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DISTRIBUTION OF WHEEL LOADS ON HIGHWAY BRIDGES

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NATIONAL COOPERATIVE HIGHWAY RESEARCH PROGRAM
REPORT

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DISTRIBUTION OF WHEEL LOADS ON HIGHWAY BRIDGES

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RESEARCH SPONSORED BY THE AMERICAN ASSOCIATION
OF STATE HIGHWAY OFFICIALS IN COOPERATION
WITH THE BUREAU OF PUBLIC ROADS

SUBJECT CLASSIFICATION:

BRIDGE DESIGN

HIGHWAY RESEARCH BOARD

DIVISION OF ENGINEERING NATIONAL RESEARCH COUNCIL

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1970

NATIONAL COOPERATIVE HIGHWAY RESEARCH PROGRAM

Systematic, well-designed research provides the most effective approach to the solution of many problems facing highway administrators and engineers. Often, highway problems are of local interest and can best be studied by highway departments individually or in cooperation with their state universities and others. However, the accelerating growth of highway transportation develops increasingly complex problems of wide interest to highway authorities. These problems are best studied through a coordinated program of cooperative research.

In recognition of these needs, the highway administrators of the American Association of State Highway Officials initiated in 1962 an objective national highway research program employing modern scientific techniques. This program is supported on a continuing basis by funds from participating member states of the Association and it receives the full cooperation and support of the Bureau of Public Roads, United States Department of Transportation.

The Highway Research Board of the National Academy of Sciences-National Research Council was requested by the Association to administer the research program because of the Board's recognized objectivity and understanding of modern research practices. The Board is uniquely suited for this purpose as: it maintains an extensive committee structure from which authorities on any highway transportation subject may be drawn; it possesses avenues of communications and cooperation with federal, state, and local governmental agencies, universities, and industry; its relationship to its parent organization, the National Academy of Sciences, a private, nonprofit institution, is an insurance of objectivity; it maintains a full-time research correlation staff of specialists in highway transportation matters to bring the findings of research directly to those who are in a position to use them.

The program is developed on the basis of research needs identified by chief administrators of the highway departments and by committees of AASHO. Each year, specific areas of research needs to be included in the program are proposed to the Academy and the Board by the American Association of State Highway Officials. Research projects to fulfill these needs are defined by the Board, and qualified research agencies are selected from those that have submitted proposals. Administration and surveillance of research contracts are responsibilities of the Academy and its Highway Research Board.

The needs for highway research are many, and the National Cooperative Highway Research Program can make significant contributions to the solution of highway transportation problems of mutual concern to many responsible groups. The program, however, is intended to complement rather than to substitute for or duplicate other highway research programs.

This report is one of a series of reports issued from a continuing research program conducted under a three-way agreement entered into in June 1962 by and among the National Academy of Sciences-National Research Council, the American Association of State Highway Officials, and the U. S. Bureau of Public Roads. Individual fiscal agreements are executed annually by the Academy-Research Council, the Bureau of Public Roads, and participating state highway departments, members of the American Association of State Highway Officials.

This report was prepared by the contracting research agency. It has been reviewed by the appropriate Advisory Panel for clarity, documentation, and fulfillment of the contract. It has been accepted by the Highway Research Board and published in the interest of an effectual dissemination of findings and their application in the formulation of policies, procedures, and practices in the subject problem area.

The opinions and conclusions expressed or implied in these reports are those of the research agencies that performed the research. They are not necessarily those of the Highway Research Board, the National Academy of Sciences, the Bureau of Public Roads, the American Association of State Highway Officials, nor of the individual states participating in the Program.

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FOREWORD

By Staff

Highway Research Board

This report contains proposed revisions to those sections of the *AASHO Specifications for Highway Bridges* having to do with the distribution of wheel loads on bridge decks. It is recommended to engineers, researchers, and members of specification writing bodies concerned with bridge design. Its most immediate importance will be to the members of the AASHO Committee on Bridges and Structures; however, others concerned with bridge design should find it interesting and informative.

Whether or not the factors currently in use for load distribution on bridge decks are adequate for the various types of floor systems currently in use is open to conjecture. Considerable research has been conducted on this question, but the results have not been correlated and evaluated in a manner such that recommendations for changes in the *AASHO Specifications* could be made. There was a definite need for review and correlation of the past analytical and experimental work and for definition of improved load distribution factors for each popular type of highway bridge deck.

Many theories have been proposed in past years for the determination of the load distribution behavior of floor systems. These include orthotropic plate theory, articulated plate theory, flexibility or stiffness methods, and many others. In a study limited to short- and medium-span bridges of the beam-and-slab, multi-beam, the box girder types, researchers at Iowa State University examined the theories applicable to bridge analysis and determined those most applicable to these bridge categories. Furthermore, they (1) verified the validity of the theories by comparing the measured behavior of actual bridges under load with the predicted moments or deflections obtained from the theoretical analysis; (2) extended existing or developed new analytical approaches applicable to the popular types of bridge floor systems; (3) determined the variables that have an important influence on load distribution; and (4) recommended specification changes that will result in designs that are realistic and yet have adequate factors of safety.

At the present time (fall 1969), the findings from this study have no direct application to practice for those bound by the *AASHO Specifications*. It is a matter for the AASHO Committee on Bridges and Structures to decide whether the recommendations concerning the distribution of wheel loads presented in this report will be adopted for practice; therefore, bridge design engineers and researchers involved with loadings on bridge structures will find the results of the study to be presently a matter of general information only.

There is considerable interest and work throughout the United States in measuring the performance of bridges under load. As these data continue to be accumulated, it will be possible to study further the reliability of the load distribution methods suggested in this report.

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ACKNOWLEDGMENTS

The research reported herein was conducted by the Structural Engineering Research Laboratory staff, Engineering Research Institute, Iowa State University. The investigation was directed by Wallace W. Sanders, Jr., Associate Professor of Civil Engineering, serving as Principal Investigator, and Hotten A. Elleby, Assistant Professor of Civil Engineering, serving as co-investigator. Graduate research assistants working on the study were James G. Arendts, Orhan Gurbuz, and Eiichi Watanabe.

The assistance and cooperation of many persons and agencies in furnishing information and making suggestions on specific details of the conduct of this study are acknowledged. Special thanks are given to the many researchers whose studies were used as the basis or stimulation for much of the work reported herein. Special appreciation also goes to Messrs. Arendts and Watanabe for their enthusiastic efforts during the entire investigation.

DISTRIBUTION OF WHEEL LOADS ON HIGHWAY BRIDGES

SUMMARY

The research reported herein was undertaken for the purpose of developing more realistic design criteria for distribution of wheel loads on highway bridges.

For more than 30 years the *Standard Specifications for Highway Bridges* of the American Association of State Highway Officials (AASHO) has included a procedure for determining this load distribution. Although several detailed studies were conducted on specific bridge types, many of the criteria have been based on extensions developed from separated limited studies. It was the purpose of this investigation to study at one time the static distribution of movable wheel loads in a broad range of bridge types used by today's designers. This approach gives a uniform approach to the development of specification criteria.

The current AASHO specifications for load distribution were essentially developed in their present format about 25 years ago. Although some minor changes in procedures have been made and several new bridge types included, the basic approach has remained unchanged since that time. Presently, the only major variables considered are beam spacing and general bridge floor system makeup. However, many other variables affect the behavior (some quite significantly), and with the many analytical tools available more realistic distribution criteria can be developed. It is for this purpose that this study was undertaken.

However, the study was limited to short- and medium-span bridges; that is, bridges with spans up to about 120 ft. In this span range, the bridge types can be classified into three general categories: beam and slab, multi-beam, and cast-in-place concrete box girder. The behavior of these bridges can be characterized by the following major variables: aspect ratio (bridge width/bridge span), relative stiffness of beams and floor, relative diaphragm stiffness, and extent of bridge continuity. The effect of these variables on the load distribution was investigated with respect to the number and position of wheel loads.

During the past 50 years many theories have been proposed and developed which are applicable to the determination of the behavior of the floor system under load. These include orthotropic plate theory, articulated plate theory, flexibility or stiffness methods, grillage method, finite element method, harmonic analysis, folded plate theory, and moment distribution procedures. Each of these theories has particular inherent assumptions which make it more applicable to a particular bridge geometry. However, because a wide variety of bridge types is considered herein, several generally applicable modifications of the plate theory have been employed in the over-all analysis. To limit complexity, the general plate theory was used and adapted to the specific bridge types previously listed. Thus, a similar set of geometric

parameters is applicable to the bridge types studied. For the beam and slab bridges, the orthotropic plate theory was used; for the multi-beam bridges, the articulated plate theory; and for concrete box girder bridges, the folded plate theory.

To verify the validity of the theories and their assumptions in predicting the behavior of an actual bridge under load, correlations were made between moments or stresses obtained from actual field tests and those computed by applicable theories using the actual bridge geometry and loading. These correlations indicate that the theories selected do adequately predict the load distribution in the particular bridge types.

Extensive numerical studies relating beam moments to the number and the lateral position of standard truck loadings for various combinations of the variables previously listed were then conducted. These results were used to determine a number of influence lines for beam moment. However, the complexity of the interrelation of the variables makes using these charts in a design office virtually impossible. Thus, an empirical equation developed from these charts was formulated and is presented in a proposed revision to the current AASHTO Specifications (279) for load distribution.

Although numerous revisions have been proposed in Section 3 on "Distribution of Loads," the major change has been recommended for Article 1.3.1(B) in distribution on bending moment in stringers and longitudinal beams. Even though these changes, in many cases, do not significantly affect current designs, they do make them more realistic and do consider the benefits derived from improving bridge properties. It is recommended that this entire article be replaced by the following new Article 1.3.1(B), recommended for inclusion in the AASHTO Specifications (279):

1.3.1—DISTRIBUTION OF LOADS TO STRINGERS, LONGITUDINAL BEAMS AND FLOOR BEAMS.

(A) Position of Wheel Loads for Shear—*unchanged*.

(B) Live Load Bending Moment in Stringers and Longitudinal Beams for Bridges Having Concrete Decks.*

In calculating bending moments in longitudinal beams or stringers, no longitudinal distribution of the wheel load shall be assumed. The lateral distribution shall be determined as follows:

(1) Load Fraction (all beams).

The live load bending moment for each beam shall be determined by applying to the beam the fraction of a wheel load (both front and rear) determined by the following relations:

$$\text{Load Fraction} = \frac{S}{D}$$

in which S is

S_a for beam and slab bridges,†

$\frac{12N_L + 9}{N_g}$ for multi-beam bridges,‡

* In view of the complexity of the theoretical analysis involved in the distribution of wheel loads to stringers, the empirical method described herein is authorized for the design of normal highway bridges. This section is applicable to beam and slab, concrete slab, multi-beam, and concrete box girder bridges. For composite steel box girder bridges, the criteria specified in Article 1.7.104 should be used.

† For slab bridges, $S = 1$ and the load fraction obtained is for a 1-ft width of slab.

‡ A multi-beam bridge is constructed with precast reinforced or prestressed concrete beams placed side by side on the supports. The interaction between the beams is developed by continuous longitudinal shear keys and lateral bolts, which may or may not be prestressed.

the maximum of the two values for concrete box girder bridges, and the value of D is determined by the following relationship:

$$D = 5 + \frac{N_L}{10} + \left(3 - \frac{2N_L}{7}\right) \left(1 - \frac{C}{3}\right)^2 \quad C \leq 3$$

$$= 5 + \frac{N_L}{10} \quad C > 3$$

in which

S_a = average beam spacing, in feet;

N_L = total number of design traffic lanes from Article 1.2.6;

N_g = number of longitudinal beams; and

C = a stiffness parameter that depends on the type of bridge, bridge and beam geometry, and material properties.

The value of C is to be calculated using the following relationships. However, for preliminary designs C can be approximated using the values given in Table 1.3.1. For beam and slab ¶ and multi-beam bridges,

$$C = \frac{W}{L} \left[\frac{E}{2G} \frac{I_1}{(J_1 + J_t)} \right]^{\frac{1}{2}}$$

For concrete box girder bridges:

$$C = \frac{1}{2} \frac{W}{L} \left(1 + N_g \sqrt{\frac{d}{W}} \right) \left[\frac{E}{2G(1 + N_d)} \right]^{\frac{1}{2}}$$

in which

W = the over-all width of the bridge, in feet;

L = span length, in feet (distance between live load points of inflection for continuous spans);

E = modulus of elasticity of the transformed beam section;

G = modulus of rigidity of the transformed beam section;

I_1 = flexural moment of inertia of the transformed beam section per unit width §;

J_1 = torsional moment of inertia of the transformed beam section per unit width § $\left(= J_{\text{beam}} + \frac{1}{2} J_{\text{slab}} \right)$;

J_t = $\frac{1}{2}$ of the torsional moment of inertia of a unit width § of bridge deck slab **;

and, for concrete box girder bridges:

d = depth of the bridge from center of top slab to center of bottom slab;

N_g = number of girder stems; and

N_d = number of interior diaphragms.

For concrete girder bridges, the cantilever dimension of any slab extending beyond the exterior girder shall preferably not exceed $S/2$.

When the outside roadway beam or stringer supports the sidewalk live load and impact, the allowable stress in the beam or stringer may be increased 25 percent for the combination of dead load, sidewalk live load, traffic live load, and impact.

¶ For noncomposite construction, the design moments may be distributed in proportion to the relative flexural stiffnesses of the beam and slab section.

§ For the deck slab and beams consisting of reinforced or prestressed concrete, the uncracked gross concrete section shall be used for rigidity calculations.

** For multi-beam bridges, the torsional moment of inertia shall be computed at the thinnest transverse cross section in the beams.

TABLE 1.3.1 VALUES OF K TO BE USED IN $C = K(W/L)$

BRIDGE TYPE	BEAM TYPE AND DECK MATERIAL	K
Beam and slab (includes concrete slab bridge)	Concrete deck:	
	Noncomposite steel I-beams	3.0
	Composite steel I-beams	4.8
	Nonvoided concrete beams (prestressed or reinforced)	3.5
	Separated concrete box-beams	1.8
Multi-beam	Concrete slab bridge	0.6
	Nonvoided rectangular beams	0.7
	Rectangular beams with circular voids	0.8
	Box section beams	1.0
	Channel beams	2.2
Concrete box girder	Without interior diaphragms	1.8
	With interior diaphragms	1.3

(2) Total Capacity of Stringers.

The combined design load capacity of all the beams in a span shall not be less than required to support the total live and dead load in the span.

(3) Edge Beams (Longitudinal).

Edge beams shall be provided for all concrete slab bridges having main reinforcement parallel to traffic. The beam may consist of a slab section additionally reinforced, a beam integral with and deeper than the slab, or an integral reinforced section of slab and curb.

It shall be designed to resist a live load moment of $0.10PS$,

where

P = wheel load, in pounds (P_{15} or P_{20}); and

S = span length, in feet.

This formula gives the simple-span moment. Values for continuous spans may be reduced 20 percent unless a greater reduction results from a more exact analysis.

CHAPTER ONE

INTRODUCTION

STATEMENT OF PROBLEM

This study was undertaken to develop a more realistic analysis of and to develop better design specifications for the distribution of live load in the floor systems of highway bridges. Numerous analytical and experimental studies have been made to help improve the methods used for highway bridge design; however, in some areas the studies have not resulted in realistic, yet simple, procedures for design. One of these areas is in highway bridge floor systems.

It has been suggested that the present specifications

(279), although giving satisfactory designs for service, are too conservative and limited in consideration of variables affecting behavior. They provide no satisfactory consideration of such important variables as the flexural and torsional stiffnesses of the floor slab and beams, the bridge span and the bridge width in the determination of the distribution of beam live loads. In addition, they do not provide consistent design criteria for all types of highway bridges. Thus, changes, where warranted, are recommended in the current specifications for distribution of wheel loads for use in design of floor systems for highway bridges.

The study outlined herein relies significantly on the theoretical methods and field test results of other investigators. These studies were used as the basis for the investigation. Modifications and extensions of the applicable theories were made so that the theories would be applicable to all bridge types considered. After correlation with the field test results, extensive analytical results were obtained relating all significant variables. From these results, proposals for appropriate specification changes have been developed and are presented.

STATE-OF-THE-ART

For more than 25 years numerous researchers have studied the behavior of bridge floor systems. Although most of these studies were limited to theoretical behavior, a significant number of field tests have been reported in the literature. An extensive bibliography of available references in both areas is given in Appendix B. An extensive report of the state-of-the-art of the analysis of most common bridge types has been presented by Reese (182). The succeeding paragraphs in this section briefly outline available theoretical and experimental studies.

The designs of the floor systems of highway bridges are quite varied and depend upon many factors. These varied designs, however, may be classified, based on their assumed behavior, into a few major categories. There are several types of structures that may be analyzed by the same theoretical methods, although their physical nature may be somewhat different. Slabs, plates, open grid frameworks, interconnected bridge girders, bridge decks and cellular plate structures, for example, may all be classified as grids. Nearly all of the floor systems of the many types of highway bridges fall, in one form or another, into one of the classifications. For this study, the various types of bridges have been classified, as shown in Figure 1, into three major categories—beam and slab type, multi-beam, and concrete box girder bridges.

Theoretical Analyses

Beam and Slab Bridges

Theoretical investigations of beam and slab bridges vary in their approach as well as in their accuracy and assumptions. The majority of the analytical approaches can be placed into the following four classifications:

1. Unit or plate analysis.
2. Redundant or grid analysis.
3. Combination of plate and grid analyses.
4. Specialized methods.

The unit method, commonly known as orthotropic plate analysis, replaces the actual structure with an equivalent orthogonally anisotropic plate (5, 65). This method is characterized by a relatively complex closed form solution. The equivalent plate has the same transverse and longitudinal torsional and flexural rigidities as the actual structure. Initial development of the orthotropic plate analysis as applied to bridge decks is due to the work of Guyon (67, 68) who found solutions for the limiting cases of torsional rigidity in the equivalent plate. His results are valid

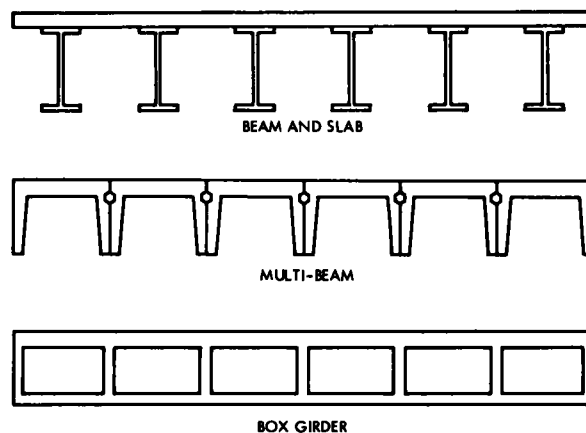


Figure 1. Intermediate-length highway bridge types.

for the no-torsion condition and the full-torsion or isotropic condition. Massonnet (129-133) extended the analysis to include intermediate values of torsional rigidity through the use of an interpolation formula. Both of these investigators assumed Poisson's ratio of the equivalent bridge materials to be zero. Rowe (195-199, 201) extended the analysis by providing for the inclusion of any value of Poisson's ratio. Another solution of the orthotropic plate equation was found by Sanders and Munse (210) and Roesli (191), who considered the applied load to be uniformly distributed over a small rectangular area. A third solution of the orthotropic plate equation has been proposed by Stein (235). In this case, singularity functions are used to represent Huber's orthotropic plate equation and the solutions found after transformations between the singularity and cartesian systems. Numerical solution of the plate equation has also been employed by various investigators. Notable are Heins and Looney (74, 75), who applied finite difference techniques to the plate equation for comparison with experimental results from tests on several different bridge types. Detailed reviews and analyses of the development of the orthotropic plate solution can be found in the references cited.

In the second general method of analysis, the actual bridge structure is replaced with an equivalent grid system. A direct solution of gridworks through the use of slope-deflection and compatibility equations has been developed by Homberg (84, 85). Lazarides (108, 122) has solved the gridwork problem by determining the deflection compatibility equations at each beam intersection and solving the resultant simultaneous equations. Numerical solutions have also been used for gridwork analysis by Leonhardt (109, 110) through the use of moment or torque distribution, while Scordelis (223) used shear distribution and Fader (50) used a reaction distribution method. The gridwork or redundant analysis usually involves a large number of simultaneous equations if solved exactly, or numerous arithmetic calculations if one of the numerical techniques is employed.

The third general analytical approach to the load distribution problem, a combination of the plate and redundant

procedures, is represented by two theories—harmonic analysis and numerical moment distribution.

The development of harmonic analysis as a technique for the determination of load distribution in highway bridges results from the work of Hendry and Jaeger (76-79). This procedure considers the same flexural and torsional rigidities as the orthotropic plate analysis, with the exception that torsional rigidity in the transverse direction is neglected. The harmonic analysis, however, requires the calculation of a number of constants, which are utilized in an infinite series summation. Preliminary calculations for the determination of these constants are somewhat lengthy. This procedure is also characterized by a relatively slow convergence of the series for the most highly stressed beam.

The second method of analysis that combines the plate and redundant member technique is the numerical moment distribution procedure developed by Newmark (152, 154-156) and Jensen (94). In this procedure the slab is first considered independent of the beams and is assumed to be isotropic. From boundary conditions and the Levy series expression for loading, a solution of the isotropic plate equation is found. A slab strip is now considered to be a beam continuous over flexible supports (the actual beams), and a Hardy Cross moment distribution procedure is carried out to determine moments in the actual beams.

The fourth general category of distribution procedures, specialized methods, contains widely differing approaches. An approximate gridwork solution was developed by Pippard and deWaele (171). This procedure requires the replacement of all transverse grid members by a single member at midspan with equivalent stiffness. This approximation results in fewer calculations than those required in the general gridwork solution. The beam on elastic foundation analogy has been proposed by Massonnet (131). Because of similarity between the plate equation when the torsional term is ignored and the beam on elastic foundation equation, the elastic foundation analogy can be used for bridges with little torsional rigidity with only a small error if the equivalent plate is assumed to have zero torsional rigidity. If a bridge system has few longitudinal beams and if transverse beams or diaphragms are ignored, another approach can be used by considering the bridge to be a complex beam. This analysis is relatively simple but has limited applicability. In fact, all of the approximate methods can be applied with reasonable accuracy to very specific beam and slab types, but chance of potential error is greatly magnified when these methods are applied to the general beam and slab bridge.

Multi-Beam Bridges

The number of methods available for the analysis of multi-beam bridges is somewhat more limited. About 10 years ago, Duberg, Khachaturian and Fradinger (45) analyzed a multi-beam bridge by assuming that it consisted of beam elements placed side by side and connected to each other along the span by hinges at the corners of the cross section at the level of the top fiber. Other investigators (80, 171) have made similar assumptions, such as:

1. No rotation of individual members at their intersections with other members.
2. Floor system prevents twist of main girders.
3. Cross girders are replaced by a continuous connecting system.

The behavior of multi-beam bridges is in many respects similar to that of the beam and slab bridges. The major difference is the elimination of the moment restraint between the individual beam units, which leads to some modifications in the applicable theories. The methods of analyses can be divided into two major categories. The first category is normally called the method of compatible deformation based on the flexibility method. The second category can be classified as a plate theory.

The first step involved in the first category is to consider the equilibrium of the mechanical system and express various mechanical quantities, such as deflection and bending moments, in terms of certain unknown forces acting on the system. The solutions are obtained by considering the compatibility conditions of the system; subsequently, the last step is to solve simultaneous linear equations for these unknown forces. Arya (6) and Pool (100, 172, 173) used this method of compatible deformation to analyze multi-beam bridges.

The second category assumes that the number of beam elements is large enough for the real structure to be replaced by an idealized plate with continuous properties so that differential calculus can be applied. The plate theory can be divided into several methods. One method assumes no flexural rigidity in the transverse direction of the bridge because of the discontinuities at the shear keys. On the other hand, another method would allow some flexural rigidity in the transverse direction, taking into account the effect of transverse prestress force and some continuity even at the location of shear keys. The first method is usually known as articulated plate theory (261); the latter is termed orthotropic plate theory, which was first studied by Guyon and Massonnet and has been extensively used in the analysis of beam and slab bridges, as previously mentioned. Roesli (189), Nasser (151), and Pama (38, 165, 166) used these theories to analyze multi-beam bridges.

Concrete Box Girder Bridges

Numerous analyses of concrete box girder bridges have been carried out by Scordelis (221, 222, 224). The method of analysis used was based on a direct stiffness solution of a folded-plate harmonic analysis based on an elasticity method (41). Scordelis used elastic plate theory for loads normal to the plane of the plates and two-dimensional plane stress theory for loads in the plane of the plates. This is the only method of analysis used extensively for this bridge type.

Field Test Investigations

There have been a number of field tests of the types of bridges considered in this study. However, most of these tests were conducted on beam and slab bridges. The most extensive single effort of field testing was conducted at the AASHO Test Road (278). Eighteen bridges of the four

general beam and slab bridge types were tested. These types were:

1. Noncomposite steel wide-flange beam bridges.
2. Composite steel wide-flange beam bridges.
3. Reinforced concrete beam bridges.
4. Prestressed concrete beam bridges.

In addition, numerous field tests of this bridge type have been reported in the literature. A summary of these tests performed up to 1965 has been prepared by Varney and Galambos (251). Numerous tests have been conducted since that time on beam and slab bridges. These include a series of tests of box beam bridges by Van Horn *et al.* (44, 62, 63, 113) and three tests in Maryland by Reilly and Looney (183). A summary of a number of these tests of beam and slab bridges has been prepared by Arendts (5).

As indicated previously, the number of tests of multi-beam and concrete box girder bridges is limited. Only three full-scale tests of the type of multi-beam bridges studied herein are reported (23, 202, 204). The first test (23) was conducted on a bridge consisting of channel sections; the second (204) on a bridge with solid sections with holes; and the third (202, 204) on a bridge composed of solid sections. The latter two tests were conducted in England. All of the tests of concrete box girder bridges have been conducted on bridges constructed by the California Department of Highways. The only field test reported to date was conducted by Davis, Kozak and Scheffey (39, 222) on the Harrison Street Undercrossing in Oakland, Calif.

Although limited in some cases, the number of tests and the types of bridges studied in the field tests are sufficient to verify the applicability of the theories used to predict the behavior of the particular bridge types included in this investigation.

SCOPE OF INVESTIGATION

The determination of beam bending moments in highway bridges requires a design procedure to predict with reasonable accuracy the maximum beam moment produced by a standard loading. This design procedure should be governed by bridge behavior characteristics or parameters that reflect the bridge behavior. The development of this procedure and of the recommendations for changes in the current AASHTO Specifications (279) were based on:

1. A thorough bibliographic search (Appendix B) for all available studies into the theoretical and experimental behavior of highway bridges.
2. The study of these references to determine the theoretical procedures most applicable to the bridges included in the study scope. These theoretical procedures were then used to predict the behavior of field-tested bridges. A comparison of these results with those actually obtained in the field tests was used to verify the applicability of the procedures. The comparisons are discussed in detail in Chapter Two.
3. An extensive study of the effect of the variation in the parameters affecting the wheel load distribution. The procedures selected were used to determine the maximum beam bending moments due to numerous possible loading condi-

tions. The results of this analytical study are given in Chapter Three.

4. The simplification of these results into a form that would still be readily usable in the design office, yet give sufficient accuracy in the prediction of load distribution. Where the difference in accuracy between these procedures and the current specification was felt to warrant changes, recommendations for new criteria were then made. The details of this simplification and the resulting recommendations are given in Chapters Four and Five.

After a thorough study of the theoretical procedures found in the bibliographic search, procedures were selected for use in the analytical studies which were felt to best predict the behavior of those bridges included in this investigation. For beam and slab bridges, the orthotropic plate theory and harmonic analysis were selected; for multi-beam bridges, the articulated plate theory; and for concrete box girder bridges, the theory of prismatic folded-plate structures. Numerous other procedures were considered and, although applicable to specific bridge geometries, it was felt that those selected were more generally applicable.

Because of the existence of a wide variety of highway bridge geometries, some bridge geometrical restrictions were specified to limit the scope of the study. The bridges studied conform to the following geometrical conditions:

1. The longitudinal axis of the bridge is at right angles to the piers or abutments.
2. The bridge spans between adjacent piers or abutments are simple or noncontinuous, although the effects of bridge continuity are considered based on other investigations.
3. The spans are of short or intermediate length (20 to 130 ft).

In addition to these constructional and geometric conditions, the study of the bridges was restricted to statically applied live loads only. This loading condition requires that the test load vehicles in the field tests be either stopped on the bridge or moving at creep speeds (less than about 5 mph) while measurements were in progress. Furthermore, the consideration of beam and slab type bridges with composite wood-concrete members or timber stringers and orthotropic plate deck type bridges were not within the scope of the study. Even though these constructional and geometric conditions may seem quite restrictive, these conditions will be satisfied for the construction and design of the majority of actual highway bridges.

Major variables or geometrical parameters considered in the study included: torsional and flexural stiffness of the beams, deck, and diaphragms; the width of the bridge; the roadway width; the span of the bridge; and the number and position of design traffic lanes. In addition, although the stiffness of the floor (such as the concrete slab or steel grid) was considered in the distribution of wheel loads in beam and slab bridges, the actual design of the floor was not considered. The details of these parameters and the theories are presented in Chapter Three.

Beam and slab, multi-beam, and box girder highway bridges are classified as different type bridges due to their differences in construction and structural behavior under load. Figure 1 shows the differences in construction.

Beam and slab bridge construction is characterized by separated longitudinal beams which support a deck slab. The beams, as shown in Figure 2, can vary in material as well as construction. Steel beams may be rolled shapes or plate girders and may have either a composite or non-composite deck slab. If the beams are prestressed concrete, composite action is generally provided for through shear connectors. Prestressed concrete beams are usually precast as I shapes; but other beam shapes are possible, such as T shapes where the beams are cast monolithically with a portion of the deck slab. Also, in reinforced concrete beam bridges, the beam shape is considered as the T formed of the beam stem and a portion of the slab. In fact, when any beam and slab bridge is compositely constructed, a portion of the slab is always considered to be a part of the beam.

Multi-beam bridges consist of several longitudinal beams placed side by side. The beams are usually precast prestressed concrete and are connected by longitudinal shear keys. In addition, the beams are usually tied together by post-stressed transverse steel cables. Although transverse prestressing may be present, it may not be of sufficient magnitude to provide transverse continuity through the loading spectrum. Beam shapes vary, but a common configuration is the concrete channel beams shown in Figures 1 and 3. Nonvoided rectangular, tee, and voided or hollow rectangular beam shapes, as shown in Figure 3, are also common.

Box girder bridges are usually made of monolithically-cast reinforced concrete, but a recent method of construc-

tion combines light-gauge steel box sections with a composite concrete deck. The reinforced concrete box girder bridge shown in Figure 1 is constructed of two continuous flanges with monolithic vertical webs. Separated box-beam and slab bridges should not be confused with concrete box girder bridges. The composite steel concrete box girder bridges are characterized by a separation of the steel boxes and are thus, in reality, beam and slab type bridges.

Structural behavior is important to the classification of beam and slab, multi-beam, and concrete box girder bridges. Both beam and slab and multi-beam bridges can be represented by an equivalent plate, but the structural models representing these plates differ. The principal difference is the ability of the bridge or equivalent plate to transmit bending moment in the transverse direction. Beam and slab bridges are flexurally continuous in the transverse direction due to the ability of the deck slab and transverse beam or diaphragm to transmit bending moment. On the other hand, the shear keys connecting the individual beams of a multi-beam bridge act as hinges. Therefore, transverse flexural continuity is not present in multi-beam bridges and the equivalent plate must be treated differently from the equivalent orthotropic plate that represents the beam and slab bridge. Concrete box girder bridges differ from the previous two bridge types in that procedures are not currently available for theoretically representing the entire structural system as a single equivalent plate. Each plate element in the concrete box girder bridge can be treated individually by using folded-plate analysis or a similar procedure. This does not, however, mean that approximate design methods could not be developed for this type of bridge.

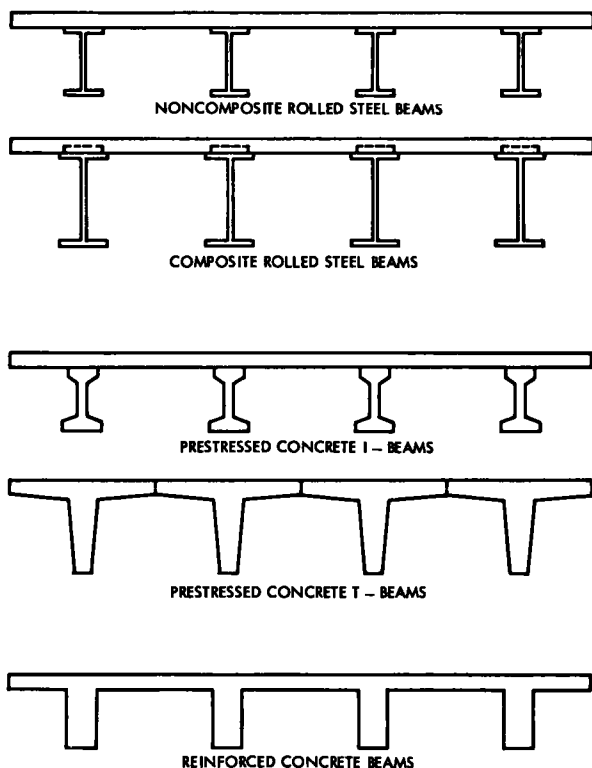


Figure 2. Beam and slab highway bridge types.

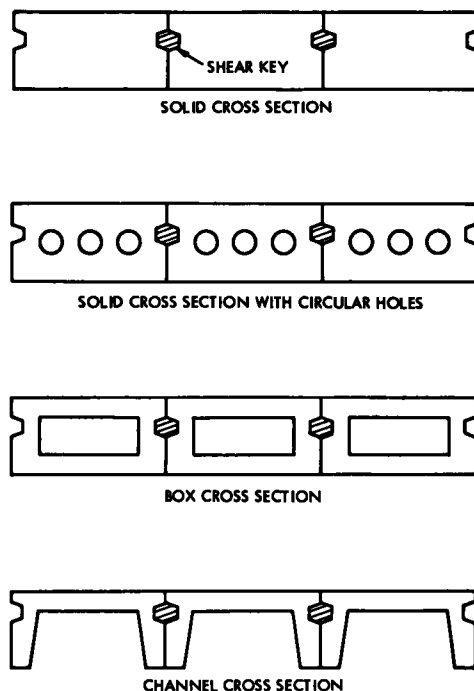


Figure 3. Multi-beam highway bridge types.

CHAPTER TWO

COMPARISON OF FIELD TEST RESULTS WITH THEORETICAL STUDIES

GENERAL

The validity of the use of any theoretical procedure for predicting load distribution characteristics can be determined by comparing the results obtained from field tests of bridges to similar results as predicted by the theory. The research outlined herein was conducted for that purpose. The results show the validity of the theories selected. In this investigation the procedures used for determining the distribution in beam and slab bridges (5) and in multi-beam bridges (261) were studied. However, the method of analysis used for studying concrete box girder bridges was not used to investigate actual bridges because of the verification provided by Scordelis in the procedure development (222, 224).

A literature search has indicated that existing work dealing with load distribution in beam and slab and multi-beam highway bridges can be separated into two categories—reports of experimental investigations on prototype and model bridge structures, and theoretical investigations on idealized structures. Although load distribution tests have been conducted on both model and actual highway bridges, only experimental research dealing with prototype bridge structures was considered for verifying the procedures for actual conditions. Much of the experimental work stems from dynamic studies of highway bridges. Only the results obtained for calibration of these bridges were considered, because results usable for predicting static load distribution are obtained only at static or creep speeds (0 to 5 mph). Another important but limited source of field test data is reports dealing solely with static or creep loading.

It can be seen in the bibliography that there have been numerous field tests of beam and slab bridges reported in the literature. These studies of highway bridges can be categorized according to the type of supporting beams. The most numerous experimental reports deal with bridges with reinforced concrete deck slabs supported by steel beams. The beam and slab system is constructed as either composite (connected by shear transfer devices) or noncomposite. Prestressed concrete beams composite with a reinforced concrete slab form the second type of bridge studied. Reinforced concrete beams monolithic with a concrete deck is the third type of bridge studied experimentally. In all beam and slab bridge types, transverse beams, bulkheads, cross-bracing, or diaphragms are usually present. A detailed study was conducted of eleven bridges covering all types of the beam and slab bridges listed. However, in this summary report, only results of three typical bridges are discussed in detail. The studies of the remaining bridges are discussed by Arendts (5).

The number of reported tests on multi-beam bridges is very limited. In addition, some of the reported results are for bridges with substantial skew and, thus, are not of significant value. In verifying the validity of the theoretical

procedure used, the results of four test bridges were analyzed (261). The study of three of these is presented herein to indicate the general trend of the results.

In each case, the theories proposed for the type of bridge being studied were used to determine the moments in each beam element for the particular loading on the bridge. The results of these analyses were then compared with the results of the field test to determine the validity of the procedure in predicting actual behavior. The comparisons are shown using moment or deflection distribution coefficients; i.e., the individual beam moment or deflection divided by the average moment or deflection. These coefficients were used to normalize the plots for ease in comparison.

BEAM AND SLAB BRIDGES

The studies of the three bridges discussed in detail cover the cross section of bridge types generally constructed. The bridge types included are as follows:

1. A prestressed concrete T-beam bridge with the top flanges forming the roadway.
2. A two-lane composite concrete deck and rolled steel I-beam bridge.
3. A simple-span structure consisting of a reinforced concrete slab composite with box-section prestressed concrete longitudinal girders.

The over-all investigation (5) also included four of the beam and slab bridges constructed and tested as part of the AASHO Test Road (278). One bridge of each of the four generally designed types was studied: 9-A, a noncomposite steel wide-flange beam bridge; 2-B, a composite steel wide-flange beam bridge; 7-A, a reinforced concrete beam bridge; and 5-A, a prestressed concrete beam bridge. In addition, four other steel beam bridges (15, 82, 101, 103) were considered. These included:

1. A 41-ft composite slab and beam, simple-span bridge.
2. Two separate, but identical, 67.5-ft bridge spans from multi-simple-span structures, each consisting of four longitudinal girders composite with a concrete slab.
3. A 45-ft simple-span portion of a four-span system composed of four rolled beams supporting a noncomposite concrete slab.

The distribution of moments in each of these bridges was analyzed using both orthotropic plate theory and harmonic analysis. These two theories were selected because of their application to the broad range of beam and slab bridges and the availability of generalized functions to predict behavior. The application of these theories to actual test bridges assumed that the properties of the cross sections conformed to the following assumptions of the theories:

1. The bridge is rectangular in plan.
2. All beams and diaphragms are evenly spaced.
3. All beams are of equal stiffness.
4. All diaphragms are of equal stiffness.
5. All beams and diaphragms are prismatic.
6. The deck slab does not contain joints or hinges.
7. The bridge behaves elastically.

In addition, for the particular solutions of the governing equations, the following support conditions were also assumed: two opposite edges are simply supported and the other two edges are free. The development of the two theories is presented in Chapter Three. However, the relationship between the theoretical and experimental results is presented in this chapter to show the validity of the theories in predicting load distribution. The procedures used for calculating the geometrical parameters used in the theories are given in Appendix A.

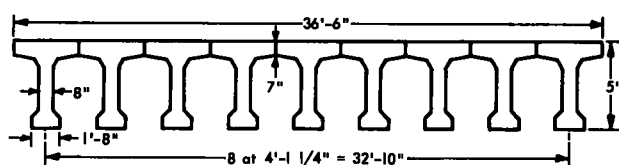
The general trend of the results can be seen by examining in some detail the comparisons of theoretical and experimental behavior for three of the bridges studied. The results are typical of all bridges studied.

Shawan Road Bridge

The Shawan Road Bridge (183), as shown in Figure 4, was built of nine prestressed concrete T beams placed side by side so that the top flanges form the roadway. This structure is not a multi-beam bridge due to the presence of full transverse prestressing cables in the top flanges of the beams and through the diaphragms. The 36.5-ft-wide bridge spans 100 ft. Interior diaphragms are located at the quarter-span points; the diaphragms are monolithic portions of the longitudinal girders and are post-stressed together.

The results of the analysis, shown in Figure 5, indicate good agreement between both theoretical procedures and the moments obtained from strain readings on the actual bridge. In the vicinity of the load the harmonic analysis does, however, predict moments about 20 percent higher than the experimental moment for the central loading, but about 10 percent lower for the eccentric loading.

In addition to the wheel positions shown in Figure 5, two other tests with eccentric loadings were conducted. Combining the results of one of these with those presented in Figure 5, a loading pattern similar to that expected from the current AASHO loading criteria can be obtained. In this case, the maximum moment coefficient (i.e., the ratio of the individual beam moment to the average beam moment)



NOTE: EXTENSIVE TRANSVERSE PRESTRESSING IN EFFECT MAKES TOP FLANGES ACT AS CONTINUOUS TRANSVERSE SLAB

Figure 4. Cross section of Shawan Road prestressed concrete beam bridge.

was 1.232 and occurred in the outside girder. The orthotropic plate theory predicted a coefficient of 1.282 (a +4.5 percent error) and the harmonic analysis one of 1.178 (a -4.4 percent error). The current AASHO distribution formula would have predicted a coefficient of 1.680 for an interior girder, or 35 percent higher than that actually obtained from a loading similar to that expected from the specifications. It should be noted that because of the positions of the test loadings on this bridge, exterior beam moments predicted by the current specifications would not be applicable.

Holcomb Test Bridge

The Holcomb Test Bridge (32), located in Ames, Iowa, is a two-lane 71-ft bridge composed of four rolled beams supporting compositely an 8-in. concrete slab. The details of the bridge cross section are shown in Figure 6. The 16WF36 transverse interior diaphragms are located at the third-span points.

The results of the comparative analyses are shown in Figure 7. It can be seen that for the central loading the two theories compare favorably with the experimental results. However, for the wheel loads in an eccentric position, the orthotropic plate theory predicts the behavior, whereas the harmonic analysis underpredicts the maximum beam moment by 21 percent (moment distribution coefficient of 1.920 vs 1.518). The results from the orthotropic plate theory are shown on both a per-foot basis and a per-beam basis. The per-beam coefficient is simply obtained by integrating the area under the distribution coefficient per-foot curve over a width of half the distance to each adjacent beam and normalizing the answer.

Five additional tests were conducted—two with a single truck and the other three with two trucks for the loading. The results for these tests were similar, with both theories predicting behavior favorably for the central loadings, whereas for eccentric loadings the harmonic analysis significantly underpredicts the maximum beam moment.

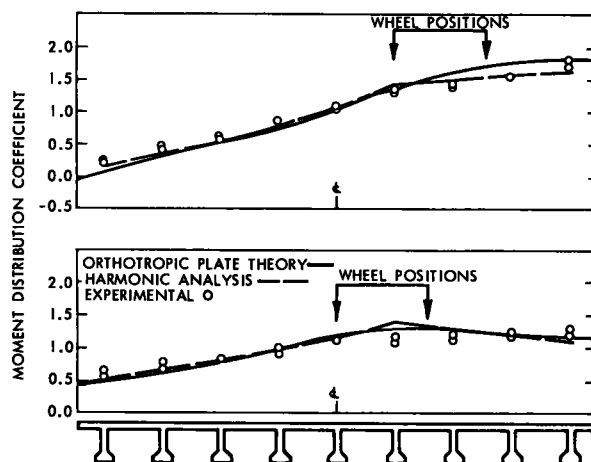


Figure 5. Transverse moment distribution in Shawan Road Bridge.

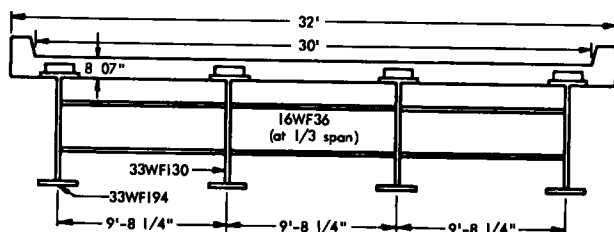


Figure 6. Cross section of Holcomb field test bridge.

For the loading that corresponds most closely to the AASHTO loading, the maximum moment distribution coefficient obtained from the field test was 1.490 for an exterior beam. Using the current AASHTO procedure the AASHTO coefficient would be 1.761 for an interior girder, whereas from the orthotropic plate theory the maximum coefficient was 1.554, and from the harmonic analysis it was 1.278. It can be seen that the results of the AASHTO procedures are 18 percent higher than the field test result and the harmonic analysis is 14 percent lower, but the orthotropic plate theory accurately predicts the maximum moment. The AASHTO coefficient for an interior girder was used because it was larger than that of the exterior girder and all girders are similar.

Dreher'sville Bridge

The Dreher'sville Bridge (44) is a simple-span structure consisting of a 6.7-in. reinforced concrete slab composite with five identical box-section prestressed concrete longitudinal girders. The over-all width of this structure, as shown in Figure 8, is 35.5 ft; the span is 61.5 ft. Each beam is 33 in. deep by 48 in. wide and consists of 5-in. vertical and bottom walls with a 3-in. top wall. A 10-in. thick cast-in-place transverse diaphragm is located at midspan.

The results of the comparisons are shown in Figure 9. For both the central loading and the eccentric loading, the coefficients predicted by both theories are in good agreement with the field test results. However, for the combination of truck loads that most nearly conform to the AASHTO loading, the maximum moment distribution coefficient from the field tests was 14 percent less than that predicted by the current specifications, but 10 percent above those from the two theories.

In the studies presented herein, three major beam and slab bridge types are represented by comparisons of experimental results with analytical predictions of beam moments. In addition, similar results were obtained from

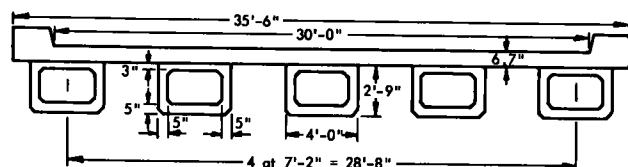


Figure 8. Cross section of Dreher'sville Bridge.

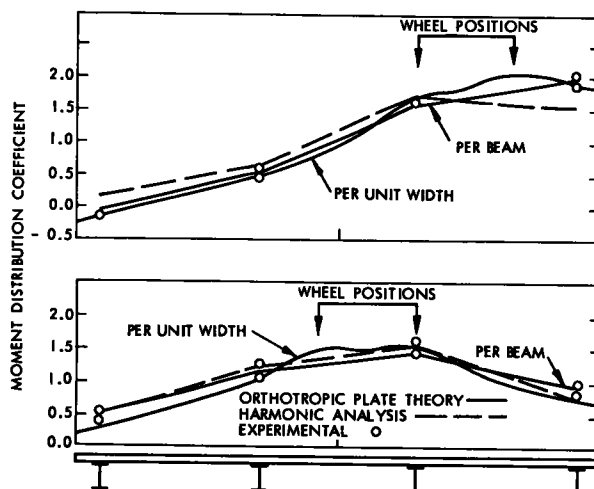


Figure 7. Transverse moment distribution in Holcomb test bridge.

comparison for eight other bridges consisting of either steel beam and concrete deck bridges or nonvoided prestressed and reinforced concrete beam and concrete deck bridges (5). A summary of the bridge properties is given in Table 1. A detailed summary of the comparisons between the maximum moment coefficients predicted by the theories and from field test results is given in Table 2. Where possible, the moment coefficients obtained using the current specifications (279) are also given. When all maximum loading conditions for all test bridges are examined, the following conclusions are found:

1. Of the total of 18 maximum beam moment cases considered, orthotropic plate theory predicted 11 moments conservatively (positive error) and harmonic analysis predicted 5 moments conservatively. The conservative har-

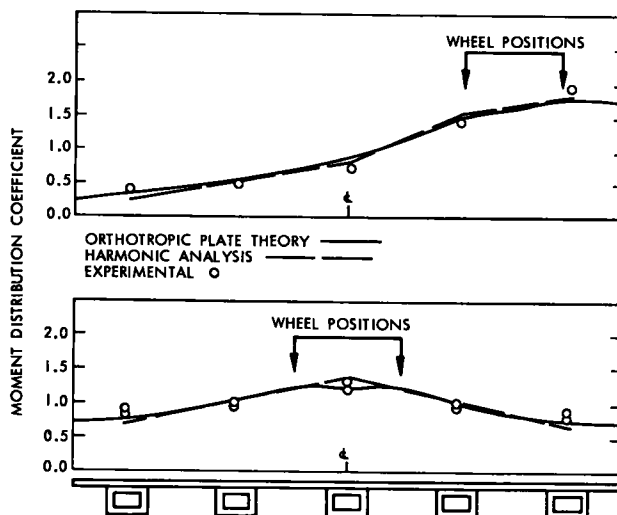


Figure 9. Transverse moment distribution in Dreher'sville Bridge.

TABLE 1
DESCRIPTION OF BEAM AND SLAB FIELD TEST BRIDGES

BRIDGE					
NO.	NAME	SPAN (FT)	OVER-ALL WIDTH (FT)	BEAMS	REF.
				NO. AND TYPE	
1	71-ft Holcomb test	71.25	32	4 Composite WF	82
2	41-ft Holcomb test	41.25	32	4 Composite WF	82
3	Weyers Cave	67.5	28	4 Composite WF	101
4	Hazel River	67.5	28	4 Composite WF	103
5	Burris Fork	45	24.67	4 Noncomposite WF	15
6	AASHO Road Test 5-A	50	15	3 Prestr. conc. I	278
7	AASHO Road Test 7-A	50	15	3 Reinf. conc. T	278
8	AASHO Road Test 2-B	50	15	3 Composite WF	278
9	AASHO Road Test 9-A	50	15	3 Noncomposite WF	278
10	Drehersville	61.5	35.5	5 Separated prestr. conc. box	44
11	Shawan Road	100	36.5	9 Prestr. conc. T	183

monic analysis predictions were all within 10 percent of the field test results; all orthotropic plate predictions were in error by less than 10 percent except two, which were 12 and 28 percent in error.

2. Harmonic analysis predicted 13 unconservative moments (negative errors), of which 7 errors were between 10 and 20 percent, with the remaining errors less than 10 percent. Orthotropic plate theory predicted 7 unconservative moments when compared to the field test results, with all less than about 10 percent in error.

It can be seen from this summary that harmonic analysis predicted unconservative maximum moments more frequently with errors of greater magnitude. The converse is true of the conservative results. Table 3 gives the maximum moment error for all bridges.

From the comparisons presented herein and those also presented by Arendts (5), it can be seen that the shape of the predicted moment coefficient curves from orthotropic plate theory is in close agreement with the experimental distributions. However, not only are some of the harmonic analysis distribution curves not consistent with the experimental distributions, but also all the maximum moments occurred at interior girders, although test results place the maximum moment at the exterior girder. In fact, most of the harmonic analysis comparisons tended to underpredict the exterior beam moments, especially for loads with large eccentricities.

From these results, it is felt that the orthotropic plate theory is the more accurate of the two theories considered. This conclusion is based on the accuracy of the prediction of the maximum beam moment, the beam location of the

maximum moment, and the general distribution curve. Furthermore, the procedures used for determining the stiffness parameters, as presented in Appendix A, are felt to accurately represent the behavior of the types of beam and slab bridges studied. It is obvious that both an accurate analytical procedure and an accurate method of computing stiffness parameters are necessary for satisfactory comparisons. The comparisons of experimental results and theoretical predictions support this conclusion. However, it is felt that the harmonic analysis is still a valuable tool to use as a check for the validity of the orthotropic plate theory in ranges of variables beyond those considered in the field test comparisons.

The results of the comparisons of the current AASHO distribution procedure with the individual test results (Table 2) and the summary in Table 3 show that the current procedures are inconsistent with experimental results. The summary comparisons show that the average errors were +16 percent for single truck loads and +25 percent for two truck loads superimposed when the maximum AASHO coefficient was considered. However, for individual bridges, the difference between the maximum values predicted by the current specifications (279) and the maximum experimental value ranged from +1.1 to +35.0 percent. If the distribution coefficient is computed by the AASHO procedure for the girder from which the maximum experimental coefficient was determined, the difference ranged from -17.7 to +35.0 percent. These results show that a new design procedure is needed to more accurately predict load distribution by considering more fully the over-all behavior.

TABLE 2

COMPARISON OF MAXIMUM BEAM MOMENT COEFFICIENTS WITH EXPERIMENTAL RESULTS

BRIDGE NO.	LOADING ^a	MAXIMUM MOMENT COEFFICIENT						
		EXPERIMENTAL (1)	O.P.T. (2)	% DIFF., (1) AND (2)	H.A. (3)	% DIFF., (1) AND (3)	AASHTO (4)	% DIFF., (1) AND (4)
1	Single truck, max. eccentricity	2.530	2.614	+3.3	2.080	-17.8	—	—
	Single truck, int. eccentricity	1.920	1.961	+2.1	1.518	-21.0	—	—
	Single truck, central	1.530	1.453	-5.0	1.576	+3.0	—	—
	2 Trucks measured, max. eccentricity	1.685	1.721	+2.1	1.379	-17.8	1.507 ^b	-10.6
	2 Trucks measured, int. eccentricity	1.520	1.544	+1.6	1.245	-18.1	1.761 ^c	+4.5
	2 Trucks measured, central	1.375	1.340	-2.5	1.440	+4.7	1.507 ^b	-0.9
	2 Trucks measured, central	1.375	1.340	-2.5	1.440	+4.7	1.761 ^c	+15.8
	2 Trucks, superimposed loads	1.490	1.554	+4.3	1.278	-14.2	1.761 ^{b, c}	+28.1
2	Single truck, max. eccentricity	2.540	2.684	+5.7	2.094	-17.6	1.507 ^b	+1.1
	Single truck, int. eccentricity	1.785	1.626	-8.9	1.859	+4.1	1.761 ^c	+18.2
	Single truck, central	1.850	1.613	-12.8	1.801	-2.6	—	—
	2 Trucks measured, max. eccentricity	1.600	1.669	+4.3	1.290	-19.4	1.507 ^b	-5.8
	2 Trucks measured, int. eccentricity	1.565	1.444	-7.7	1.603	+2.4	1.761 ^c	+10.1
	2 Trucks measured, central	1.490	1.423	-4.5	1.588	+6.6	1.761 ^{b, c}	+12.5
	2 Trucks measured, central	1.490	1.423	-4.5	1.588	+6.6	1.761 ^{b, c}	+18.2
	2 Trucks, superimposed loads	1.382	1.530	+10.7	1.200	-13.2	1.507 ^b	+9.0
3	Single truck, eccentric load	1.815	1.975	+8.8	1.736	-4.4	1.761 ^c	+27.4
	Single truck, central load	1.320	1.207	-8.6	1.260	-4.5	—	—
4	Single truck, max. eccentricity	1.690	2.164	+28.0	1.806	+6.9	—	—
	Single truck, int. eccentricity	1.440	1.429	-0.8	1.496	+3.9	—	—
	Single truck, central load	1.260	1.211	-3.9	1.257	-0.2	—	—
	2 Trucks, superimposed loads	1.152	1.138	-1.2	1.182	+2.6	1.395 ^{b, c}	+21.1
5	Single truck, eccentric load	1.816	1.866	+2.8	1.632	-10.1	—	—
	Single truck, central load	1.355	1.265	-6.6	1.350	-0.4	—	—
6	Single truck, eccentric load	1.404	1.285	-8.5	1.371	-2.3	1.500 ^{b, c}	+6.8
7	Single truck, eccentric load	1.216	1.201	-1.2	1.342	+10.4	1.500 ^{b, c}	+23.4
8	Single truck, eccentric load	1.293	1.268	-1.9	1.410	+9.4	1.500 ^{b, c}	+16.0
9	Single truck, eccentric load	1.259	1.225	-2.7	1.356	+7.7	1.500 ^{b, c}	+19.1
10	Single truck, max. eccentricity	1.918	1.746	-10.0	1.811	-5.6	—	—
	Single truck, int. eccentricity	1.540	1.405	-8.8	1.338	-13.0	—	—
	Single truck, central	1.280	1.295	+1.2	1.387	+8.4	—	—
	2 Trucks, superimposed loads	1.306	1.173	-10.2	1.186	-9.2	1.075 ^b	-17.7
11	Single truck, max. eccentricity	1.855	2.077	+12.0	1.840	-0.8	1.630 ^c	+24.8
	Single truck, int. eccentricity	1.705	1.739	+2.0	1.570	-7.9	—	—
	Single truck, central	1.235	1.168	-5.4	1.100	-10.9	—	—
	2 Trucks, superimposed loads	1.232	1.282	+4.5	1.178	-4.4	1.680 ^{b, c}	+35.0

^a int = intermediate.^b For girder with maximum measured experimental value.^c Maximum predicted by ASSHO Specifications (279), either interior or exterior girder.

TABLE 3
AVERAGE ERRORS^a IN MAXIMUM BEAM MOMENTS

	ORTHOTROPIC PLATE THEORY		HARMONIC ANALYSIS		AASHO ^b	
	POS. ERROR	NEG. ERROR	POS. ERROR	NEG. ERROR	POS. ERROR	NEG. ERROR
(a) SINGLE TRUCK LOADS						
Avg. error	10.6	4.7	8.6	8.4	16.3	—
Tests run ^c	6	5	4	7	4	0
(b) TWO TRUCK LOADS SUPERIMPOSED						
Avg. error	6.5	5.7	2.6	10.3	25.3	—
Tests run ^c	3	2	1	4	5	0
(c) TWO TRUCK LOADS AS MEASURED						
Avg. error	3.2	—	—	18.6	7.3	—
Tests run ^c	2	0	0	2	2	0

^a For all beam and slab bridges analyzed.

^b Based on maximum values.

^c Maximum case for each bridge (interior or exterior girder).

MULTI-BEAM BRIDGES

The number of field tests and large-scale laboratory tests of multi-beam bridges is limited. After an extensive literature search, reports of only four such tests that were applicable to this study were located. These are summarized in Table 4. It can be seen that, although the number of tests is small, the types of cross sections investigated do include the three most commonly used multi-beam systems. The results of the tests of the three actual bridges are discussed in some detail to show the accuracy of the theoretical procedure in predicting behavior. A complete analysis of all four tests was presented by Watanabe (261).

The behavior of each of these bridges was predicted using the articulated plate theory. This theory has also been used by other investigators to analyze similar bridges (6, 7, 8). It was selected initially because of the good relationship of the assumptions in the analysis with the structural geometry and, also, because of the similarity of parameters with the plate theory and harmonic analysis considered for beam and slab bridges. A detailed outline of the theory is given in Chapter Three. The procedures used in computing the geometrical parameters are presented in Appendix A.

The validity of the proposed procedures can be seen by examining the results of the comparison of the moment distribution coefficients obtained from the theory and parameter calculations and from the field test results for each of the three bridges.

North Carolina Bridge

The North Carolina test bridge (23) is composed of 10 precast, prestressed concrete, channel beam elements. The cross section of the bridge is shown in Figure 10. The shear connection consists of a tongue-and-groove-type key, triangular in shape, which was packed from the top with a jute fiber and grouted to prevent the asphalt seal from

entering the joints. The interior members are channel sections and prestressed longitudinally with five cables of $\frac{7}{16}$ -in. diameter in each stem; the exterior members were constructed by casting a curb to an interior beam element.

The comparison of the distribution coefficients from the theory with those from the field test results is shown in Figure 11. It should be noted that because of damage to the strain gauges during loading, the distribution coefficients from the field test are based on the deflection gauge readings. The theoretical distribution is, however, based on the beam moments. The field test experiment showed that the change of prestress force significantly affects the distribution of wheel loads, as can be seen from the widely scattered experimental values. In addition, the prestress force reduces the coefficient near the loading points. When the average experimental values are considered, the results have reasonably good correlation with the theory.

Centerport Bridge

The Centerport test bridge (204), composed of nine precast prefabricated beam elements, has a clear span of 32 ft and a width of 27 ft. These beam elements were placed side by side, as shown in Figure 12, and connected by dry-packed mortar and a steel bolt at midspan. Each beam element had a cross section of 36 × 21 in. and an over-all length of 35 ft 6 in. The rectangular cross section had two hollow circular cores of 12½-in. diameter.

The results of the comparison of theoretical and experimental moment distribution coefficients are shown in Figure 13. The maximum deflection coefficients by experiment turned out to be roughly 10 to 20 percent higher in average than the theoretical deflection values. Also, the range of the experimental coefficients at or near the loading positions was roughly 20 percent of their average ordinates. However, the experimental coefficients were in good

TABLE 4

DIMENSIONS AND CHARACTERISTICS OF THE MULTI-BEAM BRIDGES TESTED

NAME	SPAN LENGTH (FT-IN.)	NO. OF BEAMS	BRIDGE WIDTH (FT-IN.)	CROSS SECTION	SHEAR KEY	TRANSVERSE PRESTRESS	SCALE	LOADING SYSTEM	MEASUREMENT
North Carolina	30-0	10	25-6	Channel PC beams; curb, 9 in.	Mortar	At 7 locations, up to 18,900 psi each	Full	22FG Corbett truck; 18.72 t/truck	120 SR-4 strain gauges; deflection dials of 0.001-in. least reading
Centerport	32-0	9	27-0	Solid RC beams with 2 circular holes; curb, 8 in.	Dry-packed mortar	No prestressing in transverse direction; a 2-in. ϕ tie rod at c-l.	Full	Scale truck and tractor trailer truck and hydraulic jacks	Control gauges; level bar readings
Langstone	31-0	16	34-0	Solid PC beams; curb + 5'-5" footpath	Dry-packed mortar	0.2-in. ϕ Freysinet cables at 12 points	Full	Two-bogie vehicle total load: central loading 20.8—90t; eccentric loading 60.7—90t	39 Ames dial gauges of 0.001-in. least reading; level bars; strain gauges
Lab. test by Best	17-10	12	11-10	Solid PC beams	Mild-steel shear loops	None	1/4	Hydraulic jack: pads, 3 3/4 \times 1 1/2 in., up to 18 tons	6 dial gauges; 8-ft Demec strain gauges

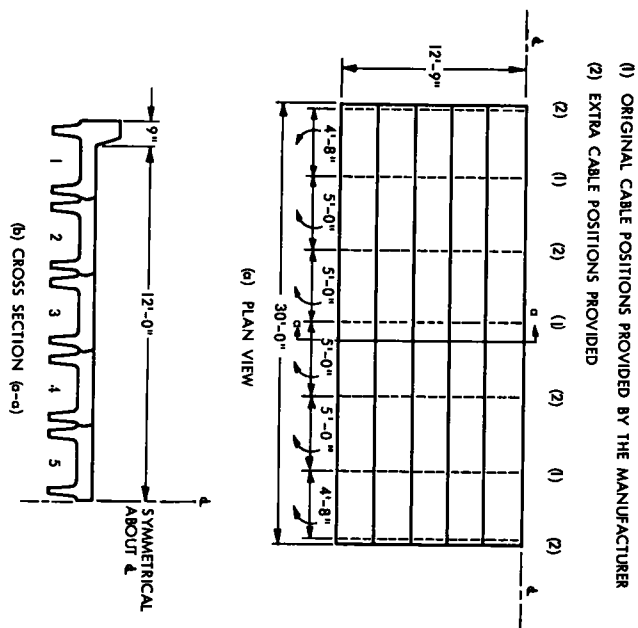
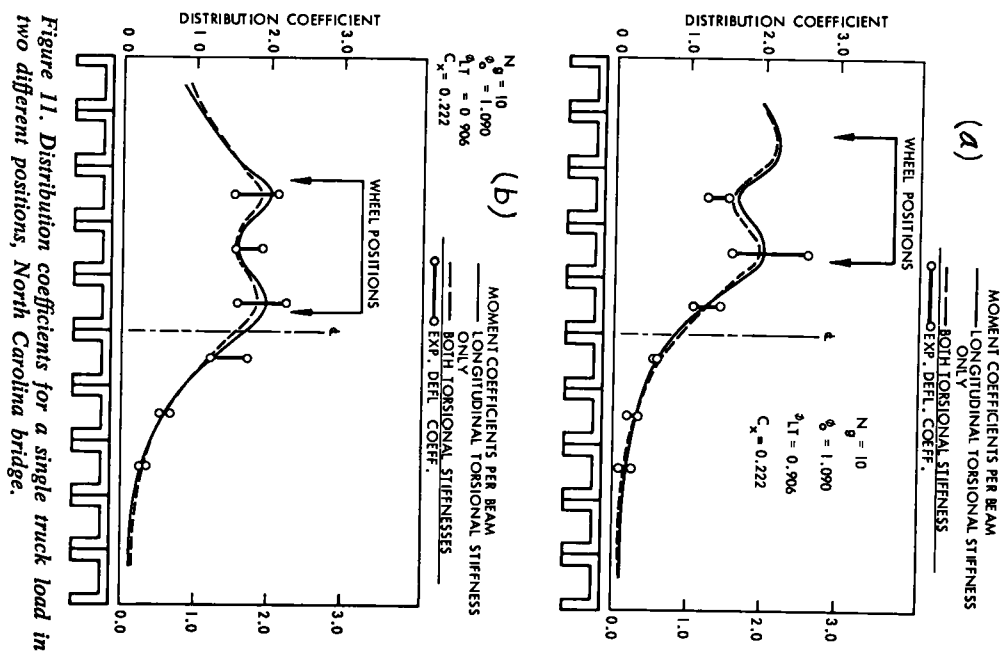


Figure 10. Details of North Carolina bridge.



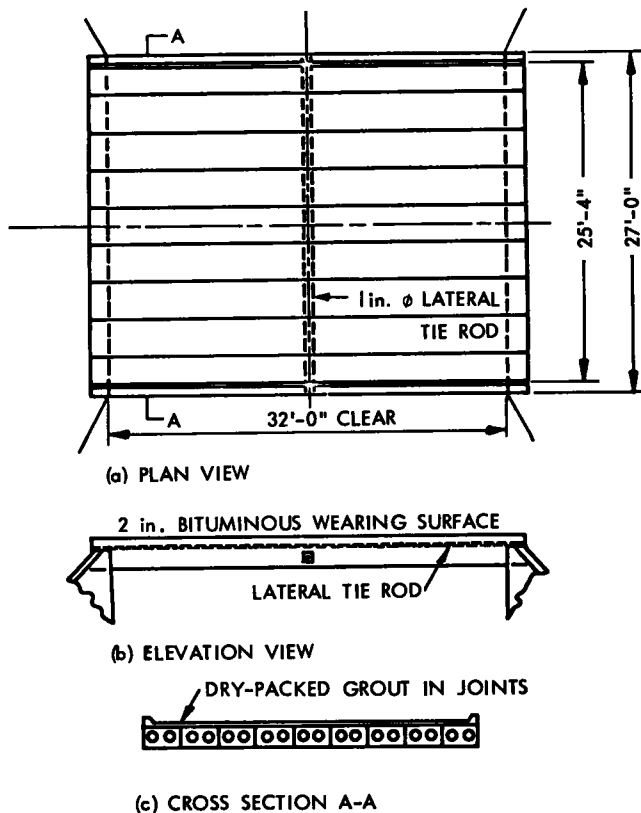


Figure 12. Details of Centerport Bridge.

correlation with the theoretical moment coefficients per beam. It should be noted that the effect of the transverse torsional stiffness was so small compared with the stiffness in the longitudinal direction that the difference between the theoretical distribution coefficients corresponding to both torsions and to longitudinal torsion only was hardly recognizable.

Langstone Bridge

The Langstone Bridge (202, 204) comprises 29 simply supported beams, each of 31-ft effective span. Each beam element, as shown in Figure 14, was 18 in. in depth and 18 in. in width. Sixteen elements were placed side by side, jointed with a dry-packed mortar, and transversely stressed with twelve cables. This bridge has two prestressed concrete "fascia" beams at the edges. The bridge was loaded with two bogie loads consisting of eight solid wheels on two axles. Two loading patterns were considered—one yielded a symmetric loading with respect to the middle line of the bridge, the other was such that the external wheels were 1 ft from the curb.

The comparisons of the distribution coefficients as predicted by applying the articulated plate theory to the properties of the actual loaded bridge with those obtained in the field test are shown in Figure 15. The theory was applied only to the 16 beam elements carrying the roadway, without regard to the edge beams. The results are in good agreement with the theory. Furthermore, when the theory is

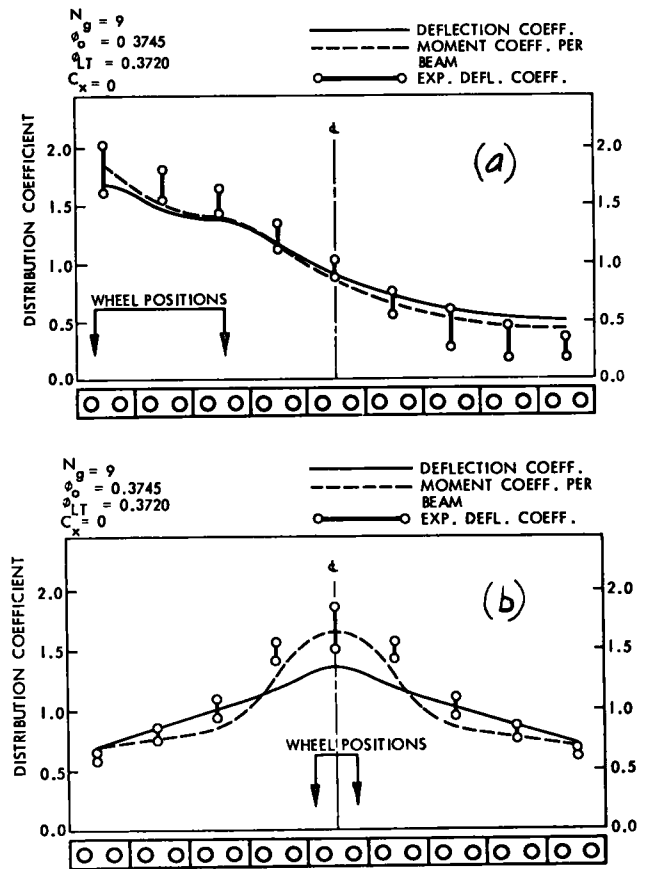


Figure 13. Distribution coefficients for Centerport Bridge: (a) for a single truck load, and (b) for two jack loads.

based on the single torsional rigidity in the longitudinal direction, the maximum error was less than 5 percent for the central loading, but 15 percent for the eccentric loading. If both the transverse and the longitudinal torsional rigidities are taken into account, the maximum error was found to be around 10 percent for the central loading and almost none for the eccentric loading.

In summary, the comparison of test results with theory for all three bridges shows relatively good correlation of the theory with the tests. The theory tends to underpredict the maximum bending moment in most cases by less than 10 percent, but in some cases the error was as much as 20 percent. However, it is felt that the articulated plate theory as developed herein has sufficiently predicted the behavior. The only other major theory considered, that proposed by Arya and others (6, 7, 8), predicted even lower moments than those obtained from the theory used in this investigation. Thus, the articulated plate theory as outlined in Chapter Three was used for the study of multi-beam bridges.

CONCRETE BOX GIRDER BRIDGES

As mentioned earlier, the theory considered for the study of concrete box girder bridges is based on the theory of pris-

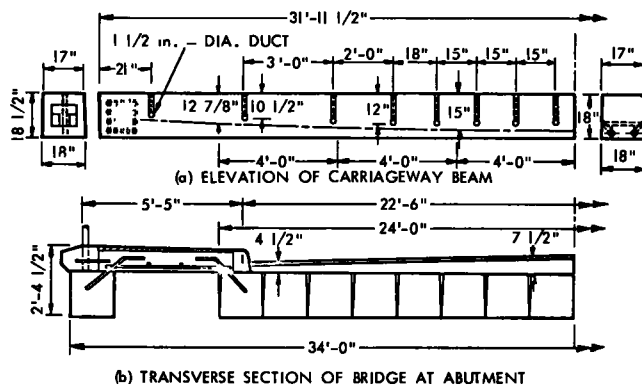


Figure 14. Details of Langstone Bridge.

matic folded plates developed by Goldberg and Leve (57). The solution procedure was developed by Scordelis, of the University of California at Berkeley (222). To indicate the validity of this theory for predicting the bridge behavior, an analysis was made by Scordelis of the results obtained from the test of the Harrison Street undercrossing (39). In addition, studies were also made of the following California bridges: the La Barranta Way Undercrossing, the College Avenue Undercrossing, and the Sacramento River Bridge and Overhead. The details of these studies are presented by Scordelis (222).

The validity of the theory to predict the behavior of concrete box girder bridges has been shown in the development of the theory.

SUMMARY

The validity of the theories proposed for the study of the behavior for each type of bridge considered in this investigation has been demonstrated in this chapter and in supporting work (5, 222, 261). Thus, for the studies of the effect of variations in loading pattern and bridge geometry on load distribution, the following theories as outlined have been used:

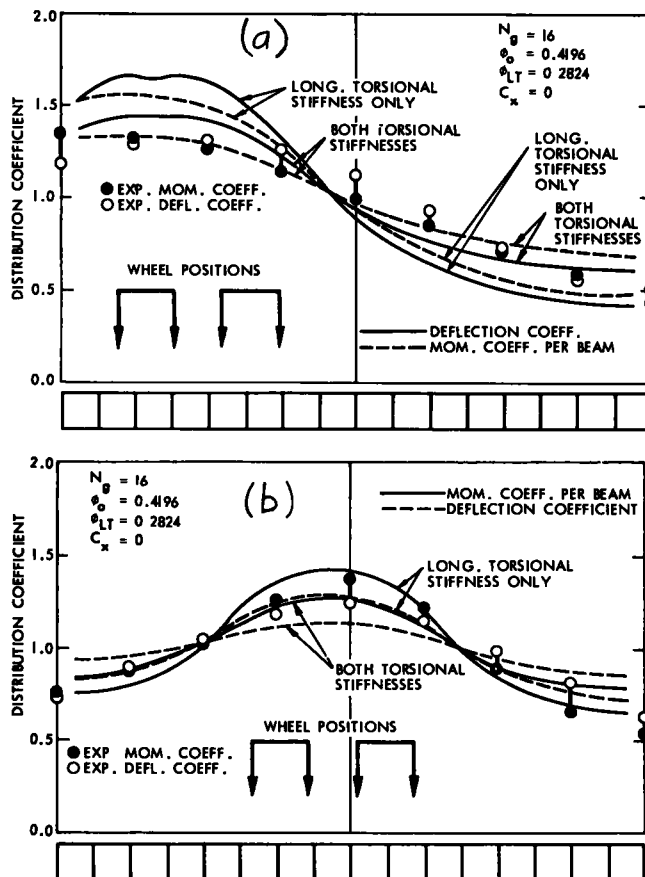


Figure 15. Distribution coefficients for two truck loads in two different positions, Langstone Bridge.

1. Beam and slab bridges: Orthotropic plate theory. However, harmonic analysis has been used to verify results when studies are made in ranges extended beyond those studied in field tests.
2. Multi-beam bridges: Articulated plate theory.
3. Concrete box girder bridges: Theory of prismatic folded-plate structures.

CHAPTER THREE

ANALYTICAL STUDIES ON EFFECTS OF VARIABLES

GENERAL

Extensive analytical studies were conducted to determine the theoretical load distribution characteristics of each type of bridge considered in the program. The studies of these

bridges encompassed the range of each of the variables that probably will occur in practice in each bridge type. The initial analytical results provided the transverse variation of the longitudinal beam moment for numerous transverse

positions of a single wheel; i.e., influence lines were generated. Thus, any combination of specific wheel positions could be considered for the determination of maximum beam moments. The use of these influence lines in combination with all of the loading conditions possible under the loading criteria yielded the maximum design moments.

The direct use of moments as a specification criterion would require significant changes in the design procedures. However, there is a direct relationship between the beam moment and the width over which a wheel load is distributed. This width is, in fact, used in the current specification in the distribution load factor equation, S/D . Thus, results of the load distribution studies were expressed in terms of D , the width of bridge over which one longitudinal line of wheels is distributed. If a satisfactory relationship between all of the variables and this width can be obtained, a more accurate and realistic distribution could be obtained without significantly altering the general distribution procedure.

In this chapter, a brief outline is provided of the analytical procedures used to develop the extensive results. In addition, summaries of the results obtained are given. Because there is significant variation in the analysis and behavior of each bridge type considered, the discussion of each bridge type is treated separately.

BEAM AND SLAB BRIDGES

Development of Theories

There are numerous theoretical methods for analyzing beam and slab bridges, as outlined in Chapter One. Each method has special features that make it more suitable for a particular cross section or loading. However, after reviewing the available methods for the analysis of beam and slab bridges under live loads, two methods were considered primary for determining the general behavior of this bridge type. These methods, as mentioned previously, are orthotropic plate theory and harmonic analysis. The reasons for this selection were:

1. These two theories seem to predict the load distribution more accurately than other methods for the entire range of geometries, configurations, and materials used (82, 191).

2. They can be used to express the load distribution properties of a bridge as a function of only a few generalized dimensionless variables so that investigation of a large variety of bridge properties becomes feasible. Both theories assume the beams and slab to be replaced by a continuous medium, which eliminates the requirement for knowing the specific bridge beam geometry in the theory formulation. Most other theories require advance knowledge of beam geometry, bridge dimensions, etc.

3. Parameters used in one method can be expressed in terms of the parameters in the other. Thus, one method can readily be compared with the other, as well as with field test results.

Extensive comparisons of theoretical results and field test results, as outlined in Chapter Two, have already shown that the plate theory can more accurately predict the behavior of

the specific bridges considered. Initially, however, analytical results from both of the theories were obtained to determine if any significant difference in the behavior of bridges as predicted by the theories could be seen. These comparisons were used particularly in the variable ranges where field test data were not available to verify the theories.

Orthotropic plate theory and harmonic analysis have been used extensively, and detailed development of the theories is given in numerous references (28, 29, 76, 77, 78, 88, 199, 245). Thus, only the basic equations are presented herein. However, a more extensive review of the development of the theories is presented in these references.

Orthotropic Plate Theory

Orthotropic plate is the common name for an orthogonally-anisotropic plate. This is a plate that has elastic properties that are uniform but different in two orthogonal directions. In bridges, this is primarily due to the different moduli of elasticity and different flexural and torsional moments of inertia along the major axes.

The governing differential equation for orthotropic plates has been known and extensively used for many years. Many methods have been devised for the solution of this basic equation. For this investigation, the approach as originated by Guyon (67, 68) and expanded by Massonnet (130, 131, 132) was used.

In this method of analysis the following assumptions, in addition to those of the thin-plate theory and small deflections, have been made:

1. Representation of the structural system with an "equivalent" orthotropic plate with uniform thickness in two orthogonal directions is sufficiently accurate. This is equivalent to stating that the effect of longitudinal edge stiffening is negligible in the over-all behavior of the bridge.
2. Poisson's ratio is equal to zero.
3. All connections can transfer the full effects of moment, torque, and shear.
4. In a beam and slab bridge, spacing of the beams, as well as the diaphragms, is uniform.

The first of these assumptions has been verified by experimental work and field test results. In effect, this permits the change of the beams to an equivalent continuous medium, which is then considered as part of the slab. Details of behavior comparisons between predictions by theory and field test results were presented in Chapter Two.

The second assumption is theoretically not correct. Poisson's ratio, if considered, tends to increase the value of the maximum moment coefficient. However, this increase is usually small and can be neglected. For beam and slab bridges, this effect was found to be within 2 to 3 percent if Poisson's ratio is assumed to be 0.15 for concrete (199).

The third assumption holds true if the connections between the various elements of the bridge are rigid. For semirigid or flexible connections, as are most bolted or riveted joints, a reduction of the corresponding rigidities is necessary. Sanders and Munse (210), for example, suggested that the effective rigidity of diaphragms of railroad bridges be taken as 25 percent of the computed value be-

cause of flexibility of the connections at the beams. Similar reductions would be applicable in highway bridges with steel diaphragms.

The fourth assumption is generally true with respect to current practice. This assumption relates to the first in that generally a nonuniform beam spacing is similar in effect to edge stiffening. If the spacing is nonuniform, the total stiffness can be spread uniformly across the cross section with sufficient accuracy.

Considering these assumptions, the behavior of the plate satisfies the following fourth-order linear differential equation (244). The equivalent plate used has the same average flexural and torsional stiffnesses in the two orthogonal directions as the actual bridge structure being studied.

Therefore,

$$D_x \frac{\partial^4 w}{\partial x^4} + 2H \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4} = p(x, y) \quad (1)$$

in which x and y are the axes of the coordinate system used (as in Fig. 16), and

$D_x = E_x I_x$, flexural rigidity per unit width in x direction;

$D_y = E_y I_y$, flexural rigidity per unit width in y direction;

$2H = D_{xy} + D_{yx}$, sum of orthogonal torsional rigidities;

I_x = moment of inertia per unit width in x direction;

I_y = moment of inertia per unit width in y direction;

E_x = modulus of elasticity in x direction;

E_y = modulus of elasticity in y direction;

D_{xy} = torsional rigidity per unit width in x direction;

D_{yx} = torsional rigidity per unit width in y direction;

and

$p(x, y)$ = function depending on live load on bridge.

If the Levy series is used to determine the solution of the differential equation for the bridge with a concentrated load acting at midspan at a distance v from the centerline of the bridge, the deflection of the bridge is

$$w = \sum_{m=1}^{\infty} \frac{H_m}{16 D_y \sqrt{2(1+\alpha)}} \left(\frac{W}{m\theta\pi} \right)^3 F(y, v, m, \theta, \alpha) \sin \frac{m\pi x}{L} \quad (2)$$

where

$$\alpha = \frac{D_{xy} + D_{yx}}{2\sqrt{D_x D_y}}, \text{ a relative torsional stiffness parameter;}$$

$$\theta = \frac{W}{2L} \sqrt{\frac{D_x}{D_y}}, \text{ a relative flexural stiffness parameter;}$$

$F(y, v, m, \theta, \alpha)$ = a function dependent on bridge parameters, location of deflection determination, location of concentrated load, and the boundary conditions of the bridge; and

H_m = Fourier constant for the concentrated load.

In the equations defining θ and α , it can be seen that θ ,

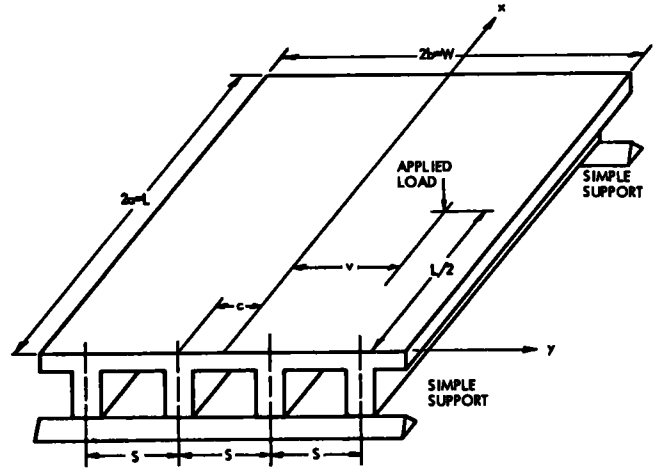


Figure 16. Bridge deck nomenclature for orthotropic plate theory.

the relative flexural stiffness parameter, primarily depends on the aspect ratio (W/L) of the bridge for its sensitivity, rather than the ratio of the flexural stiffnesses. For example, if the cross-sectional geometries remain the same and the width doubles, the parameter θ doubles. On the other hand, doubling the longitudinal stiffness (D_x) increases the parameter θ by only 19 percent, or about only one-fifth as much. It can also be seen that the aspect ratio of the bridge has no effect on the relative torsional stiffness parameter, α . Thus, if the cross-sectional geometries of the bridge remain the same, the parameter α is unchanged and, hence, is only a measure of distribution due to local torsional conditions in the bridge.

If the load on the bridge is a concentrated line load acting transversely on the bridge at midspan instead of the concentrated load represented in Eq. 2, the deflection of the bridge is

$$w = \sum_{m=1}^{\infty} \frac{H_m}{W D_x} \left(\frac{L}{m\pi} \right)^4 \sin \frac{m\pi x}{L} \quad (3)$$

The longitudinal moments in the bridge are found by differentiating either deflection equation twice with respect to x and multiplying by the longitudinal stiffness. Thus,

$$M_x = -D_x \frac{\partial^2 w}{\partial x^2} \quad (4)$$

The second derivative of Eq. 2 results in the series equation necessary to find the longitudinal moment at any point on the bridge. The second derivative of Eq. 3 results in the series equation for the mean longitudinal moment at any transverse section of the bridge. The moment distribution coefficient for this concentrated load can be found by taking the ratio of these two series equations. Thus,

$$K_m = \frac{\theta \pi \sum_{m=1}^{\infty} \frac{H_m}{m} F(y, v, m, \theta, \alpha) \sin \frac{m\pi x}{L}}{\sqrt{2(1+\alpha)} \sum_{m=1}^{\infty} \frac{H_m}{m^2} \sin \frac{m\pi x}{L}} \quad (5)$$

in which K_m is the moment coefficient for the concentrated load at midspan.

Harmonic Analysis

As mentioned previously, the second method of analysis considered was the harmonic analysis. In this method the bridge is assumed to consist of a continuous member supported by a set of elements in the longitudinal direction. In this respect, the method is quite similar to that developed by Newmark (152).

The Newmark method was developed for noncomposite beam and slab bridges and the torsional rigidity of the beams is neglected. Harmonic analysis, on the other hand, takes into account composite action of beams with the slab. The torsional rigidity of the beams is also included in the formula developed. This comparison is made to indicate only the differences between the two methods.

The harmonic analysis method was developed by Hendry and Jaeger (77) and was found to correlate with experimental results by independent researchers (86). However, as mentioned in Chapter Two, the correlation between test results and the orthotropic plate solutions was found to be better. For this reason, only the briefest review of the harmonic analysis is given here. A detailed discussion of the theory is given elsewhere (5, 65, 77).

The assumptions in this method are basically the same as those used in the orthotropic plate method, the major difference being that the effect of torsional rigidity in the transverse direction is neglected.

Harmonic analysis is used to compute bending moment coefficients by a distribution of the individual harmonics in the Fourier series expansion for concentrated loads acting on beam and slab bridge decks. The applied load is first distributed to the individual beams by assuming that the deck slab is a continuous beam over nondeflecting supports (the actual beams). Expressions for shear, moment, slope, and deflection for each beam are found by successive integrations of the load series. By using these expressions, transverse force equilibrium, and the transverse slope-deflection equations the load influence coefficients, P_{ijk} , can be found (77) which define bending moment according to the following expression (65):

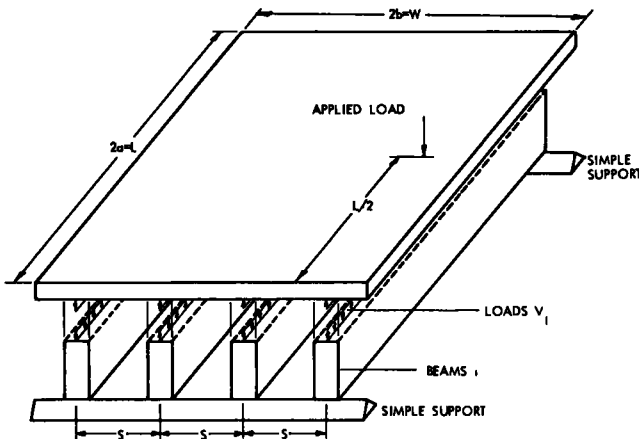


Figure 17. Bridge deck nomenclature for harmonic analysis.

$$M_i = \frac{2L}{\pi^2} \sum_{j=1}^N \left[\left(\sum_{k=1}^{\infty} \frac{1}{k^2} P_{ijk} \sin \frac{m\pi u}{L} \sin \frac{m\pi x}{L} \right) V_j \right] \quad (6)$$

in which i refers to the beam at which the moment is to be found, j refers to the loaded beam, k refers to the harmonic term, V_j refers to the concentrated unit load reaction on the j th beam when the beams are nondeflecting, N is the number of beams, and L is the span length of the bridge. Figure 17 illustrates this terminology.

According to the definition of the moment coefficient, the expression for the coefficient at midspan due to a concentrated load acting at the midspan can be written as (65):

$$K_i = \frac{8N}{\pi^2} \sum_{j=1}^N \left[\left(\sum_{k=1,3,\dots}^{\infty} \frac{1}{k^2} P_{ijk} \right) V_j \right] \quad (7)$$

in which K_i is the bending moment coefficient for the i th beam.

In Eqs. 6 and 7, P_{ijk} is a function of the number of beams, the transverse and longitudinal flexural stiffnesses, and the longitudinal torsional stiffness. The parameters associated with the determination of the P_{ijk} values can be directly related to the stiffness parameters, θ and α , used in the orthotropic plate analysis. Thus it is possible to compare the two solutions for any bridge with the use of these two parameters.

Harmonic analysis predicted higher coefficients for bridges having few beams; but as the number of beams increased, harmonic analysis tended to predict lower coefficients. From the field test comparisons (Chapter Two) and the generally unconservative results found from the harmonic analysis in a rather extensive study of the variation of beam moment with variation in the bridge parameters, it was felt that in the final analysis results of analytical studies used to generate design criteria should originate from the orthotropic plate analysis. The results, it was felt, would be more realistic over the entire range of parameters considered.

Range of Parameters

The maximum value of K_m at midspan for a concentrated load that is not placed at the midspan of the bridge is always less than the K_m value for a load at the midspan of the bridge. Thus, when multi-axle loads are considered the K_m values for the midspan load are used, and the moment at any point on the transverse section at midspan is K_m multiplied by the mean static moment of all axles. This produces slightly conservative design moments, but this was preferable to introducing an additional parameter defining the longitudinal positions of the actual concentrated loads.

In the determination of a K_m value, it was estimated that using the first nine terms of the series yields at least 97 percent of the maximum moment coefficients for a single concentrated load. This percentage was based on studies of selected ranges of the variables with up to 15 terms (65). However, other investigators (67, 68, 144, 199, 210) found that if only the first term of the moment series was used, the resulting value yielded only 85 to 90 percent of the maximum moment coefficient. Thus, the use of nine terms was felt to yield results of sufficient accuracy, because in nearly

all other instances conservative assumptions to maximize effects were used.

However, it should be noted that, because of rapid convergence, the deflection coefficients can be determined with sufficient accuracy using only the first term of the deflection series. This variation in rate of convergence between the moment and deflection coefficient series can lead to erroneous comparisons if only the first term of the respective series is used. The deflection coefficients would be quite accurate, but, as mentioned previously, the moment coefficients would be 10 to 15 percent below the true values.

In the development of the K_m influence lines for concentrated loads acting at the midspan of the bridge, 17 equally spaced points on the transverse section at midspan were considered. Making use of symmetry, nine moment coefficient curves were needed for each combination of the stiffness parameters, θ and α .

Rowe (199) stated that the range of the flexural stiffness parameter, θ , is about 0.3 to 1.13 for slab bridges, 0.5 to 1.2 for concrete T-beam bridges and 0.3 to 1.0 for box beam bridges. A study of standard bridges (281, 295) and of a number of typical bridge plans furnished by various state highway departments shows that the value of θ lies in a range from about 0.4 to 1.25 for all types of beam and slab bridges. Thus, to encompass all of the values of the parameters currently found and to consider possible changes in sections, the range of θ used in computations was 0.25 to 1.25, with an interval of 0.25.

Rowe (199) also stated that the range of the torsional stiffness parameter, α , is from about 0.05 to 1.00 for the common bridges. The parameter for standard Bureau of Public Roads bridges (281, 295) was estimated to be from about 0.045 to 0.30. To include this range, the load distribution was determined for values of $\sqrt{\alpha}$ ranging from 0.0 to 1.0 at 0.2 intervals. These values of $\sqrt{\alpha}$ correspond to α values of 0.04, 0.16, 0.36, 0.64, and 1.00.

It can be seen that there will be $9 \times 5 \times 6 = 270$ influence lines necessary to determine the K_m value for a load at any position at midspan for any of the 30 combinations of the stiffness parameters. In reality, however, not all of these combinations are possible because of design or physical limitations.

Loading System

Standard AASHO truck loading was used in the analysis for all bridge types. The criteria for its use are given in detail here for beam and slab bridges and are referred to in the discussions for other bridge types.

The current specifications (279) require (Sec. 1.2.6) that the standard truck "shall be assumed to occupy a width of 10 ft. These loads shall be placed in design traffic lines having a width of $W_L = W_r/N$ The lane loadings or standard trucks shall be assumed to occupy any position within their individual design traffic lane (W_L) which will produce the maximum stress." In addition, the number of lanes is specified for various roadway widths, with the minimum width about 10 ft and the maximum at 15 ft. However, for all practical purposes it is impossible to have normal lanes of less than 12 ft. Thus, for this study, several modifications have been made in these requirements to

make the loading more consistent with the actual maximum loading conditions. Considerable discussion has occurred in the AASHO Bridge Committee over a number of years concerning loadings, and the loading criteria used in this study seem to be a conservative consensus of the proposed changes considered by the Committee. In addition, the criteria are similar to the loading system used in the development of the distribution procedure for composite box girders in the AASHO Specifications (279), Section 1.7.104, particularly concerning the number of lanes in a roadway.

The criteria used for this study differ from the current requirements in that:

1. The number of design traffic lanes is the whole number of 12-ft lanes that can be placed within the roadway width.
2. The 12-ft lanes are placed anywhere transversely across the roadway cross section to produce maximum stress, although they may not overlap.

Furthermore, the standard trucks are assumed to be centered in a 10-ft width, which may be positioned for maximum effect anywhere within the 12-ft lanes. The maximum number of design traffic lanes is given in Table 5.

Due to the different requirements used in placing the actual driving lanes within the curb-to-curb dimensions of a bridge, the 12-ft lanes were placed within this width without consideration of lane lines, but so as to produce maximum effects. Current practice, in some instances, requires safety or shoulder lanes, which, under these criteria, produce more driving lanes than actually are used. However, considering the tremendous growth in traffic, use of the full roadway width was considered a realistic conservative assumption. In any case, only as many 12-ft lanes were loaded as was necessary to produce maximum moment.

Two conditions were considered in the placement of the 12-ft lanes. The first was the arrangement of the loads to produce maximum eccentricity with respect to the centerline of the bridge (or the eccentric loading case). This developed maximum moments in the exterior girders of the bridge. It was accomplished by arranging the 12-ft lanes side by side with the outside edge of the first lane 1.5 ft from the edge of the bridge (it was assumed that the curb width was 1.5 ft). The first 10-ft truck width was then positioned in each of the adjacent 12-ft lanes with an eccentricity of 1 ft. Therefore, the first wheel load was 3.5 ft

TABLE 5
NUMBER OF DESIGN TRAFFIC LANES

BRIDGE WIDTH, ^a W (FT)	ROADWAY WIDTH, W_r (FT)	NO. OF LANES, N_L
27 to 38.9	24 to 35.9	2
39 to 50.9	36 to 47.9	3
51 to 62.9	48 to 59.9	4
63 to 74.9	60 to 71.9	5

^a Based on a 1.5-ft curb width.

from the edge of the bridge and thus 2 ft from the edge of the curb as required by the AASHTO Specifications. Arrangements of wheel loads for eccentric loadings are shown in Figure 18.

The second consideration was the arrangement of the loads to produce minimum eccentricity with respect to the centerline of the bridge (or the central loading case). This arrangement developed maximum moments in the center girders of the bridge. Two possible arrangements can be used to produce this effect. The first is with one 12-ft lane centered over the bridge centerline, with the truck centered in this lane. The adjacent 12-ft lanes would have the 10-ft truck widths positioned eccentrically 2 ft toward the centerline of the bridge. The second arrangement is with two 12-ft lanes placed side by side on the centerline of the bridge. The 10-ft truck widths would be positioned eccentrically in the 12-ft lanes, 2 ft toward the centerline of the bridge. Arrangements of wheel loads for central loadings are shown in Figure 19.

Maximum Load Factors

To determine the maximum effect in each girder or beam, moment coefficient curves for 17 positions across the width of the bridge were obtained using orthotropic plate theory for all combinations of stiffness parameters. When finding the moment curve for a particular concentrated load that does not fall at one of the 17 positions, linear interpolation was used. The moment coefficient curve for a truck or combination of trucks is formed by summing the moment curves for each concentrated load position and dividing by the sum of the number of wheel loads. Thus, the average moment will remain unity. The moment coefficient curves are output for each combination of 12-ft lanes until the maximum number of lanes is reached for each possible combination. Table 6 gives the widths of bridges used for various numbers of lanes. Although the influence lines are non-dimensional, the use of actual truck loading dictates the use of actual bridge widths for loading considerations.

In conjunction with the determination of moment coefficient curves for each load combination, the value of D , the width of bridge over which one longitudinal line of wheels is distributed, was also obtained. These values were based on the assumption that the bridge had eight girders or beams. Thus, the moment coefficient at any beam is

$$K_{ms} = \frac{1}{6} (K_{m(i-1)} + 4K_{m(i)} + K_{m(i+1)}) \quad (8)$$

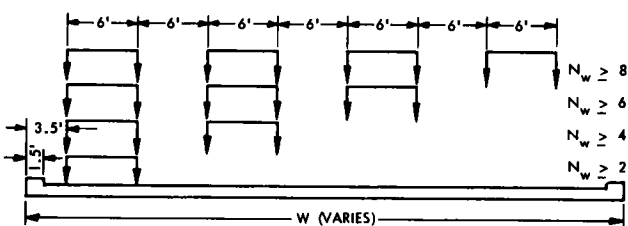


Figure 18. Loading cases considered for various bridge widths, eccentric loading cases.

TABLE 6

BRIDGES CONSIDERED FOR LOAD FACTORS

WIDTH, W (FT)			
2 LANES	3 LANES	4 LANES	6 LANES
28	39	51	75
33	41	53	—
37	45	57	—
—	49	61	—

in which the K_{ms} terms are the values from the 17 positions on the influence curves. The corresponding D value is found from

$$D = \frac{W}{N_w K_{ms}} \quad (9)$$

in which

W = width of bridge;

N_w = number of longitudinal lines of wheels; and

D = equivalent width of bridge needed to support one line of wheel load as used by the current AASHTO Specification in the load factor equation, $LF = S/D$, in which S is the spacing between girders.

In the calculation of K_{ms} , the assumption of eight beams generally leads to a conservative value, inasmuch as bridges normally have fewer beams. Because the K_{ms} value per beam is found by integrating under the K_m value per foot curve, the use of K_{ms} yields a higher value than that based on the actual number of beams as a result of the concentrated effect of the peak in the moment coefficient curve. If there are more than eight beams, the moment coefficient per foot and per beam curves are sufficiently close to cause negligible difference.

The minimum D value from all possible load combinations for a particular bridge width and stiffness parameters, θ and α , is the theoretical value that was used to determine a design criterion. These theoretical D values for various bridge widths for different combinations of stiffness pa-

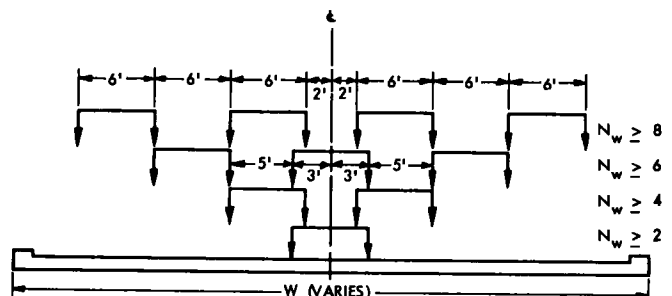


Figure 19. Loading cases considered for various bridge widths, central loading cases.

rameters, θ and α , are given in Table 7, which also indicates the critical loading case.

Use of two loading criteria (one for maximum central loading and one for maximum eccentric loading) yields two critical values of D , one for each criterion. For the central loading case the critical beam is at the center of the bridge, whereas for the eccentric loading case it is near the edge of the bridge. In the latter case the critical girder could be either the exterior girder or the first interior girder, depending on the number (or spacing) of beams. It was felt, however, that the design should be based on the absolute critical case, because the loading case which is critical varies with the bridge stiffness parameters. Thus, to determine different criteria for interior or exterior girders requires almost a complete analysis of a known bridge geometry. In addition, the difference between the two critical cases was not great enough to warrant the more complicated procedure.

In nearly every case where central loading controlled, the minimum D value (or maximum K_m) was obtained in the central girder with all lanes loaded. However, for eccentric loading, although in the majority of the cases the critical

value of D was obtained in the exterior girder with all lanes loaded, there were numerous cases where partial loadings controlled. In these cases, the difference between the D value for the partial loading case and the fully loaded cases was very small. In a very few cases where eccentric loading controlled, the critical K_m (or D) value was found slightly inside the edge of the bridge; if there were a number of girders, this could lead to the first interior girder being critical. In each of these later cases, the critical conditions always occurred with all lanes loaded.

Reduction of the data in Table 7 to a useful design criterion is treated in Chapter Four. However, there are many observations that should be made here as to the meaningfulness of these data. There are four principal variables, as follows:

1. α , the relative torsional stiffness parameter.
2. θ , the relative flexural stiffness parameter.
3. W , the actual width of the bridge.
4. N_w , the number of longitudinal lines of wheel loads.

First, it can be seen that as α increases the value of D increases, an indication that load distribution characteristics

TABLE 7

THEORETICAL VALUES OF D IN $L.F.=S/D$ FOR BEAM AND SLAB BRIDGES

		VALUE OF D FOR BRIDGE WIDTH, W , AND (N_w) , NO. OF WHEEL LOADS											
θ	α	28 FT (4)	33 FT (4)	37 FT (4)	39 FT (6)	41 FT (6)	45 FT (6)	49 FT (6)	51 FT (8)	53 FT (8)	57 FT (8)	61 FT (8)	75 FT (10)
0.25	0.00	5.66	5.23	5.16	5.17	5.20	5.27	5.13	5.10	5.08	5.08	5.12	5.04
	0.04	5.97	5.78	5.82	5.87	5.88	5.72	5.68	5.68	5.70	5.76	5.65	5.63
	0.16	6.38	6.59	6.85	6.23	6.25	6.34	6.49	6.17	6.18	6.24	6.32	6.10
	0.36	6.60	7.12	7.55	6.33	6.46	6.73	7.01	6.24	6.34	6.54	6.75	6.16
	0.64	6.71	7.42	7.97	6.38	6.58	6.95	7.32	6.27 ^a	6.43	6.71	6.99	6.18 ^a
	1.00	6.78	7.61	8.25	6.40 ^a	6.64	7.08	7.52	6.29 ^a	6.48	6.81	7.14	6.19 ^a
0.50	0.00	5.81	5.53	5.44	5.42	5.43	5.47	5.41	5.36	5.33	5.30	5.32	5.25
	0.04	5.89	5.70	5.63	5.63	5.64	5.67	5.57	5.53	5.51	5.50	5.53	5.45
	0.16	6.06 ^a	6.07	6.10	6.09 ^a	6.07	5.97	5.95	5.95	5.95	5.95	5.90	5.86
	0.36	6.17 ^a	6.47	6.62	6.15 ^a	6.24	6.29	6.36	6.03 ^a	6.16	6.20	6.24	6.03 ^a
	0.64	6.28 ^a	6.79	7.08	6.20 ^a	6.36	6.54	6.71	6.08 ^a	6.26 ^a	6.40	6.53	6.06 ^a
	1.00	6.38 ^a	7.05	7.45	6.25 ^a	6.46	6.72	6.98	6.12 ^a	6.31 ^a	6.55	6.75	6.09 ^a
0.75	0.00	5.13 ^a	5.52 ^a	5.86	5.58 ^a	5.64 ^a	5.73	5.74	5.49 ^a	5.54 ^a	5.66 ^a	5.65	5.62
	0.04	5.21 ^a	5.63 ^a	5.92	5.63 ^a	5.71 ^a	5.83	5.86	5.54 ^a	5.60 ^a	5.73 ^a	5.73	5.69 ^a
	0.16	5.41 ^a	5.89 ^a	6.09	5.76 ^a	5.86 ^a	6.00	5.99	5.65 ^a	5.73 ^a	5.90 ^a	5.96	5.77 ^a
	0.36	5.62 ^a	6.18 ^a	6.33	5.88 ^a	6.02 ^a	6.15	6.17	5.76 ^a	5.87 ^a	6.09 ^a	6.12	5.85 ^a
	0.64	5.82 ^a	6.43	6.61	5.99 ^a	6.16 ^a	6.30	6.39	5.86 ^a	5.99 ^a	6.23	6.29	5.92 ^a
	1.00	5.98 ^a	6.64	6.88	6.08 ^a	6.27 ^a	6.45	6.59	5.94 ^a	6.09 ^a	6.35	6.45	5.97 ^a
1.00	0.00	4.64 ^a	4.88 ^a	5.15 ^a	5.29 ^a	5.29 ^a	5.30 ^a	5.37 ^a	5.20 ^a	5.20 ^a	5.23 ^a	5.29 ^a	5.44 ^a
	0.04	4.73 ^a	4.98 ^a	5.26 ^a	5.35 ^a	5.36 ^a	5.40 ^a	5.48 ^a	5.25 ^a	5.26 ^a	5.31 ^a	5.39 ^a	5.48 ^a
	0.16	4.93 ^a	5.24 ^a	5.56 ^a	5.49 ^a	5.54 ^a	5.62 ^a	5.74 ^a	5.38 ^a	5.42 ^a	5.50 ^a	5.61 ^a	5.58 ^a
	0.36	5.18 ^a	5.56 ^a	5.92 ^a	5.66 ^a	5.73 ^a	5.88 ^a	6.04 ^a	5.52 ^a	5.59 ^a	5.73 ^a	5.87 ^a	5.68 ^a
	0.64	5.41 ^a	5.87 ^a	6.27 ^a	5.79 ^a	5.91 ^a	6.12 ^a	6.21	5.65 ^a	5.74 ^a	5.93 ^a	6.11 ^a	5.77 ^a
	1.00	5.61 ^a	6.14 ^a	6.54	5.91 ^a	6.05 ^a	6.29	6.36	5.76 ^a	5.87 ^a	6.10 ^a	6.32 ^a	5.85 ^a
1.25	0.00	4.42 ^a	4.58 ^a	4.79 ^a	4.91 ^a	5.04 ^a	5.15 ^a	5.16 ^a	5.06 ^a	5.05 ^a	5.06 ^a	5.10 ^a	5.28 ^a
	0.04	4.49 ^a	4.66 ^a	4.88 ^a	5.00 ^a	5.13 ^a	5.22 ^a	5.24 ^a	5.10 ^a	5.10 ^a	5.12 ^a	5.17 ^a	5.34 ^a
	0.16	4.65 ^a	4.86 ^a	5.10 ^a	5.24 ^a	5.37 ^a	5.40 ^a	5.44 ^a	5.21 ^a	5.23 ^a	5.28 ^a	5.35 ^a	5.45 ^a
	0.36	4.86 ^a	5.13 ^a	5.41 ^a	5.50 ^a	5.54 ^a	5.63 ^a	5.70 ^a	5.34 ^a	5.39 ^a	5.48 ^a	5.57 ^a	5.55 ^a
	0.64	5.08 ^a	5.42 ^a	5.96 ^a	5.64 ^a	5.71 ^a	5.85 ^a	5.97 ^a	5.47 ^a	5.54 ^a	5.67 ^a	5.81 ^a	5.64 ^a
	1.00	5.29 ^a	5.70 ^a	6.06 ^a	5.76 ^a	5.86 ^a	6.05 ^a	6.20	5.59 ^a	5.67 ^a	5.85 ^a	6.02 ^a	5.73 ^a

^a Controlled by central loading, other values controlled by eccentric loading.

improve. Second, as θ increases the value of D generally decreases, indicating a lessening of the load distribution characteristics. Third, as the bridge width increases for a specific number of wheel loads the distribution characteristics improve. Fourth, as the number of longitudinal lines of wheel loads increase the load distribution characteristics lessen.

The first observation can be explained in the following manner. Comparing a box section beam system to a steel WF-type section system supporting similar slabs where the I_x and I_y values are the same, the α value for the box-type section is larger due to the increase in the torsional stiffness of the beams. Thus, with respect to load distribution, because the box-type section is torsionally stronger, the lateral stiffness of the plate between the beams is greater. This improves the load distribution characteristics, as is demonstrated by the data in Table 7. The second observation can be explained by noting that as θ increases I_y decreases if other variables remain the same. Thus, the lateral stiffness in the transverse direction is less, reducing the load distribution characteristics of the bridge. The third observation is rather obvious, considering that the total width of the bridge is increased to support the same given total static moment. The fourth observation can be explained by considering the fact that as the number of wheels increases along with the width of the bridge, the concentrated wheel loads are relatively closer to the more critically loaded beam. As the spacing of the loads becomes relatively closer, it can be seen that the total moment on the beam increases, because the influence line for the beam is curved in the vicinity of the beam. However, as the bridge becomes increasingly wider this effect gradually diminishes.

The value of D in Table 7 could actually be used in the design of highway bridges. However, to use this table the user must employ a three-way interpolation between the three parameters involved—i.e., the bridge width, the flexural stiffness parameter θ , and the torsional stiffness parameter α . Of course, this is highly impractical and the reduction of this table to a more usable form is outlined in Chapter Four.

MULTI-BEAM BRIDGES

Development of Theory

The analysis of multi-beam bridges used in this study was basically an extension of the work using articulated plate theory undertaken by Arya, Khachaturian, and Siess (6, 7, 8). In the method as presented by Arya, the solution is found by solving the simultaneous equations found through satisfying the compatibility equations of the structural system. The effect of transverse prestressing is not considered in this analysis.

The extension of this method of analysis by Watanabe (261) uses the same basic derivations as presented by Arya. A summary of the extension of the theory is presented in this section. The significant difference in the method development by Watanabe is that the equilibrium of the system is expressed in terms of the deformations and the limit of these expressions is taken as the element size shrinks to zero. This differential equation is then similar in many

respects to the orthotropic plate equation, except that the term representing the transverse stiffness is absent and the equation includes a term for the torsional warping stiffness. Therefore, the solution will satisfy

$$D_x \frac{\partial^4 w}{\partial x^4} + 2H \frac{\partial^4 w}{\partial x^2 \partial y^2} - C_x \frac{\partial^6 w}{\partial x^4 \partial y^2} = P(x) \quad (10)$$

in which D_x , $2H$, and $P(x)$ are as defined in the previous section and C_x is the torsional warping stiffness per unit width. The basic dimensions as used for multi-beam bridges are similar to those used in beam and slab bridges, as shown in Figure 16. Using the Levy series to determine the solution of the differential equation for the bridge with a concentrated load acting at a distance v from the centerline of the bridge at midspan, the deflection of the bridge can be expressed in the following form:

$$w = \sum_{m=1}^{\infty} \frac{H_m}{16} \frac{D_x}{(D_{xy} + D_{yx})^2} \left(\frac{W}{m\phi\pi} \right)^3 F(y, v, m, \phi, g_m) \quad (11)$$

in which

$$\phi = \frac{W}{2L} \sqrt{\frac{D_x}{D_{xy} + D_{yx}}}, \text{ a combined flexural-torsional stiffness parameter; and}$$

$$g_m = \sqrt{1 + \frac{C_x}{D_{xy} + D_{yx}} \left(\frac{m\pi}{L} \right)^2}, \text{ a torsional warping parameter.}$$

The longitudinal moments in the bridge are then found by differentiating Eq. 11 twice with respect to x and multiplying by the longitudinal stiffness (Eq. 4). The moment coefficient for a concentrated load at midspan may be found by taking the ratio of the moment as determined by Eq. 4 and dividing by the moment caused by the same load distributed uniformly in the transverse direction. Thus,

$$K_m = \frac{\phi\pi \sum_{m=1}^{\infty} \frac{H_m}{m g_m} F(y, v, m, \phi, g_m) \sin \frac{m\pi x}{L}}{\sum_{m=1}^{\infty} \frac{H_m}{m^2} \sin \frac{m\pi x}{L}} \quad (12)$$

Parameters and Loading System

Moment coefficient curves were calculated using the integration of Eq. 12 for various numbers of beams, values of ϕ , and values of C_x . From the results of these calculations, it was found that the number of elements involved in the bridge did not greatly affect (maximum difference of approximately 5 percent) the value of the moment coefficients per unit width. Therefore, 16 beams were chosen for design reference. Furthermore, as in the case of beam and slab bridges, the determination of K_m was made using the first nine terms of the Levy series solution. Because this series for multi-beam bridges converges at least as rapidly as the beam and slab series, the computed K_m value is within about 1 to 2 percent of the true value.

It was also found that if the torsional warping factor (C_x) was included, there was a small difference of 3 to 4 percent in the moment coefficient values when compared with similar values for $C_x = 0$. This makes the constant $g_m = 1.0$.

The results are, therefore, conservative for open sections where the torsional stiffness is increased by resistance to torsional warping.

The values of ϕ used in determining the moment coefficients were 0.1, 0.3, 0.5, 0.7, 1.0, and 2.0. The widths of bridges chosen for the analysis were the same as those listed for beam and slab bridges, except that bridge widths greater than 53 ft were not included.

The values of D were calculated for the same truck loading combinations as listed in the previous section and shown in Figures 18 and 19.

Maximum Load Factors

Table 8 gives the results of the computations for the minimum D value for each bridge width and for ϕ , the flexural torsional stiffness parameter value. This minimum value gives the maximum load factor.

It can be seen in Table 8 that, for multi-beam bridges, the following effects on the load factor or value of D were obtained:

1. As the value of ϕ decreases, for a given bridge width, the value of D increases (lower load factor and better distribution). Thus, if the cross-sectional properties of the bridge are constant, as the length of the bridge increases the value of D increases because ϕ would be inversely proportional to that length.

2. For a given value of ϕ and number of lanes, the distribution improves (higher D value) as the bridge widens. This is obvious, because there is more total longitudinal stiffness to support the same statical moment. The effect is more significant at lower values of ϕ .

The critical loading case is given in Table 8, and in the majority of cases considered was the central loading case with all lanes loaded. However, there were several instances where the full eccentric loading controlled, with the exterior beam critical, or where a partial central loading controlled. In this last case, the difference between the critical partial and full loading cases was very small. As indicated for beam and slab bridges, it was felt that use of the absolute critical case for design, rather than consideration of the critical beam, and loading criteria, would lead to a satis-

factory design without the complications of including these factors.

As stated previously, the information presented in Table 8 can be used to design multi-beam bridges, but does not readily lend itself to such use. A reduction of these raw data to a more usable form is given in Chapter Four.

CONCRETE BOX GIRDER BRIDGES

Development of Theory

The analysis of the concrete box girder sections was carried out using a modification of the theory of prismatic folded-plate structures. Use of this theory for analyzing concrete box girder bridges was developed by Scordelis (222). The direct stiffness solution was developed using a folded-plate harmonic analysis based on an elasticity method (41). Scordelis used elastic plate theory for loads normal to the plane of the plates and two-dimensional plane stress theory for loads in the plane of the plates.

Using these theories, a computer program, MUPDI, was developed by Scordelis. This computer program can be used to analyze box girder bridges, with and without intermediate diaphragms, under concentrated or distributed loads anywhere on the bridge. The program was used as the basis for studying the parameters affecting the bridge behavior. The basic program, however, was altered slightly so that the format was compatible with the research agency's computer system. In addition, some subroutines were eliminated or changed to compute only those quantities needed for this investigation. A subroutine was also written to compute the equivalent longitudinal beam moments and the corresponding bending moment coefficients.

The basic assumptions used in the analysis are as follows (222):

1. The elements of the box girder are rectangular plates of uniform thickness and are made of an elastic isotropic and homogeneous material.
2. The force-deformation relationships are linearly elastic so that superposition is valid.
3. The bridge is simply supported at the ends.
4. Diaphragms are considered to be nondeformable in

TABLE 8

THEORETICAL VALUES OF D IN L.F. = S/D FOR MULTI-BEAM BRIDGES

ϕ	VALUE OF D FOR BRIDGE WIDTH, W , AND (N_w), NO. OF WHEEL LOADS								
	28 FT (4)	33 FT (4)	37 FT (4)	39 FT (6)	41 FT (6)	45 FT (6)	49 FT (6)	51 FT (8)	53 FT (8)
0.1	6.85	8.00	8.82 ^a	6.44	6.76	7.35 ^a	7.90 ^a	6.32	6.55
0.3	6.32	7.18	7.81 ^a	6.24	6.48	6.96	7.27 ^a	6.10	6.29
0.5	5.85	6.47	6.98	6.03	6.22	6.57	6.82 ^a	5.89	6.02
0.7	5.46	5.91	6.31	5.85	6.00	6.24	6.43	5.69	5.81
1.0	5.03	5.34	5.62	5.64	5.75	5.88	5.95	5.46	5.55
2.0	4.40	4.54	4.72	4.78 ^b	4.93 ^b	5.10 ^b	5.25	5.08	5.14

Unless indicated, central loading with all lanes loaded, controlled.

^a Eccentric loading, all lanes loaded, controlled.

^b Central loading, two lanes loaded, controlled.

their own plane, but perfectly flexible normal to their own plane.

5. Stresses and displacements in a plate element due to normal loading shall be determined by classical thin-plate bending theory as applied to plates supported on all sides.

6. Stress and displacements in a plate element due to in-plane loading shall be determined by classical thin-plate theory assuming a condition of plane stress.

Parameters and Loading System

For beam and slab bridges and multi-beam bridges, the analysis was carried out using a simple variation of only one or two parameters. However, due to the nature of the method of analysis used for the solution of the box girder problem, and the complexity of the cross section, each variable had to be specified independently. The major variables studied for the analysis of box girders were:

1. Span length.
2. Over-all width.
3. Over-all depth of the cross section.
4. Number of girders (vertical longitudinal plates).
5. Number of transverse diaphragms.
6. Thickness of webs and flanges.
7. Edge conditions.

To estimate the ranges that must be considered for these variables, the information on these ranges obtained by Scordelis (222) from 200 California box girder bridges was used, together with additional information secured from the California Division of Highways and the Iowa State Highway Commission. In summary, the variables ranged as follows:

1. Span length: The span lengths of the majority of simple-span box girder bridges fall within the range of 50 to 110 ft. Spans of 50 ft, 80 ft, and 110 ft were considered in the analysis to incorporate the entire range.

2. Over-all width: Over-all widths considered herein correspond to the widths studied for the beam and slab and multi-beam bridges, except that the narrowest width (28 ft) was not considered.

3. Depth of cross section: According to the sources supplying the data, the depth of the bridge is related to the span. The depth/span ratio ranges from 0.05 to 0.07 for reinforced concrete bridges, although a prestressed box girder bridge may have a ratio as low as 0.045. In this study, depth/span ratios of 0.05 and 0.07 were considered.

4. Number of girders: The number of girders equals the number of cells plus one. The number of cells and the width of the cells were chosen such that the transverse spacing between the vertical webs of the girders was within the normal design range of from 7 to 9 ft. Therefore, for the widths of bridges studied, bridges with 4, 6, and 8 cells were included.

5. Number of diaphragms: The geometries considered included bridges with no diaphragms and with one or two diaphragms. Because the most common design is one with no diaphragms, this case was studied in depth. Six combinations of length and depth were considered for each width of bridge. However, to determine the effect of the diaphragms on the load distribution characteristics, limited

studies of bridges with diaphragms were conducted. For the case of only one interior diaphragm (at center span), two combinations of span and depth were considered for selected widths. For the case with two diaphragms (at the third-span points), only the most critical situation of the shortest bridge with the deepest section was studied.

6. Thickness of webs and flanges: The thicknesses of the plates used in the general study were 6.5 in. for the top flange, 5.5 in. for the bottom flange, and 8.0 in. for the webs. These dimensions are felt to be typical of the designs used in practice for most box girder bridges. However, it was found that these dimensions could be increased to 7.0, 6.0, and 12.0 in., respectively, in those cases where the depth of the bridge may be limited (e.g., prestressed bridges with $d/L = 0.045$). Additional information was obtained for these greater thicknesses in a few cases to determine the over-all effect of the change in thickness.

7. Edge conditions: A cantilever overhang of 3.5 ft was assumed to exist in all cases studied for two reasons—first, this configuration is commonly used in many designs; second, this condition puts the exterior wheels directly over the exterior web, tending to maximize the moment in this section for the eccentric loading case.

Table 9 gives the range and values of each variable considered.

The loading patterns used in the computation of the maximum load factors (i.e., minimum D value) included those shown in Figures 18 and 19 for beam and slab bridges, as well as two special box girder loading cases. These two special cases, shown in Figure 20, were developed to maximize the moment coefficients due to the peaked condition of the influence lines in the region of the webs.

Maximum Load Factors

Influence lines were generated for the girder moment coefficients for each of the combinations listed in Table 9. The final moment coefficients from the actual truck loads were found by superposition using these influence lines. As in the study of beam and slab and multi-beam bridges, the final moment coefficient is reduced to D as used in the S/D load factor equation. The value of S used in this equation is modified as explained in Chapter Four. The results of these computations, together with the critical girder and loading cases, are given in Table 10, which gives the absolute minimum value of D for all loading cases considered. The reduction of these data to a proposed design equation is discussed in Chapter Four.

It should be pointed out, however, that because of the complexity of the analysis and because of the integral nature of the section, the most accurate design can only be obtained by an analysis of the complete section. However, for ordinary design purposes it is felt that a satisfactory distribution procedure can be obtained from the preceding analysis and range of variables.

The results presented in Table 10 do, however, show several significant facts about the behavior of box girder bridges, as follows:

1. For a particular bridge cross section, as the span

TABLE 9
VARIABLES IN BOX GIRDER BRIDGE STUDY

NO. OF CELLS	NO. OF WHEELS	BRIDGE WIDTH, W (FT)	CELL WIDTH (FT)	d/L RATIO	NO. DIAPHRAGMS CONSIDERED, N_d		
					50-FT SPAN	80-FT SPAN	100-FT SPAN
4	4	33	6.5	0.07	0, 1, 2	0	0
				0.05	0 ^a	0	0, ^a 1
		37	7.5	0.07	0, 1, 2	0	0
				0.05	0 ^a	0	0, ^a 1
	6	39	8.0	0.07	0, 1, 2	0	0
				0.05	0 ^a	0	0, ^a 1
		41	8.5	0.07	0	0	0
				0.05	0	0	0
		45	9.5	0.07	0, 1, 2	0	0
				0.05	0 ^a	0	0, ^a 1
6	6	45	6.3	0.07	0, 1, 2	0	0
				0.05	0	0	0, 1
		49	7.0	0.07	0, 1, 2	0	0
				0.05	0	0	0, ^a 1
	8	51	7.3	0.07	0, 1, 2	0	0
				0.05	0	0	0, 1
		53	7.7	0.07	0	0	0
				0.05	0	0	0
		57	8.3	0.07	0	0	0
				0.05	0 ^a	0	0 ^a
8	8	61	6.8	0.07	0, 1, 2	0	0
				0.05	0 ^a	0	0, ^a 1
	12	75	8.5	0.07	0	0	0
				0.05	0	0	0
				0.05	0	0	0

^a Combinations where variations in thickness were studied.

increases the distribution of loads improves because the value of D increases.

2. The inclusion of diaphragms improves the load distribution characteristics (D increases). A single diaphragm at midspan apparently is better in distributing the loads than a pair of diaphragms at the third-points of the span. However, this is probably due to the fact that all wheel loads were placed at midspan, and thus were directly over the single diaphragm. Actually, the benefits from diaphragms should be computed for the wheel loads in their true longitudinal position.

3. If the depth of the section increases (d/L increases), the load distribution is slightly worse due to the reduction in torsional stiffness. This is, however, more than offset by the increase in the longitudinal moment of inertia. Thus, the resultant extreme fiber stress decreases. Also, if the thicknesses of the section elements are held within the ranges selected for study, no significant change in distribution behavior is expected.

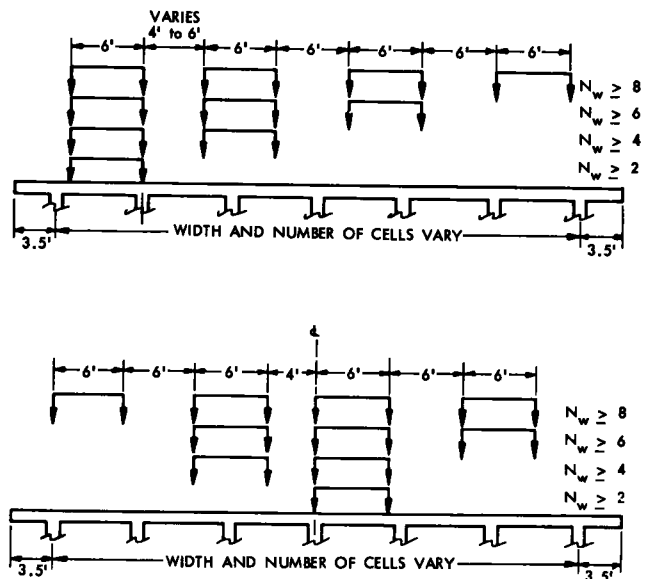


Figure 20. Additional loading cases considered for concrete box girder bridges.

^a Unless indicated, eccentric loading controlled, with the first interior girder critical; all lanes were loaded.
^b Values in parenthesis refer to special computations where thicknesses were varied.
^c Central loading controlled, with center girder critical.

DEVELOPMENT OF DESIGN PROCEDURES

GENERAL

In the development of any design procedure, the main consideration to be kept in mind is the realization that the final procedure must not be so complicated that it is totally unacceptable to the practicing engineer. In addition, the procedure must offer improvement in accuracy over previously accepted procedures. Therefore, the objective becomes one of finding the simplest procedure with the best accuracy with respect to the known theoretical and experimental behavior.

The present design criteria have remained essentially unchanged for the last 25 years, except where new bridge types have been introduced. The criteria have proved to be conservative in predicting the maximum moments in most structures considered. In addition, only a very limited number of variables is considered in the current procedures. The procedures given herein were developed so that the design moments can be more realistic in keeping with the actual behavior of the bridge, and can consider all of the significant variables affecting that behavior.

The main objection to the present design procedures has been that the design of the members is based only on the type of bridge and the spacing between longitudinal girders (as in the case of beam and slab bridges and box girder bridges). It is apparent from the results given in Chapter Three that as the aspect ratio of the bridge (width-to-length ratio) decreases, the load distribution characteristics of the bridge should improve. Secondly, there are no provisions for the flexural and torsional stiffness characteristics of the individual bridge. For example, there has been no design benefit from adding diaphragms or deepening the slab. It is necessary that the final procedure(s) eliminate these objections, to the extent that this is feasible, so that improvements in designs can be obtained.

Although the results of the studies outlined in Chapter Three showed that the critical girder or beam could be either the exterior girder or an interior girder, it was felt that, because the critical condition varied with the combination of parameters, the design criteria should be uniform for all beams and should be based on the absolute critical case. In the normal range of parameters, the differences between the critical cases for both girder positions did not warrant this consideration. Thus, a common design criterion has been developed for all beams.

BEAM AND SLAB BRIDGES

In beam and slab bridges, it was found that the major variables describing the behavior of the bridge for a particular case could be combined into a flexural stiffness parameter, θ (the relative flexural stiffness ratio multiplied by the aspect ratio of the bridge), and the torsional stiffness

parameter, α (the relative ratio of the torsional stiffness to the flexural stiffness of the bridge). Several analytical and graphical methods can be applied to the theoretical data of Table 7 to show the relationship between D (in S/D) and the stiffness parameters.

The method used to determine a possible relationship was to sketch contour lines of constant D on a coordinate system using $\sqrt{\alpha}$ as one coordinate and θ as the other. A typical example of this type of representation is shown in Figure 21. These plots clearly show that for the practical ranges of θ and α the contour lines are nearly straight lines converging on the coordinate axes. This indicates that for a particular bridge width, a parameter which could be used in the design procedure is $\theta/\sqrt{\alpha}$. This ratio, referred to as C , is used rather than considering θ and $\sqrt{\alpha}$ each as an independent parameter. Thus,

$$C = \frac{W}{(\sqrt{2})L} \sqrt{\frac{D_x}{D_{xy} + D_{yz}}} \quad (13)$$

There remains, then, the question of the influence of the deviation of the actual contours from the straight lines (use of one parameter for the two), the effect of the bridge width, and the number of wheel loads acting on the bridge. This can be seen in the plot of D vs C for all values listed in Table 7. These plots (Figs. 22, 23, and 24) show that the D values are comparatively well banded with respect to the value of C for each particular set of wheel loads. One of the simplest equations that best suits these results and incorporates all variables is in the form:

$$\begin{aligned} D &= 5.0 + \frac{N_w}{20} + \left(3.0 - \frac{N_w}{7}\right) \left(1.0 - \frac{C}{3}\right)^2 & C \leq 3 \\ D &= 5.0 + \frac{N_w}{20} & C \geq 3 \end{aligned} \quad (14a)$$

also,

$$D \leq \frac{W}{N_w}$$

in which $C = \theta/\sqrt{\alpha}$, and N_w is the number of design wheels from AASHTO Article 1.2.6, modified to conform to the criteria used in this study. Eq. 14a can also be expressed directly in terms of the total number of design traffic lanes (N_L) by changing the N_w term to $2(N_L)$. Thus, Eq. 14a becomes

$$\begin{aligned} D &= 5.0 + \frac{N_L}{10} + \left(3.0 - \frac{2N_L}{7}\right) \left(1.0 - \frac{C}{3}\right)^2 & C \leq 3 \\ D &= 5.0 + \frac{N_L}{10} & C \geq 3 \end{aligned} \quad (14b)$$

N_L in this case would be obtained from a new Article 1.2.6

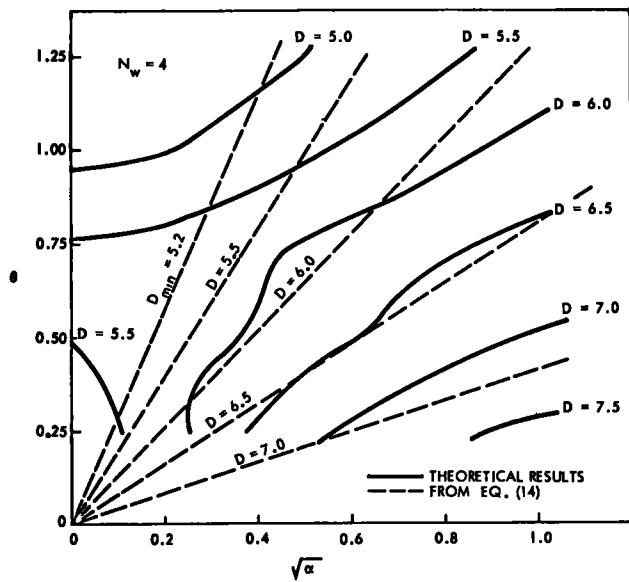


Figure 21a. Contours of D for beam and slab bridges, $W=33$ ft.

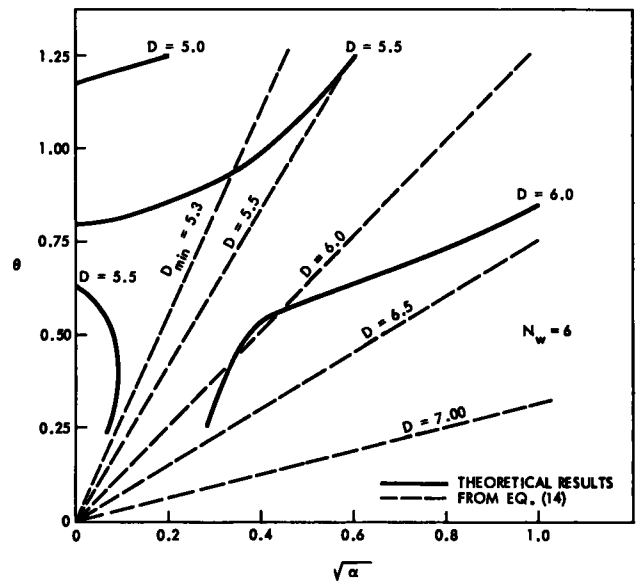


Figure 21b. Contours of D for beam and slab bridges, $W=39$ ft.

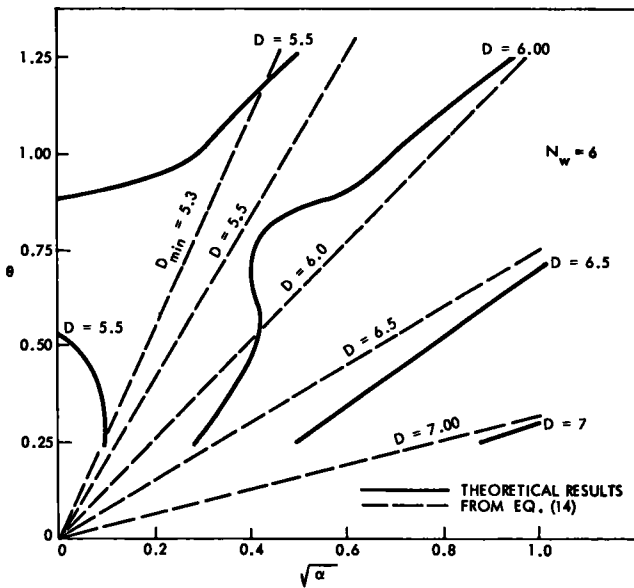


Figure 21c. Contours of D for beam and slab bridges, $W=45$ ft.

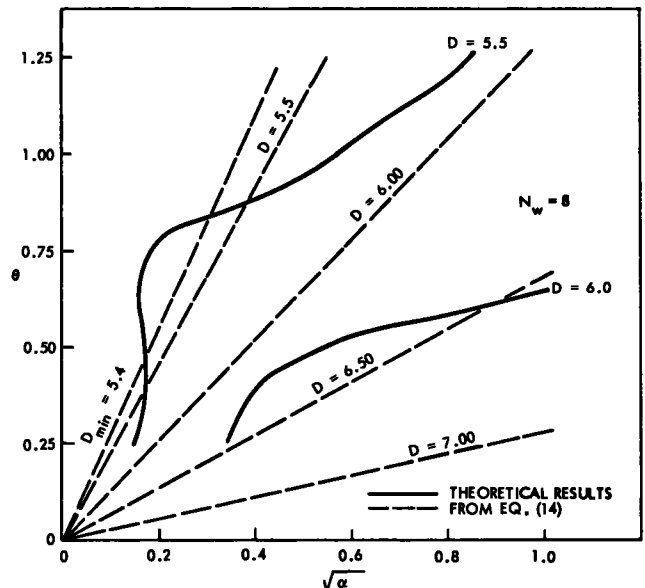


Figure 21d. Contours of D for beam and slab bridges, $W=51$ ft.

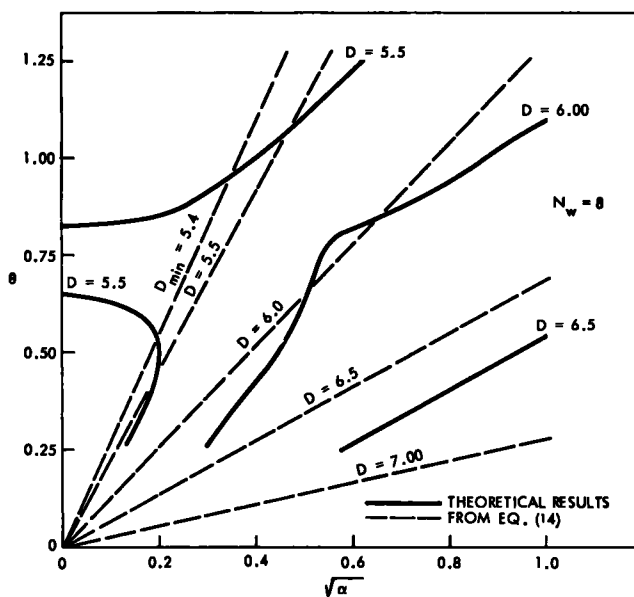


Figure 21e. Contours of D for beam and slab bridges, $W=57$ ft.

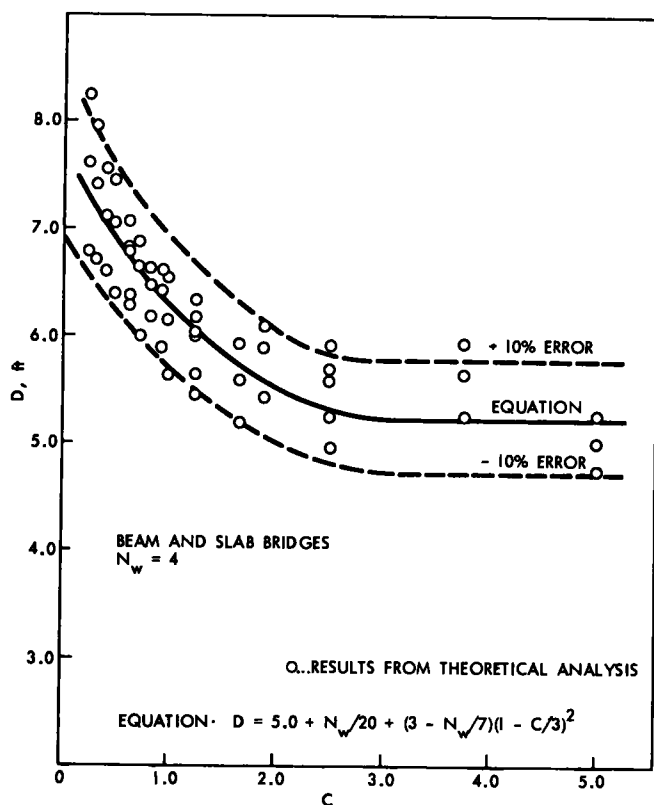


Figure 22. Variation of D with bridge stiffness parameter, C , for beam and slab bridges; $N_w = 4$.

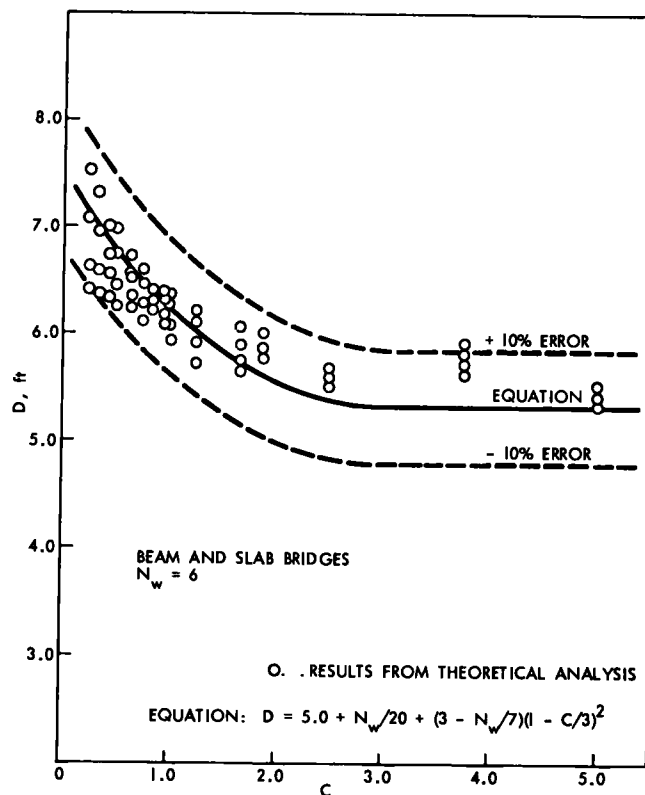


Figure 23. Variation of D with bridge stiffness parameter, C , for beam and slab bridges; $N_w = 6$.

—Traffic Lanes, which would be based on the lane criterion used in this study. This criterion is, in effect, the width of the roadway (curb to curb) in feet, divided by 12, reduced to the nearest whole number. Then N_w in Eq. 14a is just twice the number of lanes.

The details involved in the computation of C are given in Appendix A. However, in the case of composite steel box girder bridges (beam and slab bridges with separated steel box girders), because of the special nature of the cross section, the effective torsional rigidity is somewhat less than the torsional rigidity computed using standard procedures. Thus, the computed C would be less than the effective C to be used in Eq. 14a. By comparison of Eq. 14b with the extensive results of Johnston, Mattock and others (97, 135), it was found that the effective rigidity was approximately 25 percent of the indicated torsional rigidity, and, thus, for composite steel box girders, the effective C is twice the computed C . Table 11 summarizes these results, showing the relationship of D from Eq. 14a to the theoretical results (97), and the D from the new load distribution equation (97, 135) and from Article 1.7.104 of the 1966-1967 AASHTO Interim Specification (279). It can be seen that the use of Eq. 14a with the modified C gives better correlation than the new specifications. However, it is felt that the current interim specifications are more readily usable in design offices, especially considering the small differences in the D values.

Eq. 14a seems to indicate that there is no influence of

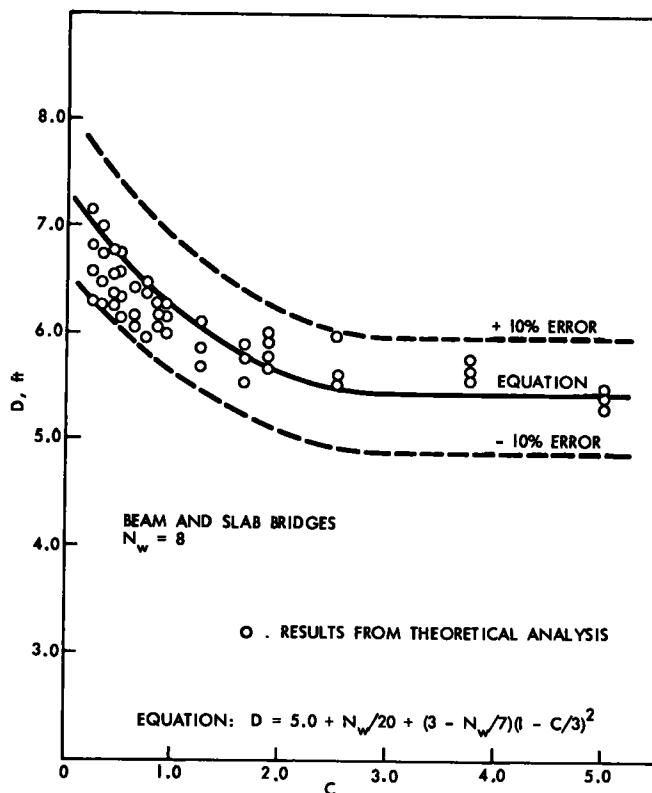


Figure 24. Variation of D with bridge stiffness parameter, C , for beam and slab bridges; $N_w = 8$.

TABLE 11

COMPARISON OF PROCEDURES FOR LOAD DISTRIBUTION
IN COMPOSITE BOX GIRDER BRIDGES

BRIDGE NO. ^a	NO. OF LANES	SPAN (FT)	NO. OF GIRDERS	GIRDER SPACING (FT)	VALUE OF <i>D</i>		
					FROM THEORY ^a	FROM BOX GIRDER EQ. 14, ^b	FROM EQ. 14A USING EFFECT. <i>C</i>
50-4	3	50	4	10.50	5.58	6.33	5.66
50-6	4	50	5	10.50	5.52	6.33	5.61
75-2	2	75	3	10.50	6.44	6.45	6.28
75-3	3	75	3	14.33	6.10	6.90	6.07
75-5	4	75	4	13.33	5.70	6.64	6.06
100-4	3	100	4	10.50	6.12	6.33	6.18
100-6	4	100	5	10.50	5.71	6.33	6.02

^a Ref. (97). ^b Ref. (135).

the transverse stiffness of the bridge, EI_y , on the load distribution characteristics of the bridge. However, it must be remembered that the ranges of the parameters θ and \sqrt{a} have been previously established. This, therefore, automatically limits the minimum value of I_y . For example, assuming that the value of θ is limited to a maximum of 1.00

$\left(\theta = \frac{W}{2L} \sqrt{\frac{D_x}{D_y}} = 1.00\right)$, the value of I_y must be

$$I_y \geq \frac{W^4}{16L^4} I_x \quad (\theta \leq 1.00)$$

or, if $\theta \leq 1.25$,

$$I_y \geq \frac{W^4}{40L^4} I_x \quad (\theta \leq 1.25)$$

For the practical range of bridge design, it is obvious that this criterion will always be satisfied. In reality, the major effect of an increase in I_y is included in a change in the torsional stiffness parameter. This change may be significant if I_y is increased due to thickening of the slab, but will probably be minimal if due to including diaphragms. Individual diaphragms are ineffective in this analysis because an equivalent plate is used to represent the actual bridge system. Therefore, the true effects of a transverse diaphragm on the distribution characteristics are not represented because the stiffness of these members has been distributed longitudinally along the bridge. Thus, in the case of bridges with transverse diaphragms, Eq. 13 yields conservative results. However, unless the diaphragms are rigid and closely spaced, their effects on distribution will usually be minor. For the general bridge system with various considerations of flexural and torsional stiffnesses, Eq. 13 will produce D values that are within ± 10 percent of the correct value with respect to the equivalent plate.

In the design of a beam and slab bridge the determination of the load factor is as follows:

$$LF_{\text{beam, slab}} = S/D \quad (15)$$

in which

S = average distance between beams in beam and slab

bridges (S will be taken as 1.0 ft for slab bridges to determine the moment per foot); and

D = width of bridge, in feet, necessary for the design of one line of wheels as determined by Eq. 13.

The design load per beam is then the load factor times the magnitude of the wheel load.

MULTI-BEAM BRIDGES

In multi-beam bridges, it was found that only one physical parameter, ϕ , is required to predict the distribution. It can be seen from Eqs. 11 and 13 that the parameter, ϕ , in terms of physical properties is identical to the C used in beam and slab bridges, except that for multi-beam bridges it becomes $C_{\text{multi-beam}} = \sqrt{2}\phi$.

Using the same methods as outlined for beam and slab bridges, it was found that the D values have the same type of relationship to the C values. It was also found that the banding in the D vs C plot is not as strong as for the beam and slab bridges, but was scattered due to the sensitivity of the D value to the width of the bridge. However, it was found that if the S value used to compute the load factor, S/D , was changed to correspond to the average width of bridge for a given number of wheel loads, the same equations for D in beam and slab bridges could also be used for multi-beam bridges. The values of D modified for the change in the definition of S are given in Table 12. The variation of the modified D with C for multi-beam bridges is shown in Figures 25, 26, and 27. Thus, for multi-beam bridges, S in Eq. 15 to determine the load factor should be taken as

$$S_{\text{multi-beam}} = (6N_w + 9)/N_g \quad (16)$$

in which

N_w = the number of longitudinal lines of wheel loads;
 N_g = the number of beam elements;

and D should be taken as given in Eq. 14a. The design load per beam is then obtained by multiplying the wheel load by the load factor.

TABLE 12
THEORETICAL VALUES OF D IN $L.F.=S/D$ FOR MULTI-BEAM BRIDGES^a

ϕ	VALUE OF D FOR BRIDGE WIDTH, W , AND (N_w) , NO. OF WHEEL LOADS								
	28 FT (4)	33 FT (4)	37 FT (4)	39 FT (6)	41 FT (6)	45 FT (6)	49 FT (6)	51 FT (8)	53 FT (8)
0.1	8.08	8.00	7.87	7.43	7.42	7.35	7.25	7.07	7.04
0.3	7.45	7.18	6.97	7.20	7.12	6.96	6.67	6.82	6.76
0.5	6.90	6.47	6.23	6.96	6.83	6.57	6.26	6.59	6.48
0.7	6.44	5.91	5.63	6.75	6.59	6.24	5.90	6.36	6.25
1.0	5.93	5.34	5.01	6.51	6.31	5.88	5.46	6.10	5.97
2.0	5.19	4.54	4.21	5.55	5.41	5.10	4.82	5.68	5.53

^a $S = (6N_w + 9)/N_g$.

CONCRETE BOX GIRDER BRIDGES

In concrete box girder bridges, no attempt was made to analyze these sections based on the equivalent plate type of approach. Therefore, the parameters involved in the study were based on the basic dimensions of the bridge itself. However, the dimensionless parameters found to affect the load distribution characteristics to the greatest extent were the aspect ratio, W/L ; the depth-width ratio, d/W ; the number of vertical webs (or girders), N_g ; and the number of diaphragms, N_d . It was found that by defining the stiffness parameter of the bridge as

$$C = 0.55 \frac{W}{L} \left(1 + N_g \sqrt{\frac{d}{W}} \right) \left(\frac{1}{\sqrt{1 + N_d}} \right) \quad (17)$$

and S in Eq. 15 as

$$S = \text{Maximum} \left(S_a \text{ or } \frac{6N_w + 9}{N_g} \right)$$

in which S_a is the actual girder spacing, N_w is the number of wheel loads, and N_g is the number of girders, D in Eq. 15 can be defined by Eq. 14a as used in the previous discussion for beam and slab and multi-beam bridges. The actual values of C and D as found from the analytical solutions, as well as the values of D as computed using C and S from Eq. 17 together with Eq. 14a are given in Table 13. This table also gives the error of the value of D from the equations with respect to the actual computed theoretical value. The results are also shown in Figures 28, 29, and 30.

The results given in Table 13 clearly indicate the validity of the proposed definitions of D and S . In nearly every case the difference between the values of D computed using the theory of prismatic folded plates (222) and those obtained using the procedure proposed in Eq. 17 is less than 5 percent. Only in two cases is the error greater than 10 percent, and these are conservative differences.

INITIAL DESIGN CONSIDERATIONS

The design of bridges as determined by the use of the equations in the previous sections presupposes an actual knowledge of the bridge geometries, which is not actually the case. The only information usually available prior to design is the value of the aspect ratio of the bridge, and possibly the

beam spacing and slab thickness. Therefore, initial values of D must be found to expedite the design procedure. To provide information in this regard, it is suggested that C be approximated for preliminary design purposes by

$$C = K(W/L) \quad (18)$$

in which

W = width of the bridge;

L = length of the bridge; and

K = a coefficient dependent on the bridge type.

Table 14 gives values of K as determined from bridges already built that conform to the present AASHTO Specifications. The K value suggested for composite steel box girder bridges does consider the effective torsional rigidity. However, the table gives an indication of the range of the stiffness parameter which can be expected for each bridge type.

It is recommended that if the current AASHTO Specifications are changed to correspond to the recommendations in this report, the values of K given in Table 14 be studied in about five years and modified to conform to the practice at that time. The reason for this statement is that present design criteria tend to make the bridge conform to the design criteria. Therefore, bridges with a very small aspect ratio now tend to be conservative; thus, the C values tend to be unnaturally high compared with an optimum design.

Figure 31 shows, for a four-lane bridge, the design moment per foot of bridge width for various bridge lengths and various values of C under HS-20 loading. The current AASHTO design equation for slabs (Section 1.3.2C) for HS-20 loading is also shown. This figure clearly shows the change in design moment due to the change in the value of C for various lengths of bridges. Therefore, an alternate initial design for a four-lane bridge would be to determine the value of C from Eq. 18 and the design moment per foot from Figure 31. The design moment per girder would then be

$$M = M_{Fig. 31} S \quad (19)$$

A more accurate value of C would then be determined from this preliminary design. Similar figures could be constructed for this procedure for bridges with different numbers of lanes.

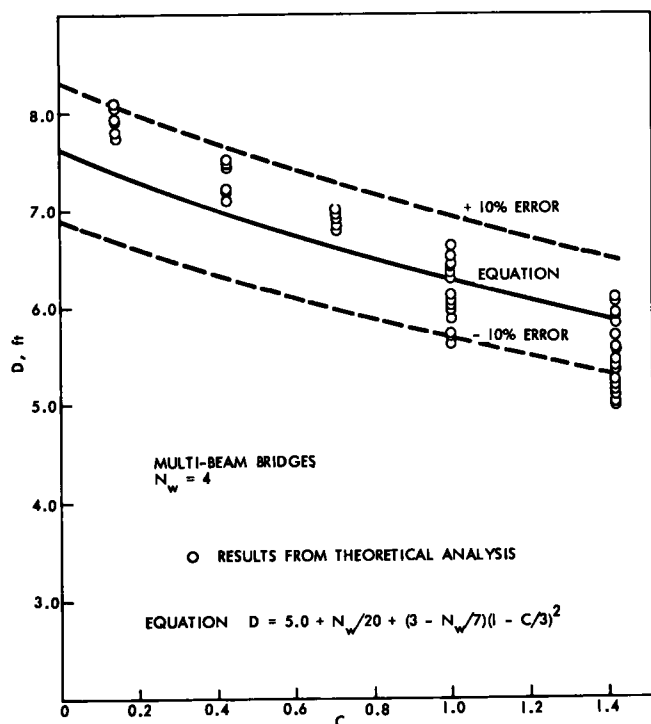


Figure 25. Variation of D with bridge stiffness parameter, C , for multi-beam bridges; $N_w = 4$.

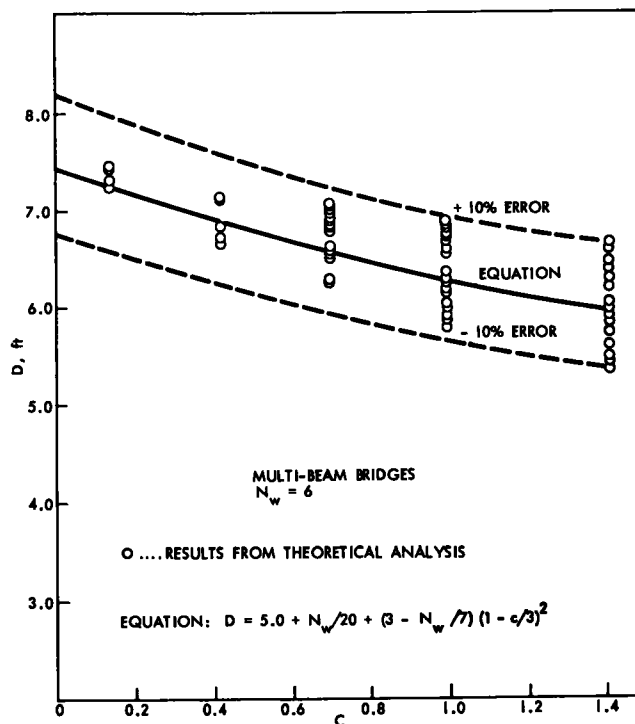


Figure 26. Variation of D with bridge stiffness parameter, C , for multi-beam bridges; $N_w = 6$.

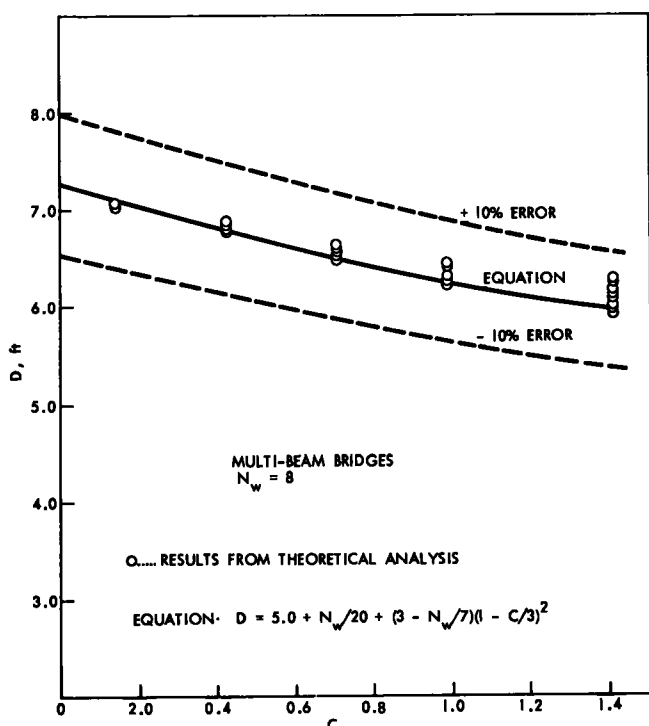


Figure 27. Variation of D with bridge stiffness parameter, C , for multi-beam bridges; $N_w = 8$.

EFFECTS OF EDGE STIFFENING

In the case of beam and slab bridges having curbs and rails, the effect of additional members may be taken into account by defining the effective width of the bridge as

$$W_e = (N_g - 2)S + 2\left(\frac{I_e}{I_i}\right)S \quad (20)$$

in which

- W_e = effective width;
- I_e = moment of inertia of exterior girder section;
- I_i = moment of inertia of interior girder section;
- S = beam spacing; and
- N_g = number of longitudinal beams.

For bridges having nominal safety curbs (up to 2-ft wide and about 1-ft deep), the effect of these additional edge members (such as built-up curbs and rails) can be neglected, a common practice. This conclusion is based on a study of actual bridges that showed the effective width obtained by Eq. 20 to be about equal to a width computed from $(N_g)S$.

However, for bridges having stiffer curbs (acting integrally with the slab), and possibly an additional longitudinal member which supports the curb, the previous definition of effective width (Eq. 20) should be used to determine the design moment for the beams. However, C should be found by using the actual over-all bridge width. Thus, the interior beams should be designed using Eq. 15 where S is the actual beam spacing, and the exterior girders

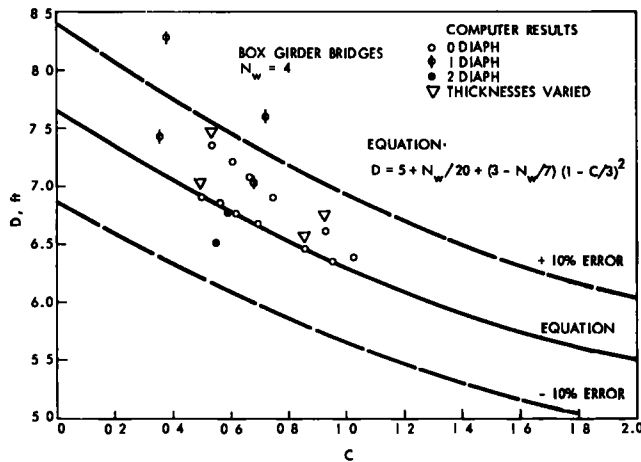


Figure 28. Variation of D with bridge stiffness parameter, C , for concrete box girder bridges; $N_w = 4$.

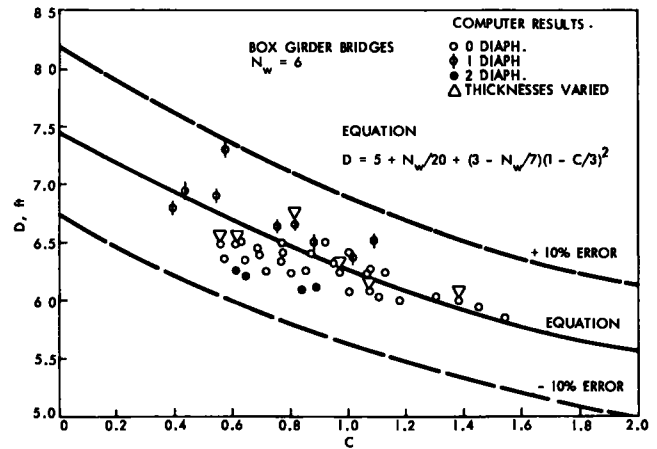


Figure 29. Variation of D with bridge stiffness parameter, C , for concrete box girder bridges; $N_w = 6$.

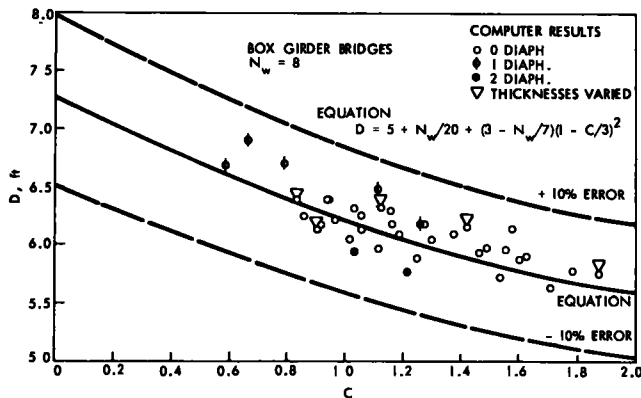


Figure 30. Variation of D with bridge stiffness parameter, C , for concrete box girder bridges; $N_w = 8$.

should be designed using Eq. 15 where S is the effective spacing, S_e , for the exterior girders as defined in Eq. 20, or

$$S_e = \frac{I_e}{I_t} S \quad (21)$$

and, thus, the L.F. = S_e/D . For low values of I_e/I_t (i.e., less than 5) it is felt that this design procedure will be sufficiently accurate.

The edge stiffening members in slab bridges are usually designed in the form of safety or sidewalk curbs. Considerable work has been done on the slab bridges with curbs. Jensen (95) presented a design procedure in which empirical formulas are used in determining the moments in curbs and in the slabs. Test results have shown this to be correct. A review of this method is felt to be unwarranted due to the length of the subject matter required.

Rowe (199) presented a method in which the effect of an edge-stiffening beam can be taken into account accurately. This method, however, would be difficult to adapt for use in a design office, due to its complexity. Pama and Cusens (165) also studied edge-beam stiffening of multi-beam

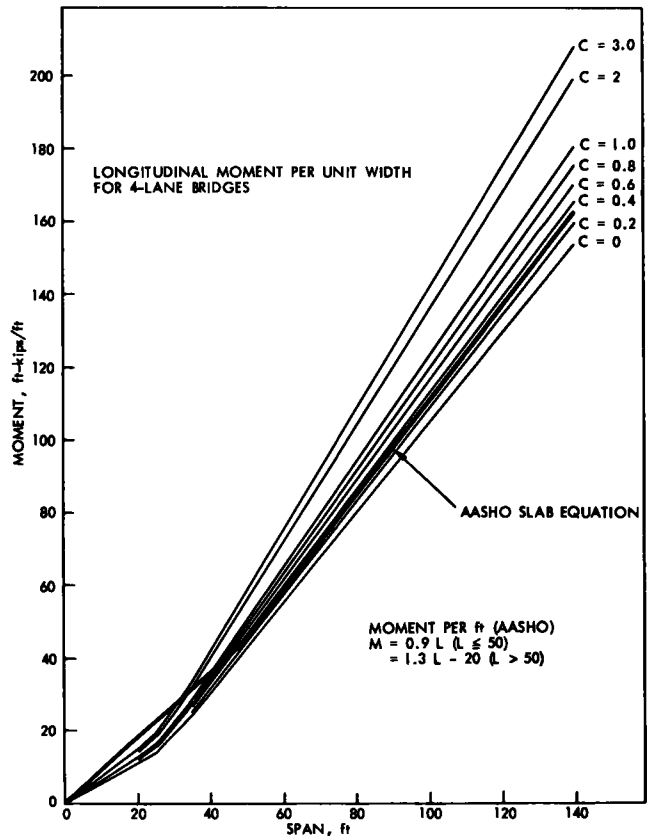


Figure 31. Comparison of design moments from Eq. 14q with current AASHO slab equation.

bridges. They concluded that as the flexural rigidity of the edge-stiffening beam increases, the absolute maximum value of K_m no longer occurs at the edge but moves to the center of the bridge. Also, the absolute value of K_m decreases at a diminishing rate. Although the study is on multi-beam

TABLE 13
CONCRETE BOX GIRDER BRIDGE PARAMETER STUDY RESULTS

N_w	W	N_g	N_s	L	d/L	C^a	VALUE OF D^b		
							TABLE 7	EQ. 14A	% DIFF.
4	33	5	0	50	0.07	0.952	6.35	6.34	+0.2
					0.05	0.858	6.47 (6.53)	6.43	+0.6 (+1.6)
				80	0.07	0.693	6.67	6.63	+0.6
					0.05	0.620	6.76	6.75	+0.1
				110	0.07	0.562	6.86	6.82	+0.6
					0.05	0.500	6.92 (7.00)	6.90	+0.3 (+1.4)
			1	50	0.07	0.674	7.04	6.65	+5.9
				110	0.05	0.354	7.43	7.11	+4.5
			2	50	0.07	0.550	6.52	6.82	-4.4
4	37	5	0	50	0.07	1.029	6.39	6.24	+2.4
					0.05	0.931	6.61 (6.69)	6.34	+4.3 (+5.5)
				80	0.07	0.747	6.90	6.58	+4.9
					0.05	0.671	7.08	6.65	+6.5
				110	0.07	0.606	7.21	6.75	+6.8
					0.05	0.540	7.34 (7.44)	6.82	+7.6 (+8.2)
			1	50	0.07	0.728	7.60	6.58	+15.5
				110	0.05	0.382	8.28	7.07	+17.1
			2	50	0.07	0.594	6.76	6.77	-0.1
6	39	5	0	50	0.07	1.068	6.22	6.20	+0.3
					0.05	0.969	6.26 (6.29)	6.28	-0.3 (+0.1)
				80	0.07	0.774	6.39	6.50	-1.7
					0.05	0.695	6.39	6.56	-2.6
				110	0.07	0.626	6.50	6.67	-2.5
					0.05	0.560	6.48 (6.53)	6.73	-3.7 (-3.0)
			1	50	0.07	0.755	6.63	6.50	+2.0
				110	0.05	0.396	6.80	6.90	-1.4
			2	50	0.07	0.617	6.25	6.67	-6.3
6	41	5	0	50	0.07	1.107	6.03	6.15	-1.9
					0.05	1.001	6.08	6.24	-2.6
				80	0.07	0.800	6.23	6.45	-3.4
					0.05	0.719	6.25	6.56	-4.7
				110	0.07	0.647	6.35	6.62	-4.1
					0.05	0.579	6.36	6.69	-4.9
			1	50	0.07	1.180	5.99	6.09	-1.6
					0.05	1.074	6.08 (6.13)	6.20	-1.9 (-1.1)
				80	0.07	0.853	6.25	6.39	-2.2
					0.05	0.767	6.33	6.50	-2.6
				110	0.07	0.689	6.43	6.58	-2.3
					0.05	0.619	6.48 (6.54)	6.67	-2.8 (-2.1)
6	45	5	0	50	0.07	0.885	6.50	6.37	+2.0
					0.05	0.436	6.93	6.86	+1.0
				80	0.07	0.646	6.20	6.62	-6.3
			1	50	0.07	1.454	5.94	5.87	+1.2
					0.05	1.305	6.03	5.98	+0.8
				80	0.07	1.071	6.23	6.20	+0.5
					0.05	0.950	6.32	6.30	+0.3
				110	0.07	0.875	6.40	6.37	+0.5
					0.05	0.774	6.48	6.50	-0.3
6	45	7	0	50	0.07	1.019	6.36	6.24	+1.9
					0.05	0.547	6.90	6.73	+2.5
				80	0.07	0.840	6.09	6.39	-4.7
			1	50	0.07	1.542	5.85	5.81	+0.7
					0.05	1.387	6.01 (6.06)	5.92	+1.5 (+2.4)
				80	0.07	1.131	6.24	6.13	+1.6
					0.05	1.006	6.41	6.24	+2.7
				110	0.07	0.924	6.50	6.35	+2.4
					0.05	0.817	6.66 (6.74)	6.43	+3.6 (+4.8)
6	49	7	0	50	0.07	1.091	6.51	6.15	+5.8
					0.05	0.578	7.29	6.69	+9.0
				80	0.07	0.891	6.11	6.37	-4.1
			1	50	0.07	1.581	6.15	5.82	+5.6
					0.05	1.428	6.17 (6.20)	5.92	+4.2 (+4.7)
				80	0.07				
				110	0.07				

TABLE 13 (Continued)

N_w	W	N_g	N_d	L	d/L	C^a	VALUE OF D^b		
							TABLE 7	EQ. 14A	% DIFF.
8	53	7	0	80	0.07	1.160	6.30	6.10	+3.3
					0.05	1.035	6.32	6.19	+2.1
				110	0.07	0.945	6.40	6.27	+2.1
					0.05	0.838	6.41 (6.44)	6.38	+0.5 (+0.9)
				1	50	0.07	1.119	6.49	+6.0
					110	0.05	0.592	6.60	+1.4
				2	50	0.07	0.915	6.18	-2.1
					50	0.07	1.627	5.91	+2.1
				80	0.05	1.464	5.95	5.90	+0.8
					0.07	1.190	6.10	6.08	+0.3
				110	0.05	1.060	6.14	6.18	-0.6
					0.07	0.969	6.23	6.25	-0.3
	57	7	0	50	0.05	0.858	6.26	6.34	-1.3
					0.07	1.706	5.65	5.75	-1.7
				80	0.05	1.535	5.73 (5.77)	5.84	-1.9 (-1.2)
					0.07	1.250	5.89	6.03	-2.3
				110	0.05	1.115	5.98	6.12	-2.3
					0.07	1.016	6.06	6.21	-2.4
				50	0.05	0.901	6.15 (6.20)	6.31	-2.5 (-1.7)
					0.07	1.788	5.79	5.69	+1.8
				80	0.05	1.610	5.89	5.80	+1.6
					0.07	1.302	6.06	5.99	+1.2
				110	0.05	1.164	6.19	6.10	+1.5
					0.07	1.060	6.27	6.18	+1.5
8	61	7	0	50	0.05	0.945	6.40	6.27	+2.1
					0.07	1.263	6.18	6.01	+2.8
				110	0.05	0.668	6.90	6.51	+6.0
					0.07	1.032	5.96	6.19	-3.7
				1	50	0.07	2.108	5.68	+2.2
					0.05	1.878	5.77 (5.81)	5.66	+1.9 (+2.7)
				80	0.07	1.555	5.97	5.84	+2.2
					0.05	1.377	6.11	5.95	+2.7
				110	0.07	1.276	6.19	6.01	+3.0
					0.05	1.128	6.33 (6.39)	6.12	+3.4 (+4.4)
				1	50	0.07	1.491	5.98	+1.7
					110	0.05	0.798	6.71	+4.8
				2	50	0.07	1.219	5.78	-4.6
	75	9	0	50	0.07	2.413	5.89	5.65	+4.2
					0.05	2.168	5.96	5.69	+4.7
				80	0.07	1.777	6.00	5.80	+3.5
					0.05	1.582	6.06	5.89	+2.9
				110	0.07	1.451	6.08	5.95	+2.4
					0.05	1.285	6.12	6.02	+1.7

^a By Eq. 17.^b Values in parentheses refer to cases where the thicknesses were varied

bridges, the same conclusions are applicable to slab bridges.

The edge member decreases the maximum longitudinal moments in two ways (199):

1. The decrease in the mean moment caused by the additional stiffness at the edges (due to increase in the effect width).

2. The reduction in the maximum distribution coefficient, due primarily to the edge shear forces.

The decrease in the longitudinal moment per unit width may be readily taken into account by using the effective width,

$$W_e = W_c + 2 \frac{I_e}{I_s} S_e \quad (22)$$

in which

W_c = curb to curb width;

I_e = moment of inertia of edge beam (curb) per unit width;

I_s = moment of inertia of slab per unit width; and

S_e = width of edge beam (curb).

Thus, for slab bridges, the load factor per foot of width is $1/D$. For the curb portion of the bridge, the effective width is

TABLE 14
VALUES OF K TO BE USED IN $C=K(W/L)$

BRIDGE TYPE	DECK MATERIAL AND BEAM TYPE	K
Beam and slab (includes concrete slab bridge)	Concrete deck:	
	Noncomposite steel I-beams	3.0
	Composite steel I-beams	4.8
	Nonvoided concrete beams (prestressed or reinforced)	3.5
	Separated concrete box-beams	1.8
	Separated steel box-beams (composite box girders)	2.6
	Concrete slab bridges	0.6
	Nonvoided rectangular beams	0.7
	Rectangular beams with circular voids	0.8
	Box section beams	1.0
	Channel beams	2.2
	Without interior diaphragms	1.8
	With interior diaphragms	1.3

$$S_e = \frac{I_e}{I_s} S_c \quad (23)$$

and

$$L.F. = S_e/D \quad (24)$$

For low values of I_e/I_s it is felt that this design procedure is sufficiently accurate.

CONTINUITY EFFECTS

Bridges that have end conditions other than simple supports as assumed in this report require special attention. When the ends of a bridge are restrained against rotation, the immediate effect is the reduction of the mean positive moment.

The secondary moments in the bridge due to its flexibility and the eccentricity of the loading are not equally reduced. This problem has been given some attention by Rowe (199). The method of design in this case can be handled by assuming that the effective length of the bridge for load distribution effects (Eq. 14a) is the distance between points of contraflexure of the bridge. Eq. 15 can then be used as before to determine the load factor per beam. It is felt that this procedure will be conservative but should be clarified through future additional theoretical work. In the case of concrete box girder bridges, considerable work on the effects of continuity has been conducted and is continuing at the University of California at Berkeley (221). However, no specific design recommendations have been published.

CHAPTER FIVE

PROPOSED REVISIONS TO AASHTO SPECIFICATIONS

GENERAL

In the previous chapters, the bases for the proposed revisions to the specifications (279) have been presented. The developments of the proposals were outlined specifically in Chapter Four.

The current procedures for distribution of loads have been shown generally to be conservative in predicting beam moments. Numerous investigators (2, 6, 135, 151, 182, 222, 228) have realized, however, that more realistic procedures are required and have proposed numerous revisions to the

specifications for specific bridge types. It was the purpose of this investigation to make an over-all study of the wheel load distribution in most of the types of short- and medium-span bridges and propose revisions where required.

Because of the extreme simplicity of the current requirements, it is obvious that any change to make them more realistic must entail some increase in complexity. The proposals presented herein are a balance between the need for an accurate distribution criterion and for a usable design office criterion. It should be noted that as the complexity

of the bridge system increases, the simplification of the theoretical procedures requires more approximations. Thus, considerations of unusual conditions are required. The use of any theory outlined herein for a total computerized analysis of the basic behavior will lead to the most accurate design and would be the optimum considerations. It is felt, though, that the changes proposed will lead to sufficiently accurate designs.

PROPOSED SPECIFICATIONS

Based on the research summarized herein, the following revisions are recommended in "Section 3, Distribution of Loads" of the 1965 edition of the AASHTO *Standard Specifications for Highway Bridges* (279) as revised by the 1966-1967 Interim Specifications.

1.3.1—DISTRIBUTION OF LOADS TO STRINGERS, LONGITUDINAL BEAMS AND FLOOR BEAMS.

- (A) Position of Wheel Loads for Shear—*unchanged*.
- (B) Live Load Bending Moment in Stringers and Longitudinal Beams for Bridges Having Concrete Decks.*

In calculating bending moments in longitudinal beams or stringers, no longitudinal distribution of the wheel load shall be assumed. The lateral distribution shall be determined as follows:

(1) Load Fraction (all beams).

The live load bending moment for each beam shall be determined by applying to the beam the fraction of a wheel load (both front and rear) determined by the following relations:

$$\text{Load Fraction} = \frac{S}{D}$$

in which S is

S_a for beam and slab bridges,†

$$\frac{12N_L + 9}{N_g} \text{ for multi-beam bridges,‡}$$

the maximum of the two values for concrete box girder bridges, and the value of D is determined by the following relationship:

$$D = 5 + \frac{N_L}{10} + \left(3 - \frac{2N_L}{7}\right) \left(1 - \frac{C}{3}\right)^2 \quad C \leq 3$$

$$= 5 + \frac{N_L}{10} \quad C > 3$$

* In view of the complexity of the theoretical analysis involved in the distribution of wheel loads to stringers, the empirical method described herein is authorized for the design of normal highway bridges. This section is applicable to beam and slab, concrete slab, multi-beam, and concrete box girder bridges. For composite steel box girder bridges, the criteria specified in Article 1.7.104 should be used.

† For slab bridges, $S = 1$ and the load fraction obtained is for a 1-ft width of slab.

‡ A multi-beam bridge is constructed with precast reinforced or prestressed concrete beams placed side by side on the supports. The interaction between the beams is developed by continuous longitudinal shear keys and lateral bolts, which may or may not be prestressed.

in which

- S_a = average beam spacing, in feet;
- N_L = total number of design traffic lanes from Article 1.2.6;
- N_g = number of longitudinal beams; and
- C = a stiffness parameter that depends on the type of bridge, bridge and beam geometry, and material properties.

The value of C is to be calculated using the following relationships. However, for preliminary designs C can be approximated using the values given in Table 1.3.1. For beam and slab ¶ and multi-beam bridges,

$$C = \frac{W}{L} \left[\frac{E}{2G} \frac{I_1}{(J_1 + J_t)} \right]^{\frac{1}{2}}$$

For concrete box girder bridges,

$$C = \frac{1}{2} \frac{W}{L} \left(1 + N_g \sqrt{\frac{d}{W}} \right) \left[\frac{E}{2G(1 + N_d)} \right]^{\frac{1}{2}}$$

in which

- W = the over-all width of the bridge, in feet;
- L = span length, in feet (distance between live load points of inflection for continuous spans);
- E = modulus of elasticity of the transformed beam section;
- G = modulus of rigidity of the transformed beam section;
- I_1 = flexural moment of inertia of the transformed beam section per unit width; §
- J_1 = torsional moment of inertia of the transformed beam section per unit width § $\left(= J_{\text{beam}} + \frac{1}{2} J_{\text{slab}} \right)$

J_t = $\frac{1}{2}$ of the torsional moment of inertia of a unit width § of bridge deck slab; **

and, for concrete box girder bridges:

- d = depth of the bridge from center of top slab to center of bottom slab;
- N_g = number of girder stems; and
- N_d = number of interior diaphragms.

For concrete for girder bridges, the cantilever dimension of any slab extending beyond the exterior girder shall preferably not exceed $S/2$.

When the outside roadway beam or stringer supports the sidewalk live load and impact, the allowable stress in the beam or stringer may be increased 25 percent for the combination of dead load, sidewalk live load, traffic live load, and impact.

¶ For noncomposite construction, the design moments may be determined in proportion to the relative flexural stiffnesses of the beam and slab section.

§ For the deck slab and beams consisting of reinforced or prestressed concrete, the uncracked gross concrete section shall be used for rigidity calculations.

** For multi-beam bridges, the torsional moment of inertia shall be computed at the thinnest transverse cross section in the beams.

TABLE 1.3.1 VALUES OF K TO BE USED IN THE
RELATION: $C = K \frac{W}{L}$

BRIDGE TYPE	BEAM TYPE AND DECK MATERIAL	K
Beam and slab (includes concrete slab bridge)	Concrete deck:	
	Noncomposite steel I-beams	3.0
	Composite steel I-beams	4.8
	Nonvoided concrete beams (prestressed or re- inforced)	3.5
	Separated concrete box- beams	1.8
Multi-beam	Concrete slab bridge	0.6
	Nonvoided rectangular beams	0.7
	Rectangular beams with circular voids	0.8
	Box section beams	1.0
	Channel beams	2.2
Concrete box girder	Without interior diaphragms	1.8
	With interior diaphragms	1.3

(2) Total Capacity of Stringers.

The combined design load capacity of all the beams in a span shall not be less than required to support the total live and dead load in the span.

(3) Edge Beams (Longitudinal).

Edge beams shall be provided for all concrete slab bridges having main reinforcement parallel to traffic. The beam may consist of a slab section additionally reinforced, a beam integral with and deeper than the slab, or an integral reinforced section of slab and curb.

It shall be designed to resist a live load moment of $0.10PS$, where

P = wheel load, in pounds (P_{15} or P_{20}); and

S = span length, in feet.

This formula gives the simple-span moment. Values for continuous spans may be reduced 20 percent unless a greater reduction results from a more exact analysis.

(C) Live Load Bending Moment in Stringers and Longitudinal Beams Supporting Timber Floors and Steel Grids.††

(1) Interior Stringers and Beams.

(This section should include those parts of current Article 1.3.1(B)(1) which are applicable to these floor systems.)

(2) Outside Roadway Stringers and Beams.

The live load bending moment for outside roadway stringers or beams shall be determined by applying to the stringer or beam the reaction of the wheel load obtained by assuming the flooring to act as a simple span between stringers or beams.

(D) Bending Moment in Floor Beams (Transverse)—*unchanged*.

(E) Dead Load for Stringers and Beams.

The dead load considered as supported by the roadway

stringer or beam shall be that portion of the floor slab carried by the stringer or beam. However, curbs, railings and wearing surface, if placed after the slab has cured, may be considered equally distributed to all roadway stringers and beams.

1.3.2—*change titles and modify:*

DISTRIBUTION OF LOADS AND
DESIGN OF FLOOR SYSTEMS

(A) Concrete Slabs.

- (1) Span Lengths—same as Article 1.3.2(A).
- (2) Edge Distance of Wheel Load—same as Article 1.3.2(B).
- (3) Bending Moment—same as Article 1.3.2(C) except that Case B is applicable only to slabs supported by transverse floor beams and the approximate formula in this case should be changed to:
HS-20 Loading: $LLM = 900S$ foot-pounds.
HS-15 Loading: *unchanged*.
The last paragraph on lateral distribution for multi-beam bridges should be deleted.
- (4) Distribution Reinforcement—same as Article 1.3.2(E).
- (5) Shear and Bond Stress in Slabs—same as Article 1.3.2(F).
- (6) Unsupported Edges, Transverse—same as Article 1.3.2(G).
- (7) Cantilever Slabs—same as Article 1.3.2(H).
- (8) Slabs Supported on Four Sides—same as Article 1.3.2(I).
- (9) Median Slabs—same as Article 1.3.2(J).

(B) Timber Flooring.

Same as Article 1.3.4 except that subsection headings changed to:

- (1) Flooring, Transverse.
- (2) Flooring, Longitudinal.
- (3) Continuous Flooring.

(C) Composite Wood-Concrete Members.

Same as Article 1.3.5 except that subsection headings changed to:

- (1) Distribution of Concentrated Loads for Bending Moment and Shear.
- (2) Distribution of Bending Moments in Continuous Spans.
- (3) Design.

(D) Steel Grid Floors.

Same as Article 1.3.6 except that subsection headings changed to:

- (1) General.
- (2) Floors Filled with Concrete.
- (3) Open Floors.

1.3.3—MOMENTS, SHEARS AND REACTIONS—same as Article 1.3.7.

1.3.4—DISTRIBUTION OF WHEEL LOADS
THROUGH EARTH FILLS—same as Article 1.3.3.

†† Article 1.3.1(B)(2) is also applicable to this article.

COMMENTARY

The major change proposed is in Article 1.3.1(B), where a complete revision is recommended. The majority of the other changes suggested are only made in order to make the entire Section 3 consistent in design approach, inasmuch as many of the systems covered were not within the scope of this study. For example, it is suggested that current Article 1.3.2(D) on "Edge Beams, Longitudinal" be moved to Article 1.3.1(B)(3) in order that the design of longitudinal beams be consolidated. The change suggested in Article 1.3.2(A)(3) is due to the inclusion of slab bridge design in Article 1.3.1(B). Thus, the slabs designed under Article 1.3.2(A)(3) will be those spanning transverse floor beams and the longer spans are no longer applicable.

It should be noted that the proposal just presented is also based on a change in Article 1.2.6—TRAFFIC LANES. Because the lanes used in the development were 12 ft wide, it is further recommended that Article 1.2.6 be changed to:

The lane loading or standard trucks shall be assumed to occupy a width of 10 ft. These loads shall be placed in design traffic lanes having a width of 12 ft, which are placed in a position to produce maximum stress. The lanes may not overlap. The lane loadings or standard trucks shall be assumed to occupy any position within these individual design traffic lanes which will produce the maximum stress. The number of design traffic lanes shall be equal to the roadway width between curbs (in feet) divided by 12, *reduced to the nearest whole number*.

If this change is not considered in conjunction with the recommendations for changes in Section 3, the following definition should be used in Article 1.3.1(B)(1):

$N_L = W_C/12$, *reduced to the nearest whole number*; and
 W_C = roadway width between curbs, in feet.

In the development of the lateral load distribution criteria for composite steel-concrete box girders, the use of Article 1.2.9—REDUCTION IN LOAD INTENSITY, was not recommended. This recommendation was included in Article 1.7.104 of the 1966-1967 *Interim Specifications* (279). It is felt, however, that the purpose of Article 1.2.9 is to consider the probability of all lanes being fully loaded simultaneously. At those occasional periods when this loading may occur, the structure could sustain the overload temporarily. This overload could be considered as a factor being included in the factors of safety in the design stresses. In the few instances where less than all lanes are loaded to obtain the critical condition, the difference between the maximum beam moment for the fully loaded condition and the critical condition is very small. Thus, continued use of the load intensity reduction is recommended. Nevertheless, this use should be restricted to those cases where the specific critical lane loading pattern is known, such as reaction shear distribution (Article 1.3.1A). However, in the case of wheel load fractions used for determination of bending moment, where the critical loading pattern is not known and may be for less than the total number of lanes, the

reduction should not be used. This restriction is currently practiced and is consistent with Article 1.107.4.

The equations for C , the stiffness parameter, given in the proposed Article 1.3.1(B)(1) include the factor $E/2G$. It should be noted that further simplification of these equations can be obtained, if desired, by using the relationship between E , the modulus of elasticity, and G , the modulus of rigidity. If Poisson's ratio is assumed to be zero, the factor becomes 1. If, in the case of concrete box girder bridges, Poisson's ratio is assumed to be 0.15, the equation for C shown becomes identical with Eq. 17.

SIGNIFICANCE OF PROPOSED CHANGES

The proposed changes, in many cases, do not significantly affect current designs. However, they do make them more realistic and do consider the benefits derived from improving bridge properties. The design of a bridge using the detailed data outlined in Chapter Three would lead to even more accurate analysis, because conservative assumptions have been made in developing the empirical equations proposed.

In general, the proposal permits consideration of the significant variables affecting load distribution. Because the present AASHTO criteria were developed on the basis of the behavior of typical bridges of the types considered, it should be expected that the average values for distribution coefficients in the proposal would be close to those in the current specifications.

The major benefit of the proposal is consideration of the effect of individual and new bridge geometries on load distribution. For example, by increasing the slab thickness the load distribution characteristics will generally improve. This improvement is reflected in the proposed specifications, whereas it is not in the current specifications. Although, to some extent the torsional rigidity of the beams is considered in the present specifications (by use of different D values for steel stringers, concrete T-beams and concrete box girders), it is an integral part of the proposal for all variations of beam geometry. In addition, the aspect ratio (W/L) has a significant effect on the distribution, but is not currently considered. Because the designer can obtain lower design live load stresses per beam with appropriate changes in the cross section, he is more likely to incorporate them. Thus, economies should result.

In the present specifications, separate design criteria are proposed for interior and exterior beams, yet the study showed that the critical beam can be either, and is a function of the bridge properties and loading. Thus, a single criterion for all beams is proposed.

The specific significance of the changes is discussed in the following for each of the bridge types considered.

Concrete Slab Bridges

The significance of the proposal can be readily seen in Figure 31. For spans less than about 50 ft, the new criterion will generally require less moment than currently

TABLE 15

COMPARISON OF PROPOSED AND CURRENT SPECIFICATIONS
FOR BEAM AND SLAB BRIDGES

BEAM TYPE	SPAN RANGE (FT)	BRIDGE WIDTH RANGE (FT)	C RANGE ^a	VALUE OF D ^b	
				CUR- RENT	PROPOSED RANGE
Composite steel I-beam	41-90	29-37	1.96-2.40	5.5	5.3-5.7
Noncomposite steel I-beams	50-70	25-30	1.50-1.60	5.5	5.9
Concrete T-beams	40-70	29-36	1.33-1.46	6.0	5.9-6.1
Prestressed concrete I-beams	35-100	29-37	1.59-3.83	5.5	5.2-5.9 ^c
Prestressed concrete box beams	61-72	33-46	0.67-0.83	5.5	6.5-6.7

^a Based on typical bridges included in field tests and provided by various state highway departments.

^b In load fraction equation: $L.F. = S/D$.

^c Typical value for longer spans, about 5.8.

specified. This span is about the upper economic limit; thus, the section required to carry the static live load in this bridge type can be expected to have a similar or smaller thickness.

Beam and Slab Bridges

The D value currently required for beam and slab bridges varies from 5.5 for steel I-beam stringers and prestressed concrete girders to 7.0 for concrete box girders. The proposal will yield D values in about the same range, but, more important, will permit an increase in D if the specific cross section has the improved distribution properties. The specific benefits can be seen in Table 15. This effect of improved distribution can be noted, in particular, for prestressed concrete beams, where the D values can vary significantly, depending on the cross section. It should be noted that the use of the new criterion should lead to more economical designs as such changes as improved beam torsional rigidity will lower design beam moments, which is not generally the case presently.

Multi-Beam Bridges

The D value currently required for multi-beam bridges is based on slab design for main reinforcement parallel to traffic. The distribution width per wheel is equal to $4.0 + 0.06L$ and varies from 5.2 for a span of 20 ft to a

maximum of 7.0. This range is essentially the same as the proposed criteria will give, although the resultant distributions will not necessarily be the same. The relationship between the proposed and current specifications can be obtained by examining the distribution widths computed for the multi-beam bridges described in Chapter Two. These widths are given in Table 16. The ratio of the resultant wheel load fractions may vary somewhat, due to the different criteria for effective beam width. However, it can be seen that the new criterion considers the wheel to be distributed over a wider area (better distribution). These bridges are quite narrow in comparison to today's standards, where the actual and effective beam widths are practically identical and the difference between E and D given in the table would be a realistic comparison of design moment per beam. Thus, it is expected that economies will result.

Concrete Box Girder Bridges

The current specifications for concrete box girder bridges indicate that a wheel load shall be distributed over a width of 7 ft. It can be seen in Table 13 that this width (D) can actually vary from about 5.7 for short-span bridges with four lanes to about 8.3 for longer-span (110 ft) bridges with two lanes. However, the proposed specifications are somewhat conservative in the higher D values and the value is limited to about 7. Of more importance, it should be noted that for most designs the actual D value is about 10 percent less than currently specified, indicating that current designs may be slightly unconservative.

TABLE 16

DISTRIBUTION WIDTHS FOR MULTI-BEAM BRIDGES

BRIDGE	SPAN (FT)	C	E ^a (FT)	D ^b (FT)
North Carolina	30	1.28	5.80	6.02
Centerport	32	0.53	5.92	6.86
Langstone	31	0.59	5.86	6.76

^a Section 1.3.2(C)—Case B: Current specifications (279).

^b Proposed specifications.

Summary

In summary, the proposed specification does provide a more realistic approach to load distribution. In some cases, significant economies may result. In other cases, such as the concrete box girder bridges, higher design moments are specified. However, in each case, the load fraction applied to the beam, and the resultant moment, will more truly represent the actual conditions.

CHAPTER SIX

CONCLUSIONS

The purpose of the research summarized in this report was to study the distribution of wheel loads in highway bridges and to recommend, where warranted, changes in the AASHO *Standard Specifications for Highway Bridges*. The study was generally limited to short- and medium-span bridges of the following types: slab, beam and slab, multi-beam and concrete box girder.

After an extensive literature search and a study of available methods of analysis, it was found that the distribution of wheel loads in these bridge types could be accurately determined using the following theories:

1. Beam and slab bridges: orthotropic plate theory.
2. Multi-beam bridges: articulated plate theory.
3. Concrete box girder bridges: prismatic folded-plate theory.

These procedures have been used to obtain extensive results relating the behavior of highway bridges with the variables affecting the behavior. From these studies it was found that:

1. Although generally predicting conservative load distribution in bridges, the current AASHO load distribution criteria (279) do not realistically consider the significant

variables affecting behavior. However, it should be noted that present criteria do give realistic values for many typical beam and slab bridges. Yet, substantial improvements in geometry can be made without a resulting change in the distribution of loads as currently specified.

2. Accurate empirical relationships between the variables which significantly affect the load distribution and the fraction of wheel loads carried by each beam can be obtained. Relationships of this type are presented herein.

3. The major variables which affect the load distribution in each of the major bridge types are: relative flexural stiffness in longitudinal and transverse directions, relative torsional stiffness in the same directions, bridge width, and effective bridge span. Each of these variables is considered in the relationships developed.

The results of the analytical studies and the development of empirical load distribution equations have been used to prepare specific recommendations for changes in the current load distribution criteria. It is felt that with these new criteria, prediction of wheel load distribution will be more accurate and will more truly indicate the behavior of the bridge types studied.

APPENDIX A

EVALUATION OF PARAMETERS FOR LOAD DISTRIBUTION IN HIGHWAY BRIDGES

In beam and slab bridges, the following two parameters are considered the most significant regarding the lateral distribution of wheel loads:

1. Flexural parameter, θ

$$\theta = \frac{W}{2L} \sqrt[4]{\frac{D_x}{D_y}} \quad (\text{A-1})$$

2. Torsional parameter, α :

$$\alpha = \frac{1}{2} \frac{D_{xy} + D_{yx}}{\sqrt{D_x D_y}} \quad (\text{A-2})$$

in which

L = span length, in feet;

W = bridge width, in feet;

$D_{xy} = G_c J_1$ = torsional stiffness in the longitudinal direction, in lb in.²/ft;

$D_{yx} = G_c J_t$ = torsional stiffness in the transverse direction, in lb in.²/ft;

D_x = flexural stiffness in the longitudinal direction, in lb in.²/ft; and

D_y = flexural stiffness in the transverse direction, in lb in.²/ft.

It was found that these two parameters can be combined into one parameter, C , to predict the wheel load distribution. This new parameter is defined by

$$C = \theta / \sqrt{\alpha} = \frac{\sqrt{2}}{2} \frac{W}{L} \sqrt[4]{\frac{D_x}{D_{xy} + D_{yx}}} \quad (\text{A-3})$$

Also, in multi-beam bridges, it was found that the most important cross-sectional parameter was ϕ as defined by

$$\phi = \frac{W}{2L} \sqrt{\frac{D_x}{D_{xy} + D_{yz}}} \quad (\text{A-4})$$

It can be easily seen that the parameters in beam and slab bridges and in multi-beam bridges are essentially identical, the only difference being the numerical constant. In other words, the relationship between parameters C and ϕ is

$$C = (\sqrt{2})\phi \quad (\text{A-5})$$

Therefore, the evaluation of the constant or parameter C can be carried out in the same manner both in beam and slab bridges and in multi-beam bridges.

The assumptions on which the parameter calculations are based may be summarized as:

1. A typical interior beam or diaphragm and its portion of the deck slab (the width of a beam or diaphragm spacing) are used for parameter calculations.
2. Full transverse flexural and torsional continuity of the diaphragms is assumed only when they are rigidly connected to the longitudinal beams.
3. The torsional rigidity of steel beams or diaphragms is ignored.
4. For flexural and torsional rigidity calculations of steel beam-concrete deck bridge types, the steel cross-sectional area should be expressed as an equivalent area of concrete.
5. The uncracked gross area of the concrete cross section may be used for rigidity calculations involving prestressed or reinforced concrete structural members.
6. Standard engineering procedures are used for computing the torsional and flexural rigidities of typical bridge systems.

Using these assumptions, the stiffness parameter, C , can be found by using bridge dimensions and the mechanical properties of materials for different cross sections as outlined in subsequent sections.

The effect of diaphragms is generally small or negligible and, thus, is normally not considered in determining the stiffness parameter, C . Thus, in subsequent sections the stiffness of the diaphragms is not indicated in the calculations. However, if consideration of the diaphragms is desired, the torsional stiffness in the transverse direction, D_{yz} , should be increased by the torsional stiffness of the diaphragm divided by its spacing (stiffness/ft).

STEEL BEAMS AND CONCRETE DECK

Noncomposite Cross Section

The moment of inertia of the total cross section of width S (see Fig. A-1a) is

$$I = n I_b + \frac{1}{12} S t_s^3 \quad (\text{A-6})$$

in which

- I_b = moment of inertia of a beam element, in in.⁴;
- n = modular ratio ($= E_s/E_c$);
- S = beam spacing, in feet; and
- t_s = thickness of slab, in inches.

Therefore, the longitudinal flexural stiffness is

$$D_x = \frac{E_c I}{S} = t_s^3 \left(\frac{n I_b}{S t_s^3} + \frac{1}{12} \right) E_c \quad (\text{A-7})$$

Also, the torsional stiffnesses are given by

$$D_{xy} = \frac{G_c}{6} t_s^3 \quad (\text{A-8a})$$

$$D_{yz} = \frac{G_c}{6} t_s^3 \quad (\text{A-8b})$$

Finally,

$$C = \frac{\sqrt{6}}{2} \sqrt{\frac{E_c}{G_c}} \frac{W}{L} \sqrt{\frac{n I_b}{S t_s^3} + \frac{1}{12}} \quad (\text{A-9})$$

For most cases, the second factor in the last square root term is negligible compared with the first, or

$$C = 1.9 \frac{W}{L} \sqrt{\frac{n I_b}{S t_s^3}}; \quad \nu = 0.2 \quad (\text{A-10a})$$

$$C = 1.7 \frac{W}{L} \sqrt{\frac{n I_b}{S t_s^3}}; \quad \nu = 0 \quad (\text{A-10b})$$

Moreover, the expression of C for slab bridges can be derived by neglecting the moment of cross section of steel beam element, I_b . Therefore, for slab bridges,

$$C = 0.55 \frac{W}{L} \quad \nu = 0.2 \quad (\text{A-11a})$$

$$C = 0.5 \frac{W}{L} \quad \nu = 0 \quad (\text{A-11b})$$

Composite Cross Section

The position of the neutral axis is (see Fig. A-1a) given by

$$\bar{d} = \frac{S t_s d_1 + n A_b d_2}{S t_s + n A_b} \quad (\text{A-12})$$

in which

- d_1 = distance indicating the position of the center of gravity of the slab portion;
- d_2 = distance indicating the position of the center of gravity of the beam portion; and
- A_b = cross-sectional area of the beam portion.

Therefore, the moment of inertia of the cross section is

$$I = n I_b + \frac{1}{12} S t_s^3 + S t_s (d_1 - \bar{d})^2 + n A_b (\bar{d} - d_2)^2 \quad (\text{A-13})$$

Thus, the flexural stiffness becomes

$$D_x = E_c t_s^3 \left[\frac{n I_b}{S t_s^3} + \frac{1}{12} + \frac{n A_b}{S t_s + n A_b} \left(\frac{d_1 - d_2}{t_s} \right)^2 \right] \quad (\text{A-14})$$

The torsional stiffnesses, as before, are given by Eqs. A-8. Finally,

$$C = \sqrt{6} \sqrt{\frac{E}{G}} \frac{1}{2} \frac{W}{L} \sqrt{\frac{n I_b}{S t_s^3} + \frac{1}{12} + \frac{n A_b}{S t_s + n A_b} \left(\frac{d_1 - d_2}{t_s} \right)^2} \quad (\text{A-15})$$

It was found that, practically, the entire last square root

term is approximately equal to $1.6 \sqrt{\frac{n I_b}{S t_s^3}}$.
Hence,

$$C = 3.0 \frac{W}{L} \sqrt{\frac{n I_b}{S t_s^3}}; \quad \nu = 0.2 \quad (\text{A-16a})$$

$$C = 2.8 \frac{W}{L} \sqrt{\frac{n I_b}{S t_s^3}}; \quad \nu = 0 \quad (\text{A-16b})$$

NONVOIDED CONCRETE BEAMS AND CONCRETE DECK

The expression for the flexural stiffness is exactly in the same form (Eq. A-7). The torsional stiffnesses are (see Fig. A-1b):

$$D_{xy} = G_c \sum_{i=1}^R K_i \frac{b_i}{S} t_i^3 + G_c \frac{1}{6} t_s^3 \quad (\text{A-17a})$$

$$D_{yx} = \frac{G_c}{6} t_s^3 \quad (\text{A-17b})$$

in which

b_i = length of the i th rectilinear portion of the concrete beam;

K_i = St. Venant's torsion constant for the i th rectilinear portion of the concrete beam; and

t_i = thickness of the i th rectilinear portion of the concrete beam.

(Eq. A-17b will be recognized as Eq. A-8b, previously given.)

Therefore,

$C =$

$$\frac{\sqrt{6}}{2} \sqrt{\frac{E_c}{G_c}} \sqrt{\frac{\frac{I_b}{S t_s^3} + \frac{1}{12} + \frac{n A_b}{S t_s + n A_b} \left(\frac{d_1 - d_2}{t_s} \right)^2}{1 + 3 \sum_{i=1}^R K_i \frac{b_i}{S} \left(\frac{t_i}{t_s} \right)^3}} \frac{W}{L} \quad (\text{A-18})$$

Use of the simplification

$$\sqrt{\frac{I_b}{S t_s^3} + \frac{1}{12} + \frac{n A_b}{S t_s + n A_b} \left(\frac{d_1 - d_2}{t_s} \right)^2} \doteq 1.6 \sqrt{\frac{n I_b}{S t_s^3}} \quad (\text{A-19})$$

gives

$$C = 1.6 \frac{\sqrt{6}}{2} \sqrt{\frac{E_c}{G_c}} \sqrt{\frac{I_b}{S t_s^3} \left[1 + \frac{h}{S} \left(\frac{t_w}{t_s} \right)^3 \right]} \quad (\text{A-20})$$

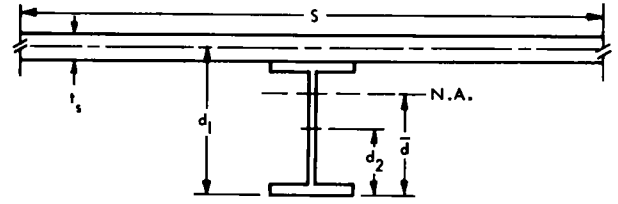
because, practically, $K_i \doteq \frac{1}{3}$ and

$$\sum_{i=1}^R K_i \frac{b_i}{S} \left(\frac{t_i}{t_s} \right)^3 \doteq \frac{1}{3} \frac{h}{S} \left(\frac{t_w}{t_s} \right)^3 \quad (\text{A-21})$$

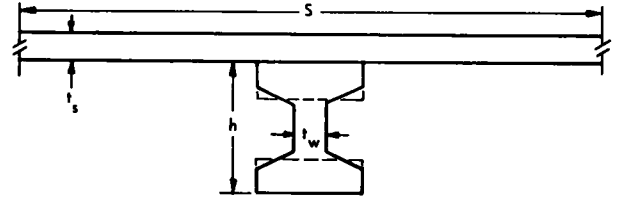
Finally,

$$C = 3.0 \frac{W}{L} \sqrt{\frac{I_b}{S t_s^3 + h t_w^3}}; \quad \nu = 0.2 \quad (\text{A-22a})$$

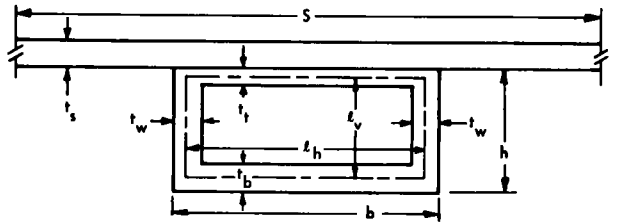
$$C = 2.8 \frac{W}{L} \sqrt{\frac{I_b}{S t_s^3 + h t_w^3}}; \quad \nu = 0 \quad (\text{A-22b})$$



A. STEEL BEAM AND CONCRETE DECK



B. NONVOIDED CONCRETE BEAM SHOWING ASSUMED RECTILINEAR SHAPES



C. SEPARATED CONCRETE BOX-BEAM

Figure A-1. Nomenclature for calculation of bridge stiffness parameter, C , for beam and slab bridges.

SEPARATED CONCRETE OR STEEL BOX BEAMS AND CONCRETE DECK

Again, the expression for the flexural stiffness is exactly in the same form (Eq. A-7) as before (see Fig. A-1c). The torsional stiffnesses at this time are as follows: *

$$D_{xy} = \frac{G_c}{6} t_s^3 + \frac{4(l_h l_v)^2 G_c}{\left(2 \frac{l_v}{t_w} + \frac{l_h}{t_t} \frac{t_t + t_b}{t_b} \right) S} \quad (\text{A-23a})$$

$$D_{yx} = \frac{G_c}{6} t_s^3 \quad (\text{A-23b})$$

in which

$$l_h = h - \frac{1}{2} (t_b + t_t) = h - \bar{t};$$

$$l_v = b - t_w;$$

$$\bar{t} = \frac{1}{2} (t_b + t_t)$$

t_b = thickness of the bottom flange of box beam, in inches;

t_t = thickness of the top flange of box beam, in inches;

t_w = web thickness, in inches;

* Equivalent transformed concrete areas shall be used for steel sections.

h = height of box beam element, in inches; and
 b = width of box beam element, in inches.

(Again, Eq. A-23b will be recognized as Eq. A-8b.)
 Therefore,

$$C = \frac{1.6}{2} \sqrt{\frac{E_c W}{G_o L}} \sqrt{\frac{I_b}{b^2 h^2} \left(\frac{h}{t_w} + \frac{b}{\bar{t}} \right)} \left(1 - \frac{t_w}{b} \right) \left(1 - \frac{\bar{t}}{h} \right) \quad (\text{A-24})$$

Here, the following approximate relationship was used:

$$\sqrt{\frac{n I_b}{S t_s^3} + \frac{1}{12} + \frac{n A_b}{S t_s + n A_b} \left(\frac{d_1 - d_2}{t_s} \right)^2} \doteq 1.6 \sqrt{\frac{n I_b}{S t_s^3}} \quad (\text{A-25})$$

Finally,

$$C = 1.2 \frac{W}{L} \sqrt{\frac{I_b}{b^2 h^2} \left(\frac{h}{t_w} + \frac{b}{\bar{t}} \right)} \left(1 + \frac{t_w}{b} \right) \left(1 + \frac{\bar{t}}{h} \right); \quad \nu = 0.2 \quad (\text{A-26a})$$

$$C = 1.1 \frac{W}{L} \sqrt{\frac{I_b}{b^2 h^2} \left(\frac{h}{t_w} + \frac{b}{\bar{t}} \right)} \left(1 + \frac{t_w}{b} \right) \left(1 + \frac{\bar{t}}{h} \right); \quad \nu = 0 \quad (\text{A-26b})$$

Note: It was assumed that the geometrical mean value is approximately equal to the algebraic mean value; i.e.,

$$\bar{t} = \frac{1}{2} (t_t + t_b) \doteq \sqrt{t_t t_b} \quad (\text{A-27})$$

Also, because t_w/b and \bar{t}/h are much smaller than unity,

$$1 / \left(1 - \frac{t_w}{b} \right) = 1 + \frac{t_w}{b}; \quad 1 / \left(1 - \frac{\bar{t}}{h} \right) = 1 + \frac{\bar{t}}{h} \quad (\text{A-28})$$

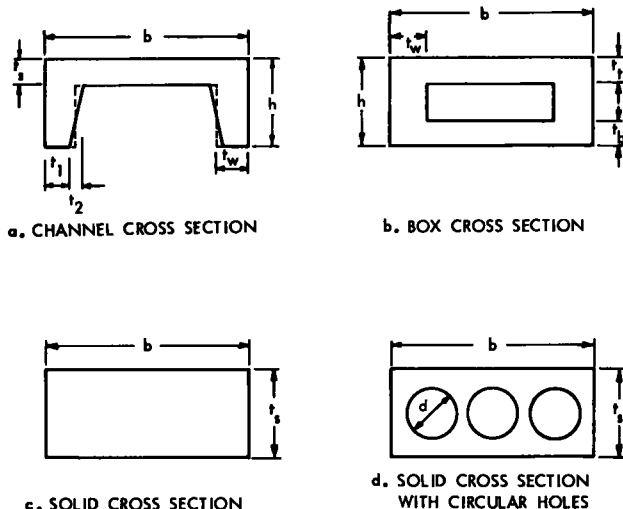


Figure A-2. Nomenclature for calculation of bridge stiffness parameter, C , for multi-beam bridges.

SOLID CONCRETE CROSS SECTION WITHOUT VOIDS FOR SLAB AND MULTI-BEAM BRIDGES

For slab bridges this type can be considered to be a special case of steel beams and concrete deck. Therefore, I_b and A_b can be set equal to zero in the expression of C for steel beams and concrete deck (refer to Figs. A-1a and A-2c). However, in the derivation of the expression for C in beam and slab bridges it was assumed that torsional stiffness D_{xy} is given by Eq. A-8a. Therefore, St. Venant's torsion constant, K_1 , for the longitudinal torsion in slabs was assumed to be $1/6$, which corresponds to the torsional resistance of a small transverse section of the slab. Hence, it may be better to use the general coefficient, K_1 , in slab bridges for the torsional stiffness in the longitudinal direction for multi-beam bridges. Now, the flexural stiffness can be obtained as follows:

$$D_x = \frac{1}{12} E_c t_s^3 \quad (\text{A-29})$$

Also, for torsional stiffness in general,

$$D_{xy} = G_c K_1 t_s^3 \quad (\text{A-30a})$$

$$D_{yx} = \frac{G_c}{6} t_s^3 \quad (\text{A-30b})$$

(Again, Eq. B-30b will be recognized as Eq. B-8b.)

Therefore, the following general expression for C is obtained:

$$C = \frac{1}{12 K_1 + 2} \frac{W}{L}; \quad \nu = 0 \quad (\text{A-31a})$$

Thus, for multi-beam bridges K_1 can be found from Table A-1 and for slab bridges K_1 can be assumed to $1/6$. Ordinarily, b/t_s is between 1.0 and 1.5 for multi-beam bridges; hence,

$$C = 0.5W/L; \quad \nu = 0 \quad (\text{A-31b})$$

It would be interesting to note that the expression for C is identical both for slab and multi-beam bridges. However, it was found necessary to revise the value of beam spacing when the same load distribution formula $L.F. = S/D$ is used in multi-beam bridges as was given in Chapter Four. That is, instead of using the real beam spacing, b , the following length should be considered for it:

$$S = (6N_w + 9)/N \quad (\text{A-32})$$

The comparison of the load fraction between slab bridge and multi-beam bridge is given in Table A-2, in which

$$D = 5 + \frac{N_w}{20} + \left(3 - \frac{N_w}{7} \right) \left(1 - \frac{C}{3} \right)^2; \quad C \leq 3 \quad (\text{A-33a})$$

$$= 5 + \frac{N_w}{20}; \quad C \geq 3 \quad (\text{A-33b})$$

and N is the number of girders.

SOLID CONCRETE CROSS SECTION WITH CIRCULAR VOIDS FOR MULTI-BEAM BRIDGES

Because the number of holes does not influence the value of C significantly, Eq. A-26 can be used (see Fig. A-2d).

BOX CONCRETE CROSS SECTION FOR MULTI-BEAM BRIDGES

The expression of C for this cross section is obtained as a special case of the cross sections dealt with under "Separated Concrete or Steel Box Beams and Concrete Deck" (see Fig. A-2b). Here, the moment of inertia of a beam portion is given by:

$$I_b = \frac{1}{12} [b h^3 - (b - 2t_w)(h - 2\bar{t})^3]$$

$$\doteq \frac{b h^3}{6} \left(1 - 2 \frac{\bar{t}}{h}\right) \left[3 \frac{\bar{t}}{h} + \frac{t_w}{b} \left(1 - 2 \frac{\bar{t}}{h}\right)^2\right]$$

$$\doteq \frac{b h^3}{6} \left(1 - 2 \frac{\bar{t}}{h}\right) \left[3 \frac{\bar{t}}{h} + \frac{t_w}{b} \left(1 - 4 \frac{\bar{t}}{h}\right)\right] \quad (\text{A-34})$$

in which

$$\bar{t} = \frac{1}{2} (t_t + t_b) \quad (\text{A-35})$$

However, ordinarily,

$$3 \frac{\bar{t}}{h} \gg \frac{t_w}{b} \left(1 - 4 \frac{\bar{t}}{h}\right) \quad (\text{A-36})$$

Therefore,

$$I_b = \frac{b h^3}{2} \left(1 - 2 \frac{\bar{t}}{h}\right) \frac{\bar{t}}{h} \quad (\text{A-37})$$

Assuming that the Poisson's ratio is zero,

$$C = 0.5 \frac{W}{L} \sqrt{1 + \frac{h}{b} \frac{\bar{t}}{t_w} \left(1 + \frac{t_w}{b}\right)}; \quad \nu = 0 \quad (\text{A-38})$$

CHANNEL CONCRETE CROSS SECTION FOR MULTI-BEAM BRIDGES

This section can be regarded as a special case of the non-voided concrete beams and concrete deck section (see

TABLE A-1
COEFFICIENTS FOR SOLID SECTIONS

$S/t_s = b/t_s$	K_t	C
1.0	0.14	0.521W/L
1.5	0.20	0.480W/L
2.0	0.23	0.459W/L
3.0	0.26	0.442W/L
10.0	0.31	0.418W/L
∞	0.33	0.408W/L

Fig. A-2a). The moment of inertia of the beam portion (actually, of the leg portion of the channel cross section) can be obtained as follows:

$$I_b = \frac{t_w h^3}{6} \left(1 - \frac{t_s}{h}\right)^3 \quad (\text{A-39})$$

in which

$$t_w \doteq \frac{1}{2} (t_1 + t_2) \quad (\text{A-40})$$

As in previous cases (in nonvoided concrete beams and concrete cross section), it can be considered that $K_t = 1/3$. Compared with continuous slab in a beam and slab bridge, the longitudinal torsional stiffness is somewhat different:

$$D_{xy} = \frac{G_c}{3} t_s^3 + \sum_{i=1}^{N-2} G_c K_t \frac{b_i}{b} t_i^3$$

$$= \frac{G_c}{3} t_s^3 + \frac{2}{3} G_c \frac{h - t_s}{b} t_w^3 \quad (\text{A-41})$$

Therefore,

$$D_{xy} + D_{yx} = G_c t_s^3 \left[\frac{1}{2} + \frac{2}{3} \frac{h - t_s}{b} \left(\frac{t_w}{t_s} \right)^3 \right]$$

$$= \frac{1}{2} G_c t_s^3 \left[1 + \frac{4}{3} \frac{h - t_s}{b} \left(\frac{t_w}{t_s} \right)^3 \right] \quad (\text{A-42})$$

TABLE A-2
LOAD FRACTIONS FOR SLAB AND MULTI-BEAM BRIDGES

W (FT)	N_w	SLAB BRIDGE		MULTI-BEAM BRIDGE	
		S (FT)	LOAD FRACTION	S (FT)	LOAD FRACTION
27	4	27/N	27/(ND)	33/N	33/(ND)
33	4	33/N	33/(ND)	33/N	33/(ND)
37	4	37/N	37/(ND)	33/N	33/(ND)
39	6	39/N	39/(ND)	45/N	45/(ND)
45	6	45/N	45/(ND)	45/N	45/(ND)
49	6	49/N	49/(ND)	45/N	45/(ND)
51	8	51/N	51/(ND)	57/N	57/(ND)
57	8	57/N	57/(ND)	57/N	57/(ND)
61	8	61/N	61/(ND)	57/N	57/(ND)
63	10	63/N	63/(ND)	69/N	69/(ND)
69	10	69/N	69/(ND)	69/N	69/(ND)
73	10	73/N	73/(ND)	69/N	69/(ND)

Then, C can be obtained as follows:

$$C = \frac{1}{\sqrt{3}} \frac{W}{L} (1.6) \sqrt{\frac{t_w (h - t_s)^3}{b t_s^3 + \frac{4}{3} (h - t_s) t_w^3}} \quad (\text{A-43})$$

in which it is assumed that

$$\sqrt{\frac{n I_b}{S t_s^3} + \frac{1}{12} + \frac{n A_b}{S t_s + n A_b} \left(\frac{d_1 - d_2}{t_s} \right)^2} = 1.6 \sqrt{\frac{n I_b}{S t_s^3}} \quad (\text{A-44})$$

Finally,

$$C = 0.92 \frac{W}{L} \sqrt{\frac{t_w (h - t_s)^3}{b t_s^3 + \frac{4}{3} (h - t_s) t_w^3}}; \quad \nu = 0 \quad (\text{A-45})$$

Also, the following expression was found to approximate the rigorous expression within small error:

$$C = 1.1 \frac{W}{L} \sqrt{\frac{t_w (h - t_s)^3}{b t_s^3 + h t_w^3}}; \quad \nu = 0 \quad (\text{A-46})$$

APPENDIX B

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