This paper deals with an application of the principles and concepts of fracture mechanics to the problem of cracking of flexible pavements under repeated loading. A brief discussion of the relevant principles of fracture mechanics is presented, and a summary is given of the theoretical and experimental work done at Ohio State University. The advantages and disadvantages of the method are discussed, and future developments are outlined.

Mechanistic Approach to the Solution of Cracking in Pavements

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The term "fatigue" is always associated with damage or deterioration under repetitive loading that eventually leads to cracking and sometimes catastrophic failure of the structural component. It implies a process of localized progressive structural change occurring in a material subjected to fluctuating stress that generally results in the lowering of the resistance of the material to subsequent stressing.

Fatigue is now recognized as a phenomenon of a highly complex nature, and it is generally accepted that no single theory can deal with all the relevant aspects of the problem. Ultimately many disciplines must be drawn together to develop a unified theory.

Existing theories tend to tackle the problem from only 1 of 3 points of view: statistical mechanics, microstructural, and continuum mechanics.

The statistical mechanics approach considers the problem on the basis of the kinetic concept of the mechanism of fracture that involves the breakage and reformation of atomic bonds by stress and thermal fluctuations, the accumulation of rupture bonds resulting in loss of stability, and eventually breakdown. That approach leads to quantitative results but suffers from the lack of consideration of the mechanics of the microstructures and the geometrical and boundary effects.

Microstructural theories describe the mechanism of crack initiation and growth. The type of microstructure of the material, crystalline or amorphous, and the loading conditions are considered. They tend to be qualitative because geometrical and boundary conditions are ignored.

The continuum mechanics approach is the most powerful. It considers the localized nature of the problem, the geometrical and boundary conditions, and conforms as closely as possible to the microstructural theories that are known to explain correctly the mechanism of fatigue.

The phenomenon of fatigue failure is associated with the concept of "damage" or those material changes that lead to formation of macroscopic cracks and subsequent structural instability. The occurrence of fatigue failure is a result of 2 separate processes:

damage initiation and damage growth. The occurrence of those 2 processes in a material system results in a gradual weakening of the structural components. However, the failure state is not reached until the damage approaches a critical level. In short, damage initiation and growth are necessary but not sufficient conditions for the occurrence of fatigue failure. In fact, damage growth in a material body can be arrested during the course of repeated loading before reaching the threshold of instability. The arrest of the damage can occur because either the applied load cannot furnish sufficient energy required for growth or other changes in the material body and boundary conditions alter the state of stress distribution in the structural component.

The processes of crack initiation and growth differ among various materials. Because of the presence of inherent flaws, it is reasonable to expect a crack to initiate at the first few cycles of load application (1) in certain alloys, plastics, polymers, and heterogeneous compositions such as asphaltic materials. The statistical distribution of such internal discontinuities can, in fact, account for statistical variations in the fatigue life.

The process of crack growth has been discussed by various theories, but fundamentally it is related to the deformation occurring at the tip of discontinuities and is associated with the energy balance in those regions. The work of external forces in the regions of discontinuity is divided into stored elastic energy, the energy required for irreversible changes in the material body as viscous or plastic flow, and the surface energy required to form a crack. The rate of crack growth then depends entirely on the energy balance, and the path it follows is governed by the minimum energy requirement.

During the cyclic deformation process, the tip of the zone of discontinuity blunts and resharpens, resulting in crack growth through the body (Fig. 1). That process continues until a crack of critical size has been reached, and the induced state of stress results in structural instability or terminal event of fracture.

Although the crack growth is discontinuous, it is assumed to be continuous to justify the use of the continuum mechanics approach. The problem of the initiation of crack growth and fracture belongs to the domain of fracture mechanics.

The development of fracture mechanics followed from Griffith's classical theory for brittle materials. Subsequently, Irwin and Kies proposed that a modified Griffith theory could be employed widely in fracture-strength analysis in the presence of substantial amounts of plastic strain as long as fracture occurred in advance of general yielding.

Modern fracture mechanics owes its development to Irwin, who proposed the concept of the stress-intensity factor. In 1957 he observed that all crack behavior could be classified into 3 distinct modes according to whether the resulting displacements contribute to the opening (mode 1), in-place sliding (mode 2), or tearing (mode 3), or modes of relative displacement of the crack surfaces (Fig. 2). The 3 modes are necessary and sufficient to describe all the possible modes of crack behavior in the most general state of elastic stress.

It follows naturally that each of the crack movements is associated with a stress field in the immediate vicinity of the crack tip. The distribution of stress in the vicinity of the crack tip is basically a problem in the mathematical theory of elasticity in which it can be shown that all crack-tip stress fields exhibit inverse square root singularities. Thus, for small-scale yielding (i.e., for a small plastic zone as shown in Fig. 1), the stresses in the vicinity of the crack tip can be expressed as follows for open mode 1:

$$\begin{pmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{pmatrix} = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2} \begin{pmatrix} 1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2} \\ 1 + \sin\frac{\theta}{2}\sin\frac{3\theta}{2} \\ \sin\frac{\theta}{2}\cos\frac{3\theta}{2} \end{pmatrix}$$

There are similar expressions for the other 2 modes involving K_{II} and K_{III}.

The parameters K_I, K_{II}, and K_{III} are called the stress-intensity factors and clearly govern the magnitude of the local stresses in the vicinity of the crack tip.

The validity of the stress field is confined to an annular zone around the leading edge of the crack. The zone lies beyond the zone of plastic and nonlinear strains but does not extend beyond distances from the crack tip smaller than the crack and specimen dimensions.

The utility of the elastic stress field analysis lies in the similarity of the near crack-tip stress distributions for all configurations with the same stress-intensity factor; i.e., 2 bodies with cracks that are of different size and have different manners of load application but are otherwise identical will have identical near crack-tip deformation fields if the stress-intensity factors are equal. Or, in the words of Irwin, "The point of view so far represented is that of an imaginary small observer looking outward from the crack edge plastic strain zone, and [being] unable to distinguish whether an increase in the surrounding stresses arises from an increase in the applied load or from an increase in the size of the crack" (1).

The stress-intensity factor K has also been shown to be related to the Griffith strain energy release rate G as follows:

$$K^2 = \frac{GE}{1 - \nu^2}$$

for the plane strain, and

$$K^2 = GE$$

for the plane stress. $G = \delta W/\delta c$, the strain energy release rate, may also be considered as the crack extension force. Thus, K is seen to be a powerful parameter for it not only governs the magnitude of the stress field in the vicinity of the crack in accordance with the load, size of crack, and geometrical and boundary conditions but also is proportional to the force tending to cause crack extension.

It was not surprising, therefore, that in 1957 Irwin wrote that a substantial fraction of the mysteries associated with crack extension might be eliminated if some estimates of the stress conditions near the loading edge of the crack were made in terms of the stress-intensity factor.

However, it was not until 1961 that Paris, Gomez, and Anderson (2) first introduced the application of the stress-intensity factor to fatigue crack propagation rates. In 1963, Paris and Erdogan (3) found from experimental data that the crack propagation rate, dc/dN, was proportional to the fourth power of ΔK for a number of materials. This law of crack growth is expressed as $dc/dN = AK^n$, where A is a material constant and $n = 4$.

Later, the fourth-power relation was justified by consideration of the energy absorption within the entire plastic zone ahead of the crack tip (4). Rice (5) also derived Paris and Erdogan's expression by a rigid plastic model, which assumes plastic deformation is limited to a strip of material ahead of the crack tip.

It is clear from the discussions given above that the stress-intensity factor is the dominant parameter controlling the crack growth in a pavement, and therefore it is important to be able to determine its value, both theoretically and experimentally, for all modes of cracking in pavements as well as 2-dimensional simplifications used for simulation in laboratory experiments. The available theoretical solutions and experimental methodology are discussed in the following section.

DETERMINATION OF THE STRESS-INTENSITY FACTOR

To determine the stress-intensity factor K for a given crack size and specimen geometry, analytical methods have been developed for various boundary conditions.

The K-value for a simply supported beam with a central load was solved by Winne and Wundt and by Gross and Srawley (6). The results of their analysis are shown in

Figure 3. Finite element method has been used to develop a similar nondimensionalized relation among stress-intensity factor K, load, and beam geometries.

The K-value for a beam supported on an elastic foundation with a central load was obtained by the boundary collocation method. The solution is given in another paper (7). Similarly, a finite element program has been developed to calculate K for any crack size.

The solution for the stresses in a slab with a semi-infinite crack supported on an elastic foundation was obtained by Williams, Ang, and Folias (8). From that solution, the K-values under a moving load were obtained as given elsewhere (7).

The stress-intensity factor K for any type of loading, crack pattern, and geometry can be determined experimentally by a very simple procedure. That is done by measuring the change in the deflection as the cracks grow and by applying the formula

$$K^2 = \frac{P^2E}{2(1 - \nu^2)} \frac{\delta L}{\delta c}$$

where

- P = load,
- E = Young's modulus,
- L = compliance, or inverse slope of the load-deflection diagram, and
- c = crack length.

EXPERIMENTAL CHARACTERIZATION OF PAVEMENT SYSTEMS

In the research work carried out at the Ohio State University, the applicability of the theory to asphaltic materials has been examined in the light of 2 very important assumptions made in the theoretical concepts:

1. The material must be homogeneous, isotropic, and essentially elastic-plastic, and
2. The size of the plastic zone at the tip of the crack must be small in comparison to the crack and specimen dimensions.

The first assumption is one that is generally accepted for asphaltic materials. With regard to the second assumption, the size of the plastic zone r_y can be calculated from the formula

$$r_y = \frac{1}{2\pi}\left(\frac{K_I}{\sigma_y}\right)^2$$

where σ_y = yield stress in tension.

Using that estimate of the size of the plastic zone, Srawley and Roberts (9) established criteria for the crack length, width, and depth of beam to ensure plane strain conditions and the applicability of linear elastic fracture mechanics. The criteria state that both the crack depth and width of the beam should exceed

$$2.5\left(\frac{K_I}{\sigma_y}\right)^2$$

For a pavement, the theory is applicable if the thickness of the asphaltic layer is greater than 1.25 $(K_I/\sigma_y)^2$. For typical highway mixes, that criterion is satisfied even for pavement layers smaller than 1 in. and for the heaviest loads; therefore, from a practical point of view, we may say that linear elastic fracture mechanics is always applicable.

The results of experiments conducted at Ohio State University were reported in other papers (10, 11, 12, 13). The following salient points were made in those papers.

1. Tests on simply supported beams of sand asphalt beams tested at 23 F (10) showed that the rate of crack propagation dc/dN correlated well with the stress-intensity factor K in accordance with Paris' law

$$\frac{dc}{dN} = AK^4$$

where A is a material property.

2. The beams failed when the crack reached the critical crack length c_f corresponding to the critical stress-intensity factor K_{IC}. K_{IC} is the failure criterion for both monotonic fracture and fatigue and is a material property.

3. The fatigue life N_f of the beam may then be expressed as

$$N_f = \int_{c_o}^{c_f} \frac{1}{AK^4} dc$$

where c_o is the starter flaw. The starter flaw is a material constant but is subject to statistical variation and is believed to be principally responsible for the statistical variation of fatigue life.

4. Tests on simply supported sand asphalt beams (11) at 77 F showed that there was considerable interaction between creep and fatigue. The amount of creep was minimized and more realistic conditions were simulated when the beams were supported on an elastic foundation. Both controlled stress and controlled strain types of loading were used.

5. The results of the tests on the beams supported on an elastic foundation provided further verification of the crack propagation law and of the fact that the starter flaw was indeed a constant for the material. Furthermore, the prediction of the fatigue life was independent of the method of loading (controlled strain or controlled stress). That effect is fully accounted for by changes in the stress-intensity factor due to changes in the load.

6. The experiments on asphalt concrete beams (12) supported on an elastic foundation also showed excellent correlation with the crack growth law. The asphalt concrete mix was a typical Ohio Department of Highways 404 mix containing 6.5 percent asphalt content with both 60-70 and 85-100 penetration asphalt.

7. Sand asphalt slabs 44 in. in diameter were tested on an elastic foundation by an MTS machine (7). The amount of cracking and the crack pattern were obtained by X-ray photography. The cracks originated at the bottom and grew radially. Eventually circumferential cracks that originate at the top appeared. The completion of the circumferential crack marked the end of the experiment. The crack pattern is shown in Figure 4.

8. The results provided further verification of the validity of the crack propagation law: $dc/dN = AK^4$. The value of A was approximately the same as the value determined for the sand asphalt beams of the same composition as the slabs, showing that A was indeed a property of the material.

The foregoing principles can now be applied to the design and analysis of pavements.

OUTLINE OF METHOD OF ANALYSIS AND DESIGN OF MULTILAYERED PAVEMENT SYSTEMS AGAINST FATIGUE DISTRESS

Inevitably pavement design against fatigue distress involves many complex and interrelated factors. The foregoing theoretical and experimental work established that the rate of crack propagation in a pavement could be expressed in terms of the properties of the component layers and the geometrical and boundary conditions. Thus, any variation in the material properties during the life of the pavement is automatically accounted for, providing that such variation can be ascertained in a quantitative sense.

Changes in Young's modulus E and Poisson's ratio ν of the asphaltic layer due to temperature, speed, and aging would also influence the fatigue behavior of the pavement with regard to the stress distribution and the effect on the crack growth constant A. Those effects must be ascertained with proper consideration to the environmental and climatic conditions. Similarly, the effects of changes in the subgrade support due to moisture content variation must be estimated.

Figure 1. Stress-deformation at crack tip.

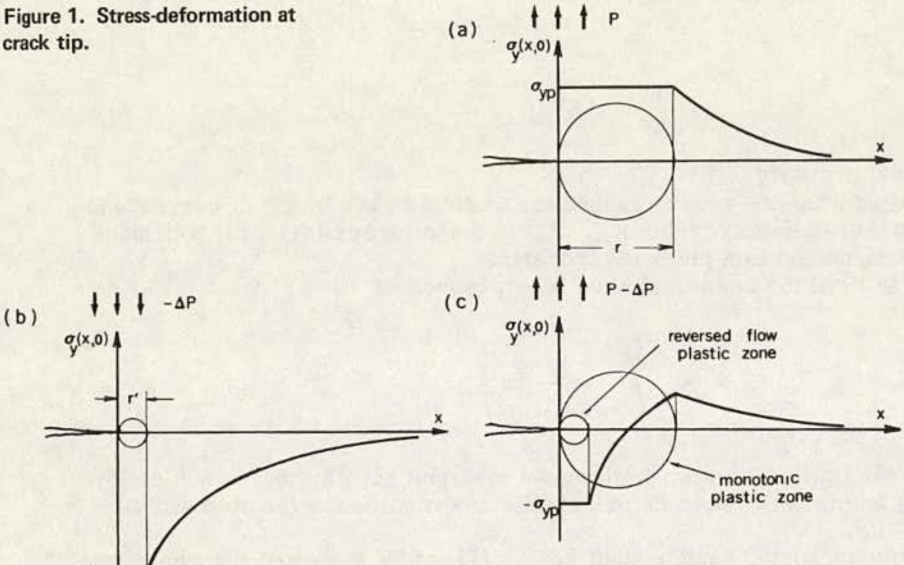


Figure 2. Modes of deformation of crack.

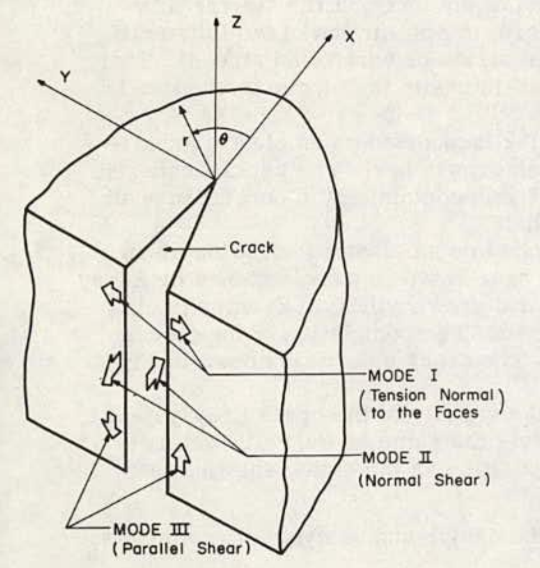


Figure 3. Dependence of square of stress-intensity coefficient on relative crack depth.

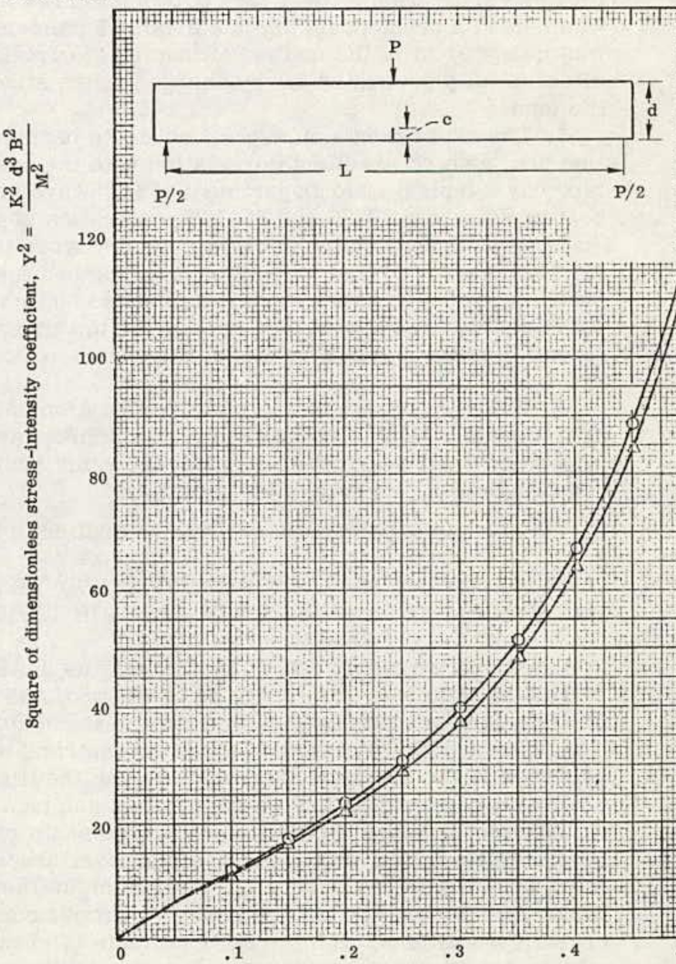
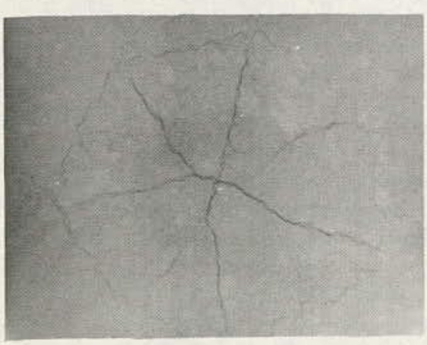


Figure 4. Crack pattern.



However, a number of important factors contributing to the fatigue life of a pavement have not been considered. Foremost among those are the cumulative effects of random loading and the effects of the interaction of rutting on cracking. Both of those topics are now the subjects of continuing research at Ohio State University. Recent results show that the effects of random loading can be largely accounted for by a simple modification of the concepts given above.

Of secondary importance is the effect of rest periods. Laboratory tests conducted at room temperature show that rest periods may be beneficial to fatigue due to healing. In practice, however, healing may not take place because of dust and water that may enter through the top cracks.

Within those concepts and within those limitations the following method of analysis and design is proposed.

Materials Characterization

1. Determine from laboratory tests which materials will be used for the base course and subgrade, and select a suitable asphaltic mix for the surface layer. Evaluate from standard test methods the properties E and ν for each layer and the modulus of subgrade reaction k as a function of temperature, frequency, aging index, and moisture content where applicable.
2. From fracture tests on beams of the same asphaltic mixture as the surface course, determine the critical stress-intensity factors K_{1c} and K_{2c} as functions of temperature, frequency, and aging index.
3. From fatigue tests on the asphaltic beams supported on an elastic solid, determine the constants A_1 and A_2 in the crack propagation law $dc/dN = A_1K_1^4 + A_2K_2^4$ as functions of temperature, frequency, and aging. Determine also the range and distribution of the experimental constant c_o.

Analysis and Evaluation

The basic principle of the fatigue analysis is that damage will be proportional to the average of the fourth power of the rises and falls in the load-time history of random loading (13) as shown in Figure 5. The exact damage per passage of axle load dc/dN is expressed by

$$\frac{dc}{dN} = A_1K_1^4 + A_2K_2^4$$

where K_1 and K_2 = average of the rises and falls of the stress-intensity factors corresponding to the peak-to-trough rises and falls in the K-time histories.

1. Obtain the components of the stress-intensity factors K_1 and K_2 for a pavement slab for any configuration of wheel loads, for any size and location of crack, and for any combination of the thicknesses and material properties of the component layers. Typical influence lines for K_1 and K_2 for a semi-infinite crack for a full-scale pavement are shown in Figure 6.
2. Determine the fatigue crack propagation in the pavement by determining the size and distribution of starter flaws c_o in the pavement. For a typical pavement the lateral distribution of wheel loads may be assumed to be as shown in Figure 7. Evidently the fastest crack growth will take place under the greatest concentration of loading because the stress-intensity factor is highest there. Thus, the cracking will be assumed to be primarily along the wheel track and perpendicular to it as shown in Figure 7.
3. Determine the size of starter cracks along the line of cracking by applying statistical analysis to laboratory tests on specimens prepared in the laboratory or cut out from the pavement at regular intervals or by cutting out continuous specimens in the wheel track over a distance equal to the shortest distance over which the distribution of c_o will be repetitive. Only the upper range of c_o values will be of practical interest because for exceedingly small values the crack will not propagate to the surface in the design period of the pavement.

Figure 5. Load-time and K factor-time history of random loading.

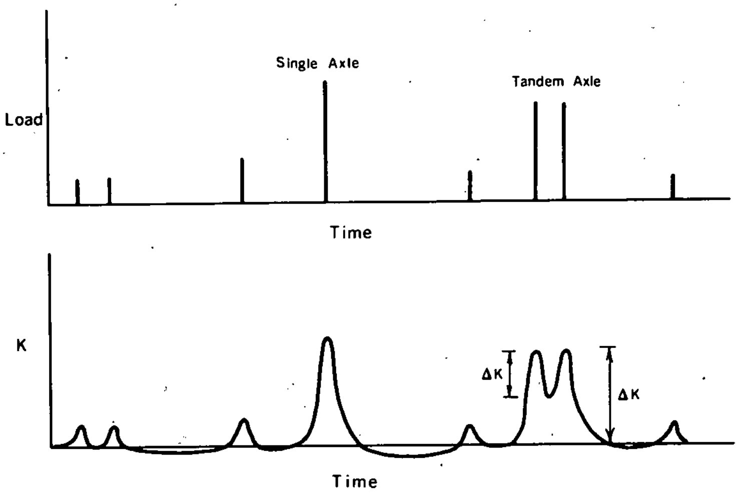
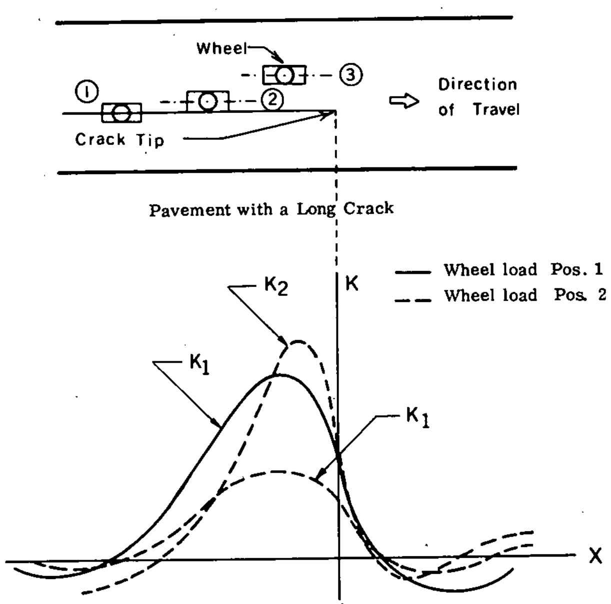


Figure 6. Typical influence lines for K_1 and K_2 for semi-infinite crack.



4. Confining attention to the growth of the cracks in a small representative section of the pavement, obtain the value of K_1 and K_2 for each crack. For simplicity it will be assumed that the distribution of the cracks is such that there is no interaction.

5. Integrate the rate of crack propagation numerically for 1 passage of the load train, and obtain the increment of damage. Damage is defined as the length of crack inclusive of the plastic zone formed at the crack tip or, for simplicity, as the length of crack. Thus, the increment of damage Δc is

$$\Delta c = \sum_{p=1}^{n} \left[A_1(\Delta K_{1_p})^4 + A_2(\Delta K_{2_p})^4\right]$$

where K_{1_p} and K_{2_p} = averages of the rises and falls of the influence lines for K_1 and K_2 respectively of the stress-intensity factor for each crack for the pth axle load.

6. Increase the length of the cracks, and compute K_1 and K_2 again. If the maximum values of K_1 and K_2 exceed K_{1c} and K_{2c}, the critical values of K, for any axle loads in the load train, the cracks will propagate rapidly and the pavement will be considered to have failed. If not, then increase each crack length by the increment of damage Δc, and repeat the procedure.

7. For each location of the assumed starter flaws, compute at reasonable intervals the length of longitudinal and transverse crack, and determine the total area of cracking as the product of the sum of the longitudinal cracks and the average length of the transverse cracks.

Thus at regular intervals the maximum values of K_1 and K_2 and the total area of cracking will be known. If K_1 and K_2 exceed the critical values of K_{1c} and K_{2c}, then the pavement will fail by rapid crack extension. On the other hand, K_1 or K_2 may never exceed K_{1c} or K_{2c} even if the length of cracks becomes exceedingly long. In such cases it is convenient to adopt the suggestion of Zube and Skog that, when the alligator type of cracking exceeds 10 percent of the total area, the pavement should be considered to have failed.

In this discussion, longitudinal and transverse cracks have been considered as the primary mode of crack propagation. In reality, of course, cracks in other directions and parallel to the main cracks will also develop rapidly into the well-known alligator pattern (hexagonal-shaped cracks). The alligator pattern may be deduced directly from the Griffith theory of fracture, which requires that the total surface energy expended to form new surfaces must be a minimum. It seems mathematically that this condition is best satisfied for hexagonal-shaped cracks rather than triangular-, quadrilateral-, or pentagonal-shaped cracks.

Once the primary mode of crack propagation has reached an advanced stage, the secondary and tertiary modes of cracking leading to the completion of the alligator pattern will follow in rapid succession. Thus, the definition of the failure criteria based on the primary mode of crack propagation is sufficiently accurate to obtain an estimate of the service life. Should the pavement design prove to be unsatisfactory, a new design must be made and the analysis for the service life repeated.

Load Equivalency Factors

From the theoretical relation $dc/dN = AK^4$ and the fact that K is proportional to the load P, the load equivalency factor for single axle loads is proportional to the fourth power of the load (but may be somewhat higher because of the effects of random loading). The load equivalency factor for tandem axles depends on the spacing of the axles and the shape of the influence line of K as the loads move across the crack.

As an illustration, consider 2 pavement sections, shown in Figures 8 and 9, that have about the same structural capacity (by the AASHO design method), but different relative stiffnesses. The influence lines for K for an 18-kip single axle and a 36-kip tandem axle load moving directly over a longitudinal crack 2 ft long are also shown in Figures 8 and 9.

Figure 7. Typical lateral distribution of wheel loads in pavement and assumed idealized random distribution of cracks.

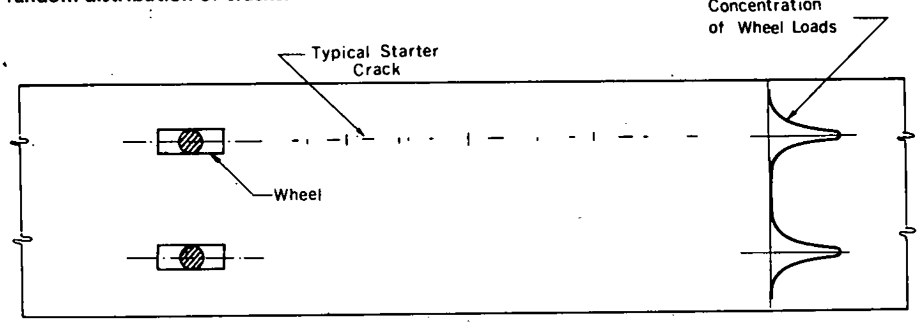


Figure 8. Load equivalency factor for high relative stiffness pavement.

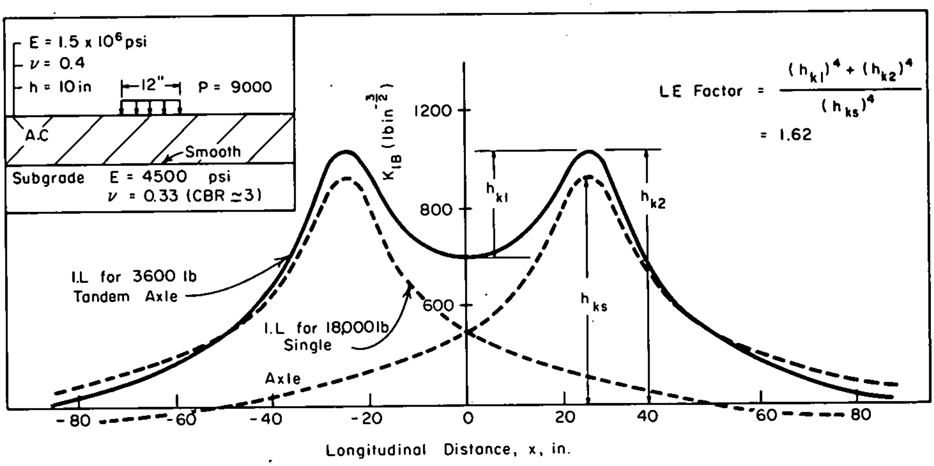
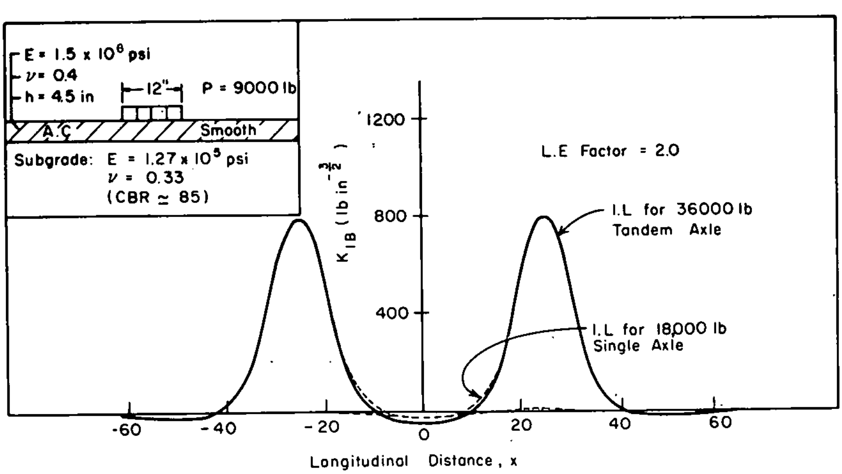


Figure 9. Load equivalency factor for low relative stiffness pavement.



The load equivalency factor, defined as the destructive ratio or crack ratio produced by 1 passage of the axle load as compared to an 18-kip single axle load can be obtained by taking the ratio of the fourth power of the rises and falls in the influence lines for K. Applying that criterion shows that the factor changes from 1.62 to 2.0 as the relative stiffness of the pavement decreases.

ADVANTAGES OF MECHANISTIC METHOD

1. The prediction of the fatigue life of pavements from laboratory tests is independent of the mode of loading or the specimen geometry. Those effects are completely accounted for by the stress-intensity factor.
2. Failure is defined realistically as follows: (a) failure by rapid crack propagation occurs when the stress-intensity factor for a particular crack approaches the critical stress-intensity factor and is most likely to occur under very heavy loads, thick asphalt layers, and brittle mixes; or (b) failure occurs when the area of cracking (obtained directly from the computations) exceeds 10 percent of the total area of the pavement.
3. Load equivalency factors for any type of loading can be obtained by a simple and reasonably accurate procedure.
4. Statistical variations are explained by the basic concepts of the method.
5. The effect of variables other than the bending stress, e.g., the modulus of the foundation and the thickness of the asphaltic layer, is accounted for.

DISADVANTAGES OF MECHANISTIC METHOD

1. To predict the area of cracking requires that the K-value be obtained for the actual crack pattern. However, solutions have been obtained only for 2-dimensional cases and certain simplified crack patterns such as a semi-infinite crack, a semicircular crack, or a very short crack for 3-dimensional cases. That represents the main difficulty in applying the method to obtain realistic crack patterns in pavements. Some type of numerical or finite element solution is required.
2. The computational time is at present lengthy, but simplifications could be introduced to reduce this to an acceptable time.

FUTURE DEVELOPMENT

1. A solution to obtain the K-value for all types of crack patterns is required.
2. There is need for the development of a theoretical solution to account for the effects of the interaction between cracking and rutting due to repeated loading.
3. The effects of random loading and of the various properties of the asphaltic mixture on fatigue need further investigation. Some of this work is being done at Ohio State University.

CONCLUSIONS

1. The crack propagation theory, $dc/dN = AK^4$, satisfactorily explains the fatigue performance of bituminous pavements; the stress-intensity factor K is the dominant parameter controlling crack growth, and A is a constant of the bituminous material. The theory applies only for plane strain conditions; that is when the thickness h of the bituminous slab exceeds $1.25\ (K_{1c}/\sigma_y)^2$, where K_{1c} is the critical value of K_1, and σ_y is the yield strength.
2. The critical stress-intensity factors K_{1c} or K_{2c} are the failure criteria for low temperatures or thick slabs. For higher temperatures or thin slabs, failure may be considered to have taken place when about 10 percent of the total surface area is cracked.
3. The load equivalency factor, defined as the destructive ratio when compared to an 18-kip single axle load, is proportional to the fourth power of the axle load for single axle loads; but, for tandem axle loads of equal magnitude, it is dependent on the spacing between the axles and the relative stiffness of the pavement. The load equivalency factor can be determined by taking the average of the fourth power of the peak-to-trough rises and falls in the K-time history.

4. The bending stress σ^* in the bituminous slab is not a sufficiently good parameter to describe the fatigue behavior of pavements, for it cannot account for the cracking and the subsequent redistribution of the stress. The stress-intensity factor K fully accounts for those effects; in general, K is a function of the bending stress, the crack length in the pavement, the relative stiffness of the pavement, and the geometrical and boundary conditions.

5. Finally, when one knows the traffic loading and the variation of material properties E, ν, A, K_{1c}, and K_{2c} due to temperature, moisture, and aging index during the service life of the pavement and also the magnitude and distribution of the starter flaws c_o, a reasonable estimate of the service life can be obtained by the method outlined.

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Discussion

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The authors have attempted to apply the theory of classical fracture mechanics for predicting the fatigue life of flexible pavements. Most of the material in the paper is concerned with the review of the elementary concept of fracture mechanics based on the stress-intensity factor parameter. The authors conclude that the stress-intensity factor is the parameter controlling crack growth in the pavement. However, the experimental data (13) quoted by the authors appear inconclusive. Therefore, the writer wishes to raise several questions with regard to the contents of the paper.

In the abstract of the earlier paper (13), the authors define the fatigue failure criterion of pavements as the time for the stress-intensity factor of the longest crack to reach its critical value at which rapid crack propagation occurs. That is in contrast to the crack growth data shown in Figure 7 of that paper (13) in which the crack tends to reach a subcritical crack length rather than rapid unstable crack propagation.

The authors use the word "damage" very loosely. It appears that damage and crack growth are used synonymously. In collecting crack growth data, it is essential to report the size of the crack opening, which could range from, say, 10^{-1} in. (nonmetals) to 10^{-6} in. (high-strength alloys). Moreover, the classical fracture mechanics theory is not a theory of cumulative damage.

Recent (14, 15) and previous (16) works on the direction of crack initiation have shown that there is only one fundamental mode of crack propagation, namely, that the crack runs in a plane perpendicular to the direction of maximum stress (16). The classical concept of mode 2 crack extension is inadequate because the crack does not run directly ahead. A more refined theory based on the stationary value of the strain energy density factor S (14, 15) indicates that the crack runs in the direction at which S reaches the critical value S_c. In pavement studies one has to treat the mixed mode crack problem involving a combination of at least k_1 and k_2 because the cracks do not run in a straight line.

The linear fracture mechanics theory is not restricted to the definition of small-scale yielding as stated by the authors where the zone of plasticity is small in comparison to the crack length. It has been shown that the elastic stress-intensity factor and the so-called "plastic stress-intensity factor" (18) do not differ significantly even though the plastic zone size may be as large as half the crack length.

The basic assumption of the crack growth relation used by the authors is an empirical power-law relation obtained from crack growth data on metal alloys (17). The fourth-power exponent is to be questioned because we know that the exponent n can vary from 2 to 100 depending on the material, the environmental conditions, and the range of cyclic loading. For example, in the range of 10^{-6} to 10^{-4} in./cycle, n normally varies from 2 to 10. Moreover, the exponent can deviate greatly from 4 if the cyclic range is varied below and above 10^{-6} to 10^{-4} in./cycle. Those observations have been made on metallic materials such as aluminum, steel, and titanium and on nonmetallic materials such as plexiglass and other thermoplastic materials. It is an open question as to whether the fourth-power exponent or any other value should be used for predicting the fatigue life of pavement structures. Experimental data shown in the references listed by the authors were based on simply supported beams of sand asphalt material tested at 23 F. It is not clear whether all the crack growth data in Figures 8 to 10 (13) were or were not taken from the beam-bending specimen where the crack is only partially through the thickness. It should be recalled that the fourth-power relation was originally established from data on metal specimens with a through crack running at both ends. Moreover, the analytical results of a cracked plate on an elastic foundation also correspond to a through crack and not an edge crack. Therefore, it is the opinion of the writer that the experimental data presented by the authors are insufficient to establish prediction relations for the fatigue life of pavements.

The authors state that "linear fracture mechanics is always applicable," based on the 1.25 $(k_1/\sigma_y)^2$ relation. One must remember that the concept of size limitation such as 2.5 $(k_1/\sigma_y)^2$ was established on the basis of a through crack in a homogeneous material with a reasonably well-defined yield stress. In the case of pavement material, the crack may not penetrate through the entire pavement, and, furthermore, one must carefully define what is meant by σ_y for a multiphased material that is also time dependent.

Based on the evidence presented by the authors, the discussion given above suggests that there is now inadequate justification for the application of fracture mechanics to the prediction of fatigue life of pavements based on a simple interpretation of the fracture mechanics theory using equations borrowed from studies on metal alloys. It should be made clear, however, that the concept of fracture mechanics can be used to analyze pavement materials as in the case of composite systems. Successful application of the theory depends on the ability of the analyst to come up with a realistic analytical model catering to the specific problem at hand. What has been developed for metals may not necessarily apply to pavement materials. There is no doubt that additional work must be done on defining the service loading and environmental conditions under which fatigue crack growth takes place in pavements.

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Closure

The authors wish to thank Fang for his review and discussion of our paper. We fully concur that fatigue analysis of pavement systems using fracture mechanics requires consideration to complex service loading and environmental conditions and is much more involved than results presented in our paper. The data we presented are for a simplified laboratory pavement model with well-defined material characteristics and geometrical conditions. We regret to note that Fang in discussing the limitation of our results apparently assumes that we used a composite or multilayered asphaltic pavement system. We have clearly indicated that the results are obtained from a pavement model resting on elastic foundation.

Fang's comment on our use of the concept of damage is irrelevant. We have never stated nor suggested that fracture mechanics is a cumulative damage law. Furthermore, we fully disagree with his statement, "There is inadequate justification for application of fracture mechanics to the prediction of fatigue life of pavements . . . using equations borrowed from studies on metal alloys."

Stress dependency of the damage rate and its sensitivity to specimen geometry is a well-known and accepted concept. The Paris equation used in our study is only another form of representing the rate of damage due to fatigue loading.

We concur with Fang that this equation is a semi-empirical formula and n might be other than 4. It is also granted that test data from metal alloys and certain paving mixtures show a variation from 2 to 5 (4, 5, 6, 7). The important fact is that, for the sand asphalt material used in the Ohio State University research projects, it was found experimentally that the fourth-power relation best fits laboratory data. Figures 8 and 9 (13) clearly show that the data were obtained from tests on slabs resting on an elastic foundation, while it is also shown in Figure 10 that the data come from beams and slabs, shown there as a comparison.

There is absolutely no contradiction between the stated criterion for fatigue failure, defined as the time for the stress-intensity factor at the tip of the largest crack to reach its critical value or the time for the total area of cracking to exceed 10 percent of the area of the pavement surface, and the crack growth curve from a laboratory test on a slab with stationary load shown in Figure 7 (13). Obviously the criterion postulated refers to a real pavement system with moving loads.

It has been proposed by the authors (2, 3) that the exact damage per passage of axle load in a pavement, dc/dN, is expressed by

$$\frac{dc}{dN} = A_1K_1^4 + A_2K_2^4$$

where K_1 and K_2 are the average of rises and falls of the stress-intensity factors corresponding to mode 1 and mode 2 of crack extension. However, as Fang indicated, the use of energy density factor S seems more appropriate.

The idea of using the plastic stress-intensity factor is a sound one. However, at the time this study was being conducted, experimental evidence was not available to compare differences between plastic and elastic stress-intensity factors.

If indeed the concept of K can be shown to be applicable to larger plastic zones, it will greatly reinforce the application of fracture mechanics to analysis of pavement systems.

We have shown evidence from observations during the test and X-ray photographs (2, 3) that the cracks do penetrate through the entire pavement. That is why the theory for part-through cracks was used. The yield stress σ_y was measured experimentally versus rate of loading, and the results were reported (2).