Resource Paper on Product Attributes

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One of the most fundamental questions that arises in both advertising and product planning is what product attributes (i.e., characteristics) are most important in the consumer decision-making process. Advertising objectives are often set based on improving consumer perception of a product in terms of the specific product attributes deemed to be most important (1). In the product-planning area, decisions must always be made that require trade-offs in that a higher level of one attribute necessitates a lower (or higher) level of some other attribute because of engineering or financial considerations or both. These trade-off decisions require inputs regarding the relative importance consumers attach to the attributes in question (2).

To be of assistance in these decisions, the marketing analyst must first obtain measures on a set of attributes that describe the alternative products from a consumer-choice point of view. The analyst must then determine which product attributes appear to be most important to consumers in their choice among alternative products. However, although there is a great deal of literature that is relevant, there is no generally accepted methodology. A full review of this literature is not possible here, but Myers and Alpert (3) and others have provided a review of many of the various approaches that have been suggested. The purpose of this paper then is to develop and illustrate a new approach for obtaining measures on a set of attributes that describe alternative products from a consumer viewpoint and for estimating the relative importance consumers attach to each of the attributes in selecting among the alternative products.

The particular consumer product choice selected to illustrate the methodology suggested is the transportation mode (product) choice decision for the journey to work. In this paper, the choice involved is between automobile and rail transit. However, Wallace, in an earlier paper (4), has applied some of the same methodology for new-car-purchase decisions with encouraging results. The approach suggested here is, generally speaking, a combined application of measurement theory (scaling), consumer demand theory, analysis of variance, and econometrics.

ATTRIBUTE DEFINITION AND MEASUREMENT

Developing a description of alternative products in terms of attributes immediately raises the problem of definition and measurement. Quantitative attributes, that is, those with natural physical units of measurement familiar to the consumer, cause little difficulty. Examples are price (dollars) and gas mileage (miles/gallon) for automobiles or travel time (minutes) and fare (dollars) for modes of travel. But qualitative attributes having no natural physical unit of measurement familiar to the consumer do cause a problem. Examples are tartness and cleaning power for tooth paste and comfort, sex appeal, dependability, and noise for automobiles. Because in practice it is seldom (if ever) possible to fully describe alternative products solely in terms of quantitative attributes, some scaling technique is required. It will later be argued that, because this is so, for consistency it seems reasonable to obtain scale ratings on all attributes.

There is considerable literature on scaling procedures and their applications. There are the traditional metric methods such as the semantic differential and the Thurstone methods and the newer nonmetric scaling methods. Each has its strengths and weaknesses, a discussion of which is beyond the scope of this paper. Green and Tull (5) provide a thorough review of these methods. The approach taken here is the semantic differential, but other procedures could have been used. However, a recent study by Green and Rao (6) indicates that the traditional metric scaling techniques appear to perform as well as the newer nonmetric methods when it comes to returning a known product group configuration.

Product-attribute definition and measurement via the semantic differential requires, first, a choice as to the number of intervals; second, a selection of polar adjectives to define the end points of the scale; and, third, a choice between a monadic and paired-comparison research design. A 7-point scale has been recommended by Osgood (7). Although some have suggested fewer intervals, the Green and Rao (6) work also supports Osgood's recommendation—so selecting a 7-point scale appears reasonable.

Regarding the definition of polar adjectives, there appear to be 2 approaches. The first is to attempt to define the end points so that attribute-rating measurements will not be value loaded, that is, will depend solely on level of the attribute and not on respondent's utility function (8). But for qualitative attributes in particular, this does not appear to be possible because it is very difficult (if not impossible) to define end points that guarantee that most respondents do not provide ratings based on a mental comparison with other actual or ideal products. If we assume that this is the case, the only alternative is to force value-loaded judgments from respondents by appropriate selection of the polar adjectives. In this way, individual measurements become attribute-satisfaction ratings rather than attribute ratings because respondents are asked to provide a measure of their satisfaction with regard to a particular attribute of a specific product.

Models are often formulated in which attribute ratings and attribute-satisfaction ratings are used interchangeably. Myers and Alpert (3, p. 18) cite a regression model in which certain attributes such as color, overall appearance, and taste of a cocktail dip mix were rated on a 7-point scale with end points "liked very much" and "disliked very much" and other attributes such as strength of flavor and spiciness were rated on a 5-point scale from "much too strong (spicy)" to "much too weak (bland)." Buying intention was used as the dependent variable. Attribute-satisfaction ratings were obtained on the first set of attributes, whereas an apparent attempt was made to obtain attribute ratings on the latter set. A more obvious attempt would have been to label the end points "very weak" to "very strong" for flavor and "very spicy" to "very bland" for spiciness. However, even with these end points, some respondents will, in general provide ratings based on a mental comparison with another actual or ideal product in the same choice category, and other respondents may provide ratings that are not value loaded. This inconsistency among respondents leads to unreliable measurements and strongly suggests the use of end-point definitions that clearly request measures of satisfaction with a particular product attribute. This implies the use of end points such as "poor/excellent," "very unsatisfactory/very satisfactory" or "highly unsatisfactory/completely acceptable," as shown in Figure 1. Although this point is of critical importance, arguing it further is beyond the scope of this paper.

The fact that obtaining reliable measures regarding the level of qualitative attributes does not appear possible and that measuring attribute-satisfaction ratings is thereby necessary has very important ramifications for building models to estimate the importance of attributes. These difficulties arise because attribute-satisfaction ratings depend not only on the level of the attribute but also on the individual consumer's utility function (9). A potential solution to this problem was suggested by Wallace (2), and a questionnaire was designed to provide the data necessary to calibrate and validate the proposed consumer-choice model. The data obtained and method of collection are described in the next section.

As stated above, a choice must also be made between a paired-comparison and a monadic research design. Greenberg's study (10) provides numerous references regarding the strengths and weaknesses of the 2 approaches. One of the major arguments against the paired-comparison approach is that it tends to magnify what are actually minor differences in attribute satisfaction. This problem is particularly relevant when these data are to be used as input to a model designed to infer the importance of attributes from consumer product-choice decisions. Another strong argument in favor of the monadic design is that it provides data in the case of quantitative attributes to test alternative hypotheses regarding the mapping from attribute to attribute-satisfaction ratings. This fact will be made use of in a later section. Because of these points, a monadic design appears most reasonable. Of course, the monadic design must be used if an attribute has a different meaning or no meaning at all for the 2 products under consideration.

THE EXPERIMENT

As mentioned above, the product-choice decision process for which data were collected is the mode-choice decision for the journey to work. The following information was obtained by a 5-page, mailed questionnaire for the respondent's first and second mode choice: attribute-satisfaction ratings based on the semantic differential for 15 different attributes (Fig. 1), attribute values for 7 quantitative attributes (Table 1), and the usual demographic data. There was a total mailing of 10,000; approximately 1,000 were returned. The statistical results in this paper are for the subsample making the choice between automobile (driver or passenger) and rail transit. The total sample was 117 (60 choosing automobile, 57 choosing transit). A detailed discussion of the questionnaire and its design is in the literature (11).

DEMAND EQUATION FORMULATION AND ESTIMATION PROCEDURES

The first objective of this section is to develop the demand side of a model for estimating the importance of product attributes. Actually a family of demand-side equations is developed. The second objective is to suggest means by which these models can be calibrated. To facilitate later discussion of the empirical results, we will describe the model in terms of the mode-choice decision. To generalize, traveler is replaced by consumer and mode by product.

The model will be confined to explaining the modal-choice behavior of individuals who actually do have a choice between alternatives (i.e., are not captive to any one mode) and to considering the modal choice as a binary decision between the 2 "best" alternatives available. The latter assumption is based simply on the hypothesis that the typical traveler is unlikely to have many more than 2 feasible alternatives and in

Figure 1.

Below is a list of phrases some people use to describe their trip to work. For each phrase, rate your overall HOME TO WORK trip by placing a check mark ☑ in the box along the scale at that point which best describes your SATISFACTION with that aspect of the overall trip. If a phrase does not apply, check the box marked "Not Applicable" (N.A.)

		COMFORT IN VEHICLE *(See Footnote)		N.A.
1.	EXCELLENT	[7] [6] [5] [4] [3] [2] [1]	POOR	[0]
		DEPENDABILITY OF ON-TIME ARRIVAL		N.A.
2.	EXCELLENT	☐☐☐☐☐☐☐	POOR	☐
		PROTECTION FROM WEATHER WHILE WAITING		N.A.
3.	EXCELLENT	☐☐☐☐☐☐☐	POOR	☐
		FREQUENCY OF VEHICLE DEPARTURE TIMES		N.A.
4.	EXCELLENT	☐☐☐☐☐☐☐	POOR	☐
		PLEASANTNESS OF TRIP		N.A.
5.	EXCELLENT	☐☐☐☐☐☐☐	POOR	☐
		ATTRACTIVENESS OF VEHICLE *(See Footnote)		N.A.
6.	EXCELLENT	☐☐☐☐☐☐☐	POOR	☐
		NOISE IN VEHICLE *(See Footnote)		N.A.
7.	COMPLETELY ACCEPTABLE	☐☐☐☐☐☐☐	HIGHLY UNSATISFACTORY	☐
		CHANCE OF ACCIDENTS		N.A.
8.	COMPLETELY ACCEPTABLE	☐☐☐☐☐☐☐	HIGHLY UNSATISFACTORY	☐
		EXPOSURE TO UNDESIRABLE BEHAVIOR OF OTHERS		N.A.
9.	COMPLETELY ACCEPTABLE	☐☐☐☐☐☐☐	HIGHLY UNSATISFACTORY	☐
		TRAFFIC		N.A.
10.	COMPLETELY ACCEPTABLE	☐☐☐☐☐☐☐	HIGHLY UNSATISFACTORY	☐
		BODILY CROWDING		N.A.
11.	COMPLETELY ACCEPTABLE	☐☐☐☐☐☐☐	HIGHLY UNSATISFACTORY	☐
		OUT OF POCKET COST OF TRIP		N.A.
12.	COMPLETELY ACCEPTABLE	☐☐☐☐☐☐☐	HIGHLY UNSATISFACTORY	☐
		TOTAL TIME SPENT RIDING		N.A.
13.	COMPLETELY ACCEPTABLE	☐☐☐☐☐☐☐	HIGHLY UNSATISFACTORY	☐
		TOTAL TIME SPENT WALKING		N.A.
14.	COMPLETELY ACCEPTABLE	☐☐☐☐☐☐☐	HIGHLY UNSATISFACTORY	☐
		TOTAL TIME SPENT WAITING		N.A.
15.	COMPLETELY ACCEPTABLE	☐☐☐☐☐☐☐	HIGHLY UNSATISFACTORY	☐

*(Consider vehicle used for LONGEST TIME during trip)

Table 1. Descriptions of quantitative attributes.

Attribute Number	Description	Units	Abbreviation
4	Frequency of vehicle departure times	Minutes	Frequency
13	Total time spent riding	Minutes	Riding time
14	Total time spent walking	Minutes	Walking time
15	Total time spent waiting	Minutes	Waiting time
16	Distance traveled	Miles (coded)	Distance
17	Daily parking cost	Cents	Parking cost
18	One-way fare	Cents	Fare

any case in the end is not likely to make a decision between more than 2; that is, he is likely to reduce a choice between 3 or more to the "best" 2 and choose between these. This process of reducing the number of alternatives as the time of actual choice nears is discussed by Nicosia (12) and others.

For a representative individual traveler from the population and for modes j = 1, 2 and attributes i = 1, 2, ..., m, the following notation is established:

y = probability that mode 1 is preferred to mode 2;
X_i^j = value or level of attribute i on mode j;
X^j = m-element vector of attribute value $[X_i^j]$ for mode j;
$Q_i^j = Q_i(X_i^j)$ = attribute-satisfaction rating for attribute value X_i^j;
Q^j = m-element vector of attribute-satisfaction ratings $[Q_i^j]$;
$Q_i = Q_i(X_i^1) - Q_i(X_i^2) = Q_i^1 - Q_i^2$ = relative attribute-satisfaction rating for attribute i for modes 1 and 2;
Q = m-element vector of relative attribute-satisfaction ratings Q_i;
$U_i(X_i^j)$ = utility associated with attribute value X_i^j; and
$U(X^j)$ = total utility associated with mode j.

We next assumed the probability that an individual chooses mode 1, that is, prefers mode 1 to mode 2, is a function f of the difference in the total utilities to him of the 2 modes.

$$y = f[U(X^1) - U(X^2)] \tag{1}$$

The probability p that he prefers mode 2 to mode 1 is then given by

$$p = 1 - y \tag{2}$$

indicating that a choice is made to travel by either mode 1 or mode 2. Generalizing to the choice between many modes is nontrivial.

We also assume the total utility of a mode is derived from the utilities of the attributes of the mode in the additive form

$$U(X^j) = \sum_{i=1}^{m} U_i(X_i^j) \tag{3}$$

for j = 1, 2. The assumption of additive utilities, i.e., the assumption that the utility of the whole is equal to the sum of the utilities of its parts, is an important one in the model formulation because of its implication that the attributes are value-wise independent. Thus, for Eq. 3 to be valid, the utility $U(X_i^j)$ must be independent of X_k^j for all $k \neq i$. Fishburn (13), for example, in the context of the factors determining the utility of a decision states this as the requirement that the "evaluator be able to make consistent value judgments about the levels of any one factor when the levels of all other factors are held fixed, and his judgments must not depend on the particular fixed levels of the other factors." This assumption implies the desirability of developing a set of attributes that fully describe the products under consideration that from a consumer point of view can be measured along orthogonal axes. This is discussed further in a later section on the adequacy of attribute description.

Combining Eqs. 1 and 3 gives

$$y = f\left\{\sum_{i=1}^{m} [U_i(X_i^1) - U_i(X_i^2)]\right\} \tag{4}$$

The function $U_i(X_i^j)$, it is assumed, is not dependent on the mode j (although obviously is dependent on the attribute i). That is to say, the utility derived from a certain value of some attribute, say, travel time, is the same whether this is the travel time by bus or by automobile. This assumption is to some extent validated in a later section on validation of assumptions.

Next, we assume that the function $U_i(X_i^j)$ is monotonic in X_i^j and has the diminishing marginal utility property (as is commonly assumed as the basis for theories of rational economic behavior of consumers). The form shown in Figure 2 is appropriate where high attribute values or levels are associated with high levels of utility, for instance, comfort, dependability, and safety. Attributes having an associated utility function of this form can be referred to as "comfort" attributes. The form shown in Figure 3 is appropriate for what may be termed "cost" attributes, i.e., those for which high values of the attribute are associated with low levels of utility such as cost of travel. The value of M_i is the maximum level of utility associated by the traveler with any value or level of the attribute, and obviously the value of M_i will not, in general, be the same for different attributes i. However, it is assumed to be not dependent on the mode j and to be finite.

The relations shown in Figures 2 and 3 are assumed to be of the form

$$U_i(X_i^j) = M_i h_i(X_i^j) \tag{5}$$

A specific form for $h_i(X_i^j)$ that seems reasonable is the exponential

$$h_i(X_i^j) = \left(1 - e^{-\lambda_i x_i^j}\right) \tag{6}$$

for comfort attributes i, and

$$h_i(X_i^j) = e^{-\lambda_i x_i^j} \tag{7}$$

for cost attributes i. These assumptions make it possible to specify demand Eq. 4 in terms of the attribute values X_i^j. However, X_i^j is not measurable for qualitative attributes so that the demand equation must be written in terms of Q_i, the attribute-satisfaction ratings.

We let the attribute-satisfaction rating $Q_i(X_i^j)$ be measured on a semantic differential scale with (k + 1) scale intervals 0, 1, ..., k and assume the following direct proportionality relation between $Q_i(X_i^j)$ and $U_i(X_i^j)$:

$$Q_i(X_i^j)/k = U_i(X_i^j)/M_i \tag{8}$$

where M_i is the maximum utility associated with attribute i, which may be illustrated for, say, a cost attribute with k = 6 (a 7-point scale) as shown in Figure 4. Combining Eqs. 5 and 8 and writing Q_i^j for $Q_i(X_i^j)$ give

$$Q_i^j = kh(X_i^j) \tag{9}$$

If the semantic differential scale does not have a 0 origin, then appropriate adjustments must be made. If, for instance, a k-point scale (1, 2, ..., k) is used (as is the case for these data), the relations (and their inverses) for the exponential form of $h(X_i^j)$ in Eqs. 6 and 7 are respectively

$$Q_i^j = 1 + (k - 1)\left(1 - e^{-\lambda_i x_i^j}\right) \tag{10}$$

for comfort attributes,

$$Q_i^j = 1 + (k - 1)\, e^{-\lambda_i x_i^j} \tag{11}$$

for cost attributes,

$$X_i^j = \frac{1}{\lambda_i} \log\left[(k - 1)/(k - Q_i^j)\right] \tag{12}$$

Figure 2.

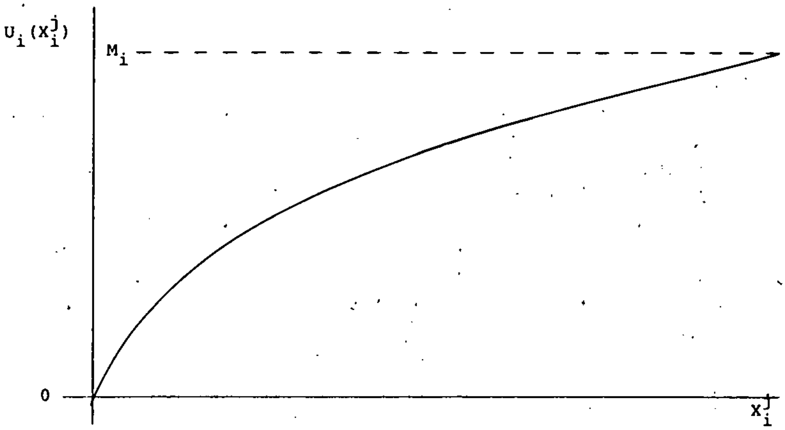


Figure 3.

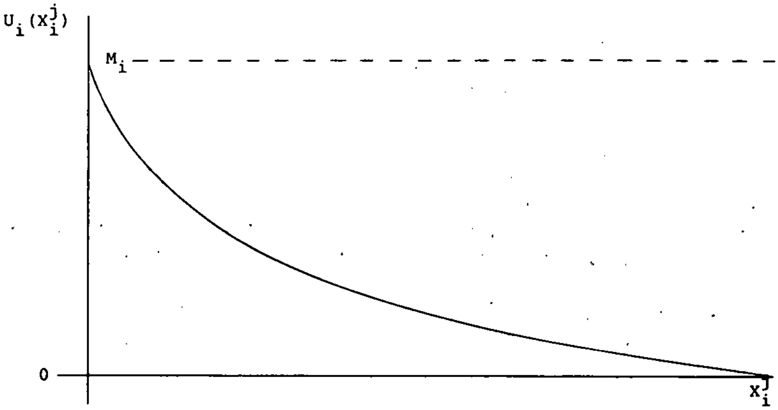
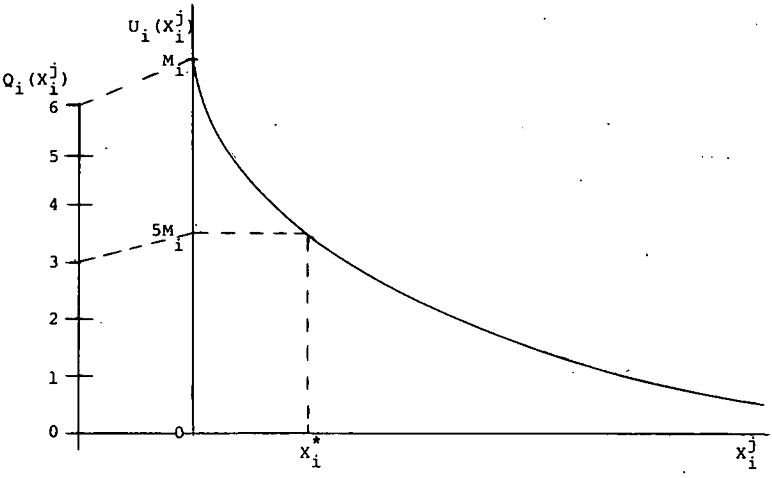


Figure 4.



for comfort attributes, and

$$X_i^j = \frac{1}{\lambda_i} \log [(k - 1)/(Q_i^j - 1)] \tag{13}$$

for cost attributes.

Finally, the Q/X relations hypothesized in, say, Eqs. 10 and 11 may be estimated and their validity investigated for those attributes for which a sample of observations on both attribute values and the corresponding attribute-satisfaction ratings is available. The results of this investigation are reported in a later section on validation of assumptions.

Now, we combine the demand Eq. 4 with Eq. 8 to obtain the demand equation

$$y = f\left\{\frac{1}{k} \sum_{i=1}^{m} M_i [Q_i(X_i^1) - Q_i(X_i^2)]\right\} \tag{14}$$

The variable $Q_i(X_i^1) - Q(X_i^2)$ (or $Q_i^1 - Q_i^2$) is the difference in the attribute-satisfaction ratings (measured on the same semantic differential scale) for attribute i. To simplify notation, we will write this variable as Q_i and refer to it as the relative attribute-satisfaction rating for attribute i. The m-element vector of relative attribute-satisfaction ratings Q_i will be written as Q. Equation 14 may then be written as

$$y = f\left(\frac{1}{k} \sum_{i=1}^{m} M_i Q_i\right) \tag{15}$$

or more concisely as

$$y = f(Q) \tag{16}$$

Any one of a number of forms may be proposed for the function f, the more straightforward being included in the class of functions h such that $f(Q) = h[g(Q)]$ and, where $g(Q)$ is a linear function of the Q_i,

$$g(Q) = a_0 + \sum_{i=1}^{m} a_i Q_i \tag{17}$$

where $a_i = M_i/k$. The following discussion will be confined to this class of functions. The simplest of this class is the case where h is the identity function so that

$$f(Q) = g(Q) \tag{18}$$

and

$$y = a_0 + \sum_{i=1}^{m} a_i Q_i \tag{19}$$

This has been referred to as the linear probability function.

Suppose (as is the case here) observations on the dependent variable y are dichotomous, taking on the value 1 if the individual prefers mode 1 and 0 if he prefers mode 2. This raises peculiar problems of estimation, which have been considered, for instance, by Warner (14) and Goldberger (15). It is possible to treat Eq. 19 as a classical linear regression model with the expected value of the regressand (the dependent variable) y specified as a linear function of nonstochastic regressors (explanatory variables) Q_i and to obtain classical least squares estimates of the parameters. The conditional expectation of y may then be interpreted as the conditional probability of modal choice given the Q_i. As shown by Goldberger, however, the basic classical least squares assumption of homoscedasticity is not fulfilled in the case of a "dummy" dependent variable because the disturbance term of the model varies systematically with the values of the regressors. Consequently, the classical least squares estimates although unbiased are inefficient (15, p. 238). Classical least squares estimation of Eq. 19 does not then yield "best" estimates of the coefficients. However, it should be mentioned that the heteroscedasticity problem can be alleviated by obtaining a probability-of-choice measure from respondents over the interval ($0 \le p \le 1$).

However, in addition to the difficulty caused by heteroscedasticity, the linear probability function of Eq. 19 itself may be objected to on the grounds that it is quite possible for predicted values of y to fall outside the 0, 1 interval, which is inconsistent with the definition of y as a probability. The function is thus "illogical at the ends."

Two methods have been widely used to take care of the problem of confining predicted values of the regressand to the unit interval. These are probit analysis and logit analysis, both of which essentially fit an S-shaped "sigmoid" curve to a linear function of the data. If g(Q) is denoted as the linear function, the general form of the sigmoid curve fitted by probit and logit analyses is as shown in Figure 5. In probit analysis the sigmoid function is given by the cumulative normal distribution function,

$$y = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{g(Q)} e^{-t^2/2}\,dt \qquad (20)$$

where

$$g(Q) = a_0 + \sum_{i=1}^{m} a_i Q_i .$$

Nonlinear estimation yields maximum likelihood estimates of the parameters of g(Q) as shown by Tobin (16).

Logit analysis, which like probit has its origins in bioassay (17), fits the logistic curve to a linear function g(Q).

$$y = \frac{1}{1 + e^{-g(Q)}} \qquad (21)$$

where

$$g(Q) = a_0 + \sum_{i=1}^{m} a_i Q_i .$$

For both probit and logit analyses, if we assume that the usual assumptions hold, both unbiased and efficient maximum likelihood estimates of the parameters of the linear function g(Q) may be obtained. No conclusive evidence has been presented to indicate that statistically one provides a better fit than the other to modal-choice data (14).

Here parameters of the demand equation will be estimated by 3 methods. First, because of its computational simplicity, ordinary least squares regression is used to estimate the linear probability function of Eq. 19. Second, nonlinear least squares regression is used to estimate the logistic function of Eq. 21 (18). Third, the demand equation is an integral part of a system of equations, and estimations are by 2-stage least squares.

DEMAND EQUATION PARAMETER INTERPRETATION AS IMPORTANCES

The concept of importance will be discussed and then defined in the context of the consumer choice model described above. As Myers and Alpert pointed out, the term importance has been used to mean many different things. It is necessary, therefore, to define carefully what is to be meant by importance here. From the point of view of the consumer, the relative importance of attributes can be said to be the ratio of the marginal utilities of attributes; that is,

$$\frac{dU_i}{dX_i} \Big/ \frac{dU_m}{dX_m} \quad \text{or} \quad \frac{dU_i}{dQ_i} \Big/ \frac{dU_m}{dQ_m} \tag{22}$$

for i = 1, 2, ..., m - 1, depending on whether the utility function is specified in terms of attribute or attribute-satisfaction ratings. From the point of view of allocating advertising funds or funds for product change, attribute importance may be defined differently. If C = dollar expenditure, importance in this case is given by

$$\frac{dU_i}{dX_i}\frac{dX_i}{dC} \Big/ \frac{dU_m}{dX_m}\frac{dX_m}{dC} \tag{23}$$

for i = 1, 2, ..., m - 1, or

$$\frac{dU_i}{dQ_i}\frac{dQ_i}{dC} \Big/ \frac{dU_m}{dQ_m}\frac{dQ_m}{dC} \tag{24}$$

for i = 1, 2, ..., m - 1, because it is assumed that preference and, therefore, sales are monotonic in U. Given the additive-utility assumption, total rather than partial derivatives are appropriate.

Methods for estimating dX/dC for quantitative attributes and dQ/dC for all attributes are outside the scope of this paper. Suffice it to say that, in the case of advertising planning, advertising pretesting procedures can be used and, in the case of product development, product clinic procedures can be applied. In the case of quantitative attributes, care must be taken to note that the model is in terms of perceived attributes and not actual attributes so that engineering and financial estimates may not suffice.

To obtain comparable importance measures for all attributes, it seems reasonable to obtain semantic differential attribute-satisfaction ratings for all attributes, even those that are quantitative. As a result, attribute importances defined in terms of Q rather than X will be considered.

Of primary interest here then is dU_i/dQ_i (i = 1, 2, ..., m). But differentiation of Eq. 8 implies that

$$\frac{dU_i}{dQ_i} = \frac{M_i}{k} = a_i \tag{25}$$

for i = 1, 2, ..., m, in Eq. 17. Thus, based on the assumption given by Eq. 8, relative importance from a consumer viewpoint is given by

$$\frac{a_i}{a_m} = \frac{M_i}{M_m} \tag{26}$$

for i = 1, 2, ..., m.

As a result, the demand model given by Eq. 15 implies that preference depends on relative attribute-satisfaction ratings and attribute importances. A validation of the assumption of Eq. 8 is provided in a later section. Means by which the a_i can be estimated have been discussed above, and results will be given below.

The hypothesis that attitude and consequently behavior are determined by the satisfaction with, and importance of, the "attitude object" has been the basis for attitudinal models in applied psychology. Fishbein (19) has suggested an additive-utility model that implies that an individual's attitude toward an object will depend on (a) how satisfactorily the object possesses certain attributes and (b) how important these attributes are to him. Confirmation of hypotheses of this kind suggests that the choice behavior of individuals can be described in terms of their satisfaction with the perceived level of modal attributes and the importance attached by them to the attributes. Bass and Talarzky (20) have used this approach in their research. Frequently, importances are estimated via regression techniques and discriminant analysis (21).

There are, therefore, precedents for this modal-choice formulation. However, this model, in contrast to the psychological models discussed above, involves no a priori specification of the coefficients as importances. The interpretation instead arises naturally as a direct consequence of the model assumptions of (a) additive utilities, (b) the particular diminishing marginal utility form of the utility function, and (c) the proportional mapping from U to Q. As mentioned previously, the assumptions b and c are validated in a later section.

SUPPLY-SIDE FORMULATION

To this point the consumer choice model has been expressed in terms of a single demand equation, and in the preceding section it was shown that estimation of the parameters of the equation would yield estimates of the relative importances of the associated attributes. However, the data that must be used to calibrate the model were "generated" by the simultaneous solution of demand and supply relations. Calibration of the single-equation demand model from such data, while ignoring the supply side, is consequently likely to yield statistically biased and inconsistent estimates of the relative importances, and these estimates may be misleading (15, pp. 280-290). It will be shown in a later section that this turned out to be the case in the illustrative example cited here. In order to obtain meaningful estimates of importances, the relevant supply-side relations must be included in the structure of the model, and the model must be calibrated by one of the techniques appropriate for systems of simultaneous equations. A discussion of these techniques is outside the scope of this paper. Goldberger provides an excellent reference textbook (15).

It is important to note that, even though the data for each of the 2 modes considered are taken via questioning the traveler, certain of the attribute values are related to other attribute values because of supply considerations. Suppose that for mode j some of these supply considerations can be expressed by an equation relating the value of attribute r to the values of other attributes i. Simple equation forms that may be thought appropriate for specific supply relations are, for instance, the additive form

$$X_r^j = b_r^j + \sum_i b_i^j X_i^j \tag{27}$$

or the multiplicative form

$$X_r^j = b_r^j \prod_i b_i^j X_i^j \tag{28}$$

where the b are coefficients to be estimated. The additive form may be, for example, appropriate for the automobile mode for, say, a relation describing the value of the attribute out-of-pocket cost as an additive function of the various other attributes such as traffic, travel time, and parking costs. Any number of other more complex equation forms may, of course, be hypothesized; these two are suggested only as possible simple forms. Moreover, as before, some of the attribute values X_i^j may be difficult or impossible to measure, in which case it is necessary to resort to the use of the corresponding attribute-satisfaction ratings Q_i^j in estimating the relations.

Assume the first p of the attributes (i = 1, ..., p) on the right side of Eqs. 27 and 28 are expressed in terms of their satisfaction ratings (Q variables) and the next q (i = p + 1, ..., p + q) are expressed in terms of attribute values (X variables). Also assume attribute r on the left side is expressed as a satisfaction rating.

As before, assume that the exponential relations given by Eqs. 12 and 13 exist between X_i^j and Q_i^j (i = 1, ..., p, r) so that

$$X_i^j = \frac{1}{\lambda_i} \log [(k - 1)/Q_i^{j*}] \quad (29)$$

where

$Q_i^{j*} = (Q_i^j - 1)$ for cost attributes, and
$Q_i^{j*} = (k - Q_i^j)$ for comfort attributes.

Substituting for X_i^j (i = 1, ..., p, r) from Eq. 29 in Eq. 27 then yields one possible form of supply relation:

$$Q_r^{j*} = \prod_{i=1}^{p} (Q_i^{j*})^{c_i^j} \exp\left(c_r^j + \sum_{i=p+1}^{p+q} c_i^j X_i^j\right) \quad (30)$$

The coefficients c are simple arithmetic combinations of the coefficients b, λ, and k.

Suppose the exponential relations of Eq. 29 may be approximated by a linear relation over the ranges of X_i^j and Q_i^j of interest; namely,

$$X_i^j = \mu_{0i} + \mu_i Q_i^j \quad (31)$$

This form also results from assuming U to be a linear function of X. Substituting for X_i^j (i = 1, ..., p, r) in Eq. 27 then yields another possible form of the supply-side relation:

$$Q_r^j = c_0^j + \sum_{i=1}^{p} c_i^j Q_i^j + \sum_{i=p+1}^{p+q} c_i^j X_i^j \quad (32)$$

where, again, the coefficients c are simply derived from the coefficients of Eqs. 27 and 31. It should be stressed that supply-side Eqs. 30 and 32, which are derived above, are only suggested as possible forms of supply-side relations that have some plausibility and are relatively simple to estimate. They will be referred to as the nonlinear and linear supply equations respectively. It is important to note that attributes included in the demand equation may be correlated because of correlation with a third variable rather than because of direct causal relations.

Table 2 gives the correlation of semantic differential ratings of dependability, out-of-pocket cost, riding time, walking time, and waiting time for automobile and transit. For the automobile, relatively high correlations exist between dependability and riding time and between out-of-pocket cost and riding time because of supply-side considerations. Supply-side relations involving these variables as well as dependability and waiting time were developed for transit. It is beyond the scope of this paper to develop these equations here. However, for this mode-choice problem, supply-side model development has been discussed by Sherret (22) and will be further discussed in a forthcoming paper.

SIMULTANEOUS-EQUATION MODEL FORMULATION

Having discussed the demand-side and supply-side relations, we can now propose a simple simultaneous model structure. We assume for purposes of illustration that both the demand and supply relations are linear; more complex equation forms (in the same variables) may be substituted without changing the basic structure of the model. Then,

$$y = a_0 + \sum_{i=1}^{m} a_i Q_i \tag{33}$$

where

$$Q_i = Q_i^1 - Q_i^2 \tag{34}$$

for i = 1, ..., m, and

$$Q_i^j = c_{0i}^j + \sum_{k=1}^{p} c_{ki}^j Q_k^j + \sum_{k=p+1}^{p+q} c_{ki}^j X_k^j \tag{35}$$

for i = 1, ..., m and j = 1, 2.

Equation 33 is the demand relation expressing the probability of modal choice in terms of the relative attribute-satisfaction variables Q_i, with the coefficients a_i (i = 1, ..., m) being the importances of the modal attributes that are to be estimated. Equation 34 is simply a set of identities defining the relative attribute-satisfaction ratings Q_i as the difference in attribute-satisfaction ratings for mode 1 minus mode 2. These identities provide the link with the supply-side relations of Eq. 35 that express for modes 1 and 2 separately the relations existing between the attributes of the modes on the supply side. Equation 35 indicates that there exist supply relations for all the m attributes of both modes 1 and 2. Although this may be true in general, it is likely that in any given model formulation some of the Q_i^j will be considered exogenous to the model—in each of which cases the coefficients of all the variables on the right side of the relevant supply relation will be 0 with the exception of the particular Q_i^j for which the coefficient is 1.

In the model the variable y and all those Q_i^j for which supply relations exist are the endogenous (i.e., jointly determined) variables. The remaining Q_i^j and the X_i^j are the exogenous (i.e., externally specified) variables of the model. As a simple example, suppose that on the demand side the probability y of preferring mode 1 to mode 2 is a function of m = 2 attributes (total travel time and comfort) expressed as their relative attribute-satisfaction ratings $Q_1 = Q_1^1 - Q_1^2$ and $Q_2 = Q_2^1 - Q_2^2$. On the supply side for mode 1 (automobile), Q_1^1 (travel time) is a function of the exogenous variables Q_3^1 (traffic) and X_4^1 (distance). On the supply side for mode 2 (transit), Q_1^2 (travel time) is a function of the exogenous variables Q_5^2 (total riding time) and X_6^2 (time between departures). The automobile comfort variable Q_2^1 is a function of traffic Q_3^1 and travel time Q_1^1, and transit comfort is exogenous. The model may then be written as

$$\left.\begin{aligned} y &= a_0 + a_1Q_1 + a_2Q_2 \\ Q_1 &= Q_1^1 - Q_1^2 \\ Q_2 &= Q_2^1 - Q_2^2 \\ Q_1^1 &= c_{01}^1 + c_{11}^1Q_3^1 + c_{21}^1X_4^1 \\ Q_1^2 &= c_{01}^2 + c_{11}^2Q_5^2 + c_{21}^2X_6^2 \\ Q_2^1 &= c_{02}^1 + c_{12}^1Q_3^1 + c_{22}^1Q_1^1 \end{aligned}\right\} \tag{36}$$

where the variables Q_1^2, Q_1^1, Q_1, Q_2^1, Q_2, and y are the jointly determined endogenous variables of the model, and the remaining ones are considered exogenous. To obtain consistent estimates of importance, we must obtain estimates of the coefficients a_0, a_1, and a_2 via an appropriate simultaneous-equation estimation procedure.

The important point to note is that, even though the data used to calibrate this model of consumer choice came from questioning consumers (the demand side), in general it is still necessary to introduce supply-side relations in order to obtain consistent estimates of the importance of product attributes. Because the nature of the supply side will vary from industry to industry, it is the objective of this paper to focus on the demand side.

Another point that needs emphasis here is that this model has been developed in terms of perceived levels of attributes. If consumer perception differs widely from, say, engineering fact, it may be difficult to validate what are a priori realistic supply-side relations. In that case, there would appear to be no choice but to work with single-equation demand models. This problem is discussed further below.

It should be mentioned that the estimated relative importance will depend on the pair of modes (or products) the traveler (or consumer) is asked to choose between. This arises, of course, from the fact that model calibration and associated statistical inference require the assumption of fixed or nonstochastic values of the explanatory variables (i.e., relative attribute-satisfaction values). If another pair of modes leads to significantly different relative attribute-satisfaction ratings, then the model will need to be recalibrated. In most cases, the supply equations will change as well. In either case, recalibration is required.

This need for recalibration is not a retraction of the assumption that U_t and, therefore, M_t are independent of the mode (or product) under consideration. It is simply a result of the fact that a given mode pair may not provide sufficient variability of the attribute values X_t. However, it does seem likely that validity of the assumption of additive utilities would depend on the range of X_t as well. For these reasons, it is likely to be necessary to recalibrate these models by using a number of different mode pairs if choice between a number of modes is of interest.

ADEQUACY OF ATTRIBUTE DESCRIPTION

A major question to be answered before we proceed to develop a simultaneous-equation, importance-estimation model is whether it will be possible to predict consumer choice based on estimated importances and relative attribute-satisfaction ratings. The issue being specifically addressed here is whether the set of (15) attributes fully (or at least adequately) describes the alternative modes from a consumer point of view. The relative importance of specific attributes is not at issue here. The argument is that, if the set of relative attribute-satisfaction ratings does not allow the prediction of mode choice with some degree of success, there would seem to be little sense in attempting to explain behavior based on the data. For this reason, it is desirable to perform a discriminant analysis (23). The discriminant analysis results are given below.

Item	Value	Item	Value
n_1	60	p_2	0.487
n_2	57	P_0	0.500
n	117	P	0.812
K	0.051	z	6.25
p_1	0.513	D^2	84.18

In all cases group 1 refers to the sample choosing automobile in preference to transit, and group 2 refers to the sample choosing transit. The notation used in the tables is as follows:

n_1 = group 1 sample size;
n_2 = group 2 sample size;
$n = n_1 + n_2$ = total sample size;
$K = \log_e(n_2/n_1)$ = classification rule criterion;
$p_1 = n_1/n$ = a priori probability of classification in group 1;
$p_2 = n_2/n$ = a priori probability of classification in group 2;
$P_0 = (p_1)^2 + (p_2)^2$ = "chance" probability of correct classification;
$P = m/n$ = proportion of sample correctly classified by discriminant-classification rule;
$z = (P - P_0)/\sqrt{P_0(1 - P_0)/n}$ = statistic to test significance of difference in proportions $(P - P_0)$; and
D^2 = Mahalonobis sample distance statistic.

In interpreting the above results, one should bear in mind that P is the proportion of individuals correctly classified within the sample by the sample discriminant-classification rule and is consequently an upward biased estimate of the correct classification rate of the population (24, 25). Comparison of P_0 and P, therefore, gives an overly optimistic view of the predictive power. However, P appears to be much better than the chance probability P_0, and the statistical test of z against the critical z value confirms that the difference is statistically significant (3.72) at better than the 0.01 percent level. The D^2 statistic also confirms a highly significant difference in the sample means.

To resolve the question of the extent of the bias in the estimates of the correct classification rates P, we used a "jackknife" estimation method. The method is similar to that of Lachenbruch (26), but to reduce the computation involved we based the estimates on 10 different discriminant functions per sample rather than the n suggested by Lachenbruch.

The resulting approximately unbiased estimates of correct classification rates, P′, are compared to the corresponding biased estimates P as follows:

Item	Value
Biased	
n	117
P	0.812
P_0	0.500
z	6.25
Unbiased	
n	110
P'	0.736
P_0'	0.505
z	4.85
Difference	
$(P - P')/P$	0.94

Also given above are the "chance" probabilities P_0 and P_0', which give the appropriate comparisons for P and P′ respectively; the z values to test the differences between P and P_0 and between P′ and P_0'; and the difference P - P′ expressed as a fraction of P. The chance probabilities P_0 and P_0' are different as a result of the slightly smaller sample sizes used in the jackknife estimates.

The results indicate that there is an appreciable upward bias in the correct classification estimates P, but the unbiased estimate P′ is still very highly significantly different from the chance correct classification rate. The conclusion that there is significant discriminatory power in the data, thus, is not changed by a knowledge of the bias in P. Moreover, because the analysis was in terms of relative attribute-satisfaction ratings (i.e., difference), these results support the view that the semantic differential technique provided interval-scaled data—a requirement of the demand model.

Interpretation of the constant term in the demand equation can assist in determining the nature of omitted attributes in the linear model

$$y = a_0 + \sum_i a_i Q_i \tag{37}$$

where y is the probability of preferring automobile, and the constant term a_0 indicates the probability that the typical individual will prefer automobile to transit if all the Q_i are zero, i.e., if his satisfactions with the 2 modes are equal for all attributes.

For the sample under study, the a priori probability of preferring automobile was 60/117 = 0.513. The estimated value of a_0 was 0.477. The null hypothesis $a_0 = 0.513$ cannot be rejected even at the 50 percent level. Based on these findings and those of the discriminant analysis, it would appear that the original set of 15 attributes provides an adequate description of the 2 modes in question from a consumer-choice point of view.

The sample discriminant function coefficients associated with the 15 attributes are as follows:

Attribute	Coefficient	Attribute	Coefficient	Attribute	Coefficient
1	-0.06	6	-0.26	11	0.12
2	0.36	7	-0.10	12	0.05
3	0.02	8	0.18	13	0.17
4	0.03	9	-0.03	14	0.51
5	0.22	10	-0.09	15	0.25

It is important to note that these weights have often been referred to as relative importances in the literature (3, p. 18; 5, p. 370). As discussed in the section on supply-side formulation, these weights are not consistent estimates of importances as defined in the preceding section on importances. It is true that in the 2-group case discriminant analysis weights will be proportional to those of a regression analysis with a dummy-dependent variable implying that the linear demand Eq. 19 could be estimated in either way. Note, however, that there is no classical linear regression model equivalent to discriminant analysis for more than 2 groups and that only the regression model permits statistical inference regarding the relative importance of attributes. Statistical inference using discriminant analysis must be confined to the statistical significance of a particular set of attributes in predicting group membership, not to the individual relative importance of the attributes.

REDUCTION OF SEMANTIC REDUNDANCY

Developing a set of attributes that fully describe a group of products from a consumer point of view is a tedious process. Potential attributes and their end points must have meanings clear to the respondent. In general, it is not possible to develop a list that does not contain some semantic redundancy. Usually the initial list will become quite long. One of the authors used 65 semantic differentials to describe automobiles in a product clinic designed to pretest Chevrolet's Vega.

Figure 1 shows the 15 attributes for which satisfaction ratings were obtained. It is clear that they may contain semantic redundancy. An equal-tails test of the null hypothesis that the true population correlation coefficient for any pair of variables is 0 gives critical points of 0.182 at the 5 percent level and 0.238 at the 1 percent level. Examination of the correlation matrix indicated that, of the 205 elements to one side of the principal diagonal, 67 were greater than 0.238, demonstrating that, statistically speaking, many highly significant correlations existed.

These high intercorrelations give rise to the problem of multicollinearity if these correlated attributes are included as explanatory variables in a multiple regression model. The problem of multicollinearity in regression analysis is a perplexing one arising frequently in econometric studies; it is discussed, for example, by Goldberger (15, pp. 192-194). The problem arises in interpreting the estimated coefficients of the regression because, if high intercorrelations exist between some, or all, of the explanatory variables, it becomes difficult if not impossible to distinguish among the separate influences of the explanatory variables and obtain a reasonably precise estimate of their relative importances. Multicollinearity has the effect of producing large standard errors of the coefficients for the explanatory variables of an equation; as intercorrelations become higher, confidence in the reliability of the coefficient estimates is reduced (15, pp. 192-194).

Because of this problem and the additive-utility assumption, it is desirable to reduce the original set of attributes to a smaller set by removing those attributes that are highly correlated to others because of semantics. Care must be taken from the outset to identify those correlations that are likely to be due to supply-side relations and those that are due to semantics. This is accomplished most simply by establishing on an a priori basis those attributes that are likely to be correlated for supply-side reasons, e.g., automobile out-of-pocket cost and traffic. The objective here is to suggest a technique for handling the problems caused by semantic redundancies and also for assisting in the development of a nearly orthogonal set of attributes.

It seems reasonable to suppose that the traveler thinks in terms of a smaller number of (orthogonal) decision "factors" or "dimensions" than the 15 attributes given in the questionnaire. In fact, the demand model is constructed on the basis of additive utilities. But several attributes, for example, comfort and pleasantness, may actually be closely related to the same dimension of the mode-choice process because comfort and pleasantness when applied to a mode of transportation may mean about the same thing to people.

This hypothesis is supported by a correlation of 0.7 between, for example, attributes 1 (comfort) and 5 (pleasantness). Hence, it seems likely that several attributes are closely related to essentially the same dimension of the modal-choice decision. The problem arising out of this hypothesis—that of analyzing the basic dimensionality of a sample of observations on a large number of variables—can be addressed by factor analysis (27, p. 4).

A principal-components type of factor analysis on all 15 relative attribute-satisfaction ratings (Q_i) was performed. Varimax rotation of the first 9 principal components was also performed as an aid to interpretation. The results are given in Table 3.

Nine of the possible 15 principal components are given in Table 3. The choice of the 9 factors can be justified by the fact that the 88 percent of variance explained is substantial, but equally important these 9 factors may be interpreted as modal-choice "decision factors" in a way that is intuitively satisfying. For example, attributes 5 (pleasantness), 1 (comfort), and 7 (noise) have the highest loading in factor 1; 10 (traffic) and 8 (accidents) have the highest loading in factor 2; 4 (frequency) and 15 (waiting time) have the highest loading in factor 3. Conversely, attribute 14 (walking time) is the only variable with a high loading in factor 4, and this is consistent with the "prior" that walking time is relatively independent of other mode attributes. It was difficult to interpret the factors beyond the ninth as "different" dimensions of the modal-choice decision. It is interesting to note that Green and Rao have suggested that at least 8 attributes be used to describe a product (6, p. 38).

In general, the principal-components analysis of a set of variables that are prospective regressors in a multiple regression equation may be used in alleviating the multicollinearity problem in 2 ways. First, the principal-components solution may be used directly as suggested, for example, by Kloek and Mennes (28). The m-element vector of observations on the original variables is replaced by the p-element vector of linear combinations of the variable (i.e., factor "scores"), which are obtained by multiplication of the original variables by the loadings given in the principal-components factor matrix. These p-factor scores are then used as the explanatory variables of the regression.

Figure 5.

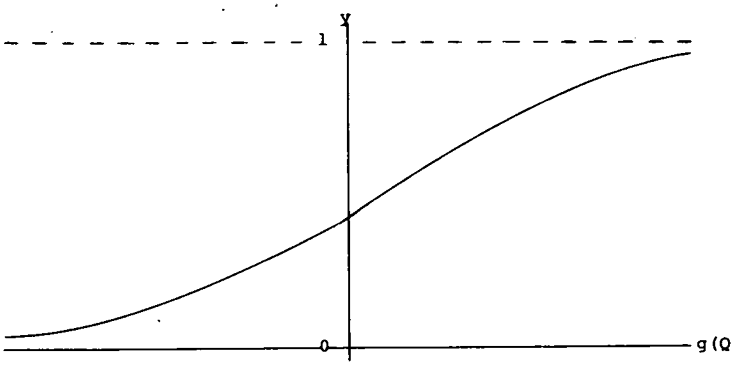


Table 2. Correlation matrix of semantic differential ratings.

Mode	Attribute	Dependability	Cost	Riding Time	Walking Time	Waiting Time
Automobile	Dependability	1.0000				
	Cost	0.3303	1.0000			
	Riding time	0.5855	0.4429	1.0000		
	Walking time	0.0671	0.0232	-0.0429	1.0000	
	Waiting time	0.2618	0.1585	0.2871	-0.0425	1.0000
Transit	Dependability	1.0000				
	Cost	0.4545	1.0000			
	Riding time	0.4463	0.4746	1.0000		
	Walking time	0.1560	0.2328	0.2468	1.0000	
	Waiting time	0.7252	0.3880	0.4615	0.3765	1.0000

Table 3. Summary of varimax rotation results.

Rotated Factor		Attribute	
Number	Description	Number	Factor Loading
1	Physical comfort	5	0.90
		1	0.87
		7	0.74
		6	0.68
		11	0.62
		13	0.33
		9	0.33
2	Congestion	8	0.87
		5	0.38
		10	0.38
3	Service frequency	4	0.90
		15	0.60
		11	0.40
4	Walking time	14	0.98
5	Weather exposure	3	0.93
6	Dependability	2	-0.81
		15	-0.45
		13	-0.40
7	Social comfort	9	0.87
		11	0.38
8	Riding time	10	0.83
		13	0.53
9	Cost	12	-0.85
		13	-0.52

Obviously, however, the regression coefficient estimates will not be the same for the 2 regressions. In fact, the difficulties involved in interpreting the coefficients of the factor score regression represent the major drawback of the use of this method in structural analysis. The interpretation of a regression coefficient as the magnitude of the effect on the dependent variable produced by a unit change in an explanatory variable (factor) becomes difficult where the explanatory variable is a linear combination of the observed variables (relative attribute-satisfaction ratings). Very often the sum of the weights for those attributes not loading heavily is higher than the sum of the larger weights that provided the factor interpretation. Moreover, the absence of well-tried means of testing the statistical significance of the coefficients estimated via principal-components analysis further complicates interpretation of the regression coefficients. Thus, although the method of using the principal-components solution directly in the multiple regression does remove the multicollinearity problem and is considered by some to introduce a certain objectivity to the estimation procedure, it is of little help where the aim is interpretation of the coefficients of the regression equation as structural parameters.

An alternative use of the principal-components analysis in reducing the effects of multicollinearity is to select a subset of p from the m original variable on the basis of their factor loadings in the p principal components (which account for "most" of the sample variance) and perform the regression on this subset of the original variables. The most obvious criterion is to select those p variables that have the highest loadings in each of the p components. The resulting set of variables will tend to have low intercorrelations, thus reducing (although not eliminating) multicollinearity, and the use of the actually observed variable in the regression simplifies interpretation of the associated coefficients. Furthermore, the method allows the inclusion or exclusion of any of the variables dictated by supply-side considerations on grounds of the model structure.

Use of principal-components analysis in this latter indirect way would then seem to be a much more appropriate method than the former in most instances where regression coefficients are to be interpreted structurally. Selection of a subset of the original variables so that highly intercorrelated variables are omitted is the standard procedure for dealing with multicollinearity in regression; the use of principal-components analysis in the way outlined here merely provides a systematic and rational basis for selection of the variable to be included. This view is supported by Green and Tull (5, pp. 422-426) in their review of the usefulness of principal-components analysis.

In this case, the principal-components analysis of relative attribute-satisfaction data indicates that fewer than 15 attributes adequately account for the dimensionality of the modal-choice decision; the first 9 factors are intuitively interpretable as "different" dimensions. These 9 are, moreover, fairly easily identified with attributes in the original list of 15 so that the method discussed above is helpful in selecting variables for subsequent regression analysis. Accordingly, on the basis of the principal-components analysis, the following 9 attributes were selected for further analysis:

Description	Number
Comfort in vehicle	1
Dependability of on-time arrival	2
Protection from weather while waiting	3
Exposure to undesirable behavior of others	9
Traffic	10
Out-of-pocket cost of trip	12
Total time spent riding	13
Total time spent walking	14
Total time spent waiting	15

Although many of the correlations among these 9 were still statistically significant, the very high correlations present in the 15-variable set of attributes were removed. The multicollinearity problem is, thus, still present, but its seriousness is lessened. Strictly speaking, it is now necessary to be sure that the reduced set of attributes is adequate, that is, to repeat the discriminant analysis procedure.

Although it is assumed that the retained attributes form a set that, in fact, represents the various dimensions of the modal choice as perceived by the traveler, there may still remain correlations among the relative attribute-satisfaction ratings because of supply relations. For example, examination of the correlations for the 9 attributes listed above revealed that the highest correlations occurred between attributes 12 (cost) and 13 (riding time), 10 (traffic) and 13 (riding time), and 2 (dependability) and 15 (waiting time). These correlations do not, however, arise for semantic reasons but for reasons that may be labeled supply-side oriented; that is, cost and riding time are correlated because there is a functional dependency between cost and riding time, not because travelers understand the same thing by out-of-pocket cost and total time spent riding. In this sense, the correlations between traffic and riding time and between dependability and waiting time also arise as a result of such supply-side relations (although the correlation matrix obviously does not indicate the direction of causality of the relations).

Also the semantic correlations are traveler-dependent and, hence, arise from relations on what have been termed the demand side, and the functional correlations arise from relations that are logically mode-dependent and on the supply side. In other words, the analyst must determine the relevant supply-side relations and provide the linkage between supply and demand.

Thus, correlations among these data arise for both semantic and supply-side reasons. It is important to appreciate that, although principal-components analysis is helpful in summarizing the data in a way that facilitates recognition of the former, it is of little help in distinguishing between the two. The analysis method is, in other words, unable to identify the underlying causalities that define the structure of the data. The factor analysis has been done in terms of relative attribute-satisfaction ratings rather than separately for each mode—automobile and transit. Because the modes have different supply-side relations, using relative attribute-satisfaction ratings tends to confound the supply sides leaving the semantic problems. Separate principal-components analyses for each mode lead to results that did not yield to logical interpretation even with varimax rotation.

In this case, the principal-components analysis supports the view that the "experiment" underlying the attribute-satisfaction and modal-choice data is not a simple "single-equation" economic process but a complex process of interrelated and simultaneous relations. The modal-choice decision experiment generates observations that reflect the equilibrium of supply and demand relations; a properly structured model of modal choice must then make explicit the simultaneous interaction of these supply and demand relations.

VALIDATION OF SOME CRUCIAL ASSUMPTIONS

The object of this section is to validate some of the important assumptions of the demand equation formulation given in an earlier section. The assumed relation for the exponential type of utility function and linear U to Q mapping is

$$Q_i^j = 1 + (k - 1)\left(1 - e^{-\lambda_i X_i^j}\right) \tag{10}$$

for comfort attributes and

$$Q_i^j = 1 + (k - 1)\, e^{-\lambda_i X_i^j} \tag{11}$$

for cost attributes, where k is the number of intervals on the semantic scale, equal to 7 for these data. An alternative relation between Q_i^j and X_i^j can be developed on the basis of a linear utility function and the linear U to Q mapping. It has the linear form,

$$Q_i^j = \mu_{0i} + \mu_i X_i^j \tag{38}$$

where the parameter μ_i is positive for comfort attributes and negative for cost attributes.

Equations 10 and 11 are central to the construction of the demand equation of the model, and the linear U to Q mapping assumption leads to the interpretation of the parameters of that equation as importances. It is desirable then to investigate the validity of both the exponential relations of Eqs. 10 and 11 and the linear Eq. 38 inasfar as the data allow.

In the data a sample of observations is given on both Q_i^j and the corresponding X_i^j for the following attributes:

Attribute-Satisfaction Rating	Attribute Value
Automobile mode	
Total time spent riding (QA13)	Total riding time, min (XA13)
Total time spent walking (QA14)	Total walking time, min (XA14)
Transit mode	
Frequency of vehicle departure times (QT4)	Headway of vehicle departures, min (XT4)
Out-of-pocket cost (QT12)	Fare (XT18)
Total time spent riding (QT13)	Total riding time (XT13)
Total time spent walking (QT14)	Total walking time (XT14)
Total time spent waiting (QT15)	Total waiting time (XT15)

From the sample of 117 individuals making a choice between automobile and rail transit, a subsample of 84 gave complete responses on all the variables listed above. This subsample is used to estimate the assumed Q/X relations in this section. The estimations of both the exponential and linear forms are given below.

All the attributes listed above are what have been termed cost attributes; i.e., increasing values of the attribute are associated with decreasing utility levels. This is the case simply because attribute-value measurements are not available for the comfort attributes, which tend to be qualitative attributes. Therefore, only the relations of the form of Eqs. 10 and 11 can be estimated.

Equation 10 rewritten as a regression equation is

$$(Q_i^j - 1) = \varkappa_i e^{-\lambda_i X_i^j} \tag{39}$$

where $\varkappa_i$ and λ_i are both parameters to be estimated. It is necessary to estimate relations of Eq. 39 directly by nonlinear regression in order to obtain estimates of $\varkappa_i$ and λ_i. The relations given in Table 4 were estimated by a nonlinear least squares algorithm described by Hartley (18). Before these results are studied, the following points should be made.

From comparison of Eqs. 11 and 39, it would be expected that the estimated value of $\varkappa_i$ would be equal to (k - 1) or 6, that is, independent of the attribute i if the hypothesized relation between Q_i^j and X_i^j fits the data exactly. The closeness of the coefficient $\varkappa_i$ to 6 in the results given is, therefore, an indication of the validity of the relation and, hence, the assumption of an exponential utility function and linear U to Q mapping.

The R^2 statistics given for the regression results are computed from 1 minus the ratio of the sum of squares about the exponential regression curve (the sum of squared residuals) to the sum of squares about the mean. This indicates the goodness of fit to the data of a regression curve of the form shown in Figure 6. However, the observations on Q_i^j being fitted are not continuous over the interval 1 to 7, as Figure 6 implies, but integer valued. This being the case, the appropriate curve by which to judge fit should really be a step function as shown in Figure 7.

If all observed points fell on the step function, it would be as good a fit as possible; the sum of squared residuals about the exponential regression curve would, however, obviously not be 0, and hence the R^2 statistic would be less than 1. In general, the sum of squares about the regression curve tends to be greater than that about the step function, and consequently the R^2 statistics tend to give conservative indications of the goodness of fit. As a supplemental measure of the fit of the data to the regression

Table 4. Summary of estimated exponential Q/X relations.

Attribute	Variable Q	Variable X	K Est.	K t Stat.[a]	λ Est.	λ t Stat.[b]	Regression R^2	Proportion Fitted ±1[c]
Automobile								
Riding time	QA13	XA13	7.09	1.49	0.0199	5.48	0.353	0.667
Walking time	QA14	XA14	6.08	0.34	0.0253	3.71	0.153	0.881
Transit								
Frequency	QT4	XT4	5.28	-2.26	0.0172	3.55	0.178	0.667
Riding time	QT13	XT13	5.59	-0.73	0.0148	3.91	0.192	0.512
Walking time	QT14	XT14	5.79	-0.44	0.0245	3.61	0.153	0.643
Waiting time	QT15	XT15	5.97	-0.09	0.0416	5.88	0.425	0.798

Note: Critical t (5 percent) = 1.989; critical t (1 percent) = 2.637; critical R^2 (1 percent) = 0.078.
[a]To test null hypothesis K = 6. [b]To test null hypothesis λ = 0. [c]Explanation given in text.

Figure 6.

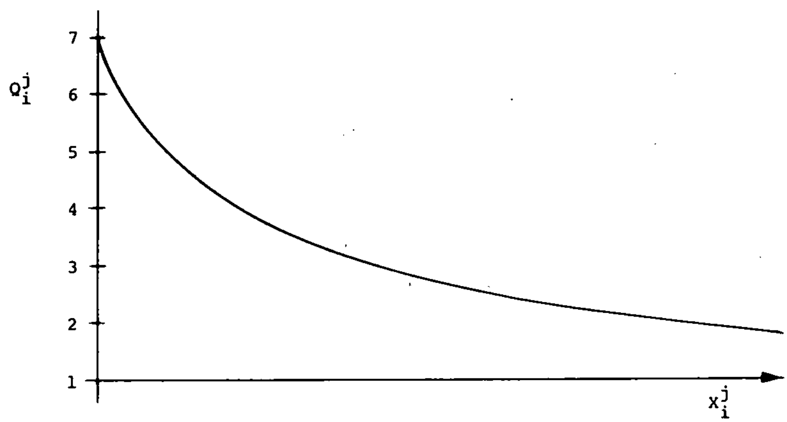
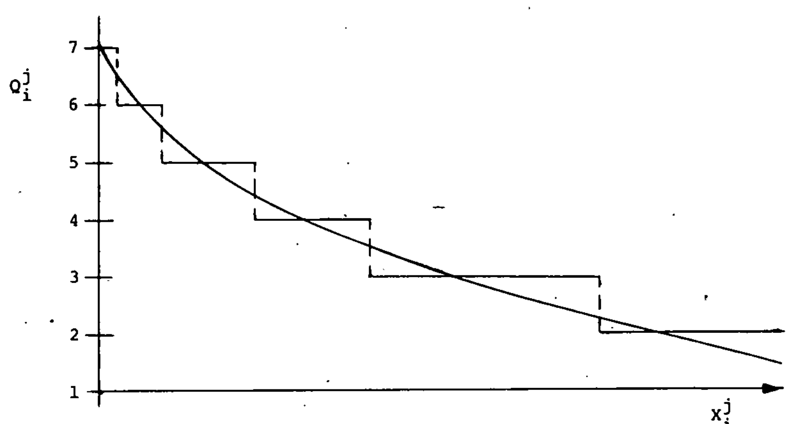


Figure 7.



curve, for each equation, the proportion of observations on Q_i^j having values within ±1 of the predicted value was computed and is given in Table 4 as "proportion fitted ±1." The t statistics given for the estimates of $\varkappa$ are the appropriate statistics to test the null hypothesis $\varkappa = 6$. The t statistic computed for the λ estimates are the familiar null t statistics.

The results given in Table 4 show a convincing fit of the exponential Q/X relation to the data for the 6 attributes included. The estimated values for $\varkappa$ are all close to 6, and the associated t statistics show that (with 1 exception) the differences from 6 are insignificant (judging significance under the usual assumptions of normality) for all estimates. The t statistics associated with λ estimates also indicate these all to be reliable. The R^2 statistics, although not very large, are in all cases highly significant and indicate reasonably close fits—given the nature of the data. For example, no stratification based on demographics has been made. The proportions of fitted values within ±1 of the observed values also indicate reasonable fits.

The regression of Eq. 39 was also performed on the data with the constant term $\varkappa$ constrained to equal 6, in order to give estimates for the parameter λ that could be compared among attributes. These results are given in Table 5. Interesting results are the values of the parameter estimates for the attributes riding time and walking time for the automobile and transit modes: viz.

	Automobile	Transit
Riding	0.015	0.017
Walking	0.024	0.027

In the demand model formulation, it was assumed that the satisfaction or utility obtained from a given modal attribute level is independent of the mode considered. The closeness of the above λ estimates provides an interesting validation of this assumption.

The results given in Table 6 are the estimated Q/X relations of Eq. 38, which may be estimated via the linear regression equation

$$Q_i^j = \mu_{0i} + \mu_i X_i^j \tag{40}$$

where μ_0 and μ_i are parameters to be estimated. For cost attributes, the regression results are for the same sample of 84 observations as were used in the nonlinear estimations. The R^2 statistics are all significant and, although not high, are close to those given for the corresponding exponential relations given in Table 5, indicating a similar fit to the data. The t statistics of μ also indicate all estimated coefficients to be significantly greater than 0 at a 1 percent confidence level. As expected for cost attributes, a μ_0 of approximately 7 was obtained. The null hypothesis $\mu_0 = 7$ is not rejected at the 1 percent level in all cases but 1.

Two important assumptions of the model have been supported by the evidence provided here. The first was that the U_i could be specified to be mode independent. The second was that of a linear mapping from U to Q. This assumption is critical to the determination of importance by estimates of a_i. The assumption appears to stand up well in connection with either an exponential or linear utility function assumption. The final basic assumption used in deriving the demand relations, viz., additive utilities, implies the need to specify an attribute description that is (nearly) orthogonal from a semantic point of view. Methods for accomplishing this were discussed in an earlier section. Correlation due to supply-side relations does not cause difficulties in this regard.

ESTIMATION OF UTILITY-FUNCTION PARAMETERS OF TARGET MARKETS

One of the areas requiring additional research is that of estimating utility functions of various consumer groups—so-called target markets. The results reported here are preliminary but encouraging. Assume a demand function of the form of Eq. 19. To estimate relative importances M_i/M_n as a function of demographics requires only that the sample be stratified into different groups and an independent analysis be performed for each group. This was not possible here because of degree-of-freedom problems given the sample size available.

However, an attempt was made to estimate the Q/X relation as a function of income. Although perhaps not obvious, it turns out that it is difficult to develop unassailable hypotheses as to how changes in income will affect the λ parameter of the utility function. Only waiting time appears straightforward. The higher income is, the larger the expected $|\lambda|$ is. The following equation yields estimates by nonlinear regression:

$$Q_i - 1 = 6e^{-\lambda X_i} \tag{40}$$

where $\lambda = A + BY + CY^2$; A, B, and C are parameters; and Y is a dummy income variable. The adjusted R^2 was 0.45 compared to 0.43 for the λ = constant model (Table 5) where the λ estimate was -0.0422. The tabulation below gives $-\lambda$ as a function of income.

Income (dollars)	$-\lambda$ Value
5,000 to 7,000	0.0218
7,000 to 10,000	0.0378
10,000 to 15,000	0.0460
Over 15,000	0.0461

As expected, $-\lambda$ increases with income, indicating that dissatisfaction with waiting time increases as income increases.

ESTIMATES OF RELATIVE IMPORTANCE AND MODEL STRUCTURE

The purpose of this section is to show that estimates of the parameters of the demand equation and, therefore, estimates of importances are highly sensitive to model structure. These estimates are not only sensitive to demand-side structure but also sensitive to the insertion of a supply side into the model. Three different models are considered.

Model 1 is the single-equation linear probability model given by Eq. 19. Estimation is by ordinary least squares. Parameter estimates are inefficient, that is, are not minimum variance because of the heteroscedasticity problem. Also the model structure is poor because the function is illogical at the ends. Of course, ignoring the supply side implies that the estimates are not only inefficient but also inconsistent. Given that this model and discriminant analysis yield identical estimates of relative importances, this is probably the most frequently applied statistical inference model for determining attribute importances.

Model 2 is also a single-equation importance-estimation model; the logistic function demand model is given by Eq. 21. Estimation is by nonlinear least squares. The estimation procedure would yield the best unbiased estimates if the data used to calibrate the model were not the result of supply and demand interaction. Hence, the estimates are inconsistent.

Model 3 is a simultaneous-equation model incorporating the supply side developed by Sherret (22). The model has the general form given by Eqs. 33, 34, and 35. Five supply equations were developed. Because the model was the linear probability demand function, parameter estimates are still inefficient. However, the estimation procedure used, essentially 2-stage least squares, yields consistent estimates of the parameters. Hence, model 3 parameter estimates are consistent but inefficient.

Table 7 gives the estimates of relative importance obtained by each of the 3 models. For comparison purposes, 4 attributes are shown: walking time (Q14), dependability

(Q2), waiting time (Q15), and riding time (Q13). The 2 single-equation models (1 and 2) yield very different results. Both yield 1 parameter estimate that is insignificant. In fact, waiting time and riding time reverse roles in the 2 models, 1 of the 2 being insignificant and, therefore, least important in both models. Both models 1 and 2 imply that walking time is most important and dependability is next most important. In terms of the t statistic, model 2 provides lower variance estimates.

Model 3 yields estimates of relative importance that are very different from those of either model 1 or model 2. All of the parameter estimates are significant, in fact, for all 4 attributes; the t statistic is highest for model 3. Moreover, dependability is found to be most important, walking time second, riding time third, and waiting time least important. The rank-order importances for the 3 models are as follows:

Attribute	Model 1	Model 2	Model 3
Dependability	2	2	1
Walking time	1	1	2
Riding time	4	3	3
Waiting time	3	4	4

Although the results of model 3 seem most sensible to the authors, the point is that they are very much different from those of the other 2 models. It seems that supply-side considerations simply cannot be ignored as well as the demand-side considerations.

As an aside, it should be mentioned here that in practice it would be wise to obtain measures of importance directly from consumers in addition to obtaining them by the statistical inference technique suggested above. This can be done via the semantic differential with end points "very important/very unimportant" (29). Paine et al. (30) measured attribute-satisfaction ratings and importances for the mode-choice decision problem via the semantic differential. Their results regarding relative importance were similar to those obtained above in that reliability of destination achievement was found to be most important and travel time was next, where travel time included expected value of travel time and dependability of on-time arrival. They also found comfort attributes way down the list in terms of importance for the work-trip mode choice. Paine et al., however, determined only rank-order importances and made no attempt to relate their results to the choices people actually make.

APPLICATION OF RESULTS

The purpose of this section is to illustrate how the estimated importances can be used along with the attribute-satisfaction data in advertising or product planning or both. Mean relative attribute-satisfaction ratings divided by first choice of automobile and transit are as follows:

Attribute	Automobile	Transit
Dependability	0.67	-1.70
Walking time	1.73	0.42
Riding time	0.78	-1.08
Waiting time	1.97	0.25

As expected, both groups give automobile the edge for walking and waiting time but disagree concerning dependability of on-time arrival and riding time.

Table 8 gives mean attribute-satisfaction ratings for automobile and transit separately by first choice of automobile and transit. Assume that the question of interest is how to improve patronage of transit by advertising.

The last column gives the difference between mean ratings of transit given by those choosing transit and those choosing automobile. Along with the automobile ratings, it provides some information for estimating $\Delta Q/\Delta C$, that is, the degree to which it may

Table 5. Summary of estimated exponential Q/X relations for κ constrained to 6.

Attribute	Variable Q	Variable X	Est. λ	Regression R^2
Automobile				
Riding time	QA13	XA13	0.0150	0.332
Walking time	QA14	XA14	0.0235	0.152
Transit				
Frequency	QT4	XT4	0.0258	0.128
Riding time	QT13	XT13	0.0171	0.186
Walking time	QT14	XT14	0.0271	0.151
Waiting time	QT15	XT15	0.0422	0.425

Note: Critical R^2 (1 percent) = 0.078.

Table 6. Summary of estimated linear Q/X relations.

Attribute	Variable Q	Variable X	μ_0 Est.	μ_0 t Stat.[a]	μ Est.	μ t Stat.[b]	Regression R^2
Automobile							
Riding time	QA13	XA13	6.98	-0.05	0.0641	6.68	0.352
Walking time	QA14	XA14	7.04	0.19	0.1319	3.84	0.152
Transit							
Frequency	QT4	XT4	6.21	-3.09	0.0722	4.45	0.194
Riding time	QT13	XT13	6.42	-1.43	0.0587	4.71	0.213
Walking time	QT14	XT14	6.67	-0.86	0.1087	3.95	0.160
Waiting time	QT15	XT15	6.57	-2.03	0.1455	8.02	0.440

Note: Critical t (5 percent) = 1.989; critical t (1 percent) = 2.637; critical R^2 = 0.078.
[a]To test null hypothesis $\mu_0 = 7$. [b]To test null hypothesis $\mu = 0$.

Table 7. Comparison of importance-estimation models.

Attribute	Model 1 Est. Coef.	Model 1 t Stat.	Model 1 Rel. Import.	Model 2 Est. Coef.	Model 2 t Stat.	Model 2 Rel. Import.	Model 3 Est. Coef.	Model 3 t Stat.	Model 3 Rel. Import.
Q14	0.0706	3.569	1.000	0.6091	3.589	1.000	0.0804	4.232	1.000
Q2	0.0525	2.712	0.744	0.4333	2.802	0.711	0.1027	2.936	1.277
Q15	0.0447	2.063	0.633	0.2283	1.547	0.375	0.0493	2.110	0.613
Q13	0.0233	1.139	0.330	0.3577	2.286	0.587	0.0647	2.903	0.804

Note: Critical $t_{0.025,111} = 1.981$.

Table 8. Mean attribute-satisfaction ratings.

Attribute	Automobile Ratings: First Choice Automobile	Automobile Ratings: First Choice Transit	Rail Transit Ratings: First Choice Automobile	Rail Transit Ratings: First Choice Transit	Difference
Dependability	5.42	4.23	4.75	5.95	1.20
Walking time	6.52	6.53	4.78	6.10	1.32
Riding time	5.05	4.45	4.27	5.54	1.27
Waiting time	6.67	6.35	4.70	6.10	1.31

be possible to change the transit ratings of people who choose automobile. Walking time may be ruled out immediately on the assumption that people know how far it is to the nearest transit stop. Given the attribute importances, the decision to advertise regarding dependability, riding time, or waiting time (frequency of service) depends on the $\Delta Q/\Delta C$ estimates. These estimates could be obtained via pretesting ads with automobile commuters. As discussed earlier, the product of relative importance $\Delta U/\Delta Q$ and $\Delta Q/\Delta C$ is the test criterion. However, because dependability is twice as important as waiting time and 50 percent more important than riding time and because commuters likely are aware of the schedules, it would seem that dependability would get the nod.

From the point of view of product planning, rail transit patronage would seem to be severely hampered because of fixed routes and the associated walking time required. This suggests the possibility of developing multimode transportation systems. Such systems are currently under study (31).

SUMMARY AND CONCLUSIONS

The object of this paper is to outline and illustrate a methodology for estimating the relative importance of product attributes. Product-attribute descriptions were developed in terms of attribute-satisfaction ratings obtained by a particular type of semantic differential. This was required because of the qualitative nature of many attributes. Satisfaction ratings, rather than attribute ratings, were obtained because of the apparently insurmountable difficulties in obtaining reliable measures of the latter for qualitative attributes.

The use of attribute-satisfaction ratings rather than attribute ratings required the development of a family of demand relations specified in terms of attribute-satisfaction ratings. It was shown that relative importances could be defined in the context of the parameters of these relations.

In a choice between 2 products, the probability of preferring product 1 to product 2 was determined to depend on the consumer's satisfaction with both products on each product attribute (relative attribute-satisfaction ratings) and the relative importance of each of these product attributes. Attribute importance was determined to be proportional to the maximum utility obtainable from any level of the attribute.

It was argued that correlations among attribute-satisfaction ratings were likely to arise for 2 reasons: The first is the existence of supply-side relations; the second is semantic redundancy in the set of attributes. Failure to explicitly specify these supply-side relations will lead to inconsistent estimates of importances. Failure to handle the semantic-redundancy problem will lead to importance estimates with unduly high variance.

It was suggested that discriminant analysis and principal-components type of factor analysis be used in an iterative fashion to develop a set of attributes that fully describe the product from a consumer point of view but are as orthogonal as possible.

The demand-side relations were developed on the basis of 3 fundamental assumptions: additive utilities, exponential utility function specified independent of the product, and linear mapping from utilities to attribute-satisfaction ratings. Empirical evidence of the validity of the latter 2 assumptions was provided. Some evidence was provided that it may be possible to estimate relative importance as a function of demographic variables.

It was shown that estimates of relative importances vary greatly depending on model specifications. It was argued that the most frequently used statistical inference model is likely to lead to importance estimates that are both inefficient and inconsistent. A methodology is suggested that can lead to estimates that are both efficient and consistent.

Finally, an attempt was made to illustrate how relative attribute-satisfaction ratings and relative importances can actually be used to facilitate advertising or product planning or both.

In conclusion, it appears that, although considerable time and money will be required to develop an importance-estimation model based on the methodology described above, the payoff in terms of improved understanding of the consumer decision-making process can be considerable.

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