This paper examines and evaluates currently available techniques to determine (a) load-deformation characteristics of the various paving components that are suitable for use in performing a practical fatigue analysis and (b) the response in terms of stress, strain, and deflection of the pavement structure when subjected to traffic loading. Emphasis is placed throughout the paper on practical aspects of material characterization and structural analysis. An extensive list of references is given. The survey of literature indicates that suitable dynamic laboratory tests are available for use in characterizing pavement material properties in analytical or numerical solutions to layered pavement systems. A detailed discussion is given of material properties, testing techniques, and testing systems. Existing layered theories are reviewed, and a number of comparisons are given between calculated and measured responses of several layered pavement systems. A table summarizes the availability and limitations of several computer programs for predicting the response of layered pavement systems. Practical recommendations are given concerning the required laboratory testing and the layered theory suitable for use in a fatigue subsystem.

Material Characterization and Layered Theory for Use in Fatigue Analyses

Richard D. Barksdale and R. G. Hicks
School of Civil Engineering, Georgia Institute of Technology

In recent years, highway engineers have shown considerable interest in developing a mechanistic basis for designing and evaluating pavement structures. Linear-elastic, non-linear-elastic, and viscoelastic layered system theories for use in predicting pavement response are all currently either in use or under development. A number of recent studies (1, 2, 5, 6, 11, 17, 24, 33, 34, 40, 49) indicate that the performance of a flexible pavement is closely related to the stresses, strains, and displacements calculated from layer pavement theories in which experimentally measured material properties are used.

Proper design and evaluation of pavement structures require consideration of many factors: environment, traffic, material properties, construction variables, maintenance variables, and economics. In the past, most widely used design methods for asphalt concrete pavements have involved empirical correlations of field performance with material properties measured in the laboratory or the field or both. Use of those methods in many instances has provided reasonably satisfactorily performing pavements; however, major difficulties arise when those methods are extrapolated beyond the conditions for which they were originally developed. Only an improved pavement design and evaluation procedure based on the mechanistic behavior of the pavements could be extended to new service conditions.

The state of the art is such that the technical knowledge necessary to develop a mechanistic pavement design procedure is now available. Unfortunately, a price must be paid to accomplish that goal. More sophisticated tests are required to characterize the pavement materials, and a computer is required to predict the pavement response. The additional cost to develop suitable design methods, retrain personnel, and purchase new testing equipment, however, will permit a better understanding of pavement behavior and lead to more efficient use of paving materials.

The purpose of this paper is to examine and evaluate currently available techniques for determining (a) load-deformation characteristics of the paving components that are suitable for use in performing a practical fatigue analysis and (b) stress, strain, and deflection responses of the pavement structure when it is subjected to traffic loadings. Rutting is not considered in this paper but has been treated in detail elsewhere (1, 4). Emphasis is placed throughout the paper on practical aspects. Specific recommendations are given concerning selection, use, and availability of laboratory testing equipment and computer programs that are suitable for application in a mechanistic pavement design method.

MATERIAL CHARACTERIZATION

Proper design of asphalt concrete pavement systems relies in part on a thorough understanding of the response of the component materials to loading. Two broad areas of material characterization must be considered in pavement design: (a) material parameters for use in establishing failure criteria and (b) load-deformation characteristics of each component for use in calculating the physical response of the system. It is not likely that a properly designed pavement would fail under a single-axle loading. As a result, the traditional concept of shear strength (1, 16), which would define failure under a single load, will not be discussed in this paper. Under a large number of repetitions, however, fatigue cracking of stabilized materials frequently occurs because of repeated bending. That is the mode of failure of concern in this symposium. The mechanism of fatigue failure and methods for predicting fatigue are given in other papers in this Special Report (14, 56).

Practical Considerations

The material properties used in any mechanistic design procedure should be evaluated in tests that simulate as closely as practical the stress conditions occurring under the action of a moving load. When a wheel load moves past an element of material located beneath the pavement system, the element is subjected to stress states similar to that shown in Figure 1 (3). Each element of material is subjected to a simultaneous buildup in both the major principal stress σ_1 and the minor principal stress σ_3. As those stresses build up, a rotation of the principal stress axis also occurs (Fig. 1). In addition to duplicating the stress or strain states, the test should also attempt to simulate the environmental conditions that exist in the field.

The CBR and the R-value, and to a somewhat lesser extent the static triaxial tests, do a relatively poor job of simulating the in situ stress conditions. The in situ stress can, however, be reasonably approximated by the use of either the repeated load triaxial test or the cyclic triaxial test. The exact duplication of the stress pulse appears to be somewhat less important in determining the dynamic modulus of elasticity than in determining the rutting characteristics of pavement materials; in fact, creep (5, 49, 52), relaxation, and free vibration tests (29) have all been used with reasonably good results to evaluate the modulus of elasticity of pavement materials.

Figure 2 (3) shows the typical haversine-shaped major principal (or vertical) compressive stress pulse to which an element of material beneath the pavement is subjected. Square, triangular, sinusoidal, and haversine-shaped pulses have all been used to simulate the actual in situ stress. Any of those wave forms should give a satisfactory approximation for estimating the elastic response provided care is taken in selecting the magnitude and duration of the pulse. For conventional flexible pavement structures and spring and summer temperatures, the duration of the stress pulse varies primarily with the location of the element with respect to the point of loading and with the velocity of the vehicle. The curves shown in Figure 3 are suggested as a practical guide to selecting the duration of the stress pulse that should be used in a dynamic laboratory test.

Other things that must be considered in the testing of pavement materials are the effects of previous stress history and loading stress path. Dehlen (17) and later Hicks (33) found that for moderate stress levels the elastic response of most subgrade soils, unstabilized granular materials, and asphalt concrete will become relatively constant after approximately 100 load repetitions. Those studies also indicated that a single test specimen could be used to characterize the non-linear-elastic response of granular materials. That can be accomplished by determining the elastic bounce at several different confining pressures or deviator stresses or both provided care is exercised to gradually increase the severity of the stress state.

Advanced Concepts

Elastic Materials

A general anisotropic, linear-elastic material can be modeled by the following linear stress-strain law (21):

$$
\begin{aligned}
\sigma_x &= C_{11}\epsilon_x + C_{12}\epsilon_y + C_{13}\epsilon_z + C_{14}\gamma_{xy} + C_{15}\gamma_{yz} + C_{16}\gamma_{zx} \\
\sigma_y &= C_{21}\epsilon_x + C_{22}\epsilon_y + C_{23}\epsilon_z + C_{24}\gamma_{xy} + C_{25}\gamma_{yz} + C_{26}\gamma_{zx} \\
& \quad \vdots \qquad\quad \vdots \qquad\quad \vdots \qquad\quad \vdots \qquad\quad \vdots \\
\tau_{yz} &= C_{61}\epsilon_x + C_{62}\epsilon_y + C_{63}\epsilon_z + C_{64}\gamma_{xy} + C_{65}\gamma_{yz} + C_{66}\gamma_{zx}
\end{aligned}
\tag{1}
$$

where

$\sigma_{x,y,z}$ = normal stress components,
ϵ_{ij} = normal strain components,
τ_{ij} = shear stress components,
γ_{ij} = shear strain components, and
C_{ij} = material constants.

From energy considerations, $C_{ij} = C_{ji}$; and, as a result, a general anisotropic linear material can be characterized by 21 independent elastic constants. For an isotropic linear-elastic material, however, it has been shown (21) that a material is completely characterized by 2 elastic constants that can be determined from material tests. Usually in pavement design, the modulus of elasticity E and Poisson's ratio ν are the 2 elastic constants evaluated in the laboratory and used in layered theory. Once any 2 elastic constants are known, all other elastic constants can be determined by the use of Table 1 (21).

The stress-strain relations for an isotropic linear material can be expressed as follows:

$$
\begin{aligned}
\epsilon_x &= \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right] & \gamma_{xy} &= \frac{2(1+\nu)}{E}\tau_{xy} \\
\epsilon_y &= \frac{1}{E}\left[\sigma_y - \nu(\sigma_z + \sigma_x)\right] & \gamma_{yz} &= \frac{2(1+\nu)}{E}\tau_{yz} \\
\epsilon_z &= \frac{1}{E}\left[\sigma_z - \nu(\sigma_x + \sigma_y)\right] & \gamma_{zx} &= \frac{2(1+\nu)}{E}\tau_{zx}
\end{aligned}
\tag{2}
$$

The proper use of those equations is quite important and will be illustrated by means of a simple example. Consider an element of material that is subjected to 3 tensile principal stresses and has the material properties shown in Figure 4. From Eq. 2 the

Figure 1. Rotation of principal stress axis of element as vehicle moves over surface.

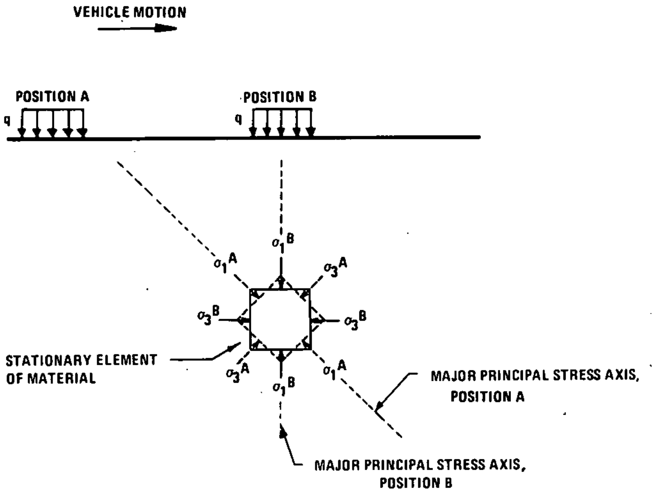


Figure 2. Equivalent sinusoidal and triangular stress pulses.

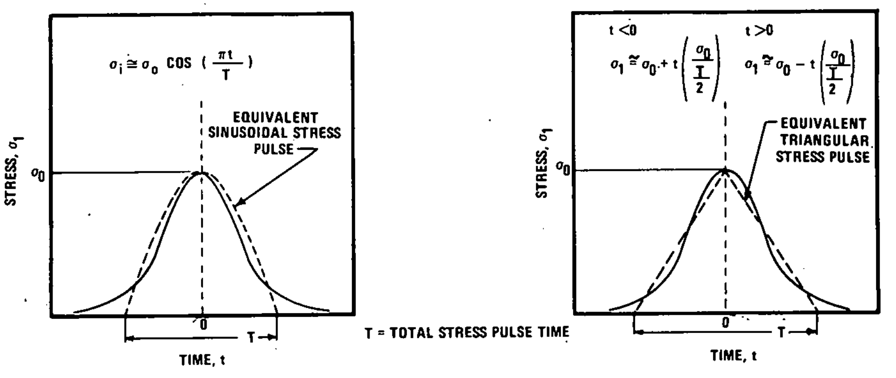
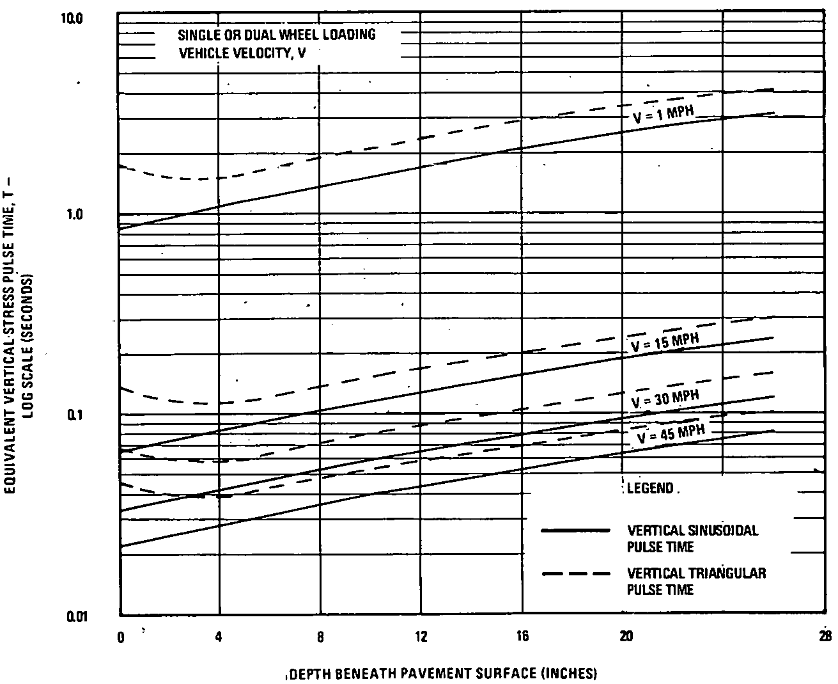


Figure 3. Variation of equivalent vertical stress pulse time with vehicle velocity and depth.



strain in the z-direction is then $\epsilon_z = (1/E)\,[\sigma_z - \nu(\sigma_x + \sigma_y)]$, or $\epsilon_z = (1/1{,}000)\,[10 - 0.4\,(30 + 20)] = -0.01$ in./in. (compression). That simple example readily illustrates that (a) $\epsilon_z \neq \sigma_z/E$ because of the effect of Poisson's ratio and (b) even though the stress in the z-direction is tensile the strain can be compressive also because of the effect of Poisson's ratio.

Viscoelastic Material Characterization

Both asphalt concrete and plastic clays exhibit strong viscous characteristics because their properties are greatly influenced by the frequency and duration of load. As a result, several investigators (5, 25, 26, 51, 52, 53) have quite logically characterized paving materials in both linear and nonlinear viscoelastic models. Usually, materials are assumed to be linearly viscoelastic to simplify their representation and make stress analyses practical. Whether a material is actually linearly viscoelastic must be determined experimentally.

The differential operator form of the linear viscoelastic stress-strain law (79) is most commonly used and can be visualized for conceptual purposes as a combination of springs and dashpots. A complete catalog of mechanical models, together with their stress-strain behavior for creep, relaxation, and constant strain-rate loadings, has been assembled by Williams, Blatz, and Schapery (78).

An isotropic, linear viscoelastic material can be represented by 2 independent linear viscoelastic operational moduli (material properties) such as $E(p)$ and $\nu(p)$, which are functions of the transform parameter p (transformed time). Since a direct analogy exists between the viscoelastic and elastic constants, the usual elastic formulas (Table 1) can be used to convert $E(p)$ and $\nu(p)$ to, for example, $G(p)$ and $K(p)$.

Laboratory Test Methods

Flexible pavement materials are to varying degrees nonhomogeneous, anisotropic, nonlinear, and nonelastic. Some of their properties are time dependent and affected by changes in the environment such as temperature or moisture content. A detailed list of variables affecting material response was previously reported by Deacon (16).

The elastic or viscoelastic material properties for use in a mechanistic pavement design procedure have been determined in several types of tests. Tests that have been used most frequently in the past and continue to be the most promising from a practical viewpoint are repeated-load triaxial, cyclic load triaxial, and creep. Other laboratory tests that either have been or could be employed to characterize materials for use in layered system theories include flexural bending, hollow cylinder, torsion, and indirect tension (5, 51, 52).

Evaluation of Elastic Constants

The repeated-load test has been most frequently used to evaluate the elastic constants E and ν of paving materials. In that test, a cylindrical specimen is placed in a conventional triaxial cell and subjected to repeated deviator stress pulses such as the rectified sinusoidal wave form shown in Figure 5a. A few researchers (17, 51, 52) have subjected the specimen to simultaneous repeated axial (Fig. 5a) and lateral stress states (Fig. 5b), which duplicate reasonably closely the stress conditions that exist in the field. Most tests, however, have used a constant cell pressure and are much simpler to perform. Careful selection of a constant confining pressure should give results that are satisfactory for use in a practical mechanistic approach. Additional work is now being conducted to determine the advantages of repeated confining pressure (8).

Resilient Modulus

The resilient modulus of elasticity E_R obtained from the repeated-load triaxial test is defined as the deviator (or repeated axial) stress divided by the recoverable strain

Table 1. Relations between elastic constants in isotropic linear materials.

Elastic Constant	λ Dimension FL^{-2}	G Dimension FL^{-2}	E Dimension FL^{-2}	ν Dimension 1	K' Dimension FL^{-2}
λ					
G	—	—	$\frac{G(3\lambda + 2G)}{\lambda + G}$	$\frac{\lambda}{2(\lambda + G)}$	$\frac{3\lambda + 2G}{3}$
E	—	$\frac{A^{*} + (E - 3\lambda)}{4}$	—	$\frac{A - (E + \lambda)}{4\lambda}$	$\frac{A + (3\lambda + E)}{6}$
ν	—	$\frac{\lambda(1 - 2\nu)}{2\nu}$	$\frac{\lambda(1 + \nu)(1 - 2\nu)}{\nu}$	—	$\frac{\lambda(1 + \nu)}{3\nu}$
K	—	$\frac{3(K - \lambda)}{2}$	$\frac{9K(K - \lambda)}{3K - \lambda}$	$\frac{\lambda}{3K - \lambda}$	
G					
E	$\frac{G(2G - E)}{E - 3G}$	—	—	$\frac{E - 2G}{2G}$	$\frac{GE}{3(3G - E)}$
ν	$\frac{2G\nu}{1 - 2\nu}$	—	$2G(1 + \nu)$	—	$\frac{2G(1 + \nu)}{3(1 - 2\nu)}$
K	$\frac{3K - 2G}{3}$	—	$\frac{9KG}{3K + G}$	$\frac{3K - 2G}{2(3K + G)}$	
E					
ν	$\frac{\nu E}{(1 + \nu)(1 - 2\nu)}$	$\frac{E}{2(1 + \nu)}$	—	—	$\frac{E}{3(1 - 2\nu)}$
K	$\frac{3K(3K - E)}{9K - E}$	$\frac{3EK}{9K - E}$	—	$\frac{3K - E}{6K}$	
ν					
K	$\frac{3K\nu}{1 + \nu}$	$\frac{3K(1 - 2\nu)}{2(1 + \nu)}$	$3K(1 - 2\nu)$	—	

$^{*}A = \sqrt{(E + \lambda)^2 + 8\lambda^2}$.

Figure 4. Properties of example material.

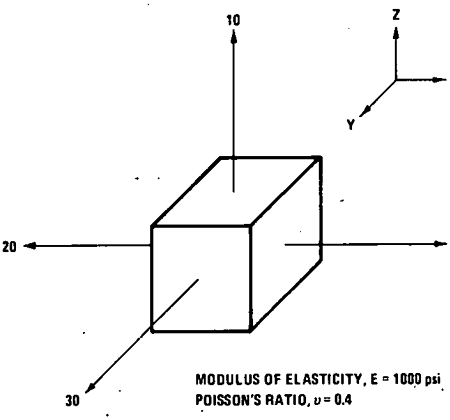
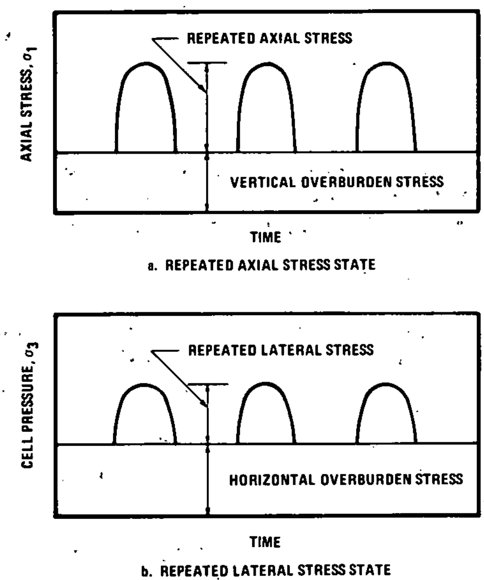


Figure 5. Stress states in repeated load triaxial tests using sinusoidal stress pulse.



associated with the bounce of the specimen. By definition, a secant modulus is obtained that corresponds to the minimum value occurring during the loading portion of the test.

In any dynamic tests, the deformation as well as loads must be measured by electronic measuring and recording equipment. Many times undesirable deformations can occur during loading in the piston and end platens and also in the associated connections between components. If those movements are included in the deformation used to calculate the recoverable strain, the calculated resilient modulus will be smaller than the actual value. Results indicate that when the resilient modulus is greater than about 15,000 psi special measuring devices should be used inside the cell to eliminate that problem. Reasonably reliable resilient deformation measurements can be obtained by attaching 2 thin, circular aluminum or plexiglass clamps around the specimen approximately at the quarter points (Fig. 6). Theoretical studies (17) indicate that the stiffening effect of the clamp should not increase the resilient modulus by more than about 10 to 15 percent.

Reliable axial deformations of the specimens can be obtained by the use of 2 linear variable differential transducers (LVDT) attached to the clamps, placed on opposite sides of the specimen, and wired so that their electrical outputs are added together (or averaged) and then recorded on a reasonably fast-responding electronic recorder. At the Georgia Institute of Technology deformations are usually measured with a dc electrical system by means of a pair of Collins SS-203 or SS-204 LVDT's. Measurements can also be obtained by ac recording systems. The advantage of using an ac measuring system is that the LVDT's are lightweight and cost only about a third as much as dc LVDT's. An ac recording system should not, however, be used without suitable correction networks when phase angle relations are to be measured between stress and strain. Of course, many other types of measuring systems, such as displacement potentiometers and optical scanners, can be successfully used.

Wire resistance strain gauges bonded directly on the specimen provide an excellent means for directly measuring the resilient strain in stabilized materials such as asphalt concrete and cement-stabilized materials. The active strain gauge length should probably be equal to or greater than twice the maximum diameter of the aggregate in order to measure the average strain in those specimens.

Poisson's Ratio

Poisson's ratio is an elastic constant that is difficult to reliably evaluate for most pavement materials. For an ideal, isotropic, cylindrical specimen of material subjected to a uniform principal stress state, Poisson's ratio is equal to

$$\nu = -\epsilon_l/\epsilon_a \tag{3}$$

where ϵ_l and ϵ_a are the lateral and axial strains respectively. Dehlen (17) has theoretically shown that, if perfectly frictionless caps and bases are used, the errors associated with uneven lateral strain of the specimen should be less than 10 percent. Therefore, Eq. 3 can be used to calculate Poisson's ratio provided that end friction is eliminated.

From physical considerations, ν for elastic isotropic materials should be between -1 and $\frac{1}{2}$. However, experimentally determined values of ν from the repeated-load test are in some instances greater than $\frac{1}{2}$ (33). Those large values of ν may at least partially be caused by the nonuniform stress and deformation conditions that exist in the triaxial specimens and also by pavement materials not behaving as ideal elastic solids.

Most researchers (1, 17, 24, 33, 37) who have attempted measurements of ν have used either wire resistance strain gauges for stabilized materials or LVDT's for nonstabilized materials. For bound materials (1, 17, 33), a pair of strain gauges can be bonded to the specimen at midheight with the gauge oriented horizontally (17, 33). Lateral deformation in clay or unbound gravel or both has been measured by 2 ac transducers fixed to aluminum or plexiglass clamps at the quarter points (33). For clays, Dehlen (17) also drilled diametral holes through the sample and used an LVDT to measure the lateral deformation. Another approach to measure lateral deformation is by the use of a lateral deflectometer (Fig. 7, 5). That consists of 3 thin metal probes that press against the specimen and are supported on an aluminum ring positioned about the center of the specimen. A strain gauge is bonded to the side of each probe to measure the strain in it as the specimen deforms.

Poisson's ratio can also be determined from the total volume change that the specimen undergoes. From the theory of elasticity, Poisson's ratio is related to the volume change by the following approximate relation:

$$\nu \simeq \frac{1}{2}\left(1 - \frac{1}{\epsilon_a}\frac{\Delta V}{V}\right) \tag{4}$$

where

ν = Poisson's ratio,
ΔV = change in volume of the specimen,
V = initial volume of the specimen, and
ϵ_a = axial strain.

The volume change can be evaluated by measuring the deformation profile of the specimen directly or by filling the cell with a fluid and measuring its change in volume (83).

Evaluation of Viscoelastic Properties

Viscoelastic material properties are usually determined experimentally by creep, relaxation, and sinusoidal stress input or sinusoidal strain input tests (5, 54). Athough low order spring and dashpot models have sometimes been used to characterize test results, they cannot realistically represent experimental data during more than 1 or 2 log cycles of time.

Probably the best method available at this time to evaluate the dynamic modulus of the asphalt concrete surfacing is to apply a sinusoidally oscillating stress. That method, which is referred to in this paper as the cyclic triaxial test, minimizes impact effects that may become important as the equivalent frequency of loading becomes relatively large as, for example, in the surfacing when a vehicle exceeds a speed of 30 to 40 mph. The dynamic modulus $|E^*|$, which can also be used in an elastic analysis, is equal to the peak sinusoidal stress σ_o divided by the peak recoverable axial strain ϵ_o (Fig. 8, 12).

A good alternative approach, which does not require expensive test equipment, is to use a creep test to evaluate the dynamic elastic or viscoelastic modulus of pavement materials. If that approach is used, the sample should probably be carefully preconditioned by methods such as statically cycling the axial load through about 4 to 6 repetitions. A conventional triaxial cell can be used together with a simple loading arrangement (12, 51, 53). Comparative studies should, however, be made if possible between the dynamic modulus obtained from the creep test and either the repeated-load or the cyclic triaxial test.

Descriptions of techniques for handling the viscoelastic properties are described elsewhere (2, 5, 59, 60, 79). From either creep or sinusoidal test results, a dynamic viscoelastic modulus for any loading frequency can be predicted from the results of a single creep or dynamic test by the use of a generalized Kelvin model (70). Coffman et al. (12) and Kallas (37) have both used this approach to evaluate the dynamic elastic properties of pavement materials. Typical values for $|E^*|$ determined by this method are shown in Figure 9 (37) for materials having 5 percent asphalt and 3.8 percent air voids.

Testing Equipment

Dynamic properties of pavement materials can be evaluated by the use of a mechanical system (12, 42), a pneumatic system (64, 65), an open-loop hydraulic system (20), and a closed-loop hydraulic servosystem (37). The advantages, disadvantages, approximate costs, and selected sources of those testing systems are as given below.

Figure 6. Circumferential clamp and LVDT for measuring axial deformation under repeated loading.

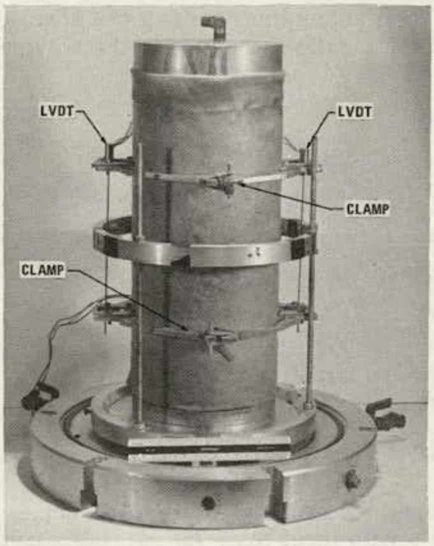


Figure 7. Diameter deflectometer positioned to measure change in diameter at center of granular base specimen.

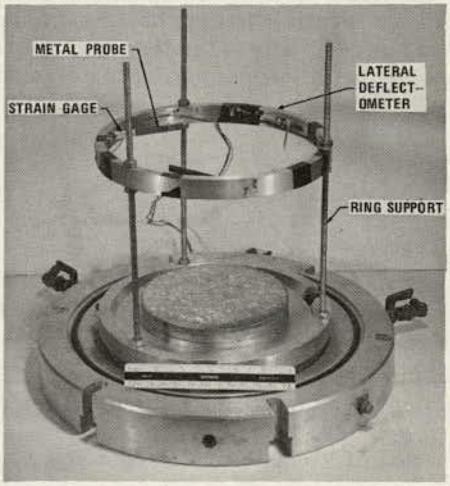
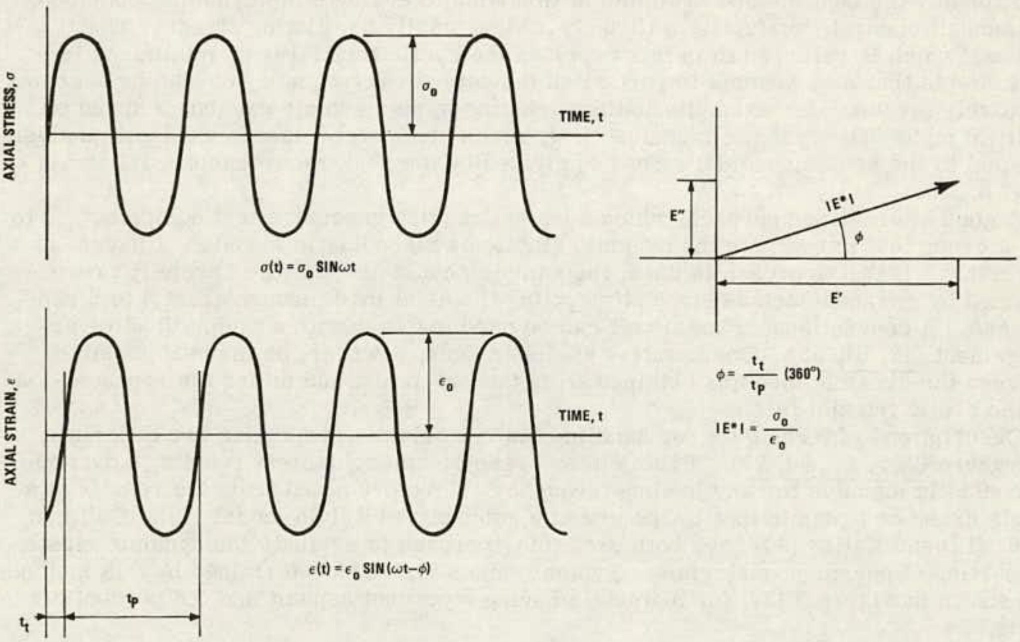


Figure 8. Dynamic test recording.



1. The mechanical system is relatively reliable but has some problems with design, balance, and operation. It can apply desired load pulse shapes by adjusting cams. Its maximum response is between 5 and 25 Hz, and its cost is between $4,000 and $6,000. It is available by special fabrication only.

2. The pneumatic system is relatively simple, cheap, reliable, and easy to design and repair. It will require periodic replacement of valves and cylinder. Its practical load limit is about 3,000 to 5,000 lb. An exact pulse shape is hard to apply. Its maximum response is between 5 and 8 Hz, and its cost is between $3,000 and $5,000. Selected sources are Geotechnical Research, Inc., 2403 Wylie Drive, S.E., Marietta, Georgia 30062; Research Engineering, 2640 Dundee Road, San Pablo, California; and Structural Behavior Engineering Laboratory, Inc., Post Office Box 9727, Phoenix, Arizona.

3. The hydraulic or open-loop control system is more complex to design and set up than the pneumatic system and requires a hydraulic pump and storage reservoir. A hydraulic system has a faster response (5 to 8 Hz) than a pneumatic one and can go to much higher loads. An exact pulse shape is hard to apply. It costs between $4,500 and $8,000 and is available by special fabrication only.

4. The hydraulic servo or closed-loop control system can have a fast response (25 to 100 Hz), high load capacity, and capability to apply any pulse shape to specimens. Disadvantages are its high initial cost (between $10,000 and $30,000) and maintenance cost and complex electronics. Some systems are very hard to balance and keep in proper operating condition. Selected sources are Hydratech, 2890 John Road, Troy, Michigan 48084; Mb Electronics, Post Office Box 1825, New Haven, Connecticut 06508; and MTS Systems Corporation, Minneapolis, Minnesota 55424.

For routine testing of pavement materials, a reliable system that is easy to maintain and repair is essential. Furthermore, if the dynamic material properties of all layers of a flexible pavement are to be evaluated, the system should have as minimum requirements a load capacity of at least 1,500 to 2,000 lb and the capability of applying a pulse to the specimen in 0.1 sec or less and at frequencies ranging from approximately 0.5 to at least 5 Hz.

The closed-loop hydraulic servosystems have by far the best overall capability. Those systems, however, can be "electronic monsters"; they are often quite expensive and time-consuming to maintain properly for even routine operations. As a result they are not considered to be suitable as a production type of testing system for use in most highway materials laboratories. The pneumatic testing system (or a slightly faster acting air-oil system) does not have nearly the overall capability as that of a closed-loop testing system. However, if properly designed, it can meet the minimum requirements for the dynamic testing of pavement materials and is very reliable and easy to maintain. Because of its relatively low cost and high degree of reliability, the pneumatic (or air-oil) type of system is recommended for routine dynamic testing. Where loads of more than 4,000 to 5,000 lb are required, such as for loading prototype pavement systems, an open-loop hydraulic system can often be used to good advantage.

Dynamic Properties of Pavement Materials

Results of dynamic tests on all materials show that the dynamic modulus, and to some extent Poisson's ratio, depends on the stress state. Because of nonlinearity, serious errors can arise if E is not evaluated by the use of a stress state that is compatible with the one that will exist in the pavement.

Typical values for the dynamic modulus of elasticity of various materials in the pavement section are given in Table 2. Those values can be used in the design of pavements in the absence of actual laboratory test results but should be considered as approximate and used with considerable caution.

Cohesive Subgrade Soils

In general, the resilient modulus E_R of cohesive soils decreases with increasing repeated stress level σ_d (Fig. 10, 70) and for some soil types is relatively unaffected

Figure 9. Dynamic modulus of asphalt concrete as function of loading frequency.

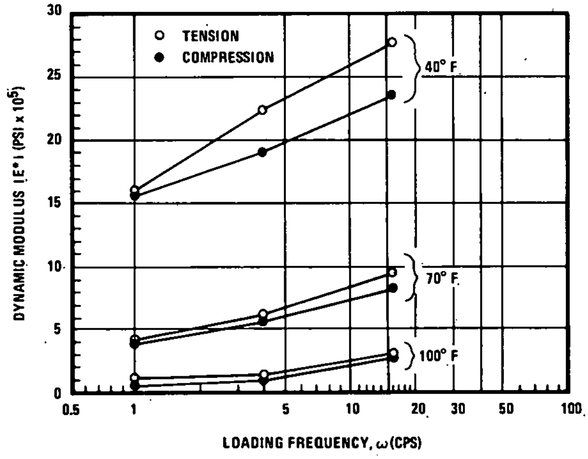


Table 2. Selected measured dynamic moduli for pavement materials.

Reference of Test Method[a]	Material Description	Frequency and Duration	Load Repetition	Dynamic Modulus
Asphalt Concrete				
33	California type B; ½ in. max med. aggr.; 85 to 100 pen. asphalt	30 cpm, 0.1 sec	100	300,000 psi, 70 F 70,000 psi, 90 F
1	Georgia standard A; 1½ in. max aggr.; 85 to 100 pen. asphalt	20 cpm, 0.1 sec	10,000	220,000 psi, 72 F 100,000 psi, 89 F
51	California type B; ⅜ in. max med. aggr.; 85 to 100 pen. asphalt	30 cpm, 0.1 sec	100	2,500,000 psi, 40 F 1,500,000 psi, 55 F 500,000 psi, 70 F 50,000 psi, 100 F
37	Asphalt Institute mix IVb; ½ in. max aggr.; 60 to 70 and 85 to 100 pen. asphalts	1 to 16 cps	250 to 300	600,000 to 2,000,000 psi, 40 F, 1 cps 150,000 to 750,000 psi, 70 F, 1 cps 50,000 to 150,000 psi, 100 F, 1 cps 1,100,000 to 3,000,000 psi, 40 F, 16 cps 350,000 to 1,300,000 psi, 70 F, 16 cps 90,000 to 450,000 psi, 100 F, 16 cps
Unbound Granular Base				
36	Colorado; standard base, ½ in. max and 8.7 percent < No. 200; standard subbase, 2½ in. max and 7.9 percent < No. 200	120 cpm, 0.2 sec	10,000	10,618 psi, σ_3 0.447, 2.4 percent w-c 10,019 psi, σ_3 0.465, 6.3 percent w-c 8,687 psi, σ_3 0.496, 8.2 percent w-c
33	California; well-graded and sub-rounded gravel ¾ in. max; class 2 aggr. base	30 cpm, 0.1 sec	100	Dry 10,000 to 13,000 psi, 3 percent < No. 200, σ_3 0.53 8,000 to 9,000 psi, 8 percent < No. 200, σ_3 0.59 Partially saturated 7,000 to 10,000 psi, 3 percent < No. 200, σ_3 0.55 5,000 to 7,000 psi, 8 percent < No. 200, σ_3 0.60
33	California; well-graded and angular crushed stone ¾ in. max; class 2 aggr. base	30 cpm, 0.1 sec	100	Dry 11,000 to 12,000 psi, 3 percent < No. 200, σ_3 0.57 14,000 to 15,000 psi, 10 percent < No. 200, σ_3 0.50 Partially saturated 9,000 to 10,000 psi, 3 percent < No. 200, σ_3 0.57 7,500 to 9,500 psi, 10 percent < No. 200, σ_3 0.57
1	Soil-aggregate of 17 percent silty sand and 83 percent crushed granite; 100 percent T-180; w-c = 5.1 percent	33 cpm, 0.1 sec	10,000	3,836 psi, $\theta^{0.53}$ 3,145 psi, $\theta^{0.55}$
65	California; well-graded and sub-rounded gravel ¾ in. max; class 2 aggr. base (dry)	30 cpm, 0.2 sec	10,000	7,000 psi, σ_3 0.55
1	Silty fine sand 100 percent AASHO T-99; 40 percent < No. 200, w-c = 13.4 percent	33 cpm, 0.1 sec	10,000	1,856 psi, $\theta^{0.61}$ 3,126 psi, $\theta^{0.37}$
Subgrade				
36	AASHO class A-6 silty clay; w-c = 14 to 18 percent; γ_d = 110 to 114 pcf	120 cpm, 0.2 sec	10,000	3,000 to 4,000 psi, 18 percent w-c 7,000 to 8,000 psi, 16 percent w-c 15,000 to 20,000 psi, 14 percent w-c
1	Micaceous silty sand subgrade	33 cpm, 0.1 sec	10,000	3,000 to 4,000 psi, wet season 1,500 to 2,000 psi, dry season
64	Silty clay (AASHO test); σ_d = 5 to 10 psi; σ_3 < 3 psi; γ_d = 110 to 115 pcf	20 cpm, 0.25 sec		13,000 psi, 13 percent w-c 10,000 psi, 14 percent w-c 8,000 psi, 15 percent w-c 7,000 psi, 16 percent w-c 2,000 to 5,000 psi, 17 percent w-c 2,000 psi, 18 percent w-c
65	Highly plastic clay (PI = 36.5) and silty clay (PI = 25.5)	30 cpm, 0.1 sec	10,000	4,150 psi, σ_d 1.0 3,200 psi, σ_d 5.2
36	AASHO class A-7-6 silty clay; w-c = 11 to 20 percent; γ_d = 102 to 105 pcf	120 cpm, 0.2 sec	10,000	7,000 to 10,000 psi, 20 percent w-c 15,000 to 16,000 psi, 18 percent w-c 14,000 to 15,000 psi, 16 percent w-c
17	AASHO class A-6 to A-7-6 silty clay	30 cpm, 0.1 sec	100	10,000 psi, 1 atm[b] 100,000 psi, 10 atm

[a]All are repeated load triaxial except reference 37, which is cyclic load triaxial. [b]Refers to soil moisture suction at time of test.

by small changes in confining pressure. The effect of confining pressure on the modulus appears to become greater as the clay decreases and the material becomes stiffer. The simplified nonlinear model also shown in Figure 10 has been used for clay subgrade soils where K_1, K_2, K_3, and K_4 are laboratory-determined constants.

The variation of ν with stress level is less clear although Hicks and Finn (34) found that it remained constant or increased slightly with increasing repeated vertical stress. Poisson's ratio, however, appears not to be significantly affected by confining stress.

Unstabilized Granular Base

For unstabilized granular materials, both E_R and ν are functions of the applied stress conditions. Numerous studies (1, 20, 33, 55, 65) of the resilient response of sands, gravels, and crushed stone have indicated that E_R significantly increases with confining pressure and is affected to a much smaller extent by the magnitude of the repeated vertical stress. As shown in Figure 11 (33), the resilient modulus of granular materials (partially crushed aggregate, low density, coarse grading) can usually be approximated by

$$E_R = \bar{k}\theta^{\bar{n}} \tag{5}$$

where $\bar{k}$ and $\bar{n}$ are constants evaluated from repeated-load triaxial test results and θ is the sum of the principal stresses ($\theta = \sigma_1 + 2\sigma_3$ in a conventional triaxial test). Studies have also shown that an expression of the form $E_R = k\sigma_3^n$ can also be used as long as the variation in the deviator stress is not too great for a given confining pressure.

Poisson's ratio of granular materials can vary considerably with the stress state and is apparently related to the principal stress ratio σ_1/σ_3 (33). The effects on E_R and ν of aggregate type, water content, density, and gradation have been described elsewhere (1, 33).

Asphalt-Bound Materials

Results of repeated-load triaxial tests have indicated that the temperature and rate (or frequency) of loading have a very significant effect on the stiffness of asphalt-bound materials (Fig. 9); asphalt content and type, air voids, and aggregate grading and type—all have a lesser effect on the stiffness. Several studies (18, 51, 55) found that the stiffness is stress dependent (i.e., the material is nonlinear), but that effect is still relatively small compared to that of temperature. Results indicate that Poisson's ratio may undergo an important increase with increasing temperature and to a lesser extent is affected by the stress level (Fig. 12, 51).

Although the results from flexural and indirect tensile tests show the same general trends, E_R determined in those tests is usually less than the value determined in the triaxial tests by as much as a half. Probably a value of E_R somewhere between those 2 extremes is appropriate to use for pavement design.

Cement-Bound Materials

The results of repeated-load tests on cylindrical specimens performed by Mitchell and Chen (44), Wang (85), and Barksdale (1) have shown that the E_R of soil-cement decreases with increasing confining pressure. Laboratory test results indicate that the resilient modulus measured in the triaxial test may be as much as 10 times greater than that obtained in repeated flexure (55), although one researcher (85) has shown results indicating the flexural modulus to be greater. The results of the flexure tests should probably be used in layered theory because that test more closely simulates conditions of bending, which occurs in very stiff base layers. Also, because of cracking with time in cement-treated bases, the effective modulus will tend to decrease significantly.

From the results of repeated-load triaxial tests on a cement-stabilized base, Fossberg (24) has shown that E_R is not very stress dependent and is essentially equal in

Figure 10. Effect of deviator stress on resilient modulus of fine-grained soil.

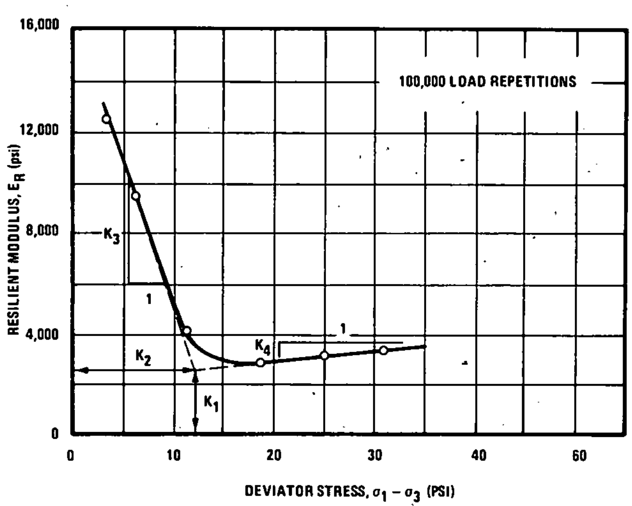


Figure 11. Variation in modulus with sum of principal stresses.

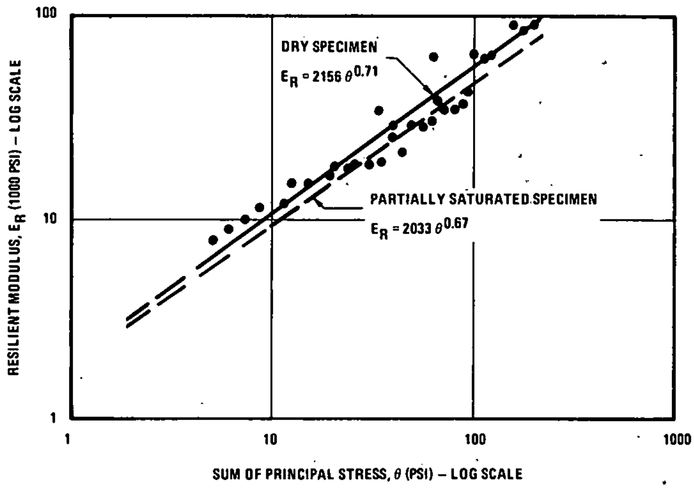
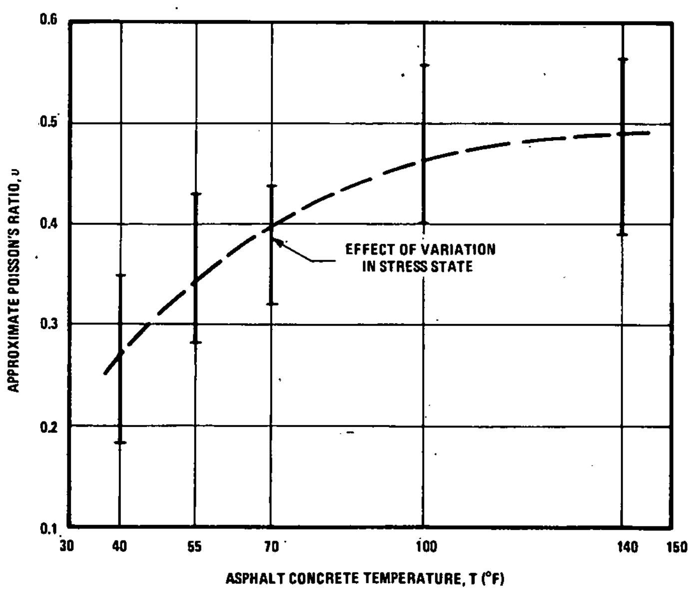


Figure 12. Approximate variation of Poisson's ratio with temperature.



tension or compression so long as cracking does not occur. Poisson's ratio was found to usually be between the limits of 0.1 and 0.2.

Empirical Correlations

In some instances it may become necessary to estimate the dynamic modulus of elasticity of pavement materials from empirical correlations with more easily measured material properties. Several such correlations have been developed by researchers for estimating the dynamic modulus of elasticity of pavement materials, but none has been developed for estimating Poisson's ratio.

Asphalt-Bound Materials

If direct measurements of stiffness are not possible, estimates may be made by the use of nomographs developed by Van der Poel (72) or Heukelom and Klomp (31) to obtain the stiffness of the bitumen as a function of time of loading and temperature (Fig. 13, 72). After the stiffness of the bitumen has been determined, the following empirical relations can be used to obtain the stiffness of the asphalt mixture:

$$\frac{S_{mix}}{S_{bit}} = \left(1 + \frac{2.5}{n}\,\frac{C_v}{1 - C_v}\right)^n$$

where

S_{mix} = mixture stiffness, kg/cm^2;
S_{bit} = bitumen stiffness, kg/cm^2;
C_v = volume of aggregate/(volume of aggregate + volume of asphalt); and
n = $0.83 \log(4 \times 10^5)/S_{bit}$.

The following correction is normally applied to mixes having void contents larger than 3 percent:

$$C_v' = \frac{C_v}{1 + \Delta H}$$

where

C_v' = modified volume concentration of aggregate,
C_v = original volume concentration of aggregate, and
ΔH = difference between air void content and 3 percent, expressed as decimal.

Fair agreement (less than 50 percent error) has been shown between the nomograph and measured value of dynamic stiffness (48).

Soil and Unbound Aggregate

A limited number of correlations have been made between the dynamic modulus of elasticity of soils and unbound base materials with routine test procedures. Heukelom and Klomp (31), for unstabilized clay and sandy soils, have proposed a relation between the dynamic subgrade modulus of elasticity and the familiar CBR value as follows:

$$E(\text{psi}) = \text{constant} \times \text{CBR} \tag{6}$$

Normally a value of E(psi) = 1,500 CBR has been used for design purposes although the constant has been shown to vary from 700 to 10,000. Kirnan and Glynn (86) developed similar relations for 2 boulder clays:

$$E(\text{psi}) = 250 \times \text{CBR} \tag{7}$$

The tremendous variations in those correlations clearly indicate the magnitude of error that can result from using such relations.

Other more generalized correlations for estimating the dynamic modulus of elasticity for the unstabilized base and subgrade materials have been summarized by Shell (87) and are shown in Figure 14. In addition, for granular materials, E_R can be crudely estimated from a knowledge of the supporting layers. Heukelom and Klomp (31) observed on the basis of field measurements that E_R of the granular layer is approximately 2 times greater than that of the supporting layer. That multiplying factor, however, can vary from less than 1 to approximately 5; the lower values appear to be associated with a dry, stiff subgrade.

Limitations

Great caution should be exercised in using generalized empirical relations for estimating the dynamic modulus because they were developed for a very limited number of soil types and conditions; even then, the scatter in test results was appreciable. For one micaceous silty sand subgrade soil, the empirical correlation given in Eq. 6 indicated a dynamic modulus of 4,500 to 6,000 psi, whereas the actual resilient modulus was found to be only 2,000 psi. It is quite possible that use of generalized empirical correlations together with a mechanistic design approach could give results that have the same limitations as those associated with currently used empirical design methods.

Practical Application

The dynamic elastic properties of pavement materials can be evaluated by the use of either the repeated-load or the cyclic triaxial test. For routine production work, the repeated-load test using a pneumatic or air-oil loading system (or a mechanical system applying a sinusoidal loading) is the most practical approach for evaluating the dynamic modulus and should be used by most highway department material laboratories.

Under field service conditions, variation in the dynamic modulus due to changes in environmental factors such as moisture content and temperature can have significant effects on the overall performance and life of the pavement. For example, a recent study of pavements in Georgia (1) indicates that approximately 75 percent of the fatigue damage to pavements occurs during the wet season of the year. Therefore, even though reasonably accurate values of the dynamic modulus of pavement materials may be obtained in the laboratory, extreme care must be exercised in incorporating those data into mechanistic design procedures. To obtain the resilient modulus of asphalt-bound layers requires that estimates be made of the variation of temperature with depth, time of day, and season. Theoretical methods of predicting temperature are now available and give reasonably good results (38).

Another difficult problem is defining the in situ degree of saturation for different soils, drainage conditions, and groundwater table locations. A good alternative approach for estimating the degree of saturation is to develop generalized procedures using the soil suction profile (88).

Correlations can be made between the resilient modulus and carefully selected pertinent tests for specific soil types and conditions. For fine-grained silt and clay subgrade soils, correlations could be made, for example, between E_R and soil suction (17). Similar correlations could be developed by the use of creep tests or possibly free vibration tests for all types of materials. Some evidence indicates that E_R does not vary greatly for unstabilized granular bases so long as the geologic source, grading, relative density, and moisture content of the materials remains about the same. Under those conditions, a good estimate of E_R can in many instances be made by the use of previously determined laboratory relations such as $E_R = \bar{k}\theta^{\bar{n}}$.

Because of the problems associated with measuring accurate values of ν as well as the fact that the response of the pavement is relatively insensitive to reasonable variations, estimated values of ν can be used as an engineering approximation for the mechanistic design procedures. For asphalt concrete mixes, Poisson's ratio appears to vary between approximately 0.25 and 0.35; for unstabilized granular subbases and bases, $0.30 \leq \nu \leq 0.4$; and for clayey subgrades, $0.4 \leq \nu \leq 0.5$. A limited number of tests could, of course, be performed to establish representative values of ν for specific materials and structural conditions.

Figure 13. Nomograph for determining stiffness of bitumens.

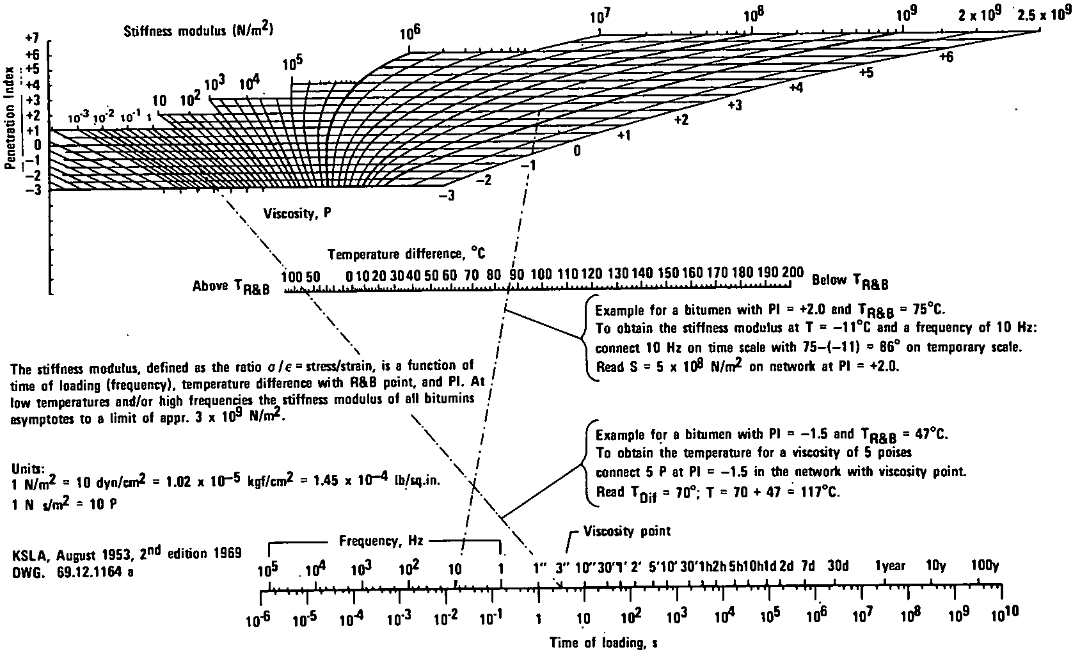
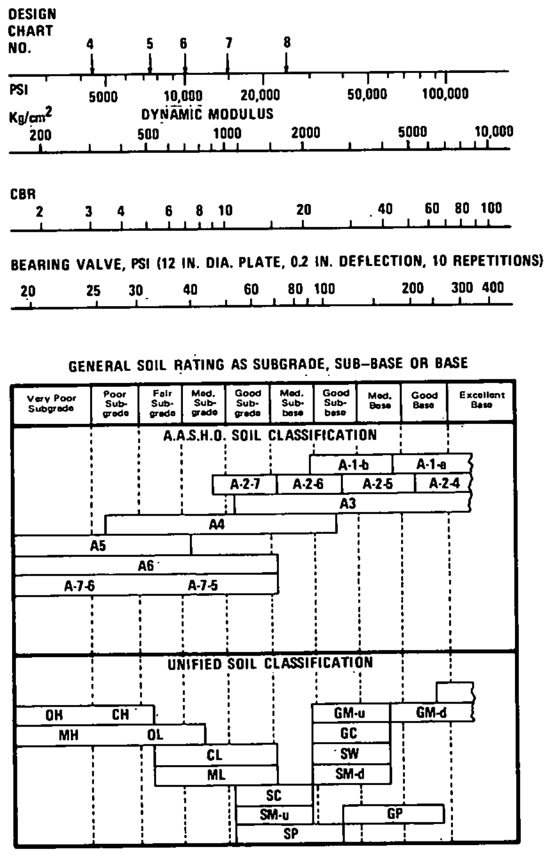


Figure 14. Crude empirical relations between dynamic modulus of elasticity and routine tests.



LAYERED SYSTEM ANALYSIS

Procedures for the prediction of traffic-induced deflections, stresses, and strains in pavement systems are based on the principal of continuum mechanics. The essential factors that must be considered in predicting the response of layered pavement systems are (a) the stress-strain behavior of the materials, (b) the initial and boundary conditions of the problem, and (c) the partial differential equations that govern the problem. Fortunately the highway engineer need only concern himself with the stress-strain behavior of the material, the physical configuration of the problem, and the general assumptions that have been made or implied in developing solutions to the layered system problem.

Reasonably good predictions of pavement response to load can be obtained provided carefully selected material properties are used with theories that make realistic assumptions. Unfortunately, the solution of the pavement system problem requires the use of a high-speed digital computer. If an engineer selects a formula that is not applicable to his set of conditions, an incorrect answer is obtained; likewise, if a computer program not suited to the particular problem is used, equally poor results are obtained. Therefore, to properly use the theoretical solutions that are now available, an engineer must thoroughly understand the assumptions and limitations associated with the use of these methods.

Solutions for Layer Systems

Elastic Layered Systems

The response of pavement systems to wheel loadings has been of interest since 1926 when Westergaard (77) used elastic layered theory to predict the response of rigid pavements. Later Burmister (9) solved the problem of elastic multilayered pavement structures (Fig. 15) using classical theory of elasticity. The assumptions that Burmister and most others (57, 76) have made in developing closed-formed solutions are as follows:

1. Each layer acts as a continuous, isotropic, homogeneous, linearly elastic medium infinite in horizontal extent;
2. The surface loading can be represented by a uniformly distributed vertical stress acting over a circular area;
3. The interface conditions between layers can be represented as being either perfectly smooth or perfectly rough;
4. Each layer is continuously supported by the layer beneath;
5. Inertial forces are negligible;
6. Deformations throughout the system are small; and
7. Temperature effects are neglected.

The partial differential equations associated with that boundary value problem can be solved by the use of integral transforms (57, 76). The response is then obtained in the form of infinite integrals that must be numerically integrated. If a sufficiently close integration interval spacing is not used, or if the integration is not taken out far enough before "chopping" it off, convergence of the integral to the correct value may not occur.

Comprehensive tables and charts of influence values for 2- and 3-layer systems subject to uniform circular loadings are given in the literature (35, 54). The use of those tables and charts can be quite tedious and time-consuming for 3-layer pavement systems, and tabulated solutions for 4-layer systems are not practical. Therefore, for general pavement design applications, the use of a computer is a necessity from a practical standpoint.

Schiffman (67) and Westmann (90) have further generalized the Burmister theory to include either vertical or horizontal asymmetric surface loading for multilayered elastic systems. Computer programs for asymmetric loading conditions, however, are not available at the present time.

Finite-Element Approaches

The continuum mechanics approaches described in the previous section satisfy exactly the governing differential equations associated with the pavement design problem. That closed-form solution can only be obtained for the case of homogeneous, isotropic, and elastic layers. The finite-element method, on the other hand, offers the capability of modeling pavements in a considerably more realistic manner. In contrast to the closed-form analytical solutions, each element in the system can be given independent anisotropic material properties, and the layers need not be infinite in width. In addition, solutions of the finite-element formulation for displacements, stresses, and strains are obtained for each element of the grid. In closed-form solutions, those quantities must be calculated individually at each desired point. The finite-element approach, however, is a numerical approximation, and as a result the cost of computer time to solve problems by that method may be as much as 2 to 5 times that for classical elastic layered solutions.

Detailed descriptions of the finite-element method have been presented by Zienkiewicz and Cheung (91). Usually the pavement system problem is approximated as one of axial symmetry, and a cylindrical coordinate system represented by r, θ, and z is used. A layered pavement structure may be idealized as an assemblage of finite numbers of discrete triangular or rectangular ring-shaped elements (Fig. 16, 3). Each adjacent element in the system comes together at a common point and is interconnected by frictionless pins called nodes.

The structural stiffness properties of each element can be determined by the application of energy principles (91) using an approximate displacement function and the usual elastic relations for displacements, strains, and stresses. The selection of the displacement function is quite important and should be chosen so as to maintain internal displacement continuity inside each element. In general, interelement displacement compatibility should also be maintained so that gaps will not open up between adjacent elements. Finally, rigid body displacement states and all uniform strain states must also be included in the element characterization. Those element displacement requirements will then ensure a monotonic convergence of the strain energy function if the element mesh size used in the structural model is repeatedly divided. If a fine enough mesh of elements is used, the approximate finite-element theory will usually give a satisfactory solution. Convergence to the correct solution with decreasing mesh size is not, however, always guaranteed.

Limitations of Layered Theory

In both classical and finite-element layered theories, the pavement structure is normally modeled as an axisymmetric solid. Axisymmetry usually means that both load and pavement geometrics are symmetrical about a common centerline. Unfortunately, the effects of wheel loads applied close to a crack or pavement edge cannot be analyzed by the use of methods that require axisymmetry. Although 3-dimensional solid models could be used with the finite-element method, that representation is not very practical for general use because of the large amount of computer time required to solve the model. An extended 2-dimensional finite-element program that approximates the loading as a Fourier series has been used to study the effects of edge loadings for multiple rectangular wheel loadings (89). Although that approach should lead to a much better understanding of pavement behavior, it requires too much computer time for general use in a design method. Caution should also be exercised in using Fourier series expansions for the loading to ensure that a sufficient number of terms are used to give accurate results. Furthermore, information is not available on the condition of slip that exists at the interface between layers. The assumption of a rough interface condition, which most investigators have used, appears to be reasonable, although varying degrees of slip can be considered.

In all of the theoretical approaches, inertial forces have been neglected. The inertial force is simply the force on a small element caused by a dynamic loading equal to the mass of the element times the acceleration. Also, none of the layered system

Figure 15. Classical linear-elastic layered pavement structure idealization.

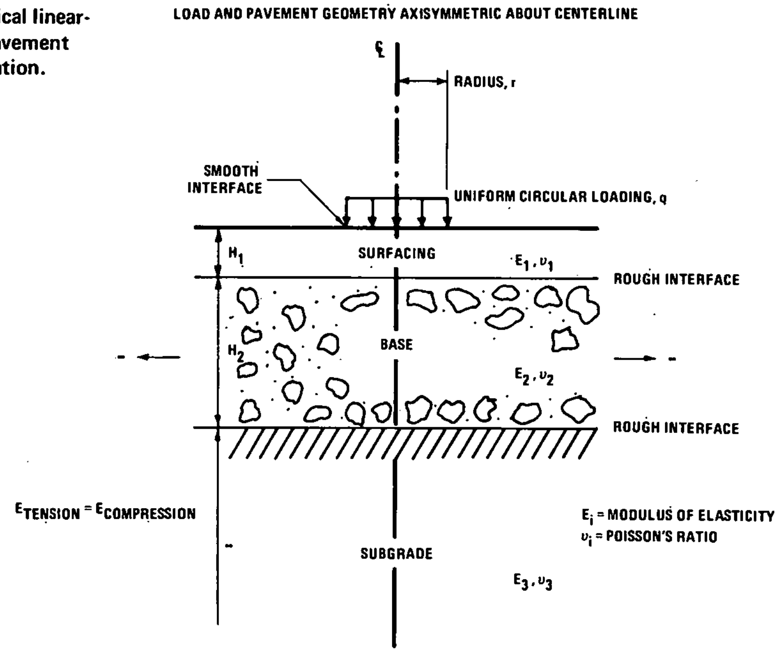
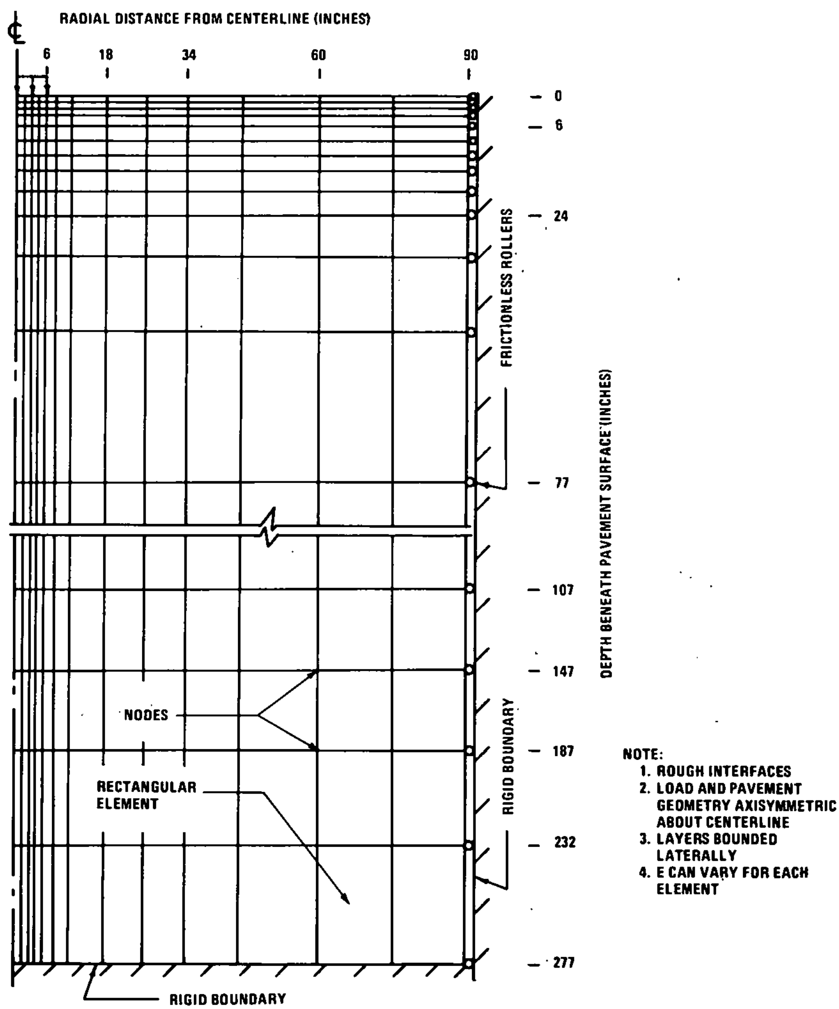


Figure 16. Finite-element idealization of pavement system.



theories considers the effects of vibrations. Neglecting vibrations is probably not a bad assumption for vehicle speeds lower than 60 mph on materials having cohesion. However, for cohesionless materials compacted to lower relative densities, neglecting vibratory effects may lead to densification that would cause rutting and changes in material properties.

Numerous laboratory tests have indicated that the dynamic modulus of paving materials varies with the confining pressure or deviator stress or both (1, 33, 55). Because of the variation in stress state that exists in each layer of the pavement system, the dynamic modulus actually changes with both depth and lateral position in each layer. Therefore, uncertainties arise in trying to determine what values of dynamic modulus to use in a linear-elastic layered analysis. Furthermore, elastic layered theory cannot consider variations in the modulus with lateral position. Those limitations for the most part can be overcome by the use of nonlinear finite-element theory (5, 17, 19, 33). In that method, the pavement response is initially calculated by using assumed moduli for each layer. The calculated stresses are then used to estimate a new stress-dependent modulus from experimentally measured material properties. Additional stress states are then calculated, and the process is repeated by either an iterative or an incremental procedure. In both cases, the modulus is matched with the stress state in each element. Those approaches, however, require considerably more computer time than does a single elastic layered solution.

An excellent alternative approach, which is a practical trade-off, is the use of a nonlinear, iterative elastic layered solution (1, 33, 38). That iterative procedure is analogous to the one used for finite-element theory. In this approach, the base and subgrade can be subdivided into several fictitious layers for better accuracy. The technique uses in each layer a modulus that is dependent on the average stress state that exists in the vicinity beneath the wheel loadings. That simplified nonlinear approach makes possible the analysis of a large number of pavement sections at a computer cost of roughly $10 for each structural section.

The bottom of each layer of a pavement structure is subjected to radial and tangential tensile stresses and strains. In nonstabilized granular materials, the application of such a stress state can result in an appreciable influence on the modulus of the material. A description of a slip model that can be used to handle that condition has been developed by the use of anisotropic finite-element theory (5). The behavior of granular materials under those stress conditions, however, certainly needs further study.

Prediction of Pavement Response

Classical Elastic Theory

One of the first attempts to verify layered theory was by Coffman et al. (12); they compared the deflections measured at the AASHO Road Test with deflections calculated from results of laboratory tests. A 3-layered elastic system was solved with tables prepared by Peattie and Jones (34, 54). Base, subbase, and subgrade materials were characterized by creep tests, and the results were transformed to a dynamic modulus. The dynamic modulus of the asphalt concrete surface was determined by a cyclic triaxial test. A typical comparison of calculated and measured deflections under normal traffic loads is shown in Figure 17 (12).

Brown and Pell (6) also verified elastic layered theory in carefully controlled laboratory experiments; they used a program developed by Jones (93). They found that elastic theory predicted vertical and maximum shear stress and maximum surface deflections satisfactorily. Klomp and Niesman (40) used the Shell Bistro program (57) to compare predicted strains in the asphalt-bound surface and base layers with values measured by wire resistance strain gauges. Flexural tests were used to determine the modulus of the asphalt-bound layers, and wave propagation techniques were used for the subgrade. In general, their study found that strain in the bound layers could be predicted with reasonably good accuracy. However, surface strain measurements traditionally have been more difficult to predict because of the effect of factors including tire profile, temperature variations, and tensile stresses in the asphalt concrete.

Seed et al. (65) also used classical elastic layered theory (35) to predict the response of layered systems consisting of asphalt concrete, granular base, and clay subgrade. Most materials were characterized by repeated-load triaxial tests although in some cases values of E for the asphalt concrete were determined by repeated flexural test (15). In each case, they were able to predict reasonably well the deflections occurring within a prototype pavement section. Kasianchuk (38) extended the approach used by Seed by developing an approximation-linear iterative technique for use with layered elastic theory. That technique has been employed by Barksdale (1), Hicks (33), and Hicks and Finn (34) to predict with varying accuracy the response of a wide variation of pavement sections. For example, Hicks (33), using classical elastic theory, was able to predict surface deflections and strains in the asphalt concrete, strains in the untreated base layer, and stresses in the subgrade with reasonably good accuracy. However, for the prototype pavement investigated, better comparisons were obtained with finite-element techniques. Hicks and Finn (34) used a similar procedure to predict the response of test sections at the full-scale San Diego test road. Typical comparisons between measured and predicted responses for one of the sections investigated (Fig. 18, 34) were reasonably good; however, other sections had ratios of predicted over measured values ranging from 0.4 to 1.4 for total deflection, 0.1 to 2.3 for surface strain, and 0.4 to 2.6 for strain within the asphalt layers.

Thrower et al. (84) have also presented evidence indicating that layered elastic theory can be used to predict stresses and strains in pavements subjected to moving wheel loads. Dynamic moduli of the asphalt concrete were measured by means of a dynamic flexural test. Wave propagation techniques were used to determine E of the base, and the subgrade E was determined by the Shell correlation procedure and the measured CBR value. They found good agreement between theory and measured response for stiff pavement sections. Serious discrepancies, however, occurred at high road temperatures partly because of difficulties in evaluating E of the asphalt concrete at high temperatures.

Finite-Element Approaches

Shifley (92) in 1967 and Duncan et al. (19) in 1968 used finite-element techniques to analyze pavement structures composed of an asphalt concrete surface, granular base, and clay subgrade. Shifley (92) conducted a series of rigid plate repeated-load tests on a full-scale test section to simulate slowly moving traffic. Transient deformations computed from the results of the repeated-load laboratory tests for multilayer pavement structures compared reasonably well with the measured test road deflections. The nonlinear behavior for both aggregate base and subgrade were approximately accounted for by an iterative finite-element procedure.

Duncan et al. (19) analyzed a pavement structure for both summer conditions (low asphalt stiffness) and winter conditions (high asphalt stiffness). Response of in-service pavements was calculated by an iterative finite-element program together with material properties determined from the repeated-load triaxial test. Predicted deflections were found to be in good agreement with those measured by the California traveling deflectometer, although some question exists concerning the convergence of the procedure.

A comprehensive study in which finite-element techniques were used has been performed by Dehlen (17) and Hicks (33). Both full-depth and conventional asphalt concrete structures were investigated. Dehlen made comparisons for the San Diego test road between measured stresses, strains, and deflection and those predicted by analytical and finite-element techniques. He found that, although the predicted and measured pavement responses were similar in shape, the quantitative agreement was only fair to good. Typical comparisons for surface deflections and strains within the asphalt concrete layer are shown in Figure 19. Hicks conducted similar analyses on both prototype (Fig. 20) and a conventional section on the San Diego test road and reached conclusions similar to those of Dehlen.

Figure 17. Comparison of measured and calculated surface deflections.

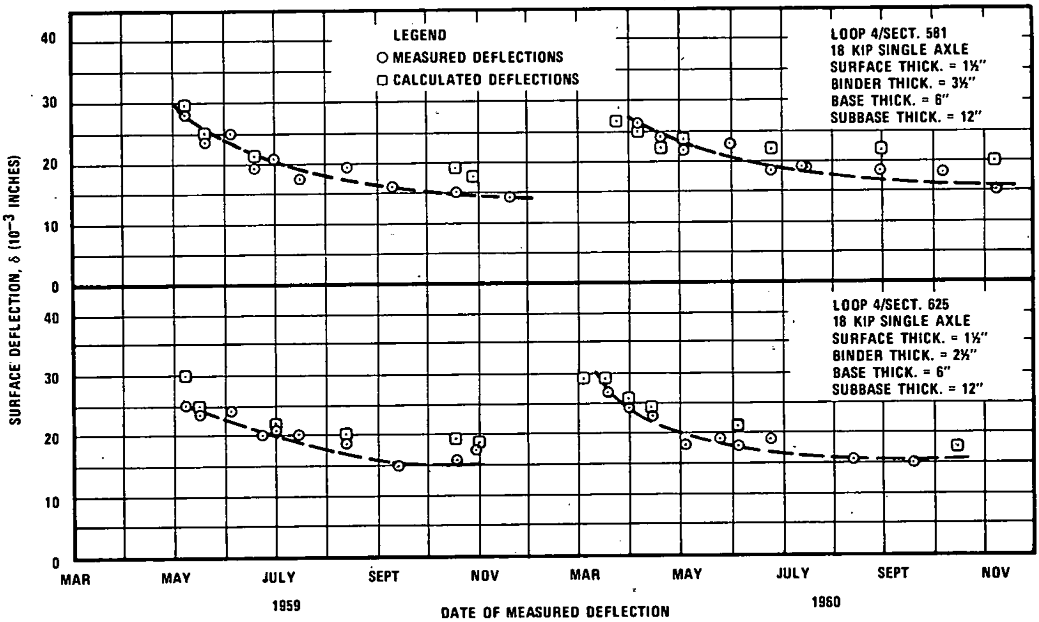
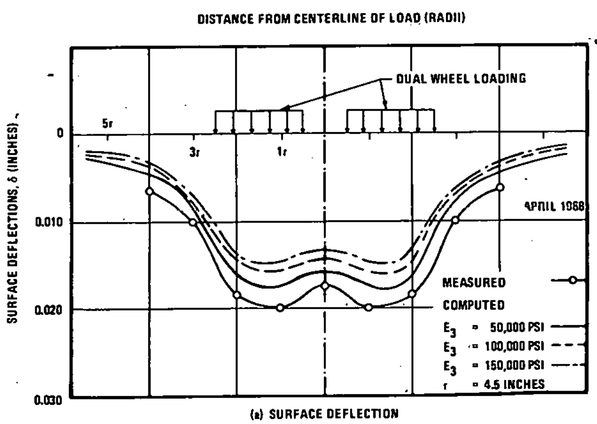


Figure 18. Comparison of measured and computed responses of test road section.



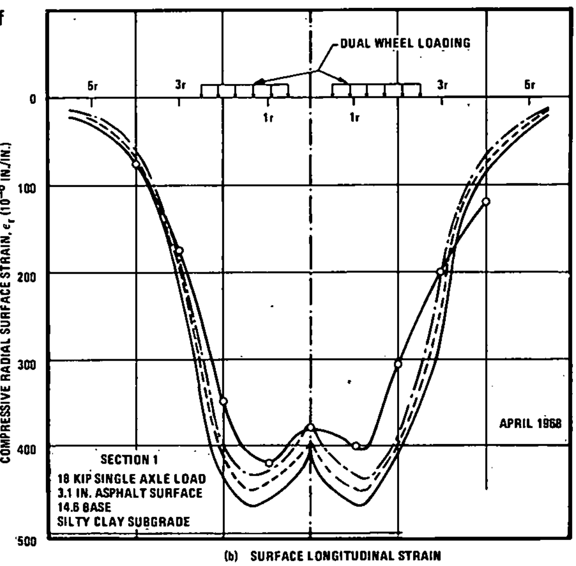


Figure 19. Comparison of measured and calculated surface deflections and radial strains.

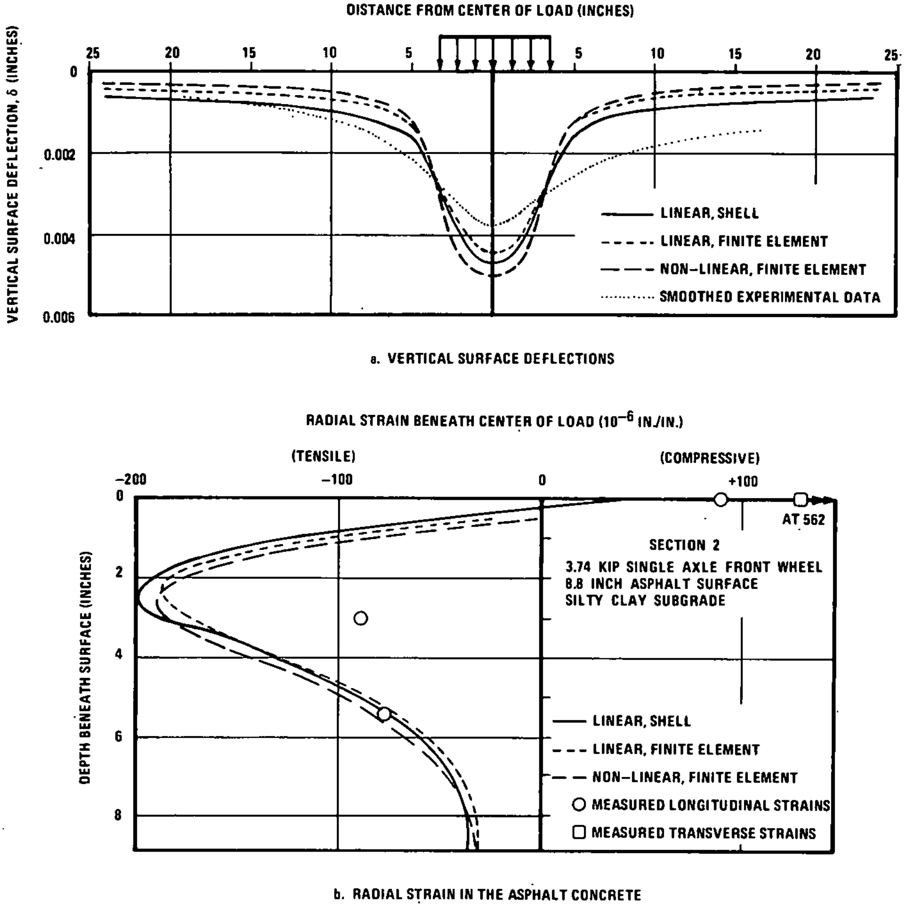
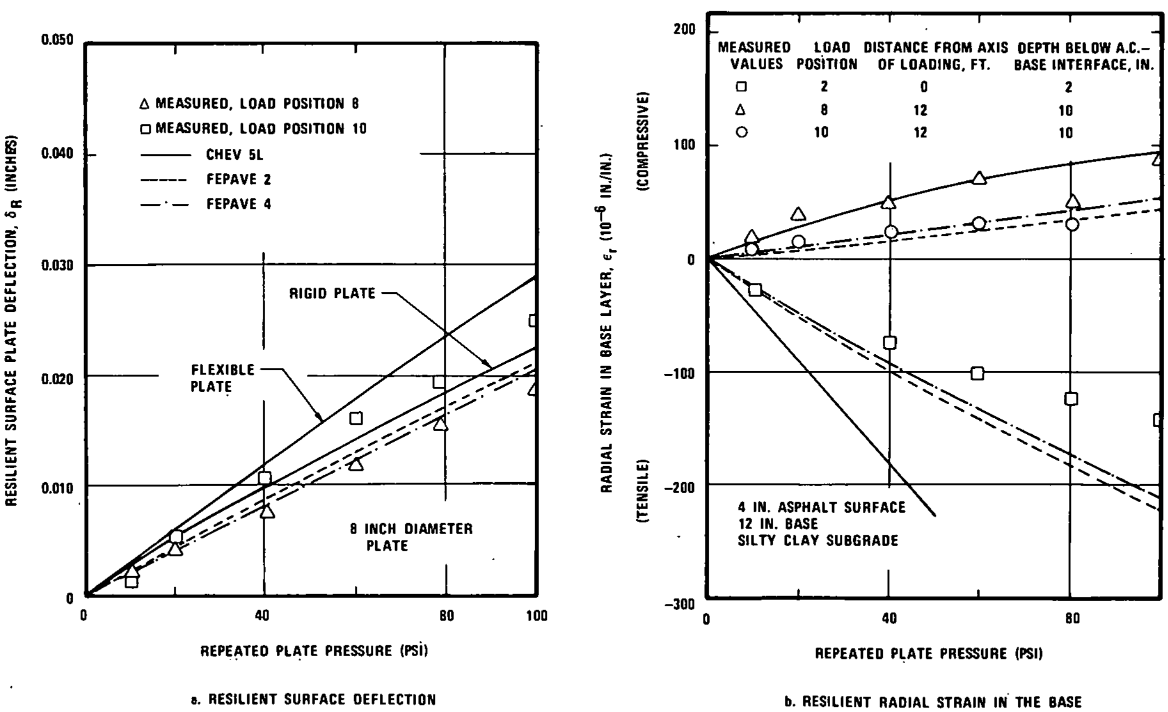


Figure 20. Comparison of measured and computed responses at surface and base.



Viscoelastic Approaches

Barksdale and Leonards (2) compared measured and computed responses by using a linear viscoelastic approach. They calculated both permanent and resilient deflections for 3-layer systems on the AASHO test road. The stress-strain relations were obtained from the results of repeated-load triaxial tests. The calculated deflections were in general close enough to suggest that viscoelastic theory can give reasonable estimates of resilient and permanent deformations. Drennon and Kenis (94) and Barksdale (5) have shown how a linear and nonlinear viscoelastic theory can be used to calculate layered system response.

Theory Selection

The studies just described show that layered theories give reasonably good predictions of deflections, stresses, and strains in layered pavement systems provided dynamic moduli are used that are compatible with the stress state in the layer. A number of elastic and finite-element computer programs that could be used in a mechanistic pavement design approach are now in operational form. Some of those programs and an indication of their availability and where they can be obtained are given in Table 3. A comparative study by Pichumani (89) of several of those programs is given in Table 4.

The selection of the most appropriate theory to use should depend on whether there is any benefit gained in using a more sophisticated approach. For example, a linear-elastic theory can usually be used with success for pavements having stabilized bases and a subgrade that exhibits a relatively linear response. In a general design procedure, however, a more flexible approach should probably be employed to account for the nonlinear behavior of some paving materials. A good example of such an approach is the modified Chevron 5-layer program, which provides for iteration to obtain moduli compatible with the stress state in the layer. Of course, use of a nonlinear finite-element approach would be more desirable and could be used if excessive computer time were not required in applying the program to a design method. Although linear-elastic theories require the least computer time and have been used by several organizations with apparently satisfactory results, considerable care must be taken in selecting appropriate material properties. For developing a general pavement design procedure, a linear-elastic program is recommended that has provisions for iteration.

SUMMARY

Suitable analytical procedures based on layered theory have been developed for predicting the response of flexible pavement structural sections. If applied, those methods have the potential for greatly improving the accuracy with which required structural sections can be selected for widely varying traffic, subgrade, and environmental conditions.

Material properties for use in solutions to layered theory should, where practical, be evaluated in a dynamic testing procedure. The dynamic loading can be readily accomplished by repeated loads to a cylindrical specimen inside a triaxial cell using a pneumatic loading system. The response of the pavement structure can then be predicted with reasonably good accuracy from the laboratory-determined material properties and either a linear-elastic computer program or an ad hoc non-linear-elastic program having iterative capability. Because the moduli of most pavement materials are stress dependent, the iterative approach is recommended.

Perhaps the most important problem in developing a mechanistic design approach is the inherent variability of material properties with depth and position along the project alignment. The ability to overcome that problem will depend to a great extent on how the properties are incorporated into the design procedure and how the procedure is related to actual field performance.

Predicting accurately the response of pavement structures can often be quite involved because of the nonlinear, inelastic, anisotropic nature of pavement materials, changes of material properties with time, uncertainties in conditions during construction,

Table 3. Operational computer programs and their availability.

Theory	Program	Availability	Source
Linear elastic (classical layered theory)	Shell BISTRO (axisymmetric)	Limited distribution (researchers only)	Shell Oil Company, One Shell Plaza, Houston, Texas 77002
	Chevron (axisymmetric)	No restrictions	R. J. Schmidt, Chevron Research Company, Richmond, California
Linear finite element	General axisymmetric (W1L67)	No restrictions	C. L. Monismith, University of California, Berkeley
	Prismatic space (2-dimensional)	No restrictions	University of California, Berkeley
	Extended 2-dimensional (AFPAV)	Under development	R. Pichumani, Kirtland Air Force Base, New Mexico
Nonlinear elastic (classical layered theory)	Chevron 5-layer elastic with iteration	No restrictions	University of California, Berkeley
Nonlinear finite element	FePave 1 (iterative)	No restrictions	University of California, Berkeley
	FePave 2 (incremental)	No restrictions	University of California, Berkeley
	Anisotropic incremental iterative	Available March 1973	R. D. Barksdale, Georgia Institute of Technology, Atlanta
Viscoelastic	Linear viscoelastic	No restrictions	F. Moavenzadeh, Massachusetts Institute of Technology, Cambridge

Table 4. Comparison of computer programs.

Description of Features	BISTRO	WIL67	VISAB3	AFPAV	Remarks
Theoretical basis	Burmister's layered system theory	Finite-element analysis of axisymmetric solids	Discrete-element analysis of pavement slabs on Winkler foundation	3-dimensional analysis of prismatic solids	AFPAV is called extended 2-dimensional finite-element code and cannot solve general 3-dimensional problems
Type of pavement analyzed	Rigid and flexible pavements	Rigid and flexible pavements	Rigid pavement only	Rigid and flexible pavements	Except for VISAB3, codes do not distinguish type of pavement
Number of pavement layers	N-layers (N = 10)	N-layers (N = 12)	2	N-layers (N = 15)	VISAB3 can treat only 2 layers, but the value of N can be changed by suitably modifying other codes
Number of applied loads	12 (can be increased)	1 (can be modified to analyze multiple loads)	Unlimited	Unlimited	BISTRO and WIL67 (when modified) use principle of superposition for multiple loads; VISAB3 and AFPAV consider multiple loads simultaneously
Contact pressure	Uniform over circular area	Uniform over circular area	Concentrated	Uniform over rectangular area	Nonuniform contact pressure can be treated approximately in AFPAV code
Joints in rigid pavement	Cannot consider	Cannot consider	Considers approximately	Can consider realistically	Joint analysis using AFPAV code is yet to be developed
Stress concentrations along joints	No	No	No	Yes	
Edge loads in rigid pavement	Cannot analyze	Cannot analyze	Can analyze	Can analyze	Edge-load analysis using AFPAV code is yet to be developed
Material properties	Linear elastic	Bilinear elastic	Linear elastic	Linear elastic	AFPAV code can be further developed to consider nonlinear material properties
Nonuniform layer thicknesses and properties	Cannot consider	Cannot consider	Cannot consider	Can consider	Changes referred to are in 1 vertical plane only and those should be constant in the longitudinal direction
Discontinuities (e.g., culverts) under pavement	Cannot analyze	Cannot analyze	Cannot analyze	Can analyze	Orientation of culvert should be normal to one horizontal axis of gear arrangement

uncertainties in environmental and subgrade conditions, and many other factors. As a result, the predicted behavior from any proposed pavement design method should be compared with the behavior observed in the field for as many pavement sections as possible. If the predicted behavior is not satisfactory, then the theory should be either revised or adjusted to give acceptable results. Provided the approach taken is theoretically sound, probably the most practical method will be to simply correlate the results with observed field performance (1, 82, 87) to develop design criteria such as limiting stresses and strains. Finally, design criteria developed by other investigators may not yield satisfactory results if either the method of material characterization or the method of calculating the response of the pavement structure is changed.

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