NATIONAL COOPERATIVE HIGHWAY RESEARCH PROGRAM REPORT **71**

ANALYTICAL STUDY OF WEIGHING METHODS FOR HIGHWAY VEHICLES IN MOTION

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NATIONAL RESEARCH COUNCIL
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SUBJECT CLASSIFICATION:

- TRANSPORTATION ADMINISTRATION
- ROAD USER CHARACTERISTICS
- TRAFFIC MEASUREMENTS

HIGHWAY RESEARCH BOARD
DIVISION OF ENGINEERING NATIONAL RESEARCH COUNCIL
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NATIONAL COOPERATIVE HIGHWAY RESEARCH PROGRAM

Systematic, well-designed research provides the most effective approach to the solution of many problems facing highway administrators and engineers. Often, highway problems are of local interest and can best be studied by highway departments individually or in cooperation with their state universities and others. However, the accelerating growth of highway transportation develops increasingly complex problems of wide interest to highway authorities. These problems are best studied through a coordinated program of cooperative research.

In recognition of these needs, the highway administrators of the American Association of State Highway Officials initiated in 1962 an objective national highway research program employing modern scientific techniques. This program is supported on a continuing basis by funds from participating member states of the Association and it receives the full cooperation and support of the Bureau of Public Roads, United States Department of Transportation.

The Highway Research Board of the National Academy of Sciences-National Research Council was requested by the Association to administer the research program because of the Board's recognized objectivity and understanding of modern research practices. The Board is uniquely suited for this purpose as: it maintains an extensive committee structure from which authorities on any highway transportation subject may be drawn; it possesses avenues of communications and cooperation with federal, state, and local governmental agencies, universities, and industry; its relationship to its parent organization, the National Academy of Sciences, a private, nonprofit institution, is an insurance of objectivity; it maintains a full-time research correlation staff of specialists in highway transportation matters to bring the findings of research directly to those who are in a position to use them.

The program is developed on the basis of research needs identified by chief administrators of the highway departments and by committees of AASHO. Each year, specific areas of research needs to be included in the program are proposed to the Academy and the Board by the American Association of State Highway Officials. Research projects to fulfill these needs are defined by the Board, and qualified research agencies are selected from those that have submitted proposals. Administration and surveillance of research contracts are responsibilities of the Academy and its Highway Research Board.

The needs for highway research are many, and the National Cooperative Highway Research Program can make significant contributions to the solution of highway transportation problems of mutual concern to many responsible groups. The program, however, is intended to complement rather than to substitute for or duplicate other highway research programs.

This report is one of a series of reports issued from a continuing research program conducted under a three-way agreement entered into in June 1962 by and among the National Academy of Sciences-National Research Council, the American Association of State Highway Officials, and the U. S. Bureau of Public Roads. Individual fiscal agreements are executed annually by the Academy-Research Council, the Bureau of Public Roads, and participating state highway departments, members of the American Association of State Highway Officials.

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FOREWORD

By Staff

Highway Research Board

This report will be of particular interest to those individuals interested in developing weighing systems to determine the static weight of any vehicle by measurement of the dynamic forces developed by the vehicle in motion. The various dynamic waveforms associated with the highway vehicles were reviewed, and several computational procedures were employed to ascertain axle weights from the expected sampled force data which would be provided by a number of platform transducers or scales. This report describes the digital or digital-analog computer systems required to make the recommended computations.

Modern techniques for highway pavement design and performance evaluation are dependent on data regarding vehicle axle loads and their frequency and time distribution. Much better estimates than are now available are required. Conventional weighing equipment interrupts traffic flow, requires excessive manpower, and may result in statistical bias in samples. Equipment is needed that automatically determines axle weight without a change in vehicle speed, or other traffic interference, or driver awareness, and that instantaneously records the weight and time of passing of all vehicles, especially axle loads in excess of 3,000 pounds.

Techniques are available to measure and record the distribution of axle loads by electronic methods. However, improved systems are desirable for automatic data processing and the presentation of summary statistics. The primary objective of this research was to design a vehicle-in-motion axle-weighing system so that every axle of significant weight which crosses the location of the device will be weighed within an accuracy of ± 5 percent of its true static weight.

The primary emphasis of this study was on the errors associated with weighing highway vehicles in motion. A review of vehicle dynamics indicated the type of waveforms associated with highway vehicles. It was assumed that sampled force data would be provided by several platforms. Consequently, the research involved the investigation of computational methods for estimating the static weight from these recorded dynamic forces.

Methods for estimating the static axle weight from sample dynamic force data included averaging, dynamic models, interlacing polynominals, and regression analysis. A preliminary system for the detection and the analysis for weighing vehicles in motion is shown diagrammatically. Estimated costs are provided for developing recommended systems.

The Franklin Institute's research presented in this report complements the studies performed by others who are developing hardware for weighing vehicles in motion. The analytical procedures and the computer programs should assist others in developing a prototype system to actually determine the static weight from dynamic measurements.

CONTENTS

ACKNOWLEDGMENTS

This study was conducted by The Franklin Institute Research Laboratories, Philadelphia, Pa., in conjunction with Project 7-3 of the National Cooperative Highway Research Program.

The analytical studies of error associated with the weighing of highway vehicles while in motion were conducted in two parts; the first extended from February 1964 to April 1965, and the second from January 1966 through February 1967. R. Clyde Herrick of the Mechanical and Nuclear Department was the principal investigator. The analyses performed required the efforts of many members of the Franklin Institute staff, and certain of these efforts must be noted here. Acknowledgment is made to Dr. M. M. Reddi for his considerable contribution toward the Dynamics Methods, Integral Model III and for additional studies not directly reported. Thanks are also extended for the assistance given by Dr. Kishor D. Doshi and Miss Diana Fackenthal.

In addition, the cooperation of the Michigan State Highway Department Research Laboratory and the University of Kentucky in discussing their experience with, and installations for, in-motion weighing of vehicles is appreciated.

ANALYTICAL STUDY OF WEIGHING METHODS FOR HIGHWAY VEHICLES IN MOTION

SUMMARY

This report describes an analytical study program for the investigation of error associated with the weighing of highway vehicles while in motion, using multi-platform electronic scales placed in the highway. The error associated with a weighing system stems from two sources: (1) the electronic equipment for the measurement of force on a highway, and the processing of data; and (2) the computational procedures that are employed to ascertain axle weight from the sampled force data provided by the platforms. Accordingly, the purposes of this study were to:

1. Investigate a number of mathematical methods by which the weight of any given axle could be computed to within 5% of the actual weight.
2. Describe (including cost and accuracy) physical equipment by which certain of these methods may be implemented.

A review of vehicle dynamics indicated the type of waveforms that are associated with highway vehicles. This may be summarized as follows. The waveform of force at any axle is composed of a static part proportional to axle weight, and a dynamic part consisting of a number of damped harmonic components reflecting the various oscillatory modes of the vehicle and dynamic excitation. Further investigation disclosed the feasibility of using an axle force model consisting of a static component plus non-damped periodic oscillatory components in the analytical studies, in the absence of adequate experimental data. The deletion of damping of the oscillatory components was warranted by the use of force records not longer than one or two basic wavelengths. The review of vehicle dynamics also indicated that a good estimate of weight could be obtained by discerning the static component of axle force over a short interval in the absence of significant excitation (a relatively smooth highway surface where weighing).

The investigation of computational methods for determining weight from the sampled force data provided by the platforms included the following:

1. *Direct averaging methods, whereby the force over platforms was simply averaged to yield an estimate of axle weight.* The study showed that for an equipment cost of $40,000 to $45,000, axle weight can be computed to within 5% most of the time where vehicles are oscillating 20 to 25% about the static weight. This includes speeds to 60 mph, although the cost and error is somewhat proportional to vehicle speed.
2. *Dynamic methods, whereby an analogous mechanical oscillator is employed to fit a sine curve through the oscillating component of force in the average.* This seemed warranted by the character of axle force where experimental data confirmed the dominance in amplitude of the low-frequency components. The methods were not fully exploited when other methods promised better accuracy.
3. *Integral Model III, whereby a regression model approximating the second integral of axle force was used to compute weight.* The method showed promise;

but the investigation was halted when the accuracy was not as good as other methods, while the amount of computation was fairly high.

4. *Method VWEIGH, an averaging process whereby interpolation is employed to yield a continuous force data record.* The interpolated record is then averaged to yield a weight estimate that is better than the average over the platforms. This weight estimate is then used as a key value to determine a basic wavelength of the data record. Finally, the average of the interpolated data record over the computed one-wavelength yields, in most cases, the best estimate of weight. Computer programs for the use of this method at a data center are included in this report. Highway equipment for recording weight data costs approximately $55,000. Data center costs are about $100 per 800 to 900 axle weights computed. With oscillation of 20 to 24% about the static weight, the accuracy of computed weight can be within 3%, with only an occasional larger error.

This study showed that simple averaging over the platforms is an economical method that can be employed, using readily available standard equipment at a highway site, with maximum error generally less than 5 to 6%, depending on vehicle oscillation. More accurate weight computation is much more expensive and can presently be done most economically at a data center with sampled force data recorded at a highway site.

CHAPTER ONE

INTRODUCTION

It has long been possible to construct highways of great endurance; but these are so costly that, even with today's budget estimates, only a fraction of the present mileage could be constructed if only the most superior materials and practices were used.

Thus, if construction of the ultimate technically possible in highways is accepted as being impractical, it is necessary to construct highways of lesser endurance at lower cost; this is done by anticipating a certain loading and the prevailing climatic and environmental conditions.

To anticipate loading requires a knowledge of the types, weights, and frequency of vehicles that will use a highway. However, the needed knowledge is not the load imposed by any single vehicle; rather, it is the load spectrum or density imposed by all vehicles that use the highway.

For burden vehicles, it is not always possible to estimate the total load, let alone the load per point of application to the highway surface (load per axle or per wheel); therefore, the practice has been to make periodic weighings of the heavier vehicles. This has been done not only for the enforcement of load limits, but, by recording the weights of all vehicles that pass over the scales, an indication of the highway load spectrum could be obtained. It has been necessary in most cases to divert the vehicle from the active highway to a weighing station, where a mechanical scale generally is employed. Even for fixed installations, the relatively slow response of mechanical weighing systems has required the vehicle to stop or to move slowly across the scale platform. Where fixed-scale installations are not feasible, portable mechanical scales have been used widely to weigh one or more wheel assemblies at a time. Although accurate weight can be determined by the mechanical scale (including those using fluid pressure), whether fixed or portable, the procedure is time-consuming and costly. When the line of vehicles waiting to be weighed extends onto the active traffic lanes, vehicles that are under the load limit, as well as all traffic on the highway, are delayed.

During more recent years the development of electronic scales has been watched closely by highway engineers. Electronic scales now compete with mechanical scales in fixed weighing installations, but the highway engineer has looked beyond this application to foresee highways incorporating built-in, accurate, automatic weighing devices. For research, it now seems possible to gain knowledge of the loading spectra (frequency of loading vs magnitude) for a given segment of a highway.

The electronic scale, simply described, is a platform or treadle containing, or supported by, various force transducers that provide an electrical signal proportional to the force on the platform. Various designs have been highly developed and serve well in fixed-scale installations. A number of such scales have also been placed in active highways and in lead-in strips to existing weighing stations; the "weight" from the electronic scale (weighing the moving vehicle) can then be compared with the "static weight" measured by the fixed-scale installation. These two "weights" seldom are in agreement when a simple one-platform electronic scale is used for weighing the moving vehicle.

The major defect of present in-highway scales is this "error" recorded for vehicles in motion that is caused by dynamic oscillation of vehicles as they cross the scales. This was the concern of this study, the purpose of which was to investigate as many aspects of the weighing problem as practicable, and to indicate the feasibility and economics of a number of methods to weigh each successive axle within an accuracy of 5%.

This study was not intentionally biased toward any particular purpose in weighing. Instead, an effort was made to cover methods and systems that could be used for making unbiased statistical studies of axle weights for highway-planning purposes, for enforcing stated load limits, or even for checking axle or vehicle weight in a particular industry.

THE PROBLEM

The vehicle can be described as a damped oscillatory system with a number of natural frequencies, the lower of which are the oscillations of the vehicle body and payload upon the vehicle suspension system (including springs and tires). This includes basically two modes—bouncing up and down, and pitching fore and aft—although it includes a small amount of flexing of the vehicle structure. For motor trucks, the frequencies range from about 2 to 8 cycles per second (cps) for the body and payload and from 6 to 15 cps for axle assemblies. Thus, the vehicle is oscillating in various modes as it moves forward, causing the force on the highway surface to vary. Experimental measurements (*1, 11*) have shown that this variation is as great as 30 to 40% of the weight at any axle. For this reason a simple scale cannot be used to weigh the vehicle. If the scale were long enough to weigh the whole vehicle while it described at least one complete cycle of the lowest natural frequency, the scale would be so long that a number of vehicles could be on it at one time. However, if the scale were made very short to weigh one axle at a time, its length would have to be on the order of 3 ft to prevent tandem and triple axles from being on the scale at the same time. The short scale will measure the force while the axle is over the scale, but, as already stated, this "force" can differ from the "weight" by as much as 30 to 40%.

Consider sinusoidal motion of a simple spring and mass system:

$$X = X_0 \sin \omega t \tag{1}$$

Maximum acceleration is

$$a = \left.\frac{d^2X}{dt^2}\right|_{\max} = \omega^2 X_0 \tag{2}$$

The force is

$$F = W\left(1 + \frac{\omega^2 X_0}{g}\right) \tag{3}$$

in which

W = weight;
ω = circular frequency;
X_0 = oscillation amplitude; and
g = acceleration due to gravity.

If a frequency of 2 cps is considered,

$\omega = 2\pi f = 2\pi(2) = 12.56$ rad/sec; and
$g = 386$ in./sec²

then

$$F = W\,(1 + 0.408\,X_0) \tag{4}$$

Similarly, for higher frequencies,

$$\left.\begin{array}{ll} 4 \text{ cps}, & F = W\,(1 + 1.63\,X_0) \\ 10 \text{ cps}, & F = W\,(1 + 10.2\,X_0) \end{array}\right\} \tag{5}$$

Thus, if an oscillation amplitude of $X_0 = 0.1$ in. is considered, the error in weight is 4.08% at 2 cps, 16.3% at 4 cps, and 102% at 10 cps. The assumption of $X = 0.1$ in. is purely illustrative, but in this trivial case the relation between the magnitude of the error, $\omega^2 X_0$, and the amplitude, X_0, is linear; therefore, the error associated with an amplitude that is *n* times as great is also *n* times the error for $X_0 = 0.1$ in. at a particular frequency. The actual amplitude of a loaded truck may be many times 0.1 in.; therefore, appreciable error is possible if only a part of the waveform is sampled by the weighing device.

Literature on the weighing of moving vehicles indicates that dynamic vehicle oscillation has been recognized. Smooth lead-in strips have been prepared on the highway preceding the weighing platform to reduce the amount of excitation transmitted to the vehicle immediately before it crosses the weighing platform. This is a good practice, but it cannot be depended on to increase the accuracy by the amount desired. Because the tolerance on a smooth section of highway is, at best, about ⅛ in. over only 16 ft, it is possible that a vehicle may be excited to a sufficient amplitude to preclude accurate weighing by these small random variations in the height of the smooth lead-in strip. The extent of the vehicle response depends on the amplitude and spacing of the highway surface variations plus the natural frequency, damping, and speed of the vehicle.

Even if a perfectly smooth lead-in strip could be obtained, it would take a number of cycles for the amplitude to decay to a level that would not seriously affect the accuracy of recorded weight. For example, assume that a vehicle is oscillating with an amplitude of X_0 in. at the start of a perfectly smooth lead-in strip. Assume further that the vehicle structure, tires, and suspension system has equivalent viscous damping of approximately 0.1 that of critical. The damping ratio in simple vibration theory is then $\zeta = 0.1$. Textbook references on logarithmic decrement show that the amplitude of free oscillation, *k* cycles after some particular time, is

$$X_k = X_0\, e^{-k\delta} \tag{6}$$

in which

X_0 = amplitude at the reference point;
k = number of cycles; and
δ = logarithmic decrement defined as

$$\delta = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}} \tag{7}$$

The ratio of the amplitudes is

$$X_k/X_0 = e^{-k\delta} \tag{8}$$

Where damping of $\zeta = 0.1$, as assumed for this illustration, the ratio of the amplitudes for successive cycles is as shown in Figure 1. Assume that without the perfectly smooth lead-in strip the error would be on the order of 20 to 50%. It would be desirable to reduce this to 5%, or even lower, if possible. Assume a reduction of error to 0.1 that associated with no lead-in strip so that the 20 to 50% error would reduce to 2 and 5%, respectively. Therefore, the amplitude of oscillation would have to be reduced to 0.1 that of the original. Figure 1 shows that for a damping ratio of $\zeta = 0.1$ nearly four cycles are required to accomplish this reduction. For vehicles with twice as much damping this can be accomplished in one-half the number of cycles (see Fig. 1). Four cycles may not seem to be many, but when the low frequencies of loaded vehicles and the speed at which they move on the open highway are considered, the length of the required perfectly smooth strip becomes more impressive.

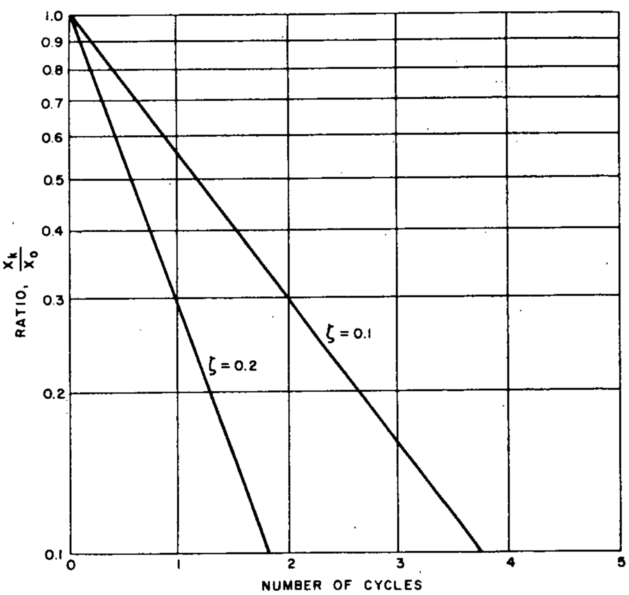

Figure 1. Decay of free damped vibration.

Consider a vehicle operating at 60 mph; it will cover 88 ft each second. If the frequency of oscillation is 2 cps, this wavelength of 44 ft would require the perfectly smooth lead-in strip to be on the order of 150 to 160 ft to effect a one-decade reduction (see Fig. 1). By comparison with the usual dimensions associated with highway engineering this is not large, but, when it is considered that this is the length that shall not contain a disturbance in the surface profile greater than 0.1 in., the problem becomes more significant.

It appears, therefore, that the smooth lead-in strip is a step in the right direction, but for speeds encountered on modern highways it is not sufficient to ensure accuracy.

Because it appears to be impractical to dissipate all oscillations before the vehicle crosses the platform, the next approach to the improvement of accuracy is to consider the data that can be recorded as the vehicle crosses the weighing platform. Again, consider a vehicle operating at 60 mph. If this vehicle has an oscillation frequency of 2 cps, a platform of 44 ft in the direction of travel would be required to gather data from one complete cycle. This is not possible for highway traffic because the record would show the effect of other axles of either the same vehicle or different vehicles. Because axle spacings are relatively close on tandem-axle vehicles, the length of the platform should be less if the recorded weight is to be the load per axle, which is the load limited by most state highway departments.

Previous electronic scale platforms have varied from 3 to 7 ft in the direction of travel. For vehicles operating at 60 mph, Table 1 gives the time that a wheel is on the platform and the fraction of a cycle measured.

The data that can be gained from one 3-ft or 7-ft platform for vehicles operating at 60 mph, or even 30 mph, are too meager to reconstruct the actual waveform of the frequencies anticipated, especially when the frequencies are not known. Some means must therefore be provided to obtain more knowledge of the waveform, particularly of the longer wavelength components that have the larger amplitude.

TABLE 1

WHEEL TIME ON PLATFORM AND FRACTION OF CYCLE MEASURED [a]

PLATFORM LENGTH (FT)	WHEEL TIME ON PLATFORM (SEC)	NO. OF CYCLES AT A FREQUENCY OF 2 CPS	5 CPS	15 CPS
3	0.03408	0.068	0.170	0.51
7	0.07952	0.159	0.398	1.19

[a] For vehicles operating at 60 mph.

CHAPTER TWO

FINDINGS AND APPRAISAL

The first step in the solution of the vehicle axle weighing problem is to consider the placing of additional force-sensing platforms in the highway, with the platforms spaced to supply force data over a greater portion of the axle force waveform.

The second and largest step is determining what to do with the force data that have been obtained, so as to extract the elusive axle weight. To begin this second step, the dynamics of vehicles were reviewed to establish the basic characteristics of axle force waveforms (see Appendix A). The vehicle dynamics appear to be generally such that the random excitations—mostly from variations in the height of a road surface—cause the vehicle to oscillate in various modes, the lowest of which tend to produce the greatest fluctuations in axle force about a long-term static component. This static component is proportional to weight—the constant of proportionality approaching unity as the angle of inclination of non-curved surfaces approaches zero.

As such, the basic requirement in weighing an oscillating vehicle is an adequate set of data from which the constant component of force can be determined within the accuracy desired. This implies that either (1) data be taken over a very long term or (2) certain conditions can be maintained during weighing that allow the real static component to be closely estimated from considerably less data. These conditions are approximated by a smooth level roadway, with little acceleration or deceleration.

One method of finding the static component is to average the data, but, as shown in this report, this is not generally very accurate, because a large number of basic cycles should be sampled continuously if low error is desired. This is discussed in Appendix C, under "Theoretical Accuracy of Averaging."

Instead of taking a long integral average over a number of cycles, a little knowledge of the probable waveform produced by an axle can allow N force-sensitive platforms of from 1 to 3 ft (length in the direction of travel) to be spaced over some optimum distance. This method has distinct possibilities of providing adequate accuracy and providing it economically, as shown by the discussion of actual physical systems (Appendix H). Instead of simple averaging, the set of force data can be analyzed to determine the basic oscillatory characteristics of the vehicle in order to compute the constant component of force. This, in a very real sense, is what was attempted in the dynamic methods (Appendix D).

The technique investigated was that of passing a sinusoidal curve through the sampled data of unknown amplitude and periodicity with the application of an equivalent one degree-of-freedom mechanical oscillator. The idea followed from the dynamics of a complex vehicle which indicated that the various modes usually combine to form what appears to be a distorted and "noisy" sine curve. This was verified by tire pressure curves from *The AASHO Road Test* (*1*), reproduced here as Figure A-6, and later by an analog strip chart tracing of axle force made by placing strain gauges on the axle of a vehicle. The axle force curves are not sinusoidal, but a damped sine wave passed through a basic cycle of the curves fits fairly well. (Consider the non-linear tire pressure characteristics, Fig. A-6.)

Limited experimental axle force data supplied by the Research Laboratory of the Michigan State Highway Department and the University of Kentucky also indicated that a large amount of the data generally can be fit by a damped sine wave. This does not mean that the data fit on a sine wave, but that a damped sine wave superimposed on the data will represent it sufficiently so that the mean of the sine wave is approximately the mean of the data. This is all that is necessary. However, the solutions employed for the dynamic methods did not bring about the desired results. When other methods appeared at the time to yield better results, the investigation was halted without fully exploiting the possibilities.

Following this, various operations were made upon the expression of axle force until it was found that a sample regression equation could be stated to satisfy closely the first terms of the second integral of force over some convenient period. This is Integral Model III (Appendix E). It showed good possibilities for two-component waveforms approximating the two basic frequencies of a simple vehicle. However, the large error that can be triggered by what appears to be an almost inevitable frequency spread in real signals caused the investigation of this model to be halted also. Physical systems to compute weight by this method would require development of a specific analog system, or would require that the force data be processed later at a computer center. No cost estimates of an analog system were attempted.

Because it had been evident that many vehicles oscillate with the basic or lowest frequency components producing the greater part of axle force variation, it was recognized that a fair approximation of the real static component could be made by first detecting one basic cycle or wavelength in the data and averaging the axle force signal over that period. Although the force signal might be such that the approximation of a basic wavelength could be poor when the actual repeat of the waveform required many wavelengths of the lower frequencies, it appeared that such a method would certainly be more accurate than the average over the platforms only. This was essentially implicit in the dynamic models, although it is not exactly the same.

When more analytical methods failed to predict a reasonable period from the rather meager data obtained from a limited number of platforms, a method was developed to attack it directly. This is method VWEIGH (Appendix G), which consists of interpolating between platforms by means of a series of polynomial curves fitted through the data of each successive group of three platforms. The whole interpolated data record was averaged to yield a key value which in itself was better than the average over the platform only. The interpolated data record was tested to see where it crossed the key value. It was then averaged between the first and third crossings. This worked quite well. Although the method was not fully developed, it appears to yield excellent accuracy. However, this method is expensive. It cannot be accomplished at a weighing site in real time in its present form unless a large computer is available there. The method appears feasible economically only if data are recorded at a site and taken to a computer center for weight computation. If this is satisfactory, the method can yield good accuracy for almost any given axle. Platforms and the recording system would cost about $55,000.

However, if the purpose for weighing is such that an average error per axle between 3 and 5% (with an occasional error beyond that) is tolerable, then the simple averaging methods can be used to advantage in real time on the site with approximately $40,000 to $50,000 worth of equipment. This includes the platforms and all electronics, but not any housing of the equipment.

The third step in the solution of axle-weighing problems is the choice of the equipment to sample force data and compute the weight. With the wide choice of very stable electronic equipment available as standard commercial components and modules, this is not particularly difficult. Some representative systems are suggested and priced (approximately) to indicate what can be accomplished and for how much. This is discussed in Appendix H.

CHAPTER THREE

RECOMMENDATIONS

The recommendations concerning this study are related to two distinct aspects: (1) weighing of vehicles just to determine a weight, and (2) weighing of vehicles as associated with further highway research.

The first aspect, where little information other than the vehicle weight is required, is important because this is desired in many situations that are related to highways and highway industries. One situation is the constant monitoring of axle weights for enforcing load limits and collecting statistical data; another is the possibility of assessing toll charges for bridges and highways, computed on-the-spot from computed weight categories. Still other applications are within the trucking industry. For those applications that do not require exact weight, a multi-platform digital or digital-analog simple averaging system as herein presented is recommended. These system installations can be engineered directly from the results of this study and assembled from commercially available components.

The second aspect—that of weighing associated with further highway research—needs further attention. There is much concerning the dynamic characteristics of highways, or, more rightly, the highway-vehicle system, that requires more investigation. Such studies are being given much consideration. In this regard, the application of "weighing systems," or, in this context, "data collection and conversion systems," can be of great assistance in experimental investigations of highway-vehicle dynamics. This project was initiated to provide a means of collecting unbiased weight data for further highway studies. However, there appears to be a greater need for research into the highway-vehicle relationships. To implement such studies, the use of platforms such as those developed by Lee (*11*) is recommended, because they are relatively thin (slightly more than 1 in. thick) and would not seriously affect the pavement-vehicle dynamics where emplaced. Data recording systems, such as those discussed in this report, could then be used to record force and other dynamic measurements for later computer processing.

Further investigation of all the methods herein presented is also recommended. This was intended as an initial study to show the feasibility of certain approaches; it could not be absolutely conclusive.

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APPENDIX A

TECHNICAL DISCUSSION OF AXLE FORCE

A highway vehicle can be described as a damped mechanical oscillatory system that has a number of natural frequencies. Consider a simple truck, as shown in Figure A-1, and consider only the plane shown. By lumping the truck body and payload together and thereby neglecting frequencies characteristic to the frame, body, cab, engine, and payload, there is still a four degree-of-freedom system: (1) translation of M_1, (2) rotation of I (mass M_1), (3) translation of M_2, and (4) translation of M_3. Assume for the present that the mass of the axle assemblies, M_2 and M_3, are small compared with M_1. Thus, the vehicle is reduced to a two degree-of-freedom system. Although the suspensions of vehicles generally do not constitute linear systems, they are sufficiently linear to be deemed as such for purposes of this study. Assuming that they are linear oscillatory systems, the motion of the truck in Figure A-1, neglecting axle masses, is described by the following equations:

$$M_1 \frac{d^2z}{dt^2} + (k_f + k_r)\, z + (l_f\, k_f - l_r\, k_r)\, \theta = k_f\, A_f(t) + k_r\, A_r(t) - M_1 g$$

$$I \frac{d^2\theta}{dt^2} + (k_f\, l_f^2 - k_r\, l_r^2)\, \theta + (l_f\, k_f - l_r\, k_r)\, z = k_f\, l_f\, A_f(t) - k_r\, l_r\, A_r(t) \qquad \text{(A-1)}$$

in which

z = linear bouncing displacement;
θ = angular pitching displacement;
k_f = combined spring rate of front suspension;
k_r = combined spring rate of rear suspension;
M_1 = mass of truck and payload;
I = mass moment of inertia about c.g.;
$A_f(t)$, $A_r(t)$ = variation in road surface elevation as a function of time, at front and rear, respectively; and
g = acceleration due to gravity.

This description of vehicle dynamics is illustrative only; damping is neglected altogether in the equations so as not to complicate the example.

Note that two equations involving the two distinct quantities, $M_1 \frac{d^2z}{dt^2}$ and $I \frac{d^2\theta}{dt^2}$, are required to describe the motion of the vehicle in the plane shown. This gives rise to two characteristic modes of oscillation and to two natural frequencies. The vehicle may be excited to a sufficient amplitude at either or both of these frequencies by repeated road irregularities so as to produce a significant force variation between the tires and the road. If the road surface irregularities are small, the variation in the wheel force due to running over any one disturbance may also be small. But, if the spacing of these disturbances and the velocity of the

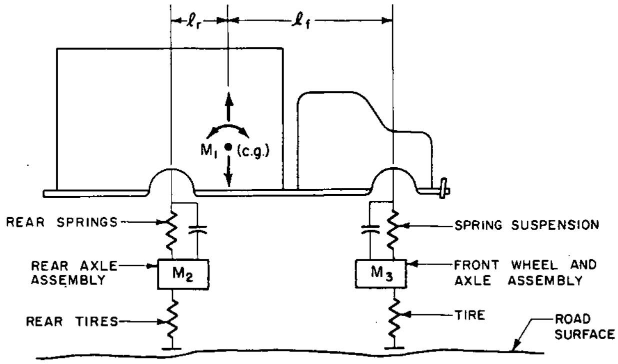


Figure A-1. Dynamic schematic of a two-axle vehicle.

vehicle are such as to effect a series of small bumps at a frequency close to a natural frequency of the vehicle, the energy of oscillation may be increased to a large value after many cycles or until the energy dissipated in friction per cycle equals the energy added from each disturbance.

Once the vehicle has been put into oscillation by a large disturbance (or by a series of small ones) the tire force on even a smooth road surface will vary so long as the oscillations last. This can be for a number of cycles. The resultant waveforms will be made up of two-component waveforms, as shown in Figure A-2. These are not necessarily those to be expected from a vehicle, but serve to illustrate a variety of two-component waveforms in this class.

In addition to the foregoing, another phenomenon can be seen from the equations of motion. Note that $(l_f\, k_f - l_r\, k_r)$ is the coefficient of θ in the bouncing motion equation and also the coefficient of z in the pitching motion equation. These are the terms that indicate coupling of bouncing and pitching motions. If this coefficient were zero, that is, if $\frac{k_f}{k_r} = \frac{l_r}{l_f}$, the bouncing and pitching motions would take place independently of each other. However, the possibility of these modes of oscillation being uncoupled is remote, although the coupling may be small. In any event, there are two characteristic frequencies that may become excited.

The general problem of the coupled modes of free bouncing and pitching of a two-axle vehicle is discussed by Timoshenko (*6*, p. 199-204). The equations for displacement in bouncing and pitching are given by Timoshenko (*6*, Eq. x, p. 203). These are:

$$\begin{aligned} z &= A_1 \cos(\omega_1 t + \alpha_1) + A_2 \cos(\omega_2 t + \alpha_2) \\ \theta &= B_1 \cos(\omega_1 t + \alpha_1) + B_2 \cos(\omega_2 t + \alpha_2) \end{aligned} \quad \text{(A-2)}$$

in which

ω_1 and ω_2 = the natural circular frequencies of the two modes of oscillation;
α_1 and α_2 = phase angles; and
A_1, A_2, B_1, and B_2 = constants governing the amplitude.

The periodic component of the force between tires and road on the front axle is then

$$F = k\, z + k\, l_f \theta \quad \text{(A-3)}$$

Substituting for z and θ from Eq. A-2 yields

$$\begin{aligned} F = {} & (A_1 k + B_1\, l_f k) \cos(\omega_1 t + \alpha_1) \\ & + (A_2 k + B_2\, l_f k) \cos(\omega_2 t + \alpha_2) \end{aligned} \quad \text{(A-4)}$$

Because the coefficients of the cosine terms are all constants, the periodic component of force may be written as

$$F = A_3 \cos(\omega_1 t + \alpha_1) + A_4 \cos(\omega_2 t + \alpha_2) \quad \text{(A-5)}$$

to show that the periodic component of force on the front suspension system can be written as a linear combination of the principal modes of oscillation.

If the modes of oscillation are coupled and if the two natural frequencies are relatively close in frequency, an interesting phenomenon occurs. By following Timoshenko's analysis it can be seen that the expressions for the bouncing and pitching displacement can be put into the form shown by Timoshenko (*6*, Eq. z, p. 204). These are

$$\begin{aligned} z &= \lambda \cos\left(\frac{\omega_1 + \omega_2}{2}\right) t \cos\left(\frac{\omega_1 - \omega_2}{2}\right) t \\ \theta &= \frac{\lambda}{i} \sin\left(\frac{\omega_1 + \omega_2}{2}\right) t \sin\left(\frac{\omega_1 - \omega_2}{2}\right) t \end{aligned} \quad \text{(A-6)}$$

in which λ is an initial displacement and i is the radius of gyration. Here $(\omega_1 - \omega_2)$ is small compared with ω_1 or ω_2. Hence, it is seen that the functions, $\cos\left(\frac{\omega_1 + \omega_2}{2}\right) t$ and $\sin\left(\frac{\omega_1 + \omega_2}{2}\right) t$, describe periodic oscillations, at the average frequency of ω_1 and ω_2 and whose amplitudes are controlled by the slowly varying functions, $\cos\left(\frac{\omega_1 - \omega_2}{2}\right)$ and $\sin\left(\frac{\omega_1 - \omega_2}{2}\right)$, respectively. Both z and θ will describe waveforms, as shown in Figure A-3. This is known as "beating."

For beating due to the coupling of modes it is interesting to note that the bouncing and pitching modes are 90 degrees out of phase: the energy of free oscillation passes back and forth between predominantly bouncing and predominantly pitching modes. In other words, the vehicle is seen to oscillate first in bouncing, then pitching, and so on.

The periodic component of force on the front or rear suspension as derived in Eqs. A-3 to A-5 is now made up of two beating oscillations, as shown by Eqs. A-2 and A-6. With z motion and θ motion described by two beat envelopes that are identical except for amplitude and phase, the linear combination of the two will still be a beat envelope of the form:

$$F = A \cos\left(\frac{\omega_1 + \omega_2}{2}\right)(t + t_0) \cos\left(\frac{\omega_1 - \omega_2}{2}\right)(t + t_0) \quad \text{(A-7)}$$

Because the periodic component of force on an axle is a linear combination of bouncing and pitching modes, the two need not be dynamically coupled to produce a beating

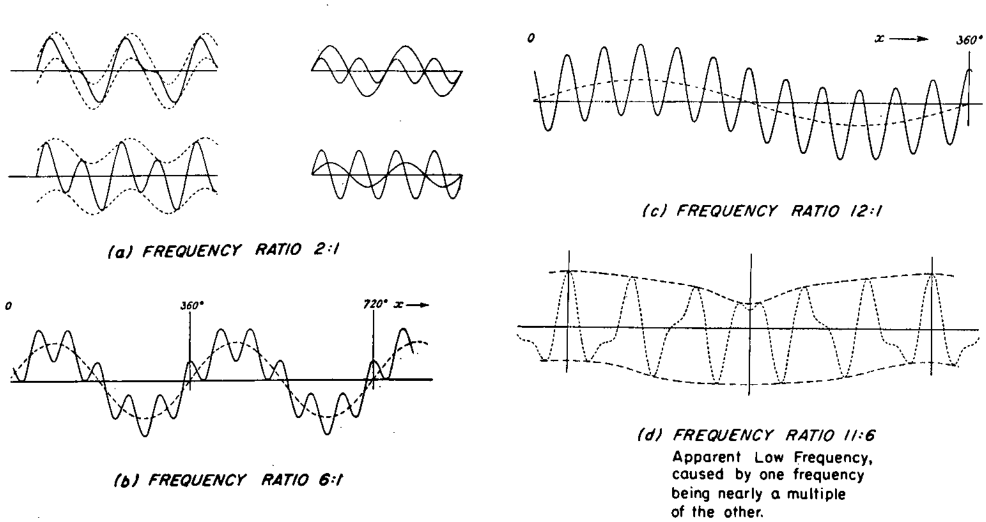


Figure A-2. Two-component waveforms.

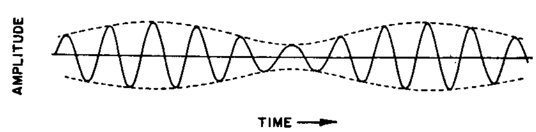


Figure A-3. Beating frequencies.

periodic force between the axle and the road. Consider the force between axle and road as taken from Eq. A-3:

$$F = k\,z + k\,l_f\theta$$

Let modes be uncoupled so that

$$z = A \cos(\omega_1 t + a_1) \qquad \text{(A-8)}$$

and

$$\theta = B \cos(\omega_2 t + a_2) \qquad \text{(A-9)}$$

Thus, F is still a linear combination of the two modes, as shown by

$$F = kA \cos(\omega_1 t + a_1) + k\,l_f B \cos(\omega_2 + a_2) \qquad \text{(A-10)}$$

This will show a similar beating phenomenon if ω_1 and ω_2 are close in frequency.

The AASHO Road Test (*1*, Table 60) gives data regarding the bounce and pitch natural frequencies of vehicles used in the Road Test. These data for free (not blocked) vehicle suspensions are given in Table A-1.

Vehicles 91 and 513, for example, have natural frequencies in bouncing and pitching that will give good evidence of beating. Although it is not known which two-axle vehicle was used to produce the oscillographic record, the trace shown in *The AASHO Road Test* (*1*, Fig. 84) of front-tire pressure in the blocked suspension condition is a good example of beating. This is reproduced as Figure A-4. Note that other effects are present, possibly arising partially from the nonlinear compression of air in the tire.

One additional condition will give rise to beat-frequencies in the force between tire and the road. Consider a vehicle that has a relatively small amount of damping or energy loss in the suspension system so that an oscillation at a natural frequency will be sustained through a number of cycles before decaying to a very small amplitude. If such a vehicle undergoes some periodic excitation at a frequency slightly different from a natural frequency, the excitation frequency and the natural frequency will beat and cause the same type of periodic force between tire and road, as discussed previously.

The foregoing discussion of vehicle dynamics indicates what is to be anticipated in the waveform of force between the tire and the road where samples of force must be taken for estimating the weight of vehicles. On a smooth highway a one- or two-component sinusoidal signal superimposed upon the static weight of the wheel could be anticipated. For a general class of functions that may be used in the analysis and evaluation of methods to estimate vehicle axle weight the following has been shown to be appropriate:

$$F = \overline{W}\,[1 + A \sin(\omega_z t + a_z) + B \sin(\omega_\theta t + a_\theta)] \qquad \text{(A-11)}$$

TABLE A-1

NATURAL FREQUENCIES OF VEHICLES, FROM AASHO ROAD TEST

VEHICLE	VEHICLE FREQUENCY (CPS) BOUNCE	PITCH
	(*a*) Two-Axle Vehicles	
91	2.3	2.6
94	2.7	3.5
A	2.0	2.4
	(*b*) Three-Axle Vehicles	
B	2.0	2.4
C	1.8	2.4
513	1.7	2.0

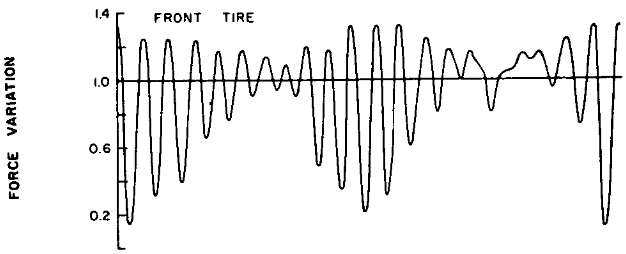


Figure A-4. Evidence of beating (from AASHO Road Test).

in which

$\overline{W}$ = static axle weight;
ω_z = natural circular frequency in the bouncing mode;
ω_θ = natural circular frequency in the pitching mode;
a_z and a_θ = phase angles; and
A and B = amplitudes of the bouncing and pitching modes, respectively, as fractions of the static axle weights.

Thus far, this discussion of the waveform that must be analyzed to estimate vehicle axle weight has considered only the waveform anticipated for a dynamic system, previously excited, but currently in free oscillation, or one in which the excitation is uniform and of small amplitude. This, however, is not always the case. Even though a road may be "smooth," certain irregularities are present. It is unlikely that surface variations are uniform in amplitude and regular in spacing to yield periodic excitation, even if it could be assumed that the vehicle maintains a constant velocity, or a fairly constant velocity in which the speed varies slowly enough to allow the system to be always in a quasi-steadystate oscillation. Rather, the surfaces of highways suggest a much more random variation in surface irregularities and a consequent random excitation to the vehicle.

The vehicle has been described as a mechanical oscillatory system that has two major modes of oscillation plus that of the axle and wheel assemblies. If a mode of oscillation is lightly damped and is subjected to a random excitation, the mechanical oscillator acts as a narrow bandpass filter admitting energy only in the neighborhood of the natural frequency. The waveform of the random excitation can be considered to be made up of many Fourier components covering a wide frequency range, many of which may be in the neighborhood of the natural frequency of the oscillator (vehicle). The phenomenon of beats was presented previously to show that the addition of two harmonic waves that are close in frequency yields a wave oscillating at the mean of the two frequencies, but with an amplitude envelope that varies at a rate equal to one-half the difference of the two frequencies. Thus, the addition of a large number of harmonic components, having frequencies in the neighborhood of the vehicle natural frequency and having randomly distributed phase angles, will produce a wave oscillating at the natural frequency of the vehicle with an amplitude envelope that varies randomly with time. However, the rapidity of the random amplitude fluctuation must be on the order of the bandwidth of the mechanical oscillator. This is known as a "narrow-band random oscillation," as distinguished from random oscillation covering a wide bandwidth. An example of narrow-band random oscillation is shown in Figure A-5.

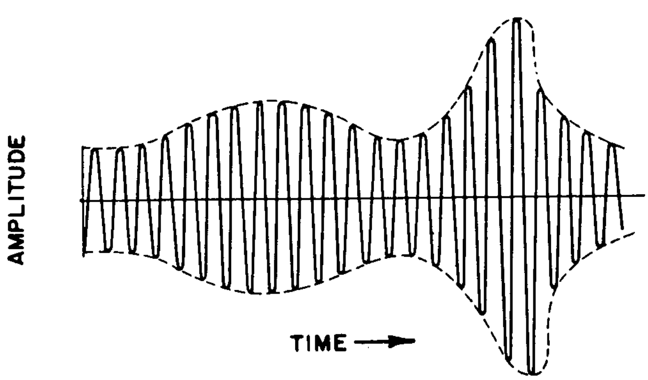


Figure A-5. Narrow-band random oscillation.

Measurements of tire pressure made in conjunction with the AASHO Road Tests (*1*, Fig. 83, 84, 85) are reproduced in Figure A-6.

Note that the curves of tire pressure in Figure A-6 bear a closer resemblance to the narrow-band random oscillation of Figure A-5 than to a limited combination of harmonic components. This suggests that even for the smooth-pavement tests shown in Figure A-6 (upper part of the figure) a random excitation still is present at a significant magnitude. However, the variation of pavement surface height for either the rough or the smooth pavement is not given quantitatively in *The AASHO Road Test* (*1*), so an absolute knowledge of the actual surface is not known. If random oscillations need to be considered to any real extent, the accurate assessment of "weight" of a moving vehicle will be even more difficult. This emphasizes the use of more carefully prepared smooth highway surfaces in the neighborhood of the force-sensing equipment of "in-highway" weighing stations.

Thus far in this discussion of the waveform to be sampled to estimate the weight, consideration has been given only to those aspects that deal with the oscillatory response and characteristics of the vehicle itself. Two other aspects will influence the waveform to be sampled: (1) electrical noise, and (2) vibration of the force-sensing equipment.

Electrical noise is usually manifested at frequencies that are integral multiples of 60 cps, and is usually due to the passing of power line frequency through the power supplies and ground loops, or by inductive pickup from nearby power transmission lines. Although electrical noise is a factor to be considered in the final waveform, the magnitude depends heavily upon the choice and arrangement of

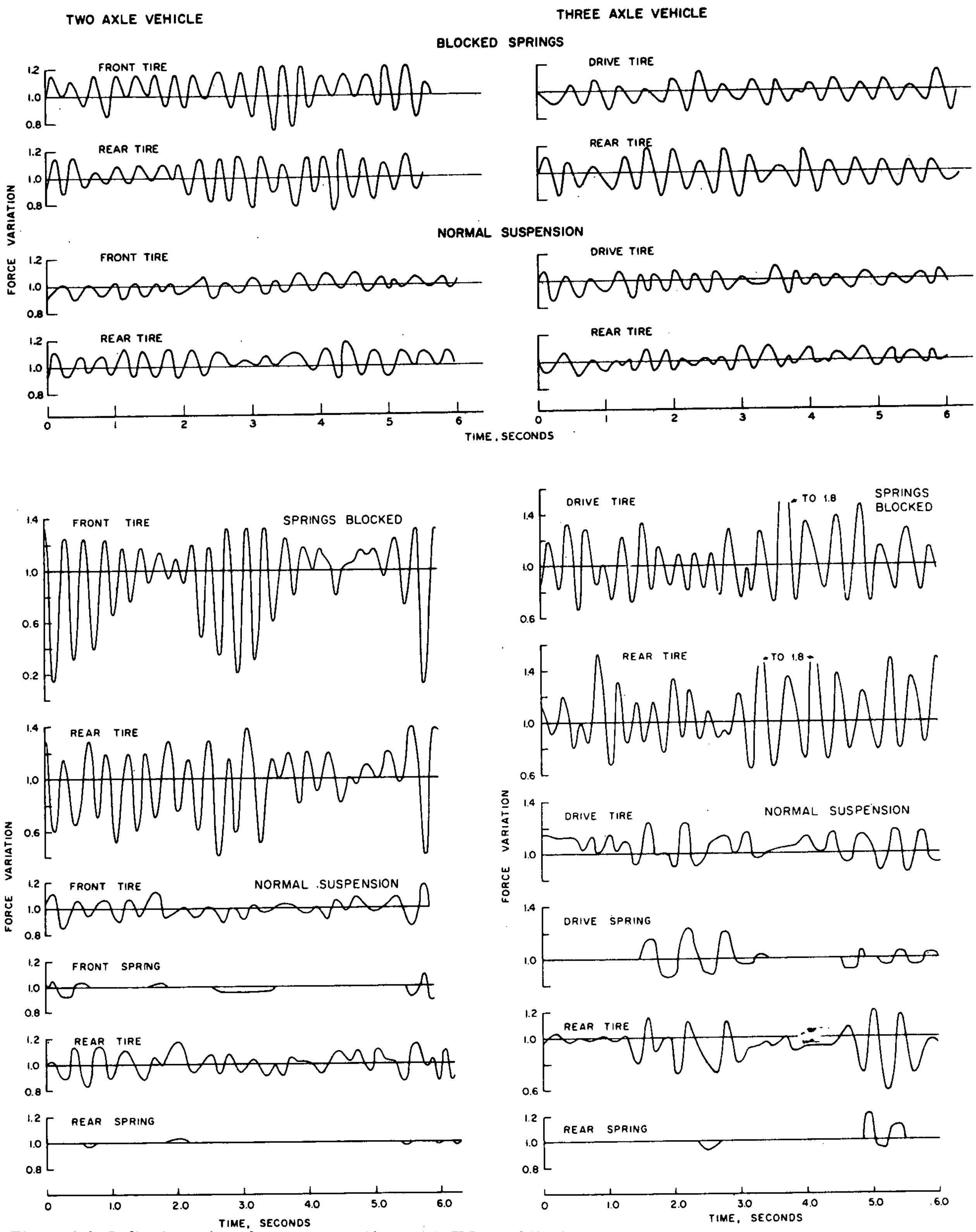


Figure A-6. Indications of random excitation (from AASHO Road Test).

instrumentation equipment, and consequently is not inherent in the basic theoretical problem of ascertaining the weight of a moving vehicle. Therefore, electrical noise is not considered further in this report.

If the magnitudes of electrical noise as a function of the choice of instrumentation equipment are ignored, it might be argued that the same would apply to the consideration of vibration in the force-sensing equipment (platforms) to be placed in the highway. However, there is a fundamental difference. Electronic instrumentation has reached a high state of development, so a wide range of equipment is applicable. However, there are few highly developed designs for highway vehicle force-sensing equipment.

The prevailing method has been the mounting of a structural steel platform, supported by strain gauge load cells, over an excavated pit spanning one lane of a highway, or the equivalent thereof, so that the surface of the platform is flush, or nearly so, with the highway surface. This design has been used to take advantage of the excellent sensitivity and linear characteristics of strain gauge load cells, but the platforms have usually exhibited very light damping, which is characteristic of low hysteresis in welded steel structures. The amplitude of platform vibration can be large compared with the average force between road and tire (see Fig. A-7). The fact that the higher frequency oscillation is that of the platform is evidenced by vibration when the wheels both enter and leave the platform.

Other types of force-sensing platforms which will not show this vibrational characteristic to such a marked extent are under development (*11*). Still, there may not be a large choice if it is desired that the weighing system function with presently emplaced force-sensing devices.

The oscillation of the platform requires that an additional harmonic component be added to those representing the dynamic vehicle and imposes additional requirements to any mathematical method for ascertaining the weight from sampled force data.

In concluding this discussion of the waveform which the weighing system must be able to average, the dynamics of a vehicle show that the waveform may contain harmonic components consistent with the normal modes of the vehicle, plus those of the force-sensing platform. In addition, the response of the vehicle and, hence, the resulting waveform, may be made to behave in a random manner if the local disturbances in the highway surface profile are sufficiently large. But, because this can be alleviated by efforts to make a fairly smooth area in the region of force measurements, the randomness can be small, locally, and the waveform can be represented satisfactorily by a constant term plus a series of harmonic components, as follows:

$$F = \overline{W}\left[1 + \sum_{j=1}^{J} a_j \sin(\omega_j t + \phi_j)\right] \qquad \text{(A-12)}$$

in which

$\overline{W}$ = axle weight;
a_j = amplitude of jth harmonic component;
ω_j = circular frequency of the jth harmonic component; and
ϕ_j = phase angle of the jth harmonic components.

This is a generalization of Eq. A-11.

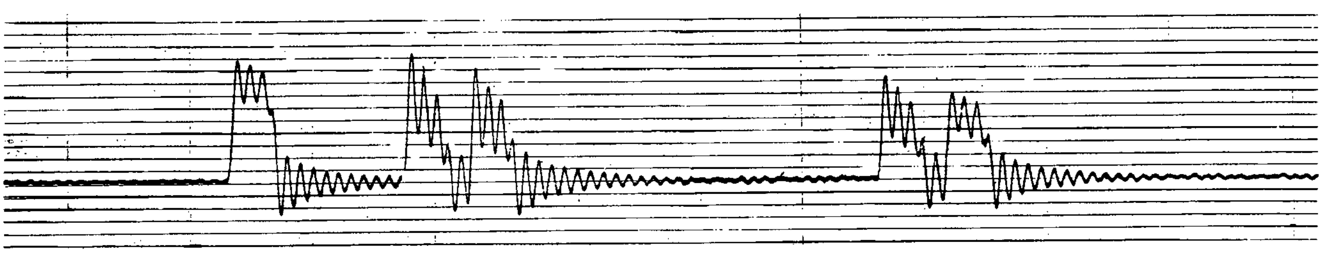


Figure A-7. Vehicle force causing platform vibration.

APPENDIX B

LIMITATIONS OF SAMPLED DATA SYSTEMS

Appendix A indicates the type of waveform with which the weighing system must be compatible, and how it may be synthesized mathematically or experimentally in the laboratory to evaluate analytical or experimental physical models of weighing systems. Basically, the waveform of force applied to the load by an axle could be represented as shown in Eq. A-12. This represents a periodic variation superimposed on a steady force.

If the waveform is to be analyzed to determine the value of the steady component, W, then any portion of at least one periodic wavelength may be inspected. In doing this, the operation is clearly that of taking a sample; it is all that is needed, because each period is the same. Although it is shown that there is also a certain amount of randomness to the oscillation of a vehicle, so that a distinct period is not present, it is obvious from practical considerations that the portion of the waveform to be analyzed must be limited. Indeed, the aim of this work is to determine how little of the waveform is required. The whole operation is based upon the sample taken and is therefore subject to the limitations of the sampled data process, which is defined as any method whereby discrete values of data are obtained for characterizing a continuous process. By contrast, a continuous or analog system has variables that are known at all instants of time.

Let $F(t)$ be a function of interest that may be described by means of an analytical expression or exist as an analog signal. If the function is sampled at equal intervals of time (say, T), the functions can be described by a sequence of numbers $F(0)$, $F(T)$, $F(2T)$ . . . . $F(nT)$. Obviously, some information is lost in the process, and whether the resulting description satisfactorily represents the original function depends on the nature of the function. If the function is reasonably smooth and well-behaved with respect to the time interval of samples, it can be interpolated between samples with good accuracy. If the function is not smooth and well-behaved with respect to the sampling interval, the possibility of large variations occurring between the samples could not be predicted. Thus, at least qualitatively, it may be seen that the sampling frequency must bear some relationship to the nature of the function being sampled lest the required information be lost in the process. If the sampling frequency is well chosen, little useful information will be lost. However, too many samples may prove a burden in information processing by providing information that could have been obtained by simple interpolation methods. The selection of the proper sampling frequency is important in any sampled data system.

Some insight into these limitations imposed by the sampling operation in sampled data systems can be seen by considering the sampling theorem attributed to Shannon (*5*, p. 16). In essence, the theorem states that for signals having a finite bandwidth, including frequency components up to but not beyond a frequency of f cps, a complete description of that signal can be obtained from samples of the signal spaced by time periods of $1/(2f)$ sec. In other words, samples must be taken at a rate at least twice the highest frequency component if the signal is to be completely reconstructed from the series of samples.

This reconstruction is accomplished by various interpolative and extrapolative procedures that have been developed. Such methods are highly advanced and are set forth in texts on statistical communication theory (*3*) and sampled-data control systems (*5*). Unfortunately, the bulk of this work deals with a continuous signal that is sampled for a sufficient time to yield, generally, a large number of samples over many periods of the signal. In the weighing of a vehicle, the problem faced is the limited number of samples that are spaced, at best, just over one period of the lowest frequency. Thus, from sampled data theory it is possible to gain the limitations of the theory, but benefit little from the major current development. However, one aspect of the sampling problem for weighing vehicles is a definite asset. That is, when the axle is over a force-sensing platform and the sample is being taken, it is possible to gain many samples, or even continuous data, for a certain interval of time. In terms of distance in the direction of travel this is the active length of each force-sensing platform.

APPENDIX C

DIRECT AVERAGING

One method of estimating the weight of an axle is to simply average the values of data taken from samples of the force between tires and road. Although the shape of the waveform of any one axle may not be known, it would be reasonable to assume that it would be made up of a number of harmonic components, as previously discussed. It may be assumed further that the relatively low frequency caused by the oscillation of the combined vehicle body and load in bouncing and pitching would most likely produce the largest variation in road force, so that the resulting waveform might appear as a distorted sine wave. With frequencies of oscillation and vehicle velocities to be expected, the bounds on wavelength of vehicles could be estimated so that force-sensing devices could be positioned in the road. In Figure C-1, F_1, F_2, ... F_N are samples of the force taken at positions x_1, x_2, ... x_N, respectively, on the standing wave produced by the tire force. These can be averaged by summing N values of the samples and dividing by the number of samples:

$$\text{Estimated axle weight} = \frac{1}{N}\sum_{i=1}^{N} F_i \qquad \text{(C-1)}$$

This would be a good estimate so long as the samples greater than and less than the average were equal in quantity and symmetrical in value about the average. Otherwise, there is error between the estimated weight and the static axle weight.

In addition, it is possible to be as much in error with N samples as for one sample (see Fig. C-2).

THEORETICAL ACCURACY OF AVERAGING

The adequacy of averaging methods in predicting vehicle weight with a specified accuracy is investigated in the following.

Let an axle with a weight, $\overline{W}$, and frequency of oscillation, f, proceed along the highway with a velocity, V. All three quantities are unknown. In the usual averaging method, the procedure would be to measure the force exerted on the road by the axle N times, add the values, and divide by N. Because interest here is in determining the minimum error, the best possible straight averaging method is considered.

To this end, measure the axle weight continuously, integrate it timewise, and divide the result by the interval of integration. This would be the best simple average possible for the selected interval of sampling. To determine the error, let a continuous sample of the axle weight, with the vehicle moving uniformly, be represented by

$$F = \overline{W}\,[1 + a \sin (2\pi ft + \phi)] \qquad \text{(C-2)}$$

in which $\overline{W}$ is the static axle weight, a is the fractional amplitude of oscillation, ϕ is an unknown phase angle, and F is the transducer reading.

Consider a treadle of length, L, over which samples are taken. If time, t, equals zero when the axle first contacts the treadle, then

$$t = x/V \qquad \text{(C-3)}$$

in which x is the distance traveled on the treadle. The average over the treadle is

$$F_{\text{avg}} = W\,(V/L) \int_0^{L/V} [1 + a \sin (2\pi ft + \phi)]\, dt$$

or

$$F_{\text{avg}} = W\,\{1 - (aV/2\pi Lf)\,[\cos (2\pi fL/V + \phi) - \cos\phi]\} \qquad \text{(C-4)}$$

Inasmuch as the product of frequency and time—shown as $(f \cdot t)$ or fL/V in the preceding equations—serves to denote the number of cycles or wavelengths, it is desirable to replace this by a parameter:

$$\zeta = ft = fL/V \qquad \text{(C-5)}$$

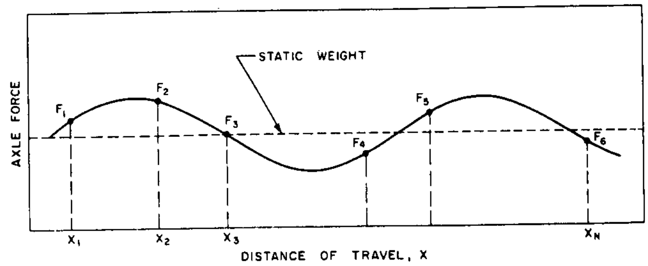


Figure C-1. Sampling of axle force on road.

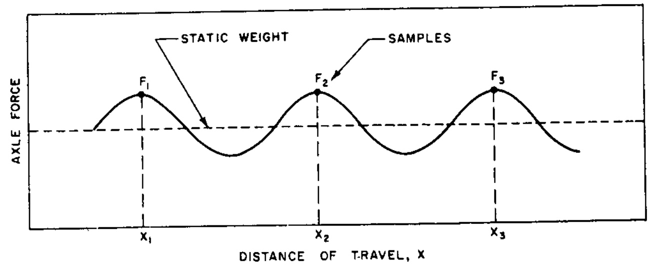


Figure C-2. Sampling rate coinciding with vehicle frequency.

In terms of the number of cycles, the force between tire and road is

$$F = W\,[1 + a \sin(2\pi\zeta + \phi)] \qquad \text{(C-6)}$$

The integral average after ζ_o cycles is

$$F_\zeta = 1/\zeta_o \int_0^{\zeta_o} F\, d\zeta = 1/\zeta_o \int_0^{\zeta_o} \left\{ \overline{W}\,[1 + a \sin(2\pi\zeta + \phi)] \right\} d\zeta$$

and

$$F_\zeta = \overline{W} \left\{ 1 - \frac{a}{2\pi\,\zeta_o} [\cos(2\pi\,\zeta_o + \phi) - \cos\phi] \right\} \qquad \text{(C-7)}$$

If F_ζ is the estimated weight, the error with respect to the actual static weight, $\overline{W}$, is

$$E = \frac{\overline{W} - F_\zeta}{\overline{W}} \qquad \text{(C-8)}$$

or

$$E = \frac{a}{2\pi\,\zeta_o} [\cos(2\pi\,\zeta_o + \phi) - \cos\phi] \qquad \text{(C-9)}$$

In any such application, the phase angle, ϕ, would be a function of the highway and the particular vehicle, and beyond the control of the operator who sets up the electronic sensing equipment. Therefore, it is meaningful here to show the maximum error, with respect to phase angle.

By setting

$$\partial E/\partial\phi = 0 \qquad \text{(C-10)}$$

the relationship between a and ζ_o is obtained that yields the values of ϕ associated with maximum error;

$$\tan\overline{\phi} = \frac{\sin 2\pi\,\zeta_o}{1 - \cos 2\pi\,\zeta_o}$$

and

$$\overline{\phi} = \arctan\left(\frac{\sin 2\pi\,\zeta_o}{1 - \cos 2\pi\,\zeta_o}\right) \qquad \text{(C-11)}$$

Now, error that is maximized with respect to phase angle, ϕ, is

$$E_{\max} = \frac{a}{2\pi\,\zeta_o} [\cos(2\pi\,\zeta_o + \phi) - \cos\overline{\phi}] \qquad \text{(C-12)}$$

A plot of $E_{\max}/a$ versus ζ_0 is shown in Figure C-3. Thus, if $a = 0.1$ (i.e., if the oscillating component of force is 10% of the static axle weight), the error in the estimated weight as a fraction of W would be 0.1 times the value of E/a from Figure C-3. For example, by averaging over 1.4 cycles, maximum error, $E_{\max}$, would be 0.0216, or 2.16%. If the oscillating component of axle force is 20% of the static weight, the error would be 4.32%.

It must be pointed out that although the error has been maximized with respect to phase angle, this is theoretically the best that can be accomplished by simple averaging. All other simple averaging methods, such as those in which discrete samples are averaged over the same period or over the same number of cycles, inherently involve greater error.

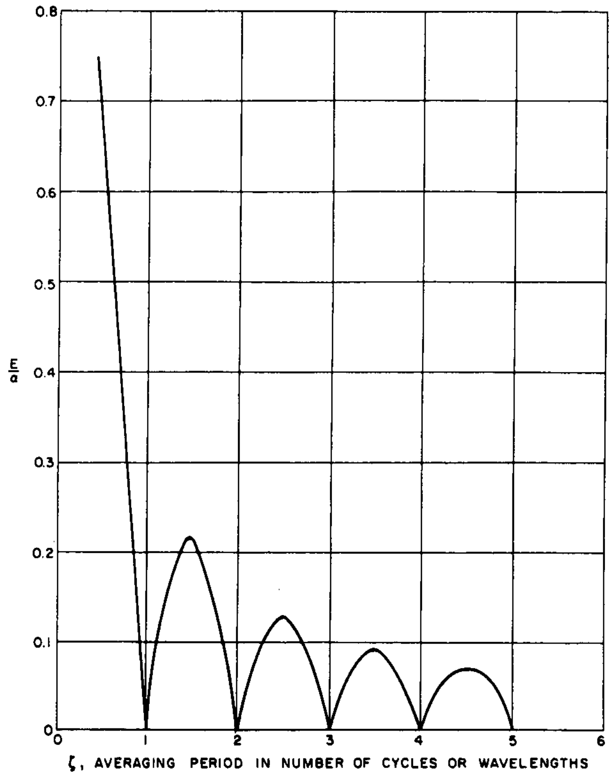


Figure C-3. Variation of integral averaging error with averaging period.

Figure C-3 illustrates one additional major point: if any great reduction in error is to be realized over that of an arbitrary single sample (single measurement of force), the averaging period must be very close to at least one full cycle or over some period comprising many cycles. Also, it may be seen that any significant reduction in error beyond that of, say, a 2-cycle averaging period, would require a period of several times longer. This is apparent from the expression for error, Eq. C-12, in which the periodic factor in brackets is multiplied by $1/\zeta_0$, the inverse of the averaging period.

For weighing vehicles in motion, simple integral averaging does not appear to be most promising for minimum error. The lower frequencies of vehicles are the hardest to cope with because they may be as low as 2 cps, or occasionally less, thus requiring a minimum averaging time of 0.5 sec. At 30 mph, the minimum averaging distance of one wavelength is 22 ft; at 60 mph it is 44 ft. Even if cost is no object, many present concepts of force-sensing platforms are not compatible with this objective. On the positive side, it can be seen that integral averaging can be useful to average out the high frequency variations in force caused by sensing-platform vibration. Consequently, integral averaging may be useful when employed with other weight-estimating schemes.

AVERAGING METHODS FOR WEIGHING VEHICLES

In addition to the method shown previously for integral averaging, simple averaging may be employed using data taken at certain intervals of time or of distance. This is shown in Figure C-4, and is much more amenable to the use of present working concepts of force-sensing platforms or treadles for weighing vehicles in motion. Here, a series of platforms or treadles are placed in a highway lane to measure, sequentially, the force of one axle. The treadles may be of arbitrary length in the direction of vehicle travel, but once the length exceeds the distance between any two axles there is a possibility that the force measured is not all from one axle. It should be remembered, though, that the longer treadle will sample a greater percentage of a given wavelength. This may or may not be relevant, depending upon the relation of treadle length to wavelength. If the ratio is quite small, the additional data might not be much more significant than just one point.

The following analyses evaluate the representative error associated with two methods of averaging with this type of sampling.

Simple Averaging Using Digital Data Systems

Digital time-rate data sampling systems have two desirable features for the sampling of force between the vehicle and the highway: (1) accuracy is high, and (2) sampling at a chosen rate of time allows more data to be taken from each treadle when the vehicle velocity is low and the corresponding wavelength shorter, whereas less data will be taken from each treadle for the same vehicle at a higher speed. The latter feature is desirable because, in general, when the wavelength is longer the change in force over one treadle is smaller and fewer samples will yield a sufficient knowledge of the force at that part of the waveform.

Let the estimated axle load be the average of all samples taken from n treadles.

If γ_{ij} is the jth sample at the ith treadle, the estimated weight is

$$W = \frac{1}{n\,m}\sum_{i=1}^{n}\sum_{j=1}^{m}\gamma_{ij} \qquad \text{(C-13)}$$

in which n is the number of treadles; and m is the number of samples per treadle.

Vehicle oscillations are essentially harmonic in character and can be described as a sum of sine and cosine functions. To test the accuracy of the axle weight as estimated by Eq. C-13, an analytic function can be chosen that describes the waveform of a vehicle axle load oscillation. For a first test it is assumed that the vehicle axle load can be approximated by

$$f(x) = \overline{W}\,[1 + \alpha\, e^{-(\zeta\, 2\pi x/\lambda)} \cos\,(2\pi x/\gamma + \phi)] \qquad \text{(C-14)}$$

(this represents the damped oscillation of a simple spring-mass system)

in which

$\overline{W}$ = axle weight;
α = ratio of oscillating component to axle weight;
ζ = damping factor;
x = distance from the origin at a coordinate system;
λ = wavelength; and
ϕ = phase angle with respect to the origin.

Using the model of the vehicle force oscillation (Eq. C-14), the sample, γ_{ij}, of the force, $f(x)$, that occurs at x_{ij} can be calculated; that is, at the distance to the jth reading on the ith scale. x_{i1} is defined as the first reading on the ith scale.

To make the problem easier it is assumed that the vehicle has constant velocity over the length of highway in which the scales are placed. This removes the integration of velocity over the time interval between samples and makes it possible to choose, instead, m equal increments in the distance over the scale.

If the scale is of width b, and m samples are to be taken, the increment of distance between samples is b/m and the distance to any sample, γ_{ij}, from the coordinate origin is

$$x_{ij} = x_{io} + b(j/m) \qquad \text{(C-15)}$$

and

$$\gamma_{ij} = f(x_{ij}) \qquad \text{(C-16)}$$

If

$$\chi_{io} = l(i/n)^{\beta} \qquad \text{(C-17)}$$

in which

χ_{io} = distance to leading edge of the ith scale;
l = distance to leading edge of last or nth scale;
i = scale number; and
β = an exponent;

then Eq. C-17 represents the distances to any of a set of n scales, with scale spacing depending upon the choice of the exponent, β. When $\beta = 1$, the scales are equally spaced.

Now, Eq. C-15 becomes

$$\chi_{ij} = l\,(i/n)^{\beta} + b\,(j/m) \qquad \text{(C-18)}$$

and

$$\gamma_{ij} = f[l\,(i/n)^{\beta} + b\,(j/m)] \qquad \text{(C-19)}$$

or

$$\gamma_{ij} = f\,(\chi_{ij}) = \overline{W}[1 + \alpha\, e^{-(\zeta\, 2\pi \chi_{ij}/\lambda)} \cos 2\pi\lambda_{ij}/\lambda + \phi \qquad \text{(C-20)}$$

and the estimated axle load is

$$W = \overline{W}\left\{1 + \frac{\alpha}{n\,m}\sum_{i=1}^{n}\sum_{j=1}^{m}[e^{-(\zeta\, 2\pi x_{ij}/\lambda)} \cos\,(2\pi x_{ij}/\lambda + \phi)]\right\} \qquad \text{(C-21)}$$

in which

$$x_{ij} = [l\,(i/n)^{\beta} + b\,(j/m)]$$

Let error, E, be defined as

$$E = \frac{W - \overline{W}}{\overline{W}} \qquad \text{(C-22)}$$

$$E = \frac{\alpha}{n\,m}\sum_{i=1}^{n}\sum_{j=1}^{m}[e^{-(\zeta\, 2\pi x_{ij}/\lambda)} \cos\,(2\pi x_{ij}/\lambda + \phi)] \qquad \text{(C-23)}$$

and

$$E/a = \frac{1}{n\,m} \sum_{i=1}^{n} \sum_{j=1}^{m} [e^{-(\zeta\, 2\pi x_{ij}/\lambda)} \cos (2\pi x_{ij}/\lambda + \phi)] \tag{C-24}$$

in which

$$x_{ij} = l\,(i/n)^{\beta} + b\,(j/m)$$

E/a is a dimensionless parameter that represents the error in the method and is independent of the amplitude of oscillation.

A phase angle, ϕ, is included in the analytical model of the vehicle force. Because a vehicle can approach the set of scales with any phase angle between 0 and 2π, the maximum value of E/a with respect to ϕ must be looked at in the evaluation of this system. The particular ϕ that makes E/a a maximum is a function of λ and must be computed for each value of λ throughout the range of wavelengths. This is done as follows:

Set

$$\frac{d\,(E/a)}{d\phi} = 0$$

$$\frac{d\,(E/a)}{d\phi} = \frac{-1}{n\,m} \sum_{i=1}^{n} \sum_{j=1}^{m} [e^{-(\zeta\, 2\pi x_{ij}/\lambda)} \sin (2\pi x_{ij}/\lambda + \phi)] = 0$$

$$\sum_{i=1}^{n} \sum_{j=1}^{m} [e^{-(\zeta\, 2\pi x_{ij}/\lambda)} (\sin 2\pi x_{ij}/\lambda \cos \phi + \cos 2\pi x_{ij}/\lambda \sin \phi)] = 0$$

$$\sum_{i=1}^{n} \sum_{j=1}^{m} \{e^{-(\zeta\, 2\pi x_{ij}/\lambda)} [\sin 2\pi x_{ij}/\lambda + \tan \phi \cos 2\pi x_{ij}/\lambda]\} = 0$$

and

$$\Phi = \arc\tan - \left[\frac{\sum_{i=1}^{n} \sum_{j=1}^{m} e^{-(\zeta\, 2\pi x_{ij}/\lambda)} \sin 2\pi x_{ij}/\lambda}{\sum_{i=1}^{n} \sum_{j=1}^{m} e^{-(\zeta\, 2\pi x_{ij}/\lambda)} \cos 2\pi x_{ij}/\lambda} \right] \tag{C-25}$$

Φ from Eq. C-25 is the value of ϕ that makes E/a a maximum. Then

$$(E/a)_{\Phi} = \frac{1}{n\,m} \sum_{i=1}^{n} \sum_{j=1}^{m} [e^{-(\zeta\, 2\pi x_{ij}/\lambda)} \cos 2\pi x_{ij}/\lambda + \Phi] \tag{C-26}$$

in which

$$x_{ij} = l\,(i/n)^{\beta} + b\,(j/m)$$

and

$$\Phi = \arc\tan \left\{ - \frac{\sum_{i=1}^{n} \sum_{j=1}^{m} e^{-(\zeta\, 2\pi x_{ij}/\lambda)} \sin 2\pi x_{ij}/\lambda}{\sum_{i=1}^{n} \sum_{j=1}^{m} e^{-(\zeta\, 2\pi x_{ij}/\lambda)} \cos 2\pi x_{ij}/\lambda} \right\}$$

Figures C-5 through C-8 show the variation in error that would occur for various combinations of the variables, n, m, l, b, β, and ζ. For this preliminary study the following parameters are fixed: ζ equals 0, no damping; l equals 60 ft, distance to last treadle from coordinate origin; and m equals 5, number of samples per treadle. These are not optimum values, but are chosen as representative values that will indicate the magnitude of error to be expected by simple averaging of "$n\,m$" number of force samples.

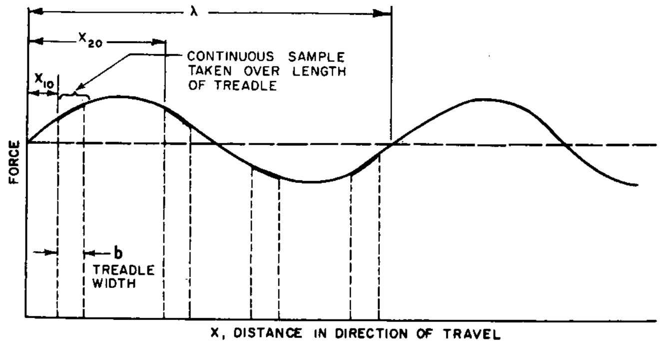


Figure C-4. Waveform and platform locations.

Note that in Figures C-5 through C-8 the "error" is shown as E/a. This is the ratio of error, E, to the amplitude factor, a, of the oscillating component of vehicle force (at the respective axle). To obtain error as a percentage of weight, the following operation must be performed: Error (in %) = (E/a) (a) (100).

Figure C-5 shows the effect of treadle (or platform) width upon E/a. Note that an increase in scale width from 1 ft to 2.5 ft produces a reduction in E/a at the very short wavelengths but has insignificant effect on longer wavelengths. This was anticipated, because as the m number of samples at each treadle is taken over a greater portion of a wavelength, the error is reduced. However, the close spacing of tandem axles precludes the use of active treadle widths much greater than 2.5 ft; therefore, little can be accomplished in reducing error by increasing treadle width. However, little overall accuracy would be lost if the treadle design were short in the direction of travel, particularly if the lower cost of the shorter design would allow the use of more treadles.

This statement is supported by the comparison shown in Figure C-6. The large values of E/a that predominate for three treadles are greatly reduced by the use of seven treadles, except for wavelengths between 7 and 11 ft. This is the region in which the wavelengths are just shorter than, equal to, or just longer than the uniform treadle spacing, so that it is possible to sample near the peak of each cycle.

To remove the chance of sampling near the peak of each cycle, one family of non-uniformly-spaced treadles is allowed for by specifying treadle spacing, as shown in Eq. C-18. Variation in non-uniform spacings is made by varying the value of β. Preliminary results representative of this family of variations are shown in Figure C-7. Note that the peak error reduces as β increases, but that the error increases for those wavelengths not associated with the original peak. For wavelengths 17 ft and greater, the increase in error with increase in β is caused by sampling too often that one part of the wave; that is, the scale spacing increases in the direction of travel.

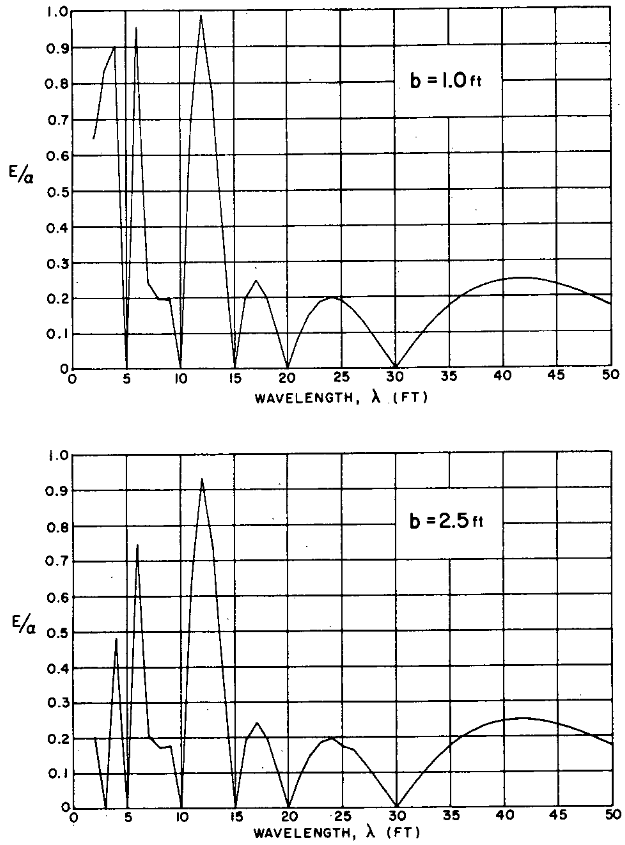


Figure C-5. Effect of scale treadle width, b, *on* E/a.

From this limited study, $\beta = 1.5$ would appear to provide the best attenuation of peak error without undue increase at other wavelengths. A further reduction in the error between 35- and 50-ft wavelengths can be made by adjusting the length, l, to some value other than 60 ft.

Although it has been indicated that $\beta = 1.5$ appears to provide good attenuation of the peak error associated with evenly spaced systems of treadles, this is only for the case of $n = 7$ treadles and $l = 60$ ft. Figure C-8 shows the error associated with $\beta = 1.5$, $l = 60$ ft and n less than 7. This again emphasizes the dependence of simple averaging systems on a larger number of treadles or sampling stations.

A number of additional computer runs were made in which the dimension l in Eq. C-18 was made both shorter and longer than the 60 ft used for the study presented. The error was not significantly changed, except that the error for the shorter dimension was somewhat less. In the case of the longer wavelengths, this is because the distance over which measurements were made was more nearly one wavelength. In addition to varying the dimension l in Eq. C-18, other families of non-uniform platform distribution were used. One of these is:

$$X_{ij} = l/2\left[1 - \cos\left(\pi\frac{i-1}{n-1}\right)\right] + b\ (j/m) \qquad \text{(C-27)}$$

No advantage in reduced error was obtained.

It should be pointed out that when the treadles were spaced closer together, the error generally was reduced, because the digital sampled averaging system more nearly approximated the limiting case of integral averaging, as previously discussed. This is a desirable practice in real systems, but it must be realized that the active portions of real physical platforms can be spaced only so close together before their inactive portions interfere.

In this analysis of error in a simulated simple averaging system with digital sampling it should be noted that the sampling rate was set at five samples per treadle. This was chosen because the mathematical model of axle force was a simple sinusoidal variation about the static or steady force. Additional samples per treadle would have made little difference, except at the shorter wavelengths of approximately 2 to 5 ft where more samples would have defined the variations in force more accurately. In a real system, the shorter wavelengths generally would be associated with slower-moving vehicles so that with time-rate sampling (sampling at fixed intervals of time) the sampling system would automatically take more samples. The example shown is illustrative only. It was more convenient to use a fixed number of samples per treadle than to simulate the real case.

It should be emphasized that the necessary sampling rate of a digital sampling system will be dictated by the highest frequency component in the waveform. As previously mentioned, the vibration of certain weight-sensing platforms (or treadles) may be of large amplitude and at a much higher frequency than that produced by the vehicle. Consequently, if platform vibration cannot be reduced by alteration or redesign, it may be the deciding factor in choosing the sampling rate. There is no trouble in achieving high sampling rates; present-day multiplexers are more than adequate. Rather, the problem is that of storing and processing the larger amount of data. The data processing must then be larger and more costly.

Simple Averaging Using Analog Systems

Instead of taking discrete samples at certain times with digital equipment (see preceding section), analog equipment can be used to achieve essentially the same results. The task here is to average the continuous sample of force over each treadle and then to average the values from each treadle. This can be accomplished with simple analog integrations, as follows.

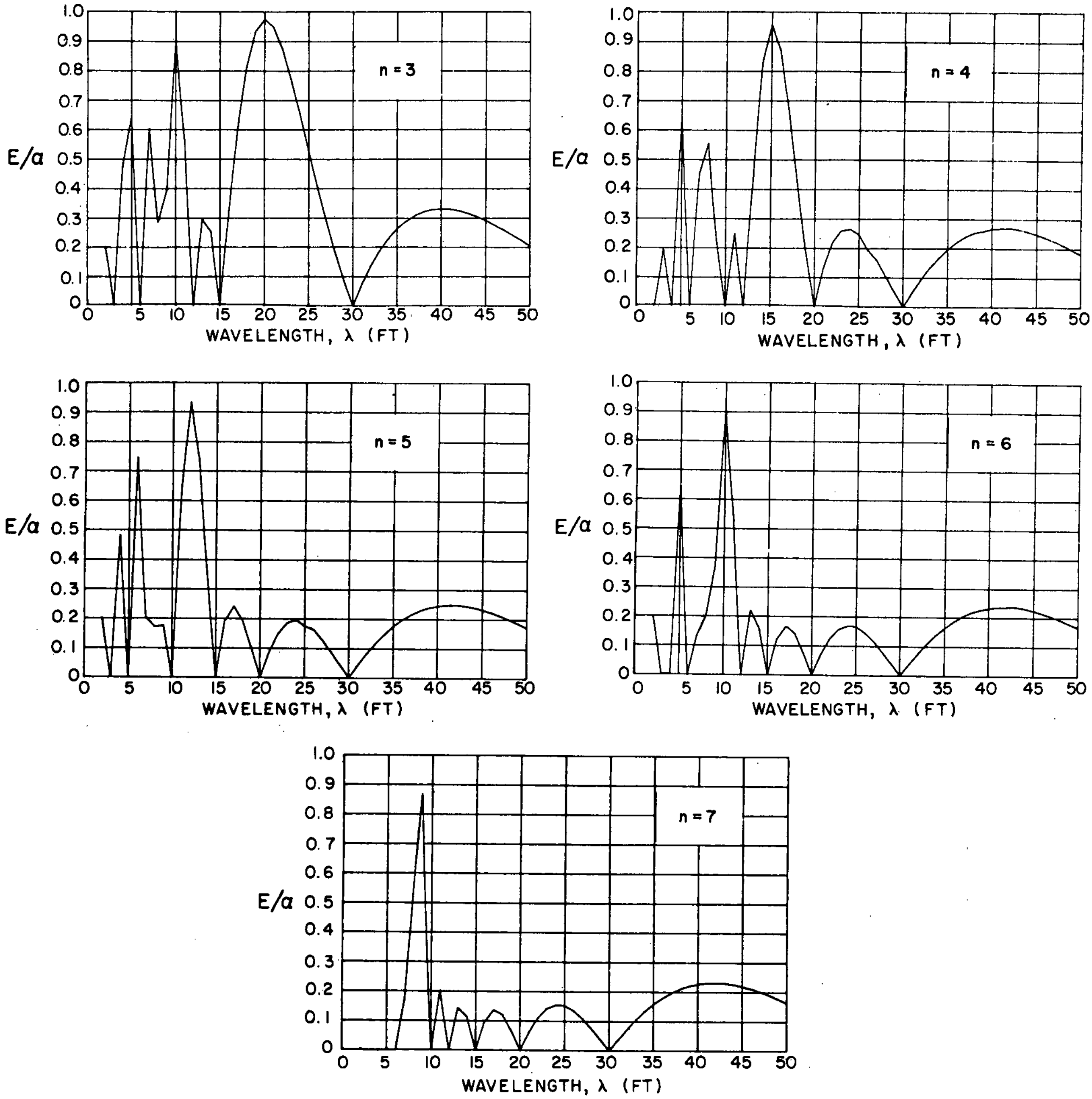
Let $f_i(t)$ be the force on treadle i; then the average value on treadle i is

$$W_i = \frac{1}{\tau_i}\int_{t_i}^{t_i+\tau_i} f(t)\ dt \qquad \text{(C-28)}$$

If average of n treadles is the estimated weight, this is

$$W = \frac{1}{n}\sum_{i=1}^{n} W_i \qquad \text{(C-29)}$$

To estimate the error in this mathematical method (ex-



b = 2.5 ft , Treadle Width
β = 1.0, equally spaced

Figure C-6. Effect of number of scale treadles on E/a.

cluding equipment errors) of assessing weight of a moving vehicle, it can be assumed that the vehicle is at constant velocity. This permits changing the form of Eq. C-28 so that when substituted into Eq. C-29 the estimated weight is analogous to that shown in Eq. C-13 of the discussion of digital systems.

For constant velocity, V,

$$x = V(t - t_o)$$

and

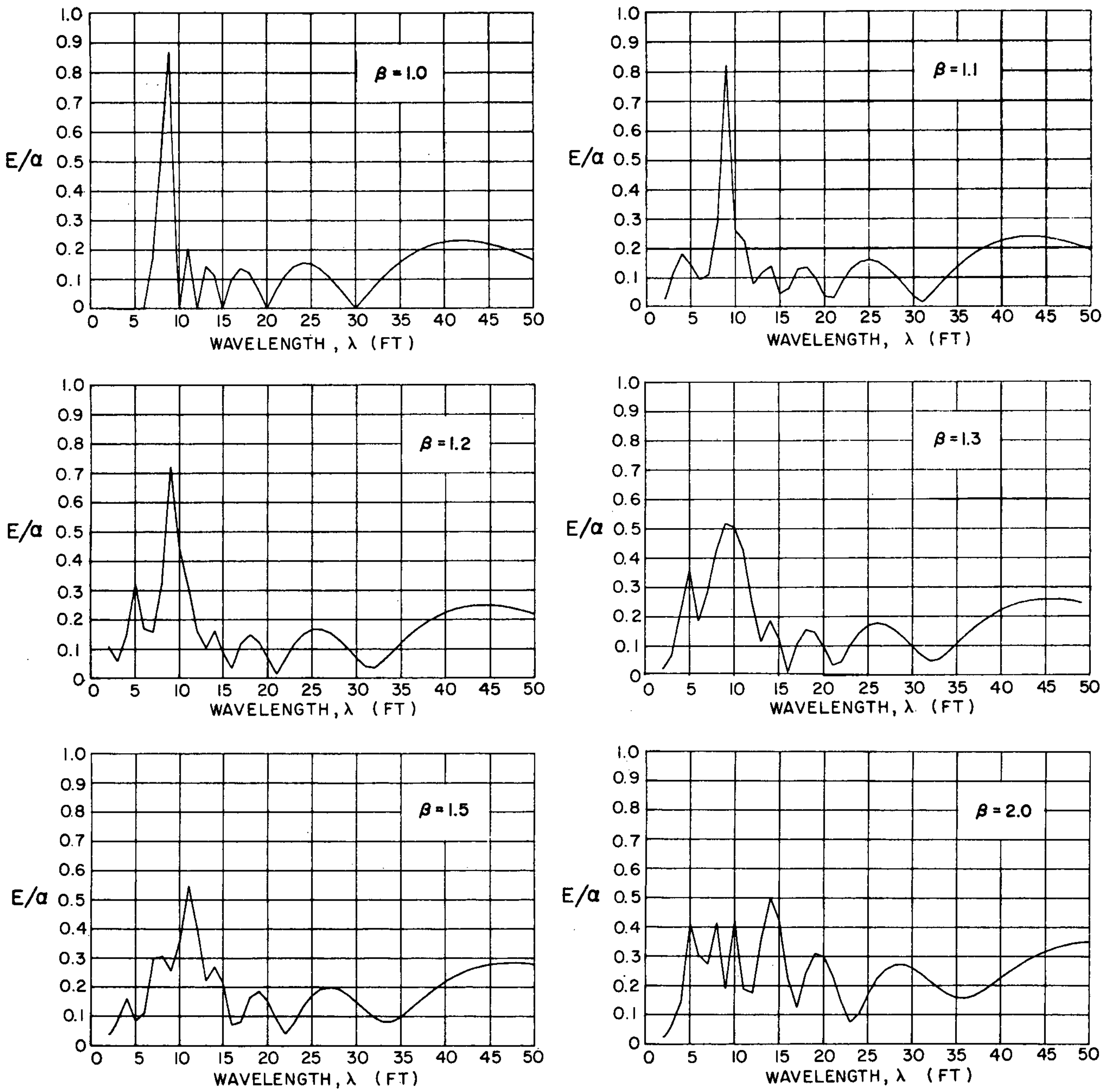
$$dx = V\,dt \tag{C-30}$$

and if the axle crosses a treadle of width b in a time period, τ, at velocity V, then

$$\tau = b/V \tag{C-31}$$

so that Eq. C-28 is equivalent to

$$W = \frac{1}{b\,n}\int_{x_i}^{x_i+b} f_i\,dx \tag{C-32}$$



$x_i = \ell(i/n)^{\beta}$

n = 7, number of treadles
i = treadle number
ℓ = 60 ft , distance to last treadle
β = an exponent
b = 2.5 ft treadle width

Figure C-7. Comparison of E/α for one family of non-uniformly-spaced treadles.

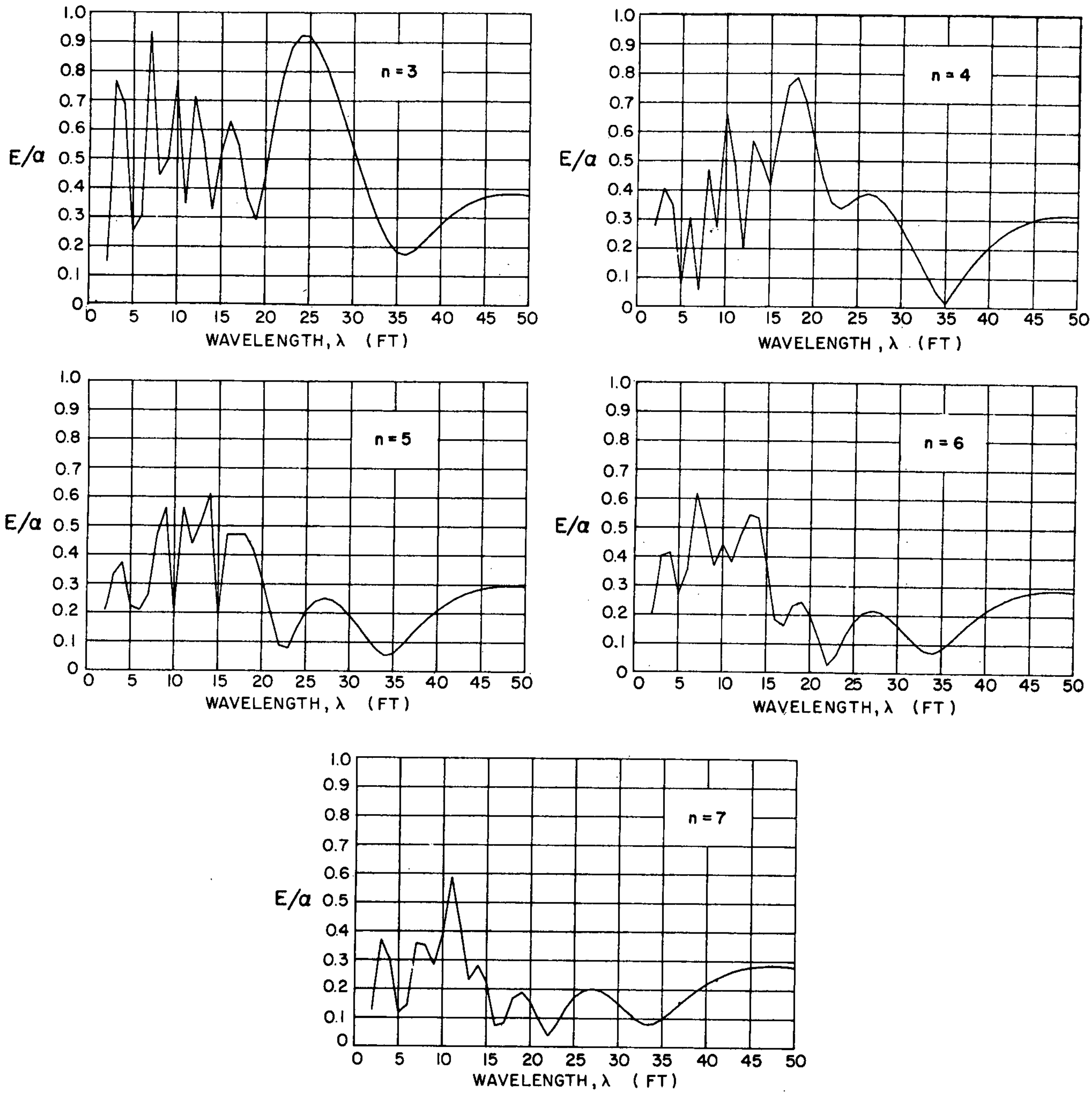


Figure C-8. Effect of number of treadles on E/a *where* $\beta = 1.5$ *and* $b = 1.0$.

The estimated weight is

$$W = \frac{1}{b\,n}\sum_{i=1}^{n}\left[\int_{x_i}^{x_i+b} f_i\,dx\right] \qquad \text{(C-33)}$$

Eq. C-33 is now equivalent to the statement of estimated weight in the discussion of digital systems (Eq. C-13) except that Eq. C-33 is based on an infinite number of samples across each treadle instead of on a finite number, *m*.

For practical purposes, the investigation of error yields the same results as previously shown for the digital systems of averaging. There is, however, both an advantage and a disadvantage of the analog method over the digital method.

The advantage is that of simplified equipment. There is no need to obtain high-speed multiplexers, because the sampling is continuous while the wheel is over the active portion of the treadle. This allows the high frequency components, such as platform vibration, to be averaged in accordance with the integral technique previously discussed.

The disadvantage is the loss of accuracy that is inherent in analog data-processing systems.

APPENDIX D

DYNAMIC MODELS

Either of the two lower-frequency oscillations characteristic of a simple vehicle (i.e., the bouncing and pitching frequencies) will produce a sinusoidal variation of the force between tires and pavement. Their combination will be in accordance with the examples of wave combinations shown in Figures A-2 and A-3. Assume that one low frequency (say, bouncing) is dominant. If, by chance, the bouncing and pitching modes produce beating frequencies, the average of the frequencies is the apparent frequency. For practical purposes this appears in graphic form as a near-sinusoidal signal of varying amplitude. Thus, for the two cases considered here, the force variation between tires and road will be a sinusoidal variation about a constant level. Although in one case—that of beating frequencies—the amplitude of the sinusoidal force varies also in a sinusoidal manner (see Eq. A-7), it is recognized that the random excitation from an uneven highway would produce a varying amplitude in a simple bouncing oscillation as well. Thus, for the cases considered here, the variation in force is a near-sinusoidal wave of varying amplitude that is similar to what may be expected from a simple mechanical oscillator, as shown in Figure D-1.

Considering the simple oscillator, let M be the "effective" mass associated with a given axle; that is,

$$M = \overline{W}/g \tag{D-1}$$

in which $\overline{W}$ is the static axle weight and g is the acceleration due to gravity. It should be emphasized that M is not the actual mass of the vehicle or any part of it; M follows only from the definition shown in Eq. D-1.

If force transducers were placed in the road, the force, $F(t)$, sensed could be expressed as

$$F(t) = M\left[\frac{d^2z}{dt^2} - g\right]$$

in which z represents the vertical displacement of mass, M. Changing to the use of superscript dots to represent differentiation with respect to time, the expression becomes

$$F(t) = M\,[\ddot{z}(t) - g] \tag{D-2}$$

Thus, at any time t_1, $F(t_1)$ can be measured by the transducer, but M and $z(t_1)$ remain two unknowns in the equation. If measurements of force are made at another time, an additional equation is obtained, but another unknown is also introduced—namely, $z(t_2)$. To generalize, if $F(t)$ were to be measured n times, n equations would be obtained relating to $n + 1$ unknowns. Consequently, Eq. D-2 is not sufficient to determine the unknowns. Additional information is needed, and this can be obtained from a further consideration of the dynamics of the simple suspension system.

Returning to the simple oscillator, the effective mass, M, associated with the axle is supported by a spring of stiffness, k, and is damped by a dashpot with a damping coefficient, D. Assuming that the road is level and smooth in the region over which force measurements are to be made, the mass will not receive significant further excitation; hence, the equation of motion can be written as

$$M\,\ddot{z} + D\,\dot{z} + k\,z = -W \tag{D-3}$$

The solution of Eq. D-3 is

$$z = -\frac{W}{k} + e^{-(D/2M)t}\,[A\cos(\omega_{nd}\,t) + B\sin(\omega_{nd}\,t)] \tag{D-4}$$

in which

$$\omega_{nd} = [k/M - (D/2M)^2]^{\frac{1}{2}}$$

This shows that the displacement of the vehicle is a constant due to the static weight on the elastic suspension plus a sinusoidal component of varying amplitude. In this equation the amplitude decreases if D is positive and increases if D is negative, both of which can occur, because this is used to approximate beating signals.

Eq. D-3 is in terms of displacement, z, normal to the road. The only measurements that can be made of an oscillating vehicle in a short time are those of force normal to the highway. Consequently, it is desirable to derive from Eq. D-3 an equation in terms of axle force based on the dynamics of this much-simplified model.

The force on the road is the sum of spring and damping forces. Thus,

$$F(t) = D\,\dot{z} + k\,z \tag{D-5}$$

Transposing,

$$z = 1/k\;F - D/k\;\dot{z} \tag{D-6}$$

Differentiating with respect to time

$$\dot{z} = 1/k\;F - D/k\;\ddot{z} \tag{D-7}$$

Substituting Eq. D-5 yields

$$\ddot{z} = -\frac{F + W}{M}$$

Substituting for $\ddot{z}$ in Eq. D-7 yields

$$\dot{z} = 1/k\;\dot{F} + D/k\,M\;(F + W) \tag{D-8}$$

Differentiating once more yields

$$\ddot{z} = 1/k\;\ddot{F} + D/k\,M\;\dot{F} \tag{D-9}$$

From Eq. D-5, $z = 1/k\;F - D/k\;\dot{z}$, and on substituting for $\dot{z}$ from Eq. D-8, yields

$$z = 1/k\, F - D/k^2\, \dot{F} - D^2/k^2\, M\, (F + W) \qquad \text{(D-10)}$$

Using Eqs. D-8, D-9 and D-10 to substitute for $\ddot{z}$, $\dot{z}$, and z, respectively, in Eq. D-3 yields

$$M/k\, \ddot{F} + D/k\, \dot{F} = -\, W$$

or

$$A\, \ddot{F} + B\, \dot{F} + F = -\, W \qquad \text{(D-11)}$$

in which $A = M/k$ and $B = D/k$.

This equation has three constants—A, B, and W—in which W is the computed weight of prime interest. If the set of quantities F, $\dot{F}$, and $\ddot{F}$ is measured at three different times, t_1, t_2, and t_3, substitution of these quantities into Eq. D-11 yields three equations for the solution of the unknowns. What this does, essentially, is fit a damped sine wave to the measured quantities so that the average level, W, can be calculated. This is the simplest form of this mathematical model. Further discussion of detailed solutions follows.

FINITE-DIFFERENCE SOLUTION

Of the quantities F, $\dot{F}$, and $\ddot{F}$, only F is measured directly from the force-sensing platforms. $\dot{F}$ and $\ddot{F}$ must be obtained by differentiation. In the following this is accomplished by a computational procedure. Let F_i, $i = 1, 2, 3, 4, 5$, be five time-rate samples from one treadle taken at intervals of Δt. Then,

$$F = F_3 \qquad \text{(D-12)}$$

$$\dot{F} = \frac{F_4 - F_2}{2\Delta t} \qquad \text{(D-13)}$$

$$\ddot{F} = \frac{F_5 + F_1 - 2F_3}{4\Delta t^2} \qquad \text{(D-14)}$$

If there were three treadles, a system of three simultaneous equations in A, B, and W could be obtained.

The finite-difference method was tested by generating transducer readings from

$$F(t) = \overline{W}\, \{1 + (e^{-xt})\, (\alpha)\, \sin[\omega\, (t + t_1)]\} \qquad \text{(D-15)}$$

in which $\overline{W}$ is the actual axle weight.

Time-rate sampling from a platform was simulated by taking five values of F at intervals of 0.01 sec. Time-rate platform spacings of 0.1 to 1 sec were studied with amplitudes ranging from 0.1 to 0.8 of the static axle weight. The factor x in Eq. D-15 controls damping or decay of the waveform; it was varied from 0 to 1.5. The axle weight was solved from Eqs. D-8, D-10, D-11, and D-12. The error was computed as

$$E = \frac{\overline{W} - W}{\overline{W}} \qquad \text{(D-16)}$$

in which W is the computed axle weight. The error was normalized as (E/α), as done in the presentation of simple averaging. The maximum absolute value of (E/α) was found to be 6×10^{-5}. Thus, for an amplitude ratio of 0.1, the maximum percent error is 6×10^{-4}.

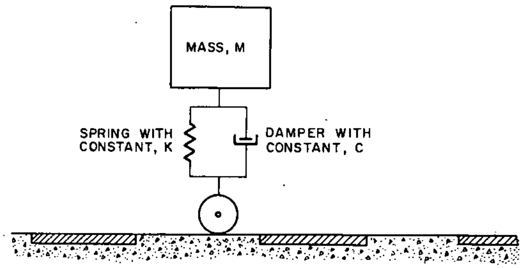


Figure D-1. Equivalent simple oscillator.

This type of theoretical accuracy is not surprising because the sine and cosine functions are defined by their power series as solutions of the differential equation, Eq. D-11, with the coefficient B set to zero. Because, in the limit, this example is exact, results of high accuracy can be expected. However, at this stage it must be emphasized that exact numerical differentiation generally is difficult to perform in practice. Random variations in the function being differentiated can give rise to significantly large errors unless caution is exercised to minimize these errors through various smoothing techniques. Moreover, the three platforms must be arranged in such a manner that no two of them yield the same information. This means that the platform spacing should not be equal to any of the wavelengths in the range of interest.

Because Eqs. D-13 and D-14 represent numerical differentiation, the function F would necessarily have to be smooth. To minimize errors due to random fluctuations, Rutledge's approximations obtained by differentiation of least squares polynomials rather than interpolation polynomials may be used. There are other means to smooth the function F; however, other solutions appeared more practical and less prone to error in actual practice.

Least Squares Fit of the Model, $A\ddot{F} + B\dot{F} + F = W$

Consider the differential equation representing the wheel forces on the road surface:

$$A\, \ddot{F} + B\, \dot{F} + F = W \qquad \text{(D-17)}$$

The coefficients A, B, and W are not known. However, an approximate solution for F (identified as F_p) is known in some interval, $t_1 \leq t \leq t_2$. Because F_p is an approximate solution when substituted into Eq. D-17, it will yield some error, ϵ. Thus,

$$\epsilon = A\, \ddot{F}_p + B\, \dot{F}_p + F_p - W \qquad \text{(D-18)}$$

A, B, and W will now be determined so as to make the square of the error a minimum in the interval (t_1, t_2). Thus,

$$\int_{t_1}^{t_2} \epsilon^2\, dt = \int_{t_1}^{t_2} [A^2\ddot{F}^2 + B^2\dot{F}^2 + (F - W)^2 + 2AB\, \ddot{F}\dot{F} + 2B\dot{F}\, (F - W) + 2\, A\ddot{F}\, (F - W)]\, dt \qquad \text{(D-19)}$$

in which subscript p has been omitted for convenience. For this integral to be a minimum it is necessary to have

$$
\begin{aligned}
\frac{\partial}{\partial A}\int_i \epsilon^2\,dt &= 0\\
\frac{\partial}{\partial B}\int_i \epsilon^2\,dt &= 0 \qquad \text{(D-20)}\\
\frac{\partial}{\partial M}\int_i \epsilon^2\,dt &= 0
\end{aligned}
$$

in which i indicates a definite integral in (t_1, t_2). Expressing conditions in Eq. D-20 in matrix form:

$$
\begin{bmatrix}
\int_i \ddot{F}^2\,dt & \int_i \ddot{F}\dot{F}\,dt & -\int_i \ddot{F}\,dt\\
\int_i \ddot{F}\dot{F}\,dt & \int_i \dot{F}^2\,dt & -\int_i \dot{F}\,dt\\
-\int_i \ddot{F}\,dt & -\int_i \dot{F}\,dt & \int_i dt
\end{bmatrix}
\begin{Bmatrix} A\\ B\\ W \end{Bmatrix}
=
\begin{Bmatrix}
-\int_i \ddot{F}F\,dt\\
-\int_i \dot{F}F\,dt\\
\int_i F\,dt
\end{Bmatrix}
\qquad \text{(D-21)}
$$

Solving Eq. D-21 for W,

$$W = \Delta W/\Delta \qquad \text{(D-22)}$$

in which

$$
\Delta \equiv
\begin{vmatrix}
\int_i \ddot{F}\,dt & \int_i \ddot{F}\dot{F}\,dt & -\int_i \ddot{F}\,dt\\
\int_i \ddot{F}\dot{F}\,dt & \int_i \dot{F}\,dt & -\int_i \dot{F}\,dt\\
-\int_i \ddot{F}\,dt & -\int_i \dot{F}\,dt & \int_i dt
\end{vmatrix} \neq 0
\qquad \text{(D-23)}
$$

and

$$
\Delta W \equiv
\begin{vmatrix}
\int_i \ddot{F}^2\,dt & \int_i \ddot{F}\dot{F}\,dt & -\int_i \ddot{F}\dot{F}\,dt\\
\int_i \ddot{F}\dot{F}\,dt & \int_i \dot{F}^2\,dt & -\int_i \dot{F}F\,dt\\
-\int_i \ddot{F}\,dt & -\int_i \dot{F}\,dt & \int_i F\,dt
\end{vmatrix}
\qquad \text{(D-24)}
$$

Instead of using the three equations with three unknowns, as was done previously, the estimation of weight is here calculated by minimizing the least squares error of as many points of measurement (platforms) as possible.

Nothing was said regarding the method of measuring the quantities $\dot{F}$ and $\ddot{F}$. The transducer provides a signal proportional to F; thus, the signal must be differentiated twice. Although this could be accomplished with either digital or analog equipment, error is associated with each. With digital systems, error is associated with the sampling process wherein discrete samples are taken sequentially. This requires the use of finite difference or polynomial techniques to obtain $\dot{F}$ and $\ddot{F}$ from the sampled data. If the sampling period and frequency are not well chosen with respect to signal frequency, much error can be introduced. Analog systems, on the other hand, may take data from the signal continuously from each platform. Nevertheless, there is inherent error in processing a voltage that is proportional to force. Although the analog system is applicable here, it was not included in this particular analysis. The anticipated error is possibly on a par with that of the digital system shown in the following analyses.

In making the analysis of error of a digital sampling system using finite-difference techniques it is better to return to the basic equation (Eq. D-18) than to use the symbolic representation shown by Eqs. D-21, D-23 and D-24. This is done to incorporate the required notation more clearly.

Where $F(t)$ are known at discrete time intervals, Δt, the error at $t = i\,\Delta t$ can be expressed as

$$\epsilon_i = A\left[\frac{F_{i-1} - 2\,F_i + F_{i+1}}{(\Delta t)^2}\right] + B\left[\frac{F_{i+1} - F_{i-1}}{2(\Delta t)}\right] + (F_i - W) \qquad \text{(D-25)}$$

denoting

$$q_i = \left.\frac{\Delta^2 F}{\Delta t^2}\right|_{i\Delta t} = \frac{F_{i-1} - 2\,F_i + F_{i+1}}{(\Delta t)^2}$$

and

$$r_i = \left.\frac{\Delta F}{\Delta t}\right|_{i\Delta t} = \frac{F_{i+1} - F_{i-1}}{2(\Delta t)}$$

Then, upon substituting, Eq. D-25 becomes

$$\epsilon_i = A\,(q_i) + B\,(r_i) + (F_i - W) \qquad \text{(D-26)}$$

The summation of the squared error sampling interval is

$$\sum_{i=1}^{N-1} \epsilon_i^2 \qquad \text{(D-27)}$$

in which N is the total number of samples from time $t = 0$ to $t = N(\Delta t)$.

Expanding Eq. D-27 yields

$$
\begin{aligned}
\Sigma\,\epsilon_i^2 = {} & A^2\,[\Sigma\,q_i^2] + B^2\,[\Sigma\,r_i^2] + W\,[N-1]\\
& + 2A\,B\,[\Sigma\,q_i\,r_i] - 2B\,W\,[\Sigma\,r_i] - 2A\,W\,[\Sigma\,q_i]\\
& + 2A\,[\Sigma\,q_i\,F_i] - 2W\,[\Sigma\,F_i] + [\Sigma\,F_i^2]\\
& + 2B\,[\Sigma\,r_i\,F_i] \qquad \text{(D-28)}
\end{aligned}
$$

The squared error over the internal $t = 0$ to $t = N(\Delta t)$ is made a minimum by partially differentiating Eq. D-28 with respect to A, B, and W and setting each differential equal to zero.

$$
\begin{aligned}
\frac{\partial}{\partial A}\sum \epsilon_i^2 &= 0\\
\frac{\partial}{\partial B}\sum \epsilon_i^2 &= 0 \qquad \text{(D-29)}\\
\frac{\partial}{\partial W}\sum \epsilon_i^2 &= 0
\end{aligned}
$$

Eq. D-29 yields a set of three simultaneous linear algebraic equations, which may be expressed in matrix form as

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{Bmatrix} A \\ B \\ C \end{Bmatrix} = \begin{Bmatrix} a_1 \\ a_2 \\ a_3 \end{Bmatrix} \quad \text{(D-30)}$$

in which

$$a_{11} = \sum_{i=1}^{N-1} q_i^2 = \frac{1}{(\Delta t)^4}\Big[\Big(6\sum_{i=1}^{N-1} F_i^2 - 8\sum_{i=1}^{N-1} F_i F_{i+1} + 2\sum_{i=1}^{N-1} F_{i-1} F_{i+1}\Big) + (F_0^2 - F_1^2 - F_{N-1}^2 + F_N^2) - 4(F_0 F_1 - F_{N-1} F_N)\Big] \quad \text{(D-31)}$$

$$a_{22} = \sum_{i=1}^{N-1} r_i^2 = \frac{1}{4(\Delta t)^2}\Big[2\Big(\sum_{i=1}^{N-1} F_i^2 - \sum_{i=1}^{N-1} F_{i-1} F_{i+1}\Big) + (F_0^2 - F_1^2 - F_{N-1}^2 + F_N)\Big]$$

$$a_{33} = \sum_{i=1}^{N-1} r_i = \frac{1}{2(\Delta t)}[-F_o - F_1 + F_{N-1} + F_N]$$

$$a_{12} = \sum_{i=1}^{N-1} q_i r_i = \frac{1}{2(\Delta t)^3}[(-F_o^2 - F_1^2 + F_{N-1}^2 + F_N^2) + 2(F_o F_1 - F_{N-1} F_N)]$$

$$a_{13} = \sum_{i=1}^{N-1} q_i = \frac{1}{(\Delta t)^2}[F_o - F_1 - F_{N-1} + F_N]$$

$$a_{23} = \sum_{i=1}^{N-1} r_i = \frac{1}{2(\Delta t)}[-F_o - F_1 + F_{N-1} + F_N]$$

$$a_1 = \sum_{i=1}^{N-1} q_i F_i = \frac{1}{(\Delta t)^2}\Big[2\Big(\sum_{i=1}^{N-1} F_i F_{i+1} - \sum_{i=1}^{N-1} F_i^2\Big) (F_o F_1 - F_{N-1} F_N)\Big]$$

$$a_2 = \sum_{i=1}^{N-1} r_i F_i = \frac{1}{2(\Delta t)}[-F_o F_1 + F_{N-1} F_N]$$

$$a_3 = \sum_{i=1}^{N-1} F_i$$

The foregoing is the digital method for the least squares approximation of the equation $A\ddot{F} + B\dot{F} + F = W$, to the measured data. To test the error of this method, the vehicle force was represented by

$$F(t) = \overline{W}[1 + \alpha \sin(\omega t + \varphi) + \mu \sin(\nu t + \theta) + \alpha/\beta \sin(\gamma t)] \quad \text{(D-32)}$$

in which α, β, and μ represent amplitudes, and ω, ν, and γ represent frequencies.

Because the force samples, $F(i\Delta t)$, at time intervals, Δt, are averaged over $\Delta\tau$, the force sample may be written as

$$F(i\Delta t) = \frac{1}{\Delta\tau}\int_{(i\Delta t - \Delta\tau)}^{(i\Delta t)} F(t)\, dt \quad \text{(D-33)}$$

from which

$$F_i = F(i\Delta t) = \frac{\overline{W}}{\Delta\tau}\Big[\Delta\tau + \frac{2\alpha}{\omega}\sin\frac{\omega\Delta\tau}{2}\sin\Big(i\omega\Delta t + \varphi - \frac{\omega\Delta\tau}{2}\Big) + \frac{2\mu}{\nu}\sin\Big(\frac{\gamma\Delta\tau}{2}\Big)\sin\Big(i\nu\Delta t + \theta - \frac{\nu\Delta\tau}{2}\Big) + \frac{2\alpha}{\beta\gamma}\sin\frac{\gamma\Delta\tau}{2}\sin\Big(i\gamma\Delta t - \frac{\gamma\Delta\tau}{2}\Big)\Big] \quad \text{(D-34)}$$

in which i varies from 0, 1, 2, . . . N, and N is the number of samples.

The error is $E = \dfrac{\overline{W} - W}{\overline{W}}$ in which W is the weight computed by minimizing the least squares error and $\overline{W}$ is the actual weight.

Figures D-2 and D-3 are representative of the error described. The curves show the error, maximized with respect to phase angle, of the least squares fitting of the equation $A\ddot{F} + B\dot{F} + F = W$. Also shown for comparison purposes is the error associated with simple averaging. Figure D-2, for 20 mph, shows that there is some gain in accuracy over simple averaging at 4 and 6 cps; but Figure D-3 shows that the error associated with both simple averaging and the least squares fitting of the equation $A\ddot{F} + B\dot{F} + F = W$ is the same, for practical purposes. Additional computer studies were made which indicate that if the noise frequency is very high or the noise amplitude very low, or zero, the accuracy of the least squares fit of $A\ddot{F} + B\dot{F} + F = W$ yields excellent results. This was also shown in the preceding finite-difference solution without least squares. The reason is obvious. The model to which the data are fitted is a differential equation for which the solutions describe a sine curve. However, the method appears to be more sensitive in fitting a damped sine wave to data than was anticipated; little deviation from a pure sine wave causes significant error in the computed weight. The deviation from a pure sine wave that produced the error in Figures D-2 and D-3 was a noise signal at 120 cps with amplitude 0.1 that of the major dynamic vehicle oscillation amplitude denoted as α.

INTEGRATION SOLUTION I

To preclude the necessity of differentiating the signal to obtain $\dot{F}$ and $\ddot{F}$, as done in the preceding applications of the differential equation $A\ddot{F} + B\dot{F} + F = W$, the equation was integrated twice, term for term, before fitting the data. This is shown in the following. Integrating once yields

$$A(\dot{F} - \dot{F}_o) + B(F - F_o) + \int_0^t F\, d\eta = Wt \quad \text{(D-35)}$$

Integrating again yields

$$A(F - F_o - \dot{F}_o t) + B\Big[\int_0^t F\, d\eta - F_o t\Big] + \int_0^t \int_0^\eta F(\xi)\, d\xi d\eta = W\frac{t^2}{2} \quad \text{(D-36)}$$

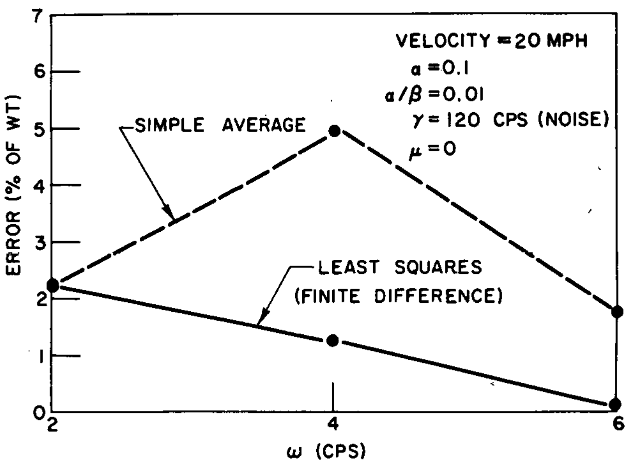


Figure D-2. Error for 20 mph. Least squares fit of dynamic model.

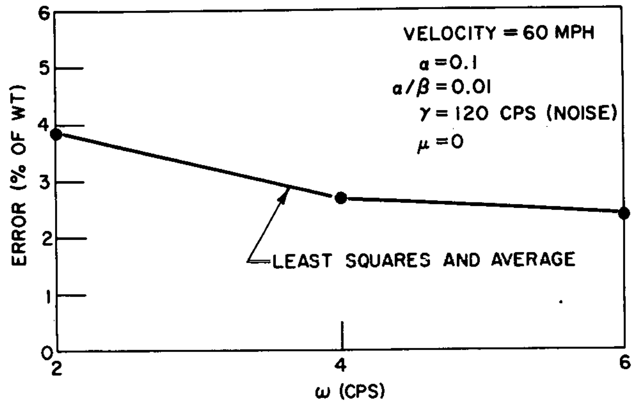


Figure D-3. Error for 60 mph. Least squares fit of dynamic model.

Let

$$\begin{aligned} L_1' &= F - F_0 - \dot{F}_0 t \\ L_2' &= \int_0^t F\, d\eta - F_0 t \\ L_3' &= -\frac{t^2}{2} \\ L_4' &= \int_0^t \int_0^\eta F(\xi)\, d\xi d\eta \end{aligned} \qquad \text{(D-37)}$$

Substituting Eq. D-37 into Eq. D-36 yields

$$L_1'\, A + L_2'\, B + L_3'\, W + L_4' = 0 \qquad \text{(D-38)}$$

If $L_1, \ldots L_4$ are the observed values, Eq. D-38 becomes

$$L_1\, A + L_2\, B + L_3\, W + L_4 = \epsilon$$

in which ϵ is again the error due to imperfect data. The squared error is

$$\begin{aligned} \epsilon^2 = L_1^2 A^2 + L_2^2 B^2 &+ (L_3^2 W^2 + L_4^2 + 2\, L_3\, L_4\, W) \\ &+ 2\, L_1\, L_2\, A\, B + 2\, L_2\, B\, (L_3\, W + L_4) \\ &+ 2\, L_1\, A\, (L_3\, W + L_4) \end{aligned} \qquad \text{(D-39)}$$

Integrating this over the sampling interval, $t = 0$ to $t = T$, and taking partial derivatives with respect to A, B, and W to find the least squares solution, yields the three following equations:

$$\frac{\partial}{\partial A} \int_0^T \epsilon^2\, dt = 2A \int_0^T L_1^2\, dt + 2B \int_0^T L_1\, L_2\, dt + 2 \int_0^T L_1\, (L_3\, W + L_4)\, dt = 0$$

$$\frac{\partial}{\partial B} \int_0^T \epsilon^2\, dt = 2A \int_0^T L_1\, L_2\, dt + 2B \int_0^T L_2^2\, dt + 2 \int_0^T L_2\, (L_3\, W + L_4)\, dt = 0$$

$$\frac{\partial}{\partial W} \int_0^T \epsilon^2\, dt = 2A \int_0^T L_1\, L_3\, dt + 2B \int_0^T L_2\, L_3\, dt + 2 \int_0^T L_3\, L_4\, dt + 2W \int_0^T L_3^2\, dt = 0$$

Rearranging, these are:

$$A \int_0^T L_1^2\, dt + B \int_0^T L_1\, L_2\, dt + W \int_0^T L_1\, L_3\, dt = -\int_0^T L_1\, L_4\, dt$$

$$A \int_0^T L_1\, L_2\, dt + B \int_0^T L_2^2\, dt + W \int_0^T L_2\, L_3\, dt = -\int_0^T L_2\, L_4\, dt$$

$$A \int_0^T L_1\, L_3\, dt + B \int_0^T L_2\, L_3\, dt + W \int_0^T L_3^2\, dt = -\int_0^T L_3\, L_4\, dt \qquad \text{(D-40)}$$

Eq. D-40 is now three equations for the three unknowns, A, B, and W, of which W is the quantity of interest. Further comments regarding this scheme are included in the latter part of the next section.

INTEGRATION SOLUTION II

Instead of integrating the basic differential equation twice, which was done so that the signal $F(t)$ could be used without differentiation, it is here integrated once. Following a procedure similar to the steps between Eqs. D-36 and D-40 of Integration Solution I yields a set of three equations that are the same as Eq. D-40, except that

$$L_1 = (\ddot{F} - \dot{F}_0)$$
$$L_2 = (F - F_0)$$
$$L_3 = (-t) \qquad \text{(D-41)}$$
$$L_4 = \int_0^t F\, d\eta$$

Note that the signal must now be differentiated once to obtain L_1. A signal from any real weighing system will contain a "noise" component in addition to the meaningful and necessary information. The differentiation of such a signal only enhances the noise. However, in these methods the quantities $F(t)$ and $\dot{F}(t)$ are averaged over an interval, T, that may be continuous or—in the case of presently installed weighing platforms—may be discontinuous. Thus, even if the noise component is highly random it will not seriously affect accuracy so long as the signals $F(t)$ and $\dot{F}(t)$ from each platform are integrated over a sufficient time. This "sufficient time" depends upon the probability density of the random noise component and the time an axle is over a platform. If the time over the platform is very large compared with the periods of the dominant frequency components of the random signal, the induced error from noise should be small. Note that taking advantage of this fact to reduce error imposes a limit on vehicle speed while weighing.

The evaluation of the integration methods was made by determining, for a pure sinusoidal force variation, the number of cycles or fraction of a cycle of continuous sampling and integration that is required to reduce the error to some acceptable value consistent with 5% overall error.

In this case, $F(t) = W\,[1 + \alpha \sin(\omega t + \phi)]$, in which $\alpha = 0.1$; $\omega = 2\pi$ (5 cps); and $\varphi = 0.7$ radian.

The error evaluated for Integration Solution II for these values is shown in Figure D-4. Note that almost one-half a cycle is required to reduce the error in the mathematical method to 1% for a pure sinusoidal oscillation with the amplitude, α, of the oscillating force equal to 0.1 of the axle weight. To compare this with integral averaging, refer to Figure C-3. Note that all certainty of less than 1.0% error is not obtained until 2.6 cycles are integrated. This is also for $\alpha = 0.1$. In this respect the integration method has a definite advantage over integral averaging, but at considerably more expense. However, all this is based upon a pure sinusoidal oscillation. The method will not converge to a small error nearly so quickly if there is noise present. A full assessment of noise averaging capability was not made, nor was a study made of this method when subjected to two- or three-component waveforms.

Figure D-4 represents the one-integration method that is more sensitive and converges to low error for a sinusoidal disturbance more quickly than with double integration (Integration Solution I).

Polynomial Smoothing of Force Data with Integration Solution II

The integration solutions just discussed consist of integrating the equation $A\ddot{F} + B\dot{F} + F = W$ to fit it to the measured data in the average without computing derivatives.

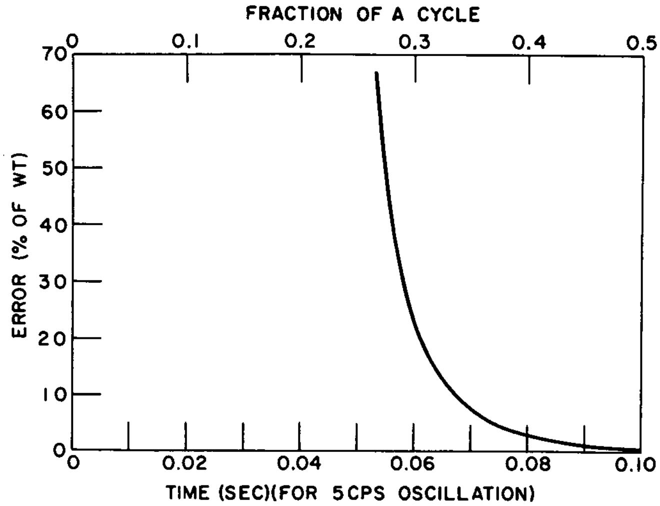


Figure D-4. Error of integration solution II.

This reduced, but did not remove, the vulnerability to noise and other irregularities, especially when the equation was integrated only once. Consequently, when the noise component was large, further smoothing was necessary before applying the integration solution or any other solution so far discussed. Polynomial smoothing by least squares fit of platform data was contemplated.

To test the added smoothing approach the decision was made to use a typically noisy signal representing two components of vehicle oscillation, plus a much higher frequency equal to that observed as platform oscillation of a heavy steel structure platform. The axle force was represented as follows:

$$F = \overline{W}\,[1 + \alpha \sin(\omega t + \varphi) + \beta \sin(\nu t + \theta) + \gamma \sin \rho t] \qquad \text{(D-42)}$$

This was sampled by force-sensitive platforms of length b, placed at a distance, X_n, from the origin. Constant vehicle velocity, v, was assumed so that time to any platform, t_n (from the origin), was

$$t_n = X_n / v \qquad \text{(D-43)}$$

Time on the platform was

$$\tau_n = b / v \qquad \text{(D-44)}$$

For any platform, the force was

$$Fn = \overline{W}\,\{1 + \alpha \sin[\omega (t_n + \tau) + \varphi] + \beta \sin[\nu (t_n + \tau + \theta) + \gamma \sin[\rho (t_n + \tau)]\} \qquad \text{(D-45)}$$

This was then measured and recorded for time periods τ_n, according to the definition of Eq. D-44.

A third-order polynomial

$$F_n' = a_0 + a_1\tau + a_2\tau^2 + a_3\tau^3 \qquad \text{(D-46)}$$

was then fit to the data by the method of least squares error in an effort to remove the high-frequency component shown by γ and ρ of Eqs. D-42 and D-45. Thus,

$$F_n' = a_0 + a_1\tau + a_2\tau^2 + a_3\tau^3 \qquad \text{(D-47)}$$
$$\dot{F}_n' = a_1 + 2\,a_2\tau + 3\,a_3\tau^2$$
$$\ddot{F}_n' = 2\,a_2 + 6\,a_3\tau$$

The basic equation of this discussion is $AF' + BF' + F' = W$. Integrating once and rearranging,

$$A\left[\frac{1}{\tau_n}\int_0^{\tau_n}\ddot{F}'\,d\tau\right] + B\left[\frac{1}{\tau_n}\int_0^{\tau_n}\dot{F}'\,d\tau\right] - W = -\frac{1}{\tau_n}\int_0^{\tau_n}F'\,d\tau \qquad \text{(D-48)}$$

The outcome was that the polynomial smoothing of data over each platform was excellent (see Fig. D-5) but the dynamic model $(A\ddot{F}' + B\dot{F}' + CF' = W)$ appeared to converge on the medium-frequency component rather than the lowest vehicle frequency. The accuracy of computed weight was not good.

Because this had already required the use of polynomials for smoothing, no further study was made of the simple dynamic method. Polynomials were thought to be more useful for smoothing over the whole record as well as just the platforms.

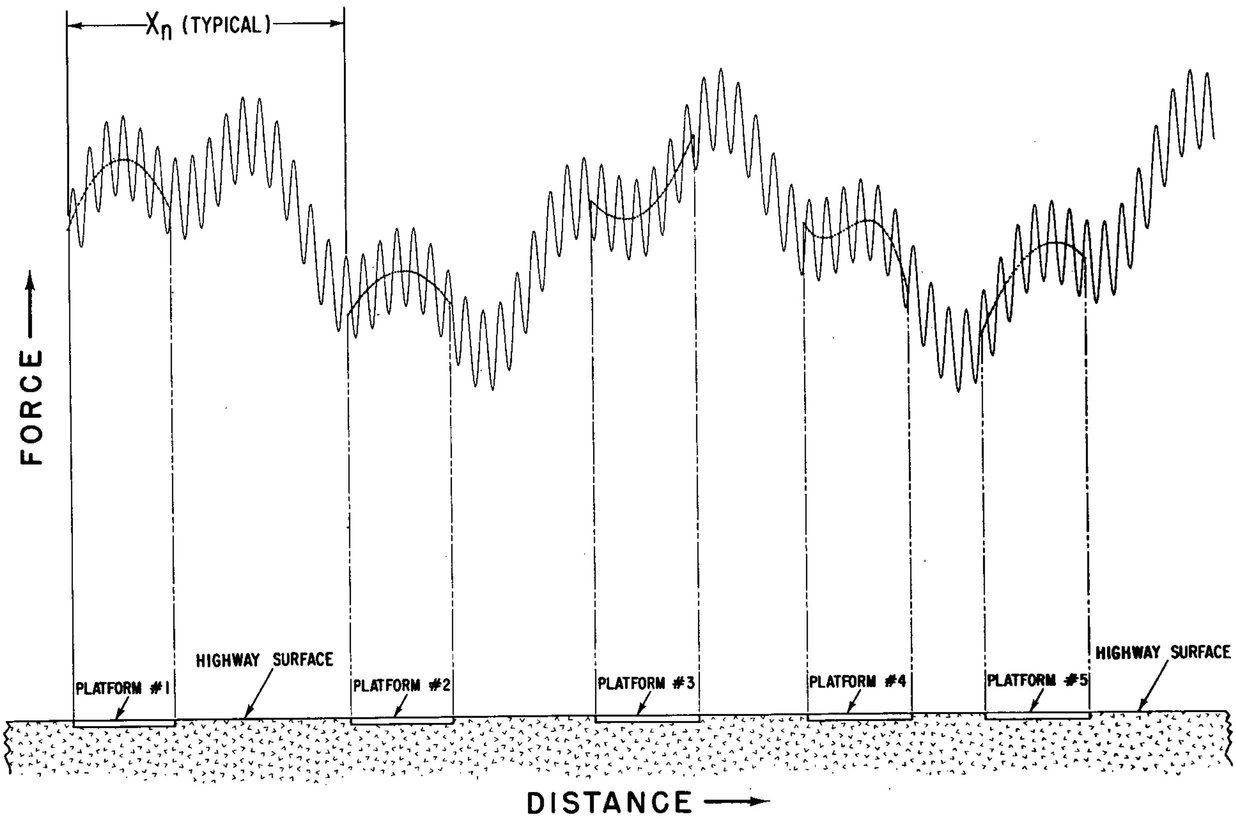


Figure D-5. Three-component waveform with polynomial smoothing over platforms.

APPENDIX E

INTEGRAL MODEL III

Often the need arises for predicting the value of dependent variable Y for any given values of one or more variables, X_i. The variables X_i may be related in an explicit functional form which may be hypothesized from past theoretical or experimental knowledge without necessarily specifying the numerical values of the constants entering into the equation. Regression analysis provides a systematic approach for estimating the unspecified constants from a set of observations of the variables entering the equation. It also allows computation of confidence limits for the range of estimates provided by the analysis.

The general assumption underlying regression analysis is that the dependent variable is representable by a linear function of other variables. The general method used in estimating a regression relation from sample data is based on least squares; the sample regression relation is that which yields a minimum for the sum of squares of the deviations of the observed values from the predicted values of the regression relation.

Even though regression analysis is hypothesized on a linear relation among the variables, it is not restricted to the analysis of linear problems alone. Many nonlinear classes of problems can be dealt with by appropriate transformation of the variables into those that are related linearly.

In using multiple regression analysis for predicting axle weight the two important tasks are: (1) selection of the dependent variable and (2) selection of the most suitable sample regression equation. Starting with the expression for the force exerted by an axle on the road surface as

$$F = \overline{W}\left\{1 + \sum_{i=1}^{K} a_i \sin(\omega_i t + \phi_i)\right\}$$

this expression was operated on in various manners to find a suitable sample regression model. The following worked fairly well.

Integrating the force expression once yields

$$\int_0^T F dt = \overline{W}\left\{T - \sum_{i=1}^{K} \frac{a_i}{\omega_i}[\cos(\omega_i T + \phi_i) - \cos\phi_i]\right\} \tag{E-1}$$

Integrating again yields

$$\int_0^T \int_0^\eta F dt d\eta = \overline{W}\left\{1/2\, T^2 - \sum_{i=1}^{K} \frac{a_i}{\omega_i^2}[\sin(\omega_i T + \phi_i) - \sin\phi_i] + \sum_{i=1}^{K} \frac{a_i T}{\omega_i}\cos\phi_i\right\} \tag{E-2}$$

Recognizing that

$$\frac{1}{\omega_j^2}(F - F_o) = \sum_{i=1}^{K} \frac{a_i}{\omega_j^2}[\sin(\omega_i T + \phi_i) - \sin\phi_i] \tag{E-3}$$

this is added and subtracted from Eq. E-2 to yield

$$\int_0^T \int_0^\eta F dt d\eta = \overline{W}\left\{1/2\, T^2 + T\sum_{i=1}^{K} \frac{a_i}{\omega_i}\cos\phi_i - \frac{1}{\omega_j^2}(F - F_o) + \sum_{i=1}^{K} a_i \frac{1}{\omega_j^2} - \frac{1}{\omega_i^2}[\sin(\omega_i T + \phi_i) - \sin\phi_i]\right\} \tag{E-4}$$

Dividing by $2/T^2$ puts the expression in a better working form:

$$\frac{2}{T^2}\int_0^T \int_0^\eta F dt d\eta = \overline{W}\left\{1 + \frac{2}{T}\sum_{i=1}^{K} \frac{a_i}{\omega_i}\cos\phi_i - \frac{2}{T^2}\left(\frac{1}{\omega_j^2}\right)(F - F_o) + \frac{2}{T^2}\sum_{i=1}^{K} a_i\left(\frac{1}{\omega_j^2} - \frac{1}{\omega_i^2}\right)[\sin(\omega_i T + \phi_i) - \sin\phi_i]\right\} \tag{E-5}$$

The dependent variable was chosen as

$$Y = \frac{2}{T^2}\int_0^T \int_0^\eta F dt d\eta \tag{E-6}$$

The predicted value of Y was then expressed by a sample regression equation of the form

$$Y' = A + B_1X_1 + B_2X_2 + B_3X_3 + \ldots B_nX_n \tag{E-7}$$

in which, clearly,

$$X_1 = \frac{2}{T}$$
$$X_2 = \frac{2}{T^2}(F_o - F) \tag{E-8}$$

and in which other variables may be assigned to approximate the terms

$$\frac{2}{T^2}\sum_{i=1}^{K} a_i\left(\frac{1}{\omega_j^2} - \frac{1}{\omega_i^2}\right)[\sin(\omega_i T + \phi_i) - \sin\phi_i]$$

Trial variables were considered in the form

$$X_3 = 2/T^2$$
$$X_4 = 2/T^3 \tag{E-9}$$

and so on, to certain variations of these.

The computed weight, as seen from Eqs. E-5 and E-7, is given by

$$W = A \tag{E-10}$$

This was convenient when, in the subsequent test runs of the method, $\overline{W}$ (the assumed weight) was normalized to the value 1.0 to allow the computed weight, W, to be compared directly with 1.0.

The error in the method is evident from the last sum of terms in Eq. E-5 in which error decreases in an inverse square manner as the total sampling time increases. It is evident also from these terms that the error is of the order

$$\sum_{i=1}^{K} a_i\left(\frac{1}{\omega_j^2} - \frac{1}{\omega_i^2}\right)[\sin(\omega_i T + \phi_i) - \sin\phi_i] \tag{E-11}$$

Thus, increasing spread between the lowest and highest frequencies in the sampled waveform leads to increasing errors. Consequently, it is essential that the amplitude of the highest frequency waveform (which is generally the platform's natural frequency) be minimized by proper design.

In testing the assumed model by the foregoing regression analysis, the following statistical parameters were computed:

1. The total sum of the squares of the deviations of all data from the average of the data:

$$\text{Total } SS = \Sigma\,(Y - \overline{Y})^2 \qquad \text{(E-12)}$$

When divided by the number of observations, the total SS gives the mean square deviation.

2. The sum of the squares of the deviations of the actual values from the predicted values:

$$\text{Error } SS = (Y' - Y)^2 \qquad \text{(E-13)}$$

This quantity is a measure of the scatter of the actual values from those predicted by the regression equation.

3. The regression sum of squares, which is the difference of the total sum of squares and the error sum of squares:

$$\text{Reg } SS = \text{Total } SS - \text{Error } SS \qquad \text{(E-14)}$$

This quantity is a measure of the part of total spread accounted for by the model.

4. The degrees of freedom of the system, which is the difference between the number of observed data points and the number of parameters in the regression equation:

$$DF = \text{Number of observations} - \text{Number of parameters} \qquad \text{(E-15)}$$

The more the degrees of freedom, the more accurate the estimate of the parameters is likely to be.

5. The error variance, which is the ratio of the error sum of squares to the number of degrees of freedom:

$$\text{Error Var} = \text{Error } SS/DF \qquad \text{(E-16)}$$

If the deviation between the actual and predicted values is taken at any point and squared, the most likely value of the resulting quantity is given by the error variance.

6. The standard deviation, defined as the square part of the error variance:

$$\text{Std Dev} = \text{Error Var} \qquad \text{(E-17)}$$

It is the root mean square error of the observed values from the predicted values. If the distribution of the observed values over the predicted values is normal, 67% of the distribution will fall within $\pm$ 1 standard deviation and 95% within $\pm$ 3 standard deviations.

7. Multiple Rho, which is the fraction of the total spread of the system accounted for by the model:

$$\text{Mult Rho} = \text{Reg } SS - \text{Total } SS \qquad \text{(E-18)}$$

8. T, which is given by the value of the parameter divided by its standard deviation. This quantity is of interest in the t-test.

In testing the method, the double integral of the dependent variable was computed for 160 milliseconds continuously and the analysis was carried out. Results for selected runs and the frequencies and amplitudes representing the axle are shown in Figure E-1.

Section I of Figure E-1 shows the results for an input of a pure sine wave superimposed on a constant value of 1. The prediction of this value, W, is given by the parameter A as $1.00000064 \pm 2 \times 2.64 \times 10^{-5}$. This is remarkably accurate inasmuch as the sampling was conducted over less than a waveform. Regarding the suitability of the model, it was found from the t-test that variables X_3 and X_4 are really unnecessary as long as only one component is involved, because the absolute values of T in the computer output for parameter X_3 and X_4 are found to be less than 1.96. However, in the multi-component tests the importance of these parameters will be noticed.

Section II of Figure E-1 shows the results for a two-component waveform with frequencies of 5 cps and 80 cps. The prediction is found to be in error by about 3%. From the t-test it is found that whereas no significance can be attached to B_2 parameter, B_3 and B_4 assume considerable importance in the analysis.

Section III of Figure E-1 shows the results for a two-component waveform with frequencies of 5 and 10 cps. The prediction is found to be in error by only 2%. It can be noticed from the t-test that all the parameters have assumed significance.

Section IV of Figure E-1 shows the results for a three-component waveform with frequencies of 5, 10, and 80 cps. Error in the prediction is found to be only 0.1%. Except for B_2, all the other parameters are again found to be significant. The reduction in error is due to the nearly double amplitude of the low-frequency components compared with that of the high-frequency component.

However, after many computer runs simulating many conditions of vehicle speed, the number of harmonic components comprising the vehicle axle force, and the relative frequency and phase angles of these components, it is concluded that the simple regression model

$$Y' = A + B_1\,(1/T) + B_2\left(\frac{F - F_o}{T^2}\right) \qquad \text{(E-19)}$$

works as well as those with added terms of $B_3\,(1/T^2)$, $B_4\,(1/T^3)$, . . . $B_K\,(1/T^{K-1})$.

In cases where the ratio of frequencies is less than two, the error appears to be relatively small for two-component signals approaching equal amplitudes of the harmonic components. However, for signals having two harmonic components with frequency ratios greater than 2 or 3 there are combinations of phase angles that produce significant error—even greater error than that of a simple average of force over the platforms. Figure E-2 shows the results of a study made upon signals with two harmonic components. The lower frequency was held constant. The amplitude of the lower frequency was also held constant at 0.1 of the static force component (weight). The integration period was 0.8 of the lower frequency. This is a relatively long period when the amount of highway that must be instrumented is considered.

These results are representative of the limited investigation that was performed for this report. Although analyses following this regression scheme may have merit, it appears that the complexity of analysis and the susceptibility to large error under more than occasional conditions do not make them more attractive than some of the less-complex averaging techniques.

$$Y' = A + B_1 X_1 + B_2 X_2 + B_3 X_3 + B_4 X_4$$

I. $\omega_1 = 5$ cps, $\alpha_1 = 0.1$, W = 1

TOTAL SS=	0.165063694E-01	REG SS=	0.165062318E-01		
ERR SS=	0.137600000E-06	DF=	0.710000000E 02		
ERR VAR=	0.193802816E-08	STD DEV=	0.440230412E-04		
N=	0.760000000E 02	MULT RHO=	0.999995831E 00		
A=	0.100000064E 01	ERR A=	0.264154278E-04	T=	0.378566891E 05
B 1=	0.243376905E-02	ERR B=	0.108960000E-05	T=	0.223363532E 04
B 2=	0.101251820E-02	ERR B=	0.628377525E-06	T=	0.161132147E 04
B 3=	-0.148900000E-08	ERR B=	0.437718891E-07	T=	-0.340172661E-01
B 4=	0.901900000E-11	ERR B=	0.284679204E-09	T=	0.316812744E-01

II. $\omega_1 = 5$ cps, $\alpha_1 = 0.1$, $\omega_2 = 80$ cps, $\alpha_2 = 0.01$, W = 1

TOTAL SS=	0.169744834E-01	REG SS=	0.120308772E-01		
ERR SS=	0.494360620E-02	DF=	0.710000000E 02		
ERR VAR=	0.696282563E-04	STD DEV=	0.834435476E-02		
N=	0.760000000E 02	MULT RHO=	0.841880332E 00		
A=	0.102975842E 01	ERR A=	0.355403787E-02	T=	0.289743232E 03
B 1=	0.226242164E-02	ERR B=	0.239559620E-03	T=	0.944408594E 01
B 2=	0.600785900E-04	ERR B=	0.635766897E-04	T=	0.944978266E 00
B 3=	-0.460684310E-04	ERR B=	0.910315208E-05	T=	-0.506071200E 01
B 4=	0.269108840E-06	ERR B=	0.665420943E-07	T=	0.404418951E 01

III. $\omega_1 = 5$ cps, $\alpha_1 = 0.1$, $\omega_2 = 10$ cps, $\alpha_2 = 0.1$, W = 1

TOTAL SS=	0.125980336E 00	REG SS=	0.123174439E 00		
ERR SS=	0.280589700E-02	DF=	0.710000000E 02		
ERR VAR=	0.395196760E-04	STD DEV=	0.628646769E-02		
N=	0.760000000E 02	MULT RHO=	0.988801042E 00		
A=	0.101944630E 01	ERR A=	0.457003822E-02	T=	0.223071723E 03
B 1=	0.368991600E-02	ERR B=	0.422689046E-03	T=	0.872962295E 01
B 2=	0.217605000E-03	ERR B=	0.420691625E-04	T=	0.517255364E 01
B 3=	-0.491503400E-04	ERR B=	0.122459934E-04	T=	-0.401358537E 01
B 4=	0.289397800E-06	ERR B=	0.672808707E-07	T=	0.430133850E 01

IV. $\omega_1 = 5$ cps, $\alpha_1 = 0.1$, $\omega_2 = 10$ cps, $\alpha_2 = 0.1$, $\omega_3 = 80$ cps, $\alpha_3 = 0.01$, W = 1

TOTAL SS=	0.128816424E 00	REG SS=	0.124989913E 00		
ERR SS=	0.382651100E-02	DF=	0.710000000E 02		
ERR VAR=	0.538945211E-04	STD DEV=	0.734128879E-02		
N=	0.760000000E 02	MULT RHO=	0.985035457E 00		
A=	0.999017280E 00	ERR A=	0.633541980E-02	T=	0.157687621E 03
B 1=	0.586274500E-02	ERR B=	0.574300376E-03	T=	0.102084993E 02
B 2=	-0.101323000E-04	ERR B=	0.478090811E-04	T=	-0.211932540E 00
B 3=	-0.112416610E-03	ERR B=	0.170111346E-04	T=	-0.660841340E 01
B 4=	0.625154500E-06	ERR B=	0.102454433E-06	T=	0.610178087E 01

Figure E-1. Summary of significance tests ($Y' = A + B_1X_1 + B_2X_2 + B_3X_3 + B_4X_4$).

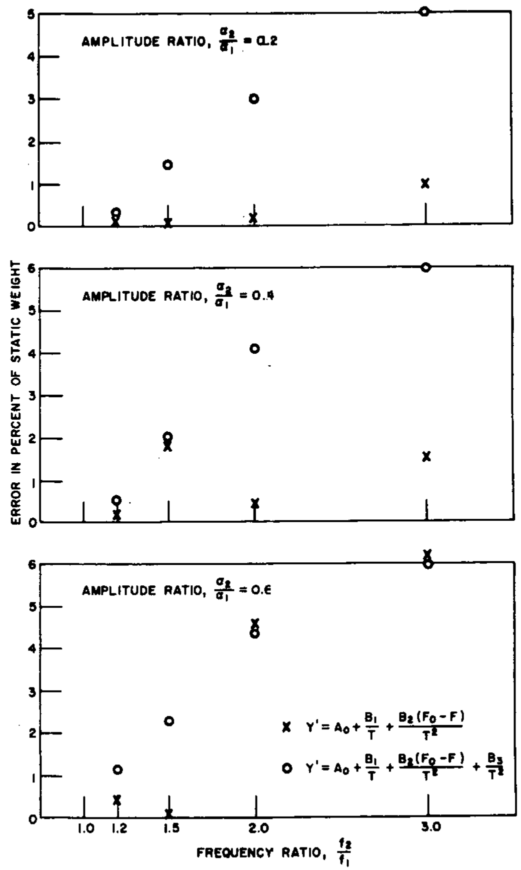


Figure E-2. Comparison of regression equations.

APPENDIX F

INTERPOLATION OF AXLE FORCE BETWEEN PLATFORMS

Although interpolation of the axle force between spaced platforms has not been explicitly considered previously in this report, it has been inherent in all methods other than the simple averaging over platforms. In the so-called "dynamic methods," interpolation is implied in that one of a family of continuous functions was fitted to the data. Direct application of interpolation by fitting polynomial curves is considered here.

Interpolation as needed for representing axle-force data between spaced platforms requires the use of well-chosen functions, because the cost of placing force-sensing platforms in the highway surface dictates that the force-sensitive areas generally are widely spaced relative to their dimension in the direction of travel. Thus, the chosen functions must satisfy approximately, or in the average, the local group of data obtained from each force-sensitive area and predict a good estimate of axle force between platforms. The basic method employed to accomplish this was least squares fitting with truncation of the polynomial to enhance the fitting of the longer wavelength components.

Specific polynomial systems investigated and reported here include the Fourier series, general trigonometric polynomials, general algebraic polynomials, and simple algebraic polynomials fitted over successive groups of force-sensing platforms. These were studied with the use of digital computers and FORTRAN programming. The programs are included in Appendix I.

TRIGONOMETRIC POLYNOMIALS

Eq. A-12, the function used to represent axle force, suggests the use of trigonometric polynomials in reconstructing the function from platform-sampled data. Because this was a limited study, only one function representing one phase relationship of a vehicle with free suspension and with tire hop at 12 cps was used for the computer study. The function providing representative axle force data was:

$$\overline{F} = \overline{W}\left[\, 1.0 \text{ x} \sum_{i=1}^{3} a_i \sin\,(2\pi f_i t + \phi_i)\,\right] \qquad \text{(F-1)}$$

in which

$\overline{W} = 10{,}000$ lb;
$a_1 = a_2 = 0.1$;
$a_3 = 0.4$;
$f_1 = 2.4$ cps;
$f_2 = 2.6$ cps;
$f_3 = 12.0$ cps;
$\phi_1 = 0$; and
$\phi_2 = \phi_3 = \pi/2$.

The trigonometric polynomial used to reconstruct a continuous estimate of axle force from the sampled data was of the form

$$F = A_0 + \sum_{k=1}^{K}\left[A_k \cos\left(\frac{k\pi t}{T}\right) + B_k \sin\left(\frac{k\pi t}{T}\right)\right] \quad \text{(F-2)}$$

and was fitted to the sampled data by the least squares method.

The computer program used to study this application is shown in Appendix I as Digital Computer Program TRIGFT.

Using platform spacing in accordance with Figure F-1, Figures F-2 through F-4 compare the fitted truncated polynomial to the curve representing axle force from which the sampled data were taken. It must be remembered that the sampled data were taken only in the relatively small shaded areas representing the location (in time) of the force-sensitive platforms. The fit is much better at the left-hand side of each curve. This is to be expected because the spacing of platforms in this model places them closer together, as shown at the left-hand side. To have spaced them evenly would have been to invite the same source of error previously discussed under simple averaging methods—that of sampling at the peaks of successive cycles if a wide range of velocities is anticipated.

Much more could have been done in the study of trigonometric polynomial fitting. The wide swings in the fitted curve possibly could have been reduced by a direct solution approach instead of the least squares approach, but other considerations dictated that that the investigation procede to what appeared to be a more successful method at that time.

ALGEBRAIC POLYNOMIALS

Following the investigation of trigonometric polynomials, the same study method with the same platform spacing and axle model was used to fit a general algebraic polynomial of the form

$$F = \sum_{k=0}^{K} A_k t^k \qquad \text{(F-3)}$$

For practical purposes the results are the same as that shown for trigonometric polynomials. The least squares procedure produced a fair fit where the platforms were closer together, and wide swings where the platforms were spaced farther apart. Again, the investigation was brief. Much more can be done with such polynomial approximation.

This study was terminated because it was recognized that a large number of terms was needed to reconstruct the continuous axle force curve even approximately. When relatively small force-sensitive areas are placed in an internal of 1.0 to 1.5 times the longest wavelength anticipated, the significant value in fitting a curve is the average platform force. The curvature of the force func-

PLATFORM SPACING (TIME IN SECONDS, DISTANCE OR WIDTH IN FEET)

PLATFORM NO.	TIME TO PLATFORM	TIME ON PLATFORM	DISTANCE TO PLATFORM	PLATFORM WIDTH
1	0.01131	0.01420	0.9948	1.2500
2	0.06337	0.01420	5.5765	1.2500
3	0.14061	0.01420	12.3736	1.2500
4	0.23580	0.01420	20.7508	1.2500
5	0.34596	0.01420	30.4442	1.2500
6	0.46923	0.01420	41.2923	1.2500
7	0.60434	0.01420	53.1818	1.2500

PLATFORM SPACING (TIME IN SECONDS, DISTANCE OR WIDTH IN FEET)

PLATFORM NO.	TIME TO PLATFORM	TIME ON PLATFORM	DISTANCE TO PLATFORM	PLATFORM WIDTH
1	0.00420	0.02841	0.3698	2.5000
2	0.05627	0.02841	4.9515	2.5000
3	0.13351	0.02841	11.7486	2.5000
4	0.22870	0.02841	20.1258	2.5000
5	0.33885	0.02841	29.8192	2.5000
6	0.46213	0.02841	40.6673	2.5000
7	0.59724	0.02841	52.5568	2.5000

Figure F-1. Platform spacing for trigonometric polynomial interpolation: platform widths, 1.25 ft and 2.50 ft.

tion over the platform yields relatively little leverage on fitting an expression. Consequently, attention was directed to a method whereby the change in value of force over the platform could play a greater part.

MULTIPLE POLYNOMIAL FITTING OVER SUCCESSIVE PLATFORM GROUPS

Recognition of the problems associated with fitting curves to functions with many points of inflection (by means of sampled data, no less) led to fitting a more simple polynomial, trigonometric or algebraic, over each successive group of three platforms. Thus, where a system is comprised of N platforms, the interpolation is provided by $(N-2)$ polynomial approximations. This proved to be attractive in that the platform became larger relative to the reduced interval covered by each simple polynomial.

Successive overlapping groups of three platforms are described as follows: 1st interpolation interval, platforms 1, 2 and 3; 2nd interpolation interval, platforms 2, 3 and 4; and so on to the last interval, $(N-2)$nd interpolation interval: platforms, $(N-2)$, $(N-1)$ and N.

The method used was to read in the axle force data from the simulated force-sensitive areas by means of a platform positioning function, plus an expression representing axle force for a vehicle with certain dynamic characteristics. Algebraic polynomials of the same form as Eq. F-3,

$$F=\sum_{k=0}^{K} A_k t^k$$

but with K ranging only from 2 through 5, were fitted by least squares in each interpolation interval. Thus, a reasonable continuous estimate of axle force was achieved. Where the interpolation intervals overlapped, the average of the interpolation functions was taken at that point.

This interpolation method, in particular, was developed to implement a more "brute force" than analytical method of determining axle weight from the sampled force data. That weighing method appears in Appendix G. For this reason, the computer program for interpolation by multiple polynomial fitting over successive platform groups is combined in computer program VWEIGH (see Appendix I) for that weighing method.

Figure F-5 is plotted from computer output shown in Appendix I. It indicates the relatively good curve following that is achieved by fitting polynomials to successive groups of platforms. Although the actual interpolation of axle force is not really good (bottom half of Fig. F-5), this is an example of a trial yielding an excellent weight estimate. The reasons for this are discussed later.

The upper half of Figure F-5 represents an example of non-"noisy" axle force generated from a vehicle with beating pitch and bounce frequencies of 2.1 and 2.4 cps of equal amplitude approximated by polynomials of second degree (parabolic curves). The bottom half of Figure F-5 represents a vehicle with blocked suspension and with tire hop. Frequencies are 3.7, 4.0, and 12.0 cps. The interpolation curves are algebraic polynomials of fifth degree.

Additional markings on Figure F-5 have to do with the method of computing weight whereby approximately one wavelength is determined so that averaging can be done over that period. This is discussed later.

FOURIER SERIES

The following investigation was given limited attention as a side application of trigonometric polynomial interpolation.

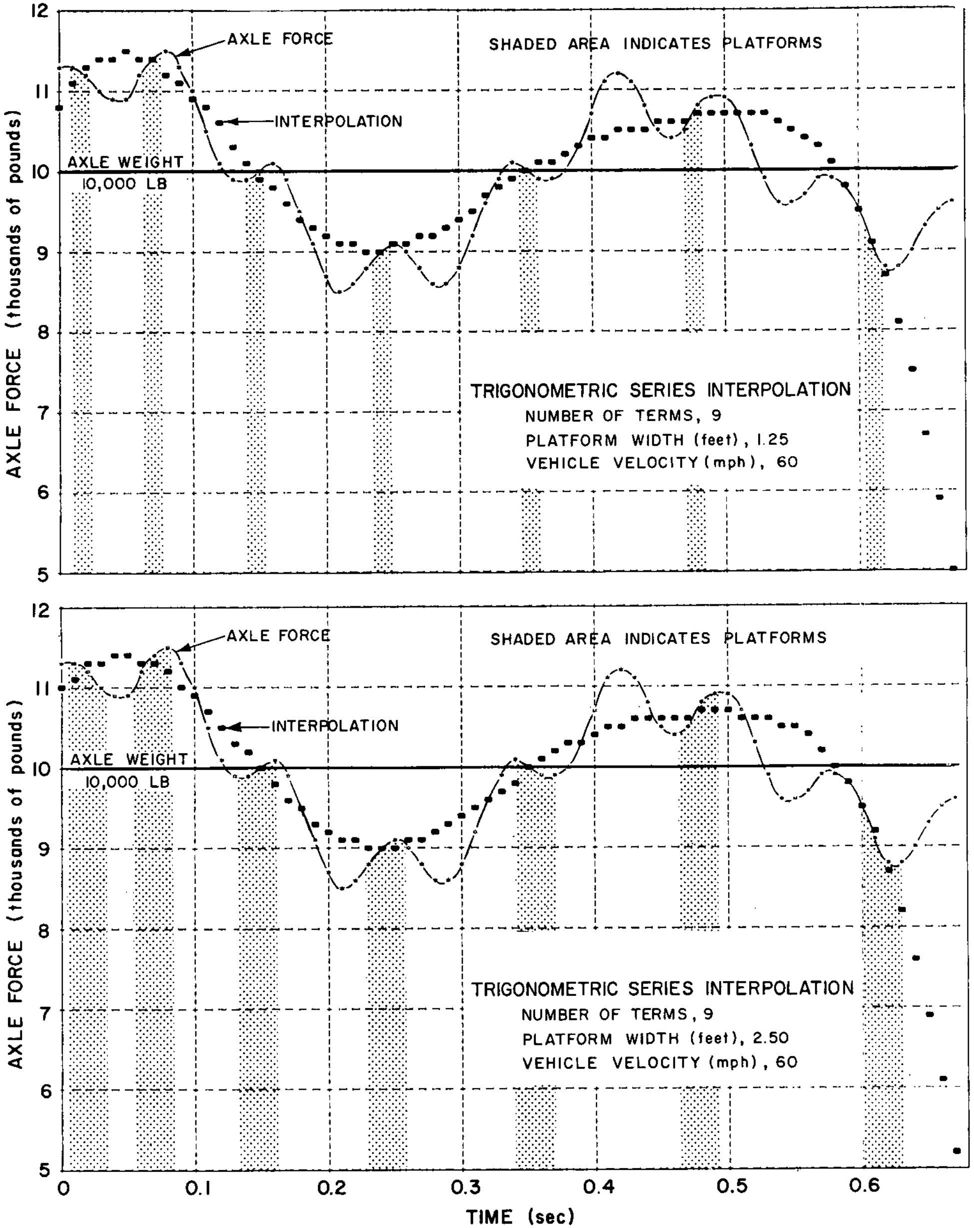


Figure F-2. Trigonometric series interpolation: number of terms, 9; platform widths, 1.25 ft and 2.50 ft.

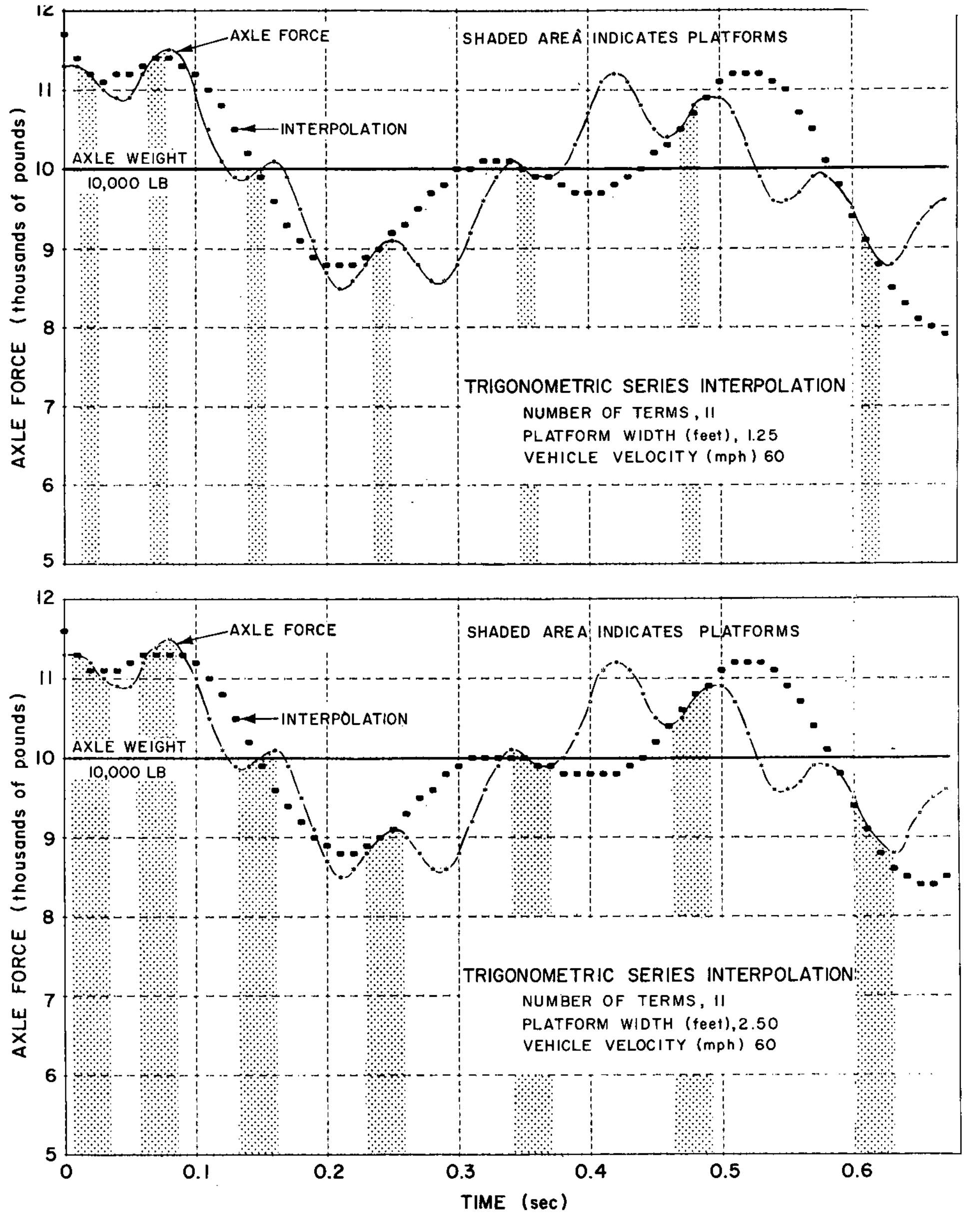


Figure F-3. Trigonometric series interpolation: number of terms, 11; platform widths, 1.25 ft and 2.50 ft.

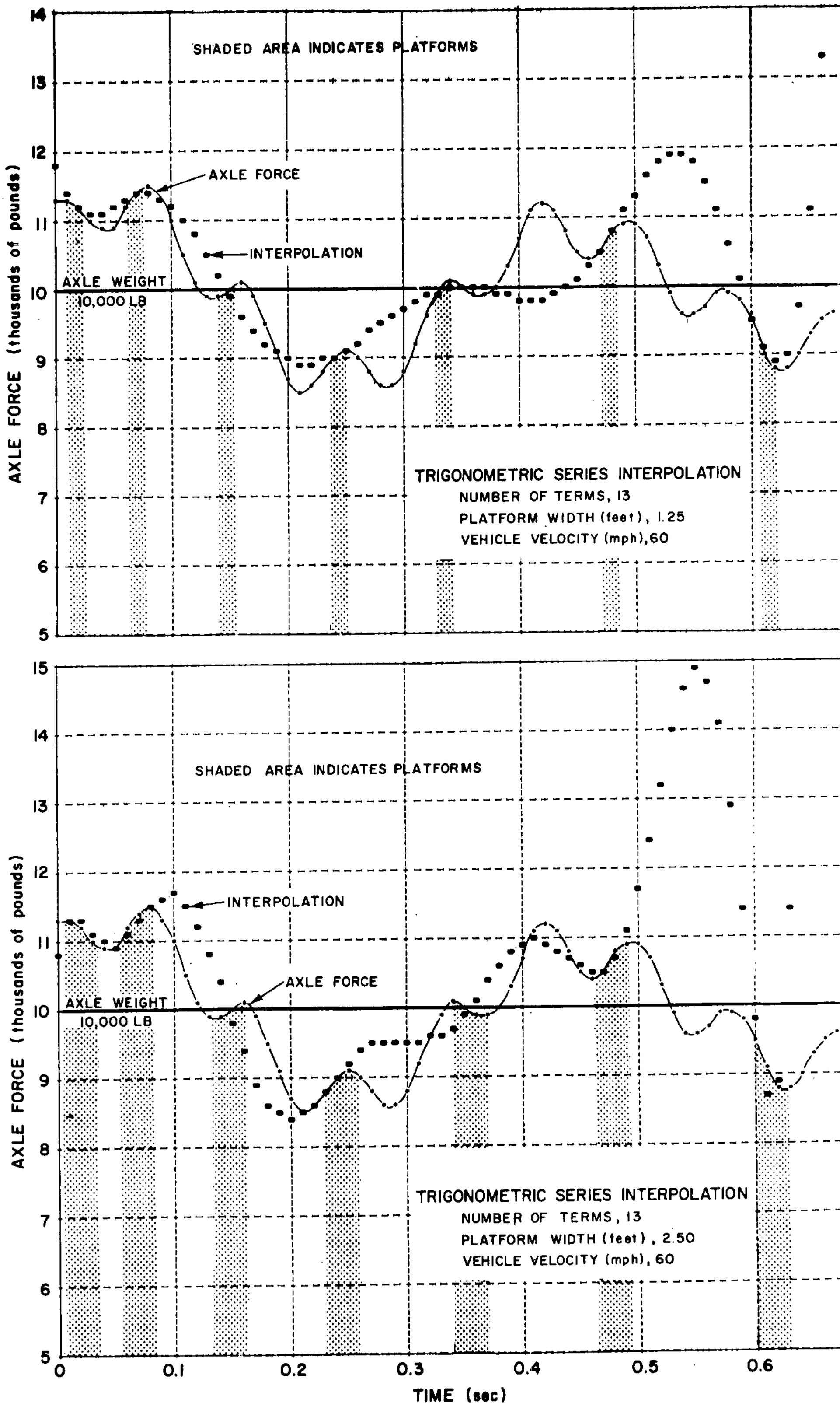


Figure F-4. Trigonometric series interpolation: number of terms, 13; platform widths, 1.25 ft and 2.50 ft.

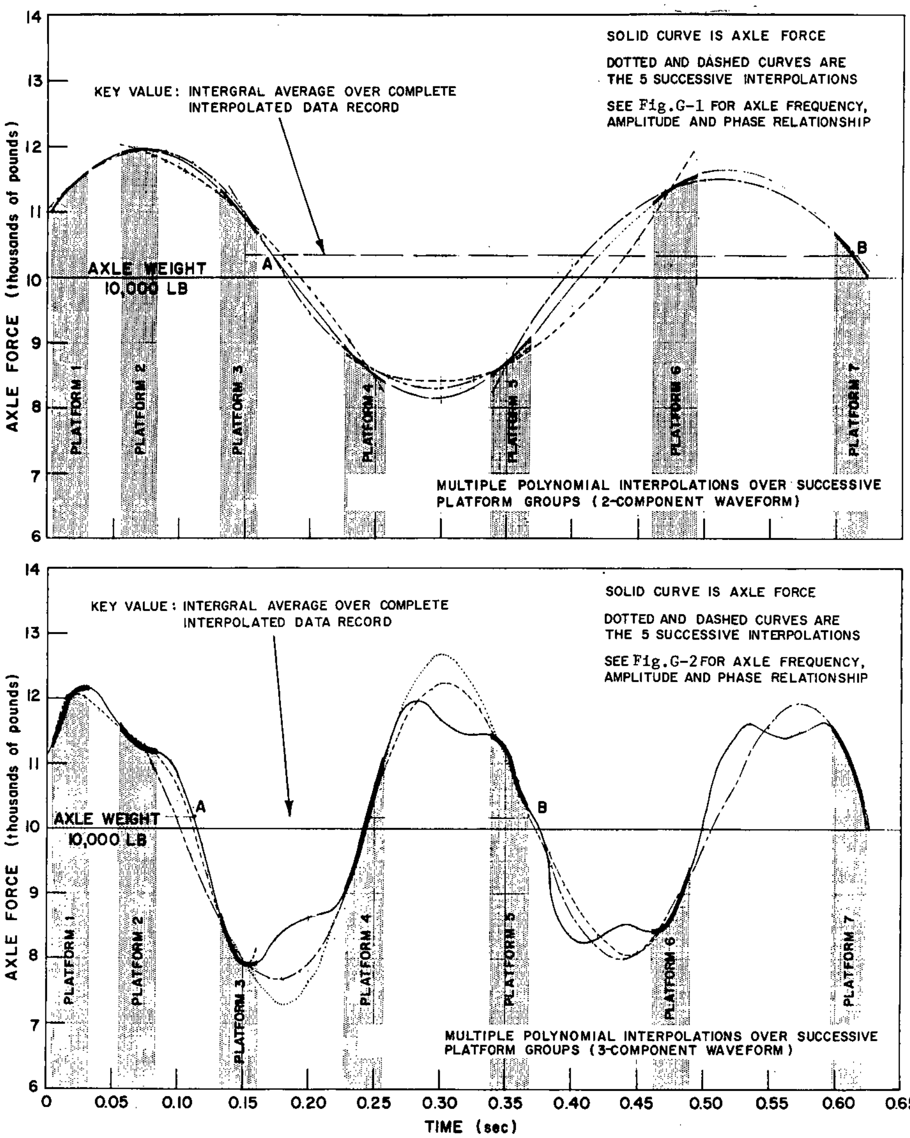


Figure F-5. Multiple polynomial interpolations over successive platform groups (two-component waveform and three-component waveform).

Although the Fourier series is not generally applicable to the reconstruction of a continuous function from unevenly spaced groups of sampled data, the series has one unique feature that is of great value for computation at a site along a highway; that is, only summation and limited division are required to evaluate the coefficients of the series' representation of axle force:

$$F = \tfrac{1}{2} A_0 + \sum_{k=1}^{K} \left[A_k \cos\left(\frac{k\pi t}{T}\right) + B_k \sin\left(\frac{k\pi t}{T}\right) \right] \quad \text{(F-4)}$$

in which

$$A_k = 2/T \int_0^T f(t) \cos\left(\frac{k\pi t}{T}\right) dt; \text{ and}$$
$$B_k = 2/T \int_0^T f(t) \sin\left(\frac{k\pi t}{T}\right) dt. \quad \text{(F-5)}$$

With the coefficients evaluated by integrating over only the platforms of the weighing installation, the Fourier series, if taken to a larger number of terms, would reproduce the spiked force diagram shown as the solid curve, instead of the dotted interpolation, over each platform in Figure F-6. As such, it is not an interpolation and gives no useful information not already given by the platforms alone. However, by severely truncating the series to a small number of terms, a relatively smooth continuous function is generated. This is still not an approximation to the axle force between platforms, inasmuch as the Fourier series is generated from a least squares application of a linear trigonometric series followed by the imposition of orthogonality relationships. Thus, the relatively smooth curve produced by truncated Fourier series is at a level much below the static component of force in quest. What is interesting and possibly useful is the fact that the Fourier series, truncated as described here, can be used to yield the basic wavelength or periodicity of the sampled force data. This is within limits, admittedly, but the dominant low frequencies of loaded vehicles appear to be close enough to make this feasible. The computer program that was used appears as Program FOUFIT in Appendix I. It can be seen that the Fourier series was generated about the average platform force in order to yield more descriptive visual readout plotted by the high-speed printer of the computer. Typical results are shown in Figure F-7. Platform spacing is in accordance with Figure F-1.

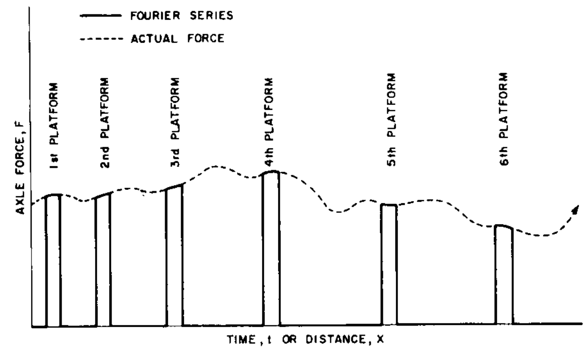


Figure F-6. Fourier series representation of sample data.

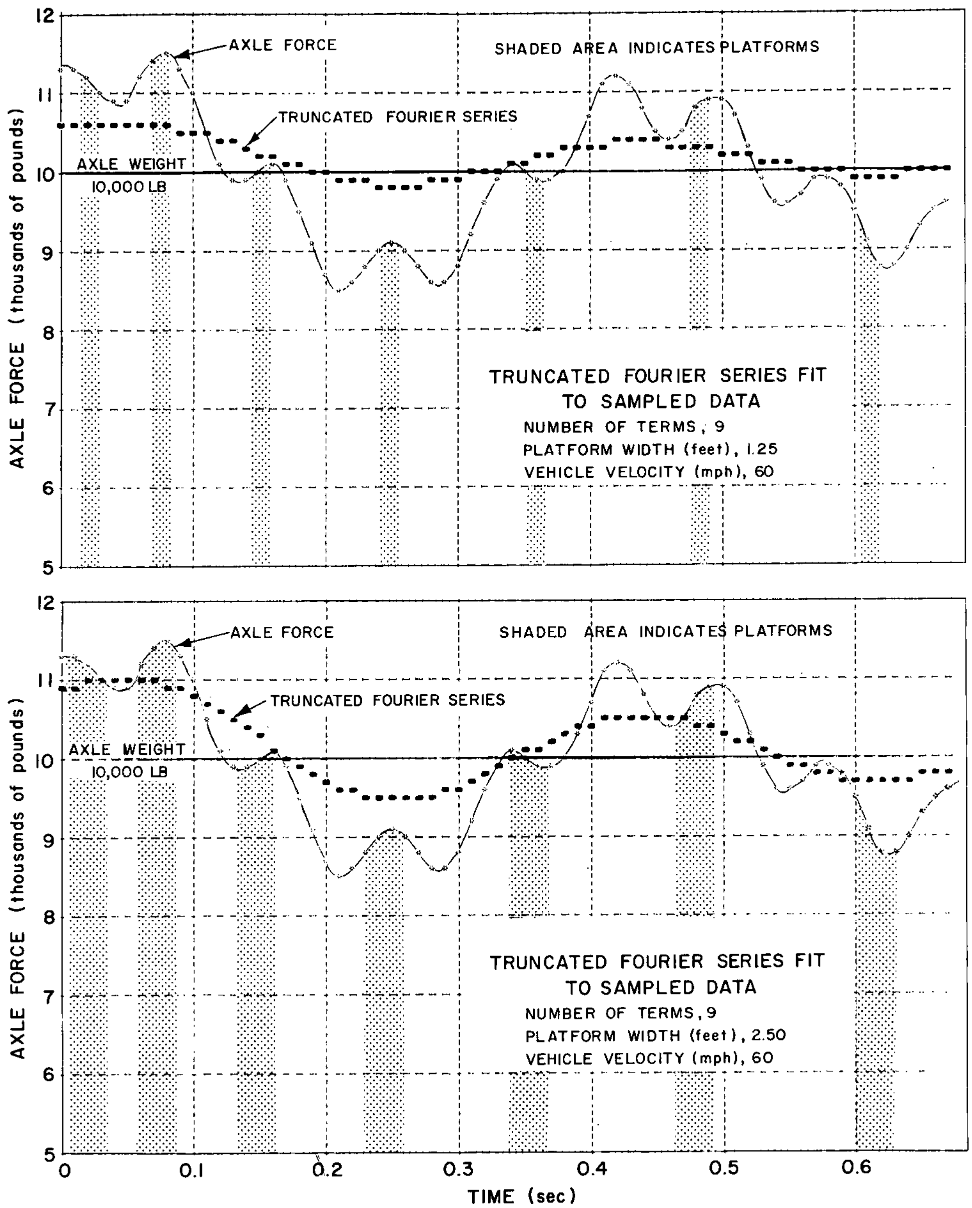


Figure F-7. Truncated Fourier series fit to sampled data: number of terms, 9; platform widths, 1.25 ft and 2.50 ft.

APPENDIX G

METHOD VWEIGH

After investigating a number of methods for ascertaining weight from sampled force data, attention was returned to a method previously conceived but temporarily set aside while "more analytical" methods were investigated.

Following the basic assumption that the force signal is made up of a static component plus dynamic components of an oscillatory nature, it can easily be seen that if one basic cycle of the oscillatory component could be deduced, averaging methods could be employed to remove the oscillatory component and leave only the static component proportional to weight. This cannot be an exact procedure because the oscillatory component of a real axle force signal is too complex in harmonic content and the data economically gathered are too sparse. However, for highway vehicles, the significant magnitude of dynamic content appears to be in the lower frequencies. Thus, an approximate procedure could be to deduce what appears to be a basic wavelength and average the signal over that period of time. This is the essence of what follows.

DESCRIPTION

Assume a series of non-uniformly-spaced platforms placed in a highway in such a way that the total length of the installation is greater than 1.5 times the longest wavelength (that described by the lowest anticipated frequency of a vehicle moving at the highest anticipated speed). Although a single wavelength is all that is necessary, an installation length of somewhat more than 1.5 wavelengths allows flexibility in the following procedure. Employing an interpolative method to reconstruct data between the discrete platforms allows an approximate continuous force record over the total installation to be available for analysis. A first approximation to the weight of an axle can be obtained from the integral average over the interpolated record. In accordance with the preceding statement, a second and better approximation to the weight can be obtained by integrating just over that portion that constitutes essentially one wavelength. The following is a simple way of deducing the one wavelength.

The integral average is used as a key or test value while the interpolated record is researched at successive increments in time to see where the interpolated record crosses the key value. When a crossing is encountered, an integration of the record is begun and continued until the third crossing of the test value is reached. At this point the integration is halted. This now approximates one basic cycle of a good many vehicles and permits a closer estimate of weight to be made from limited groups of sampled data.

This method was given limited investigation and appears to be a fairly powerful approach; it yields the lowest overall error of any method considered in this study program.

Vehicle Representation

The dynamic characteristics of a specific, but representative, test vehicle were used in this study. This was Vehicle 91 of the AASHO Road Test (*1*, p. 122). Both the blocked and free suspension conditions were considered in subsequent computer simulations of the method. In addition to the bounce and pitch frequencies of the vehicle, a frequency of 12 cps was chosen to represent the tire-hop frequency or axle oscillation frequency. One level of vehicle excitation was used to facilitate direct comparison of error readout, as follows: bounce amplitude, 10% of static weight; pitch amplitude, 10% of static weight; and tire-hop amplitude, 4% of static weight. These amplitudes of oscillation combine at one phase relationship to make the simulated oscillatory component of force 24% of the static weight. This is a significant vehicle oscillation; that, or more, would be likely over a prepared weighing installation. Damping was not included in the representation of axle force because on the order of a few wavelengths, at most, are over the scale installation. Thus, the representation of axle force for this model is Eq. A-12, but with

$\overline{W} = 10{,}000$ lb (chosen arbitrarily);
$a_1 = 0.1$;
$a_2 = 0.1$;
$a_3 = 0.04$

$$\left.\begin{aligned} \omega_1 &= 2\pi\,(2.1) \\ \omega_2 &= 2\pi\,(2.4) \\ \omega_3 &= 2\pi\,(12.0) \end{aligned}\right\} \text{free suspension; and}$$

$$\left.\begin{aligned} \omega_1 &= 2\pi\,(4.0) \\ \omega_2 &= 2\pi\,(3.7) \\ \omega_3 &= 2\pi\,(12.0) \end{aligned}\right\} \text{blocked suspension.}$$

Platform Spacing

For ease in conducting this investigation, the platform spacing model (Eq. C-18), that was used in the direct averaging study, was modified to be:

$$X_i = \tfrac{1}{2}\,\{L\,[(i/N)^{\beta} + (i - 1/N)^{\beta}] - B_i\} \qquad \text{(G-1)}$$

in which

i = platform number;
N = total number of platforms used;
X_i = distance to the leading edge of platform i;
L = installation length;
B_i = length of platform i in the direction of travel; and
β = a skewing factor for non-uniform spacing of platforms.

This platform spacing model divides the total installation length into N spaces, into each of which a platform of width b is centered. This was desired in certain of the studies (not represented here) and, as seen from the expression, is accomplished with $L\ (i/N)^{\beta}$ and $L\ (i-1/N)^{\beta}$ defining, respectively, the distances from the origin to the right- and left-hand boundaries of space i.

β is the skewing factor. $\beta = 1.0$ spaces the N platforms uniformly throughout the installation length, L. Values of β greater than 1.0 cause the lower-numbered platforms to be closer together; that is, platforms 1 and 2 are closest together while platforms $N-1$ and N are farthest apart. The advantage in such skewing is discussed in Appendix C, but there is a further advantage. If the direction of travel is such that platform 1 is encountered first (as implied in all these studies), then a positive β somewhat greater than 1.0 places the platforms first encountered closer together so that data from slower-moving vehicles could be taken from only the first K platforms. This is desirable because the shorter wavelengths that are generally associated with slower wavelengths can be detected better by closer platforms. Even though it is usually desirable to get all the information that can be obtained, that recorded for shorter wavelengths from the remaining wider-spaced platforms toward the end of the installation might allow too much of a shorter wavelength to fall between platforms. This could hinder rather than help in detecting a basic cycle. Although such conjectures are of great interest and are practical, the investigation was beyond the scope of this study. This would be part of an actual installation engineering study instead.

Platform installation lengths of 60 ft were used in all the computer runs. This was very close to the 1.5-wavelength criterion previously discussed.

Interpolating and Searching for One Wavelength

The interpolation used for the study of this weighing procedure was that of multiple polynomial fitting over successive platform groups, as described previously. What is needed from this interpolated data record is the continuous representation of force as well as knowledge of the beginning and ending of one basic cycle. This was accomplished in the computer program by using the integral average from the interpolated record as a key value and researching the interpolated record with the use of LOGIC IF statements to test for the first and third crossings of the key value. This can be seen from Program VWEIGH (Appendix I). Because the program showed promise of better weighing accuracy, care was taken to document it with a more liberal use of comment cards for identification of subsequent operations.

COMPARATIVE ACCURACY

Two partial sets of computer output (Figs. G-1 and G-2) show the information that was generated in the study of method VWEIGH. Plots from this particular output are shown in Figure F-5. Integration was begun at points A and ended at points B. As shown in the computer outputs, axle weight was computed three ways: (1) by averaging force over the platforms only, (2) by using the interpolation and averaging the force over the whole installation length, and (3) by averaging over what is detected to be approximately one wavelength. Figures G-1 and G-2 yield relative values of computed weight that are representative of a majority of the computer runs; that is, the integral average was in general better than the average over the platforms, while the method under discussion here gave the best estimate. Table G-1 compares three computed axle weights for 44 separate combinations of velocity, degree of the interpolation equation, and the number of platforms. In certain cases in Table G-1 the third method failed to compute an axle weight. This was caused by such conditions as (1) polynomial interpolation expression of insufficient degree, (2) ill-spaced platforms for the waveform subjected and the number of platforms used, and (3) the installation length was insufficient to permit three crossings of the key force level.

Figure G-3 shows the effect of the polynomial degree on the computed weight (the method of averaging over one apparent wavelength is implied, if not stated, in this discussion of method VWEIGH). The outer bounds, indicated by the dashed lines, show that the spread of computed weight for seven platforms spaced over 60 ft is inversely proportional to the degree of the interpolation. The apparent convergence to a value less than the 10,000-lb actual weight is produced by the limited values of phase angles used in the study. Because a number of parameters may be varied, it would be meaningful in any further investigations either to maximize the error due to the phase relationships (as was done in the simple averaging studies) or to use random number generators to spread the phase relationships randomly as might be anticipated for a weighing installation without significant bumps.

When the computed weight from six platforms was plotted in a manner similar to Figure G-3, the increased accuracy for all speeds from more complex polynomial interpolation expressions was not present. The limitations on this effort prevented investigation of the cause. The subject was dropped because the initial spacing of six platforms was not given consideration; the study of seven platforms was just changed to six. For this reason no comparisons are shown.

Regarding the seven-platform data of Table G-1, the effects of polynomial degree and the speed of Vehicle 91 are shown in Figure G-4. The fifth degree interpolation polynomial weighings is singled out from the computed weights of all polynomials shown. These are limited data, but are representative of the accuracy that can be obtained from axle-force readings that are varying at a maximum of 24% about the static component of force. The axle force sampled could have varied by ± 2,400 lb about the 10,000-lb static component (a possible variation between 7,600 and 12,400 lb).

The major limitation of this method is that it requires a fairly sophisticated digital computer to perform all the operations. A discussion of computers follows.

USE OF THE DIGITAL COMPUTER

Digital computers can be used in several ways. Small computers could be set up at a number of weighing stations and

```
INTERPOLATION EQUATION
    POLYNOMIALS IN T OF DEGREE  2

    PROBLEM 1,  VEHICLE 91,  FREE SUSPENSION,  WITH NO TIRE HOP
        NUMBER OF PLATFORMS IS  7,      INSTALLATION LENGTH,L, IS  60.0 FEET,      BETA IS  1.5
        VEHICLE VELOCITY IS  60.0 MILES PER HOUR
        PLATFORM SPACING (TIME IN SECONDS, DISTANCE OR WIDTH IN FEET)
```

PLATFORM NO.	TIME TO PLATFORM	TIME ON PLATFORM	DISTANCE TO PLATFORM	PLATFORM WIDTH
1	0.00420	0.02841	0.3698	2.5000
2	0.05627	0.02841	4.9515	2.5000
3	0.13351	0.02841	11.7486	2.5000
4	0.22870	0.02841	20.1258	2.5000
5	0.33885	0.02841	29.8192	2.5000
6	0.46213	0.02841	40.6673	2.5000
7	0.59724	0.02841	52.5568	2.5000

```
        SAMPLING TIME IS 0.003000 SECONDS

        FORCE COMPONENT DATA
```

FREQUENCY (CPS)	AMPLITUDE	PHASE (RAD)
2.10	0.100	0.400
2.40	0.100	0.600
-0.00	-0.000	-0.000

```
      COEFFICIENTS OF INTERPOLATION EQUATIONS
```

INTERPOLATION INTERVAL	A1	A2	A3	A4	A5	A6
1	0.11056E 05	0.25224E 05	-0.17698E 06			
2	0.12035E 05	-0.46273E 04	-0.71891E 05			
3	0.11530E 05	-0.39488E 05	0.12129E 06			
4	0.88169E 04	-0.12136E 05	0.90791E 05			
5	0.82308E 04	0.37742E 05	-0.10869E 06			

```
  AXLE WEIGHT COMPUTED FROM FORCE DATA

        1.  AVERAGE OF FORCE OVER ALL PLATFORMS
                COMPUTED WEIGHT IS 10503.2 LBS.
                ACTUAL WEIGHT IS 10000.0 LBS.

        2.  MEAN VALUE OF INTERPOLATED DATA RECORD
                COMPUTED WEIGHT IS 10359.9 LBS.

        3.  MEAN VALUE OF INTERPOLATION EQUATION BETWEEN FIRST AND THIRD CROSSING OF THE OVERALL MEAN
                COMPUTED WEIGHT IS  9938.7LBS.
                TIME INTERVAL BETWEEN FIRST AND THIRD CROSSING IS 0.444 SEC.
```

Figure G-1. Program VWEIGH: computer output of problem 1.

the vehicles checked on a real-time basis. If desired, overweight vehicles could be apprehended immediately on an overweight indication—a signal light or printed weight. If there is a computer which is both fast and inexpensive, this method of finding the truck weights would be suitable. The computed weights could be stored on tape for "historical" purposes and the tape could be processed off-line.

Another approach would be to record the converted digital data at the weighing stations and process it off-location at a data center or on an agency-owned computer. In this case, the internal speed of the computer would not be a factor, but the cost of processing the data would be important.

The idea of using one large computer on a time-shared basis among the remote weighing stations would not be feasible for this problem because the rates of transfer of data are too slow. A fast transfer rate for a 200 series data set is only about 300 characters a second. (The face readings would be about 8,000 or more characters from the seven platforms.) Very fast transfer rates could be achieved if special cables were laid from remote channels to the main computer, but the cost of installing special cables would be prohibitive.

```
INTERPOLATION EQUATION
    POLYNOMIALS IN T OF DEGREE  5

    PROBLEM 16, VEHICLE 91, BLOCKED SUSPENSION, WITH TIRE HOP AT 12CPS
        NUMBER OF PLATFORMS IS  7,     INSTALLATION LENGTH,L, IS  60.0 FEET,     BETA IS  1.5
        VEHICLE VELOCITY IS  60.0 MILES PER HOUR
        PLATFORM SPACING (TIME IN SECONDS, DISTANCE OR WIDTH IN FEET)
```

PLATFORM NO.	TIME TO PLATFORM	TIME ON PLATFORM	DISTANCE TO PLATFORM	PLATFORM WIDTH
1	0.00420	0.02841	0.3698	2.5000
2	0.05627	0.02841	4.9515	2.5000
3	0.13351	0.02841	11.7486	2.5000
4	0.22870	0.02841	20.1258	2.5000
5	0.33885	0.02841	29.8192	2.5000
6	0.46213	0.02841	40.6673	2.5000
7	0.59724	0.02841	52.5568	2.5000

```
SAMPLING TIME IS 0.003000 SECONDS

FORCE COMPONENT DATA
```

FREQUENCY (CPS)	AMPLITUDE	PHASE (RAD)
4.00	0.100	0.400
3.70	0.100	0.600
12.00	0.040	-0.000

```
COEFFICIENTS OF INTERPOLATION EQUATIONS
```

INTERPOLATION INTERVAL	A1	A2	A3	A4	A5	A6
1	0.11265E 05	0.10016E 06	-0.42924E 07	0.71529E 08	-0.53780E 09	0.14359E 10
2	0.11511E 05	0.47528E 04	-0.13749E 07	0.14535E 08	-0.58693E 08	0.97922E 08
3	0.85353E 04	-0.22820E 05	-0.92140E 06	0.24364E 08	-0.14703E 09	0.26594E 09
4	0.90474E 04	0.70356E 05	-0.37969E 05	-0.69364E 07	0.36364E 08	-0.50332E 08
5	0.11607E 05	-0.27728E 05	-0.87156E 06	0.11746E 08	-0.43581E 08	0.49463E 08

```
AXLE WEIGHT COMPUTED FROM FORCE DATA

    1.  AVERAGE OF FORCE OVER ALL PLATFORMS
            COMPUTED WEIGHT IS 10297.8 LBS.
            ACTUAL WEIGHT IS 10000.0 LBS.

    2.  MEAN VALUE OF INTERPOLATED DATA RECORD
            COMPUTED WEIGHT IS 10182.8 LBS.

    3.  MEAN VALUE OF INTERPOLATION EQUATION BETWEEN FIRST AND THIRD CROSSING OF THE OVERALL MEAN
            COMPUTED WEIGHT IS  9941.9LBS.
            TIME INTERVAL BETWEEN FIRST AND THIRD CROSSING IS 0.267 SEC.
```

Figure G-2. Program VWEIGH: computer output of problem 16.

Computer Limitations

Floating-Point Hardware

Computer speeds and cost are the most critical factors for the real-time, on-location approach to weighing the trucks. Most small, inexpensive computers are relatively slow. They are not built to handle non-integer (floating-point) arithmetic which is required in the curve-fitting part of the program. In larger computers the floating-point arithmetic capability is often built into the hardware so that computation time can be a minimum. When it is not built in, floating-point arithmetic is done using a specially called routine. This is timely, but for this application the cost of built-in hardware may be too expensive. Computer purchase prices range from $10,000 to $7,000,000, and the least expensive machine with floating hardware costs a minimum of $92,000 (SEL 840A); floating point is not common in computers of much less than $250,000. It is therefore necessary to examine computer speeds of the less-expensive computers to estimate the program time for machines without floating-point hardware.

Memory Cycle Time

The speeds of the computers can be estimated by storage cycle time, the fixed-point add time, and the times for floating-point arithmetic operations. A number of computers of less than $50,000 have fast cycle times that are under 3 μsec (see Table G-2). However, even with fast cycle

TABLE G-1

COMPARISON OF COMPUTED WEIGHTS FROM PROGRAM VWEIGH

				COMPUTED WEIGHT (LB)		
PROBLEM NO.	VEL. (MPH)	SUSPENSION [a]	DEGREE OF INTERPOLATION POLYNOMIAL	1. AVERAGE OVER PLATFORMS	2. MEAN VALUE OVER INTERPOLATED RECORD	3. MEAN VALUE BETWEEN 1ST & 3RD CROSSING
			(*a*) Seven Platforms			
1	60	Free, no hop	2	10503.2	10359.9	9938.7
2	60	Free, no hop	3	10503.2	10376.3	9955.2
3	60	Free, no hop	4	10503.2	10339.0	9908.6
4	60	Free, no hop	5	10503.2	10351.6	9921.3
5	60	Free	2	10478.2	10287.4	—
6	60	Free	3	10478.2	10303.8	—
7	60	Free	4	10478.2	10316.8	9944.8
8	60	Free	5	10478.2	10337.2	9957.5
9	45	Free	2	10249.8	10028.8	9719.0
10	45	Free	3	10249.8	9965.9	9747.0
11	45	Free	4	10249.8	9956.7	9762.3
12	45	Free	5	10249.8	9875.9	9863.7
13	60	Blocked	2	10297.8	9965.8	9826.2
14	60	Blocked	3	10297.8	10058.2	9873.0
15	60	Blocked	4	10297.8	10119.0	9850.2
16	60	Blocked	5	10297.8	10182.8	9941.9
17	45	Blocked	2	10337.7	10040.6	10035.1
18	45	Blocked	3	10337.7	9985.7	9955.7
19	45	Blocked	4	10337.7	10047.1	9993.9
20	45	Blocked	5	10337.7	9860.6	10018.9
21	30	Free	3	10207.1	10206.3	9822.8
22	30	Free	4	10207.1	10132.0	9868.5
23	30	Free	5	10207.1	9958.8	9934.8
24	30	Blocked	3	9678.2	9387.2	10169.8
25	30	Blocked	4	9678.2	9414.2	10055.8
26	30	Blocked	5	9678.2	9355.6	9938.5
			(*b*) Six Platforms			
27	30	Free	3	10285.1	10296.5	9906.9
28	30	Free	4	10285.1	10243.3	9966.7
29	30	Free	5	10285.1	10158.6	10137.1
30	30	Blocked	3	9995.2	10098.0	9212.5
31	30	Blocked	4	9995.2	10091.2	9466.4
32	30	Blocked	5	9995.2	9485.9	10241.3
33	45	Free	3	10297.6	10124.3	9874.1
34	45	Free	4	10297.6	10120.8	9869.0
35	45	Free	5	10297.6	10239.7	10010.4
36	45	Blocked	3	10516.3	10396.0	—
37	45	Blocked	4	10516.3	10369.4	9695.4
38	45	Blocked	5	10516.3	10294.8	9694.9
39	60	Free	3	10694.9	10531.0	—
40	60	Free	4	10694.9	10485.2	—
41	60	Free	5	10694.9	10451.7	—
42	60	Blocked	3	10519.9	10138.4	9983.2
43	60	Blocked	4	10519.9	10233.4	10048.8
44	60	Blocked	5	10519.9	10233.0	10166.3

[a] With tire hop unless otherwise indicated.

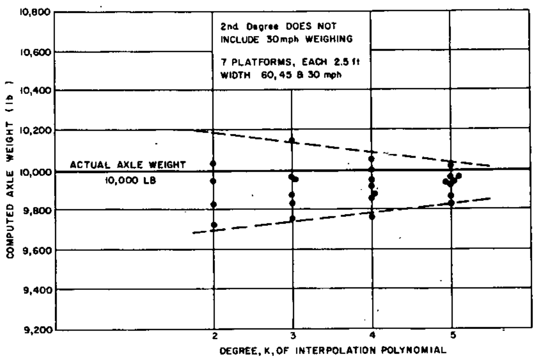


Figure G-3. Effect of polynomial degree on computed weight.

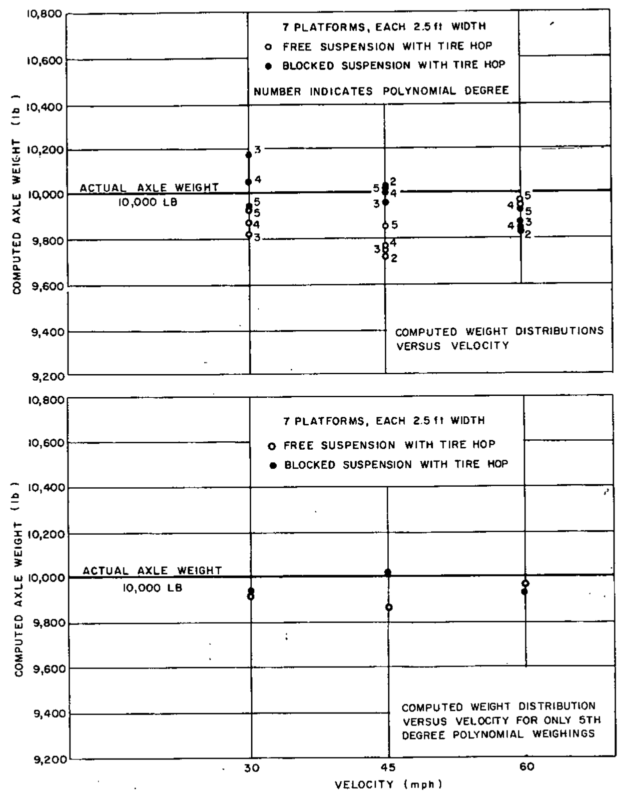


Figure G-4. Computed weight distributions vs velocity.

times the floating-point arithmetic routines sometimes take 50 times longer than floating-point arithmetic using built-in hardware. It is difficult to estimate exactly how long the program will take on the computers without floating point inasmuch as the routines for this kind of arithmetic differ greatly from machine to machine; however, for a 10-axle truck, the speed might be 3 to 10 min. instead of 5 to 20 sec.

Accuracy

The class of inexpensive, fast machines without floating-point hardware has a word length of 16 bits (PDP 8 and PDP 9 computers are exceptions). Sixteen bits will allow an accuracy to four or five significant decimal digits. For this problem, that will give an accuracy of about 1%. A longer word length (24 bits) on a fast machine is not available for less than $92,000 (SEL 840A). Twenty-four bits gives an accuracy of 0.01% for the truck weight. Fast computers with word lengths greater than 24 bits are in the $400,000 (EAI 8400, 32 bits) and higher price range.

If more accuracy than 16 bits is necessary it is possible to use double precision arithmetic on some of the computers of less than $50,000. Double precision gives double the accuracy, but it would more than double the program running time, so the time would be in the range of 6.6 to 20 min. for a 10-axle truck. Sixteen-bit accuracy may be enough for this problem inasmuch as there would be some uncertainty in the force readings after conversion to decimal data.

Input/Output

The input/output transfer rate is not a critical factor for any of the computers. The analog-to-digital conversion may be accomplished at rates faster than the program is run, and the writing of the results on tape or printing on-line is much faster than is necessary (see Table G-2). All the computers in Table G-2 have the capability at least of reading or writing tape while the computations are in operation, so an insignificant amount of time is lost for input/output operations.

Memory Size

The program is small and, as it now stands, uses about 1,000 words of core storage. The force readings, stored in a buffer area, may take about 2,500 words. Depending on the amount of core required by the system, a 4K or 8K memory capacity would be required. With skilled programming, a 4K memory probably would be sufficient, because elaborate system monitors would not be necessary for the special problem being considered.

Computer Specifications

Table G-2 gives specifications of available computers that are most suited to the truck-weighing problem. The computers are arranged according to cost, beginning with the least expensive. The more expensive computers are those which have floating-point hardware, and are included for comparison purposes.

The costs (Col. 1) for the first eight computers are for a minimum useful configuration. That includes 4K, teletype, and paper tape read and punch. For a magnetic tape deck, add $10,000 to $25,000. Additional 4K of memory costs about $8,000. The speeds given are the memory cycle time (Col. 2); the add time (Col. 3), which is the time required to acquire from memory and execute one fixed-point add instruction; and the time for floating-point arithmetic instructions (Cols. 4, 5, 6), either using the built-in hardware

TABLE G-2

SPECIFICATIONS OF COMPUTERS MOST SUITED TO VEHICLE WEIGHING PROBLEM

COMPUTER	(1) COST [a]	(2) TIME (MICROSEC) STORAGE CYCLE	(3) FIXED ADD	(4) FLOATING ADD	(5) FLOATING MULTIPLY	(6) FLOATING DIVIDE	(7) WORD SIZE (BITS)	(8) CORE AVAILABLE (1,000 WORDS)	(9) TAPE 1,000 CHAR/SEC MINIMUM	(10) TIME (SEC) 1 AXLE	(11) 10 AXLES
				(*a*) Without Floating-Point Hardware							
DDP 516	A	0.96	1.9	100	225	642	16	4-32	1.5[b]	20	200
DDP 116	A	1.7	3.4	190[b]	425	1070	16	4-32	25	60	600
EAI 640	B	1.65	3.5[b]	382	609	703	16	4-32	6	60	600
PDP 8	A	1.5	3.0	250	500	600	12	4-32	15	40	400
IBM 1800	B[c]	2.0	6.0	—	—	—	16	4-32	15	60	600
PDP 9	B	1.0	2.0	686	914	914	18	8-32	15	30	300
CDC 1700	C	1.1	2.2	—	—	—	16	4-32	7	30	300
ASI 6130	C	0.9	1.8	—	—	—	16	4-32	22.5	20	200
				(*b*) With Floating-Point Hardware							
SEL 840A	D	1.75	3.5	—	—	—	24	4-32	9	1.5	15
ASI 6050	D	1.9	3.8	7.6	13.3	12	24	4-32	22.5	1.5	15
SDS 9300	E	1.75	1.75	—	—	—	24	4-32	1.5	1.5	15
PDP 6	F	1.75	4.4	9.3	20	20	36	16-262	15	1.5	15
CDC 3500	F	0.8	1.3	—	—	—	24	7-120	7	0.5	5
SDS Sigma 7	F	1.2	2.0	3.9	5.0	12.3	32	4-131	15	1.0	10
CDC 6800	G	0.25	0.1	0.1	0.25	0.73	60	32-131	30	0.05	0.5
CDC 6600	G	1.0	0.3	0.4	1.0	2.9	60	32-131	30	0.1	1

[a] Computer system cost: A. Less than $25,000. B. $25,000 to 40,000. C. $40,000 to $50,000. D. $90,000 to $120,000. E. $190,000 to $210,000. F. $475,000 to $625,000. G. $3,000,000 to $5,000,000.
[b] Estimate.
[c] Estimate from monthly rental price.

or a software routine. The first seven computers listed have no floating-point hardware. Col. 9 gives the minimum tape transfer rate in thousands of characters per second.

The time it takes the program to compute the weight of one axle (Col. 10) was estimated for a truck traveling at 44 ft/sec over the seven platforms in a time interval of 1.5 sec. Force readings on the platforms of 2.5-ft width were taken at intervals of 0.002 sec. A FORTRAN program was approximated in machine language for the IBM 7044. The estimated time for this was 2 sec, half of which was accounted for by the floating-point arithmetic instructions. For other computers, time estimates were made from a comparison of the computer times for arithmetic instructions and memory cycle times. Col. 11 is the time estimate for 10 axles. The program computes one axle at a time; therefore, each additional axle requires the full program time again. The estimated times for the computers without floating-point hardware were made assuming the floating-point routine slowed down the program by a factor of 40 times.

Special Characteristics of Computers Listed

The first eight computers in Table G-2 are about the same size and in the same price range. Some are a little faster than others and some have special features which may be suitable to the truck-weighing problem.

The EAI 640 advertises an interval timer which generates timing pulses in intervals of 1, 10, or 1,000 millisec. Four interval timers may be used with the system.

The PDP 8 and PDP 9 computers have associated analog-to-digital converters which may be added to the system inexpensively, inasmuch as the computers are pre-wired for this optional equipment. A converter-type 138 E will convert 6 to 12 bits to an accuracy of $\pm 0.8\%$ to $\pm 0.025\%$ in 9 to 35 μsec. In addition, the PDP 9 has a slightly larger word length than the other computers in this class (18 bits) and a real-time clock and timing control which governs the timing of interval processor operations and the synchronization of core memory and input/output devices to these operations.

The ASI 6130 allows simultaneous communication and control of multiple central processors, multiple I/O channels and multiple memory modules. Several central processors would reduce computation time considerably. Of course, the price goes beyond the range of the small computers when any of these additional computer units are used.

Data Center Computing Costs

The costs of running the program off-location at several local data centers are compared in Table G-3.

The number of axles able to be processed in an hour is reduced from the estimate given in Table G-2 to allow time for tape mounting and loading the program into the computer. The number of axles that can be processed for $100 is given for comparison purposes.

The time to compute the program is prohibitively long on machines without the floating-point hardware. A real-time application could only be done on a sampling basis at a rate of one truck every 200 sec. For trucks traveling at 44 ft a second, a sample could not take trucks closer together than about a mile and a half.

The slow machines in the $25,000 to $45,000 price range give an accuracy of 1%.

Fast computation time begins in machines that cost more than $92,000. In the price range of $92,000 to $400,000, it is possible to get a computation time of about 15 sec for 10 axles within an accuracy of 0.01%. For greater accuracy and faster computation times it is necessary to go beyond $400,000. An extremely fast computer is needed to sample every truck. A CDC 6600 (in the $3,000,000 price range) could calculate weights of trucks every second. CDC will soon have a faster computer, the CDC 6800 ($5,000,000), on which trucks could be weighed every 0.5 sec, or faster.

The program for which the computer time estimates have been made is a fairly sophisticated one for computing vehicle weights. It is also probably inefficient in that it was developed in FORTRAN II and later changed to FORTRAN IV, with as little change as possible. At the time the program was developed for study of the method, the actual efficiency of the program was not of real importance.

This feasibility of digital computation, whether the program is efficient or not, dictates that if this method is employed it must be employed to study sample axle-force data at a computer center instead of locally on-the-spot. Another insight gained is that the added percentage in accuracy is costly and would not usually be warranted over that of a simple averaging system.

While the studies of computer application were being conducted for this complex averaging method, some attention was given to the simplest program possible—one that merely averages the force readings for each axle over each platform and stores the information on tape. For this simple averaging system the computer time can be reduced considerably. Floating-point arithmetic need not be used. Therefore, the program time for computing 10 axle weights would be less than 0.5 sec for any of the first eight computers of Table G-2.

TABLE G-3

COMPARISON OF COMPUTER COSTS AT SEVERAL DATA CENTERS

DATA CENTER AND COMPUTER	COST PER HR ($)	NO. OF AXLES PROCESSED PER HR	NO. OF AXLES PROCESSED PER $100
Commercial:			
IBM 360/40	115	900	783
PDP-6	231	2,000	865
University:			
IBM 7040	100	800	800

APPENDIX H

OUTLINE AND DISCUSSION OF PHYSICAL SYSTEMS

Preceding sections deal with investigations of methods to determine axle weight from sampled data obtained from force-sensitive platforms placed in the highway surface. It is proper now to show how certain of these analytical methods may be physically instrumented to determine an estimate of weight either at the weighing site or later at a computer center. The specific weighing methods considered herein are averaging over a computed one-wavelength and simple averaging over the platforms. A system for computing according to Integral Model III is included for reference only.

HARDWARE

Hardware designates the actual pieces of instrumentation that may be assembled to measure and record or compute. This includes such specific items as the force-sensitive platforms, the signal conditioning equipment used to obtain a signal from the platforms, and all further recording and computational equipment necessary. It also includes the equipment at a computer center, if data are collected and taken there for weight computation and classification. Before examining specific systems, the general characteristics of hardware common to all these systems are discussed.

Platforms

The platform is the force-sensitive device (a transducer) that, with the aid of signal conditioning equipment, produces an electrical signal proportional to the force applied to the platform. Most of the platforms used in highway vehicle weighing have been concrete slabs or steel structures mounted on strain gauge load cells. Later designs have incorporated the strain gauges into the platform structure to save space, reduce weight, and increase the dynamic response. Still other types have operated on the principle of compressing an hydraulic fluid where the fluid pressure can be measured electrically by present highly developed pressure transducers or by a mechanical gauge, although the mechanical gauge does not have much application for weighing in motion.

The basic requirements of platforms (in order of importance) are:

1. Dynamic response.
2. Accuracy.
3. Good long-term stability.
4. Weatherproof.
5. Low cost.

Dynamic response is imperative, but relative to the vehicle speed at which one wishes to weigh. If the platform is not sufficiently responsive, no other characteristic at any cost can supplant this characteristic. Fast response dictates that a platform has, ideally, low mass and a high spring constant in deflection. These characteristics, in addition to yielding good response, provide two other attributes: (1) a high natural frequency so that the energy imparted to the platform by the wheel does not result in low-frequency high-amplitude oscillation or ringing of the platform itself, and (2) the platform is stiff enough so that any deflection does not constitute a bump as the vehicle crosses.

Although older platform installations using concrete slabs or steel structures mounted on load cells utilized the accuracy and stability available in strain gauge load cells, the dynamic response was lowered by the large mass of the slab or structure. This produced large amplitude oscillation, as shown in Figure A-7.

More recent developments of force-sensing platforms have led to designs in which strain gauges are distributed within concise platform packages that are both thin and stiff. Two such platforms are available. One has been developed by Dr. Clyde E. Lee(*11*) and the other by the Taller-Cooper Company of Brooklyn, New York.

The platform featured in Lee(*11*) has been tested and shown to exhibit many desirable characteristics, including relative ease of emplacement in a highway. However, it was not being manufactured commercially at the time of this study, so estimates of price and accuracy were not final. Dr. Lee has estimated that a pair of these platforms—which would constitute one platform as described throughout this report—would cost on the order of $3,000, emplaced in the highway. Up to seven such pairs have been featured in these studies. This places an upper limit of $21,000 on the cost of platforms (in the highway) for a seven-platform installation.

The Taller-Cooper Company reported a price of $3,000 per pair of units, as required to provide one platform across a 10-foot lane. No estimate of the natural frequency of this platform was given, but it can be assumed to be high compared with the concrete slab or heavy steel structure platforms.

Precise statements of linearity, repeatability, or accuracy were not given for either, but it appears that both may be in the range of 0.5% or better, and certainly within 1.0% under extreme conditions.

Platform Spacing

The number and spacing of platforms are the parameters that have the greatest influence in minimizing the error of computed weight estimates. This depends, too, upon the method of computation, the speed and oscillatory characteristics of vehicles anticipated, and the overall degree of accuracy desired.

The philosophy that has evolved from this study is that as many spaced platforms as possible should be placed over one basic wavelength of the anticipated force signal. There is one exception. Integral Model III as presently written anticipates a continuous force signal that, without interpolation procedures, dictates a continuous force platform. Although this, too, may best be done with spaced platforms and simple interpolative techniques in a computer program, the continuous platform could be made up of a number of separate platforms butted together, plus the use of platform signal-switching techniques that would allow one or two platforms to be read at one time. What is meant is that as a wheel entered upon platform 1, only that platform would be sensed. As the wheel approached the end of platform 1 the switching logic would switch the platform signals so that the combination of 1 and 2 would be read. Thus, a continuous signal could be obtained without waiting for a wheel to roll completely onto a platform before reading could begin. The major drawback is the expense of literally paving the road with force-sensitive platforms, because this would necessarily cover a large distance if the vehicle were moving fast. However, if the vehicle were moving slowly, as it would be in approaching a toll booth, an entrance ramp, or vehicle parking area, the arrangement would be most practical.

Spaced platforms should usually be non-uniformly spaced, with those contacted first spaced the closest, as discussed elsewhere in this report.

Any platform configuration can be used with any of the following weighing system designs as dictated by the specific and unique parameters of that installation.

Signal Conditioning Equipment

Signal conditioning equipment designates a package of such separate items as highly stable power supplies; strain gauge bridges; calibration, balancing, and test circuits; temperature compensation circuits; intermediate amplifiers; as well as filter circuits such as band pass, band reject, low pass, and high pass. The signal conditioning equipment has two functions. First, it powers the force transducers which produce electrical signals proportional to force. Second, it conditions these signals to the user's needs by eliminating unwanted effects.

High-grade signal conditioning has the following characteristics:

1. Stability: voltage mode—0.01% due to changes in line and load; current mode—0.01% due to changes in line only.
2. Drift: 0.05% of output during 30 hr after ½-hr warmup.
3. Temperature sensitivity: 0.005% per degree F.
4. All silicon solid state.

Computational Equipment

Computational equipment can range from simple analog devices to large and complex computers, analog or digital. Consequently, the cost and accuracy of such an installation depend on the amount of computation required. Only general comments can be made regarding accuracy and cost.

First, an analog system must be switched to the proper platform if axles are to be properly separated, and so what might seem to be a simple analog integration scheme requires complex logic and switching circuitry to keep track of the axles. Such switching circuitry can be built up using pulse switches on the platforms and digital transistor logic circuitry; or, if high speed is not a prerequisite, the system can be composed of latching reed relays or even stepping switches. The greatly lowered price, high speed, and reliability of solid state logic components make them particularly attractive. A specific system for analog integration that incorporates digital transistor logic for switching is described herein. Assuming that the logic components for switching are of sufficient speed, it can be stated that the error of a computational method would be the error of the analog devices alone. The error of these, in turn, depends on the quality procured; integrators range in price from $15 to 50 times that, depending largely upon the long-term stability and temperature sensitivity that can be tolerated. For vehicle weighing there is one favorable factor—the vehicle is not over the platforms long enough to allow much drift of an integrator or multiplier. The major error of an analog system would be the summation of the series error of a complex system of computation. Simple integration over the platforms can be performed with a computation error of 0.2% with moderate- to low-priced equipment.

In digital computational systems the analog signal from the transducers (platforms) and signal conditioning equipment is multiplexed and directed to an analog-to-digital converter. There is error in the conversion process; a 12-bit binary conversion yields a resolution of 1 part in 2^{12}, or 1 part in 4,096. This is considered satisfactory for analog-to-digital conversion of force data inasmuch as the analog signal from the platforms and signal conditioning equipment will be accurate only to within approximately 0.5%

Digital computer processing of data is desired, because once the conversion is made from analog to digital each item is handled as an entity, with no further error in value representation. There is, however, a further source of error in digital computation. This is the accumulation of roundoff error that occurs from a long complex series of mathematical operations or from small differences of large numbers. Computation with a large computer is relatively accurate because each value is represented by 32, 36, 48, or even 60 bits. Small low-cost computers offer generally 12- or 16-bit accuracy. Inasmuch as previous discussion has indicated that small, low-cost computers are too slow for complex computation anyway, there should be little cause for concern. Simple averaging by small digital computers is not a complex process which will accumulate roundoff error. Consequently, digital computer computation may be regarded as highly accurate for averaging in real time at a weighing site.

Digital computers range in price from $10,000 up for commercial general purpose units. The PDP 8/S, for example, costs approximately that with a teletype input/output console, and a 4,000-word memory. The unit is slow, but it has possibilities in simple averaging systems.

SYSTEM FOR RECORDING DATA FOR LATER COMPUTATION

The more comprehensive methods to determine weight require a large computer, which can best be provided at a computer center. The weighing site is merely a data collection point where data may be taken and stored for later processing.

A representative system is shown in Figure H-1. This consists of seven platforms and the associated signal conditioning equipment to provide the analog signal. The analog signal is directed to a unity gain multiplex, to an analog-to-digital converter, and finally through formatting equipment to a digital magnetic tape recorder. At convenient intervals the tape may be taken to a computer center for processing.

To present a most representative and compatible data collection and conversion system, the recording system shown in Figure H-1 was recommended and priced by Radiation, Inc., of Melbourne, Florida.

The muliplexer is an all solid-state device that may be programmed for sampling of 2 to 16 channels. Here the sampling is directed, say, to seven platforms. These seven selected channels are sampled sequentially at a rate set by the sample rate generator countdown logic. For purposes of this presentation, a sampling rate of 7,000 samples per second will allow samples to be taken every 0.001 sec from each platform. This may be easily increased or decreased, depending upon the vehicle speed and dynamic characteristics. The sampling rate generator is based upon a crystal-controlled oscillator for stability considerations.

When a selected channel has been sampled by the multiplexer, the multiplexer output is held by the sample and hold circuitry within the analog-to-digital converter which has an aperture time of 0.1 μsec to approximate an instantaneous sample. The sample is held for the coding interval of approximately 20 μsec.

Data from the A-to-D converter appear as 12 bits. This is formatted into two 6-bit characters and entered into the memory that serves as a temporary storage buffer between the A-to-D converter and the tape recorder. This allows the data to be placed on tape in a gapped format, which becomes necessary when large amounts of data must be read from tape into the computer. The gapped format allows this to be read into the computer in blocks or groups instead of all at once.

Lateral parity is added to each 6-bit character by the Write Electronics, and data are written on tape as two 7-bit characters per sample in IBM compatible format.

The system can be used as shown to place sequential data from each platform on tape, or additional logic circuitry can be added to sort axles and place the data on tape in

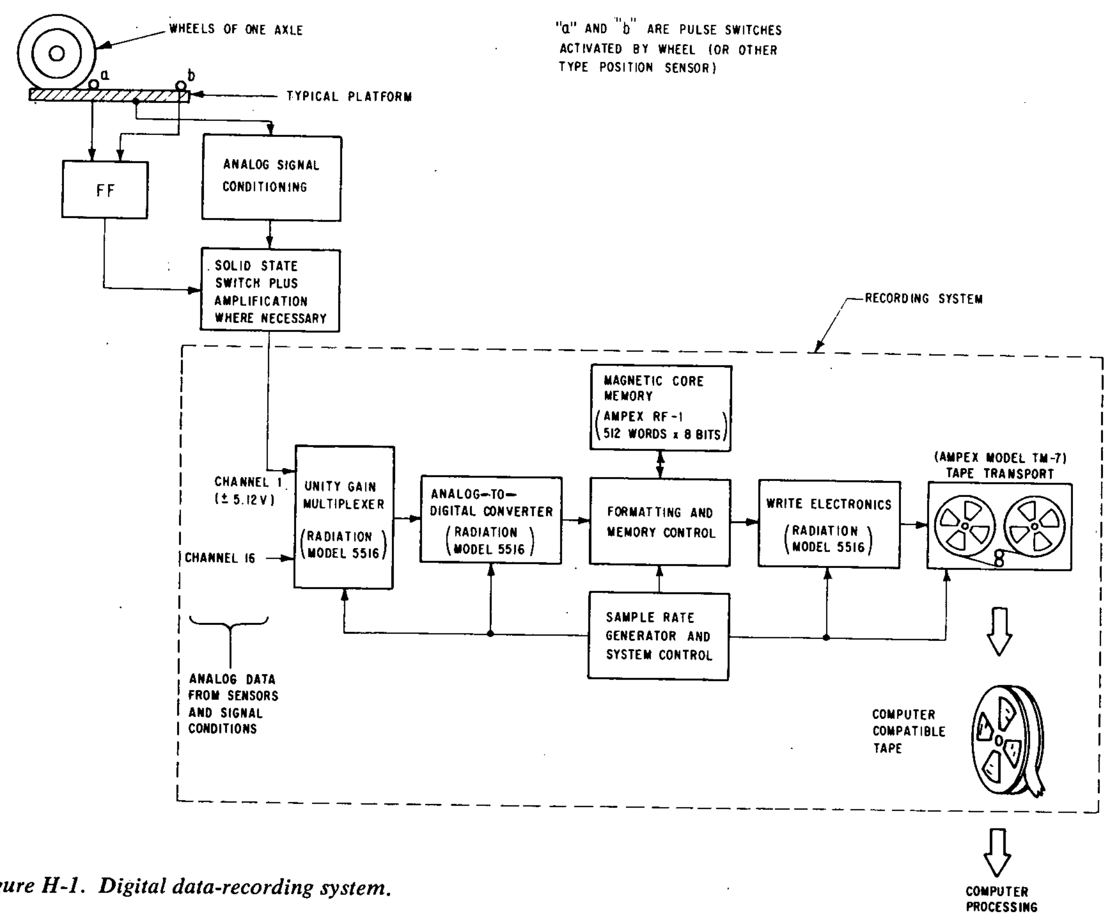


Figure H-1. Digital data-recording system.

blocks corresponding to axle groups. The method shown assumes that the sorting and identification of axles will be done at the computer center.

The approximate cost of this data collection system is:

7 platforms installed in a highway [7 platform pairs from Lee(*11*)]	$21,000
14-channel high-grade signal conditioning	4,200
Data conversion and recording package per Radiation, Inc.	30,000
Total cost	$55,200

This cost estimate includes all items except housing of the system at a remote site and the 110-volt utility power lines.

The error associated with the system is as follows:

Source	Error (%)
Platforms (depends on present development of platforms)	0.5 to 1.0
Signal conditioning (worse case)	0.1
Error of recording equipment and computer	0.1
Error of analytical weight estimation procedure for vehicle (program VWEIGH)	2.5
Overall maximum error	3.2 to 3.7

This estimate of error is the maximum error that can be expected in all but an occasional extreme case. This is based on a well-spaced system of platforms for weighing vehicles oscillating 20 to 25% or more about their static weight. The RMS error should be substantially less.

SYSTEMS FOR COMPUTING WEIGHT ON-THE-SPOT IN REAL TIME

As these studies show, the computation of axle weight by analytical methods other than simple averaging is complex, time-consuming, or expensive. Because of this, two systems are outlined in the following paragraphs to exploit the simple techniques of averaging over N platforms. One system employs electronic solid state digital circuitry to identify an axle and switch one channel of analog integration to follow the respective axle across all N platforms. The other system uses a small digital computer to effectively do the same thing, but with digital summation instead of analog integration. Logic diagrams and cost estimates are given for both.

Hybrid Digital-Analog Averaging System

Figure H-2 is a logic diagram of a system to average the force of each axle over a set of N platforms in accordance with the studies of simple averaging systems cited previously. No estimation of axle force is made between platforms. The analog integrators for each channel are stepped from platform to platform by digital solid state logic as the respective axle being followed arrives at that platform. When all platforms have been traversed, the logic circuitry switches the output of each axle integrator in turn to an analog divide circuit where the sums of the integrals of force are divided by the sums of the integrals of time:

$$F_{\text{avg}} = \frac{\left[\int_0^{\Delta t_1} F dt + \int_0^{\Delta t_2} F dt + \cdots + \int_0^{\Delta t_N} F dt\right]}{\left[\int_0^{\Delta t_1} dt + \int_0^{\Delta t_2} dt + \cdots + \int_0^{\Delta t_N} dt\right]} \tag{H-1}$$

In this system, force-sensitive platforms are installed flush with the surface of the highway. If the system uses platforms that are not butted together to form a continuous surface, it is necessary to determine when the tire is completely on the surface so that acceptance of the force signal may begin. This may be accompanied with a number of sensors, such as light beams or tape switches, mounted in the surface of the platform to yield a pulse when the tire is completely on the platform. A similar switch senses when the tire is about to roll off the platform.

When the wheels of one axle enter on platform 1, a pulse from switch "a" sets a flipflop memory and indexes a ring counter to the next position. Each respective position of the counter is connected to one input of a logic gate (shown as an AND gate) of a respective computing channel. As many positions on the counter and respective computing channels as are deemed necessary to handle all axles on the system at once can be provided. There is no limit. The output of the platform flipflop is connected to the logic (AND) gates interfacing all channels and that platform. Thus, with the counter indexed to #4 and with the flipflop set, only channel #4 receives the message to pass the analog signal from platform 1 to the integrator. When the wheel contacts switch "b" the flipflop changes state so that the logic gate of channel #4 is no longer satisfied, thus turning off the analog force signal.

With all counters set initially to the same value, each axle indexes each platform counter by a count of one as it passes, so that the same axle indexes each platform counter to the same position to call the same respective integration channel. As switch "b" of the last platform is actuated, the pulse ends the data record to the integrator as on each preceding platform, but also signals that the axle has passed all platforms so that the data are ready for readout. This is accomplished by directing the pulse of switch "b" to another counter, the output of which sets a flipflop memory in that respective channel with information that the axle channel is ready for division and will stand by until the divide and readout circuit can accept it. When the readout device signals it is ready for that particular channel through another counter and logic (AND) gate the integrator outputs are directed to the analog divide circuit where F_{avg} is computed. This, then, is ready for any analog readout. However, a printed copy may be desired in addition to a digital display or paper tape recording. Figure H-2 shows provision for this with a digital voltmeter and automatic printer.

The logic and analog circuits are simple and can be readily fabricated from off-the-shelf standard components. The digital voltmeter and printer can be standard items of medium performance.

The cost of fabrication (less the platforms and signal conditioning) for a first unit at The Franklin Institute Research Laboratories has been estimated at $27,000. This is for the largest and most complex unit anticipated. The actual components, including all cabinetry, can be purchased in lots of one for $9,000 or less. No estimate was obtained for subsequent manufacture. Thus, the total price of a system with six platform pairs is approximately as follows:

Source	First Cost	Subsequent Units
Platforms installed [7 per Lee(*11*)]	$21,000	$21,000
Signal conditioning, 14 channels	4,200	4,200
Hybrid digital-analog system	27,000	16,000[a]
Total cost	$52,200	$41,200[a]

[a] This is a rough approximation to the cost with very limited production of units.

These are approximate costs of the working equipment only, and do not include the cost of housing the equipment along a highway or the cost of installation and initial checkout.

The worst case error of the installation is approximated as follows:

Source	Error (%)
Platforms	0.5 to 1.0
Signal conditioning (extreme case)	0.05 to 0.1
Analog computation equipment	1.0 to 1.5
Total equipment error	1.55 to 2.62

The error of a composite waveform is not shown in the previous studies of simple averaging, but those studies can be used to estimate the maximum error of a three-component waveform [Vehicle 91 (*11*, p. 122)] by summing the error of each harmonic component. Representative values of error are given in Table H-1. This represents the maximum error that can occur if all phase angles are "worst case." The probability that this would occur is small. The use of an oscillating component of force totaling a maximum of 24% of the static component is considered conservative for reasonably well-chosen and maintained sites.

For a seven-platform system the maximum analytical error is approximately 5 to 6%. Combining this with the equipment error yields a total maximum error of 6.6 to

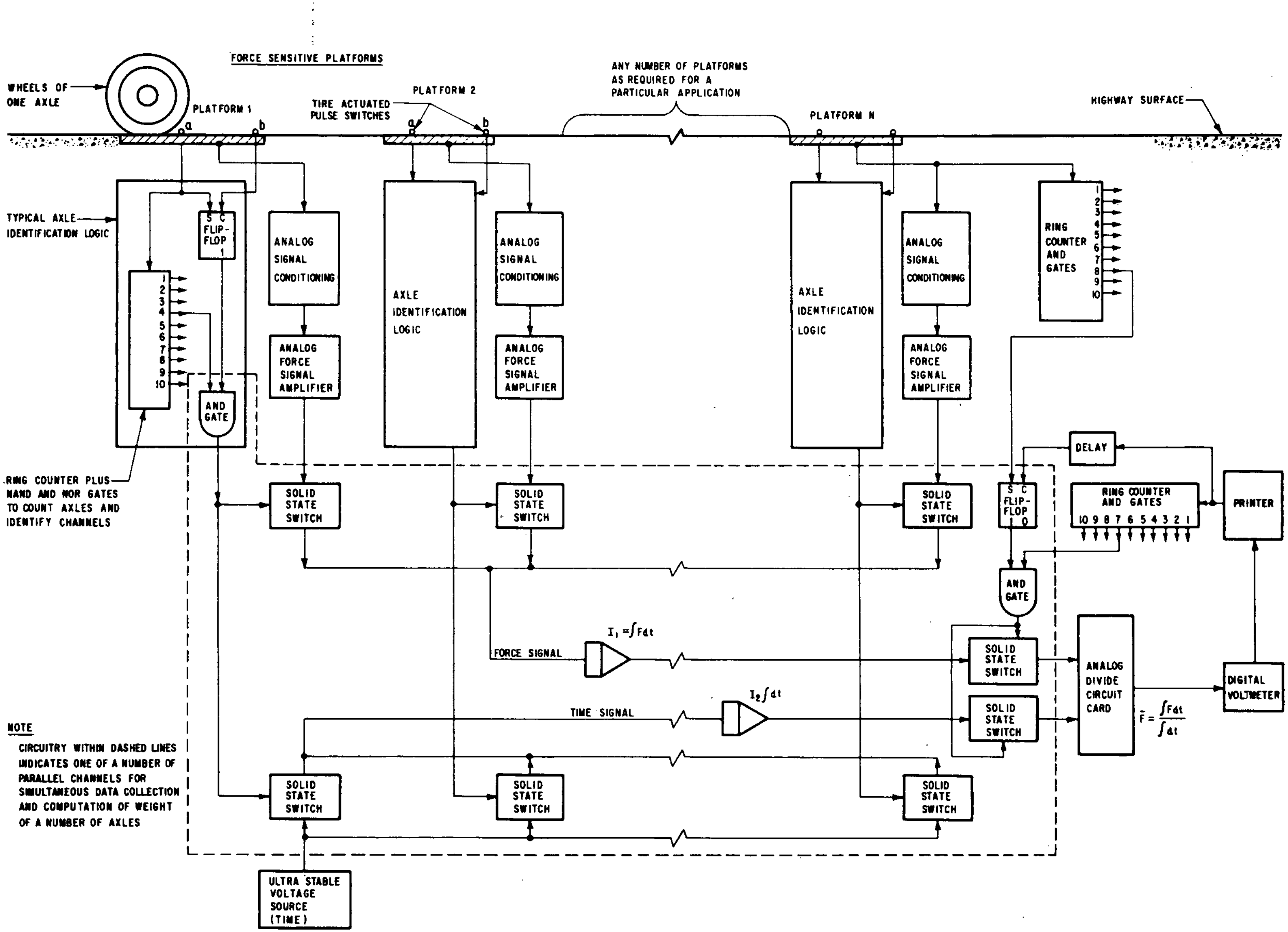


Figure H-2. Hybrid digital analog vehicle axle weighing system.

TABLE H-1

ERROR OF SIMPLE AVERAGING FOR AN AXLE WITH THREE-COMPONENT WAVEFORM TOTALING 24% OSCILLATION ABOUT STATIC WEIGHT

						E/α FROM FIG. C-8			ERROR PER COMPONENT		
						NO. PLATFORMS			NO. PLATFORMS		
SUSPENSION	VEL. (MPH)	FORCE COMPO-NENT	FREQ. (CPS)	RELATIVE AMPLI-TUDE	WAVE-LENGTH Δ (FT)	5	6	7	5	6	7
Free, with tire hop	60	1	2.1	0.1	41.9	0.24	0.24	0.24	0.024	0.024	0.024
		2	2.4	0.1	36.6	0.10	0.12	0.16	0.010	0.012	0.016
		3	12.0	0.04	7.33	0.35	0.55	0.35	0.014	0.022	0.014
Total error as a fraction of static axle weight									0.048	0.058	0.054
In percentage of axle weight									4.8	5.8	5.4
Blocked, with tire hop	60	1	4.0	0.1	22.0	0.09	0.05	0.06	0.009	0.005	0.006
		2	3.7	0.1	23.8	0.08	0.07	0.07	0.008	0.007	0.007
		3	12.0	0.04	7.33	0.35	0.55	0.35	0.014	0.022	0.014
Total error as a fraction of static axle weight									0.031	0.034	0.027
In percentage of axle weight									3.1	3.4	2.7
Free, with tire hop	45	1	2.1	0.1	31.4	0.14	0.13	0.11	0.014	0.013	0.011
		2	2.4	0.1	27.5	0.25	0.22	0.20	0.025	0.022	0.020
		3	12.0	0.04	5.5	0.22	0.30	0.13	0.009	0.012	0.005
Total error as a fraction of static axle weight									0.048	0.047	0.036
In percentage of axle weight									4.8	4.7	3.6
Blocked, with tire hop	45	1	4.0	0.1	16.5	0.47	0.17	0.08	0.047	0.017	0.008
		2	3.7	0.1	17.8	0.45	0.24	0.18	0.045	0.024	0.018
		3	12.0	0.04	5.5	0.22	0.30	0.13	0.008	0.012	0.005
Total error as a fraction of static axle weight									0.100	0.053	0.031
In percentage of axle weight									10.0	5.3	3.1

8.8%. The RMS error of this system for any one axle would probably be about 4.5%, or less, if the oscillating component is less than 24%.

Simple Averaging at a Weighing Site with a Digital Computer

Instead of building a special-purpose, digital-controlled, analog computer to weight on-the-spot by simple averaging, essentially the same logic and averaging method presented in the preceding section can be accomplished by a small, low-cost digital computer. Here the logic functions and summation are done in the computer, leaving only the platforms and the signal conditioning as separate components. Figure H-3 is a symbolic diagram of this system.

The system using a small commercial digital computer is most attractive because it has the least equipment error and is already assembled into a concise package. The latter is important if only one or a few units are desired.

A survey of the digital computer market indicates that computers for simple averaging can be purchased for as low as \$15,000, and in one case for as low as \$10,000, including typewriter and paper tape input/output. Although the speed of operation of this unit is slow compared with higher-priced units, it can handle the logic operations to identify axles and accumulate sums of force samples taken from each of seven platforms every 0.002 sec. This is adequate for simple averaging of the force signals evidenced by axles of highway vehicles. The equipment cost of such an installation is:

7 platforms [those of Lee (*11*), installed]	\$21,000
14-channel high-grade signal conditioning	4,200
Multiplexer	2,500
A-to-D converter (12-bit)	2,000
Crystal clock	800
Interfacing	500
Digital computer with input/output	10,000
Total cost	\$41,000

Again, housing, power lines, installation and checkout are not included.

The equipment error for such a system is:

Source	Error (%)
Platforms	0.5 to 1.0
Signal conditioning (extreme case)	0.05 to 0.1
Multiplexer, A-to-D converter (12-bit) and computer	0.2
Total equipment error	0.75 to 1.3

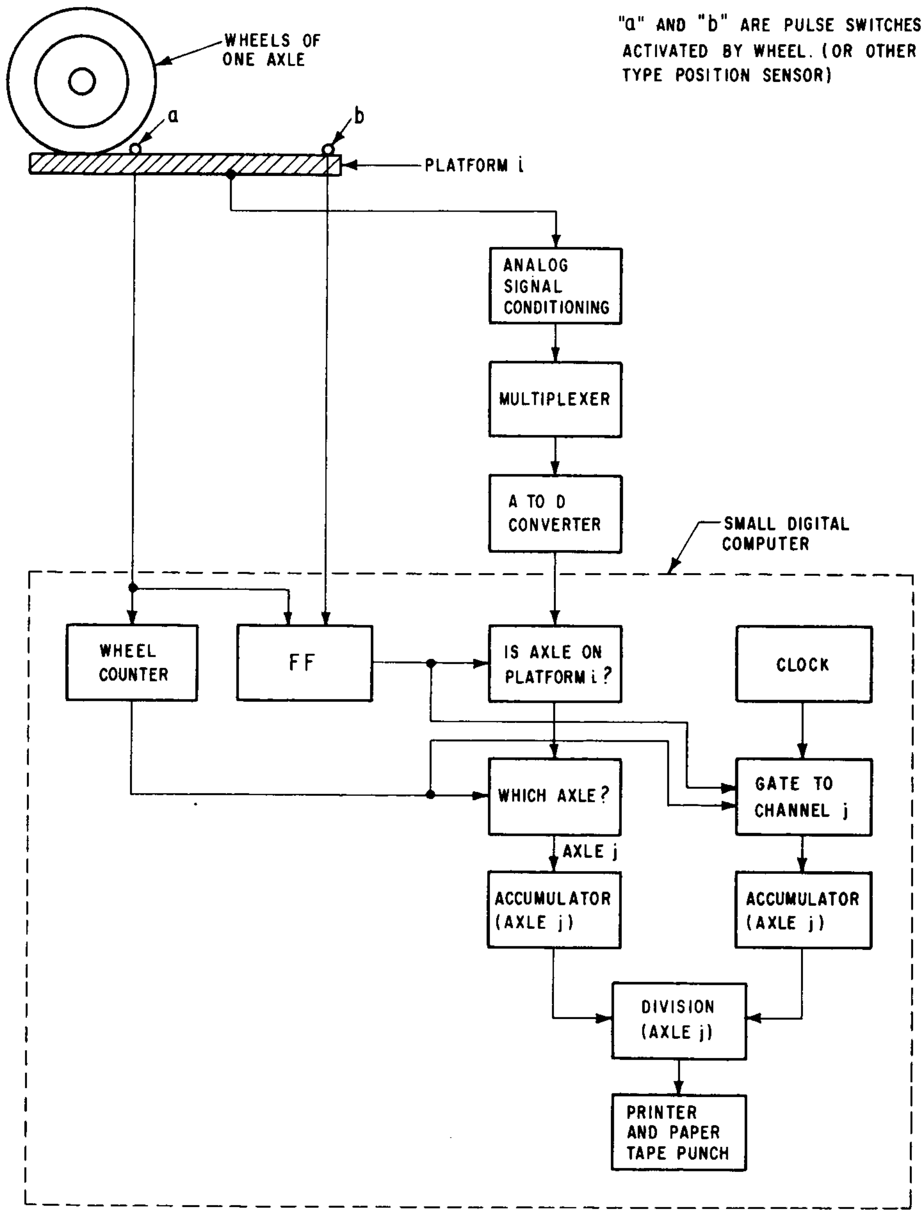


Figure H-3. Simple averaging with a small digital computer.

As stated, the usual high error anticipated for a seven-platform simple averaging system is on the order of 5 to 6%, assuming a vehicle with an oscillating component of 20 to 24% of the static force component. This combines with the equipment error to yield an overall high error or 5.8 to 7.4%. The average error would possibly be more like 3.5%, or less, for most weighing.

System for Computing Weight by Integral Model III

Figure H-4 shows an analog method for computing the weight in real time at a weighing site. Although in retrospect this method seems to have more merit than it did at the time the investigation of Integral Model III was terminated, it has not been fully developed and is presented here to show another variation of what can be done. The method is more attractive now, partly because it is possible to physically adapt separate platforms rather than the continuous long force-sensitive area made by butting individual platforms, as shown in Figure H-4.

As in the preceding physical systems, parallel channels are provided to handle each axle in the process of being weighed at any given time. Three such channels are shown, although more probably would be required for most general highway installations.

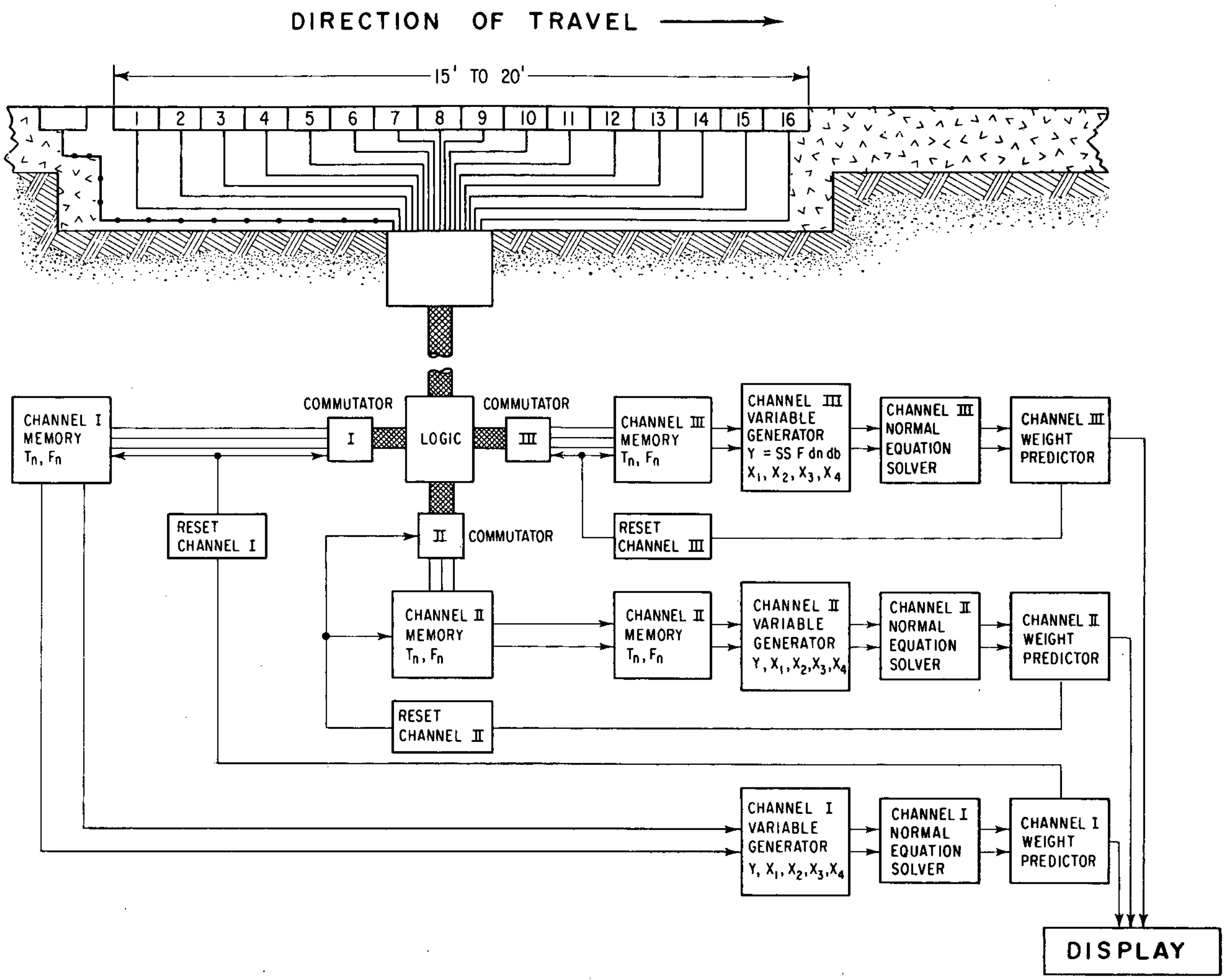


Figure H-4. Schematic for a weighing system using Integral Model III.

DIGITAL COMPUTER PROGRAM VWEIGH

Input Card Description

CARD NO.	INFORMATION		FORMAT
1	KSTØP, TITLE		(I5, 1x, 11A6)
	KSTØP;	Any number greater than 1 appearing in the first 5 columns will stop the program: To be used at end of data in place of title.	
	TITLE;	Any title or heading that is desired but limited to 66 characters.	
2	DELTA, NP, SL, BETA		(F10.0, I5, 2F10.0)
	DELTA;	Time increment between samples from any given platform.	
	NP;	Number of platforms (limited to 10).	
	SL;	Installation length.	
	BETA;	Skewing factor for non-uniform positioning of platforms.	
3	B(I), I = 1, NP		(10 F 7.3)
	B(I);	Dimension (in feet) of active portion of each of the NP platforms. Up to 10 platforms.	
4	V, W, ND		(2 F 10.0, I5)
	V;	Vehicle velocity in mph.	
	W;	Static component of axle force (static axle weight).	
	ND;	Degree of algebraic interpolation polynomials $\left(F = \sum_{k=0}^{ND} A_k t^k\right)$	
5	ØM1, ØM2, ØM3, AL1, AL2, AL3, PHI1, PHI2, PHI3		(9 F8.5)
	ØM1, ØM2, ØM3;	Frequencies (cps) of axle force waveform.	
	AL1, AL2, AL3;	Relative amplitude of 3 components of waveform as fractions of static component.	
	PHI1, PHI2, PHI3;	Phase angles of the 3 components of waveform.	

Note: Any number of weighings may be simulated in one computer run by placing the respective groups of 5 data cards into the data deck in the order of solution desired. The run is stopped by placing a KSTØP card at the end of the deck in place of what would have been the next TITLE card.

```
                                        FORTRAN SOURCE LIST
ISN        SOURCE STATEMENT

  0 $IBFTC VWEIGH
    C
    C   INCORPORATING LEAST SQUARES  POLYNCMIAL FIT OVER THREE PLATFORMS
    C
  1       COMMON AL1,AL2,AL3,OM1,OM2,OM3,PHI1,PHI2,PHI3,W
  2       DIMENSION TP(10),TR(10),TITLE(11),B(10),X(10),FV(10,40),JJ(10),AX(
         16,7),AXM(6,7),A(6,6)
  3    11 READ(5,1)KSTOP,TITLE
  5       IF(KSTOP)13,13,12
  6    12 WRITE(6,110)
  7   110 FORMAT(1H1/5X, 8HEND RUN. )
 10       STOP
 11    13 READ(5,2)DELTA,NP,SL,BETA
 13       READ(5,3)(B(I),I=1,NP)
 20       READ(5,4)V,W,ND
 22       READ(5,5)OM1,OM2,OM3,AL1,AL2,AL3,PHI1,PHI2,PHI3
 23       WRITE(6,90)
 24       WRITE(6,91)ND
 25       N = ND+1
 26       WRITE(6,93)TITLE,NP,SL,BETA,V
 27       VS = V*88.0/60.0
 30       DO 50 I=1,NP
 31       GI = FLOAT(I)/FLOAT(NP)
 32       GI1 = FLOAT(I-1)/FLOAT(NP)
 33       X(I) = 0.5*(SL*(GI**BETA + GI1**BETA) - B(I))
 34       TP(I) = X(I)/VS
 35       TR(I) = B(I)/VS
 36    50 CONTINUE
 40       WRITE(6,94)
 41       DO 96 I=1,NP
 42       WRITE(6,95)I,TP(I),TR(I),X(I),B(I)
 43    96 CONTINUE
 45       WRITE(6,97)DELTA
 46       WRITE(6,98)OM1,AL1,PHI1,OM2,AL2,PHI2,OM3,AL3,PHI3
 47     1 FORMAT(I5,1X,11A6 )
 50     2 FORMAT(F10.0, I5, 2F10.0)
 51     3 FORMAT(10F7.3)
 52     4 FORMAT(2F10.0,I5)
 53     5 FORMAT(9F8.5)
 54    90 FORMAT(1H1 /// 5X, 64HVEHICLE WEIGHING STUDY  --  B2189-02  --  HI
         1GHWAY RESEARCH BOARD //// 5X,22HINTERPOLATION EQUATION /)
 55    91 FORMAT(10X, 26HPOLYNOMIALS IN T OF DEGREE ,I3 )
 56    93 FORMAT(///10X,11A6,// 15X,22HNUMBER OF PLATFORMS IS I3, 31H,      I
         1NSTALLATION LENGTH,L, IS F6.1, 18H FEET,      BETA IS F5.1 // 15X,
         219HVEHICLE VELOCITY IS F6.1,15H MILES PER HOUR /)
 57    940FORMAT(15X,61HPLATFORM SPACING (TIME IN SECONDS, DISTANCE OR WIDTH
         1 IN FEET)// 22X, 96HPLATFORM NO.      TIME TO PLATFORM      TIME ON
         4PLATFORM     DISTANCE TO PLATFORM     PLATFORM WIDTH /)
 60    95 FORMAT(25X,I3,13X,F9.5,12X,F9.5,13X,F9.4,12X,F9.4 /)
 61    97 FORMAT(//15X, 16HSAMPLING TIME IS F9.6, 8H SECONDS  ////15X, 20HFO
         1RCE COMPONENT DATA // 21X, 51HFREQUENCY (CPS)      AMPLITUDE
         4     PHASE (RAD) )
 62    98 FORMAT(23X,F7.2,13X,F6.3,15X,F6.3//23X,F7.2,13X,F6.3,15X,F6.3//23X
         1,F7.2,13X,F6.3,15X,F6.3)
    C
```

```
ISN       SOURCE STATEMENT

    C  GENERATING FORCE DATA
    C
 63        T = 0.0
 64        TQ = 0.0
 65        FSUM = 0.0
 66        DO 15 I=1,NP
 67        J = 0
 70        F = 0.0
 71        T = TP(I)
 72     10 J = J+1
 73        F11 = F
 74        F = FORCE(T)
 75        FV(I,J) = F
 76        JJ(I) = J
 77        T = T + DELTA
100        IF(J-1)10,10,14
101     14 FSUM = FSUM + 0.5*DELTA*(F11 + F)
102        TQ = TQ + DELTA
103        IF(T-TP(I)-TR(I))10,10,15
104     15 CONTINUE
106        AV1 = FSUM /TQ
    C
    C  COMPUTATION OF WEIGHT FROM FORCE DATA
    C
107        WRITE(6,120)
110    120 FORMAT(1H1/////  7X, 36HAXLE WEIGHT COMPUTED FROM FORCE DATA ///)
111        WRITE(6,122)AV1
112    122 FORMAT(15X, 39H1.  AVERAGE OF FORCE OVER ALL PLATFORMS // 25X,18HC
          1OMPUTED WEIGHT IS F8.1, 5H LBS. /)
113        WRITE(6,119)W
114    119 FORMAT(25X, 16HACTUAL WEIGHT IS F8.1, 5H LBS. ///)
    C
    C  LEAST SQUARES FIT OF POLYNOMIAL OVER EACH THREE ADJACENT PLATFORMS
    C
115        NI = NP-2
116        M = N+1
117        DO 200 K=1,NI
120        DO 170 I=1,N
121        DO 150 J=1,N
122        IE = I+J-1
123        K1 = K+1
124        K2 = K+2
125        TRK = DELTA*FLOAT(JJ(K)-1)
126        TRK1 = DELTA*FLOAT(JJ(K1)-1)
127        TRK2 = DELTA*FLOAT(JJ(K2)-1)
130        AX(I,J) = TRK**IE   + (TP(K1)+TRK1   - TP(K))**IE -(TP(K1)-TP(K))
          1**IE
131        AX(I,J) = AX(I,J)+(TP(K2)+TRK2  -TP(K))**IE - (TP(K2)-TP(K))**IE
132        AX(I,J) = AX(I,J)/FLOAT(IE)
133    150 CONTINUE
135        AX(I,M)=0.0
136        DO 160 L=K,K2
137        T = TP(L)-TP(K)
140        J = 1
141        IF(I-1)152,152,153
```

FORTRAN SOURCE LIST VWEIGH

```
ISN       SOURCE STATEMENT

142    152 G = FV(L,J)
143        GO TO 154
144    153 G = FV(L,J)*(T**(I-1))
145    154 JJL = JJ(L)
146        DO 161 J=2,JJL
147        GO = G
150        T = T+DELTA
151        IF(I-1)156,156,157
152    156 G = FV(L,J)
153        GO TO 158
154    157 G = FV(L,J)*(T**(I-1))
155    158 AX(I,M) = AX(I,M)+(G+GO)*DELTA/2.0
156    161 CONTINUE
160    160 CONTINUE
162    170 CONTINUE
    C
    C  SOLUTION FOR COEFFICIENTS BY CROUTS METHOD
    C
164        DO 175 I=1,N
165        J=1
166    175 AXM(I,J)=AX(I,J)
170        DO 177 J=2,M
171    177 AXM(1,J) = AX(1,J)/AXM(1,1)
173        DO 179 I=2,N
174        DO 181 J=2,I
175        SUM = 0.0
176        LL = J-1
177        DO 183 L=1,LL
200    183 SUM = SUM + AXM(I,L)*AXM(L,J)
202    181 AXM(I,J) = AX(I,J)-SUM
204        II = I+1
205        DO 185 J=II,M
206        SUM =0.0
207        LL = I-1
210        DO 187 L=1,LL
211    187 SUM = SUM + AXM(I,L)*AXM(L,J)
213    185 AXM(I,J) = (AX(I,J)-SUM)/AXM(I,I)
215    179 CONTINUE
    C
    C CONTINUING CROUT METHOD --- SOLVING FOR COEFFICIENTS
    C
217        A(K,N) = AXM(N,M)
220        LLL = N-1
221        DO 189 L=1,LLL
222        I = N-L
223        SUM =0.0
224        I1 = I+1
225        DO 191 J=I1,N
226    191 SUM = SUM + AXM(I,J)*A(K,J)
230    189 A(K,I) = AXM(I,M) - SUM
232    200 CONTINUE
    C
    C THIS COMPLETES INTERPOLATION POLYNOMIALS
    C                    POLYNOMIAL COEFFICIENTS ARE A(K,I)
    C
```

ISN SOURCE STATEMENT

```
C GOING NOW TO INTEGRATION OF INTERPOLATED RECORD
C
      DO 250 K=1,NI
      K1 = K+1
      K2 = K+2
      T1 = TR(K)
      T2 = TP(K1) - TP(K)
      T3 = TP(K1)+TR(K1)  - TP(K)
      T4 = TP(K2) - TP(K)
      TQ = TQ + T2-T1
      IF(K-1)252,252,256
  252 DO 254 J=1,N
  254 FSUM = FSUM + A(K,J)*(T2**J-T1**J)/FLOAT(J)  + A(K,J)*(T4**J-T3**J
     1)/FLOAT(2*J)
      GO TO 250
  256 IF(K-NI)257,259,259
  257 DO 258 J=1,N
  258 FSUM = FSUM +A(K,J)*(T2**J-T1**J)/FLOAT(2*J) +A(K,J)*(T4**J-T3**J)
     1/FLOAT(2*J)
      GO TO 250
  259 DO 253 J=1,N
  253 FSUM = FSUM + A(K,J)*(T2**J-T1**J)/FLOAT(2*J) +A(K,J)* (T4**J-T3**
     1J)/FLOAT(J)
      TQ = TQ+T4-T3
  250 CONTINUE
      AV2 = FSUM/TQ
      WRITE(6,291)AV2
  291 FORMAT(15X, 41H2. MEAN VALUE OF INTERPOLATED DATA RECORD //25X, 18
     1HCOMPUTED WEIGHT IS F8.1, 5H LBS. ///)
C
C  THE FOLLOWING SEARCHES FOR THE TIME INCREMENTS IN WHICH FC CROSSES
C  THE MEAN, FCM, THEN TAKES MEAN OF FC BETWEEN THE FIRST AND THIRD
C  SUCCESSIVE CROSSING.
C
      WRITE(6,301)
  301 FORMAT(15X, 93H3.  MEAN VALUE OF INTERPOLATION EQUATION BETWEEN FI
     1RST AND THIRD CROSSING OF THE OVERALL MEAN /)
      TQA = 0.0
      FCINT = 0.0
      IK = 1
      T = TP(1)-DELTA
      TQ = 0.0
      K= 1
      K1=K+1
  260 T=T+DELTA
      IF(T-TP(NP-1))262,274,274
  262 IF(T-TP(K1))264,263,263
  263 K=K+1
      K1=K+1
      KO1=K-1
  264 IF(K-1)265,265,268
  265 FC=A(K,1)
      TQ=T-TP(1)
      DO 266 J=2,N
  266 FC=FC+A(K,J)*TQ**(J-1)
      GO TO (409,413,426),IK
  268 FCO1=A(KO1,1)
      FCO2=A(K,1)
      TQ=T-TP(KO1)
      DO 270 J=2,N
  270 FCO1 = FCO1 + A(KO1,J)*TQ**(J-1)
      TQ = T - TP(K)
      DO 272 J=2,N
  272 FCO2 = FCO2 + A(K,J)*TQ**(J-1)
      FC = (FCO1+FCO2)/2.0
      GO TO (409,413,426),IK
  274 K=NP-2
      KO1=NP-3
      K1=NP-1
      IF(T-TP(K1)-TR(K1))276,276,282
  276 FCO1=A(KO1,1)
      FCO2=A(K,1)
      TQ=T-TP(KO1)
      DO 278 J=2,N
  278 FCO1 = FCO1 + A(KO1,J)*TQ**(J-1)
      TQ=T-TP(K)
      DO 280 J=2,N
  280 FCO2 = FCO2 + A(K,J)*TQ**(J-1)
      FC = (FCO1+FCO2)/2.0
      GO TO (409,413,426),IK
  282 IF(T-TP(NP)-TR(NP))284,284,450
  284 FC=A(K,1)
      TQ=T-TP(K)
      DO 285 J=2,N
  285 FC=FC+A(K,J)*TQ**(J-1)
      GO TO (409,413,426),IK
  409 FD = FC - AV2
      IF(FD)410,260,412
  410 KEY = -1
      IK = 2
      GO TO 260
  412 KEY = 1
      IK = 2
      GO TO 260
  413 FD = FC - AV2
      KEY1 = KEY
      IF(FD)414,415,416
  414 KEY = -1
      GO TO 420
  415 GO TO 423
  416 KEY = 1
  420 IF(KEY+KEY1)260,423,260
  423 KEY2 = 1
      F1 = FC
      IK = 3
      GO TO 260
  426 FCINT = FCINT + DELTA*(FC+F1)/2.0
      F1 = FC
      TQA = TQA + DELTA
      FD = FC - AV2
```

FORTRAN SOURCE LIST VWEIGH

```
ISN       SOURCE STATEMENT

          KEY1 = KEY
          IF(FD)431,435,432
      431 KEY = -1
          GO TO 433
      432 KEY = 1
      433 IF(KEY+KEY1)260,435,260
      435 KEY2 = KEY2+1
          IF(KEY2 - 3)260,440,440
      440 FCI = FCINT/TQA
          WRITE(6,442)FCI,TQA
      442 FORMAT(25X, 18HCOMPUTED WEIGHT IS F8.1, 5HLBS. //25X, 49HTIME INTE
         1RVAL BETWEEN FIRST AND THIRD CROSSING IS F6.3, 5H SEC./)
          GO TO 460
      450 WRITE(6,452)
      452 FORMAT(25X, 82HWEIGHT NOT COMPUTED --- INTERPOLATION EQUATION DOES
         1NOT CROSS ITS MEAN THREE TIMES / 96X, 19HBETWEEN ZERO AND L' /)
      460 WRITE(6,462)
      462 FORMAT(1H1//// 10X, 39HCOEFFICIENTS OF INTERPOLATION EQUATIONS ///
         1 3X, 13HINTERPOLATION /  6X, 16HINTERVAL       A1,15X,2HA2, 15X,2HA
         23,15X,2HA4, 15X,2HA5, 15X,2HA6 /)
          DO 464 K=1,NI
      464 WRITE(6,466)K,(A(K,I),I=1,N)
      466 FORMAT(7X,I3,6E17.5 /)
C
C  COMPARISON OF MEASURED FORCE AND INTERPOLATED DATA
C
          WRITE(6,470)
      470 FORMAT(1H1 // 10X, 42HCOMPARISON OF FORCE AND INTERPOLATION DATA /
         1//)
          DO 480 K=1,NI
          WRITE(6,482)K
      482 FORMAT(15X, 22HINTERPOLATION INTERVAL,I3 // 24X, 55HTIME (SEC)
         1 FORCE (LBS)       INTERPOLATED FORCE (LBS)/)
          TQ = 0.0
          T = TP(K)
      484 F = FORCE(T)
          FC = A(K,1)
          DO 486 I=2,N
      486 FC = FC + A(K,I)*TQ**(I-1)
          WRITE(6,488)T,F,FC
      488 FORMAT(24X, F8.5, F15.1, F21.1)
          K2 = K+2
          IF(T - TP(K2)-TR(K2))490,490,480
      490 T=T+2.0*DELTA
          TQ = TQ + 2.0*DELTA
          GO TO 484
      480 CONTINUE
          GO TO 11
          END
```

FORTRAN SOURCE LIST

```
ISN       SOURCE STATEMENT

  C $IBFTC FRCE
          FUNCTION FORCE(T)
          COMMON AL1,AL2,AL3,OM1,OM2,OM3,PHI1,PHI2,PHI3,W
          TWOPI = 2.0*3.1415926
          AR1=TWOPI*OM1*T+PHI1
          AR2=TWOPI*OM2*T+PHI2
          AR3=TWOPI*OM3*T+PHI3
          ARG1=AMOD(AR1,TWOPI)
          ARG2=AMOD(AR2,TWOPI)
          ARG3=AMOD(AR3,TWOPI)
          FN=1.0+AL1*SIN(ARG1)+AL2*SIN(ARG2)+AL3*SIN(ARG3)
          FORCE=FN*W
          RETURN
          END
```

DIGITAL COMPUTER PROGRAM TRIGFT
(Fitting of Trigonometric Series to Sampled Axle-Force Data)

Input Card Description

CARD NO.	INFORMATION	FORMAT
1	B, V, KK, NP B; Platform width in ft. V; Vehicle velocity in mph. KK; Number of harmonic components before truncation of series (limited to 7). NP; Number of platforms (limited to 10).	(2 F 10.0, 2 I 5)
2	A1, A2, A3, A4, A5, A6 These are symbols for creating the output charts and for plotting the curves. A1 = * A2 = 0 A3 = + A4 = — A5 = (Blank) A6 = I	(6X, 6A 1)

FORTRAN SOURCE LIST

```
ISN       SOURCE STATEMENT
  0 $IBFTC TRIGFT
  1       0DIMENSION TP(10),X(10),AA(10),BB(10),ALPHA(15,16),AETA(15),FT(200)
         4,FF(200),E(101),G(101)
  2       COMMON AL1,AL2,AL3,OM1,OM2,OM3,PHI1,PHI2,PHI3,W
  3       PI=3.1415926
  4       TWOPI=2.0*PI
  5       DELTA=0.001
  6       SL=60.0
  7       BETA=1.5
 10       W=10000.0
 11       AL1=0.1
 12       AL2=0.1
 13       AL3=0.04
 14       OM1=2.4
 15       OM2=2.6
 16       OM3=12.0
 17       PHI1=0.0
 20       PHI3=1.5707
 21       PHI2=1.5707
 22     1 CONTINUE
 23       READ(5,20)B,V,KK,NP
 26    20 FORMAT(2F10.0,2I5)
 27       KTERM = 2*KK+1
 30       IF(NP)196,196,197
 31   197 WRITE(6,22)KTERM,V
 32    220FORMAT(1H1 /10X,104HTRIGONOMETRIC SERIES REPRESENTATION OF VEHICLE
         1 AXLE FORCE FROM SAMPLED PLATFCRM DATA (LEAST SQUARES FIT),// 20X,
         228HNUMBER OF TERMS IN SERIES IS,I4,//20X, 19HVEHICLE VELOCITY IS ,
         4F5.1, 4H MPH ///)
 33       VS=V*88.0/60.0
 34       TT=SL/VS
 35       TR=B/VS
 36       DO 50 I=1,NP
 37       GI=FLOAT(I)/FLOAT(NP)
 40       GI1=FLOAT(I-1)/FLOAT(NP)
 41       X(I)=0.5*(SL*(GI**BETA+GI1**BETA)-B)
 42       TP(I)=X(I)/VS
 43    50 CONTINUE
 45       WRITE(6,94)
 46       DO 96 I=1,NP
 47       WRITE(6,95)I,TP(I),TR,X(I),B
 50    96 CONTINUE
 52    95 FORMAT(25X,I3,13X,F9.5,12X,F9.5,13X,F9.4,12X,F9.4  /)
 53       FPA=0.0
 54       MTR=INT(TR/DELTA)
 55       MTR1=MTR+1
 56       KKK=2*KK+1
 57       KKK1=KKK+1
 60       ANP=FLOAT(NP*MTR)
 61       ALPHA(1,1)=ANP
 62       DO 120 I=1,KKK
 63       AETA(I)=0.0
 64       DO 120 J=1,KKK1
 65   120 ALPHA(I,J)=0.0
 70       DO 135 I=1,NP
```

```
       SOURCE STATEMENT

       T=TP(I)
       DO 135 J=1,MTR
       FA=FORCE(T)
       DO 133 L=1,KK
       AR=FLOAT(L)*PI*T/TT
       ARG=AMOD(AR,TWOPI)
       CSL=COS(ARG)
       SNL=SIN(ARG)
       ALPHA(2*L,1)=CSL + ALPHA(2*L,1)
       ALPHA(2*L+1,1)=SNL+ALPHA(2*L+1,1)
       DO 131 K=1,L
       AR=FLOAT(K)*PI*T/TT
       ARG=AMOD(AR,TWOPI)
       CSK=COS(ARG)
       SNK=SIN(ARG)
       ALPHA(2*L,2*K)=CSK*CSL+ALPHA(2*L,2*K)
       ALPHA(2*L,2*K+1)=SNK*CSL+ALPHA(2*L,2*K+1)
       ALPHA(2*L+1,2*K)=CSK*SNL+ALPHA(2*L+1,2*K)
       ALPHA(2*L+1,2*K+1)=SNK*SNL+ALPHA(2*L+1,2*K+1)
   131 CONTINUE
       ALPHA(2*L,KKK1)=ALPHA(2*L,KKK1)+FA*CSL
       ALPHA(2*L+1,KKK1)=ALPHA(2*L+1,KKK1)+FA*SNL
   133 CONTINUE
       T=T+DELTA
   135 CONTINUE
       DO 129 I=1,KKK
       DO 128 J=1,I
       ALPHA(I,J)=ALPHA(I,J)/ANP
   128 CONTINUE
       ALPHA(I,KKK1)=ALPHA(I,KKK1)/ANP
   129 CONTINUE
 C
 C     THIS COMPLETES THE DIAGONAL AND LOWER HALF OF THE MATRIX PLUS THE
 C     TERMS ON THE RIGHT HAND SIDE OF THE EQUATIONS
 C
 C     THE UPPER HALF OF THE SEMMETRIC MATRIX,ALPHA(KKK,KKK), FOLLOWS
 C
       KKK01=KKK-1
       DO 127 I=1,KKK01
       LL=I+1
       DO 127 J=LL,KKK
   127 ALPHA(I,J)=ALPHA(J,I)
 C
 C     THIS COMPLETES THE MATRIX FOR SOLUTION OF KKK EQUATIONS
 C
       CALL CROUT(KKK,ALPHA,AETA)
 C
 C     SOLUTION IS AETA(KKK)
 C
 C     COEFFICIENTS OF TRIGONOMETRIC TERMS FOLLOW
 C
       AO=AETA(1)
       DO 124 K=1,KK
       AA(K)=AETA(2*K)
   124 BB(K)=AETA(2*K+1)
```

FORTRAN SOURCE LIST TRIGFT

```
       SOURCE STATEMENT

       WRITE(6,161)FPA
   161 FORMAT(10X,48HAVERAGE LEVEL ABOUT WHICH SERIES IS GENERATED IS,F8.
      41, 5H LBS./)
       WRITE(6,160)
   160 FORMAT(/// 10X, 26HTRIGONOMETRIC COEFFICIENTS /)
       WRITE(6,162)AO
   162 FORMAT(10X,5HAO = ,E14.7, // 20X,1HK,11X,5HAA(K),15X,5HBB(K)  /)
       DO 165 K=1,KK
   165 WRITE(6,167)K,AA(K),BB(K)
   167 FORMAT(18X,I3,2E20.7, /)
       DEL5=DELTA*10.0
       MM=INT(TT/DEL5)
       T=0.0
       WRITE(6,180)
       DO 170 M=1,MM
       FT(M)=FORCE(T)
       FF(M)=AO+FPA
       DO 175 K=1,KK
       AR=FLOAT(K)*PI*T/TT
       ARG=AMOD(AR,TWOPI)
       FF(M)=FF(M)+AA(K)*COS(ARG)+BB(K)*SIN(ARG)
   175 CONTINUE
       WRITE(6,185)T,FT(M),FF(M)
       T=T+DEL5
   170 CONTINUE
       WRITE(6,100)
   100 FORMAT(1H1)
       READ(5,101)A1,A2,A3,A4,A5,A6
   101 FORMAT(6X,6A1)
       DO 102 I=1,101
   102 E(I)=A5
       DO 103 I=1,101,10
   103 E(I)=A6
       DO 104 I=1,101
   104 G(I)=A4
       DO 106 I=1,101,10
   106 G(I)=A3
       WRITE(6,107)(I,I=5,15)
   107 FORMAT(48X 33HAXLE FORCE IN THOUSANDS OF POUNDS //5X 11I10)
       KEY=10
       T=0.0
       DO 115 M=1,MM
       Y1=FT(M)
       Y2=FF(M)
       YI1=(Y1-5000.0)/10000.0
       YI2=(Y2-5000.0)/10000.0
       I1=INT((YI1+0.0049)*100.0)
       I2=INT((YI2+0.0049)*100.0)
       IF(KEY-10)112,110,112
   110 B1=G(I1)
       B2=G(I2)
       G(I1)=A1
       G(I2)=A2
       WRITE(6,111)T,G
   111 FORMAT(6X,3HT =,F5.1,101A1)
```

FORTRAN SOURCE LIST TRIGFT

ISN SOURCE STATEMENT

```
      G(I1)=B1
      G(I2)=B2
      KEY=0
      GO TO 114
  112 B1=E(I1)
      B2=E(I2)
      E(I1)=A1
      E(I2)=A2
      WRITE(6,113)E
  113 FORMAT(14X,101A1)
      E(I1)=B1
      E(I2)=B2
  114 T=T+DEL5
      KEY=KEY+1
  115 CONTINUE
      GO TO 1
  180 FORMAT(1H1//10X,20HCOMPARISON OF FORCES //19X,1HT,13X,1HF,12X,2HFF
     1,/)
  185 FORMAT(17X,F6.3,5X,F9.1,5X,F9.1,   )
  940FORMAT(15X,61HPLATFORM SPACING (TIME IN SECONDS, DISTANCE OR WIDTH
     1 IN FEET)// 22X, 96HPLATFORM NO.      TIME TO PLATFORM      TIME ON
     4PLATFORM     DISTANCE TO PLATFORM     PLATFORM WIDTH /)
  196 STOP
      END
```

FORTRAN SOURCE LIST

ISN SOURCE STATEMENT

```
$IBFTC CRUT
      SUBROUTINE CROUT(N,AX,A)
      DIMENSION AX(15,16),A(15),AXM(15,16)
      M=N+1
      DO 175 I=1,N
      J=1
  175 AXM(I,J)=AX(I,J)
      DO 177 J=2,M
  177 AXM(1,J) = AX(1,J)/AXM(1,1)
      DO 179 I=2,N
      DO 181 J=2,I
      SUM = 0.0
      LL = J-1
      DO 183 L=1,LL
  183 SUM = SUM + AXM(I,L)*AXM(L,J)
  181 AXM(I,J) = AX(I,J)-SUM
      I1 = I+1
      DO 185 J=I1,M
      SUM =0.0
      LL = I-1
      DO 187 L=1,LL
  187 SUM = SUM + AXM(I,L)*AXM(L,J)
  185 AXM(I,J) = (AX(I,J)-SUM)/AXM(I,I)
  179 CONTINUE
C
C CONTINUING CROUT METHOD --- SOLVING FOR COEFFICIENTS
C
      A(N)    = AXM(N,M)
      LLL = N-1
      DO 189 L=1,LLL
      I = N-L
      SUM =0.0
      I1 = I+1
      DO 191 J=I1,N
  191 SUM = SUM + AXM(I,J)*A(J)
  189 A(I)    = AXM(I,M) - SUM
      RETURN
      END
```

FORTRAN SOURCE LIST

ISN SOURCE STATEMENT

```
$IBFTC FRCE
      FUNCTION FORCE(T)
      COMMON AL1,AL2,AL3,OM1,OM2,OM3,PHI1,PHI2,PHI3,W
      TWOPI = 2.0*3.1415926
      AR1=TWOPI*OM1*T+PHI1
      AR2=TWOPI*OM2*T+PHI2
      AR3=TWOPI*OM3*T+PHI3
      ARG1=AMOD(AR1,TWOPI)
      ARG2=AMOD(AR2,TWOPI)
      ARG3=AMOD(AR3,TWOPI)
      FN=1.0+AL1*SIN(ARG1)+AL2*SIN(ARG2)+AL3*SIN(ARG3)
      FORCE=FN*W
      RETURN
      END
```

DIGITAL COMPUTER PROGRAM FOUFIT
(Fitting of Truncated Fourier Series to Sampled Data)

Input Card Description

CARD NO.	INFORMATION	FORMAT
1	B, V, KK, NP	(2 F 10.0, 2 I 5)
	B; Platform width in ft.	
	V; Vehicle velocity in mph.	
	KK; Number of harmonic components before truncation of series (limited to 7).	
	NP; Number of platforms (limited to 10).	
2	A1, A2, A3,	
	A4, A5, A6	(6X, 6A 1)
	These are symbols for creating the output charts and for plotting the curves.	
	A1 = *	
	A2 = 0	
	A3 = +	
	A4 = −	
	A5 = (Blank)	
	A6 = I	

FORTRAN SOURCE LIST

```
ISN       SOURCE STATEMENT

$IBFTC FOUFIT  LIST
      DIMENSION TP(10),X(10),AA(10),BB(10),SN1(10),CS1(10),SNO(10),CSO(1
     40),F(10,70),FT(200),FF(200),E(101),G(101)
      COMMON AL1,AL2,AL3,CM1,CM2,CM3,PHI1,PHI2,PHI3,W
      PI=3.1415926
      TWOPI=2.0*PI
      DELTA=0.001
      SL=60.0
      BETA=1.5
      W=10000.0
      AL1=0.1
      AL2=0.1
      AL3=0.04
      CM1=2.4*TWOPI
      CM2=2.6*TWOPI
      CM3=12.0*TWOPI
      PHI1=0.0
      PHI3=1.5707
      PHI2=1.5707
    1 CONTINUE
      READ(5,20)B,V,KK,NP
   20 FORMAT(2F10.0,2I5)
      KTERM = 2*KK+1
      IF(NP)196,196,197
  197 WRITE(6,22)KTERM,V
   22 FORMAT(1H1,/10X, 79HFOURIER SERIES REPRESENTATION OF VEHICLE AXLE
     1FORCE FROM SAMPLED PLATFORM DATA ,//20X, 28HNUMBER OF TERMS IN SER
     4IES IS,I4,//20X,19HVEHICLE VELOCITY IS,F5.1,4H MPH///)
      VS=V*88.0/60.0
      TT=SL/VS
      TR=B/VS
      DO 50 I=1,NP
      GI=FLOAT(I)/FLOAT(NP)
      GI1=FLOAT(I-1)/FLOAT(NP)
      X(I)=0.5*(SL*(GI**BETA+GI1**BETA)-B)
      TP(I)=X(I)/VS
   50 CONTINUE
      WRITE(6,94)
      DO 96 I=1,NP
      WRITE(6,95)I,TP(I),TR,X(I),B
   96 CONTINUE
   95 FORMAT(25X,I3,13X,F9.5,12X,F9.5,13X,F9.4,12X,F9.4  /)
      TC=0.0
      AO=0.0
      DO 120 K=1,KK
      AA(K)=0.0
      BB(K)=0.0
  120 CONTINUE
      FPA=0.0
      TC=0.0
      MTR=INT(TR/DELTA)
      MTR1=MTR+1
      DO 140 I=1,NP
      T=TP(I)
      DO 140 M=1,MTR1
```

ISN SOURCE STATEMENT

```
      F(I,M)=FORCE(T)
      IF(M-1)142,142,143
  142 F1=F(I,M)
      T=T+DELTA
      GO TO 140
  143 FPA = FPA+0.5*(F1+F(I,M))
      T=T+DELTA
      TO=TO+DELTA
      F1=F(I,M)
  140 CONTINUE
      FPA=FPA*DELTA/TO
      DO 150 I=1,NP
      T=TP(I)
      F1=F(I,1)-FPA
      DO 125 K=1,KK
      AR=FLOAT(K)*PI*T/TT
      ARG=AMOD(AR,TWOPI)
      SN1(K)=SIN(ARG)
  125 CS1(K)=COS(ARG)
      DO 130 M=2,MTRI
      T=T+DELTA
      F2=F(I,M)-FPA
      AO=AO+0.5*(F1+F2)
      DO 127 K=1,KK
      AR=FLOAT(K)*PI*T/TT
      ARG=AMOD(AR,TWOPI)
      SNO(K)=SIN(ARG)
      CSO(K)=COS(ARG)
      AA(K)=AA(K)+0.5*(F1*CS1(K)+F2*CSO(K))
  127 BB(K)=BB(K)+0.5*(F1*SN1(K)+F2*SNO(K))
      DO 128 K=1,KK
      SN1(K)=SNO(K)
      CS1(K)=CSO(K)
  128 CONTINUE
      F1=F2
  130 CONTINUE
  150 CONTINUE
      AO=2.0*DELTA*AO/TT
      DO 155 K=1,KK
      AA(K)=2.0*DELTA*AA(K)/TT
      BB(K)=2.0*DELTA*BB(K)/TT
  155 CONTINUE
      WRITE(6,161)FPA
  161 FORMAT(10X,48HAVERAGE LEVEL ABOUT WHICH SERIES IS GENERATED IS,F8.
     41, 5H LBS./)
      WRITE(6,160)
  160 FORMAT(/// 10X, 20HFOURIER COEFFICIENTS  /)
      WRITE(6,162)AO
  162 FORMAT(10X,5HAO = ,E14.7, // 20X,1HK,11X,5HAA(K),15X,5HBB(K)  /)
      DO 165 K=1,KK
  165 WRITE(6,167)K,AA(K),BB(K)
  167 FORMAT(18X,I3,2E20.7, /)
      DEL5=DELTA*10.0
      MM=INT(TT/DEL5)
      T=0.0
      WRITE(6,180)
      DO 170 M=1,MM
      FT(M)=FORCE(T)
      FF(M)=0.5*AO+FPA
      DO 175 K=1,KK
      AR=FLOAT(K)*PI*T/TT
      ARG=AMOD(AR,TWOPI)
      FF(M)=FF(M)+AA(K)*COS(ARG)+BB(K)*SIN(ARG)
  175 CONTINUE
      WRITE(6,185)T,FT(M),FF(M)
      T=T+DEL5
  170 CONTINUE
      WRITE(6,100)
  100 FORMAT(1H1)
      READ(5,101)A1,A2,A3,A4,A5,A6
  101 FORMAT(6X,6A1)
      DO 102 I=1,101
  102 E(I)=A5
      DO 103 I=1,101,10
  103 E(I)=A6
      DO 104 I=1,101
  104 G(I)=A4
      DO 106 I=1,101,10
  106 G(I)=A3
      KEY=10
      T=0.0
      DO 115 M=1,MM
      Y1=FT(M)
      Y2=FF(M)
      YI1=(Y1-5000.0)/10000.0
      YI2=(Y2-5000.0)/10000.0
      I1=INT((YI1+0.0049)*100.0)
      I2=INT((YI2+0.0049)*100.0)
      IF(KEY-10)112,110,112
  110 B1=G(I1)
      B2=G(I2)
      G(I1)=A1
      G(I2)=A2
      WRITE(6,111)T,G
  111 FORMAT(1X,3HT =,F5.1,101A1)
      G(I1)=B1
      G(I2)=B2
      KEY=0
      GO TO 114
  112 B1=E(I1)
      B2=E(I2)
      E(I1)=A1
      E(I2)=A2
      WRITE(6,113)E
  113 FORMAT(9X,101A1)
      E(I1)=B1
      E(I2)=B2
  114 T=T+DEL5
      KEY=KEY+1
  115 CONTINUE
```

```
                                            FCRTRAN SCURCE LIST FOUFIT
ISN        SCURCE STATEMENT

266          GO TC 1
267      18C FORMAT(1H1//10X,20HCCMPARISON OF FORCES //19X,1HT,13X,1HF,12X,2HFF
            1,/)
270      185 FCRMAT(17X,F6.3,5X,F9.1,5X,F9.1,  )
271       94CFCRMAT(15X,61HPLATFORM SPACING (TIME IN SECONDS, DISTANCE OR WIDTH
            1 IN FEET)// 22X, 96HPLATFORM NC.      TIME TO PLATFORM      TIME ON
           4PLATFORM     DISTANCE TO PLATFORM     PLATFORM WIDTH /)
272      196 STOP
273          END
```

```
                                            FCRTRAN SOURCE LIST
ISN        SCURCE STATEMENT

  0  $IBFTC FRCE     LIST
  1          FUNCTICN FCRCE(T)
  2          COMMON AL1,AL2,AL3,OM1,OM2,OM3,PHI1,PHI2,PHI3,W
  3          TWOPI = 2.0*3.1415926
  4          AR1=OM1*T+PHI1
  5          AR2=OM2*T+PHI2
  6          AR3=OM3*T+PHI3
  7          ARG1=AMOD(AR1,TWOPI)
 10          ARG2=AMOD(AR2,TWOPI)
 11          ARG3=AMOD(AR3,TWOPI)
 12          FN=1.C+AL1*SIN(ARG1)+AL2*SIN(ARG2)+AL3*SIN(ARG3)
 13          FCRCE=FN*W
 14          RETURN
 15          END
```

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