRADATION STUDY

#### INTRODUCTION AND RESEARCH APPROACH

As a result of the relatively large variations found in the gradation of aggregates at portland cement concrete plants in the initial phase of this project (published as *NCHRP Report 34*) (6), it was recommended that a similar investigation be conducted at several bituminous hot-mix plants. If the same level of variation were found to exist in the production of hot plant mixes this could have a significant effect on the essential performance characteristics of the resulting bituminous mixtures.

It is generally accepted that variation in gradation of hot aggregates in the bins of an asphalt plant is one of the most important factors in controlling the uniformity of asphaltic paving mixtures. The magnitude of these variations must be known and given due consideration if realistic specification limits and operating tolerances are to be established. Estimates of several related parameters were also determined, such as within-batch variation, range, and skewness.

This research investigation was designed to estimate such parameters by securing samples at critical points in the production stream (cold feed, hot bins, and mixer discharge) of several asphalt plants and making a statistical evaluation of the results of tests on these samples. These results can be used as a guide for establishment of control limits for bituminous plant mix specifications until such time as a broader investigation is made.

Four asphalt batch plants were selected for obtaining samples for analysis. These plants were located in the general vicinity of Raleigh, N. C.; Hickory, N. C.; Baltimore, Md.; and Rion, S. C. The plants included two manufactured by Hetherington and Berner, one by Cedarapids, and one by Barber-Greene Company. Throughout this report the plants are identified as Plant No. 1, Plant No. 2, Plant No. 3 and Plant No. 4—listed simply in the order in which they were sampled.

The general sampling approach was the same at each of the four plants. The first series of samples was obtained

from the cold-feed belt leading from the discharge of the cold-feed bins. At this point the aggregates were theoretically blended in the proportions which would produce the required job mix formula. At three of the four plants the belt was stopped and a section of the aggregate was removed for the total depth of the layer. At one plant the belt could not be stopped; therefore, samples were obtained by passing a container (pan) through the belt discharge stream as the material was being dumped into the bucket elevator feeding the drier.

The second sampling point was the hot-bin discharge. At this location the aggregate had been dried, brought to mixing temperature, and screened into various size ranges for reportioning in accordance with the job mix formula. At two of the plants the aggregate was screened into three hot-bins and at two plants into four hot-bins. The hot aggregate was discharged from these bins directly into the pugmill mixer and samples were obtained by cutting this stream of flowing aggregate with some type of sampling tool. In one case (Plant No. 1), a special device suggested by The Asphalt Institute was used so that duplicate test portions could be obtained at once by sliding the sampling pans together into the stream. Due to bin arrangement and configuration, this device could not be used at the other three plants. Plant No. 2 was equipped with an automatic hot-bin sampling device. In this case pans were pushed under the hot-bins immediately preceding discharge. When they were filled they would automatically return to the exit port of the protective cover surrounding the hot-bin assembly. At one plant (No. 3) a specially made "dolly" mounted on wheels was used. This dolly rode a track extending underneath the bins and had a trapdoor in the bottom which, when opened, discharged the material into a chute leading to a platform at ground level. This setup obviated the necessity of carrying hot samples down the steps leading from the tower. The remaining plant (No. 4) was equipped with standard

sampling pans that were supported by a track extending underneath the hot-bins. In only one of the cases (Plant No. 1) could replicate portions be obtained from a single batch; in the other three cases successive batches were sampled and treated as a single batch for estimation of within-batch variation.

The third sampling point was from the trucks as the completed mix was discharged from the pugmill. Analysis of these samples included a complete extraction test; i.e., the measurement of bitumen content as well as aggregate gradation.

Thus, variability of the aggregates from raw materials through the finished product was recorded.

Prior to beginning the field work, a random sampling plan was developed. It was estimated that 36 replicate samples would provide sufficient data for a gradation accuracy of about  $\pm 1.25\%$  at the 95% confidence level. The sampling program extended over a three-day operating period at each plant with an equal number (12) of replicate samples secured each day. The daily samples were taken on a random time basis insofar as continuity of plant operation would permit. Each plant operator was cautioned against exercising any special control measures that might influence the outcome of results and observation indicated that only normal, routine procedures were followed. All four of the plants were equipped with automatic controls, but only one (Plant No. 2) was operated automatically for batching and mixing.

A comparison of the average gradation of the cold-feed blend, the combined hot-bin samples, and the extracted aggregates is shown in the aggregate grading charts for each plant (see Appendix C). The gradation envelope in each case depicts the  $\pm 2\sigma_0$  limits from the average and will include 95% of the measurements made under similar conditions. The width of the envelope is an indication of the variability—the wider the envelope, the greater the variation in gradation. One convenient feature of these charts is that gradation variability from point to point can be compared very quickly.

The two North Carolina plants, although widely separated geographically, were both producing the same specification type mix, N. C. 1-2 surface course; the Maryland plant was producing Maryland PC-1 surface mixture; and the South Carolina plant was producing S. C. Type 3 binder course. The first three mixes were roughly comparable in gradation, whereas the fourth was considerably coarser. Results from each plant are given later in this chapter.

Samples were bagged at the respective sampling areas, identification tags attached, and the bags transported to the laboratory for analysis. Asphalt extraction tests were performed by a commercial laboratory, but all gradation testing, including that of the extracted aggregate, was performed in the research agency's laboratory at Raleigh, N. C.

Coarse aggregate gradations were performed with a Gilson Shaker and fine aggregate gradations with a Newark Sieve Shak. The latter device is similar to a Ro-Tap and will accommodate a nest of standard 8-in. sieves.

The entire weight of coarse aggregate samples was

sieved in every case; for mixtures of coarse and fine aggregate (cold-feed blend) the entire sample was passed through the Gilson sieves and the minus No. 8 fraction subsequently reduced to 150-200 gm by riffing. The resultant portion was then sieved with the Sieve Shak. Fine aggregate from the No. 1 hot-bin was initially reduced to a convenient test portion size of 150-200 gm and the remainder saved for further testing as required. The raw gradation data (amount retained on each sieve size) were entered on IBM cards for data analysis.

Prior to starting gradation testing, varying periods of shaking time were evaluated to determine the optimum time for screening to refusal. All testing was performed on the basis of "dry shakes" except for a special series of dry sieving versus washed analysis made on several randomly selected bin No. 1 samples from each of the four plants.

In certain types of aggregates the very fine particles have a tendency to cling to the larger particles or to conglomerate into balls that give the appearance of coarse aggregate particles. With this type of material a dry screen analysis does not always present a true gradation picture, because some portion of these fines will not pass through the No. 200 sieve. Aggregates containing clay are particularly susceptible to misleading results unless a washed analysis is performed.

Because the aggregates involved in the plant study contained a portion of local sand, it was not known initially whether there would be any significant difference between results from dry sieving and washed analysis. Practical limits of time and available funds precluded washing every sample, so a random selection was made of a representative portion of the material from bin No. 1 from each plant. If a significant loss were indicated, a correction factor could be applied to each test to adjust for the loss of fines.

Results of these tests are presented in Table C-1. The screen size indicating the greatest change is the passing No. 200 and this average increase amounts to less than 1%. A correction factor was calculated by dividing the percentage of minus 200 determined on a dry basis by the indicated percentage on a washed basis. An over-all average value of about 0.9 was obtained, which means that about 90% of the minus No. 200 went through the sieve in a dry state. Gradation changes in the sizes above No. 200 were very minor and average only about 0.36%.

It was therefore concluded that the time required for a correction for each test would not be justified.

The component of the total variance of the results of gradation tests and aggregates due to the test procedure is identified as testing error and given the symbol  $\sigma_t^2$  in this report. Although differences in test results may be due in part to segregation of particles during the part of the test procedure involving preparation of the test portion, such as splitting or quartering, this source of variation is not included herein as part of the testing error. Also not included are such sources of variation as differences in sieving efficiency and actual errors, such as the loss of aggregate particles from the test portion during testing, inaccurate weighing of groups of separated particles, or incorrect observations or calculations.

Aggregate particles are usually of irregular shape. Dur-

ing one test they may be favorably positioned for passing through a sieve opening, whereas during another test the same particles may not be so oriented. With some types of shaking equipment, particles that have passed through the openings of one sieve may even return to that sieve after prolonged shaking. As used in this report,  $\sigma_t^2$  is the within-test variance of the gradation test as affected by these random causes, and a measure of the repeatability of the gradation test using the *same* test portion, the same equipment, and with the same operator. It is computed from

$$\sigma_t^2 = \frac{\Sigma(X_1 - X_2)^2}{2n} \quad (11)$$

in which

- $\sigma_t^2$  = variance due to lack of repeatability of the test (i.e., testing variance);
- $X_1$  = result of first test on test portion;
- $X_2$  = result of second test on same portion; and
- $n$  = number of test portions (two measurements or tests were made on each test portion).

Because some aggregates are subject to degradation during sieving,  $\sigma_t^2$  was determined by retesting randomly selected test portions one time only, rather than by making multiple tests on the same test portion. A total of 85 retests were made on aggregates from each of the four hot-plant bins with samples being distributed among the four plants investigated. The tests were all made by the same technician, in the same laboratory, using the same sieving equipment. The tests on plus No. 8 aggregate were made using the Gilson Shaker and those on minus No. 8 aggregate were made using the Newark Sieve Shak.

The results of these tests are given in Table 9. Because one measure of repeatability is the difference two-sigma limits ( $\pm 2\sqrt{2}\sigma$ ), these results are shown in terms of the standard deviation ( $\sigma_t$ ) of the percentages passing the sieves, rather than in terms of the variance ( $\sigma_t^2$ ).

These results are of about the same order of magnitude as the values determined for testing error in the previous NCHRP Project 10-3/1 (published as *NCHRP Report 46*) (11).

TABLE 9  
SUMMARY OF RESULTS OF REPEATABILITY TESTS<sup>a</sup>

SIEVE SIZE	TESTING ERROR, $\sigma_t$
1 in.	1.0
¾ in.	0.8
¾ in.	0.2
No. 4	0.3
No. 8	0.4
No. 16	0.5
No. 30	0.4
No. 50	0.3
No. 100	0.2
No. 200	0.1

<sup>a</sup> Based on total percent passing.

A computer program, written for use in a previous NCHRP project, was employed for calculation of the required statistical parameters. A supplement to the program has been written which combines the gradations of the contents of the hot-bins in proportion to the weight drawn from each bin. This combined gradation is used as the basis of comparison with the (target) job mix formula. The raw gradation data, consisting of weights of aggregates retained on each of the selected sieve sizes, was punched into IBM cards for processing on a 1410 computer. The actual card punching and data processing was handled by the computer section of the North Carolina State University at Raleigh under the general supervision of Dr. Arnold Grandage.

The print-out sheets include a tabulation of the weight on each sieve and the gradation on both a passing-retained basis and a total percent passing basis. Following each group of data a summary sheet is provided giving the various parameters previously discussed. These summary sheets have served as a basis for most of the tables, charts, and graphs presented in this hot-bin study portion of the report.

The use of this program resulted in considerable savings of both time and money, as well as minimizing the possibility of error in the myriad of mathematical calculations required for determination of the desired values.

A copy of the complete computer print-out has been furnished NCHRP headquarters and other copies are on file with the research agency for those who may wish to examine the data in their entirety. A copy of a typical data summary sheet is shown in Figure 9. Sieve numbers refer to the sequence of sieve sizes used to define each gradation.

## FINDINGS

Results of the sampling program conducted at each plant are given in the following sections. Analyses of gradation test data are presented in summary tables and are also shown graphically for purposes of comparison.

The data contained in the tables are a summary of the average values determined at each sampling point for gradation, over-all standard deviation ( $\sigma_o$ ), within-batch standard deviation ( $\sigma_b$ ), and between-batch standard deviation ( $\sigma_t$ ). These values provide the basic information for determining the uniformity of production and the degree of compliance with specifications.

### Plant No. 1

#### Description

The first plant sampled was a 4,000-lb manually operated batch plant located near Raleigh, N. C. The bituminous mixture being produced during this period of study was designed to meet North Carolina Standard Specifications for Type I-2 surface course (100% passing ½-in. sieve) and involved crushed-stone coarse aggregate, stone screenings, and local sand. These aggregates were separately loaded into cold-feed bins, from which they were dropped onto a conveyor belt terminating at a bucket elevator charging the drier. It was not practical to stop the belt at

Average  $\bar{A} = 3.993$   
 Variance of  $\bar{A} = .0775$   
 Standard Deviation of  $\bar{A} = .2785$   
 Number of Tests = 72

Sieve Number	Average Percent Passing	Variance	Standard Deviation	Coefficient of Variation
1	100.00	.000	.000	.00
2	100.00	.000	.000	.00
3	88.18	5.277	2.297	2.60
4	59.42	41.675	6.455	10.86
5	45.73	34.140	5.842	12.77
6	36.47	24.392	4.938	13.53
7	30.48	16.867	4.106	13.47
8	19.36	6.885	2.623	13.55
9	10.15	1.825	1.351	13.30
Pan	4.77	.769	.877	18.38

Sieve Number	Maximum	Minimum	Range	Skewness	Kurtosis
1	.0	.0	.0	.00	.00
2	100.0	100.0	.0	.00	.00
3	95.3	82.4	12.8	.26	.49
4	78.9	43.9	34.9	.52	.59
5	63.0	32.4	30.5	.52	.48
6	50.1	27.1	23.0	.49	.15
7	41.1	22.5	18.6	.45	.09
8	25.9	14.1	11.8	.44	.04
9	13.4	7.1	6.2	.33	-.06
Pan	8.1	3.3	4.8	1.35	3.35

Sieve Number	Within Batch Variance	Within Batch Standard Deviation
1	.000	.000
2	.000	.000
3	3.191	1.786
4	24.129	4.912
5	18.416	4.291
6	14.203	3.768
7	10.151	3.186
8	4.458	2.111
9	1.272	1.128
Pan	.871	.933

Within Batch Variance of  $\bar{A} = .0462$   
 Within Batch Standard Deviation of  $\bar{A} = .2149$

Plant No. 3 - Cold Feed Blend

Figure 9. Typical data summary sheet.

this plant; therefore, replicate samples were not taken, and determination of within-batch variance on the cold-feed blended material was not made.

After passing through the drier, the aggregate was screened over a triple-deck screen arrangement such that bin No. 1 contained aggregate passing a  $\frac{5}{32}$ -in. screen; bin No. 2 contained aggregate passing  $\frac{1}{4}$ -in. and retained on  $\frac{5}{32}$ -in.; and bin No. 3 contained aggregate passing  $\frac{1}{2}$ -in.

and retained on  $\frac{1}{4}$ -in. The batch proportions were 74% of the total aggregate from bin No. 1 and 13% from each of the other two bins.

The plant was equipped with a hot-bin sampling pan, supported by metal tracks extending underneath the entire length of the hot bins. To provide replicate portions from the same batch, two special sampling devices were made (Fig. 10), which fitted into the standard sampling pan.





Figure 10. Asphalt Institute sampling device.

The sampling tools conformed to The Asphalt Institute design (7). This arrangement permitted a flowing stream of aggregate to be cut into two parts for determination of within-batch variation.

Samples were bagged and identification tags attached immediately upon collection, and subsequently transported to the research agency laboratory for analysis.

#### Findings

The parameters obtained from analysis of tests on samples from Plant No. 1 are given in Table 10. The relative amount of indicated variation in gradation at the various sampling points is shown graphically in Figures 11, 12, and 13.

#### Plant No. 2

##### Description

The second plant sampled was an 8,000-lb batch plant operated on a fully automatic basis. The bituminous mixture being produced during the sampling period was designed to meet North Carolina Standard Specifications for Type I-2 surface course. The plant was equipped with six cold-feed bins; however, only three were used in the blending of crushed-stone coarse aggregate, stone screenings, and local sand. The blend was controlled by varying the speed of the short feeder belts immediately beneath each bin, which discharged onto the 170-ft conveyor belt leading to the drier.

After passing through the drier the hot aggregate was discharged onto a 1¼-in. scalping screen and then through a ¼-in. screen for bin No. 3 material, a ⅜-in. screen for bin No. 2 material, and a ⅝-in. screen for bin No. 1 material. This screening set-up is similar to that of Plant No. 1, except that slightly larger screens are used over bins No. 2 and 3. Batch proportions were 72% of the total

aggregate from bin No. 1, 15% from bin No. 2, and 13% from bin No. 3.

The plant is equipped with an automatic sampling device for each bin, consisting of pans 9¼ × 9¼ × 5 in., which slide on metal tracks extending beneath each bin. A pan is inserted into the discharge stream from each bin and, when filled, is automatically delivered to the edge of the bin opening. Each pan holds approximately 20 lb of aggregate. The total time required for discharge of aggregate from each bin is only 3 to 5 sec, consequently successive batches were sampled for calculation of within-batch variation because there was not enough time for two samples to be taken from a single discharge.

After all samples were collected they were trucked to the research agency laboratory in Raleigh, N. C., for gradation tests.

#### Findings

The parameters obtained from analysis of tests on samples from Plant No. 2 are given in Table 11. The relative amount of indicated variation in gradation at the various sampling points is shown graphically in Figures 14, 15, and 16.

#### Plant No. 3

##### Description

The third plant sampled was an 8,500-lb batch plant located near Baltimore, Md. Although the plant was equipped with automatic controls, it was manually operated during the sampling program. The bituminous mixture produced during this period of study was designed to meet specification requirements for Maryland PC-1 surface mixture. It was made by blending two coarse aggregates, stone screenings, and natural sand. A cold-feed tunnel-hopper arrangement permits the blending of up to 12 aggregates on the conveyor belt system that charges the drier.

After passing through the drier, the aggregate was screened over a four-deck screening unit such that bin No. 1 contained material which passed a No. 8 × 2-in. slotted screen; bin No. 2 contained material retained on the bin No. 1 screen and passing a ⅜ × 2-in. slotted screen; bin No. 3 contained material retained on the bin No. 2 screen and passing a ½ × ⅜-in. slotted screen; and bin No. 4 contained material retained on the bin No. 3 screen and passing a ⅝-in. screen. A 1¼-in. scalping screen was used over the entire screen deck assembly. Even though this particular bituminous mixture was only slightly coarser than the two preceding mixtures, the aggregates were separated into four hot-bins for proportioning. Such an arrangement should theoretically provide a greater degree of control with less variability. Batch proportions were 46% of the total aggregate from bin No. 1, 18% from bin No. 2, 20% from bin No. 3, and 16% from bin No. 4.

A unique hot-bin sampling system is used at this plant. It consists of a container, capable of holding approximately 100 lb of aggregate, equipped with metal grooved wheels which fit a track extending beneath the bins (Fig. 17). After a sample is deposited in the container it is rolled to

a position over a chute extending to a platform at ground level, a trap door in the bottom of the container is opened, and the sample flows down the chute into a collection bucket (Fig. 18). This system makes carrying samples down from the mixing tower unnecessary and is the simplest of the methods employed at the four plants. All samples were transported to the research agency laboratory in Raleigh, N. C., for testing.

### Findings

The parameters obtained from analysis of tests on samples from Plant No. 3 are given in Table 12. The relative amount of indicated variation in gradation at the various sampling points is shown graphically in Figures 19, 20, and 21.

### Plant No. 4

#### Description

The fourth and final plant sampled was a 12,000-lb batch plant located near Rion, S. C. As in two of the previous cases, the plant was equipped for automatic operation, but was operated manually. The bituminous mixture sampled was designed to meet 1964 South Carolina Standard Specifications for Type 3 binder course. This mixture is considerably coarser than any of those previously sampled, having a top size of 100% passing the 1½-in. sieve. The mixture is made by blending three coarse aggregates and screenings used as fine aggregates through a cold-feed-bin system.

After passing through the drier the aggregates were separated over a screening unit so that bin No. 1 contained material passing a No. 6 slotted screen; bin No. 2 contained material passing a 5/16-in. screen and retained on a No. 6 screen; bin No. 3 contained material passing a 1/2-in. screen and retained on a 5/16-in. screen; and bin No. 4 contained material passing a 7/8-in. screen and retained on a 1/2-in. screen. A split scalping screen consisting of 1¼-in. and 1¾-in. screen cloth was used. The combined aggregate grading was obtained by drawing 42.8% from bin No. 1, 9.6% from bin No. 2, 14.0% from bin No. 3, and 33.6% from bin No. 4.

This plant is equipped with a built-in hot-bin sampling unit which provides easy access to the bins. It was necessary to sample successive batches for determination of within-batch variation; the hot-bin discharge was so rapid that there was time for only one sample per batch. The pan shown in Figure 22 was used for securing the sample.

All samples were transported to the research agency laboratory in Raleigh, N. C., for testing.

### Findings

The parameters obtained from analysis of tests on samples from Plant No. 4 are given in Table 13. The relative amount of indicated variation in gradation at the various sampling points is shown graphically in Figures 23, 24, and 25.

### Asphalt Content of Completed Mix

Although the primary objective of the plant sampling program was to study variations in the gradations of the aggregates, it was realized that valuable additional information could be obtained by analyzing samples of completed mix for asphalt content. Accordingly, a supplementary appropriation was requested and authorized for this purpose.

Random samples were obtained from the completed mix immediately after it was dropped from the pugmill into the trucks. The samples were obtained in general accordance with AASHTO T 168 except that only one side of the batch was sampled. Extraction testing was accomplished by the Rotarex method using a test portion of approximately 1,100 gm of the wearing course mixture, and two test portions of 1,100 gm each of the binder mixture. For the binder, the two test portions were combined to produce a single sample for gradation testing. The asphalt extraction test values are given in Appendix C and summarized in Table 14.

### Discussion of Findings

#### General Observations Pertaining to All Plants

This study indicates that most of the variation among gradation test results is due to within-batch variation, whereas actual batch-to-batch variations are slight. It was found that this local segregation within the batch averaged about 92% of the over-all variation. The results were not significantly affected by any of the sampling methods employed. Consequently, the width of the envelope shown on the aggregate gradation charts is largely dependent on the within-batch variance ( $\sigma_b^2$ ) existing at the various sampling points. As shown in the summary tables, the actual batch-to-batch variance ( $\sigma_t^2$ ) is quite small and in most cases is insignificant. This means that results of tests on individual hot-bin samples should be interpreted with the realization that companion or subsequent samples may indicate an entirely different gradation. The most practical solution appears to be to combine samples taken from aggregates representing at least five different batches, split this combined sample into a test portion, and use the results for the purpose of establishing initial mix proportions. Results of successive tests obtained in the same manner should be plotted in control chart form (9) and adjustments in mix proportions made in accordance with trends indicated by this chart rather than on the basis of individual tests.

In the absence of a control chart the degree of accuracy of the indicated average percentage of aggregate passing a given sieve can be calculated by the use of one of the methods given in ASTM E 122, "Choice of Sample Size to Estimate the Average Quality of a Lot or Process."

The results of this study indicate also that under normal operations efficient screening of the aggregate before placing into hot-bins insures against wide variations in the gradation of the aggregate in the completed mix. However, because of the relatively large variation indicated in grada-



TABLE 10  
SUMMARY OF AVERAGES AND STANDARD DEVIATIONS, PLANT NO. 1

SAMPLING POINT	NO. OF SAMPLES, $n$	IDENTIFICATION OF VALUES	¾ IN.	⅜ IN.	NO. 4	NO. 8	NO. 16	NO. 30	NO. 50	NO. 100	NO. 200	$\bar{A}$
Cold feed	36	$\bar{X}$	100.0	99.0	80.0	65.0	54.0	41.0	22.0	10.0	5.1	5.76
		$\sigma_0$		1.1	8.4	9.6	9.2	6.9	3.6	2.4	1.5	
		$\sigma_b$										
		$\sigma_i$										
Bin No. 1	36	$\bar{X}$	100.0	100.0	100.0	84.0	68.0	52.0	29.0	13.0	6.4	6.52
		$\sigma_0$				6.2	8.8	7.9	4.9	2.8	1.6	
		$\sigma_b$				5.6	7.9	8.3	5.7	3.0	1.3	
		$\sigma_i$				2.6	3.9	NS	NS	NS	0.9	
Bin No. 2	36	$\bar{X}$	100.0	100.0	20.0	6.0						3.31
		$\sigma_0$			5.6	3.6						
		$\sigma_b$			3.7	2.4						
		$\sigma_i$			4.2	2.8						
Bin No. 3	36	$\bar{X}$	100.0	95.0	5.0	3.0						3.06
		$\sigma_0$		1.5	1.7	1.5						
		$\sigma_b$		1.4	1.0	0.7						
		$\sigma_i$		NS	1.4	1.4						
Mathematically combined gradation	36	$\bar{X}$	100.0	99.0	77.0	64.0	50.0	39.0	21.0	10.0	4.7	5.65
		$\sigma_0$		0.2	0.8	4.9	6.5	5.9	3.6	2.1	1.2	
		$\sigma_b$		0.2	0.6	4.3	5.8	6.1	4.2	2.2	1.0	
		$\sigma_i$		NS	NS	2.2	2.9	NS	NS	NS	0.6	
Completed mix	24	$\bar{X}$		99.4	85.6	71.0	56.0	42.7	23.8	11.4	5.5	5.95
		$\sigma_0$		0.3	3.6	4.7	5.7	4.5	1.7	0.9	0.6	

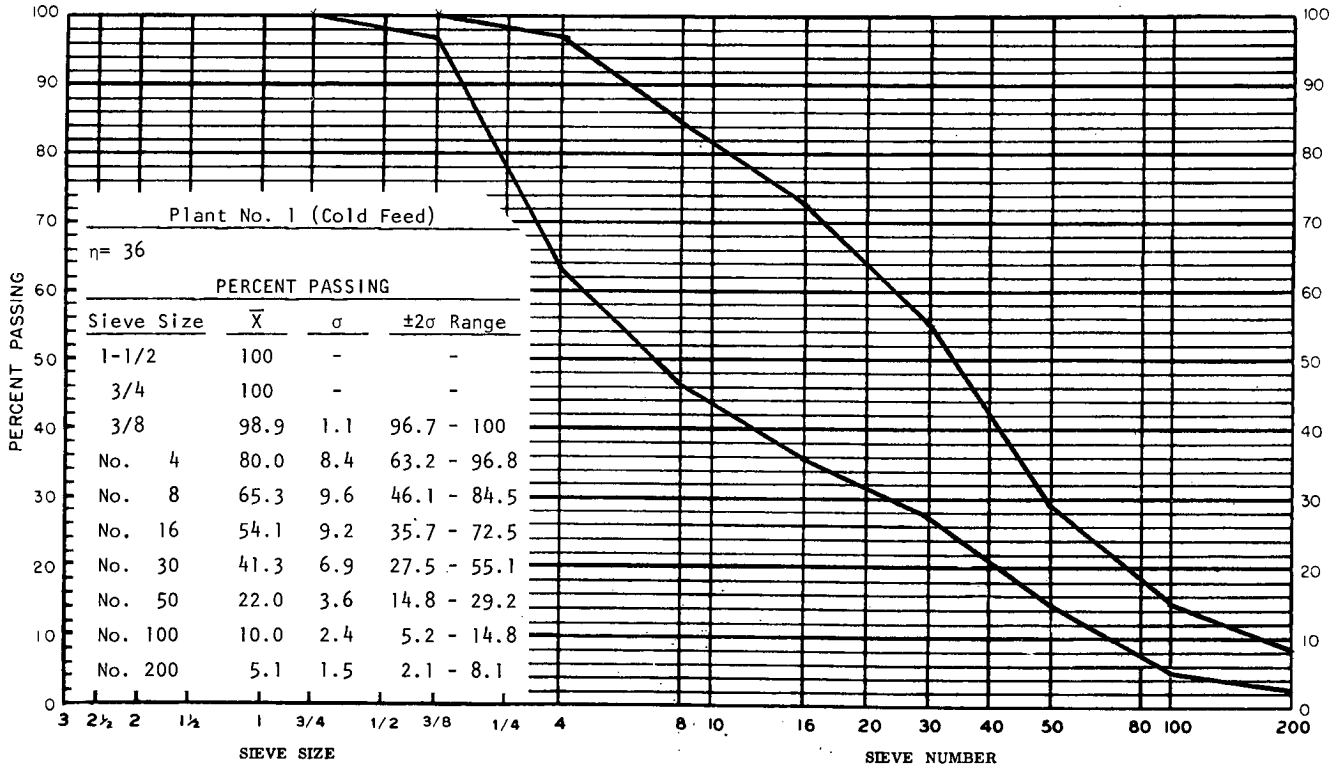


Figure 11. Aggregate gradation, Plant No. 1, cold feed.

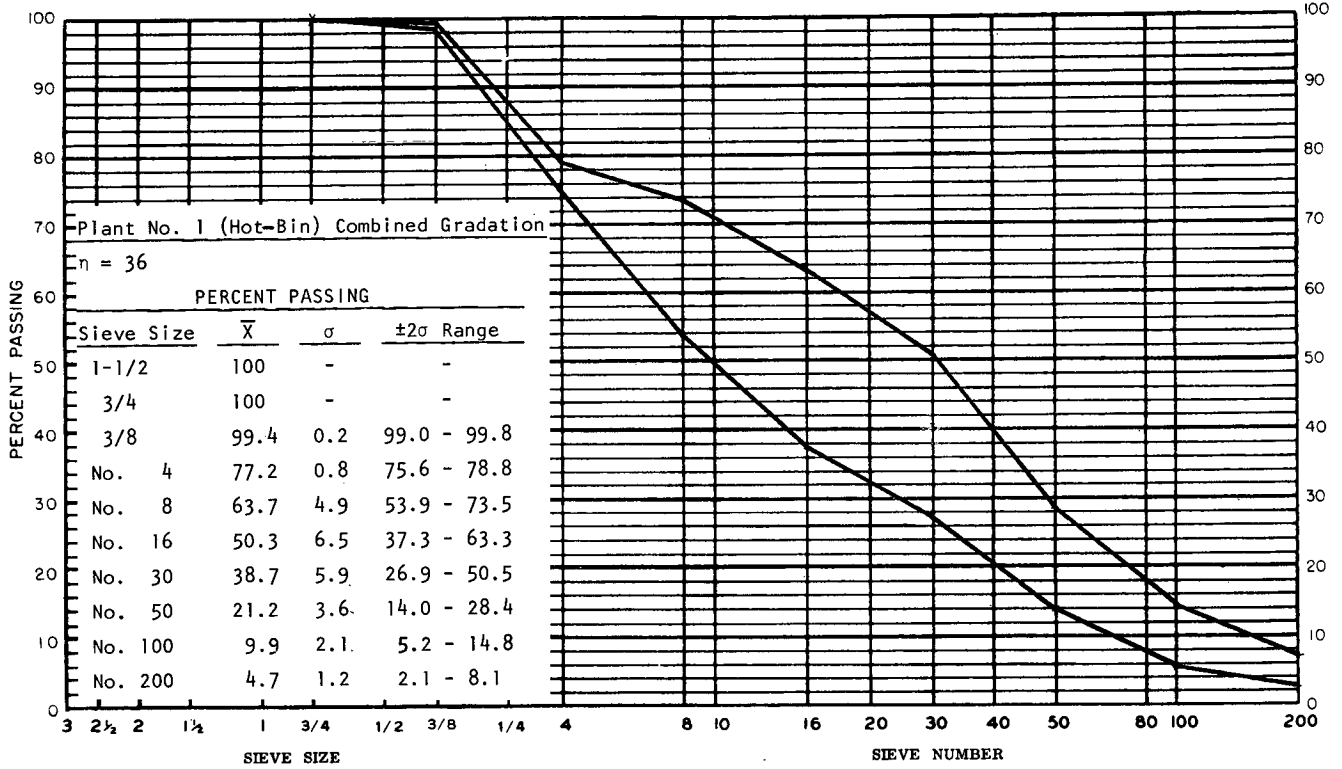


Figure 12. Aggregate gradation, Plant No. 1, hot-bin.

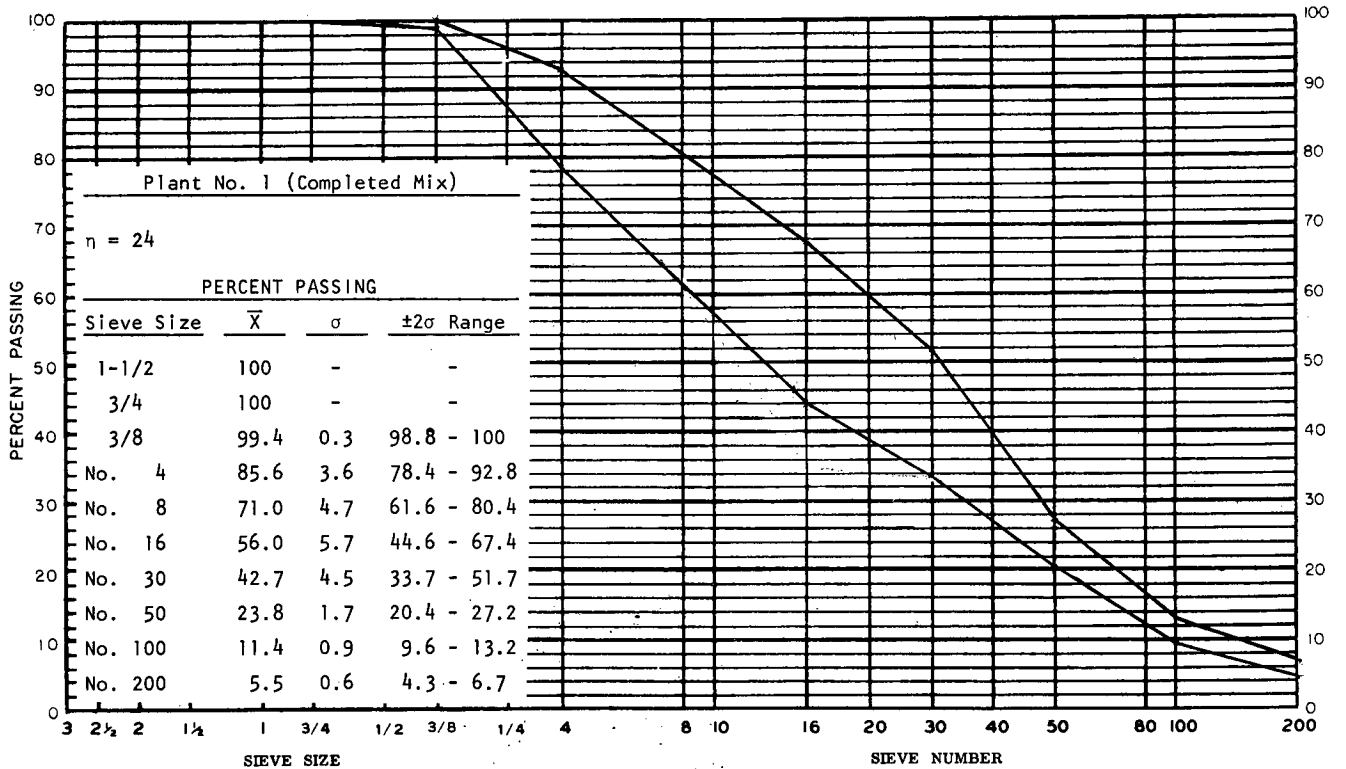


Figure 13. Aggregate gradation, Plant No. 1, completed mix.

TABLE 11  
SUMMARY OF AVERAGES AND STANDARD DEVIATIONS, PLANT NO. 2

SAMPLING POINT	NO. OF SAMPLES, <i>n</i>	IDENTIFICATION OF VALUES	¾ IN.	¾ IN.	NO.	NO.	NO.	NO.	NO.	NO.	NO.	$\bar{A}$
					4	8	16	30	50	100	200	
Cold feed	36	$\bar{X}$	100.0	96.4	81.5	68.4	49.6	31.7	17.3	9.8	5.2	5.60
		$\sigma_o$		0.8	3.9	4.7	5.7	4.3	3.9	2.8	1.5	
		$\sigma_b$		0.6	3.1	3.4	4.5	4.0	3.8	2.8	1.5	
		$\sigma_i$		NS	2.4	3.2	3.4	1.7	1.1	NS	NS	
Bin No. 1	36	$\bar{X}$	100.0	100.0	100.0	88.3	63.3	41.1	23.3	13.4	7.0	6.36
		$\sigma_o$				4.2	7.8	6.4	4.5	3.0	1.9	
		$\sigma_b$				3.8	7.8	6.6	4.3	2.6	1.5	
		$\sigma_i$				1.8	NS	NS	1.4	1.6	1.1	
Bin No. 2	36	$\bar{X}$	100.0	99.3	42.1	9.2						3.56
		$\sigma_o$		1.7	13.3	5.0						
		$\sigma_b$		0.5	4.9	2.5						
		$\sigma_i$		1.7	12.4	4.3						
Bin No. 3	36	$\bar{X}$	100.0	75.9	9.7	4.8						2.94
		$\sigma_o$		10.7	4.3	1.9						
		$\sigma_b$		6.7	2.1	1.1						
		$\sigma_i$		8.3	3.8	1.6						
Combined gradation	36	$\bar{X}$	100.0	96.8	79.6	65.6	45.6	29.6	16.7	9.7	5.0	5.49
		$\sigma_o$		1.4	2.4	3.3	5.7	4.6	3.2	2.2	1.4	
		$\sigma_b$		0.9	0.8	2.6	5.6	4.8	3.1	1.9	1.1	
		$\sigma_i$		1.0	2.2	2.0	NS	NS	1.0	1.1	0.8	
Completed mix	18	$\bar{X}$	100.0	96.3	83.5	70.9	50.0	32.1	18.1	10.8	5.6	5.67
		$\sigma$		1.7	2.2	2.0	3.3	1.7	1.3	1.1	0.9	

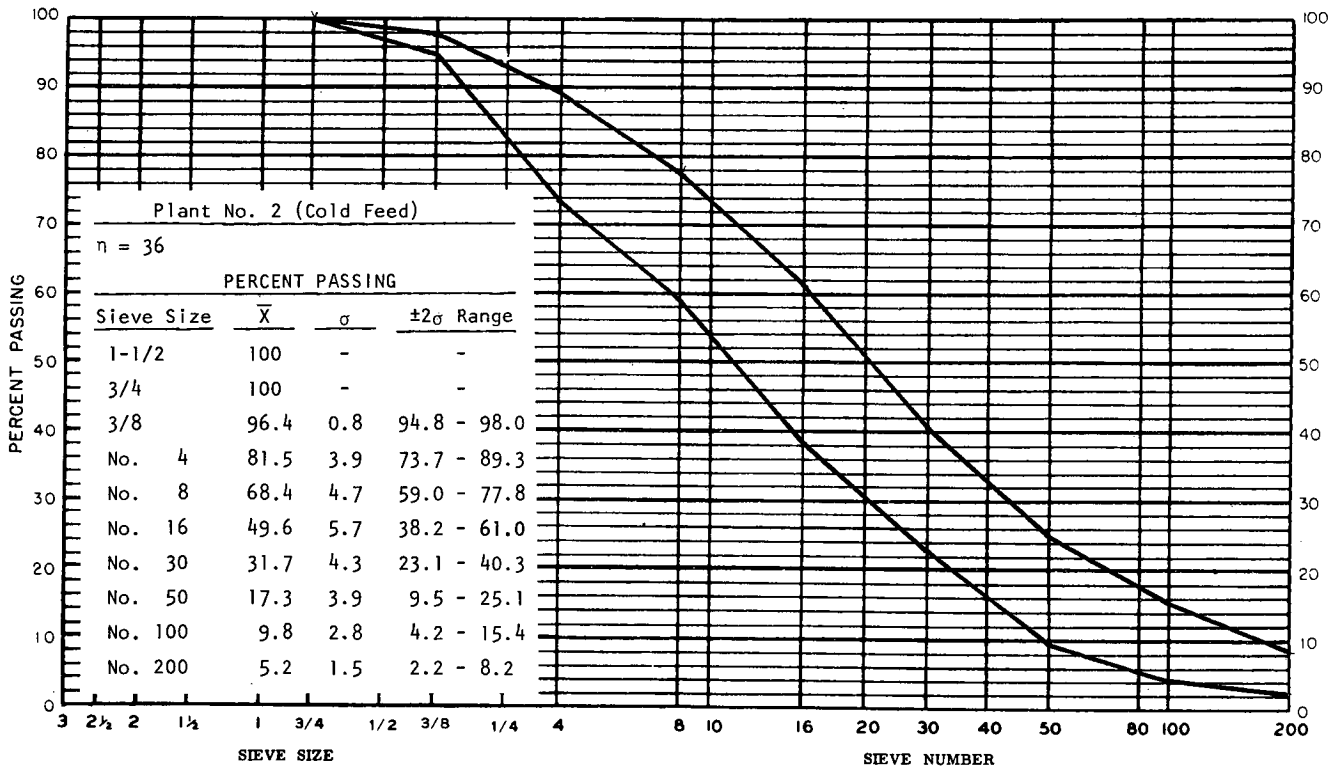


Figure 14. Aggregate gradation, Plant No. 2, cold feed.

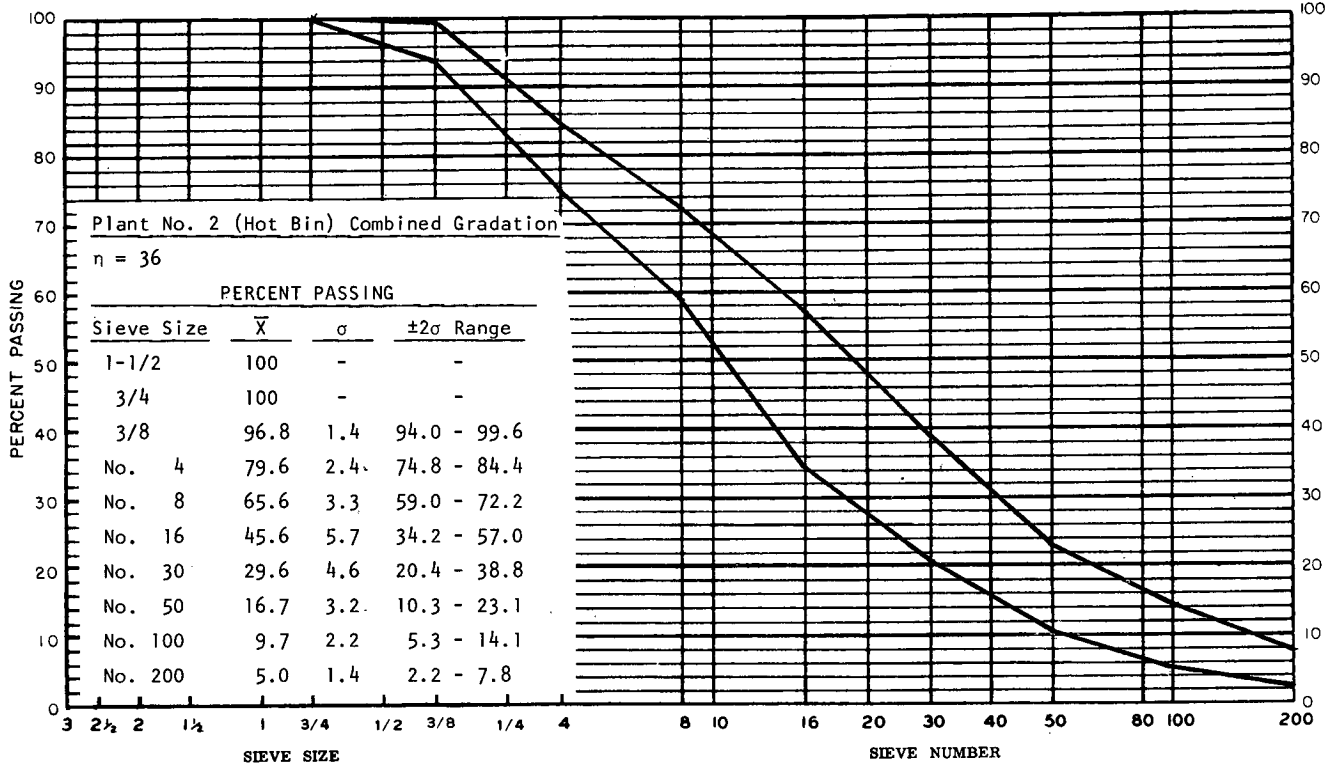


Figure 15. Aggregate gradation, Plant No. 2, hot-bin.

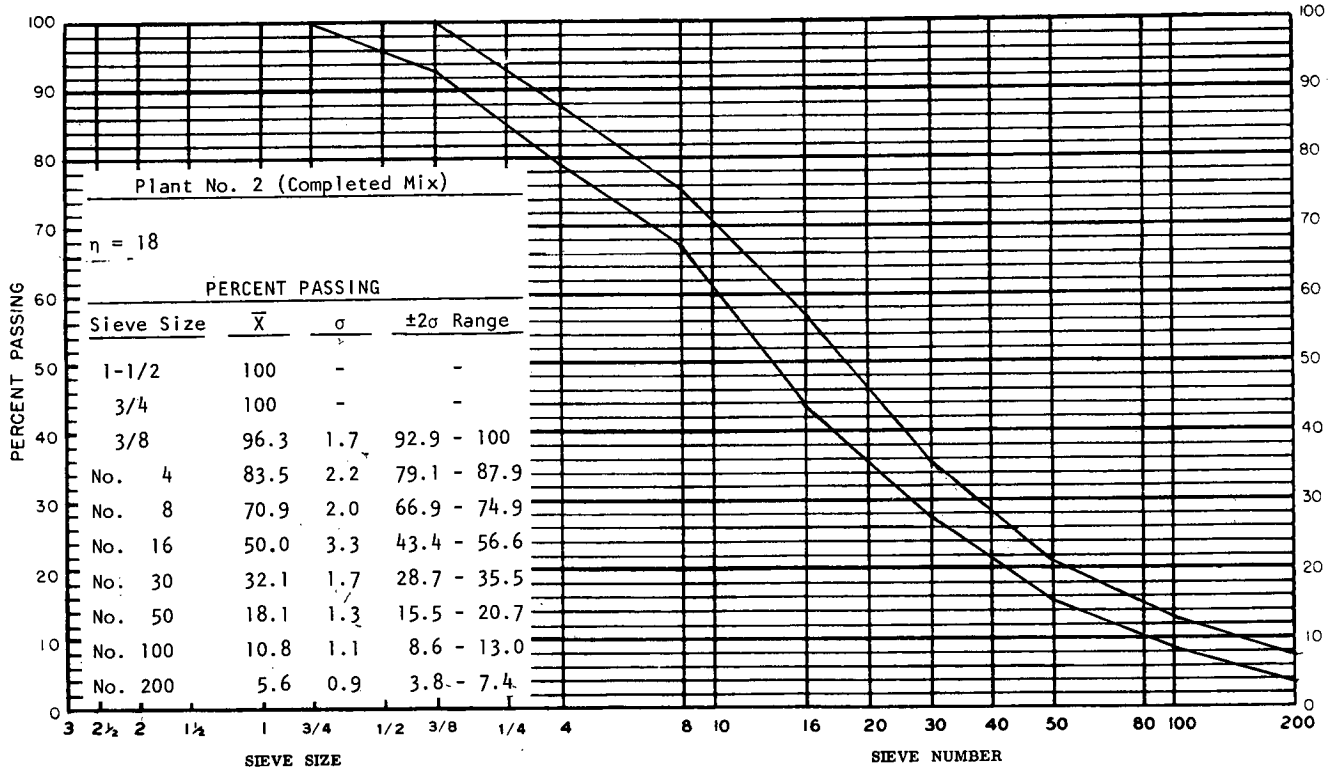


Figure 16. Aggregate gradation, Plant No. 2, completed mix.

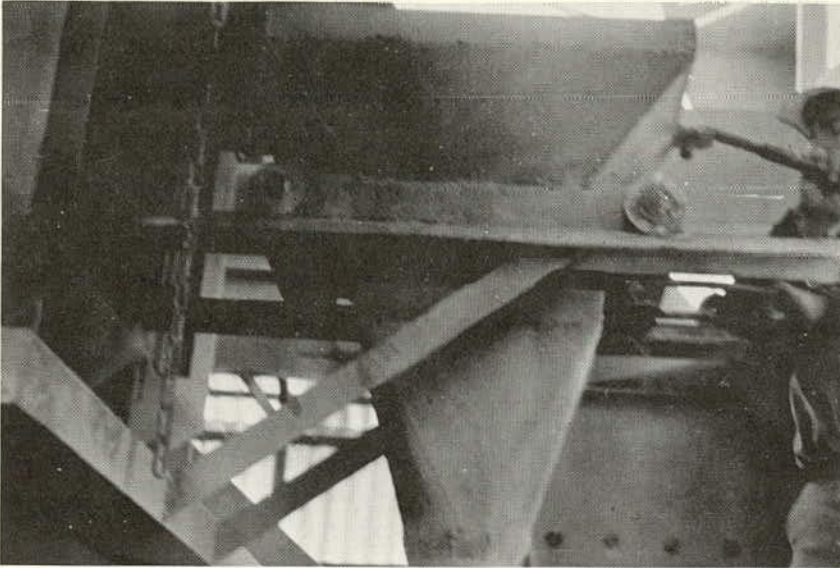


Figure 17. Sampling dolly, Plant No. 3.

tion due to within-batch variation, enforceable acceptance procedures must be based on realistic tolerances of the average of a number of tests (see Chapter Five for discussion of job-mix tolerances).



Figure 18. Sampling chute and containers, Plant No. 3.

#### Evaluation of Controls

Each of the four plants investigated was equipped with automatic controls, but only one was operated on an automatic basis during the study period. All involved reasonably constant high-volume production operations. At each location the same basic sampling scheme (Fig. 26) was employed—the first sample series was taken from the cold-feed belt, representing the cold-aggregate blend; the second series was taken from the discharge of each of the hot-aggregate bins. The results of tests on the second series of samples were used to compute a theoretical gradation by combining the individual bin gradations in the same proportions as were used for the scale settings at the plant. The final sample series was taken from the completed hot-mix after dumping into the transport truck. The results of the gradation tests on these samples are in Tables 10, 11, 12, and 13, and in Figures 11 to 16, 19 to 21, and 23 to 25. In general, it will be noted from the tables that the batch-to-batch standard deviation ( $\sigma_1$ ) is small compared to the within-batch standard deviation ( $\sigma_0$ ), and in many cases the batch-to-batch variation is not significant, as indicated by NS. A comparison of the indicated average gradation test results from samples taken from the cold feed, combined hot-bin samples, and samples of (extracted) completed mix can be made by reference to the series of three aggregate grading charts for each plant. The gradation envelope shown on these charts defines the  $\pm 2\sigma_0$  limits from the average and will include 95% of the individual results of tests made on samples obtained under similar conditions. These limits may be compared to the job-mix tolerances given in Table C-3. Examination of results from each plant indicates the following major points.

*Plant No. 1.*—Table 10 presents a summary of the average values and corresponding standard deviations as

TABLE 12  
SUMMARY OF AVERAGES AND STANDARD DEVIATIONS, PLANT NO. 3

SAMPLING POINT	NO. OF SAMPLES, <i>n</i>	IDENTIFICATION OF VALUES	1½ IN.	¾ IN.	⅜ IN.	NO. 4	NO. 8	NO. 16	NO. 30	NO. 50	NO. 100	NO. 200	$\bar{A}$
Cold feed	36	$\bar{X}$	100.0	100.0	88.2	59.4	45.7	36.5	30.5	19.4	10.2	4.8	4.95
		$\sigma_o$			2.3	6.5	5.8	4.9	4.1	2.6	1.4	0.9	
		$\sigma_b$			1.8	4.9	4.3	3.8	3.2	2.1	1.1	0.9	
		$\sigma_i$			1.4	4.2	4.0	3.2	2.6	1.6	NS	NS	
Bin No. 1	36	$\bar{X}$	100.0	100.0	100.0	100.0	92.9	75.1	62.4	39.6	19.4	8.3	6.98
		$\sigma_o$					3.3	7.2	7.1	5.1	3.3	3.0	
		$\sigma_b$					3.4	7.3	7.4	5.6	3.7	3.2	
		$\sigma_i$					NS	NS	NS	NS	NS	NS	
Bin No. 2	36	$\bar{X}$	100.0	100.0	99.9	65.0	4.1						3.72
		$\sigma_o$			0.2	7.1	1.9						
		$\sigma_b$			0.2	6.2	1.6						
		$\sigma_i$			NS	3.3	1.0						
Bin No. 3	36	$\bar{X}$	100.0	100.0	96.8	3.0	0.9						3.02
		$\sigma_o$			0.9	1.7	0.2						
		$\sigma_b$			0.9	0.7	0.2						
		$\sigma_i$			NS	1.6	NS						
Bin No. 4	36	$\bar{X}$	100.0	100.0	20.9	1.0	0.8						2.23
		$\sigma_o$			3.4	0.3	0.3						
		$\sigma_b$			2.3	0.3	0.3						
		$\sigma_i$			2.5	NS	NS						
Combined gradation	36	$\bar{X}$	100.0	100.0	86.7	58.5	43.8	34.5	28.7	18.2	8.9	3.8	4.83
		$\sigma_o$			0.6	1.3	1.6	3.3	3.3	2.3	1.5	1.4	
		$\sigma_b$			0.4	1.1	1.6	3.4	3.4	2.6	1.7	1.5	
		$\sigma_i$			NS	NS	NS	NS	NS	NS	NS	NS	
Completed mix	24	$\bar{X}$	100.0	100.0	86.8	59.2	44.7	36.4	30.8	20.5	11.3	5.6	4.95
		$\sigma$			2.4	2.5	2.0	1.8	1.8	1.4	1.3	1.2	0.11

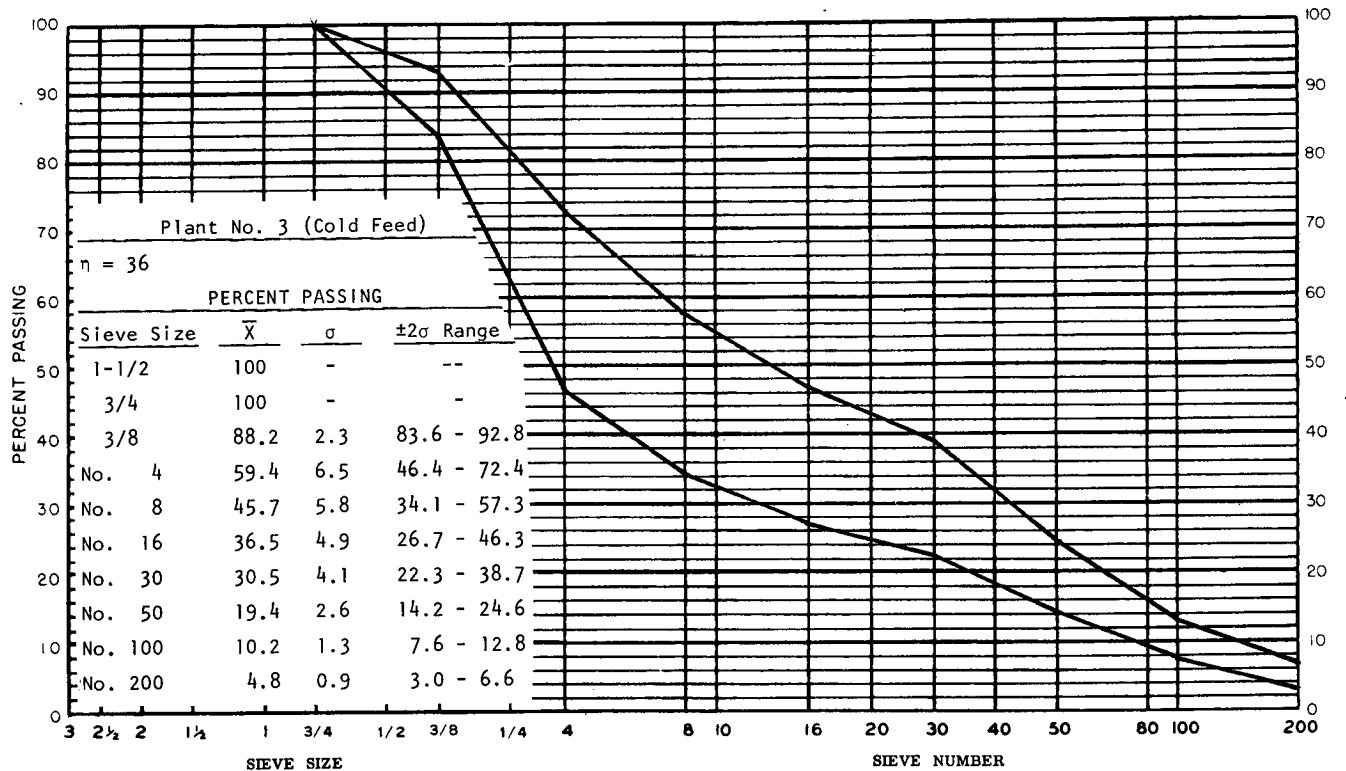


Figure 19. Aggregate gradation, Plant No. 3, cold feed.



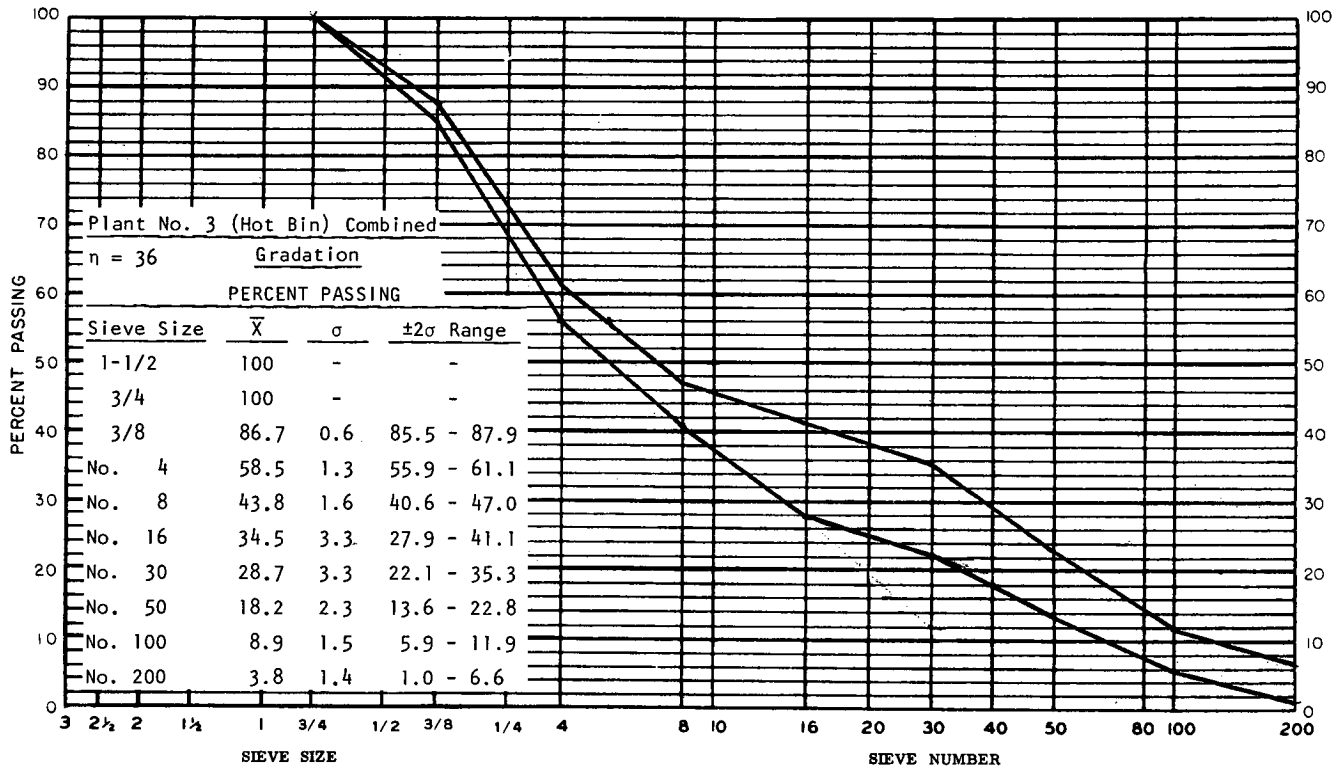


Figure 20. Aggregate gradation, Plant No. 3, hot-bin.

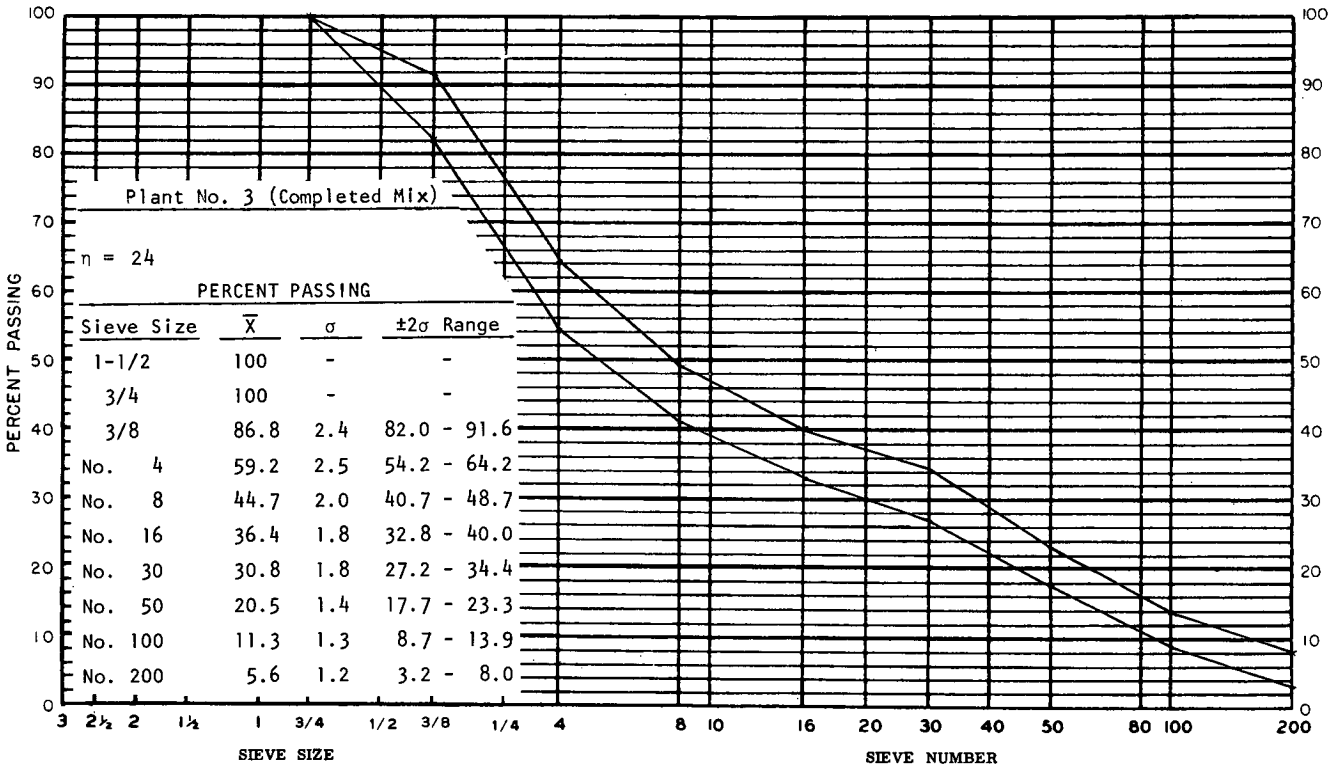


Figure 21. Aggregate gradation, Plant No. 3, completed mix.

sociated with Plant No. 1. The cold-feed gradation was slightly more variable than the theoretical combined hot-bin gradation. The maximum standard deviation (9.6%) occurred in the percentage passing No. 8 sieve for the cold-feed material, closely followed by 9.2% for the material passing the No. 16 sieve. These values decreased to 4.9 and 6.5%, respectively, in the hot-bin combined aggregate and to 4.7 and 5.7%, respectively, in the completed mix. The largest standard deviations found for aggregates in individual hot-bins were: in hot-bin No. 1 a maximum standard deviation of 8.8% was indicated for the aggregate passing the No. 16 sieve; in bin No. 2 a maximum standard deviation of 5.6% was indicated on the material passing the No. 4 sieve; bin No. 3 indicated a maximum standard deviation of 1.7% on the material passing the No. 4 sieve.

*Plant No. 2.*—Table 11 presents a summary of the average values and corresponding standard deviations associated with Plant No. 2. A maximum standard deviation for cold-feed material of 5.7% was found for the material passing the No. 16 sieve. This same sieve size in Plant No. 1 produced a standard deviation of 9.2%. Most of the other values from Plant No. 2 cold feed are also lower than comparable values from Plant No. 1, which tend to indicate a more uniform operation at this plant. This could be due to more uniformity of the raw aggregate gradations, coupled with the use of an electronic control system for proportioning the raw aggregates. Although the hot-bin blended aggregate showed a reduction in standard deviation when compared to cold feed for all sizes (except passing the No. 30) the amount of reduction was not as great as in the case of Plant No. 1. This may be attributed to the very accurate blending of cold-feed aggregates. It appears that the value of hot-screening into separate bins is questionable at this plant because of the small reduction in gradation variability from cold feed to completed mix. This observation must be made with some reservation because the effect of segregation produced by single-bin storage of a very large mass of hot, unsized aggregate has not been evaluated. With aggregates in the size range of minus ½-in., as used here, one might expect this segregation effect to be relatively small.

In bins No. 2 and No. 3, batch-to-batch standard deviation ( $\sigma_t$ ) proved to be considerably larger than within-batch standard deviation ( $\sigma_b$ ), whereas the reverse is true in the case of cold-feed, bin No. 1, and combined hot-bin gradations.

Referring to the aggregate grading charts (Figs. 14, 15, and 16), it can be seen that there is no practical difference between the variability of the cold feed and hot-bin combined gradation.

*Plant No. 3.*—Table 12 presents a summary of the average values and standard deviations associated with Plant No. 3.

The cold-feed blend shows a maximum standard deviation of 6.5% for the material passing the No. 4 sieve, which drops to 1.3% in the combined gradation. The next higher value of 5.8%, found for the No. 8 sieve, drops to 1.6% in the combined gradation. Corresponding drops are shown for most of the other size fractions, which is an indication of the value of screening and reportioning in

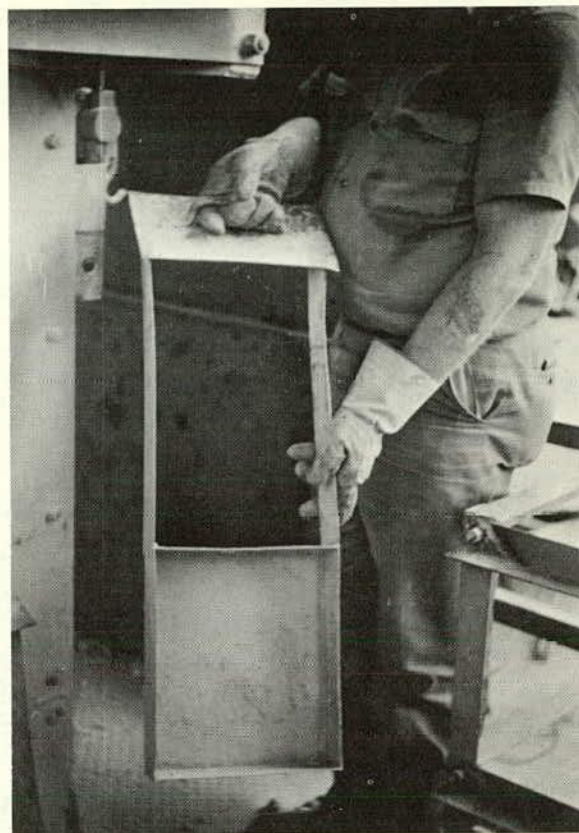


Figure 22. Sampling device used in Plant No. 4.

this case. Four hot-bins were used for storage and reportioning of the aggregates, which resulted in a very narrow range of variation in the mathematically combined gradation mix. Bins No. 2, No. 3, and No. 4 are predominately one-sized material. The within-batch variation accounts for about 80% of the over-all variability. In effect, this means that sampling error is responsible for a substantial portion of the variation indicated by tests on single samples.

The aggregate grading charts (Figs. 19, 20, and 21) show a reduction in gradation variability between the cold-feed blend, the hot-bin combined aggregates, and the completed mix. It can be seen that a significant reduction in variability has occurred in the material retained above the No. 8 sieve. This amount of reduction reflects the use of three hot-bins for separation of aggregate finer than ⅝ in. and retained on the No. 8 sieve. The hot-bin combination was determined by blending 46% from bin No. 1, 18% from bin No. 2, 20% from bin No. 3, and 16% from bin No. 4.

*Plant No. 4.*—Table 13 presents a summary of the average values and standard deviations associated with Plant No. 4.

The highest over-all standard deviation was found to be 9.4% for the cold-feed material passing the ⅜-in. sieve; this value decreased to the surprisingly low level of 0.8% in the combined material, but went to 4.0% in the com-



TABLE 13  
SUMMARY OF AVERAGES AND STANDARD DEVIATIONS, PLANT NO. 4

SAMPLING POINT	NO. OF SAM- PLES, <i>n</i>	IDENTI- FICA- TION OF VALUES	1½ IN.	1 IN.	¾ IN.	⅜ IN.	NO. 4	NO. 8	NO. 16	NO. 30	NO. 50	NO. 100	NO. 200	$\bar{A}$	
			$\bar{X}$	$\sigma_o$	$\sigma_b$	$\sigma_i$	$\bar{X}$	$\sigma_o$	$\sigma_b$	$\sigma_i$	$\bar{X}$	$\sigma_o$	$\sigma_b$		$\sigma_i$
Cold feed	36	$\bar{X}$	100.0	94.8	82.5	62.7	48.7	41.8	33.0	25.0	15.4	7.4	2.7	4.19	
		$\sigma_o$		2.4	7.4	9.4	8.4	7.9	6.1	4.6	2.9	1.5	0.5		
		$\sigma_b$													
		$\sigma_i$													
Bin No. 1	36	$\bar{X}$	100.0	100.0	100.0	100.0	100.0	89.5	70.8	54.1	34.2	17.1	6.5	6.72	
		$\sigma_o$						2.9	4.7	4.2	3.5	3.0	2.3		
		$\sigma_b$						2.6	4.1	3.5	2.7	2.1	1.7		
		$\sigma_i$						1.5	2.3	2.4	2.2	2.1	1.6		
Bin No. 2	36	$\bar{X}$	100.0	100.0	100.0	99.9	30.8	10.1						3.48	
		$\sigma_o$					4.9	4.1							
		$\sigma_b$					2.9	2.5							
		$\sigma_i$					3.9	3.3							
Bin No. 3	36	$\bar{X}$	100.0	100.0	99.9	37.7	2.3	1.5						2.43	
		$\sigma_o$			0.7	5.6	1.2	1.0							
		$\sigma_b$			0.7	4.0	1.1	0.9							
		$\sigma_i$			NS	4.0	NS	NS							
Bin No. 4	36	$\bar{X}$	100.0	87.2	44.5	0.9	0.3	0.3						1.46	
		$\sigma_o$		3.4	4.6	0.4	0.3	0.3							
		$\sigma_b$		3.4	4.0	0.3	0.3	0.2							
		$\sigma_i$		NS	2.2	NS	NS	NS							
Combined gradation	36	$\bar{X}$	100.0	95.7	81.3	58.0	46.1	39.5	30.3	23.1	14.6	7.3	2.8	4.03	
		$\sigma_o$		1.1	1.6	0.8	0.5	1.5	2.0	1.8	1.5	1.3	1.0		
		$\sigma_b$		1.1	1.3	0.6	0.4	1.2	1.8	1.5	1.2	0.9	0.7		
		$\sigma_i$		NS	NS	NS	NS	NS	NS	1.0	NS	NS	0.7		
Combined mix	20	$\bar{X}$	100.0	95.7	84.1	63.0	48.5	41.6	33.1	25.1	15.7	7.8	2.9	4.22	
		$\sigma$		4.4	3.6	4.0	2.7	2.2	1.7	1.2	0.9	0.7	0.3	0.85	

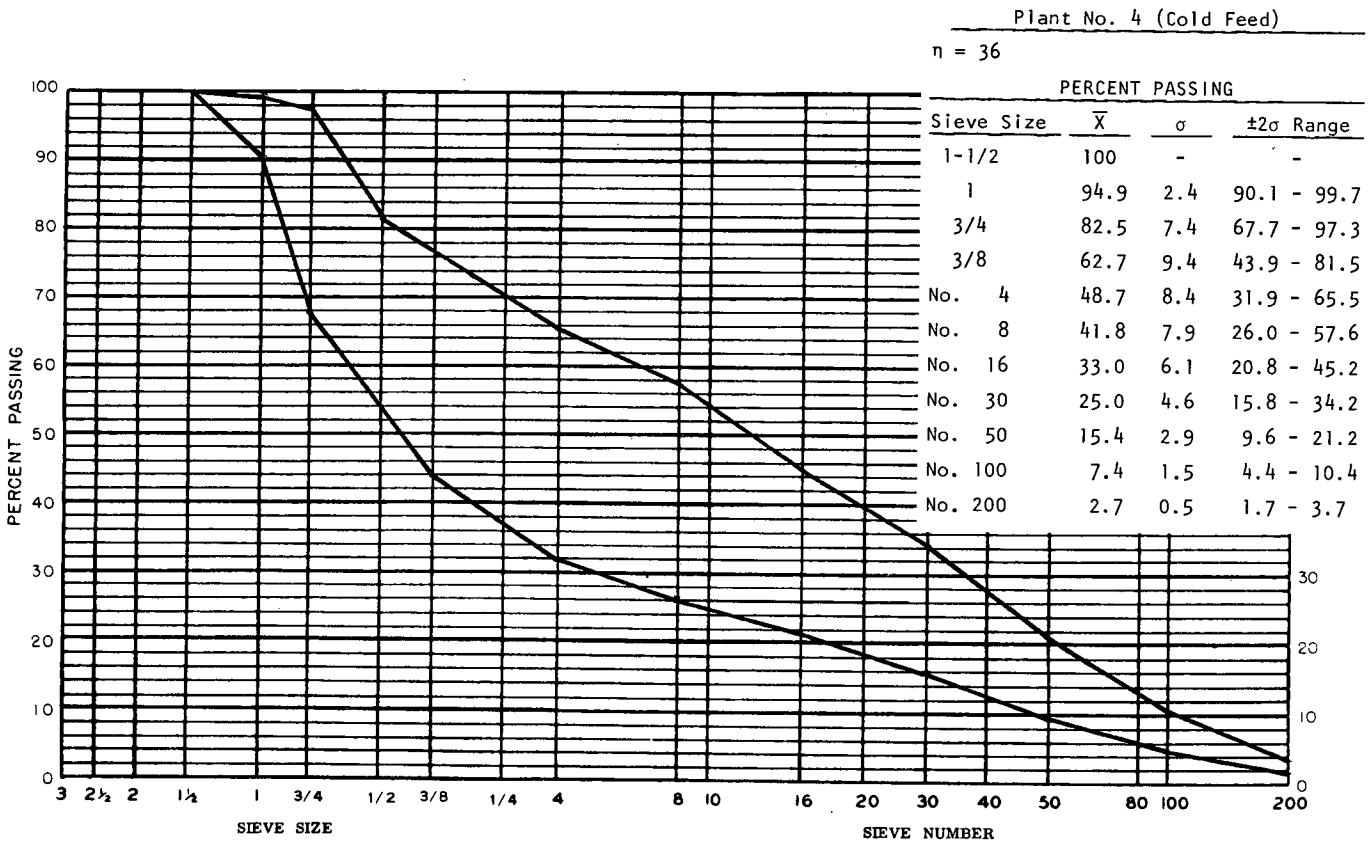


Figure 23. Aggregate gradation, Plant No. 4, cold feed.

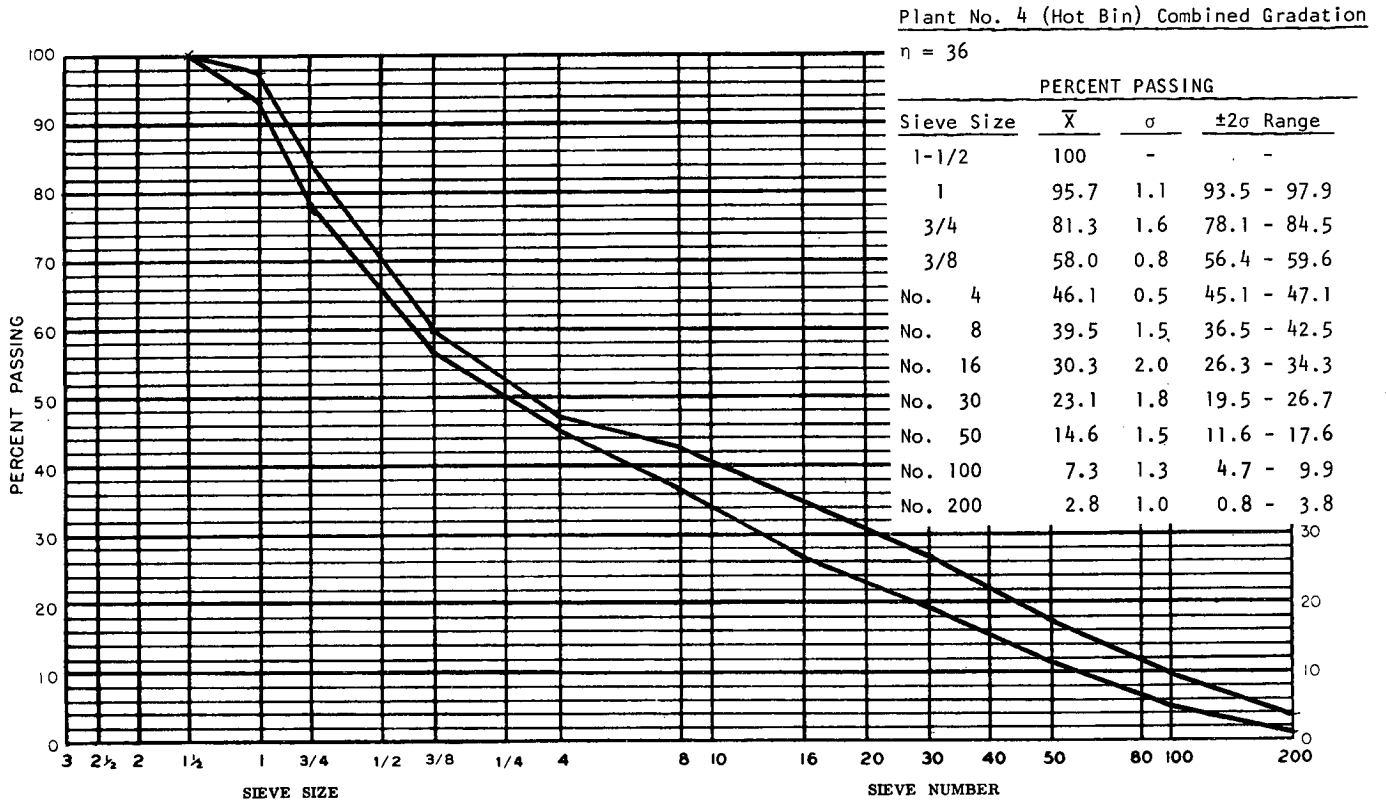


Figure 24. Aggregate gradation, Plant No. 4, hot-bin.

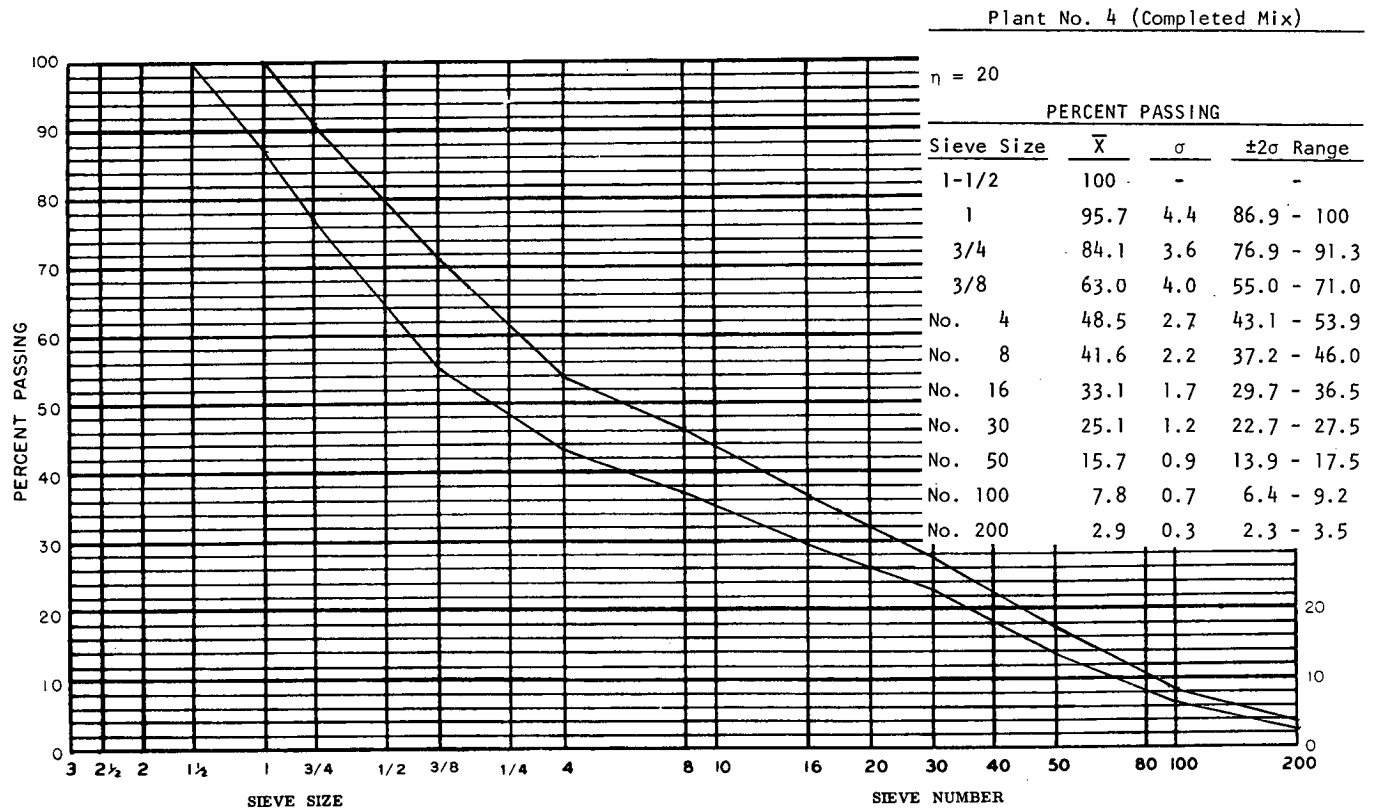


Figure 25. Aggregate gradation, Plant No. 4, completed mix.

TABLE 14  
SUMMARY OF ASPHALT CONTENT OF PLANT SAMPLES BY  
EXTRACTION TEST

PLANT NO.	TYPE OF MIXTURE	ASPHALT CONTENT				
		TARGET VALUE (%)	AVG., $\bar{X}$ (%)	STD. DEV., $\sigma_o$ (%)	RANGE (%)	% WITHIN $\pm 5\%$ OF TARGET VAL.
1	N.C. 1-2 W.S.	7.50	6.90	0.40	5.83-7.57	40.0
2	N.C. 1-2 W.S.	7.00	6.85	0.26	6.06-7.24	90.0
3	Md. PC-1	6.00	6.00	0.26	5.58-6.58	95.0
4	S.C. Type 3 binder course	4.65	5.52	0.84	3.43-7.48	72.0

pleted mix. The next highest cold-feed standard deviation is 8.4% for material passing the No. 4 sieve, which decreased to 0.5% in the combined blend and was 2.7% in the completed mix.

Although there was a high degree of variability of gradation at the cold feed of this plant, the variability of gradation of the aggregates in the completed mix was within acceptable limits, indicating a high degree of screening efficiency.

The large reduction in variability is shown by the aggregate grading charts (Figs. 23, 24, and 25). This plant should have little or no difficulty in keeping within allowable gradation tolerances.

*Asphalt Content.*—The test results given in Table 14 (based on data given in Appendix C) indicate that the

standard deviation of the indicated asphalt content for the wearing course mixtures (Plants No. 1 to 3, inclusive) was within the range of 0.14 to 0.50 found by other investigators (8).

The standard deviation of the indicated asphalt content of the binder mixture (Plant No. 4) was exceptionally large, being about twice the average value reported for similar mixtures (8). Inasmuch as erratic results were anticipated—due to the presence of large coarse-aggregate particles—in the mixture, the tests for determination of asphalt content were made on duplicate test portions from each sample. Analyses of the resulting data indicated a testing error or within-sample standard deviation of 0.44. Applying this correction reduces the standard deviation of

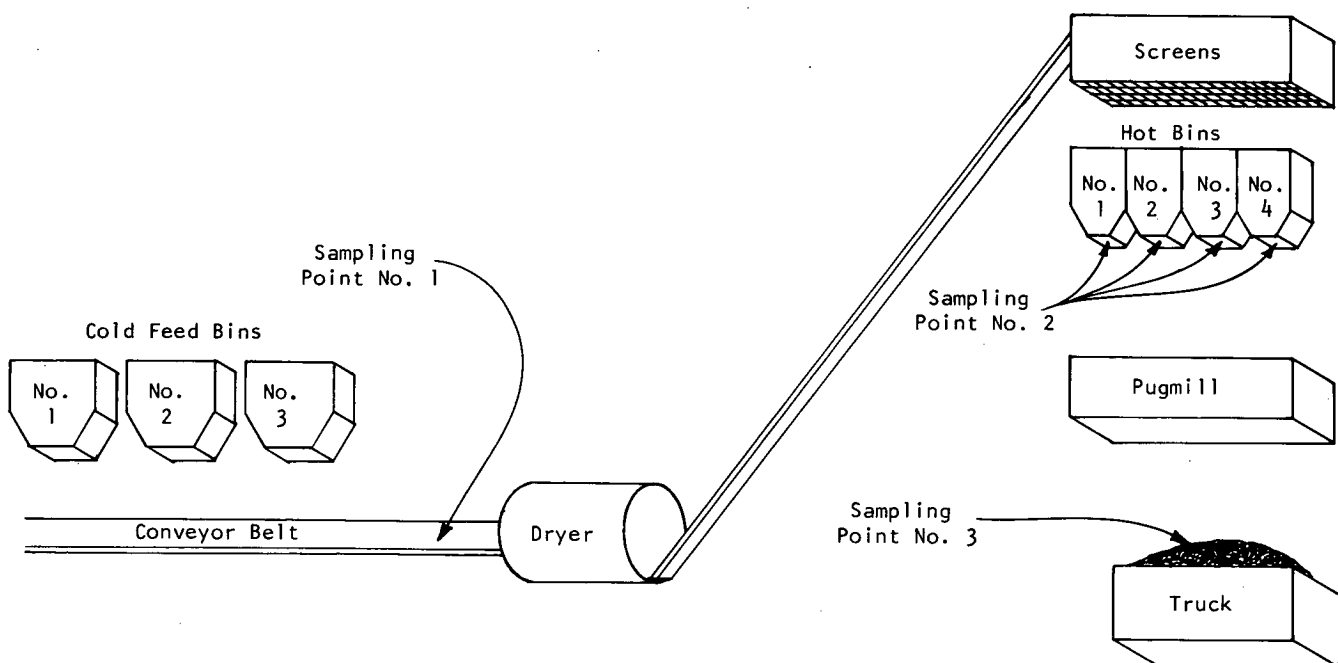


Figure 26. General location of sampling points.

the indicated asphalt content to 0.72. The actual standard deviation was probably much smaller than this value, because considerable sampling error is usually found when sampling mixtures of this type. However, because duplicate samples were not taken from each truckload, it is not possible to estimate the sampling error.

The effectiveness of controls at the four plants is evaluated in Table 14, by comparing the estimates of the percentage of test results that would fall within a range of plus or minus 0.5 percent of asphalt content from the target, or job-mix formula value of asphalt content. These percentages were computed from the normal curve defined by the average and standard deviation computed from the test results.

This comparison indicates that plants where the average actual asphalt content is close to the target value, and where the over-all standard deviation of indicated asphalt content does not exceed the average of 0.27 reported by the BPR (8), can comply substantially with a specification tolerance of plus or minus 0.5 percent of asphalt.

A suggested requirement would be that 80 percent of the test results fall within this range. The fact that up to 20 percent of the test results may be outside the range does not mean that 20 percent of the pavement has an improper asphalt content, this allowance is made to provide for the extreme values of test results due to random variations associated with sampling and testing.

An easy method of estimating the percent within tolerance is described on pages 29-31 of *NCHRP Report 17*. This method can be used when only a relatively few test results are available, and it is not necessary to know, or compute, the standard deviations.

## CONCLUSIONS, APPLICATIONS AND SUGGESTED RESEARCH

### Conclusions and Applications

The findings resulting from this research study indicate the following:

1. Modern plants are capable of controlling the actual gradation of the aggregates in the mixture within very narrow limits.

2. The large indicated variability of gradation of the combined aggregates based on samples taken from individual bins, at some plants, appears to be mainly due to within-batch variance. This means that conclusions based on results of a single sampling are not reliable, because an immediate resampling may indicate a widely different gradation. The use of control charts or quality history charts is recommended as the basis of acceptance decisions or adjustments in proportions.

3. The effect of screening the aggregates into separate bins at most operating asphalt plants appears to reduce the average over-all standard deviation of the percentage passing a sieve to about 37% of the corresponding standard deviation of unscreened aggregates at operating concrete plants.

4. Under normal operating conditions variations in the gradation of the aggregate at the cold feed do not affect variations in the gradation of the coarse aggregates after screening into bins. Variability in gradation of the minus No. 8 fraction of the aggregate at the cold feed does not appear to be directly related to variation in the gradation of this size fraction after screening and binning.

5. The indicated over-all standard deviation of each size aggregate in a calculated combined gradation can be estimated by adding the products of the standard deviation of that size in each bin multiplied by the proportion of the total aggregate taken from the bin.

6. Analysis of extraction test results made in connection with this study confirm the findings of other researchers with respect to the large standard deviation of the indicated asphalt content determined by ASTM D2172—Method A, "Bitumen Content by Centrifuge." It is recommended that acceptance specifications be based on the average of the number of tests necessary to establish required confidence limits, or that acceptance decisions be based on quality history or control charts.

7. Of the several approaches to determining the actual gradation of the aggregate in the completed paving mixture, sampling at the cold feed is not effective because, although fairly well correlated with the combined gradation under normal operating conditions, it will not detect within-plant variations due to assignable causes. These assignable causes result in trends of changes in gradation due to blinding or excessive wear of screens.

Although, in theory, the best indication of the actual gradation should be obtained by sampling the completed mixture and determining the gradation of the aggregate after extraction of the asphalt, this is not the case in practice. As shown by results of tests on the combined mix from each of the four plants, the indicated variation in gradation at this sampling point exceeded the estimate of the actual variation. This variation in the gradation of the extracted aggregate is largely due to the small size of the test portion used for the gradation test, as is discussed in the following chapters.

The most practical sampling point, with respect to obtaining the best estimate of the actual gradation of the aggregate in the completed mixture, appears to be at the discharge of the hot-bins into the weigh hopper. The chief disadvantage is the large sampling error ( $\sigma_b$ ) found at all plants. The size of this standard deviation is an indication of within-batch variation; that is, differences of gradation between small volumes of aggregate in the same bin that will be proportioned into the same batch. These within-batch variations have little, if any, effect on batch-to-batch uniformity, but cause individual test results to indicate false variations in changes in gradation, and may lead to unnecessary and incorrect changes in bin proportions. If it were possible to sample the aggregate flowing from the bin into the weigh hopper in such a way that the sample would be made up of a number of small increments from different parts of total volume, much of the false variation would

be averaged out and a more nearly correct indication of the actual gradation of the aggregate in the bin would be obtained.

Because all of the sampling devices investigated failed to accomplish this averaging, the most practical approach under present conditions appears to be to average gradations obtained by sampling successive batches. This can be accomplished by the use of the recommended quality history charts. Such charts can be constructed by simply plotting the running average of five successive values of

the percentage passing the master sieve for each bin. Decisions as to changes in the proportioning of the aggregates from the various bins can then be more reliably based on this running average.

#### Suggested Research

Based on the findings of this study, research is needed on methods of obtaining samples from hot-bins that will be more nearly representative of the average gradation of the aggregate in a batch.

## CHAPTER FOUR

# STUDY OF EFFECT OF INCREMENT SIZE ON SAMPLING ACCURACY

### INTRODUCTION AND RESEARCH APPROACH

Testing the coarse aggregate for compliance with gradation requirements involves estimating the average gradation and the variations from this average. In addition, it may also be desirable to determine batch-to-batch or unit-to-unit variation, because it is a major potential source of variations in properties of finished products or of construction.

To make a sufficiently accurate estimate of the average gradation of a LOT of aggregate, such as the tonnage batched from a concrete plant in one day, a sample of a certain weight must be passed through the specified sieves. The required weight of this sample depends on both the gradation of the aggregate and the desired accuracy, and may be estimated by the use of the nomograph (Fig. 27) developed during the course of previous work (6). This sample may be made up of a number of test portions, each taken from a batch selected by the use of a table of random numbers. The number of test portions may be simply the number of samplings of a convenient size that will result in the total required weight. However, if an estimate of the batch-to-batch variation is also desired, a minimum number of test portions must also be determined for the predetermined degree of accuracy. The required number of test portions may be found by the use of a nomograph (Fig. 28). Each test portion is made up of a number of increments, such as small scoops taken from different parts of the batch. Although the guidelines previously described are available for estimating total sample weight and number of test portions, there have been no such guidelines for determining the number of increments.

Because within-batch variations in gradation can be expected to be greatly reduced during subsequent mixing, these variations in gradation in different parts of the batch should be averaged out during sampling. This averaging can be mechanically accomplished by taking a large num-

ber of increments; however, the total weight of the increments from a batch should not exceed the proper size of test portion. For example, if the proper test portion weight is about 25 lb and the minimum increment weight is 10 lb, only two increments can be taken unless the combined increments are to be mixed and split or quartered to test portion size.

Previous investigators have been interested chiefly in the sampling of coal and mineral ores and have been concerned with the biasing of results if large particles are excluded from the increment. Various rules of thumb apparently based on judgment have been given, such as that the sampling tool should have an opening at least ten times as wide as the diameter of the largest particle in the mass to be sampled. Increments of from 1 to 6 lb have been specified for sampling coal having a lump size of  $\frac{1}{2}$  to  $2\frac{1}{2}$  in., but this requirement is based on the desired accuracy of ash determination rather than gradation.

The over-all objective of this study was to determine the effect of increment size on variations of the percentages passing each sieve used in determining the gradation of test portions of coarse aggregate. The experiment designed to accomplish this objective involved the use of five metal scoops varying in capacity from about 1 lb to about 18 lb. These scoops were used to sample gravel coarse aggregate of up to 2-in. maximum size. The batches of aggregate to be sampled were mixed in a specially designed revolving bin and samples were taken directly from this bin after mixing. To minimize segregation, gradation containing equal parts of each size fraction were used for the major portion of the work. Results were checked by use of a gradation typical of those found in actual practice.

Two separate testing programs were conducted to achieve the objectives of the study: (1) The multi-increment testing program involved sieve analyses of test portions of about 25 lb each, acquired in increments by

means of scoops of 1-, 3-, and 6½-lb capacity. (2) The single-increment testing program involved sieve analyses of single-scoop test portions using the three scoops used in the multi-increment tests plus scoops of 12- and 18-lb capacity. The multi-increment and single-increment programs were used to evaluate the effect of increment size on sampling accuracy, the primary objective of the study. Additional benefits were achieved by using the single-increment testing data to check the validity of the equation for inherent variance previously developed and to evaluate the adequacy of test portion weights specified by AASHTO and ASTM.

#### Multi-Increment Program

In the multi-increment study, increments of various sizes were taken from a well-mixed mass of aggregate containing known proportions of different sized particles. Three gradations having maximum sizes of 1 in., 1½ in., and 2 in. were used. After mixing, increments were removed from the mass of aggregate by the use of different sized scoops. Sufficient scoopfuls were taken to form test portions of a size required to attain a degree of accuracy sufficient for estimating the variability of gradation which could be attributed to increment size. The number of test portions of aggregate of each gradation was based on the requirement to form a total sample weight sufficient to serve as a basis for estimating bias with the desired degree of accuracy. Statistical methods were used to test the significance of differences among averages of percentages passing the various sieves, and of variations from these averages.

A method for mixing and storing the aggregate batches while minimizing within-batch segregation was developed. The equipment (Fig. 29) consisted of a metal-lined rotating bin, having interior dimensions of 2 x 2 x 4 ft, equipped with flights to increase mixing action and a hinged door so arranged as to provide convenient access for sampling the mixed aggregate.

To define increment size, five scoops, adapted from a design used in a Japanese study (11), were constructed. Dimensions of these scoops are given in Table 15.

Figure 30 shows these scoops in relation to aggregate size. The aggregate used in this experiment was an uncrushed gravel having a Los Angeles abrasion loss of 38 to 40%.

Initial experimentation indicated that a well-graded aggregate should be used to minimize segregation within the mixer box, and that even a slight segregation variance would obscure any differences among increments of different sizes. Accordingly, the first three experimental samplings were made on well-graded aggregate having an equal percentage of each size fraction using scoops 1, 2, and 3. Results of these samplings were later checked by sampling a large maximum-size aggregate having a gradation typical of those commonly employed in concrete production. These four initial gradations are given in Table 16. To determine the extent of possible changes in gradation due to degradation, the entire contents of the bin were re-sieved at the completion of the experiment. These results are shown in parenthesis.

It was found that the most practical batch size for efficient mixing in the revolving bin was about 240 lb. This amount of aggregate, prescreened and recombined to the desired gradation, was placed in the bin and the bin rotated until the contents were thoroughly mixed. The door of the bin was then opened and the aggregate sampled by inserting the designated scoop into the mixed aggregate at about mid-height of the mass in the manner shown in Figure 31.

In this program, the number of scoopfuls taken for each of the 50 gradation determinations was such as to approximate a test portion weight of 25 lb. This amount of aggregate was tested for gradation by shaking for 5 min on the screens of a Gilson sieve shaker. The separated fractions were weighed to the nearest 0.01 lb and returned to the revolving bin for remixing. From these weights the average percentages and the standard deviations were computed.

To estimate the constants of Eq. 12, several thousand of the aggregate particles used in the study were counted and weighed. To increase the reliability of these estimates the particle counts and weights were analyzed by regression analysis. The resulting equation is

$$\bar{w} = 1.01 \bar{d}^{2.83} \quad (12)$$

in which

$\bar{w}$  = average particle weight, in pounds; and

$\bar{d}$  = average diameter of aggregate particles in a size fraction. The values used in the computations are given in Table 17.

#### Single-Increment Program

The data obtained from the single-increment and the multi-increment programs were used to evaluate the effect of increment size on sampling accuracy. The single-increment program involved sieve analysis of single-scoop test portions used in the multi-increment tests plus scoops of 12- and 18-lb capacity. Fifty scoopfuls with each of five scoops were taken from the pregraded and mixed aggregate used in the previous program. Gradation No. 3 (Table 16) was used for this work and each increment was individually tested for gradation.

Data from the single-increment study were also used to check the validity of Eq. 13, developed in the course of previous work (9):

$$\sigma_a = \sqrt{\frac{P(100 - P)g}{454W}} \quad (13)$$

in which

$P$  = percent by weight of the aggregate passing a designated sieve;

$\sigma_a$  = the inherent standard deviation of that percentage;

$\bar{g}$  = the average particle weight, in grams, of all particles larger than the openings in the designated sieve; and

$W$  = the total weight, in pounds, of aggregate passed through all the sieves (i.e., the total weight of the test portion).

CHART FOR ESTIMATING WEIGHT OF AGGREGATE SAMPLE FOR GRADATION TEST

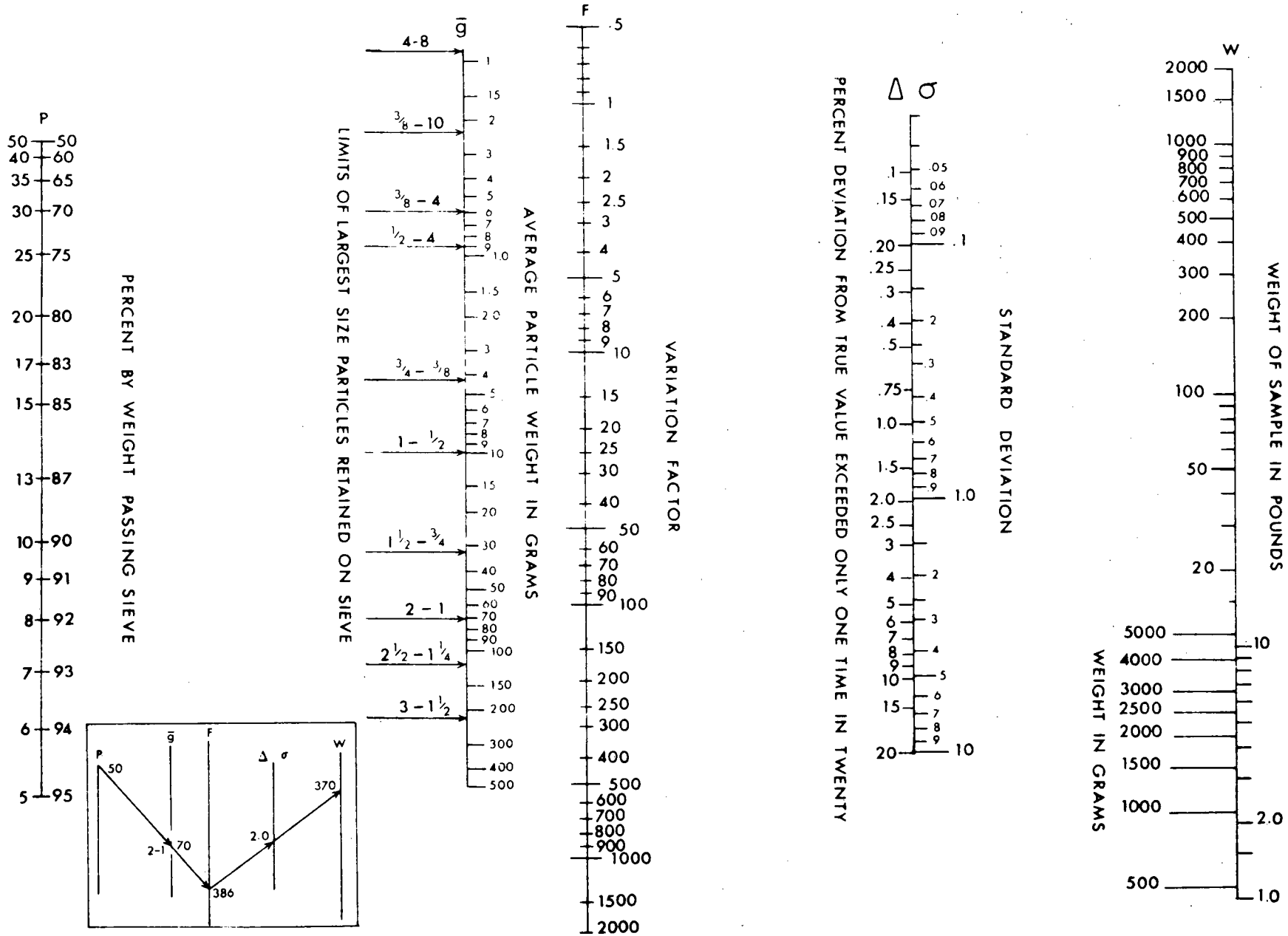
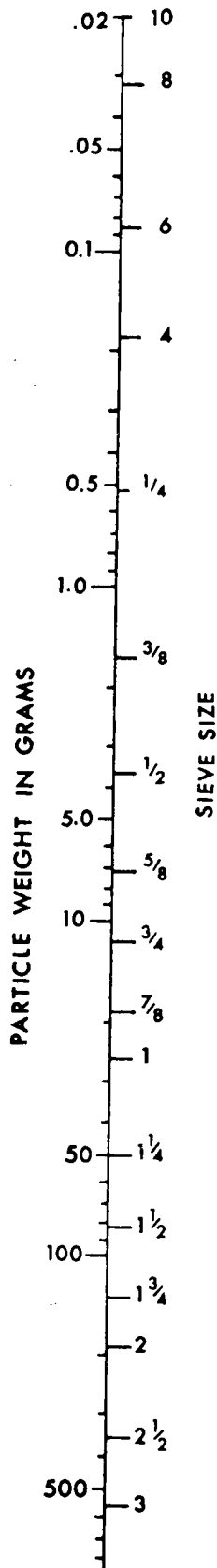


Figure 27. Nomograph for estimating weight of aggregate sample for gradation test.



### USE OF NOMOGRAPH TO ESTIMATE SAMPLE WEIGHT

1. Select the critical sieve size. This is usually that sieve which passes 35-65 percent of the aggregate.

For Example: If 99 percent passes the 2 inch sieve, 80 percent passes the  $1\frac{1}{2}$  inch sieve, and 50 percent passes the 1 inch sieve, the 1 inch sieve is the critical size.

2. Determine the average particle weight ( $\bar{g}$ ) of all particles retained on the critical sieve. If this is unknown, it can be estimated by the use of the scale on this page. First find the weight opposite the mid-point of the distance between the sieve size that the particles pass and the sieve size on which they are retained. Then divide 454 by the particle weight in grams to obtain the number of particles per pound. Calculate a weighted average particle weight ( $\bar{g}$ ) for the total material retained on the designated sieve as shown below.

Sieve Size	Percent Passing-Retained	Particle Weight ( $\bar{g}$ )	Particles Per Pound ( $n$ )
2 - $1\frac{1}{2}$	20	125	3.6
$1\frac{1}{2}$ - 1	30	48	9.5

$$\bar{g} = 454 \left( \frac{P_1 + P_2}{n_1 P_1 + n_2 P_2} \right) = 454 \left( \frac{20 + 30}{20(3.6) + 30(9.5)} \right) = 65$$

3. From the percentage passing the critical sieve on scale P, project a line through the average particle weight on scale  $\bar{g}$  to scale F.

For Example: If 50 percent passes the 1 inch sieve and the average particle weight of the aggregate retained on the sieve is 65 grams, project a line from 50 on scale P through 65 on scale  $\bar{g}$  to 350 on scale F.

4. From the point on scale F, project a line through the desired degree of accuracy on scale  $\Delta$  to the required total sample weight on scale W.

For Example: With an F factor of 350 and a desired degree of accuracy of  $\pm 2$  percent, the line projected through these points indicates a required sample weight of about 350 pounds on scale W.

5. The accuracy obtained by the use of a larger or smaller sample can be found by projecting a line from the F factor to the actual sample weight and reading the result on scale  $\Delta$ .

For Example: With an F factor of 350 and an actual sample weight of 50 pounds, the percent passing the 1 inch sieve will be correct to within  $\pm 5$  percent, 95 times in 100 determinations



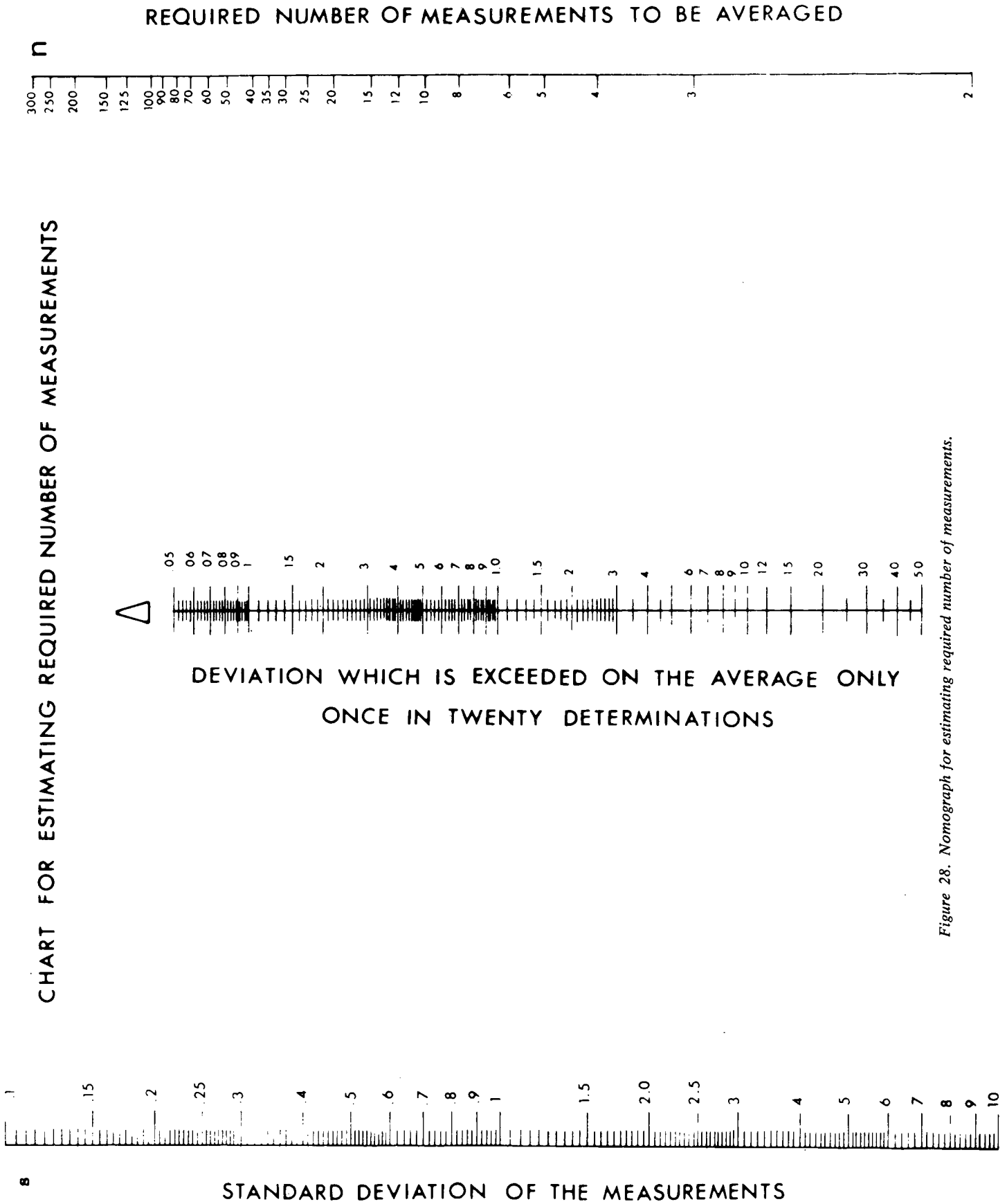


Figure 28. Nomograph for estimating required number of measurements.

To make this validity check, the various sizes of increments were considered to be test portions and tested individually.

## FINDINGS

### Summary of Test Results

The results of the multi-increment program are summarized in Table 18, which shows the following:

1. The actual aggregate gradation used in each test series.

TABLE 15  
SAMPLING SCOOP DIMENSIONS

SCOOP NO.	LENGTH AND WIDTH (IN.)	HEIGHT (IN.)	CAPACITY (LB)
1	3	2	1.2
2	4½	2½	3.2
3	6	3	6.4
4	7½	3½	11.8
5	9	4	18.0

TABLE 16  
AGGREGATE GRADATIONS USED IN INCREMENT STUDY

GRADATION NO.	PASSING AND RETAINED <sup>a</sup> (%)				
	2 IN.— 1½ IN.	1½ IN.— 1 IN.	1 IN.— ¾ IN.	¾ IN.— ½ IN.	½ IN.— ⅜ IN.
1	—	—	33.4	33.3	33.3
2	—	25.0	25.0	25.0	25.0
3	25.0 (25.7)	25.0 (25.5)	25.0 (22.4)	25.0 (26.4)	—
4	10.0	27.0	26.0	27.0	10.0

<sup>a</sup> Parentheses give values for re-sieving of entire bin contents at end of experiment.

### USE OF CHART

The purpose of this nomograph is to furnish an approximate solution of the equation,

$$n = \left[ \frac{t\sigma}{\Delta} \right]^2$$

where  $t$  depends on the number of degrees of freedom ( $n - 1$ ) associated with  $n$ .

1. To use, project a straight line from the standard deviation of the measurement on the left hand ( $\sigma$ ) scale through the desired degree of accuracy on the center ( $\Delta$ ) scale. This line will intercept the right hand ( $n$ ) scale at the approximate value of  $n$  indicated by the equation.
2. To obtain a more precise value of  $n$ , enter the  $t$  table with the number of degrees of freedom ( $n - 1$ ) associated with the chart value, and opposite this value find  $t$  in the column which has  $t = 1.96$  opposite  $df = \infty$ .

Insert this  $t$  in the equation and solve for  $n$ . Use this value of  $n$  to find a new  $t$ , and continue to iterate until the value of  $n$  found by solving the equation is nearly the same as the value of  $n$  used to find  $t$ .

2. The average gradations of 25-lb test portions accumulated by use of each of three different sized scoops.

3. The experimental standard deviation ( $\sigma_e$ ) of the percentages passing the sieves (among-test portion, within-lot variation) for each of the three scoops.

4. The theoretical inherent standard deviation ( $\sigma_a$ ).

5. The true gradation modulus ( $\bar{A}$ ) of each gradation and the average value of modulus of the test portions accumulated by use of each of the three scoops.

6. The standard deviations of the gradation moduli.

7. As indicated by asterisks, those rows which have statistically significant differences among the average percentages passing the sieves, or the average value of  $\bar{A}$ .

In the single-increment testing program all five of the

TABLE 17  
PARTICLE SIZE CONSTANTS

SIEVE SIZE	NO. PARTICLES PER POUND	AVG. WT. PER PARTICLE	
		(LB)	(GM)
2 In.—1½ In.	3.1	0.324	147.0
1½ In.—1 In.	9.8	0.102	46.3
1 In.—¾ In.	22.0	0.0455	20.6
¾ In.—½ In.	70.5	0.0142	6.44
½ In.—⅜ In.	159.0	0.0063	2.86

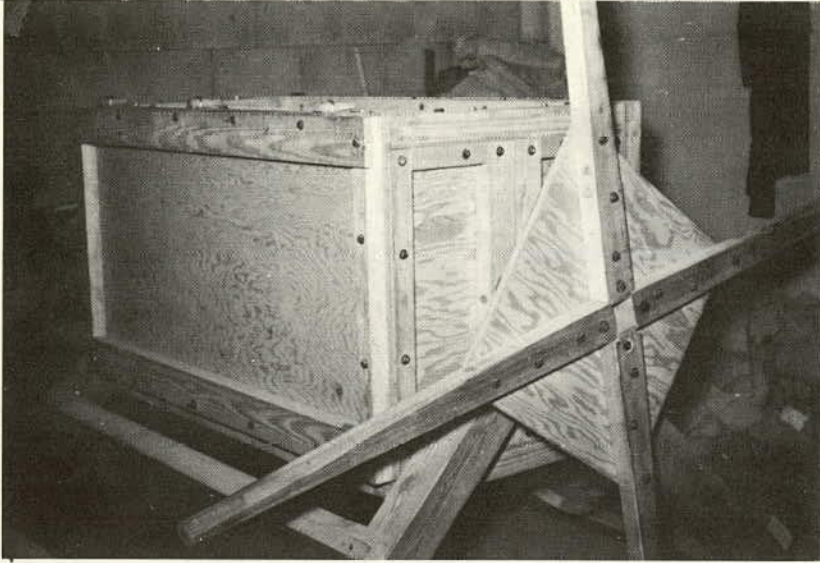


Figure 29. Aggregate mixer for increment size study.

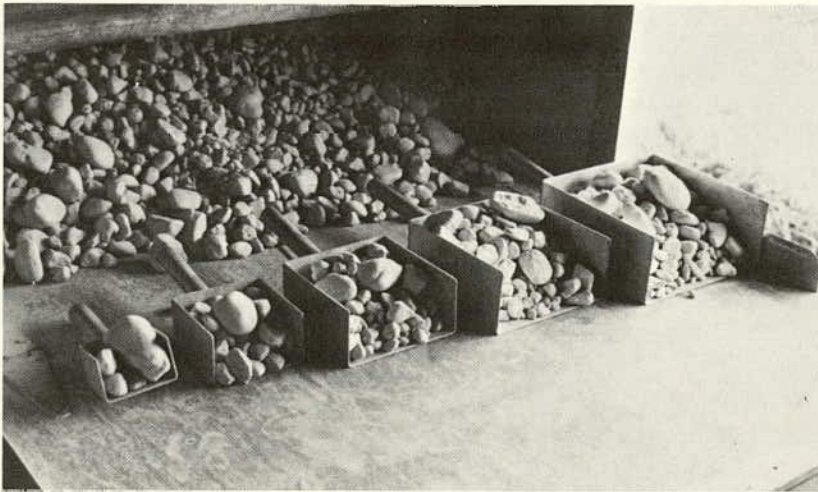


Figure 30. Sampling scoops in relation to aggregate size.



Figure 31. Method of sampling aggregate.

TABLE 18  
SUMMARY OF MULTI-INCREMENT TESTING PROGRAM

GRAD- ATION NO.	SIEVE SIZE	PASSING SIEVE <sup>a</sup> (%)									
		ACTUAL GRADATION	SCOOP 1 (1.2 LB)			SCOOP 2 (3.2 LB)			SCOOP 3 (6.4 LB)		
			$\bar{X}$	$\sigma_e$	$\sigma_a$	$\bar{X}$	$\sigma_e$	$\sigma_a$	$\bar{X}$	$\sigma_e$	$\sigma_a$
1	1 In.	100.0	100.0	—	—	100.0	—	—	—	—	—
	¾ In.	66.7	69.9	3.4	2.3	69.1	2.8	2.0	69.3	2.9	2.0
	½ In.	33.4	33.5	4.5	1.4	32.9	3.4	1.4	33.9	3.5	1.4
	¼ In.	0	0.0	—	—	0.0	—	—	0.0	—	—
	$\bar{A}$	1.81	1.83	0.039	—	1.83	0.030	—	1.83	0.031	—
2	1½ In.	100.0	100.0	—	—	100.0	—	—	100.0	—	—
	1 In.**	75.0	76.8	2.6	2.7	74.8	2.5	2.8	75.0	2.4	2.8
	¾ In.	50.0	52.0	3.2	2.5	50.6	2.9	2.5	51.3	2.8	2.5
	½ In.*	25.0	23.9	2.6	1.4	23.5	2.5	1.6	24.7	2.4	1.5
	¼ In.	0.0	0.0	—	—	0.0	—	—	0.0	—	—
	$\bar{A}$	1.56	1.58	0.040	—	1.56	0.037	—	1.57	0.034	—
3	2 In.	100.0	100.0	—	—	100.0	—	—	100.0	—	—
	1½ In.**	75.0	78.6	4.2	5.0	72.6	4.4	5.1	71.8	3.8	5.1
	1 In.**	50.0	48.9	4.3	3.8	45.1	4.0	3.9	44.9	3.7	3.9
	¾ In.	25.0	22.9	2.2	2.4	22.3	1.9	2.4	22.4	2.3	2.4
	½ In.	0.0	0.0	—	—	0.0	—	—	0.0	—	—
	$\bar{A}$ **	1.02	1.03	0.056	—	0.97	0.054	—	0.96	0.052	—
4	2 In.	100.0	100.0	—	—	100.0	—	—	100.0	—	—
	1½ In. **	90.0	93.3	2.7	2.8	89.5	3.0	3.5	90.0	2.3	3.4
	1 In. **	63.0	59.7	3.0	3.3	57.5	3.3	3.4	59.6	3.3	3.4
	¾ In. **	37.0	32.2	2.4	2.5	31.9	2.4	2.5	34.3	2.9	2.6
	½ In.	10.0	6.0	1.0	0.9	6.8	0.9	0.9	8.1	1.3	1.0
	¼ In.	0.0	0.0	—	—	0.0	—	—	0.0	—	—
	$\bar{A}$ **	1.30	1.26	0.044	—	1.22	0.050	—	1.25	0.044	—

\* Significant at 95% level;  $F_{0.95} = 3.06$ .

\*\* Significant at 99% level;  $F_{0.99} = 4.75$ .

<sup>a</sup> 25-lb test portions,  $n = 50$ ; between-test portion =  $\sigma_e$  (experimental); standard deviation =  $\sigma_a$  (theoretical).

scoops listed in Table 15 were used. The pre-graded aggregate was mixed and sampled as in the previous work, but 50 one-scoop test portions were taken with each scoop and individually passed through the sieves. Gradation No. 3 was used for this experiment. The results are summarized in Table 19. This table shows the average percentage passing each sieve ( $\bar{X}$ ), the standard deviations from this average determined by experiment ( $\sigma_e$ ), and the standard deviation predicted by Eq. 13 ( $\sigma_a$ ). Also shown is a computed standard deviation ( $\sigma_c$ ) obtained by the use of

$$\sigma_c = \frac{\sigma_e}{\sqrt{\frac{25}{W_s}}} \quad (14)$$

in which  $W_s$  is the average weight of increment obtained by the use of the different scoops. This computation was made to eliminate the effect of test portion weight and make possible the results given in Table 21 (discussed later in this chapter).

Table 18, which lists the average gradations of 25-lb samples, and the variations from these averages, does not show any firm correlation between increment size and accuracy or variability. However, there is a general tendency for variability to decrease as increment size is increased. Table 19, which gives the average gradation of

five increment sizes and the increment-to-increment standard deviation ( $\sigma_e$ ), shows a definite correlation between increment size and the correspondence of the average values of the percentages passing the sieves and the true value. The increment-to-increment standard deviation ( $\sigma_e$ ) also decreases as increment size increases, but this is mostly due to the change in value of the inherent standard deviation ( $\sigma_a$ ). When this effect is removed by adjusting the standard deviations to equal test portion weight, the resulting standard deviation ( $\sigma_c$ ) shows a reversal of trend between the percentages passing the 1½-in. sieve and the percentages passing the ¾-in. sieve.

#### Increment Size and Sampling Accuracy

The evaluation of the effect of increment size on sampling accuracy involves both the average percentage ( $\bar{X}$ ) and the standard deviation ( $\sigma_e$ ) given in Tables 18 and 19 and determined in the multi-increment and single-increment testing programs. One index of accuracy is the difference between the indicated average ( $\bar{X}$ ) of the percentage passing each sieve obtained by sampling and the true average ( $\bar{X}'$ ). The other index is the variability as measured by the standard deviation ( $\sigma$ ). The variance ( $\sigma_e^2$ ) is made up of several other variances related to the experimental condi-

TABLE 19  
SUMMARY OF SINGLE-INCREMENT TESTING PROGRAM

SCOOP NO.	IDENTIFICATION OF VALUES	PERCENT PASSING			WEIGHT (LB)	$\bar{A}$
		1½ IN.	1 IN.	¾ IN.		
—	Orig. $\bar{X}$	75	50	25	—	1.02
1	$\bar{X}$	69.6	44.9	19.3	1.16	0.92
	$\sigma_e$	22.4	14.6	8.1	0.15	0.23
	$\sigma_a$	24.3	18.6	11.0	—	—
	$\sigma_c$	4.74	3.15	1.74	—	—
2	$\bar{X}$	71.6	45.5	20.5	3.18	0.95
	$\sigma_e$	13.1	9.4	6.2	0.28	0.15
	$\sigma_a$	14.4	11.1	6.75	—	—
	$\sigma_c$	4.66	3.35	2.21	—	—
3	$\bar{X}$	71.2	45.9	21.6	6.41	0.95
	$\sigma_e$	9.1	8.6	4.6	0.32	0.13
	$\sigma_a$	10.2	7.9	4.9	—	—
	$\sigma_c$	4.60	4.35	2.32	—	—
4	$\bar{X}$	71.6	44.8	21.9	11.8	0.96
	$\sigma_e$	5.6	5.7	4.0	0.65	0.08
	$\sigma_a$	7.5	5.7	3.6	—	—
	$\sigma_c$	3.84	3.90	2.74	—	—
5	$\bar{X}$	74.4	46.7	23.3	18.0	0.99
	$\sigma_e$	4.6	4.2	3.2	0.85	0.06
	$\sigma_a$	5.9	4.6	3.0	—	—
	$\sigma_c$	3.89	3.55	2.71	—	—

$\bar{X}$  = average values;  $\sigma_e$  = standard deviation from average, by experiment;  $\sigma_a$  = standard deviation from average, predicted by Eq. 13;  $\sigma_c$  = standard deviation from average for 25-lb test portion, computed by Eq. 14.

tions. In addition to any variance due to effect of increment size there is a variance due to particle arrangements produced by the mixing action, a sampling variance that is related to the method of removing the increment from the mass of mixed material, a testing variance related to the repeatability of the sieving operation, and a trend related to degradation of the aggregates during repeated mixing.

If it is assumed that, under conditions of the experiment, all variances, and any bias except that due to increment size, remain substantially constant, the effect of increment size can be assessed by computing the percentage of times that the indicated value lies within certain limits symmetrically placed about the true value. The results of this type of analysis of the data in Tables 18 and 19 are given in Tables 20 and 21, which bring out trends that are not discernible from an examination of the individual parameters.

These tables show that under the conditions of the experiment there is a general tendency for a greater number of test results to fall within the arbitrary limits as larger increments are taken by the use of larger scoops. An extrapolation of this trend, as shown in Figure 32, would indicate that a test portion should be taken as a single increment. This, of course, is not true, because under practical conditions there is considerable local segregation or within-batch variance in a mass of aggregate to be sampled. To obtain the best estimate of the true average

gradation, increments must be taken from different parts of the batch to average out the effects of this within-batch variation.

Previous work (9) has indicated that the average within-batch standard deviation under normal plant sampling standard conditions is in the order of about 5 percentage units. This limits the theoretical accuracy of a test on a single-increment 25-lb test portion to about  $\pm 10$  percentage units. To increase this to a more acceptable accuracy of  $\pm 5$  percentage units requires that the test portion be made up of at least four increments, each of which would have a maximum average weight of about 6 lb.

#### Theoretical Standard Deviation

The standard deviations computed from the results of these tests are compared with the theoretical standard deviations computed by the use of Eq. 13 in Tables 18 and 19 and Figure 33. In general, the experimental standard deviation ( $\sigma_e$ ) was less than the theoretical standard deviation ( $\sigma_a$ ) predicted by Eq. 13. As shown by the regression equation of Figure 34, very good correlation between experimental and theoretical standard deviations was obtained, but the mixing of the aggregate did not result in complete randomization of the particles of different sizes.

In the previous work, using a very efficient method of mixing and sampling, the regression relationship was

TABLE 20  
PERCENT OF TEST RESULTS WITHIN PRECISION LIMITS,  
MULTI-INCREMENT PROGRAM

GRADATION NO.	SIEVE	PERCENT WITHIN $\bar{X}' \pm 5$		
		SCOOP 1 (1.17 LB)	SCOOP 2 (3.18 LB)	SCOOP 3 (6.41 LB)
1	1 In.	—	—	—
	¾ In.	69.4	76.3	74.7
	½ In.	73.4	85.4	84.2
	⅜ In.	—	—	—
	$\bar{X}$	71.4	80.8	79.4
2	1½ In.	—	—	—
	1 In.	88.6	95.4	96.2
	¾ In.	81.2	90.8	89.5
	½ In.	92.4	91.5	96.2
	⅜ In.	—	—	—
$\bar{X}$	88.7	92.6	94.0	
3	2 In.	—	—	—
	1½ In.	60.9	67.6	66.7
	1 In.	74.1	78.9	82.3
	¾ In.	90.7	98.7	85.1
	½ In.	—	—	—
$\bar{X}$	75.2	81.7	78.0	
4	2 In.	—	—	—
	1½ In.	73.6	90.0	97.0
	1 In.	71.3	44.0	67.9
	¾ In.	54.9	48.4	78.3
	½ In.	84.1	97.7	99.1
	⅜ In.	—	—	—
	$\bar{X}$	71.0	70.0	85.6
$\bar{\bar{X}}$	76.2	80.4	84.7	

$\sigma_e = 0.09 + 0.94 \sigma_a$  with a correlation coefficient of 0.95 and a standard error of 0.35. However, due to the necessity of making provision for sampling with different size scoops this method could not be used in the increment size study, and consequently a different regression coefficient was obtained. In view of these limitations imposed by the primary objectives of the increment study, this result does not invalidate the usefulness of Eq. 13.

Eq. 13 can be used to evaluate the adequacy of test portion weights specified by ASTM C136 on the basis of the maximum possible error under the worst possible combination of conditions. This combination occurs when 100% of the aggregate is passing the sieve representing the nominal maximum size in the gradation and 50% is retained on the next smallest sieve, as shown in Table 22. This table also shows the average particle weight of these size fractions and the maximum possible error at the 95% confidence level when the specified test portion weights are used. The maximum possible error due to inherent variation when using other test portion weights can be estimated by

$$\Delta_{\max} = 2a/w^{\frac{1}{2}} \quad (15)$$

in which

$a$  = value of standard deviation of 1-lb test portion as shown in Table 22; and

$w$  = test portion weight, in pounds.

#### CONCLUSIONS AND APPLICATIONS

The findings of this experiment indicate the following:

1. The accuracy of the percentages of aggregate passing

TABLE 21  
PERCENT OF TEST RESULTS WITHIN PRECISION  
LIMITS, SINGLE-INCREMENT PROGRAM

SCOOP NO.	AVG. WT. OF INCREMENT (LB)	PERCENT WITHIN $\bar{X}' \pm 5$		
		1½ IN.	1 IN.	¾ IN.
1	1.2	45.4	48.8	34.0
2	3.2	59.7	55.7	59.1
3	6.4	57.5	56.5	75.5
4	11.8	64.9	47.6	75.3
5	18.8	79.7	67.8	88.2

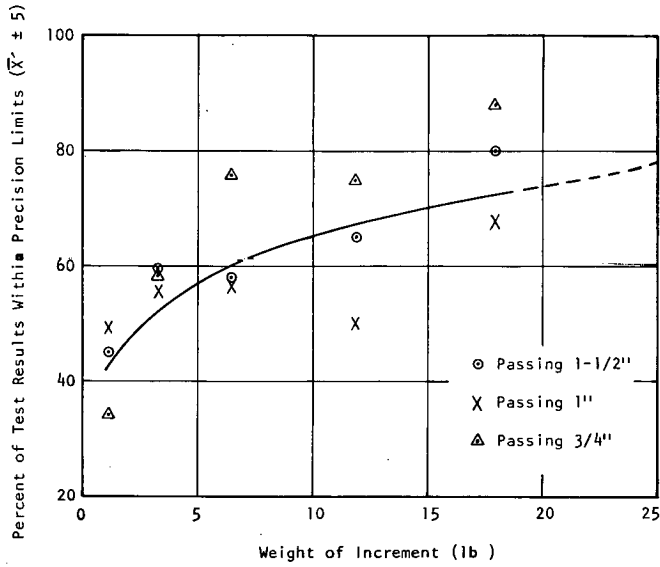


Figure 32. Effect of increment weight on accuracy of gradation test.

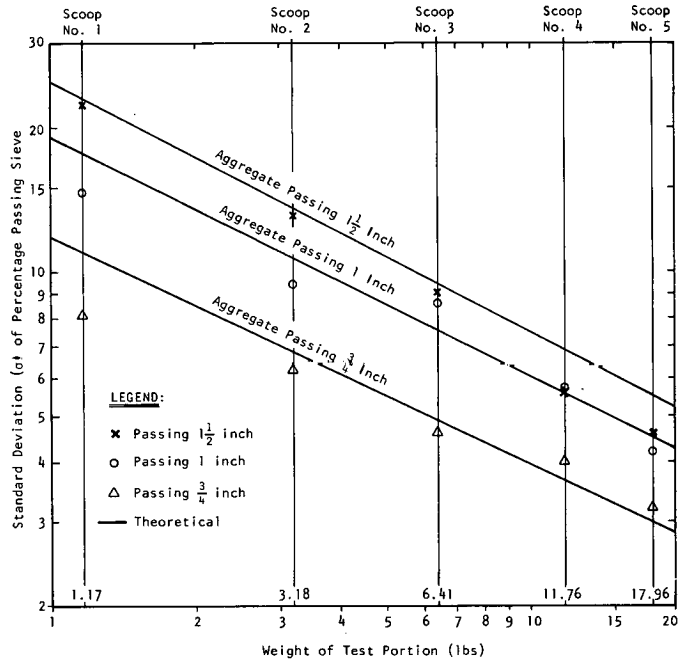


Figure 33. Effect of single-scoop test portion size on standard deviation of percentages passing sieves (actual and theoretical).

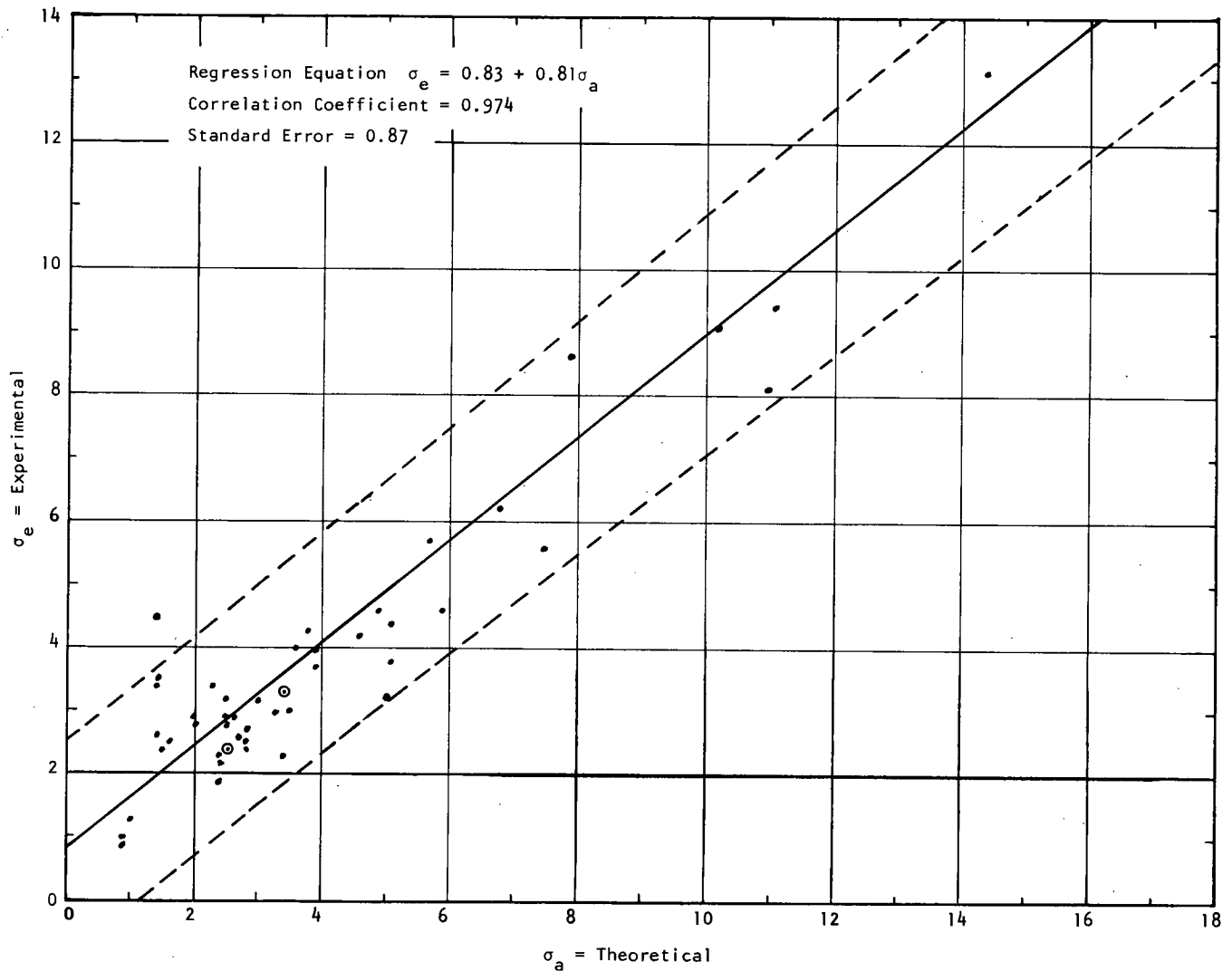


Figure 34. Correlation between experimental standard deviation ( $\sigma_e$ ) and theoretical standard deviation ( $\sigma_a$ ) of percentage of aggregates passing sieves.



TABLE 22  
 MAXIMUM ERROR ASSOCIATED WITH USE OF  
 MINIMUM ASTM SAMPLE WEIGHTS.

MAXIMUM NOMINAL SIZE OF AGGREGATE	PARTICLE WEIGHTS <sup>a</sup> (GM)	ASTM WEIGHT <sup>b</sup> OF TEST PORTION (LB)	MAX. POSSIBLE ERROR AT 2 $\sigma$ LEVEL <sup>c</sup> (LB)	$a^d$
2 In.—1½ In.	180	45	± 9.34	31.2
1½ In.—1 In.	48	35	± 5.38	15.9
1 In.—¾ In.	18	25	± 4.00	10.0
¾ In.—½ In.	7	15	± 3.20	6.2
½ In.—¾ In.	2.5	10	± 2.34	3.7
¾ In.—No. 4	0.5	5	± 1.50	1.7

<sup>a</sup> From Figure 27.

<sup>b</sup> AASHTO T-27 minimum weights are slightly smaller.

<sup>c</sup> Computed by use of Eq. 13.

<sup>d</sup> From Eq. 15.

sieves as determined by gradation tests is primarily a function of the weight of test portion used.

2. In sampling a mass of perfectly mixed aggregate, the accuracy of sampling and testing for gradation is increased by the use of large increments. However, under practical conditions where there is local segregation, optimum overall accuracy is obtained by a compromise between increment size and the number of increments. Although the many factors involved preclude a definite optimization of increment size, an increment weight of about 6 lb appears to be satisfactory for sampling aggregates having gradations similar to those used in the experiment, when a 25-lb test portion is used. For this size of increment a scoop or sampling tool having a width of about 6 in., a height of about 3 in., and a length of about 6 in. should be used.

3. Eq. 13 provides a sufficiently accurate estimate of the weight of test portion required to obtain a desired degree of accuracy.

4. The maximum possible error due to inherent variation at the 95% confidence level can be calculated by the use of Eq. 15. This equation indicates that the use of test portions of the minimum size specified by ASTM C136 or AASHTO T-27 may result in large errors in the reported results of gradation test (Table 22).

5. The essential parts of a complete aggregate specification as outlined in *NCHRP Report 34* (9) should also include a sampling plan that would state the approximate size of the increment, size and number of the test portions, and a description of the sampling tool.

## CHAPTER FIVE

# MATHEMATICAL STUDY OF PATTERN OF VARIATIONS IN GRADATION OF AGGREGATES

## INTRODUCTION

The purpose of this study was to evaluate parameters of a previously designed mathematical model that describes the extent of aggregate segregation, and to fit a mathematical model, or models, to the observed pattern of magnitude of over-all standard deviation ( $\sigma_o$ ) with respect to the percentages passing the specified sieves. Knowledge of this pattern is essential to the establishing of realistic job-mix formula tolerances. The pattern also determines the critical sieve in a coarse aggregate gradation, so that this one

sieve can be used for a quick check on compliance with gradation specification requirements.

Mathematical studies conducted during the course of work on NCHRP Projects 10-3/1 (11) and 10-2 (6) indicated that the size of the over-all variance ( $\sigma_o^2$ ) of the percentage of aggregate passing a given sieve was essentially the sum of four variances. This relationship can be expressed by

$$\sigma_o^2 = \sigma_t^2 + \sigma_a^2 + \sigma_s^2 + \sigma_l^2 \quad (16)$$

in which

$\sigma_o^2$  = the over-all variance of the percentage of aggregate passing a given sieve among test results obtained by repeated samplings of a single LOT of aggregate;

$\sigma_t^2$  = the variance due to testing error (i.e., the lack of repeatability of the gradation test);

$\sigma_a^2$  = the inherent variance due to the random distribution of particles within an aggregate mass;

$\sigma_s^2$  = the variance due to within-batch variation (i.e., the differences in gradation between different parts of a small unit of aggregate such as a batch); and

$\sigma_l^2$  = the batch-to-batch variance due to differences in the average gradation of units or batches of aggregate within the same LOT.

The Project 10-2 Interim Report (6) indicates that the size of the over-all variance ( $\sigma_o^2$ ) for an entire gradation could be expressed by a single index number. This number was obtained by plotting on a log-log graph the standard deviations of the percentage passing each sieve ( $\sigma_o$ ) against the values of  $\sqrt{P(100-P)}$ , where  $P$  was the percentage passing the same sieve. A line, drawn by eye through the plotted points, was extrapolated to the intercept with  $\sqrt{P(100-P)} = 50$ . The corresponding value on the  $\sigma$  scale is an estimate of the standard deviation for a fictitious size of sieve which would pass exactly 50% of the total aggregate ( $\sigma_{50}$ ). This derived index ( $\sigma_{50}$ ) was used to express over-all variability as a percentage of the standard deviation which is associated with complete segregation ( $\sigma = 50$ ). This percentage was called the degree of variation (DOV) and was computed by

$$\text{DOV} = \frac{\sigma_{50}}{50} \times 100 = 2\sigma_{50} \quad (17)$$

Studies were made to determine the relative significance of the terms of Eq. 16. Experiments (NCHRP Project 10-2) to determine the repeatability of the gradation test (AASHTO T-27) indicated that the testing variance ( $\sigma_t$ ) term of this equation contributed very little to the total variance and could be neglected for all practical purposes.

With regard to the second term of Eq. 16 it was found that the size of inherent variance ( $\sigma_a$ ) of the percentage passing a designated sieve due to randomization of particle sizes in a perfectly mixed mass of aggregate was a function of the percentage passing the sieve, the average particle weight of all particles larger than the sieve opening, and the weight of the test portion. In the case of gradations containing coarse aggregates and test portions of the size required by AASHTO T-27, this inherent variance contributes significantly to the total variation of the percentages passing the larger sieves. The size of this term can be estimated by

$$\sigma_a^2 = \frac{P(100-P)\bar{g}}{454W} \quad (18)$$

in which

$P$  = the percent by weight of the aggregate passing a designated sieve;

$\bar{g}$  = the average particle weight of all particles larger than the openings in the designated sieve, in grams; and

$W$  = weight of test portion, in pounds.

The remaining two terms of Equation 16 represent micro-segregation ( $\sigma_s^2$ ) and macro-segregation ( $\sigma_l^2$ ); that is, point-to-point differences in gradation of aggregates within a small unit such as a batch, and point-to-point differences in a large mass of aggregate such as a stockpile or an extensive area of aggregate base. These components of the over-all variance can be expressed by

$$(\sigma_s^2 + \sigma_l^2) = K [P(100-P)]^t \quad (19)$$

or

$$(\sigma_o^2 - \sigma_a^2) = K [P(100-P)]^t \quad (20)$$

in which

$K$  = the value of  $(\sigma_o^2 - \sigma_a^2)$  when  $P(100-P) = 1$ ; and

$t$  = the slope of the regression line through the various values of  $(\sigma_o^2 - \sigma_a^2)$ .

Substituting Eqs. 17 and 19 in Eq. 16 gives

$$\sigma_o^2 = \sigma_t^2 + \frac{P(100-P)\bar{g}}{454W} + K [P(100-P)]^t \quad (21)$$

Mathematical and graphical methods of obtaining values for the constants of the expression  $K [P(100-P)]^t$  are given in Appendix B of *NCHRP Report 46* (1967) 102 pp.

On the basis of the previous studies it had been theorized that the value of  $K$  depended on the relative amount of segregation due to sampling error ( $\sigma_s^2$ ) and batch-to-batch or unit-to-unit segregation ( $\sigma_l^2$ ). It was also theorized that the value of the slope ( $t$ ) depended on the range and distribution of particle sizes in the gradation and possible interactions or additional factors.

In this study these theories were re-examined in the light of the additional data obtained in the course of the current investigation and data obtained from another source (12). This re-examination consisted of calculating regression lines, by the method of least squares, for the part of the over-all variance represented by  $K [P(100-P)]^t$  and determining the values of  $K$  and  $t$ .

These lines represent a refinement of the previous graphs of a "best line" drawn by eye through the data points. This method was used to obtain estimates of the maximum and minimum variation of gradation due to segregation which can be expected in practice.

The data points obtained by the use of Eq. 21 were compared with data obtained in the course of previous work, and a mathematical expression was obtained which described the pattern of variation.

## FINDINGS

The results of a rigorous analysis of the data from NCHRP Projects 10-3 (13), 10-3/1 (11), 10-2 (6), and this project, indicate that the value of the standard deviation ( $\sigma_{50}$ ) depends mostly on the value of the standard deviations of the percentages passing the sieves within the range

of 20 to 80%. The value of  $\sigma_{50}$  also depends on the value of the slope parameter,  $t$ , and the coefficient,  $K$ .

The value of  $t$  depends on the ratio of the standard deviations of the percentages within the 1-to-10 and the 90-to-100 ranges to those within the 20-to-80 range. If the percentages passing the sieves for very large or very small particles are below 10% or above 90% and the standard deviations of these percentages are large, the value of  $t$  will be relatively small. If the standard deviations of these percentages are small, the value of  $t$  will be relatively large. In the cases studied, the effect of  $t$  on the value of  $\sigma_{50}$  is not large.

The value of the coefficient  $K$  depends on both the value of  $\sigma_{50}$  and the value of  $t$ , with a large value of both  $\sigma_{50}$  and  $t$  leading to a small value of  $K$ . Due to these interactions, the available data are not sufficient to establish a firm correlation between the constants of Eq. 21 and the relative amount of segregation, gradation, or aggregate type. However, some trends are apparent and the studies have confirmed that there is a definite pattern of variation that has direct application to realistic limits of variations of gradation as defined by specifications.

#### The Pattern of Variation

A characteristic pattern of variation of the percentages of aggregate passing a series of sieves can be noted in the results of studies made by this research agency and in the results obtained by other researchers. This pattern is illustrated by Figure 35, which is a plot of the over-all standard deviations reported by Meaman and Laguros (12, Table 2).

The lower curve in Figure 35, showing the over-all standard deviations of the percentages of fine aggregate passing the sieve sizes  $\frac{3}{8}$  in. to No. 100 inclusive, is quite symmetrical, whereas the upper curve, that shows the percentages of coarse aggregate passing the sieve sizes  $2\frac{1}{2}$  in. to No. 8, is negatively skewed.

The reason for the shapes of these curves can be deduced from Figure 36. In this figure the lower curve represents the plot of the third part of Eq. 21 (i.e., the component of the over-all variance due to segregation), whereas the upper curve is the plot of the over-all standard deviation. For all practical purposes this represents the standard deviation indicated by the sum of the second and third parts of Eq. 21. As predicted by the equation, the standard deviations due to segregation form a symmetrical plot. However, the standard deviations due to inherent variation are strongly influenced by particle size. Because the largest particles in a gradation are those retained on those sieves having the highest percentages passing, the combined effect is to produce an unsymmetrical curve having a maximum value at about 60 to 70% passing. This accounts for the shape of the upper curve in Figure 35, which can be approximated by the curve defined by

$$Y = a + b x + \pi^{dx} \quad (22)$$

The fit of the curve defined by

$$\sigma = 0.3 + 0.09P - 0.10\pi^{0.04P} \quad (23)$$

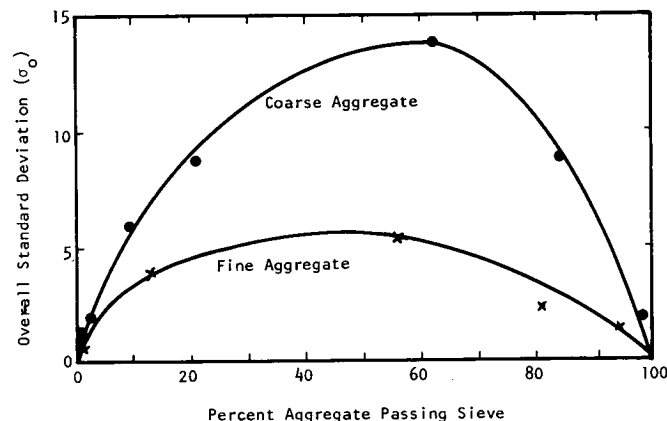


Figure 35. Characteristic pattern of variation of gradation.

to the data points of Figure 36 is shown in Figure 37. The practical use of such a curve would be to estimate the maximum standard deviation in cases where data are not available as to the variability of the percentages passing intermediate sieves which would retain 30 to 50% of the aggregate. The lower curve in Figure 35, representing the over-all standard deviation of the percentages of fine aggregate passing the sieves, is not distorted by inherent variation because the weights of the largest particles are small compared to the test portion weight. The practical significance of these patterns lies in their application to specification of realistic tolerances for gradation of aggregates. For fine aggregate the tolerance band should be widest in the vicinity of the sieve that passes about 50% of the aggregate. Tolerance bands for coarse aggregates, or mixtures containing coarse aggregate, should be widest in the vicinity of those sieves which pass 50 to 70% of the total aggregate.

These observations apply only to aggregates in situations where the proportions of the different sized particles are not controlled but are due to chance. When a gradation is created by combining different sizes of aggregates in definite proportions, as is the case of the combined gradation of Plant No. 4 (Chapter Three), the pattern may be quite different. In this case the standard deviation of the percentage of combined aggregate passing a sieve will depend on the size of the standard deviation of the percentages of the individual aggregates passing the same sieve, and on the proportions of these aggregates.

#### Range of Variation of Gradation

Review of the data accumulated in the course of studies made in connection with NCHRP Projects 10-3, 10-3/1, 10-2, and this study, indicates that the largest degree of segregation was found in stockpiled coarse aggregate. As shown in Figure 38, the standard deviation at the 50% point, which is at the maximum of the segregation curve of Figure 36, was of the order of 20 to 25 percentage points. In one case the stockpile was of irregular layer construction built up by truck-dumping a  $1\frac{1}{2}$ -in. to  $\frac{3}{8}$ -in.

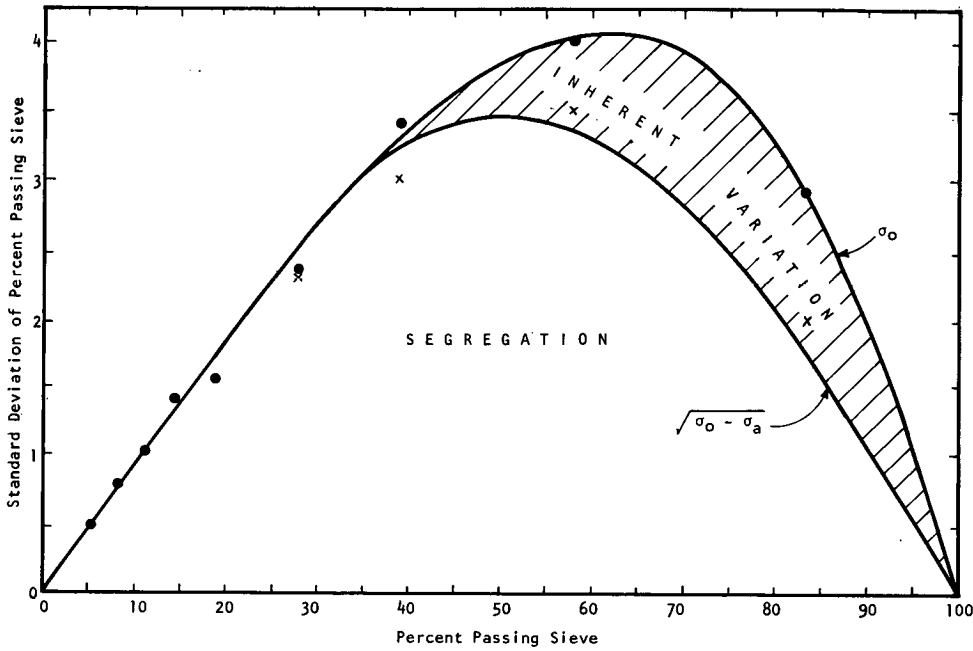


Figure 36. Characteristic pattern of variation of gradation of combined aggregate.

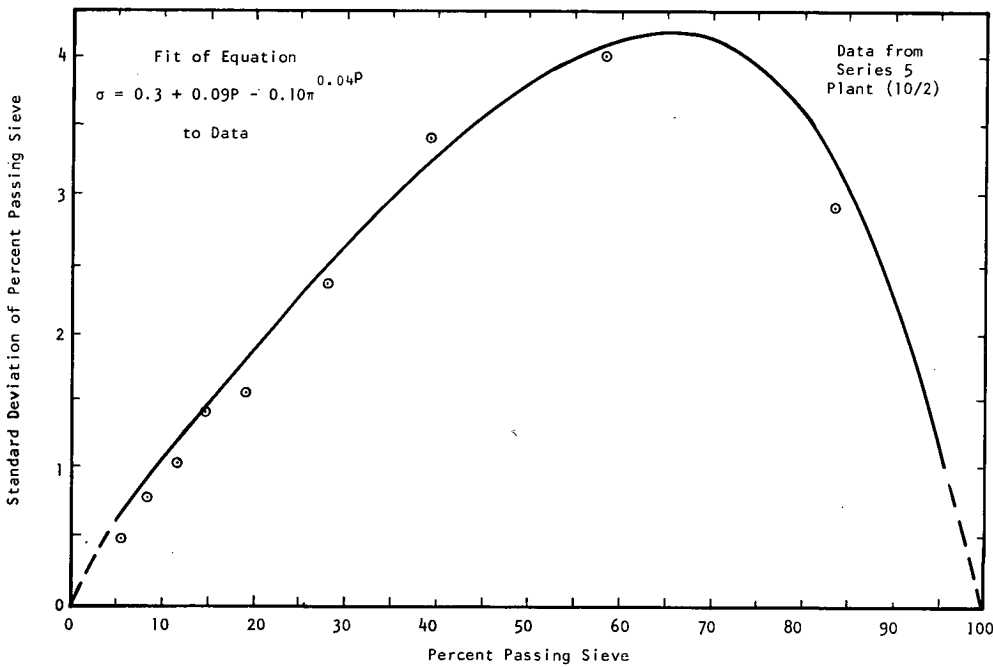


Figure 37. Theoretical pattern of variation of gradation.

crushed stone. In the other case the stockpile was formed in the shape of a cone of 1-in. to No. 4 uncrushed gravel. The constants  $K$  and  $t$  of Eq. 20 were nearly the same in both cases. This indicates that the maximum degree of segregation may be independent of the type or gradation of coarse aggregate within the range of sizes studied.

The large values of the standard deviations also indicate

less than normal kurtosis or degree of peakedness of the distribution curve. The practical significance of this is that it may not be possible, by the use of normal theory, to make accurate estimates of the percentage of test results that will be within given limits in the case of highly segregated stockpiles.

The least amount of segregation of uncombined coarse

aggregate was found in crushed aggregate sampled at the crusher. This indicates that an accurate estimate of the average gradation of crushed aggregate can be obtained with a minimum number of tests by sampling at this point. One aggregate was a 1-in. to No. 4 crushed stone, the other was a 3/4-in. to No. 4 crushed gravel. As shown in Figure 39, the two aggregates had somewhat different values of the constants  $K$  and  $t$  in Eq. 20, but the maximum standard deviations at the 50% point were about the same (i.e., 4 to 4.7 percentage points). This means that a realistic gradation tolerance band for crushed coarse aggregate should have a minimum width of about 18 percentage points ( $\pm 2\sigma$ ) in the vicinity of those sieves passing 50 to 70% of the aggregate. To this must be added an allowance for testing error, inherent standard deviation, and some variation of the average from the target value. These findings also indicate that the ASTM D-693 tolerance band of 35 to 70% passing the 2-in. sieve for No. 2 stone is not excessively wide. This tolerance is more restrictive than the 40 to 75% passing the 3/8-in. sieve specified for No. 78 stone, because of the much smaller inherent standard deviation of this smaller size if the minimum specified weights of test portions are used in each case.

In general, the data studied indicate that the standard deviations for the percentages passing other sieves are proportional to the maximum standard deviation; i.e., the

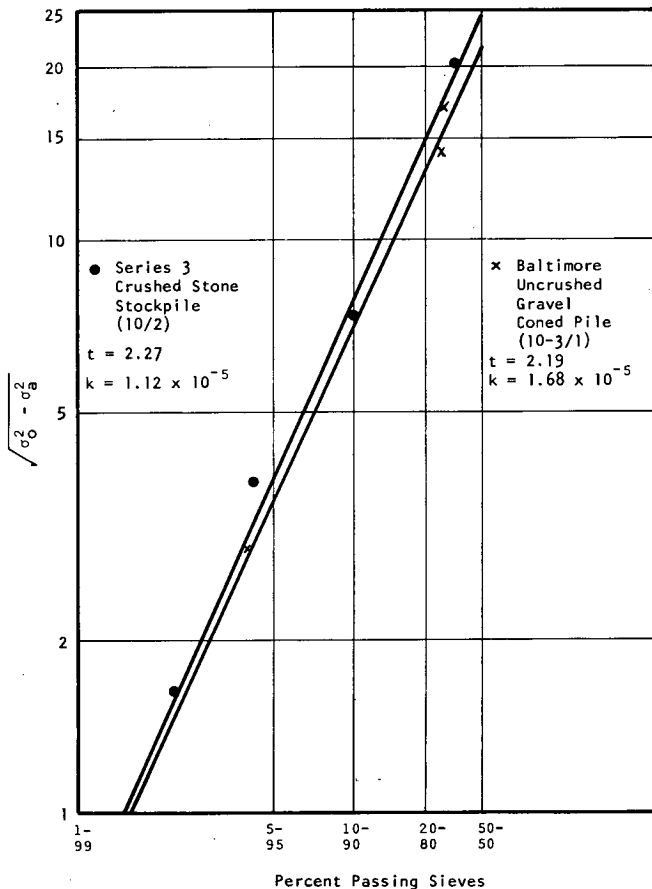


Figure 38. Maximum degree of segregation.

standard deviation of the percentage in the 50-to-70 range. This means that, if the percentage within this range complies with specification requirements, there is a high probability that the percentages passing other sieves will also comply. This leads to the possibility that a quick check of an aggregate gradation could be made by passing the aggregate through only the one sieve that most nearly passed 50 to 70% of the aggregate.

**CONCLUSIONS AND APPLICATIONS**

1. There is a definite pattern of variation of aggregate gradation. The existence of this pattern can be used as a guide for establishing realistic gradation specification tolerance limits, with the broader tolerances for coarse aggregate, and mixtures containing coarse aggregate, applied to those sieves passing 50 to 70% of the aggregate. In the case of fine aggregate (minus 3/8 in.) the broadest tolerances should be applied to those sieves passing 40 to 60%.
2. The minimum width of the gradation tolerance band for coarse aggregates, in the vicinity of those sieves passing 50 to 70% of the coarse aggregate, should be 18 percentage points.
3. The minimum variation of gradation of production crushed aggregate occurs at the crusher; the best estimate of the average gradation, with the fewest number of tests, is obtained by sampling at this point.
4. Although the theoretical maximum value of the standard deviation of the percentage of aggregate passing a sieve is 50, the highest value found in practice was about

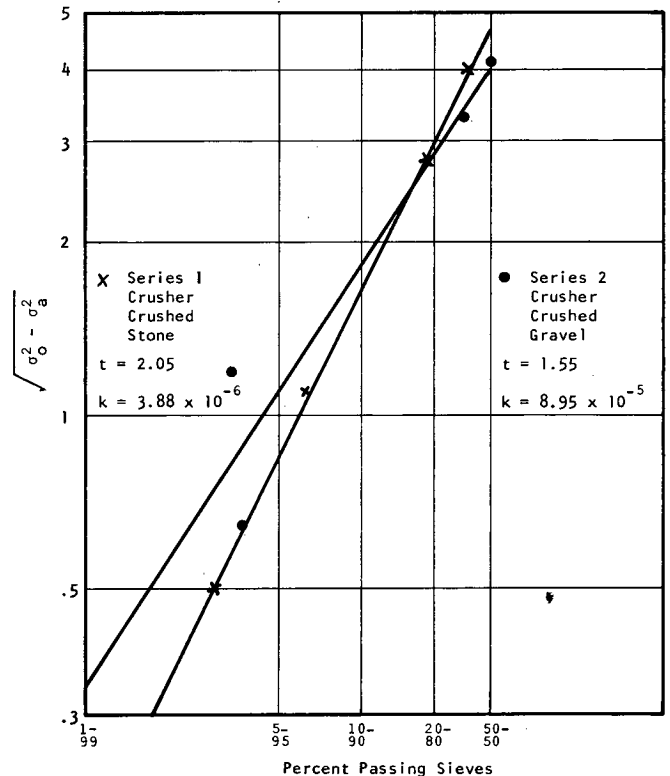


Figure 39. Minimum degree of segregation.

25. This value may be a more practical reference for the purpose of estimating the degree of segregation.

5. Standard deviations of percentages of aggregate passing sieves are usually proportional to the standard deviation of the percentage in the 50 to 70% range. This indicates that a gradation can be checked quickly by the use of the single sieve that passes a percentage of aggregate nearest this range.

6. In the case of a restrictive specification, such as the gradation of aggregate for some types of paving mixtures, a tolerance band can be constructed so that there is an equal probability of conformance for all sieve sizes. Such a tolerance band is obtained when plus or minus two standard deviations (experimentally obtained) are applied to the percentages of the desired, or target, job-mix formula.

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## APPENDIX A

### DERIVATION AND USE OF THE GRADATION MODULUS $\bar{A}$

#### BACKGROUND

Literature research and mathematical studies have indicated that previously developed aggregate gradation moduli are related mathematical expressions of the specific-surface/particle-size ( $S, \bar{d}$ ) interaction. These moduli, such as Ebbert's Bitumen Index (14), Abrams' Fineness Modulus (15), dos Santo's Soil Constant (16), and Turnbull's Soil Classification (17) are essentially based on variations of the relationship

$$Q = f(S, \bar{d}) \quad (\text{A-1})$$

That is,

$$Q = f\left[\frac{c}{\bar{d}}; \bar{d}\right] \quad (\text{A-2})$$

$$Q = \frac{c \bar{d}^\kappa}{\bar{d}} + C = c \bar{d}^{\kappa-1} + C \quad \kappa - 1 = b \quad (\text{A-3})$$

$$Q = c \bar{d}^{-b} + C \quad (\text{A-4})$$

$$Q = \frac{c}{\bar{d}^b} + C \quad (\text{A-5})$$

in which

$Q$  = quantity or percentage of liquid required for optimum film thickness of the liquid coating of soil or aggregate particles;

$S$  = surface area of particles having an average size  $\bar{d}$ ;

$\bar{d}$  = average size of particles in an aggregate fraction;

$b$  = slope of line of relationship;

$c$  = a constant depending on the assumptions made as to the relationships existing in liquid/particulate-solid system and on the specific gravity, shape, and surface texture of the particles; and

$C$  = a constant depending on the minimum value of  $Q$ .

These moduli have been used to establish the relationship of asphalt demand to the gradation of the aggregates in bituminous paving mixtures, the effect of water on the properties of soils, and the relationship of water content and the gradation of the aggregate or total solids to the properties of portland cement concrete.

In general, these moduli have shown excellent correlation in their various applications. Abrams, in particular, was able to demonstrate experimentally that gradation, *per se*, was not the causative factor in variations of the strength and consistency of concrete and that these properties remained constant over a wide range of aggregate gradation and surface area, providing the gradation modulus (FM) was maintained at a fixed value.

#### DEVELOPMENT OF HUDSON $\bar{A}$

In connection with statistical studies and designed experiments involving aggregate gradations, a factor was needed that would characterize a gradation by a single number.

The study of the advantages and deficiencies of the previously developed gradation moduli indicated that for most practical application the parameter should meet the following requirements:

1. It should be applicable to the entire range of gradation of aggregates or other particulate mixtures generally used in highways or structures.

2. It should be easy to compute, but it should not be restricted to any particular series of sieve sizes; i.e., it should be applicable to any graded aggregate, regardless of the sieves used to define the gradation.

3. It should be as nearly compatible as possible with other parameters to make possible comparisons with previous research.

4. Because most surface area effects appear to be correlated with the specific-surface/particle-size interaction, the effective mean particle diameter of a size group of particles should be taken as that which has the mean surface area.

With these requirements in view, the gradation modulus ( $\bar{A}$ ) is defined as the logarithm, to the base 2, of the ratio of an arbitrary constant (54.8) to the effective mean diameter in millimeters of a collection of aggregate particles; i.e.,

$$2^{\bar{A}} = \frac{54.8}{\bar{d}} \quad (\text{A-6})$$

The constant, 54.8, is the effective mean diameter of the largest size group (3 in. — 1½ in.) in the series of size groups under consideration. The smallest size group is the material between the No. 200 and No. 400 sieves, (practically, the effective material finer than the No. 200 sieve openings). The symbol  $\bar{A}$  was selected because the modulus is the grand average of the effective mean diameter of a series of groups of particles.

The effective mean diameter of the particles contained between two sieves is calculated by

$$\bar{d} = \frac{0.4343 (d_1 - d_2)}{\log (d_1/d_2)} \quad (\text{A-7})$$

in which

$\bar{d}$  = mean diameter of aggregate particles in gradation fraction;

$d_1$  = size of openings in largest sieve; and

$d_2$  = size of openings in smallest sieve.

When standard sieves are used,  $d_1/d_2 = 2$ , and Eq. A-7 simplifies to



$$\bar{d} = 1.443 (d_1 - d_2) \tag{A-8}$$

Because previously developed moduli resolve to some function of the average particle size ( $\bar{d}$ ), and because  $\bar{A}$  is a similar function, these moduli can be expressed in terms of the more flexible and more easily calculated  $\bar{A}$ . For example, Abram's Fineness Modulus (FM) is practically an inversion of  $\bar{A}$  with the exception of the exclusion of the minus No. 100 particles. In the case of aggregate gradations where this fine material is not of interest, FM values may be interchanged with  $\bar{A}$  by means of the relationship

$$\bar{A}^* = 9.00 - \text{FM} \tag{A-9}$$

**METHOD OF COMPUTING  $\bar{A}$**

$\bar{A}$  is most easily computed by using standard sieves. Then, to find  $\bar{A}$ , add the percentages passing sieves 1½ in., ¾ in., ¾ in., No. 4, No. 8, No. 16, No. 30, No. 50, No. 100, and No. 200 (including all 100 percent passing) and divide by 100. In the example given in Table A-1,  $\bar{A} = 5.75$ .

The value of  $\bar{A}$  can also be computed from the individual size fractions between standard sieves, as given in Table A-2, in which the value of  $\bar{A}$  is found by summing the results of multiplying the fraction of the total aggregate by the  $\bar{A}$  value for the effective mean diameter of the particles in that fraction.

However, since

$$2^{\bar{A}} = \frac{54.8}{\bar{d}} \tag{A-6}$$

or

$$\bar{A} = \frac{\log 54.8 - \log \bar{d}}{\log 2} \tag{A-10}$$

an  $\bar{A}$  factor for any size group, defined by any two sieves,

\* Gradation modulus of particles larger than No. 100 sieve.

TABLE A-1  
METHOD OF COMPUTING  $\bar{A}$

SIEVE SIZE	% PASSING
1½ In.	100
¾ In.	100
¾ In.	97
No. 4	82
No. 8	66
No. 16	51
No. 30	36
No. 50	24
No. 100	13
No. 200	6
All	575

having openings of known size, can be computed. The same gradation as in Table A-2, but defined by different sieves, is given in Table A-3. Note that the value of  $\bar{A}$  is practically the same, the small difference being due to rounding errors.

To compute the  $\bar{A}$  factor for a size group not shown in the foregoing tables (for example, the ½-in. to ¾-in. size) the first step is to find the effective mean diameter.

$$\text{That is, } \bar{d} = \frac{0.4343(12.7 - 9.51)}{\log (12.7/9.51)} = 11.06 \text{ mm.}$$

The value of  $\bar{A}$  is then found by use of Eq. A-6 and the Ln scales on a K & E Deci-log slide rule, or  $2^{\bar{A}} = 54.8/11.06 = 4.955$  and  $\bar{A} = 2.31$ . Alternately, using logarithms,  $\bar{A} = \log 54.8 - \log 11.06/\log 2 = 2.31$ .

The  $\bar{A}$  value of a combination of aggregates is found by multiplying the  $\bar{A}$  value of each by its decimal proportion and adding the results. For example, suppose 20 percent of ¾-in.-½-in. aggregate, which is to be added to a gradation having an  $\bar{A} = 5.19$ . The  $\bar{A}$  value of this size aggregate is 1.81. Then the  $\bar{A}$  value of the new gradation is

$$\begin{aligned} 0.20 \times 1.81 &= 0.362 \text{ (¾ in.-½ in.)} \\ 0.80 \times 5.19 &= 4.152 \text{ (½ in.-No. 200)} \\ \bar{A} &= 4.51 \end{aligned}$$

**PRACTICAL APPLICATION**

The gradation modulus  $\bar{A}$  of aggregates and aggregate mixtures has been found experimentally, and by comparison with the findings of other investigators, to be highly correlated with the optimum asphalt content of paving mixtures. Also, an extremely high correlation has been established between the  $\bar{A}$  value of the total solids and the consistency (slump) of concrete as interrelated to water content. The mathematical relationship of  $\bar{A}$  to other established moduli indicates a corresponding high correlation of the  $\bar{A}$  value of soil mixtures and the effects of their properties.

Accordingly, the Hudson  $\bar{A}$  gradation modulus should be appropriate for the following:

1. Characterizing a gradation by a single number that can be used in mathematical and statistical applications.
2. As a measure of the size and grading of fine and coarse aggregates and aggregate or soil mixtures.
3. As a factor in an equation for predicting the required asphalt content for different gradations of a specific aggregate in bituminous paving mixtures.
4. As a factor in an equation for predicting the effect of changes in gradation on the strength and consistency of portland cement concrete.
5. As an index for measuring changes in gradation due to segregation or degradation of aggregates.
6. As a means of defining the limits of gradations that will have essentially equivalent properties.
7. As a factor for use in predicting the effects of variations in moisture content on soil properties.

TABLE A-2  
COMPUTATION OF  $\bar{A}$  USING STANDARD SIEVES

SIZE GROUP							
SIEVE SIZE				MEAN DIAM., $\bar{d}$ (MM)	GRAD. MODULUS, $\bar{A}$	SEPARATE FRACTION <sup>a</sup> OF AGGREGATE, $f$	GRADATION MODULUS OF FRACTION, $\bar{A}$ ( $f$ )
MAXIMUM		MINIMUM					
NO.	SIZE (MM)	NO.	SIZE (MM)				
3	76.1	1½	38.1	54.8	0.00	0.00	0.00
1½	38.1	¾	19.0	27.6	1.00	0.00	0.00
¾	19.0	⅜	9.51	13.7	2.00	0.03	0.06
⅜	9.51	4	4.76	6.85	3.00	0.15	0.45
4	4.76	8	2.38	3.43	4.00	0.16	0.64
8	2.38	16	1.19	1.72	5.00	0.15	0.75
16	1.19	30	0.595	0.859	6.00	0.15	0.90
30	0.595	50	0.297	0.430	7.00	0.12	0.84
50	0.297	100	0.149	0.214	8.00	0.11	0.88
100	0.149	200	0.074	0.108	9.00	0.07	0.63
200	0.074	400	0.037	0.053	10.00	0.06	0.60
Total						1.00	5.75 = $\bar{A}$

<sup>a</sup> Differences between percentages passing sieves.

TABLE A-3  
COMPUTATION OF  $\bar{A}$  USING ASPHALT SERIES SIEVES

SIZE GROUP							
SIEVE SIZE				MEAN DIAM., $\bar{d}$ (MM)	GRAD. MODULUS, $\bar{A}$	SEPARATE FRACTION <sup>a</sup> OF AGGREGATE, $f$	GRADATION MODULUS OF FRACTION, $\bar{A}$ ( $f$ )
MAXIMUM		MINIMUM					
NO.	SIZE (MM)	NO.	SIZE (MM)				
2½	64.0	1½	38.1	49.9	0.14	0	0
1½	38.1	1	25.4	31.3	0.81	0	0
1	25.4	¾	19.0	22.0	1.32	0	0
1	25.4	½	12.7	18.3	1.58	0	0
¾	19.0	¼	12.7	15.6	1.81	0.02	0.04
½	12.7	4	4.76	8.09	2.76	0.16	0.44
4	4.76	10	2.00	3.18	4.11	0.20	0.82
10	2.00	20	0.841	1.34	5.35	0.19	1.02
20	0.841	40	0.420	0.606	6.50	0.13	0.84
40	0.420	80	0.177	0.281	7.61	0.14	1.07
80	0.177	200	0.074	0.118	8.86	0.10	0.89
200	0.074	400	0.037	0.053	10.00	0.06	0.60
Total						1.00	5.72 = $\bar{A}$

<sup>a</sup> Differences between percentages passing sieves.

## APPENDIX B

### DATA RELATED TO THE CONCRETE STUDY

TABLE B-1a  
MATERIALS USED IN CONCRETE STUDY

ITEM	SOURCE	TYPE	ABSORP- TION (%)	BULK SP. GR.	LOS ANGELES ABRASION B (% LOSS)
Crushed stone	Liberty Corp., <sup>a</sup> Philadelphia, Pa.	Argillaceous limestone	0.72	2.73	21
Gravel	Liberty Corp., <sup>a</sup> Philadelphia, Pa.	Siliceous	0.95	2.60	28
Fine aggregate	Liberty Corp., <sup>a</sup> Philadelphia, Pa.	Natural siliceous concrete sand	1.21	2.61	—
Cement	Allentown Corp., Pa.	1	—	3.5	—
Water	Philadelphia city water supply	—	—	—	—
Air-entraining admixture	—	Darex	—	—	—

<sup>a</sup> Commercial quarry.

TABLE B-1b  
GRADATION OF AGGREGATES

FINE AGGREGATE		COARSE AGGREGATE (% PASSING-RETAINED)			
SIEVE	% PASSING	SIEVE	GRADATION A, COARSE	GRADATION B, NORMAL	GRADATION C, FINE
No. 4	95.8	1 In.—¾ In.	32	5	0
No. 8	81.0	¾ In.—½ In.	43	50	35
No. 16	55.9	½ In.—¾ In.	8	10	15
No. 30	38.9	¾ In.—No. 4	17	30	40
No. 50	11.5	No. 4—No. 8	0	5	10
No. 100	2.5				
$\bar{A}$	5.86		1.90	2.30	2.58

TABLE B-1c  
EQUIPMENT USED IN CONCRETE STUDY

ITEM	DESCRIPTION AND MANUFACTURER
Mixer	Gilson, 3½-cu. ft., Model S-T
Air meter	Concrete Specialties Co., Type B Press-ur-meter
Compaction factor apparatus	Soil test, No. CT 208
Compression testing machine	Forney, 350,000-lb capacity, calibrated and certified Nov. 27, 1963
Cylinder molds	Paraffin-coated paper, metal bottom, Platt Corp., Baltimore, Md. (ASTM C470)
Sieves	Weston, rotary
Scale	Howe, platform, 0.01-lb graduations

TABLE B-2  
MIX PROPORTIONS, TOTAL SOLIDS BY WEIGHT AND VOLUME

TYPE OF MIX	PERCENT BY WEIGHT			PERCENT BY VOLUME			GRADATION MODULUS, <sup>a</sup> $\bar{A}$		
	CEMENT	FINE AGGR.	COARSE AGGR.	CEMENT	FINE AGGR.	COARSE AGGR.	MIX-TURE A,	MIX-TURE B,	MIX-TURE C,
							COARSE	NORMAL	FINE
Crushed stone	15.00	33.15	51.85	13.09	34.90	52.01	4.32	4.55	4.70
Gravel	15.0	54.4	30.6	12.73	31.34	55.93	4.17	4.40	4.55

<sup>a</sup> Total solids.

TABLE B-3  
COMPOSITION OF CONCRETE MIXTURES

TYPE OF MIX	SLUMP (IN.)	AGGREGATE (%)		CEMENT (BAG/YD <sup>3</sup> )	W/C RATIO (GAL/BAG)	EFFECT. WATER (GAL/YD <sup>3</sup> )	ENTRAINED AIR (%)
		COARSE	FINE				
Crushed stone	1.9	61	39	6.0	5.36	32.2	4.0
	4.4	61	39	5.8	5.72	33.2	4.3
Gravel	1.4	64	36	5.9	4.92	29.0	3.8
	3.5	64	36	5.9	5.28	31.2	4.0

TABLE B-4  
BATCH WEIGHTS AND YIELDS

TYPE OF MIX	GRADATION	COARSE AGGREGATE (LB)					SAND (LB)	CEMENT (LB)	ADDED WATER (LB)		AIR EN-TRAIN. AGENT (CC)	YIELD (CU FT)	
		1 IN.- ¾ IN.	¾ IN.- ½ IN.	½ IN.- ⅜ IN.	⅜ IN.- NO. 4	NO. 4- NO. 8			STIFF MIXES	PLASTIC MIXES		STIFF	PLASTIC
Crushed stone	A, coarse	34.49	46.34	8.62	18.32	0.0	69.44	31.33	16.50	17.50	2.8	1.51	1.54
	B, normal	5.39	53.89	10.78	32.33	5.39	69.44	31.33	16.50	17.50			
	C, fine	0.0	37.72	16.17	43.11	10.78	69.44	31.33	16.50	17.50			
Gravel	A, coarse	36.27	48.73	9.00	18.27	0.0	63.90	31.33	15.50	16.50	2.8	1.52	1.53
	B, normal	5.67	56.67	11.33	34.00	5.67	63.90	31.33	15.50	16.50			
	C, fine	0.0	36.67	17.00	45.33	11.33	63.90	31.33	15.50	16.50			

TABLE B-5  
SEQUENCE CONTROL

IDENTIFICATION OF BATCHES				SEQUENCE OF BATCHES					
BATCH NO.	COARSE AGGREGATE		APPROX. SLUMP (IN.)	RUN NO.	BATCH NUMBER				
	TYPE	GRADATION			FIRST DAY	SECOND DAY	THIRD DAY	FOURTH DAY	FIFTH DAY
1	Cr. stone	A, coarse	1½	1	9	2	9	6	4
2		B, normal	1½	2	2	3	5	3	10
3		C, fine	1½	3	5	10	12	1	6
4	Gravel	A, coarse	4	4	3	9	2	4	12
5		B, normal	4	5	8	1	8	5	1
6		C, fine	4	6	1	11	10	11	8
7		A, coarse	1½	7	4	4	3	9	2
8		B, normal	1½	8	11	8	1	2	11
9		C, fine	1½	9	12	6	11	10	5
10		A, coarse	4	10	7	7	7	8	7
11		B, normal	4	11	10	5	6	7	3
12		C, fine	4	12	6	12	4	12	9

TABLE B-6  
7-DAY COMPRESSIVE STRENGTH OF EXPERIMENTAL CONCRETE

COARSE AGGR. TYPE	W/C RATIO	COARSE AGGR. GRADATION	7-DAY COMPRESSIVE STRENGTH (PSI)					AVG.
			BATCH 1	BATCH 2	BATCH 3	BATCH 4	BATCH 5	
Cr. stone	5.36	A, coarse	2540	2945	2650	2670	2785	2718
		B, normal	2705	2745	2960	2715	2405	2706
		C, fine	2950	3080	3140	2925	2980	3015
	5.72	A, coarse	2150	2480	2480	2085	2380	2315
		B, normal	2235	2525	2610	1995	2840	2441
		C, fine	2570	2905	2740	2475	2910	2720
Gravel	4.92	A, coarse	2705	3050	3160	2980	2965	2972
		B, normal	3355	3070	2730	3105	3800	3212
		C, fine	2580	3455	3175	3080	3120	3082
	5.28	A, coarse	2430	2960	2635	2540	2610	2635
		B, normal	2730	2815	3095	2650	2805	2819
		C, fine	2750	2910	3380	3115	3050	3041

TABLE B-7  
28-DAY COMPRESSIVE STRENGTH OF EXPERIMENTAL CONCRETE

COARSE AGGR. TYPE	W/C RATIO	COARSE AGGR. GRADATION	28-DAY COMPRESSIVE STRENGTH (PSI)					AVG.
			BATCH 1	BATCH 2	BATCH 3	BATCH 4	BATCH 5	
Cr. stone	5.36	A, coarse	3365	3755	3320	3390	3160	3398
		B, normal	3410	3250	3230	3180	3385	3291
		C, fine	3630	3790	3965	3515	3960	3772
	5.72	A, coarse	3080	2990	3170	2575	2940	2951
		B, normal	3110	2810	3135	2490	3530	3015
		C, fine	3950	3560	3510	3140	3700	3572
Gravel	4.92	A, coarse	3960	3780	3815	3560	3890	3801
		B, normal	4215	3550	3545	3760	4490	3912
		C, fine	3310	4240	3920	3820	3950	3848
	5.28	A, coarse	3460	3500	3305	2940	3645	3370
		B, normal	3780	3490	3970	3200	3605	3609
		C, fine	3810	3485	3705	3835	3740	3715

TABLE B-8  
SLUMP OF EXPERIMENTAL CONCRETE

COARSE AGGR. TYPE	W/C RATIO	COARSE AGGR. GRADATION	SLUMP (IN.)					AVG.
			BATCH 1	BATCH 2	BATCH 3	BATCH 4	BATCH 5	
Cr. stone	5.36	A, coarse	2.00	2.50	4.00	2.00	3.00	2.70
		B, normal	1.50	2.00	1.75	2.00	2.00	1.85
		C, fine	0.50	0.75	1.50	1.00	1.50	1.05
	5.72	A, coarse	5.50	5.00	6.00	6.50	6.50	5.90
		B, normal	4.50	4.50	3.50	5.50	4.50	4.50
		C, fine	3.75	2.50	3.00	2.50	3.00	2.95
Gravel	4.92	A, coarse	2.50	2.00	2.50	2.50	2.50	2.40
		B, normal	1.00	1.50	2.50	0.75	0.00	1.15
		C, fine	0.50	0.00	0.00	1.50	1.25	0.65
	5.28	A, coarse	6.00	5.00	5.00	4.50	6.00	5.30
		B, normal	4.00	4.00	2.00	3.00	2.50	3.10
		C, fine	3.50	1.50	1.00	2.50	2.00	2.10

TABLE B-9  
LOOSE WEIGHT OF EXPERIMENTAL CONCRETE

COARSE AGGR. TYPE	W/C RATIO	COARSE AGGR. GRADATION	LOOSE WEIGHT (LB/CU FT)					AVG.
			BATCH 1	BATCH 2	BATCH 3	BATCH 4	BATCH 5	
Cr. stone	5.36	A, coarse	137.94	137.74	135.80	130.80	139.31	136.32
		B, normal	133.76	132.65	132.75	133.92	132.39	133.09
		C, fine	132.54	126.69	129.34	129.95	131.27	129.96
	5.72	A, coarse	141.86	143.33	142.77	141.15	144.50	142.72
		B, normal	140.59	138.85	139.52	139.41	140.18	139.71
		C, fine	138.91	134.83	134.78	135.90	137.18	136.32
Gravel	4.92	A, coarse	131.17	133.51	133.56	132.34	135.09	133.13
		B, normal	125.93	128.67	133.26	123.79	121.50	126.63
		C, fine	131.68	124.20	123.12	131.07	132.39	128.49
	5.28	A, coarse	134.22	139.82	138.45	138.60	141.45	138.51
		B, normal	136.00	137.43	130.92	132.59	134.53	134.29
		C, fine	132.49	132.59	125.21	130.92	133.31	130.90

TABLE B-10  
COMPACTED WEIGHT OF EXPERIMENTAL CONCRETE

COARSE AGGR. TYPE	W/C RATIO	COARSE AGGR. GRADATION	COMPACTED WEIGHT (LB/CU FT)					AVG.
			BATCH 1	BATCH 2	BATCH 3	BATCH 4	BATCH 5	
Cr. stone	5.36	A, coarse	142.21	148.27	146.80	139.16	147.81	144.85
		B, normal	146.64	147.66	146.24	146.29	148.17	147.00
		C, fine	143.95	146.69	147.25	147.00	145.98	146.17
	5.72	A, coarse	145.63	146.39	146.74	146.08	146.69	146.31
		B, normal	145.67	144.04	146.74	141.96	146.80	145.04
		C, fine	143.44	145.37	146.24	137.84	146.23	143.82
Gravel	4.92	A, coarse	144.00	145.62	146.03	145.98	146.13	145.55
		B, normal	144.25	145.68	145.01	146.08	147.71	145.75
		C, fine	140.08	145.88	145.52	143.33	145.12	143.99
	5.28	A, coarse	141.45	145.17	145.78	144.50	144.86	144.35
		B, normal	141.60	144.71	146.19	142.62	144.61	143.95
		C, fine	143.64	144.35	146.24	143.49	145.83	144.71

TABLE B-11  
UNIT WEIGHT OF EXPERIMENTAL CONCRETE

COARSE AGGR. TYPE	W/C RATIO	COARSE AGGR. GRADATION	UNIT WEIGHT (LB/CU FT)					AVG.
			BATCH 1	BATCH 2	BATCH 3	BATCH 4	BATCH 5	
Cr. stone	5.36	A, coarse	140.0	149.1	145.3	148.2	145.9	145.70
		B, normal	146.5	147.8	145.8	145.6	147.1	146.56
		C, fine	146.2	146.3	144.7	146.1	145.5	145.76
	5.72	A, coarse	144.0	147.1	145.6	144.8	145.5	145.40
		B, normal	145.7	143.7	145.5	140.2	145.2	144.06
		C, fine	143.1	145.1	145.7	145.2	144.4	144.70
Gravel	4.92	A, coarse	144.9	144.7	146.0	144.6	145.8	145.20
		B, normal	147.0	145.3	144.6	145.0	144.8	145.34
		C, fine	142.7	144.9	144.1	143.0	144.7	143.88
	5.28	A, coarse	142.5	145.1	143.3	143.0	145.0	143.78
		B, normal	142.0	144.8	144.7	141.4	143.8	143.34
		C, fine	143.2	143.1	144.5	143.0	143.7	143.50

TABLE B-12  
WEIGHT OF EXPERIMENTAL CONCRETE IN CYLINDERS

COARSE AGGR. TYPE	W/C RATIO	COARSE AGGR. GRADATION	CONCRETE WEIGHT IN CYLINDERS (LB/CU FT)					AVG.
			BATCH 1	BATCH 2	BATCH 3	BATCH 4	BATCH 5	
Cr. stone	5.36	A, coarse	138.17	144.78	143.04	145.46	144.62	143.21
		B, normal	146.56	146.44	142.18	143.16	143.78	144.42
		C, fine	143.15	145.65	143.82	144.19	142.25	143.81
	5.72	A, coarse	139.44	143.23	143.08	138.83	141.11	141.14
		B, normal	140.28	140.05	142.85	136.69	142.47	140.47
		C, fine	140.95	143.41	142.59	142.76	141.92	142.33
Gravel	4.92	A, coarse	142.47	142.57	143.02	143.59	143.41	143.01
		B, normal	143.74	142.60	140.58	143.02	145.61	143.11
		C, fine	140.64	143.77	143.01	140.83	142.14	142.08
	5.28	A, coarse	139.95	142.85	140.92	140.88	141.44	141.21
		B, normal	139.36	144.02	141.86	138.75	140.93	140.98
		C, fine	140.15	141.22	142.51	141.69	141.88	141.49

TABLE B-13  
PERCENT OF ENTRAINED AIR IN EXPERIMENTAL CONCRETE

COARSE AGGR. TYPE	W/C RATIO	COARSE AGGR. GRADATION	ENTRAINED AIR (%)					AVG.
			BATCH 1	BATCH 2	BATCH 3	BATCH 4	BATCH 5	
Cr. stone	5.36	A, coarse	4.8	3.5	4.5	3.5	4.1	4.08
		B, normal	4.0	3.5	3.5	4.5	3.6	3.82
		C, fine	4.0	4.4	4.5	3.7	4.5	4.22
	5.72	A, coarse	4.5	3.9	4.5	4.5	4.5	4.38
		B, normal	4.5	5.0	4.2	4.5	4.4	4.52
		C, fine	4.0	4.0	3.8	3.5	4.5	3.96
Gravel	4.92	A, coarse	4.2	3.5	3.2	3.5	3.9	3.66
		B, normal	4.1	3.5	3.5	3.2	3.7	3.60
		C, fine	4.0	3.6	4.0	4.3	4.5	4.08
	5.28	A, coarse	4.5	3.8	4.3	4.5	3.5	4.12
		B, normal	4.3	4.2	3.5	4.5	3.7	4.04
		C, fine	3.7	4.3	3.5	3.7	4.5	3.94



## APPENDIX C

## DATA RELATED TO THE HOT-BIN GRADATION STUDY

TABLE C-1  
WASH TEST RESULTS

PLANT NO.	SAMPLE NO.	TEST TYPE	% PASSING-RETAINED							COR-RECTION FACTOR
			NO. 4- NO. 8	NO. 8- NO. 16	NO. 16- NO. 30	NO. 30- NO. 50	NO. 50- NO. 100	NO. 100- NO. 200	NO. 200- PAN	
1	5-A	Dry	17.30	18.03	15.15	25.42	13.58	5.71	4.81	0.84
		Washed	17.25	17.97	15.03	25.05	13.34	5.65	5.71	
	24-A	Dry	11.38	18.09	15.26	26.14	16.92	7.60	4.61	0.75
		Washed	10.60	17.48	15.60	26.19	16.70	7.27	6.16	
	34-B	Dry	17.20	18.41	17.40	20.98	14.39	5.73	5.89	0.85
		Washed	16.10	18.51	17.65	21.03	14.54	5.23	6.94	
	15-A	Dry	7.91	15.17	15.97	28.01	17.55	8.78	6.61	0.83
		Washed	7.37	15.11	15.93	27.74	17.39	8.50	7.96	
	2-A	Dry	17.61	12.86	17.53	23.20	13.44	5.01	10.35	0.84
		Washed	16.94	12.60	17.46	22.87	13.36	4.42	12.35	
Avg.	Dry	14.28	16.51	16.26	24.75	15.18	6.57	6.45	0.82	
	Washed	13.65	16.33	16.33	24.59	15.07	6.21	7.82		
	% Change		-0.63	-0.18	+0.07	-0.16	-0.11	-0.36	+1.37	
2	43-A	Dry	16.34	37.55	18.15	13.56	6.24	4.31	3.86	0.86
		Washed	15.60	30.74	24.67	14.41	6.35	3.74	4.48	
	28-A	Dry	7.46	26.94	21.12	19.24	11.84	7.39	6.02	0.88
		Washed	6.83	19.80	27.57	20.55	12.28	6.14	6.83	
	13-B	Dry	7.92	19.31	25.46	21.29	12.02	7.14	6.86	0.97
		Washed	6.93	18.46	27.72	20.79	12.02	7.00	7.07	
	25-A	Dry	10.15	20.43	25.34	20.30	11.20	5.96	6.61	0.88
		Washed	10.28	27.51	19.58	18.66	10.35	6.09	7.53	
	17-B	Dry	11.87	22.70	24.16	18.26	9.62	6.15	7.24	0.94
		Washed	11.56	29.94	18.44	16.80	9.01	6.51	7.73	
Avg.	Dry	10.75	25.38	22.85	18.53	10.18	6.19	6.12	0.91	
	Washed	10.24	25.29	23.60	18.24	10.00	5.90	6.73		
	% Change		-0.51	-0.09	+0.75	-0.29	-0.18	-0.29	+0.61	
3	18-A	Dry	5.03	18.90	12.51	23.82	20.52	12.99	6.23	0.84
		Washed	4.50	15.24	15.07	25.13	21.47	11.15	7.44	
	33-A	Dry	7.07	23.79	12.76	22.29	16.88	10.56	6.65	0.83
		Washed	6.22	23.26	12.96	22.51	16.67	10.40	7.98	
	32-A	Dry	12.33	15.13	13.07	22.70	19.42	10.10	7.25	0.84
		Washed	11.38	15.08	13.07	22.70	19.21	9.95	8.62	
	31-B	Dry	5.24	16.02	14.45	24.76	20.89	10.94	7.70	0.86
		Washed	4.97	15.50	14.40	24.66	20.68	10.84	8.95	
	4-B	Dry	4.71	19.49	10.46	16.61	13.40	13.57	21.76	0.93
		Washed	4.15	18.94	10.52	16.78	13.23	13.01	23.37	
Avg.	Dry	6.88	18.67	12.65	22.03	18.22	11.63	9.92	0.88	
	Washed	6.24	17.60	13.20	22.37	18.25	11.07	11.27		
	% Change		-0.64	-1.07	+0.55	+0.34	+0.03	-0.56	+1.35	
4	6-B	Dry	7.64	24.06	15.51	20.01	16.81	10.29	5.68	0.88
		Washed	6.91	23.72	17.04	20.12	15.57	10.17	6.47	
	1-A	Dry	5.56	22.56	17.16	21.27	16.78	10.25	6.42	0.93
		Washed	5.34	22.13	17.37	21.64	16.68	9.93	6.91	
	1-B	Dry	12.63	14.57	17.44	19.43	17.39	10.11	8.43	0.94
		Washed	12.00	14.46	17.65	19.64	17.18	10.06	9.01	
	2-A	Dry	4.52	20.00	13.90	18.30	16.60	13.84	12.84	0.96
		Washed	4.28	19.59	14.19	18.42	16.60	13.61	13.31	
	16-A	Dry	5.94	14.09	19.10	20.02	16.95	11.06	12.84	0.96
		Washed	5.61	14.03	19.26	20.08	16.73	10.96	13.33	
Avg.	Dry	7.26	19.06	16.62	19.80	16.91	11.11	9.24	0.94	
	Washed	6.83	18.79	17.10	19.97	16.55	10.95	9.81		
	% Change		-0.43	-0.27	+0.48	+0.17	-0.36	-0.16	+0.57	

TABLE C-2  
INDIVIDUAL ASPHALT EXTRACTION TEST VALUES

SAMPLE NO.	EXTRACTED ASPHALT (%)			
	PLANT NO. 1	PLANT NO. 2	PLANT NO. 3	PLANT NO. 4
1	6.81	6.83	6.58	5.00
2	7.49	6.74	6.36	5.00
3	5.83	6.74	6.00	5.71
4	7.54	6.06	6.13	5.31
5	6.95	6.68	5.76	4.99
6	6.76	6.99	5.74	5.50
7	6.64	6.56	5.65	5.59
8	6.60	7.05	5.58	5.61
9	6.70	6.94	5.72	5.82
10	6.49	6.77	5.90	6.07
11	6.74	7.03	6.32	5.80
12	6.51	6.95	6.31	6.27
13	7.15	6.84	6.07	5.63
14	6.96	7.08	6.02	5.44
15	6.34	7.14	5.87	4.18
16	7.03	7.24	5.84	3.43
17	6.94	6.89	5.81	7.48
18	6.86	6.69	6.04	6.70
19	7.35		6.18	5.74
20	6.91		6.09	5.25
21	7.08		5.84	
22	7.57		5.69	
23	7.06		6.21	
24	7.30		6.26	
$\bar{X}$	6.90	6.85	6.00	5.53
$\sigma_s^2$	0.1628	0.07025	0.0675	0.7098
$\sigma_s$	0.404	0.265	0.260	0.842

TABLE C-3  
JOB-MIX FORMULA GRADATIONS AND TOLERANCES

PLANT NO.	PERCENT PASSING										
	1½ IN.	1 IN.	¾ IN.	½ IN.	⅜ IN.	NO. 4	NO. 10	NO. 20	NO. 40	NO. 80	NO. 200
1	—	—	—	100	95±5	87±5	67±4	—	31±3	13±3	5±1
2	—	—	—	100	98±5	83±5	65±4	—	27±3	16±3	5±1
3	—	—	100 <sup>a</sup>	98±7	86±5	61±5	45±3	33±3	24±3	11±3	4±2
4	100±7	93±7	80±7	65±7	—	42±6	34±6 <sup>b</sup>	—	—	—	—

<sup>a</sup> ⅝-In. sieve.    <sup>b</sup> No. 8 sieve.

## APPENDIX D

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## APPENDIX E

### SUMMARY OF APPENDIX ITEMS NOT PUBLISHED

Complete IBM 1410 computer print-out summary sheets of the data from the asphalt hot-bin study (Chapter Three) were submitted with the research agency's report. A print-out of the FORTRAN program used for analysis of the data was also included. They are not published here, but

may be obtained on a loan basis by qualified researchers by writing to: Program Director, NCHRP, Highway Research Board, 2101 Constitution Avenue, Washington, D.C. 20418.

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