AN AMPLITUDE-FREQUENCY DESCRIPTION OF ROAD ROUGHNESS

Arthur D. Brickman, James C. Wambold, and John R. Zimmerman,
Department of Mechanical Engineering, Pennsylvania State University

This paper deals with a method for reducing roughness records obtained from a continuous-output road profilometer to a compact and useful form for highway engineers. It is assumed that a voltage signal representing vertical irregularities in the road surface as a function of distance has been recorded on magnetic tape. An analog method is described for processing this signal and reducing the roughness description of the road to a simple table relating roughness heights and wavelengths. Analog computer requirements are stated for the proposed signal-processing method. Results obtained by this method are presented for both ideal and actual road profile examples.

The development of the surface dynamic profilometer (SDP) has made it possible to obtain a magnetically taped record of any road surface profile in the direction of vehicle travel. For a given section of road, this record is a randomly varying analog voltage that represents true road roughness within the accuracy limitations of the SDP system. Having obtained a profile record, the highway engineer is faced with the problem of transforming this random signal into an index, graph, or set of numbers he can use to specify quantitatively the roughness of the road. This report describes a practical method for analyzing SDP records and for reducing each of these records to a simple form usable in highway operations.

Measurement of the profile roughness of a road is a preliminary step in finding its performance as a riding surface for vehicles and in judging its surface geometry. In recent years the trend in road profilometer design has been toward instruments capable of sensing undulations in the road surface with wavelengths as high as 25 ft, and doing this accurately at speeds of 30 mph or higher (1).

All commercially available high-speed profilometers can be divided into 2 groups. The output signal of the first type is a total number or discrete sets of counts related in some empirical way to the road surface conditions. In contrast, the output of the second type is a continuous graph or voltage signal that gives height variations of the road surface with respect to a reference system. The surface dynamics profilometer (SDP), originally developed by the General Motors Research Laboratory, is a device of the second category and employs a unique arrangement of motion transducers (2, 3). It is capable of measuring the roughness profile in a way that is virtually independent of profilometer vehicle characteristics. The measured profile appears as a continuous voltage signal that can be stored as a paper strip chart or an FM magnetic tape recording.

The major problem in applying the profilometer system is that of extracting useful roughness data from SDP tapes so that visual evaluation of the taped profiles becomes

Sponsored by Committee on Surface Properties-Vehicle Interaction.

unnecessary. The methods selected for reduction of road profile data depend on the ultimate uses for which the roughness measurements are intended and on the inherent limitations of the SDP equipment. Potential uses of roughness data include specification of surface profile limits in new road construction, prediction of wear and loss of serviceability in existing roads, establishment of maintenance and replacement criteria, correlation with vibrational response and fatigue damage in vehicles, development of passenger comfort criteria, and evaluation of roughness effects on vehicle steering and braking. Some of these applications will require highly sophisticated data processing methods leading to an entirely mathematical representation of the profile record such as its power spectral density. Other applications may require only an averaging or summing method to establish a single roughness criterion such as the BPR roughness index. Highway departments having SDP equipment are able, in effect, to bring the road surface into the laboratory and to seek the most useful data processing method.

Measured road profile records must be recognized as random signals of finite duration. As such, they can be viewed and described in terms of 3 basic "domains": time, amplitude, and frequency.

The time domain description is the unprocessed elevation-versus-time (or distance) trace itself. Any attempt to rate such records by visual examination alone is hopeless. However, magnetically taped profiles serve ideally as direct input signals for simulated vehicle systems where the objective is to study vibration response.

All types of amplitude domain descriptions of a road profile reduce the observed roughness measurement to a single number or table of values. Typically, single-number descriptions represent the total vertical excursion of a wheel axle with respect to the vehicle chassis. When tabular values are obtained from profilometer measurements, they show the total excursion of the wheel resolved into the number of motions occurring at each discrete level of magnitude. This is mathematically equivalent to computing a type of amplitude probability distribution for the roughness signal.

Frequency domain representations of roughness data are generally considered to be the most useful (4, 5). They are based on the concept that any observed random signal can be reconstructed by adding together a number of different sine waves. The graph showing the particular combination of sine wave amplitudes and frequencies required for this becomes the basic frequency description.

The amplitude and frequency descriptions of random signals, as well as their measurement and application to mechanical vibrations, are covered in a general way by textbooks and instrument manufacturers' literature (6, 7). Where road profiles are used in vehicle and passenger studies, the frequency method known as power spectral density (PSD) has gained wide acceptance because of its importance as an analytical tool. Despite this fact, there is an urgent need for methods that (a) display both the amplitude and wavelength features of road roughness, and (b) relate in some direct way to the physical appearance of the road surface.

FREQUENCY AND AMPLITUDE ANALYSIS

Power Spectral Density

A periodic function can be represented as a sum of sinusoids of different amplitudes and discrete frequencies. A nonperiodic function can also be represented by a collection of sinusoids, but their frequencies vary continuously over a wide range. The strength or content of a random function in a narrow frequency interval can be found experimentally by passing the sample record through a filter having a narrow pass band Δf at frequency f. The output of the filter is then the sum of all sinusoidal constituents of the original function having frequencies in the range $(f \pm \Delta f/2)$.

The PSD of a random function describes its frequency composition in terms of its mean square value. If the sample record is passed through a band-pass filter and then the average of the squared value of the filter output is computed (Fig. 1), an estimate is obtained for the mean square value of the content of the sample record in the frequency band for which the filter is set. The PSD corresponds to the mean square

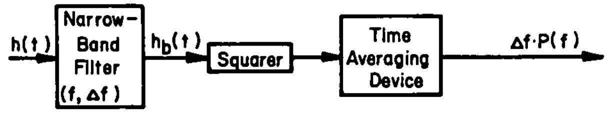

Figure 1. Analog scheme for computing power spectral density.

value of the filter output per unit of frequency bandwidth. For an infinitely long test record and an infinitely narrow filter bandwidth, the PSD of a random signal depends only on frequency and can be defined as follows (1):

$$P(f) = \lim_{\substack{T \to \infty \\ \Delta f \to 0}} \int_0^T \frac{h_b^2}{\Delta f} dt \tag{1}$$

where h_b is the output response of a band-pass filter set at frequency f and bandwidth Δf, and having the random signal h(t) as input. The method of representing PSD is shown in Figure 2. The total area under the curve represents the mean square of the signal, while the area of the crosshatched strip shows how much the components found in the Δf frequency band contribute to the mean square. The PSD value itself thus gives the intensity of the signal's mean square (i.e., power) associated with each frequency component.

Although a PSD graph provides a very compact way of describing a random signal and is useful in analytical studies to predict the random-input response of linear systems, the PSD representation does have a practical shortcoming. It does not show what particular combination of wave amplitudes in the signal produces the PSD reading at a given frequency. This is apparent, as shown in Figure 1 where the filter output signal $h_b(t)$, although restricted in frequency content, can have a variety of amplitude distributions and still result in the same computed value P(f) for the power spectral density.

Amplitude Distribution

If the frequency aspect of a random signal is disregarded and only variations in magnitude are considered, 2 important statistical characteristics of the signal can be determined. These are the amplitude probability density and the peak probability density. Figure 3a shows a random signal h(t) as it passes intermittently through a narrow band of values B just above amplitude A. The ratio of the total time $(t_1 + t_2 + t_3 + \dots)$ that the signal amplitude falls within the band B to the total duration of the signal is the probability of finding the signal level near amplitude A. A plot of this ratio for all positive and negative values of A gives the complete distribution of amplitudes, that is, the amplitude probability density.

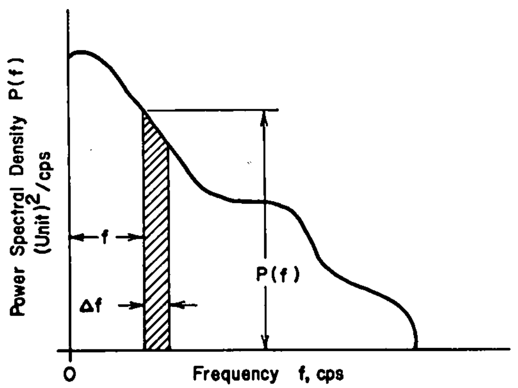

Figure 2. Representation of power spectral density function.

Figure 3b shows a random signal passing through a number of peaks $(P_1, P_2, P_3, \dots)$ that fall within the narrow band B and hence occur at amplitude A. The total number of such peaks, expressed as a fraction of the total number of peaks displayed by the signal over a long time period, can be plotted versus the amplitude. The resulting curve is the peak probability density for the random signal. A similar curve can be obtained for the occurrence of negative peaks such as N_1 shown in Figure 3b.

Amplitude and peak density curves offer a compact way of characterizing some of the important features of random data. Such curves are widely used in the study of fatigue and wear phenomena, and have provided satisfactory correlations in cases where PSD analysis failed. The computation of amplitude and peak distributions from tape recordings of random signals is a standard operation by either analog or digital methods, and it can be carried out to any degree of accuracy.

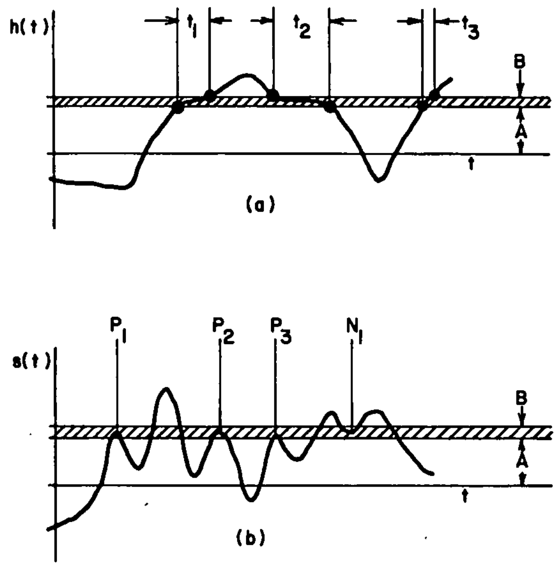

Figure 3. Occurrence of a random signal at (a) an amplitude level and (b) a peak value.

At first glance, it would appear that the PSD of a road profile contains a complete description of amplitude variations, thus making any amplitude-distribution calculations superfluous. Actually, however, the ordinate of a PSD curve indicates only the average roughness amplitude at a particular frequency. A large PSD value can conceivably be produced either by a few irregularities of large amplitude or a large number of small size: the distinction cannot be made from the PSD curve alone. On the other hand, it would not be possible to extract any information about roughness wavelength distribution in the road profile even though the amplitude density curves were known. It is evident from these considerations that the PSD and amplitude density curves each contain unique information about the road profile, and a simple method of combining the 2 representations would be desirable.

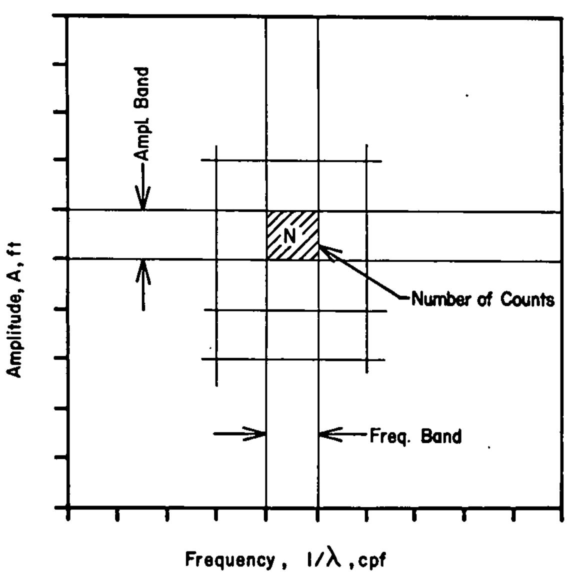

Figure 4. Basic elements of the amplitude-frequency description of a road profile.

AMPLITUDE-FREQUENCY PROCESSING OF PROFILOMETER RECORDS

An effective method for combining the information contained in both the PSD and the amplitude representations is to reduce the road profile signal to a simple tabular array that displays both the height and the length features of the surface roughness (Fig. 4). Here the coordinates, amplitude in feet versus frequency in cycles per foot, are divided up into a number of finite bands as shown. (The roughness "frequency" is the number of roughness cycles per foot of travel. This is the same as the

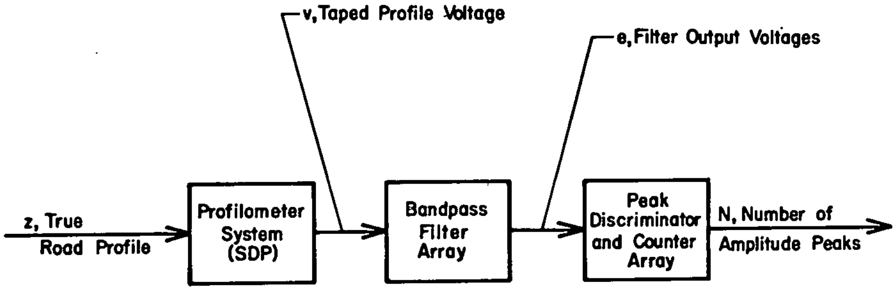

Figure 5. Profile data processing scheme to obtain the AFD tabulation.

reciprocal of the wavelength λ.) Numbers like N are computed and entered at each window-like intersection of the bands. Each of these numbers expresses the total count of road profile "bumps" having the height and length combination of that location in the table. The complete array of numbers thus identifies all the surface irregularities in a given length of road and shows its roughness as a combined amplitude and frequency distribution (AFD).

Computational Scheme

Reduction of a road profile to AFD form can be accomplished by the processing components shown in Figure 5. First, the SDP profilometer transforms the true road profile z into a tape-recorded voltage v. The accuracy with which the voltage reproduces the profile is affected by (a) the geometry and dynamics of the profilometer's trailing wheel, and (b) the deliberate amount of high-pass filtering used to remove the unwanted "hill and valley" features of the profile. Voltage signal v is then fed into a parallel array of band-pass filters, each of which is adjusted to span one of the frequency bands shown in Figure 4. Finally, the resulting filter voltages e are passed into an array of discriminator-counter units, each adjusted to cover a separate amplitude band and to produce one of the N values for the AFD tabulation.

It is important to note that quantities z, v, and e are random signals of finite duration; they can be visualized in graphical form as shown in Figure 6. Here L and T represent the length and traverse time of the test road respectively. Voltage v will have zero average value as shown in Figure 6b because of the de-trending effect of the on-board profilometer filter. Because of the band-pass filter characteristics, voltage e will appear to be a succession of sinusoidal cycles of nearly identical periods but randomly varying amplitude as shown in Figure 6c.

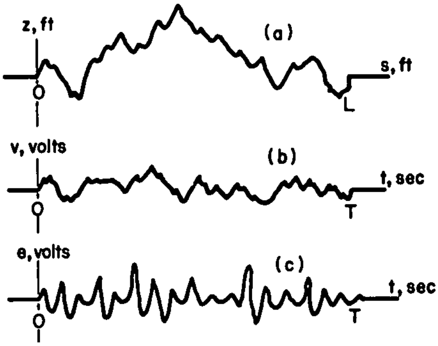

Figure 6. Appearance of random signals in AFD computer: (a) road profile, (b) taped profile voltage, and (c) filter output voltage.

An AFD array of reasonable simplicity was obtained by establishing the 6 amplitude and 8 frequency bands shown in Figure 7. Each frequency band spans a 1-octave logarithmic section of frequency scale, i.e., the lowest band includes wavelengths from 256 ft down to 128 ft, the next band extends from 128 to 64 ft, and so on to a wavelength of 1 foot. The total amplitude range extends from 0.070 to 1.00 ft (or 7 to 100 percent of the maximum value), and each band covers an equal logarithmic section of amplitude. Logarithmic rather than linear subdivisions of the AFD scales were selected to give a better resolution of roughness counts in the small-amplitude, short-wavelength section of the table. The extreme amplitude and wavelength combinations covered by the AFD array were selected to be within the measurement capability of the SDP profilometer (3).

Computer Implementation

A general-purpose analog computer equipped with digital and hybrid components was used to obtain the AFD table of roughness counts from available profilometer signals. A standard combination of second-order circuits was used to simulate the band-pass filter component. The circuit parameters were adjusted to give the attenuation characteristic shown in Figure 8. Within the 1-octave pass band, the filter attenuation curve is flat except for some rounding at the band limits. Outside the pass band, the curve drops off steeply at about 24 dB per octave down to an amplitude ratio of 0.1.

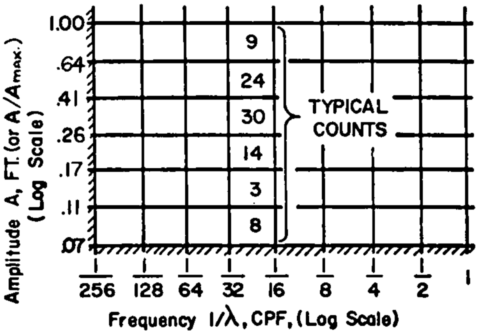

Figure 7. Logarithmic amplitude-frequency grid for display of profile AFD.

A simultaneous calculation of all 48 AFD roughness counts would require 8 filters each followed by an array of 6 discriminator-counter units. The large amount of equipment required for this was avoided by programming on the computer a single filter and its associated 6-stage counting array. A taped road profile signal was then played into the computer consecutively at each of the 4 available tape-recorded speeds. This resulted in the count numbers required for the low-frequency half of the columns of the AFD tabulations. The remaining counts were obtained by repeating this operation with the center frequency of the filter increased by 4 octaves.

There are several possible circuits for detecting the occurrence of instantaneous peaks in a randomly varying voltage. One stage of the particular circuit used in this study for peak detection in the filter output signal is shown in Figure 9. The potentiometers on the left are adjusted to produce 3 constant voltage levels corresponding to 3 contiguous amplitude levels of the AFD table. Each of these voltage levels, along

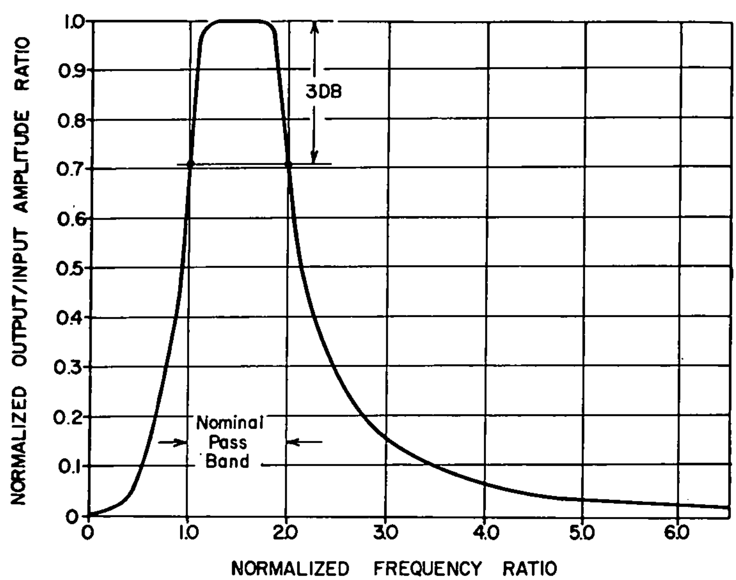

Figure 8. Amplitude attenuation characteristic of 1-octave band-pass filter.

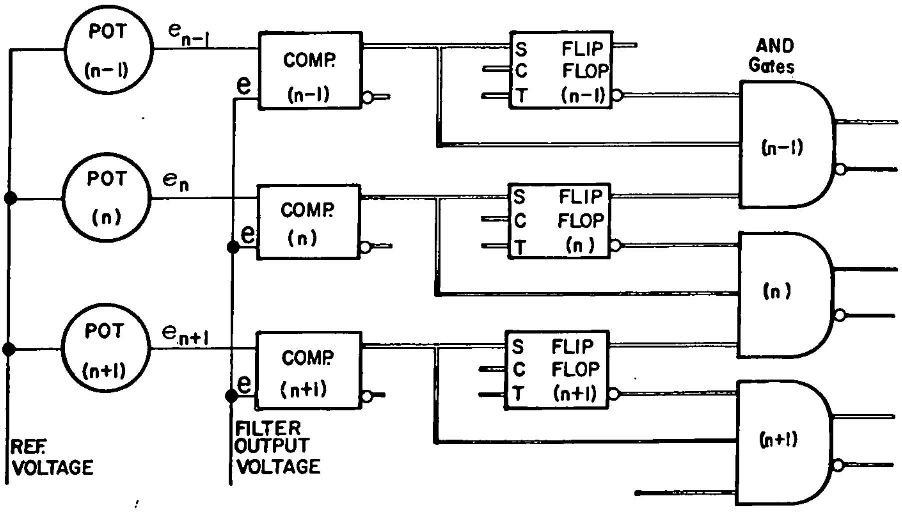

Figure 9. Hybrid computer circuit for detection of voltage peaks.

with the varying filter voltage e, is fed into a comparator whose output is a logic (on-off) voltage. Operation of the comparators can be inferred from the graph shown in Figure 10 in which voltage e passes through a peak spanned by the upper 2 pot voltages. As voltage e rises initially from zero, all comparator outputs are off; then, during interval i, comparator n - 1 turns on followed by comparator n during interval j. Both comparators switch off as e returns to zero.

The comparator outputs are fed to an interconnected set of logic flip-flops and AND gates as shown. These components have unique input-output characteristics and, like the comparators, produce logic output signals. When voltage e goes through a peak and reaches time interval k, the combined operation of the logic elements causes gate n to turn on and the other 2 gates to remain off. As e decreases to zero, the flip-flops are all reset, restoring the entire circuit to the off condition. This sequence of events repeats itself for every positive half-cycle variation in voltage e, producing 1 pulse

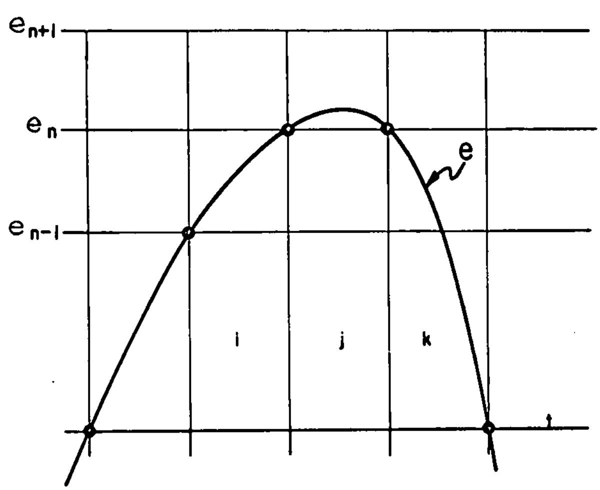

Figure 10. Typical half-cycle variation in filter output voltage.

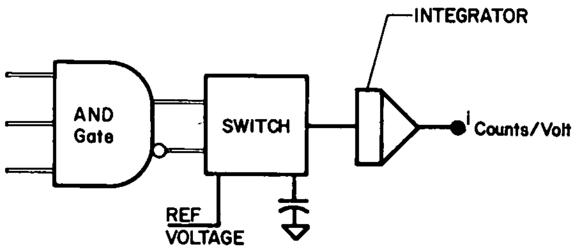

Figure 11. Hybrid computer circuit for peak counting.

output at the AND gate corresponding to each voltage peak. The circuit used to count the number of such pulses is shown in Figure 11. It consists of a logic-controlled switch that alternately charges a capacitor and discharges it into an integrator with the occurrence of a pulse from an AND gate. The voltage level ultimately appearing at the integrator output is proportional to the number of observed counts in the road profile signal.

Sources of Error

The analog filter introduces a number of inaccuracies into the computed AFD table. Each filter has a finite frequency bandwidth rather than the infinitely narrow characteristic on which the mathematical definitions are based. This means that each computed count value will actually be an average reading over a discrete section of the frequency spectrum, not a point value for individual frequencies. Greater accuracy could be gained by decreasing the filter bandwidths, but only at the expense of increasing the number of "windows" and overall complexity of the AFD tabulations.

As noted earlier, the filters do not have an ideally sharp and infinitely steep attenuation characteristics at the cutoff frequencies. This shortcoming will permit spurious AFD peaks to be counted and accumulated in locations outside the nominal frequency of each filter. However, even in the relatively unsophisticated filter used in this study, these peaks were found to be greatly attenuated and many of them fell below the 7 percent cutoff limit at the lower end of the amplitude scale.

Relation of AFD Results to PSD

The computed count numbers found in the amplitude-frequency distribution of a road profile bear a direct relation to the PSD of the profile. As shown in Figure 7, each count N can be multiplied by the square of its corresponding amplitude A, and a sum of such products can be obtained in one given frequency band. If this sum is divided by the total counts in that band, the ratio is theoretically equal to twice the mean square amplitude of the band-pass filter output. Thus by dividing the mean square by the bandwidth, an expression for PSD is obtained as follows:

$$P(1/\lambda) = \frac{\Sigma NA^2/\Sigma N}{2\Delta f/V} = \frac{\lambda \Sigma NA^2}{2L\Delta(1/\lambda)} \qquad (2)$$

A more direct calculation for PSD can be made by using the filter output voltage shown in Figure 5, as follows:

$$P(1/\lambda) = \int_0^T \frac{e^2 dt}{T\Delta(1/\lambda)} \qquad (3)$$

An integrator was added to the AFD computer circuit to obtain PSD values by this method.

Comparative tests of these computation methods, presented later in this report, show the errors inherent in the analog implementation to be of the order of a few percent. These errors can be kept under control through the use of high-quality electronic components having accurately specified performance characteristics.

TEST PROGRAM

In order to test the analog and digital procedures developed for data processing, 3 "idealized" road profiles were put on tape. These signals consisted of nonrandom voltages whose original properties were known exactly and whose PSD, AFD, and other computed properties could, therefore, be predicted with accuracy. By processing these idealized profiles and comparing the results obtained with the theoretical prediction, it was possible to check out the effectiveness of the computer circuitry and programming.

The specific idealized profile signals selected are shown in Figure 12 and itemized as follows. For all the signals a profilometer speed of V = 58.6 ft/sec (40 mph), a test road of length L = 2,640 ft (½ mile), and a corresponding signal duration T = 45 sec were assumed.

1. Test signal A is a sinusoidal voltage of constant amplitude having a frequency centered in the $\frac{1}{32}$ to $\frac{1}{16}$ cpf band shown in Figure 7.

$$v = A \sin 2\pi ft$$

where

λ = 22.6 ft,
$f = V/\lambda$ = 2.60 cps, and
A = 1.0.

2. Test signal B is a sinusoidal voltage of constant amplitude and gradually increasing frequency.

$$v = A \sin 2\pi ft$$

where

λ = 1 to 256 ft,
$f = V/\lambda$ = 0.228 to 58.4 cps,
$f = 0.228(10)^{0.0535t}$, and
A = 1.0.

3. Test signal C is a sinusoidal voltage of constant frequency having a gradually increasing amplitude.

$$v = A \sin 2\pi(2.60)t$$

where A = 0.0222t

In addition to the foregoing nonrandom signals, the profilometer records for 4 actual road profiles were recorded by means of the SDP and used for computational testing. The designation and identity of these signals is given in Table 1 where V, T, and L represent the profilometer speed, test duration, and road length respectively. Other profilometer data refer to the displacement calibration and the cutoff frequency of the high pass filter.

Computed AFD for Idealized Profile Signals

Figure 13 shows the AFD count pattern obtained for test signal A, a unit-amplitude sinusoid of 22.6-ft wavelength. The count of 124 in the top window of the band containing this wavelength is to be compared with the total of 118 actual peaks in the test signal. A theoretical filter having a perfect (rectangular) attenuation characteristic would have

TABLE 1

ACTUAL ROADS USED IN TEST

Test Signal Designation	County	Route and Start Marker	Pavement Type	Profilometer (SDP) Data				
				V (mph)	T (sec)	L (mi)	Calibr. Scale (in./v)	Filter Setting (rad/sec)
D	Perry	LR 1033/0+00	Rigid	40	45	0.5	5.0	1.0
E	Lebanon	LR 10125/408	Rigid	40	45	0.5	1.67	0.6
F	Perry	Rte 15-11/180	Flexible	40	45	0.5	5.0	1.0
G	Perry	Tre 274/482	Flexible	34	53.5	0.5	5.0	3.0

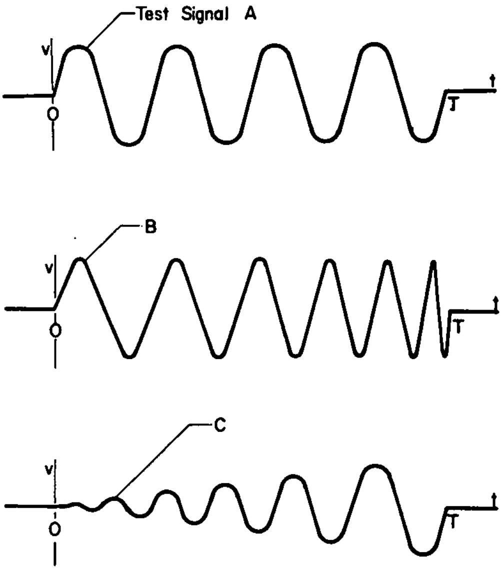

Figure 12. Idealized test profile signals.

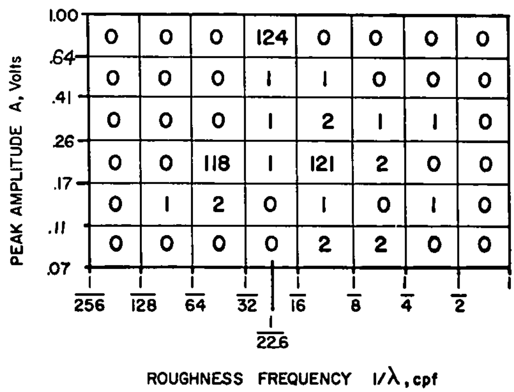

Figure 13. Computed AFD for test signal A, a constant sinusoid.

produced zero counts everywhere else. Although the actual filter does display some large counts in the adjoining frequency bands, these counts are seen to be at a lower amplitude level and could be reduced by a filter with better side band characteristics.

For test signal B, a constant amplitude sinusoid whose frequency doubles 8 times in the record duration, the theoretical count pattern should read 2, 4, 5, 15, 31, 123, and 247 along the top amplitude band and zero elsewhere. The computed AFD values shown in Figure 14 reflect this upper band count very well, but there is a large scatter of lower counts at lower amplitudes again due to filter roll-off.

Figure 15 shows the AFD pattern for test signal C, which lasts 45 sec and has 118 equally spaced peaks increasing linearly with time. A perfect discriminator set for the given logarithmic amplitude bands would produce counts of 35, 33, 21, 11, 6, and 4 in the major frequency column and zero counts elsewhere. The actual AFD computer yields a reasonably close version of the ideal count pattern but displays some side band counts of low amplitude.

Test Results for Road Profile Signals

The computed AFD tables for the 4 test roads (signals D, E, F, and G) are shown in Figures 16, 17, 18, and 19. Although the actual profiles vary randomly, the count values give a representation of the equivalent number and wavelength of roughnesses present in a measured sample of each road surface. The counts appearing in a given column

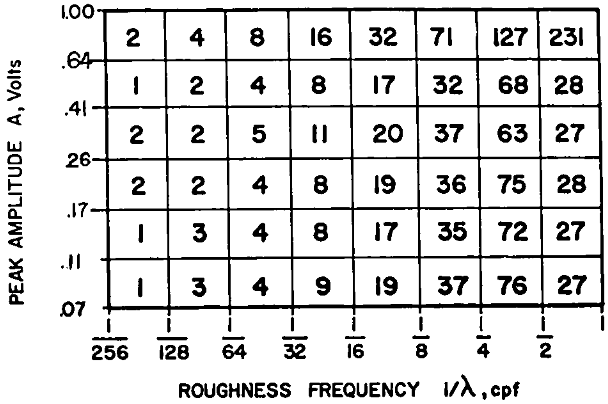

Figure 14. Computed AFD for test signal B, a variable frequency sinusoid.

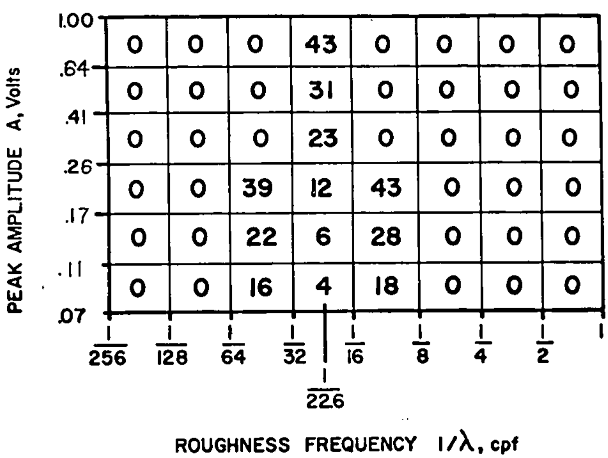

Figure 15. Computed AFD for test signal C, a variable amplitude sinusoid.

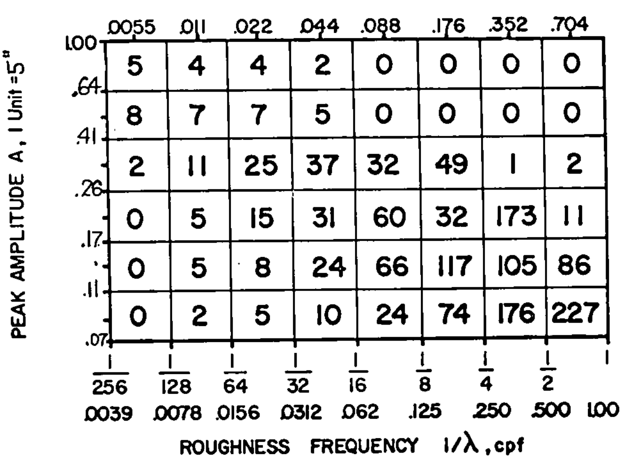

Figure 16. Computed AFD for road profile D.

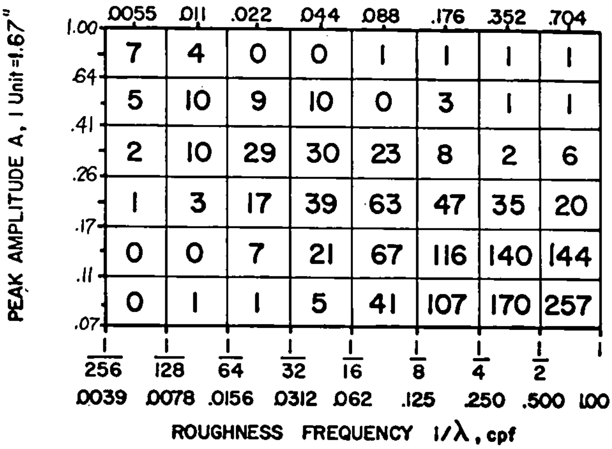

Figure 17. Computed AFD for road profile E.

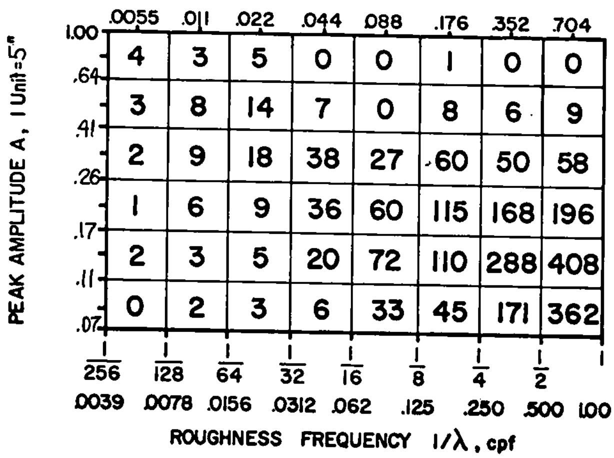

Figure 18. Computed AFD for road profile F.

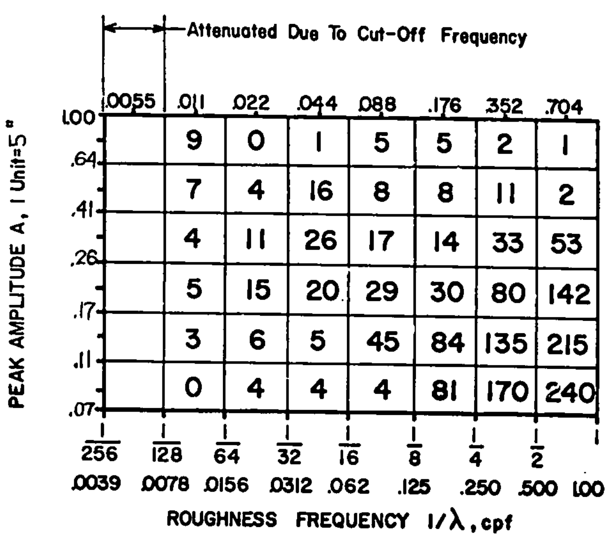

Figure 19. Computed AFD for road profile G.

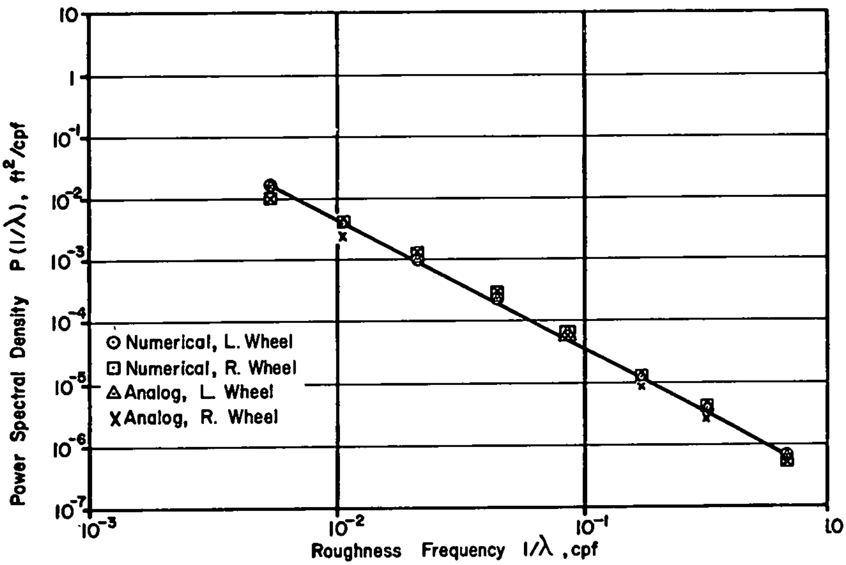

Figure 20. Computed PSD for road profile D.

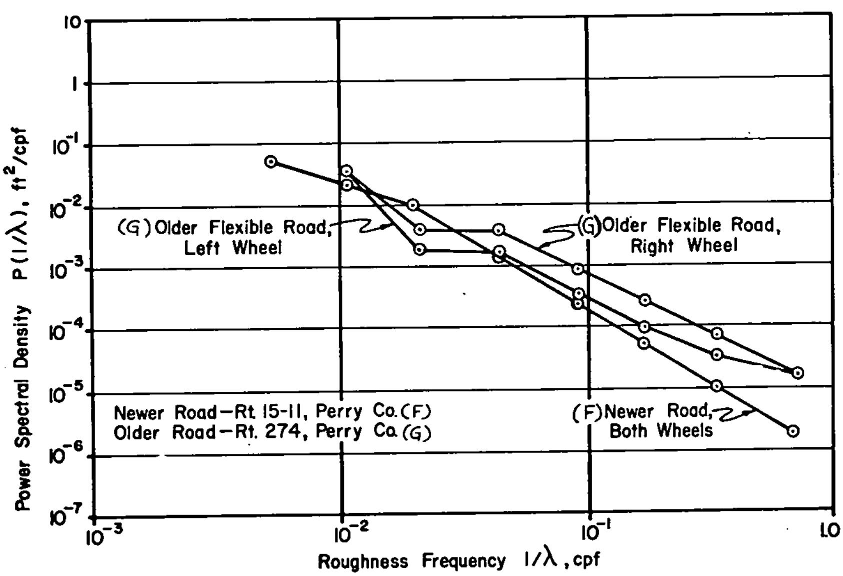

Figure 21. Computed PSD for road profiles F and G.

indicate the amplitude distribution of all roughnesses having the same wavelength, whereas the counts appearing in a given line show the wavelength variations of roughnesses having the same height. For each column of the array, a single value on the PSD curve for the road profile can be computed from Eq. 2 and compared with the PSD results from other methods.

For the sample hard-surface road profiles, the AFD results shown in Figures 16 and 17 show roughness counts that roughly double along a diagonal from upper left to lower right across each successive frequency band. A comparison of these AFD's with those of the flexible-pavement surfaces, shown in Figures 18 and 19 indicates that the latter have a broader amplitude distribution contributing to the roughness in any given wavelength band.

Figure 20 shows the computed PSD values for road profile signal D versus the mid-band value of each of the 8 wavelength increments. There is very little spread between the numerical points computed from the AFD counts via Eq. 2 and the analog points computed from Eq. 3. It is seen that, for the log-log coordinates employed, the PSD is practically a straight line, a characteristic of many hard-surface roads noted by other investigators. Figure 21 shows PSD values computed from the AFD arrays for the F and G flexible-pavement profiles. Except for the discontinuities in the 25- to 50-ft wavelength range, these curves show the generally linear trend of the hard roads.

CONCLUSIONS

The test results computed for both idealized and actual road profile records indicate that the AFD method holds considerable promise as a means of representing the roughness present on a road surface. Specific features of the AFD method are as follows:

1. The AFD tabulation is a method of representing the roughness profile of a road surface in a quantitative way: It not only shows what each wavelength contributes to the overall roughness but also gives the distribution of roughness heights associated with each amplitude.
2. The AFD of a roughness signal can be computed electronically by using standard analog components for filtering, comparing, and counting. The precision of the computed AFD is greatly affected by the shape factor of the filter.
3. The count values appearing in the AFD tabulation relate theoretically to the PSD of the road profile, and discrete values of roughness PSD can be computed from the AFD counts.
4. By reducing the roughness profile of a road to a table of heights and wavelengths, the AFD table describes the equivalent physical appearance of a road surface in a more understandable way than PSD.

ACKNOWLEDGMENTS

This research was conducted under a 1968-69 contract with the Pennsylvania Department of Transportation and the Federal Highway Administration. The work was supervised by Leo D. Sandvig, William G. Weber, Jr., and Robert Nicotera, of the Pennsylvania Department of Transportation. William H. Park, a colleague of the authors at Pennsylvania State University, played a leading role in formulating the objectives and scope of the project.

REFERENCES

Surface Dynamics Profilometer

1. Hudson, W. R. High-Speed Road Profile Equipment Evaluation. Highway Research Record 189, 1967, pp. 150-164.
2. Spangler, E. B., and Kelly, W. J. GMR Road Profilometer—A Method for Measuring Road Profile. Highway Research Record 121, 1965, pp. 27-54.
3. Darlington, J. R., and Milliman, P. A Progress Report on the Evaluation and Application Study of the General Motors Rapid Travel Profilometer. Highway Research Record 214, 1968, pp. 50-67.

Random Effects in Road-Vehicle Systems

4. Butkunas, A. A. Random Vibration Analysis and Vehicle Development. Society of Automotive Engineers, SAE Paper 690109, Jan. 1969.

5. Jaeckel, H. R., and Swanson, S. R. Predicting Service Life of Automotive Parts Calls for Random Load Test. SAE Jour., Nov. 1969, pp. 42-47.

Random Signal Processing

6. Bendat, J. S., and Piersol, A. G. Measurement and Analysis of Random Data. John Wiley and Sons, New York, 1966.
7. Keller, A. C. Considerations in the Analysis of Arbitrary Waveforms. Spectral Dynamics Corp., Tech. Publ. PS-2, Dec. 1966.